

# Public Review for Competing Memes Propagation on Networks: A Case Study of Composite Networks

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“Friend me on Facebook, Follow me on Twitter, Connect me on LinkedIn, Enter my Google Circles.” As we are multiplying social channels of communication these start to interact! In fact, it becomes increasingly difficult in the study of information propagation on social networks to be sure that a given individual who was not yet exposed to a meme has actually not heard about it in another way. In other words, we may very well enter an era where **no social networks operate in isolation**. This, is what this paper proposes to study with a simple example of “composite networks”.

Imagine all cases where memes are competing. Being early is often all that counts so one can easily imagine a speed race between a rumor and its denial, where all individuals act according to their first exposure (clicking on a link or in contrast be careful if warned). Or the competition between two substitutable products that are promoted at similar time on different media. Previous works have considered this angle only from a single network point of view. This work suggests that as different channels become viral differently, one need to study this propagation on the composite network.

In this paper, you will find these dynamics formulated as an **extension of classical epidemiological model**, a characterization of the limit regime where individuals **both** become susceptible and recover slowly. This allow one to use a simple decomposition of this competition that yields some conditions for stability, generalizing previous epidemiological studies with differential equations. The main merit of these results is to introduce the eigenvalues of interest for these systems, that are also proved numerically to characterize well this system. Reviewers were unanimous in finding this problem timely and novel, and the contribution substantial, which by itself justify its appearance in CCR. We leave it as a general question to be discussed of what is “computer networking,” especially since this paper deals with

this problem in abstract, with no reference to a practical implementation of social dissemination. It seems clear to us that we should welcome paper that broaden our horizon and pose new problems of interests in related fields, such as the one predicting how information gets diffused on social applications. Most concerns and criticism relate to the narrowness of the model, with a curious mix of repetitive susceptibility and insensitivity to other meme, simultaneous start of epidemics, not to mention that in reality the networks may not be entirely disjoint – although this last point could probably be accounted for. Another important limitation is that the analysis uses simultaneously two simplifications: On the one hand, infinitely slow propagation/recovery, and on the other hand, the asymptotic version of stability – in other words, resistance to infinitesimal change – found in previous article on non-linear dynamical system). Without being contradictory, this significantly constrains the analysis to a regime with minimal (but not inexistent) interaction. You may also question the use of data set from calls and SMS as competing, but truly this serves only the purpose to test if such theory fits well the behavior of a real topology. All the concerns above are important, but do not diminish the potential impact of this article to inspire more works and get us to think differently about our multiple connected work. Following CCR’s tradition, we privileged impact and timeliness, which is really what 6 pages can be about.

If you are now desperately trying to choose which social media to ask me to connect in order to best influence me - who knows? maybe for future CCR issues - this paper will not entirely solve your problem. However, you may want to read it and start following one of the open directions that it creates to find ultimately the answer.

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# Competing Memes Propagation on Networks: A Case Study of Composite Networks

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## ABSTRACT

If a false rumor propagates via Twitter, while the truth propagates between friends in Facebook, which one will prevail? This question captures the essence of the problem we address here. We study the intertwined propagation of two competing “memes” (or viruses, rumors, products etc.) in a composite network. A key novelty is the use of a composite network, which in its simplest model is defined as a single set of nodes with two distinct types of edges interconnecting them. Each meme spreads across the composite network in accordance to an SIS-like propagation model (a flu-like infection-recovery). To study the epidemic behavior of our system, we formulate it as a non-linear dynamic system (NLDS). We develop a metric for each meme that is based on the eigenvalue of an appropriately constructed matrix and argue that this metric plays a key role in determining the “winning” meme. First, we prove that our metric determines the tipping point at which both memes become extinct eventually. Second, we conjecture that the meme with the strongest metric will most likely prevail over the other, and we show evidence of that via simulations in both real and synthetic composite networks. Our work is among the first to study the interplay between two competing memes in composite networks.

## Categories and Subject Descriptors

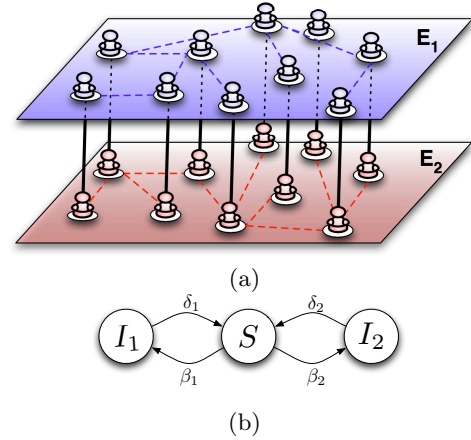
J.4 [Computer Applications]: Social and Behavioral Sciences

## Keywords

Competition, Propagation, Composite networks

## 1. INTRODUCTION

Epidemic spreading models and techniques are used to model and analyze many network phenomena across various disciplines. Such network phenomena include the spread of social information, computer viruses, fashion trends, religious beliefs, market penetration and product adoption [8, 18]. Epidemic models initially described virus and disease propagation [1]; however, due to the broad applicability of such propagation schemes, we use the generic term **meme** to represent any entity that spreads over a network [21]. Thus, a meme may represent a piece of data, information, a rumor, a computer virus, a strand of flu or a new product.



**Figure 1: (a) Example Composite Network topology: a single set of nodes  $N$  with two distinct edge sets  $E_1$  and  $E_2$ . (b) The  $SI_1I_2S$  State Transition Diagram, where  $S$  represents the susceptible state and  $I_{1,2}$  indicate the infected state for memes  $M_1$  and  $M_2$ . The transitions between states are indicated by the directed edges labeled  $\beta_{1,2}$  and  $\delta_{1,2}$ .**

We focus on the little-studied problem of two competing memes that propagate over different links across the same set of nodes. We use the term **composite network** to describe such a topology (illustrated in Figure 1). Individual agents are represented across the two planes, as indicated by the vertical lines, yet a meme may only propagate across links corresponding to a single plane. We assume mutual exclusivity of the memes, meaning that once an agent is affected by one meme, that agent is not susceptible to the cognate meme. This concept of mutual exclusivity is captured by our propagation model  $SI_1I_2S$ , and exemplified in Figure 1(b). Furthermore, memes not only propagate along *different topologies*, but they also have different propagation properties that describe their *viral strength* ( $\beta$ ) and *persistence* ( $\delta$ ).

To motivate our model, consider the following example. According to popular reports [19], the 2011 Egyptian revolution was partly coordinated using Twitter. In response, a tech-savvy government may inject bogus and/or contradictory information using a malicious Facebook application

Symbol	Definition	Symbol	Definition
$M_1, M_2$	Meme #1, #2	$\mathbf{A}_1, \mathbf{A}_2$	Adjacency matrices
$\delta_1, \delta_2$	Meme persistence of $M_1, M_2$	$\beta_1, \beta_2$	Meme strength of $M_1, M_2$
$S$	Susceptible state	$I_1, I_2$	Infected state for $M_1, M_2$
$\mathbf{S}_1, \mathbf{S}_2$	System matrix for $\mathbf{A}_1, \mathbf{A}_2$ , where $\mathbf{S} = (1 - \delta)\mathbf{I} + \beta\mathbf{A}$	$\lambda_1, \lambda_2$	Largest eigenvalue of $\mathbf{S}_1, \mathbf{S}_2$ in absolute value.

Table 1: Terminology

that spreads to someone’s friends. The central question is: which propagation will win? A related question is what is the relative intensity of a propagation to ensure dominance. Several cases could be modeled like this, at least at some abstract level. For example, we can think of the spread of a rumor in an enterprise or army via interpersonal communications versus some information distributed via official memos across the hierarchical structure of the enterprise.

The problem is very timely and relevant to many different applications and disciplines. First, the Internet, smartphones, and the emergence of online social networks have increased the communications modes among people. Furthermore, these new modes of communication enable interactions to be fast and to spread in an epidemic fashion. There is ample evidence for this sort of communication: viral YouTube videos that become highly popular in a week, the epidemic spread of news over Twitter, and the distribution of malware on Facebook. Second, a number of real-world scenarios may be modeled as described above, including the propagation of virus/anti-virus software, the spread of information/mis-information, or competing marketing campaigns (e.g., influencing a person to buy an iPhone or an Android phone).

We find that previous works have not focused on this problem. Most previous efforts study a single epidemic on a single topology [10, 16]. Those that have evaluated two competing pathogens focus on spread across a single topology [13, 15]. We discuss previous work in more detail in section 6. In this paper, we provide the first theoretical and experimental study of the competing memes problem in a composite network. Our work can be summarized in the following key points:

1. We provide a rigorous formulation of competing memes on composite networks using a modified susceptible-infected-susceptible (SIS) propagation mechanism. The full process is stochastic but at the same time very complex to analyze for real, general networks.
2. We propose a Non-Linear Dynamic System (NLDS)-based solution for the epidemic threshold (defined in Section 3) that determines the phase transition of the behavior of the system. Our analysis suggests that the first eigenvalue of an appropriately-constructed system matrix for each meme is a critical metric that determines system behavior.
3. We provide further evidence of the importance of our eigenvalue-based metric via simulations using: (1) synthetic composite networks with up to 50,000 of nodes, and (2) real-world composite network of mobile phone calls and text messages of 235 users.

To the best of our knowledge, our work is one of the first steps in understanding the interplay between two compet-

ing memes in a composite network. The power of the eigenvalue analysis is that it condenses the topological information. Furthermore, by bringing forward this less-studied, but interesting problem and its variations, we hope to spur research activity in this direction. Of course, other meme propagation and interaction models, e.g., one epidemic spread to a large population, nodes infected by both memes, or cross-contamination between the layers are possible; we leave their exploration to future work (Section 5).

## 2. MODEL AND DEFINITIONS

Our model is described by two components: (1) a composite network and (2) a propagation mechanism. We define a composite network as  $C = (N, E_1, E_2)$ , a single set of nodes  $N$  with two distinct edge sets  $E_1$  and  $E_2$ . Each layer of the composite network corresponds to a single adjacency matrix  $\mathbf{A}_1, \mathbf{A}_2$ .

The propagation mechanism is based on the popular “flu-like” SIS (Susceptible-Infected-Susceptible) model [10]. We name our model  $SI_1I_2S$  (Susceptible – Infected<sub>1</sub> – Infected<sub>2</sub> – Susceptible). Each node in the composite network is in one of three states: Susceptible (healthy),  $I_1$  (infected by  $M_1$ ), or  $I_2$  (infected by  $M_2$ ). The state transitions are shown in Fig. 1(b). Note that this is one of the several meme propagation models that one could consider, others being SI, SIR, etc. We believe that our model is a reasonable starting point, and we leave the analysis of other models as future work.

**Meme persistence:**  $\delta$ . If a node is in state  $I_1$  (or  $I_2$ ), it recovers on its own with probability  $\delta_1$  (or  $\delta_2$ ). This parameter captures the persistence of the meme in an inverse way: a high  $\delta$  means low persistence. For example, a very convincing rumor that sticks to one’s mind will be modeled with a low  $\delta$  value.

Note that we assume that while a node is infected by one meme, it cannot be infected by the other. We do not anticipate that allowing a meme to preempt each other (i.e., infect and subsume an infected node) would change the results from a qualitative point of view: this would be equivalent to having a node skip the recovery state and go straight to a new infection. As we will see later on, our metrics and methods consider the  $\delta$  values explicitly.

**Meme strength:**  $\beta$ . A healthy node gets infected by infected neighbors, and the meme strength is captured by  $\beta_1$  and  $\beta_2$ . Specifically, this parameter is the probability that an infected neighbor would pass the infection to a healthy neighbor in the absence of any other interaction. We refer to this potential infection-in-isolation as an **attack**. In the presence of multiple infected neighbors, we need to decide which infection succeeds (infects a susceptible node  $i$ ) as follows. Let  $C_1$  be the number of attacks (each happening with probability  $\beta_1$  independently) by node  $i$ ’s neighbors which are in state  $I_1$  (infected by  $M_1$ ); similarly, let  $C_2$  be the num-

ber of neighbors infected by  $M_2$ . Then, we have three possible scenarios for a node in the Susceptible state: (1) node  $i$  remains in the Susceptible state if  $C_1 = 0$  and  $C_2 = 0$ ; (2) node  $i$  gets infected with  $M_1$  with probability  $\frac{C_1}{C_1+C_2}$ ; (3) node  $i$  gets infected with  $M_2$  with probability  $\frac{C_2}{C_1+C_2}$ .

It is easy to see that this is a natural generalization of the SIS model to a competing-memes scenario. Moreover, note the competition between the memes: each meme has to compete with each other for healthy victims.

### 3. THE EPIDEMIC THRESHOLD

We want to determine the epidemic threshold (that determines viral dominance) analytically. First, we approximate the infection process by a discrete-time Non-Linear Dynamical System (NLDS) whose general form is  $\mathbf{p}_{t+1} = g(\mathbf{p}_t)$ . The NLDS gives the evolution of the system with time, as we explain below.

First, we see the probability that node  $i$  is infected by neighbor node  $j$  with meme  $M_1$  at time  $t$  is  $\beta_1 P_j^1(t-1)$ . This is what we referred to as *attack* earlier or infection by a neighbor in absence of other influences. Then, we have the probability  $\zeta_i^1(t)$  that node  $i$  does not receive the infection of  $M_1$  from its neighbors (assuming the neighbors are independent) as:

$$\zeta_i^1(t) = \prod_{j \in i's \text{ neighbors}} (1 - \beta_1 P_j^1(t-1)) \quad (1)$$

Thus, the probability that node  $i$  receives the infection of  $M_1$  at time  $t$  from its neighbors is:

$$1 - \zeta_i^1(t) = 1 - \prod_{j \in i's \text{ neighbors}} (1 - \beta_1 P_j^1(t-1)) \quad (2)$$

Using the same reasoning, we can obtain the probability of that node  $i$  receives the infection of  $M_2$  from its neighbors at time  $t$  is:

$$1 - \zeta_i^2(t) = 1 - \prod_{j \in i's \text{ neighbors}} (1 - \beta_2 P_j^2(t-1)) \quad (3)$$

Now, the probability that node  $i$  is infected by  $M_1$  from its neighbors at time  $t$  is the probability that node  $i$  receives the infection of  $M_1$  and does not receive infection of  $M_2$  from its neighbors at time  $t$ . Here we assume that the  $\beta$  and  $\delta$  values are all extremely small (or, equivalently, the time between state transitions is extremely small). We focus on the case when the epidemics are mutually-exclusive, i.e., a node cannot be infected with both viruses, but both compete for healthy victims. Other extensions are also possible where the viruses interact more strongly, which we leave as future work. This ensures that in any given timestep, the probability of having two or more events is vanishingly small. Thus, we get:

$$T_i^1(t) = (1 - \zeta_i^1(t)) \cdot \zeta_i^2(t) \quad (4)$$

With the same reasoning, the probability that the node is infected by  $M_2$  at time  $t$  is:

$$T_i^2(t) = (1 - \zeta_i^2(t)) \cdot \zeta_i^1(t) \quad (5)$$

Hence the probability that node  $i$  is in state  $I_1$  is:

$$P_i^1(t) = (1 - \delta_1) \cdot P_i^1(t-1) + T_i^1(t) \cdot S_i(t-1) \quad (6)$$

and the probability that it is in state  $I_2$  is:

$$P_i^2(t) = (1 - \delta_2) \cdot P_i^2(t-1) + T_i^2(t) \cdot S_i(t-1) \quad (7)$$

and the probability that it is in state  $S$  (Susceptible) is:

$$S_i(t) = (1 - T_i^1(t) - T_i^2(t)) S_i(t-1) + \delta_1 P_i^1(t-1) + \delta_2 P_i^2(t-1)$$

As mentioned before, for  $M_1$  we define the vector  $\vec{P}^1(t) = (P_1^1(t), P_2^1(t), \dots, P_N^1(t))'$  where  $P_i^1(t)$  is the probability that

node  $i$  is infected by meme  $M_1$  at time  $t$ . Similarly, for  $M_2$ , we have  $\vec{P}^2(t) = (P_1^2(t), P_2^2(t), \dots, P_N^2(t))'$ . Let  $\vec{V}(t) = (\vec{P}^1(t), \vec{P}^2(t))$  be the concatenation of two vectors. Using the NLDS formulation, we can now describe the whole infection process evolution as  $\vec{V}(t) = f(\vec{V}(t-1))$ , with:

$$f_i(\vec{V}(t-1)) = \begin{cases} (1 - \delta_1) P_i^1(t-1) + T_i^1(t) S_i(t-1) & \text{if } i \leq N \\ (1 - \delta_2) P_i^2(t-1) + T_i^2(t) S_i(t-1) & \text{if } i > N \end{cases} \quad (8)$$

Substituting  $T_i^1(t)$ ,  $T_i^2(t)$  and  $S_i(t-1)$  into equation 8, we find that  $f_i(\vec{V}(t-1))$  is equal to the following:

$$= \begin{cases} (1 - \delta_1) P_i^1(t-1) + (1 - \zeta_i^1(t)) \zeta_i^2(t) (1 - P_i^1(t-1) - P_i^2(t-1)) & \text{if } i \leq N \\ (1 - \delta_2) P_i^2(t-1) + (1 - \zeta_i^2(t)) \zeta_i^1(t) (1 - P_i^1(t-1) - P_i^2(t-1)) & \text{if } i > N \end{cases}$$

We make use of the following theorem about the asymptotic stability of an NLDS at a fixed point:

**THEOREM 1** (HIRSCH AND SMALE, 1974 [11]). *The system given by  $\mathbf{p}_{t+1} = g(\mathbf{p}_t)$  is asymptotically stable at an equilibrium point  $\mathbf{p}^*$ , if the eigenvalues of the Jacobian  $J = \nabla g(\mathbf{p}^*)$  are less than 1 in absolute value, where*

$$J_{k,l} = [\nabla g(\mathbf{p}^*)]_{k,l} = \frac{\partial p_{k,t+1}}{\partial p_{l,t}}|_{\mathbf{p}_t = \mathbf{p}^*}$$

The fixed point we are interested in for analyzing the threshold is the point where no node is infected (all nodes are healthy), i.e.,  $\vec{V}^* = \vec{0}$ . Using this, we have the following theorem:

**THEOREM 2.** *The system is asymptotically stable at  $\vec{V}^* = \vec{0}$  if the first eigenvalue of the system matrices for both memes as defined in Table 1, are less than 1, i.e.,  $\lambda_1 < 1$  and  $\lambda_2 < 1$ , where  $\lambda_1$  is the largest eigenvalue of matrix  $S_1 = (1 - \delta_1)I + \beta_1 A_1$  (and similarly for  $\lambda_2$ ).*

**PROOF.** Recall that we are interested in the stability of the fixed point  $\vec{V}^* = \vec{0}$ . Let the Jacobian at this point be  $\nabla(f)$  (a  $2N \times 2N$  matrix). Then

$$[\nabla(f)]_{ij} = \frac{\partial f_i(\vec{V}(t-1))}{\partial V_j(t-1)}$$

We can write it into a block matrix composed of the system matrices:

$$\nabla(f) = \begin{bmatrix} S_1 & S_3 \\ S_4 & S_2 \end{bmatrix}$$

In order to find the first eigenvalue of  $\nabla(f)|_{\vec{V}_j}$ , we define  $\vec{X}$  as  $2N$  elements vector:

$$\vec{X} = \begin{bmatrix} \vec{X}_1 \\ \vec{X}_2 \end{bmatrix}$$

where  $\vec{X}_1$  and  $\vec{X}_2$  have  $N$  elements respectively. We then have:

$$\nabla(f)|_{\vec{V}_j} \vec{X} = \begin{bmatrix} S_1 & S_3 \\ S_4 & S_2 \end{bmatrix} \cdot \begin{bmatrix} \vec{X}_1 \\ \vec{X}_2 \end{bmatrix} = \lambda_{\nabla(f)|_{\vec{V}_j}} \begin{bmatrix} \vec{X}_1 \\ \vec{X}_2 \end{bmatrix}$$

Doing the matrix multiplications, we get:

$$S_1 \vec{X}_1 + S_3 \vec{X}_2 = \lambda_{\nabla(f)}|_{v_j} \vec{X}_1$$

$$S_4 \vec{X}_1 + S_2 \vec{X}_2 = \lambda_{\nabla(f)}|_{v_j} \vec{X}_2$$

with  $S_1 = (1 - \delta_1)I + \beta_1 A_1$ ,  $S_2 = (1 - \delta_2)I + \beta_2 A_2$  and  $S_3 = S_4 = 0$  (where  $I$  is the  $N \times N$  identity matrix), as we show in Table 1 and as discussed below.

Hence, the Jacobian  $\nabla(f)$  is a block diagonal matrix and its eigenvalues are the same as the eigenvalues of  $S_1$  and  $S_2$ . So the largest eigenvalue of  $\nabla(f)$  can be either  $\lambda_1$  or  $\lambda_2$ .  $\square$

**Discussion: adjacency versus system matrix.** We can understand how the eigenvalue of the system matrix is the key parameter, if we consider the definition of the system matrix. At the same time, it is useful to stress the difference between the adjacency matrix,  $\mathbf{A}$ , and the system matrix,  $\mathbf{S}$ . One such matrix exists for each meme but here we drop the meme subscript.

The key point that we make is the following: the system matrix for a meme and thus the related eigenvalue are a function of the topology *and* the properties of the meme. The eigenvalues of the adjacency matrix  $\lambda_{\mathbf{A}}$  are related to the eigenvalues of the system matrix  $\lambda_{\mathbf{S}}$ . Recall that the system matrix is defined as  $\mathbf{S} = (1 - \delta)\mathbf{I} + \beta\mathbf{A}$ , where  $\mathbf{A}$  is the adjacency matrix. Therefore, if we consider an eigenvector for  $\mathbf{A}$ , that would also be an eigenvector for  $\mathbf{S}$  and the following will hold for the eigenvalues:

$$\lambda_{\mathbf{S}} = 1 - \delta + \beta \lambda_{\mathbf{A}} \quad (9)$$

In conclusion, the system eigenvalue  $\lambda_{\mathbf{S}}$  increases with the meme strength,  $\beta$  and the adjacency eigenvalue. Naturally,  $\lambda_{\mathbf{S}}$  decreases as the meme persistence  $\delta$  increases.

## 4. SIMULATION STUDY

We use a discrete-time simulation of our system that simulates the stochastic behavior of competing meme on several different synthetic and real composite networks.

### 4.1 Small-scale Data Sets ( $N < 1,000$ )

**Real-world enterprise composite network (ENT).** We have obtained a composite network dataset that represents the phone call and SMS text message communications within an urban branch of a large Chinese corporation [20]. Each node is an employee ( $|N| = 235$ ), the edges in  $E_1$  correspond to SMS messages exchanged between employees, and edges in  $E_2$  correspond to phone calls made between employees. The data was captured over the course of six months. Among all communicating pairs of users, 31% communicate via calls alone, 28% via SMS alone, and 41% via both calls and SMS.

**Synthetic composite networks.** We have created two synthetic graphs with 1,000 nodes: the first one is an Erdős-Rényi graph, whereas the second one is a scale-free graph; we use the Barabási-Albert model [3]. We have experimented with several different combinations of topologies. Here, we focus on these two, because: (a) we would like to have significantly different topologies, in order to show that our methods are not tailored to a particular family of graphs, and (b) scale-free graphs are known to emerge in complex human and communication networks [3].

### 4.2 Large-scale Data Sets ( $1,000 < N < 50,000$ )

To further stress-test the accuracy of our model, we conducted experiments on synthetic social networks with  $1,000 < N < 50,000$  nodes using the **forestFire**, **randomWalk**, and **nearestNeighbor** graph generation models provided by Sala et al. [17]. These synthetic models are informed by real world measurements of social networks (e.g, Facebook) and provide graph structures that resemble such networks.

### 4.3 Simulation Experiments

All experiments on real and synthetic composite networks were conducted using a combination of Matlab and Python. All values are averaged over 100 simulation runs. In each experiment, each meme infects a unique set of nodes  $Ini_1$  and  $Ini_2$ , each with the same size, selected uniformly at random from  $N$ , subject to the constraint  $Ini_1 \cap Ini_2 = \emptyset$  (i.e., mutually exclusive). We run each simulation until it reaches a relatively stable state, at which point, we determine the average number of nodes infected by  $M_1$  and  $M_2$  and report the outcome, which then gets averaged across 100 runs. Note that: the definition of “relatively stable state” hides several subtleties, which have to do with the asymptotic behavior of the system, namely what happens as time goes to infinity [16]. However, as we are reliant on simulations, we are forced to adopt a more practical definition. First, we examine the behavior, after a sufficient warm-up period, when the system converges to some relatively stable state (with only small fluctuations of its infected nodes).

### 4.4 Analysis of Results

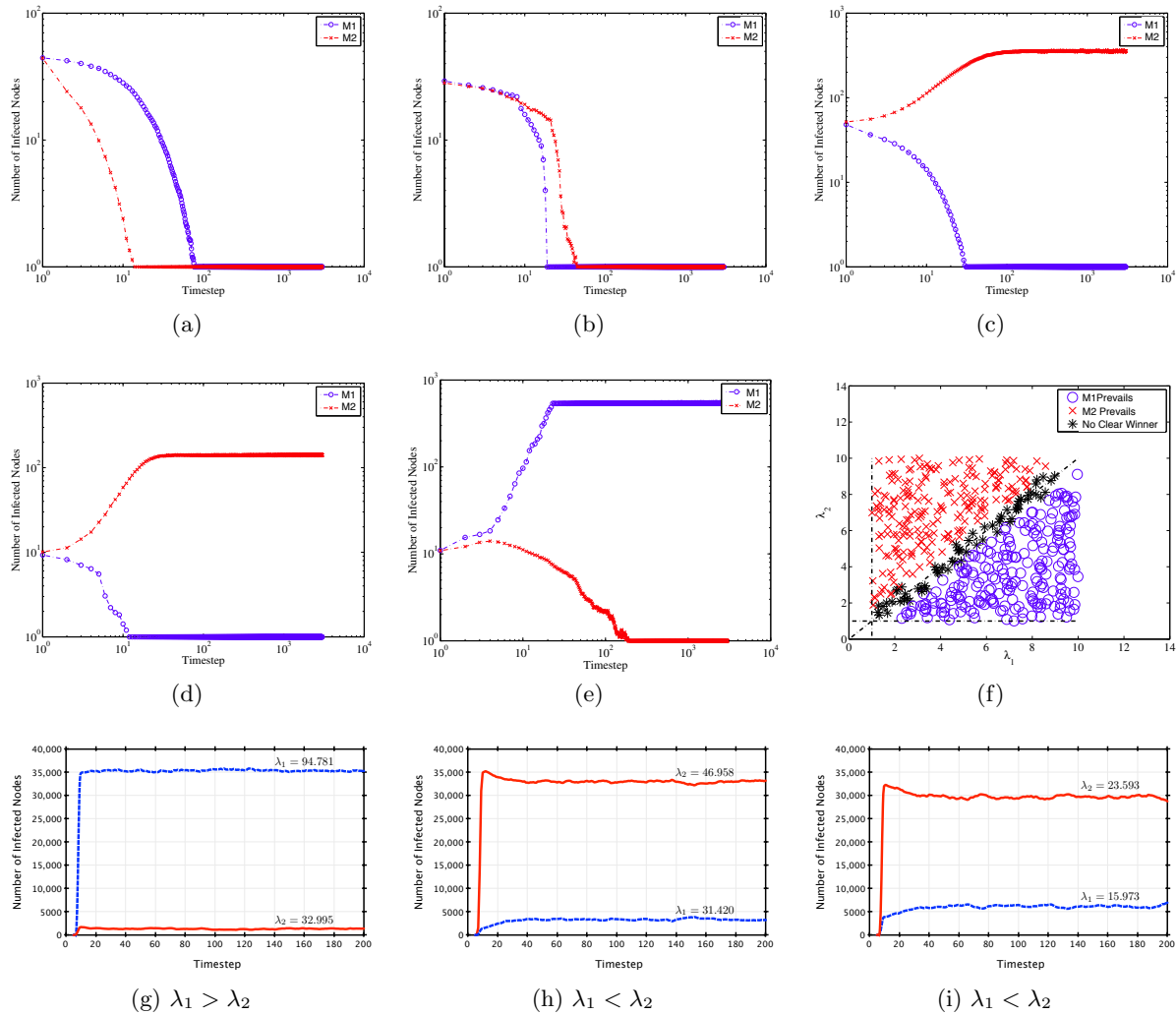
From Section 3, we know that if the system matrix’s first eigenvalue of one meme is less than 1, the corresponding meme will die-out eventually. Therefore, in this scenario, we can predict which meme prevails eventually using the following three rules:

- (i) if  $\lambda_1 < 1$  and  $\lambda_2 > 1$ , then  $M_2$  tends to prevail eventually in the composite networks;
- (ii) if  $\lambda_1 > 1$  and  $\lambda_2 < 1$ , then  $M_1$  tends to prevail eventually in the composite networks;
- (iii) if  $\lambda_1 < 1$  and  $\lambda_2 < 1$ , then both memes will die out and none of them can be said to prevail.

Figures 2(a)-(e) demonstrate the proposed rules on both synthetic and real composite networks. The infection starts by infecting 30 nodes for each meme in Figure 2(a), Figure 2(b) and Figure 2(c), and 10 nodes for each meme in both Figure 2(d) and Figure 2(e). The outcomes of below- and above-threshold from these rules can be distinctly seen in these figures. These results show that, though simple, our proposed rules are very effective for predicting which meme tends to prevail eventually in the composite networks.

This is the more interesting case in terms of competition: each meme in isolation would not die-out, so it is a “fight for dominance.” As shown in Figure 2(f), we find again that the system eigenvalues play a critical role: the meme whose first eigenvalue is larger tends to prevail eventually in the composite networks. No clear winner is the case where the difference is less than  $\theta$ , which here is 10%. Note that we experimented with other  $\theta$  values (5%, 10%, and 15%) and the results were qualitatively similar.

In addition, Figures 2(g)-(i) show experimental results of large scale epidemic simulations using **aForestFire** and **nearestNeighbor** synthetic graph models. For brevity, we only show the graphs for  $N = 40,000$  nodes, but we found



**Figure 2: Simulation Results: Infection plot over time (log-log) in Figure(a)-(e). 2(a): Synthetic Composite Networks:  $\lambda_1 = 0.97, \lambda_2 = 0.96$ ; 2(b): Real Composite Networks:  $\lambda_1 = 0.9, \lambda_2 = 0.94$ ; 2(c): Synthetic Composite Networks:  $\lambda_1 = 0.91, \lambda_2 = 1.63$ ; 2(d): Real Composite Networks:  $\lambda_1 = 0.99, \lambda_2 = 1.4$ ; 2(e):  $\lambda_1 = 4.5, \lambda_2 = 1.7$ ; 2(f): The outcomes for different combinations of system eigenvalues:  $1 < \lambda_1 < 10$  and  $1 < \lambda_2 < 10$ ; black dotted lines represent three lines  $\lambda_1=1$ ,  $\lambda_2=1$ , and  $\lambda_1=\lambda_2$ . When the eigenvalues are roughly equal there is no clear winner.**

similar experimental results for 10,000 to 50,000 nodes. Unlike smaller-scale experiments, these results show that the weaker meme may retain some endemic population, yet the meme with the larger eigenvalue clearly dominates the simulation.

## 5. DISCUSSION AND FUTURE WORK

In this section, we discuss the limitations of our work and possible future directions.

**Choice of epidemic model.** The flu-like SIS (Susceptible-Infected-Susceptible) epidemiological model is simple, yet illustrative, and has been extensively studied in past literature in a single-virus setting. Therefore, we chose to extend SIS in order to gain fundamental insights into the dynamics of competing memes. We leave the investigation of other epidemic models as future work.

**Future exploration.** Our paper is the first attempt to study competing epidemic propagations on composite net-

works. We leave some tasks to further explorations: (a) in this paper we focus on the case when the epidemics are mutually exclusive, but other extensions are also possible where the viruses interact and infect in more nuanced ways, e.g., full competition, no competition and in-between models, of memes propagation on composite network, and cross-contamination between the layers of the composite network; (b) our analysis focuses on the long-time nature of the epidemics, i.e., “what happens in the end?”; analyze the exact transient fluctuations, would also be interesting; (c) constructing more effective and accurate predictors based on our findings as shown in Figure 2(f) and immunization methods to control the outcome of this competition, and finding the extent of foot-prints, proving performance bounds for inoculation policies; and (d) running the simulation on other real-world data sets [6, 2] besides our enterprise phone call/SMS network.

## 6. RELATED WORK

**Single-meme propagation.** Compartmental models like SIS, SIR, etc., have been well-studied in many epidemiological texts [1]. Information cascades models are proposed to study the meme propagation in word-of-mouth communications [8]. Numerous studies exist on virus propagation on the Internet based on the basic epidemic models of infection [18]. A fundamental question in epidemiology is the presence of a threshold, under which an epidemic is guaranteed not to happen. Pastor-Satorras et al. [14] proposed an epidemic threshold condition for random power law networks, which uses the “mean-field” approach. Ganesh et al. [7] provided epidemic threshold for the single-virus on single topology. Prakash et al. [16] gave the epidemic threshold condition for almost all single-virus epidemic models on a single static network.

**Multiple memes and interdependent networks.** Newman [13] studied multiple viruses on a single, special random graph and provided the epidemic threshold for the case when the second virus propagates over the residual network after the propagation of the first virus has completed. This scenario is close to the dynamics of propagation of a single virus—one virus passed over the network, the second virus starts to pass over the residual network. Models for multiple cascades have been studied as extensions of the independent cascade model, where once a node is infected with a cascade, it never change its state [4]. The effects of cascades in inter-dependent networks (e.g., Internet router and power electricity networks) were investigated by Buldyrev et al. [5].

**Game theory.** Kostka et al. [12] studied competing campaigns as a game-theoretical problem and showed that being the first player was not always advantageous. Goyal et al. [9] proposed a game-theoretical framework to reveal the competition among firms who try to maximize product adoption by consumers.

## 7. CONCLUSION

In this paper, we have formulated the scenario of competing memes on composite networks as an  $SI_1I_2S$  model and showed that the epidemic threshold depends on the largest eigenvalues of the system matrices of both memes. Extensive simulations on different datasets demonstrate that the epidemic threshold and the largest eigenvalue are effective predictors of the eventual outcome.

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