NEURON AS AN AGENT

Anonymous

ABSTRACT

We propose *Neuron as an Agent* (NaaA) as a novel framework for reinforcement learning (RL), and explain its optimization method. NaaA incorporates all neural network units as agents and optimizes the reward distribution as a multi-agent RL problem. First, showing optimization of NaaA, this report describes the negative result that the performance decreases if we naively consider the units as agents. To resolve that difficulty, we introduce a mechanism from game theory. As a theoretical result, we demonstrate that the agent obeys the system to maximize its *counterfactual return* as the Nash equilibrium of the mechanism. Subsequently, we show that learning counterfactual returns leads the model to learning optimal topology among units. We propose *adaptive dropconnect*, a natural extension of dropconnect. Finally, we confirm that optimization with the framework of NaaA leads to better performance of RL, with numerical experiments. Specifically, we use a single-agent environment from Open AI gym, and a multi-agent environment from ViZDoom.

1 INTRODUCTION

Deep reinforcement learning (DRL) succeeds in many areas. Deep Q-Network (DQN) (Mnih et al., 2015; Silver et al., 2016) finds the optimal action from a screen sequence from Atari, and selects the move closest to win from a face of a board of Go. Deep Deterministic Policy Gradient (DDPG) (Lillicrap et al., 2015) realizes the multiple-join control considering conditions such as friction and gravity factors in a physical space. The applicability of DRL is becoming wider year by year. Reasonable performance is reported for 3D games such as Doom (Dosovitskiy & Koltun, 2016).

A neural network is workable for DRL because a neural network abstracts the implicit state in an environment and obtains an informative state representation. From a micro perspective, the abstraction capability of each unit contributes to the return of the entire system. Therefore, we address the following question.

Will reinforcement learning work even if we consider each unit as an autonomous agent?

The contribution of this paper is that we propose *Neuron as an Agent* (NaaA) as a novel framework for RL, and explain its optimization method. NaaA incorporates all neural network units as agents and optimizes the reward distribution as a multi-agent RL problem. In the of NaaA reward design, a unit distributes its received reward to other input units, passing its activation to the unit as cost. Consequently, the actual reward is profit, defined as the difference between inflow (received reward) and outflow (paid cost). In the setting, the economic metaphor can be introduced: profit is the balance of revenue and cost. Therefore, a unit should address tradeoffs between optimization of cumulative revenue maximization and cumulative cost minimization.

This paper is organized as presented below. First, showing the optimization of NaaA, this report describes the negative result that the performance decreases if we naively consider units as agents. As a solution to this difficulty, we introduce a mechanism of auction which applies game theory. As a theoretical result, we demonstrate that the agent obeys to maximize its *counterfactual return* as the Nash equilibrium. The counterfactual return is that by which we extend counterfactual reward, the criterion proposed for multi-agent reward distribution problem (Agogino & Tumer, 2006), along a long time axis.

Subsequently, we present that learning counterfactual return leads the model to learning optimal topology between the units. In addition, we propose *adaptive dropconnect*, a natural extension of

dropconnect (Wan et al., 2013). Adaptive dropconnect combines dropconnect, which pure-randomly masks the topology, with adaptive algorithm, which prunes the connection with less counterfactual return with higher probability. It uses ε-greedy as a policy, and is equivalent to dropconnect in the case of $\varepsilon = 0$. It is equivalent to counterfactual return maximization, which constructs the topology deterministically in the case of $\varepsilon = 1$.

Finally, we confirm that optimization with the framework of NaaA leads to better performance of RL, with numerical experiments. Specifically, we use a single-agent environment from Open AI gym, and a multi-agent environment from ViZDoom.

Although considering all the units as agents might be simplistic at first glance, it has a wider applicable area. From the perspective of optimization for a single neural network, it can be applied to pruning by optimizing the topology. Furthermore, introducing the concept of reward distribution divides the single neural network to numerous autonomous parts. It enables us not only to address sensor placing problem in IoT for partially observed Markov decision process (POMDP): arbitrary incentivized participants can join the framework.

2 Related Work

NaaA belongs to a class of partially observable stochastic game (POSG) (Hansen et al., 2004) because it processes multiple units as agents. POSG, a class of reinforcement learning with multiple agents in a POMDP environment, presents several research issues, one of which is communication. Commnet (Sukhbaatar et al., 2016), which exploits the characteristics of a unit that is agnostic to the topology of other units, employs backpropagation to train multi-agent communication. Another one is credit assignment. Instead of reward $R(a_t)$ of an agent i for actions at t a_t, QUICR-learning (Agogino & Tumer, 2006) maximizes counterfactual reward $R(a_t) - R(a_t - a_{it})$, the difference in the case of the agent i takes an action a_{it} (a_t) and not $(a_t - a_{it})$. COMA (Foerster et al., 2017) also maximizes counterfactual rewards in an actor–critic setting. In the setting, all actors have common critics, which improves both actors and critics with time difference (TD)-error of a counterfactual reward. This paper unifies both issues: communication and credit assignment. The main proposal is a framework to manage the agents to maximize the *counterfactual return*, the extended counterfactual reward along the time axis.

Training a neural network with a multi-agent game is an emerging methodology. Generative adversarial nets (GAN) (Goodfellow et al., 2014) have the goal of obtaining true generative distribution as a Nash equilibrium of a competitive game that includes two agents with contradictory rewards: a generator and a discriminator. In game theory, the outcome maximizing overall reward is named Pareto optimality. Nash equilibrium is not guaranteed to converge to Pareto optimality. The difference between them is designated as a dilemma. Because the existence of a dilemma depends on the reward design, methods to resolve dilemmas with good reward design are being investigated: mechanism design (Myerson, 1983) is also known as inverse game theory. Mechanism design is applied to auctions (Vickrey, 1961) and matching (Gale & Shapley, 1962). GAN and our proposal, NaaA, are outcomes from mechanism design. NaaA applies a digital goods auction (Guruswami et al., 2005) to reinforcement learning with a multi-agent neural network, to obtain a maximized return by units as a Nash equilibrium.

3 Background

First, we consider a POMDP environment in which a single agent acts. The POMDP environment is a seven-tuple $(\mathcal{S}_{\mathrm{H}}, \mathcal{A}, \mathcal{T}, \mathcal{R}, \mathcal{S}_{\mathrm{O}}, \mathcal{O}, \gamma)$, where $\mathcal{S}_{\mathrm{H}}$ represents a set of states, $\mathcal{A}$ stands for a set of actions, $\mathcal{T}$ denotes a transitive probability, $\mathcal{S}_{\mathrm{O}}$ represents a possible set of observations, $\mathcal{O}$ denotes a set of observation probability, and γ is the discount rate. An agent partially predicts state $h \in \mathcal{S}_{\mathrm{H}}$ through an observation $s \in \mathcal{S}_{\mathrm{O}}$. Generally, s has higher dimensions than h, and is complex. For example, although Atari 2600 has a read only memory (RAM) as the true state, which contains 128 bytes, the generated image from that s has more than 10,000 dimensions. Therefore, DQN and DRQN abstract s, and create original state representation to predict good action efficiently. (Although the original paper of DQN assumes MDP, the paper of DRQN pointed out that the environment is POMDP). Although DQN does not address the state transition directly because it is model-free method, some interpretations hold that the hidden state representation is learned in the

previous layer of the output layer (Zahavy et al., 2016) Using the method below, we assume that the agent chooses an action through a neural network.

The design of NaaA is inspired by neuroscience. A neuron in a neurocircuit consumes adenosine triphosphate (ATP) supplied from connected astrocytes. The astrocyte is a glia cell, which forms the structure of a brain. It supplies fuel from the vessel. Because the amount of ATP is constrained, the discarded neuron will become extinct with execution of apoptosis. Also, because apoptosis of a neuron is restrained by neurotrophins (NTFs) such as nerve growth factor (NGF) and brain-derived neurotrophic factor (BDNF), neurons which can obtain much NTF will live. The perspective of interpreting a neuron as an independent living object is known as neural Darwinism (Edelman, 1987).

4 NEURON AS AN AGENT

A typical artificial neural network is a directed graph $\mathfrak{G} = (\mathcal{V}, \mathcal{E})$ among the units. $\mathcal{V} = \{v_1, \ldots, v_N\}$ is a set of the units. $\mathcal{E} \subset \mathcal{V}^2$ is a set of edges representing connections between two units. If $(v_i, v_j) \in \mathcal{E}$, then connection $v_i \to v_j$ holds, indicating that v_j observes activation of v_i. We denote activation of the unit v_i at time t as $x_{it} \in \mathbb{R}$. Additionally, we designate a set of units which unit i connects to as $N_i^{\text{out}} = \{j|(v_i, v_j) \in \mathcal{E}\}$ and a set of units which unit i is connected from as $N_i^{\text{in}} = \{j|(v_j, v_i) \in \mathcal{E}\}$. We denote $N_i = N_i^{\text{in}} \cup N_i^{\text{out}}$.

NaaA interprets v_i as an agent. Therefore, $\mathfrak{G}$ is a multi-agent system. An environment for v_i comprises an environment that the multi-agent system itself touches and a set of the unit to which v_i directly connects: $\{v_i \in V|i \in N_i\}$. We distinguish both environments by naming the former as an external environment, and by naming the latter as an internal environment. v_i will receive rewards from both environments. We add the following assumption for characteristics of the v_i.

- N1: (Selfishness) Instead of minimizing the global training error, at each timing t, v_i acts to maximize toward maximizing its own return (cumulative discounted reward) $G_{it} = \sum_{k=0}^{T} \gamma^k R_{i,t+k}$, where $\gamma \in [0, 1]$ is the discount rate and T is the terminal time.
- N2: (Conversation) The summation of a reward by which $\mathcal{V}$ will receive both an internal and external environment R_{it} over all the units are equivalent to reward R_t^{ex}, which the entire multi-agent system receives from the external environment.
- N3: (Trade) The v_i receives internal reward ρ_{jit} from $v_j \in \mathcal{V}$ in exchange of activation signal x_i before transferring the signal to the unit. At the same time, ρ_{jit} is subtracted from the reward of v_j.
- N4: (NOOP) v_i has NOOP (no operation), for which the return is $\delta > 0$ as an action. With NOOP, the unit inputs nothing and outputs nothing.

In terms of neuroscience, N1 states that the unit acts as a cell. N2 and N3 state the distribution of NTF. N4 corresponds to apoptosis. NOOP is selected when the expected returns of the other actions are non-positive. In the following, we construct the framework of NaaA from the assumptions.

4.1 CUMULATIVE DISCOUNTED PROFIT MAXIMIZATION FRAMEWORK

We denote the external reward by which unit v_i receives at time step t as R_{it}^{ex}, where $\sum_{i=1}^{n} R_{it}^{\text{ex}} = R_t^{\text{ex}}$ holds. From N3, reward R_{it}, which v_i receives at t can be written as the following.

$$R_{it} = R_{it}^{\text{ex}} + \sum_{j \in N_i^{\text{out}}} \rho_{jit} - \sum_{j \in N_i^{\text{in}}} \rho_{ijt}. \tag{1}$$

The equation is divided into positive terms and a negative term, we name the former as revenue, and the latter as cost, and denote them respectively as $r_{it} = R_{it}^{\text{ex}} + \sum_{j \in N_i^{\text{out}}} \rho_{jit}$, $c_{it} = \sum_{j \in N_i^{\text{in}}} \rho_{ijt}$. We name R_{it} as profit.

In this case, v_i maximizes the cumulative discounted profit G_{it} represented as

$$G_{it} = \sum_{k=0}^{T} \gamma^k R_{i,t+k} = \sum_{k=0}^{T} \gamma^k (r_{i,t+k} - c_{i,t+k}) = r_t - c_t + \gamma G_{i,t+1}. \tag{2}$$

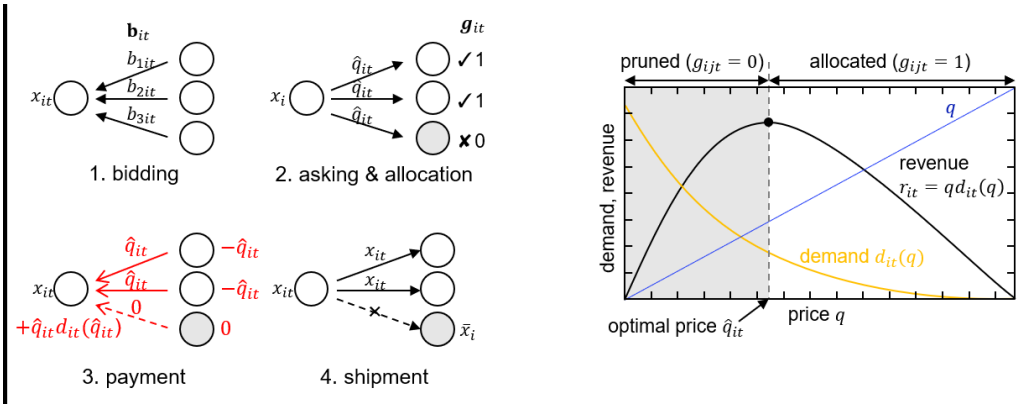

Figure 1: **Left**: The process of trade in an envy-free auction. **Right**: A price determination curve for a unit. Revenue of a unit is a product of monotonically decreasing demand and price. The price maximizing the revenue is the optimal price.

G_{it} is unobserved unless the time is reached at the end of the episodes. Because prediction based on the current value is needed to select the optimal actions, we approximate G_{it} with value function $V_i^{\pi_i}(s_{it}) = \mathbb{E}_{\pi_i}[G_{it} \mid s_{it}]$ where $s_{it} \in \mathcal{S}_{\mathrm{O}}$. In this case, the following equation holds.

$$V_i^{\pi_i}(s_{it}) = r_{it} - c_{it} + \gamma V_i^{\pi_i}(s_{i,t+1}), \tag{3}$$

Therefore, we need only consider maximization of revenue, the value function, and cost minimization. $R_{it} > 0$, i.e., $r_{it} > c_{it}$ indicates that the unit gives the additional value to the obtained data. The unit acts NOOP because $V_i^{\pi_i}(s_{it}) \leq 0 < \delta$ if $R_{it} \leq 0$ for all t.

5 Optimization

To maximize the cumulative discounted profit in a framework of NaaA, it is important to balance the two contradicting criteria: revenue r_{it} and cost c_{it}.

To achieve that, we use the mechanism design. We introduce mechanism design because, unlike several existing studies (Sukhbaatar et al., 2016), NaaA assumes that all agents are not cooperative but selfish. If we naively optimize the optimization problem of NaaA, then we obtain the trivial solution that the internal rewards will converge to 0, and that all the units become NOOP. Therefore, the multi-agent system should select the action with no information. It is equivalent to taking an action randomly. For that reason, the external reward R_t^{ex} shrinks markedly.

5.1 Envy-free Auction

To achieve Pareto optimality, we borrow the idea from the digital goods auction. The auction theory belongs to mechanism design. It is intended to unveil the true price of goods. Digital goods auction is one mechanism from auction theory. It is target to copyable goods without cost, such as digital books and music.

Although several variations of digital goods auctions exist, we use an envy-free auction (Guruswami et al., 2005) because it requires a simple assumption: the same goods have one price simultaneously. In NaaA, it can be represented as the following assumption:

N5: (Law of one price) If $\rho_{j_1,i,t}, \rho_{j_2,i,t} > 0$, then $\rho_{j_1,i,t} = \rho_{j_2,i,t}$.

Therefore, v_i has an intrinsic price at the same timing t. We denote the price as q_{it}.

We present the envy-free auction process at the left of Figure 1. It shows the negotiation process between one unit in sending activation and a group of units that buy the activation. The negotiation performed per time step in RL. We name the unit in sending activation as a seller, and units in buying activation as buyers. First, the buyer bids the unit in bidding price b_{jit} (**1**). Next, the seller decides

the optimal price $\hat{q}_{it}$, and performs allocation (**2**). After allocation, the buyers perform payment as $\rho_{jit} = g_{jit}\hat{q}_{it}$ (**3**). The seller only sends activation x_i to the allocated buyers (**4**). A buyer which cannot receive the activation approximates x_i with $\mathbb{E}_\pi[x_i]$.

In the following, we discuss revenue, cost, and value functions based on Eq:(3).

Revenue: The revenue of a unit is given as

$$\begin{aligned} r_{it} &= \sum_{j \in N_i^{\mathrm{out}}} g(b_{jit}, q_{it}) q_{it} + R_i^{\mathrm{ex}} = q_{it} \sum_{j \in N_i^{\mathrm{out}}} g(b_{jit}, q_{it}) + R_i^{\mathrm{ex}} \\ &= q_i d_{it}(q_t) + R_i^{\mathrm{ex}}, \end{aligned} \tag{4}$$

where $g(\cdot,\cdot)$ is allocation. It is defined using a step function $H(\cdot)$ as $g(b,q) = H(b-q)$. $d_{it}(q_{it})$ is a count of units for which the bidding price for q_{it} is greater than or equal to q_{it}, designated as demand. q_{it} maximizing the equation is designated as the optimal price. It is denoted as $\hat{q}_{it}$. Because the second term in the equation is independent of q_t, the optimal price $\hat{q}_{it}$ is given as

$$\hat{q}_{it} = \underset{q \in [0,\infty)}{\operatorname{argmax}}\, q d_{it}(q). \tag{5}$$

We present the curve of q_{it} on the right side of Figure 1.

Cost: The cost is an internal reward that the unit should pay to other units. It is represented as shown below.

$$c_{it} = \sum_{j \in N^{\mathrm{in}}} g(b_{ijt}, q_j) q_j \tag{6}$$

Although c_{it} itself is minimized when $b_{ijt} = 0$, this represents a tradeoff with the following value function.

Value Function: Activation x_i depends on input from the units in N_i^{in}. It affects the bidding price from units in N_i^{out}. If we minimize b_{ijt} and let $b_{ijt} = 0$, then the purchase of activation fails, and the reward the unit can obtain from the units to which the unit connects becomes lower in the future.

Then, we designate the allocation as $\mathbf{g}_{it} = (g_{i1t}, \ldots, g_{iNt})^{\mathrm{T}}$, and consider effects for value functions in the cases when a unit succeeds in purchasing v_j or not. The value function can be written as the equation using a state-value function $Q(s_{i,t+1}, \mathbf{g}_{i,t+1})$.

$$\begin{aligned} V_i^{\pi_i}(s_{it}) &= Q_i^{\pi_i}(s_{it}, \mathbf{g}_{it}) \\ &= \sum_{j \in N_i^{\mathrm{in}}} g_{ijt}(Q_i^{\pi_i}(s_{it}, \mathbf{e}_j) - Q_i^{\pi_i}(s_{it}, \mathbf{0})) + Q_i^{\pi_i}(s_{it}, \mathbf{0}) \\ &= \sum_{j \in N_i^{\mathrm{in}}} g_{ijt} o_{ijt} + Q_i^{\pi_i}(s_{it}, \mathbf{0}) \\ &= \mathbf{g}_{it}^{\mathrm{T}} \mathbf{o}_{it} + Q_i^{\pi_i}(s_{it}, \mathbf{0}) \end{aligned} \tag{7}$$

We designate $o_{ijt} = Q_i^{\pi_i}(s_{it}, \mathbf{e}_j) - Q_i^{\pi_i}(s_{it}, \mathbf{0})$ as the *counterfactual return*, which is equivalent to the cumulative discount value of counterfactual reward (Agogino & Tumer, 2006). That is, the cost the unit will pay is $\hat{q}_{it}$ in success of purchasing data, and o_{it} otherwise.

Therefore, the optimization problem is presented below.

$$\max_{\mathbf{b},q} \mathbb{E}_{\hat{\mathbf{q}}_t}\left[V_i^{\pi_i}(s_{it})\right] = \max_q q d_{it}(q) - \min_{\mathbf{b}} \mathbb{E}_{\hat{\mathbf{q}}_t}\left[\mathbf{g}_{it}(\mathbf{b})^{\mathrm{T}}(\hat{\mathbf{q}}_t - \gamma \mathbf{o}_{i,t+1})\right] + \mathrm{const.}. \tag{8}$$

We take the expectation $\mathbb{E}_{\hat{\mathbf{q}}_t}[\cdot]$ because the asked price $\hat{\mathbf{q}}_t$ is unknown for v_i, except for $\hat{q}_{it}$, and $g_{iit} = 0$.

Then, what is bidding price b_{it} to maximize return? The following theorem holds.

Theorem 5.1. *(Truthfulness) the optimal bidding price for maximizing return is* $\hat{\mathbf{b}}_{it} = \mathbf{o}_{it}$.

See the Appendix for the proof.

That is, the unit should only consider its counterfactual return (!). Consequently, in the mechanism of NaaA, the unit obeys as if performing valuation to the other units, and declares the value truthfully.

Then, the following corollary holds:

Corollary 5.1. *The Nash equilibrium of an envy-free auction* $(\mathbf{b}_{it}, q_{it})$ *is* $(\mathbf{o}_{it}, \operatorname{argmax}_q q d_{it}(q))$.

6 EXPERIMENT

To confirm NaaA widely work with machine learning tasks, we confirm our method supervised learning tasks as well as reinforcement learning tasks. As supervised learning tasks, we use typical machine learning tasks such as image classification using CIFER-10, NORB and SVHN. In the experiment, we ignore MNIST because the accuracy is in saturant, and there are no meaning of comparison.

As reinforcement tasks, we confirm single- and multi-agent environment. The single-agent environment is from OpenAI gym. We confirm the result with a simple reinforcement task, CartPole.

6.1 CLASSIFICATION

We confirm NaaA works well in our method.

6.2 SINGLE-AGENT RL

6.3 MULTI-AGENT RL

As multi-agent RL, we use ViZDoom, an environment using Doom.

6.3.1 SETUP

We used a scenario based on Defend the Center (DtC) provided by ViZDoom platform. In DtC, players are placed in a center of a field of circle, and attack enemies which will come from the wall. The players are two mans: a main player and a cameraman. Though the main player can attack the enemy with bullets, the cameraman does not have the way to attack, and only scout for the enemy. The action space for main player is the combination of { attack, turn left, turn right } (total amount is 8), and { turn left, turn right } for the cameraman. Although, the players only can move the direction, they cannot move on the field. The enemy will die if have the attack (bullet) from the main player once. The amount of ammo is 26 as a default on an episode. The main player will die if have attacks from the enemy, and the health become 0. The cameraman won't die if have attacks from the enemy. The episode will terminate when the maim player dies, or 525 steps elapsed.

DtC is standard scenario which prepared by ViZDoom platform. In DtC, up to 5 enemies appear can exists in the same time. The main player will receive reward every time defeat the enemy. After the enemy dies, the player receives 1, if the player die, he receives -1.

6.3.2 MODEL

We compared three models: one is the proposed method, and the remaining are comparing target.

Baseline DQN without communication. The main player learns standard DQN with vision which he watching. Since the cameraman does not learn, he continues to move randomly.

Comm DQN with communication. The main player learns DQN with two visions of him and cameraman. The communication vector is learnt with a feed-forward neural network . The method is inspired by CommNet.

NaaA The proposed method. The main player learns DQN with two visions of him and cameraman. The transmission of reward and communication will be performed by proposed method.

6.3.3 RESULTS

The training is performed in 10 million steps.

7 DISCUSSION

7.1 DISADVANTAGE

7.2 SHORTCOMING

An important shortcoming is its computational complexity. Because the envy-free auction uses a sort operation for computing demand, several parts should be serialized and should be improved through approximation.

Regarding the optimization method, although envy-free auction guarantees truthfulness if the buyer prices are sealed, in cases where buyers can mutually communicate and share price information, the buyer can fake the price with lower demand in a process of collusion. To address the issue, several solutions such as random sample auction Goldberg et al. (2006) are proposed.

Adoptive dropconnect has difficulty for implementation for several neural networks. Although we published source code on GitHub for Linear and CNN, implementation for RNN remains as a subject for future work.

7.3 APPLICATION

NaaA is applicable to learning distributed environments on a computer network such as a peer-to-peer network, and controlling the sub-modules of robots such as multiple cameras. Specifically, it is applicable to various methods as described below.

- Hyperparameter tuning. Several algorithms have been proposed such as neuroevolution using genetic algorithms. In the case, profit or counterfactual return is useful for a fitness function.
- Pruning. Computing costs can be reduced by downsizing a neural network.
- Attention control. Research of attention is using reinforcement learning to control attention.
- Ensemble. Our method is applicable to mixed multiple models.

These applications illustrate the direction of our research.

8 CONCLUSION AND FUTURE WORKS

This paper proposed NaaA, a reinforcement learning framework that treats each unit on a neural network as an agent. First, we pointed out there are dilemma problems if we naively optimize NaaA. We proposed an optimization method with auction. Consequently, an action by which units evaluate the counterfactual return of other units is obtained as a Nash equilibrium. Furthermore, we proposed Q-learning based algorithm, adaptive dropconnect, to optimize the neural network topology dynamically with evaluation of counterfactual return. For the evaluation, we performed experiments based on single-agent and multi-agent platforms, demonstrating that our experimentally obtained results improve existing methods.

As a direction of future research, we use on-policy methods to perform adaptive dropconnect, and consider applications combining genetic algorithms.

APPENDIX

A.1 PROOF OF THEOREM 5.1

As for a buyer, the asking price q for a seller is unknown,[we address q which has support $[0, \infty)$, and consideration to maximize $\mathbb{E}_q[G(b,q)]$, In this case, the following equation holds.

$$\begin{aligned}
\frac{\partial}{\partial b}\mathbb{E}_q[G(b,q)] &= \frac{\partial}{\partial b}\int_0^\infty (H(b-q)\cdot(v-q)+G_0)p(q)dq \\
&= \frac{\partial}{\partial b}\left[\int_0^b (v-q)p(q)dq + G_0\int_0^\infty p(q)dq\right] \\
&= \frac{\partial}{\partial b}\int_0^b (v-q)p(q)dq \\
&= (v-b)p(q=b),
\end{aligned}$$

Therefore, the condition to maximize $\mathbb{E}_q[G(b,q)]$ is $b = v$.

REFERENCES

A. K. Agogino and K. Tumer. QUICR-learning for multi-agent coordination. AAAI'06, 2006.

A. Dosovitskiy and V. Koltun. Learning to act by predicting the future. *ICLR'17*, 2016.

G. M Edelman. *Neural Darwinism: The theory of neuronal group selection.* Basic books, 1987.

J. Foerster, G. Farquhar, T. Afouras, N. Nardelli, and S. Whiteson. Counterfactual multi-agent policy gradients. *arXiv:1705.08926*, 2017.

David Gale and Lloyd S Shapley. College admissions and the stability of marriage. *The American Mathematical Monthly*, 69(1):9–15, 1962.

Andrew V Goldberg, Jason D Hartline, Anna R Karlin, Michael Saks, and Andrew Wright. Competitive auctions. *Games and Economic Behavior*, 55(2):242–269, 2006.

I. Goodfellow, J. Pouget-Abadie, M. Mirza, B. Xu, D. Warde-Farley, S. Ozair, A. Courville, and Y. Bengio. Generative adversarial nets. In *Advances in neural information processing systems*, pp. 2672–2680, 2014.

Venkatesan Guruswami, Jason D Hartline, Anna R Karlin, David Kempe, Claire Kenyon, and Frank McSherry. On profit-maximizing envy-free pricing. In *ACM-SIAM symposium on Discrete algorithms*, 2005.

Eric A Hansen, Daniel S Bernstein, and Shlomo Zilberstein. Dynamic programming for partially observable stochastic games. In *AAAI*, volume 4, pp. 709–715, 2004.

T. P. Lillicrap, J. J. Hunt, A. Pritzel, N. Heess, T. Erez, Y. Tassa, D. Silver, and D. Wierstra. Continuous control with deep reinforcement learning. *ICLR'16*, 2015.

V. Mnih, K. Kavukcuoglu, D. Silver, A. A. Rusu, J. Veness, M. G. Bellemare, A. Graves, M. Riedmiller, A. K. Fidjeland, G. Ostrovski, et al. Human-level control through deep reinforcement learning. *Nature*, 518(7540):529–533, 2015.

Roger B Myerson. Mechanism design by an informed principal. *Econometrica: Journal of the Econometric Society*, pp. 1767–1797, 1983.

D. Silver, A. Huang, C. J. Maddison, A. Guez, L. Sifre, G. Van Den Driessche, J. Schrittwieser, I. Antonoglou, V. Panneershelvam, M. Lanctot, et al. Mastering the game of go with deep neural networks and tree search. *Nature*, 529(7587):484–489, 2016.

S. Sukhbaatar, R. Fergus, et al. Learning multiagent communication with backpropagation. In *NIPS'16*, 2016.

William Vickrey. Counterspeculation, auctions, and competitive sealed tenders. *The Journal of finance*, 16(1):8–37, 1961.

Li Wan, Matthew Zeiler, Sixin Zhang, Yann L Cun, and Rob Fergus. Regularization of neural networks using dropconnect. In *Proceedings of the 30th international conference on machine learning (ICML-13)*, pp. 1058–1066, 2013.

Z. Wang, T. Schaul, M. Hessel, H. Van Hasselt, M. Lanctot, and N. De Freitas. Dueling network architectures for deep reinforcement learning. *arXiv:1511.06581*, 2015.

T. Zahavy, N. Ben-Zrihem, and S. Mannor. Graying the black box: Understanding DQNs. In *ICML'16*, 2016.