Cox likelihood derivation

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Cox model:

$$h(t) = \underbrace{h_0(t)}_{\text{non parametric b/c we don't assume distr.}} \underbrace{\exp(\beta^T X)}_{\text{parametric (regression)}}$$

→ some linear form of the predictors

⇒ semiparametric

as opposed to parametric, where $h(t) = \lambda$ or sth
$h(t) = \ldots\ldots$

Likelihood:

Let's say we have $T^* = t^*$, $\delta^* = 1$ (fail) ⇒ $\mathcal{L}_i = f(t^*)$
$= h(t^*)\,S(t^*)$

Let's say we have $T^* = t^*$, $\delta^* = 0$ (censor) ⇒ $\mathcal{L}_i = P(T > t^*)$
$= S(t^*)$

$$\mathcal{L}_i = h(t^*)^{\delta^*} S(t^*)$$

Write out full likelihood as product of individual likelihoods

$$\mathcal{L}_i = \prod_{i=1}^{n} \mathcal{L}_i$$

$$= \prod_{i=1}^{n} \underbrace{h(t_i^*)}_{\text{now for cox Model, sub in appropriate } h(t)}{}^{\delta_i^*} S(t_i^*)$$

$$= \prod_{i=1}^{n} \left[h_0(t_i^*) \exp(\beta^T X_i)\right]^{\delta_i^*} S(t_i^*)$$

$$= \underbrace{\prod_{i=1}^{n} \left[\exp(\beta^T X_i)\right]^{\delta_i^*} S(t_i^*)}_{\text{this is the partial likelihood}} \cdot \prod_{i=1}^{n} \left[h_0(t_i^*)\right]^{\delta_i^*}$$