Appendix: Efficient Stochastic Routing in Path-Centric Uncertain Road Networks

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1 NAIVE STOCHASTIC ROUTING IN PACE

Algorithm 1: Naive Stochastic Routing in PACE

Input: Source v_s; Destination v_d; Budget B; PACE graph $\mathcal{G}^{\mathcal{P}}$;
Output: The path with the largest probability of getting v_d within B;

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Priority queue Q ← ∅;
Double maxProb ← 0; Path P* ← ∅;
Q.push(⟨v_s, v_s⟩, 0);
while Q ≠ ∅ do
    Path P̂ ← Q.peek();
    if P̂ reaches v_d and Prob(D(P̂) ≤ B) > maxProb then
        maxProb ← Prob(D(P̂) ≤ B); P* ← P̂;
    for each outgoing edge or T-path e do
        Extend P̂ by edge e;
        Q.insert(P̂, Expectation(D(P̂)));
return P*;
```

2 COMPLEXITY ANALYSIS

2.1 Algorithm 1

Given graph $\mathcal{G}^{\mathcal{P}}_{rev} = (\mathbb{V}, \mathbb{E}', \mathbb{P}', \mathbb{W}')$, the time complexity for building the binary heuristics is $O((|\mathbb{E}'| + |\mathbb{P}'|)lg|\mathbb{V}|)$. Specifically, in the worst case, we visit all edges and T-paths, thus having $|\mathbb{E}'| + |\mathbb{P}'|$. For each visit, we need to update the priority queue, which at most has $|\mathbb{V}|$ elements, and thus the update takes at most $lg|\mathbb{V}|$.

The space complexity is $O(|\mathbb{V}|^2)$. For each destination, each vertex maintains a binary heuristic value.

2.2 Algorithms 2 and 3

Given graph $\mathcal{G}^{\mathcal{P}} = (\mathbb{V}, \mathbb{E}, \mathbb{P}, \mathbb{W})$, the time complexity for Algorithms 2 and 3 is $O(|\mathbb{V}| \cdot \eta^2 \cdot |out|)$, where η is the number of columns in the heuristic table and $|out|$ is the largest outdegree of a vertex. We apply dynamic programming to build the heuristics table using Eq. (4). The heuristics table has in total $\mathbb{V} \cdot \eta$ elements. To fill in an element in the table, we need to visit at most $|out| \cdot \eta$ other elements because we may visit up to $|out|$ rows (see $Z \in ON(v_i)$ in Eq. (4)) and each row we may visit up to η elements (see $\sum_{k=1}^{x}$ in Eq. (4)).

The space complexity is $O(|\mathbb{V}|^2 \cdot \eta)$. For each destination, we maintain a heuristic table taking $|\mathbb{V}| \cdot \eta$, therefore a total of $|\mathbb{V}| \cdot |\mathbb{V}| \cdot \eta$.

2.3 Building V-paths

Given graph $\mathcal{G}^{\mathcal{P}} = (\mathbb{V}, \mathbb{E}, \mathbb{P}, \mathbb{W})$, the time complexity for building V-paths is $O(|\mathbb{P}| \cdot |\mathbb{V}|)$.

Recall that V-paths are generated iteratively. Each iteration can at most generate $|\mathbb{P}|$ new V-paths and we can at most have $|\mathbb{V}|$ iterations. In the first iteration, overlapping T-paths are combined into V-paths. Thus, the number of generated V-paths must be smaller than the number of T-paths because the generated V-paths must have higher cardinality than the corresponding T-paths and they both represent the same path. In the next iteration, the same principle applies. Thus, each iteration can at most generate $O(|\mathbb{P}|)$ new V-paths.

Next, we explain why we can get most have $|\mathbb{V}|$ iterations. At each iteration, the cardinality of V-paths are at least one more than the V-paths in the previous iteration. In a graph with $|\mathbb{V}|$ vertices, the longest simple path (i.e., without loops) in the graph can at most have $|\mathbb{V}|$ vertices. Otherwise, there must be at least one vertex that appears twice in the path making a loop and thus the path is not a simple path anymore. Thus, we only need to perform at most $|\mathbb{V}|$ iterations as we need to ensure that V-paths are simple paths.

Space complexity is $O(|\mathbb{P}| \cdot |\mathbb{V}| \cdot |St|)$ where $|St|$ is the storage used for the longest V-path.

3 OFFLINE PRE-COMPUTATION COSTS

We report the overall offline pre-computation cost for building binary heuristics for D_1 and D_2 in Table 1, where run time and storage is reported by hours (H) and megabytes (MB), respectively.

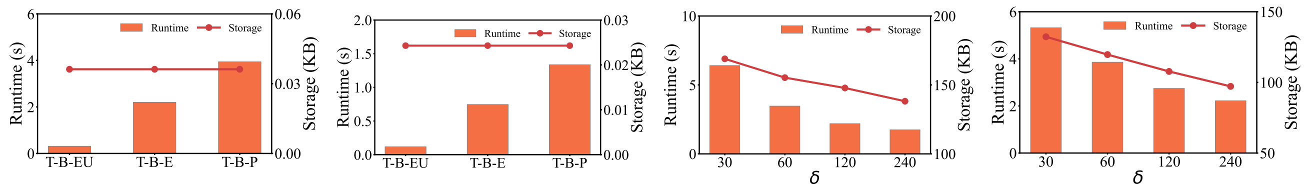
Data Set D_1						
Methods	Off-Peak Hours			Peak Hours		
	T-B-EU	T-B-E	T-B-P	T-B-EU	T-B-E	T-B-P
Run time (H)	1.8	17.6	32.3	2	19.2	34.7
Storage (MB)	1.14	1.14	1.14	1.14	1.14	1.14
Data Set D_2						
	Off-Peak Hours			Peak Hours		
	T-B-EU	T-B-E	T-B-P	T-B-EU	T-B-E	T-B-P
Run time (H)	0.56	5.1	8.9	0.67	5.5	10
Storage (MB)	0.66	0.66	0.66	0.66	0.66	0.66

Table 1: Binary Heuristics Pre-computation

When building budget-specific heuristics, we did not use parallelization, which, however, is possible because building the heuristics for different vertices is independent. The offline pre-computation cost for D_1 and D_2 is reported in Table 2, where run time and storage is reported by hours (H) and gigabytes (GB), respectively.

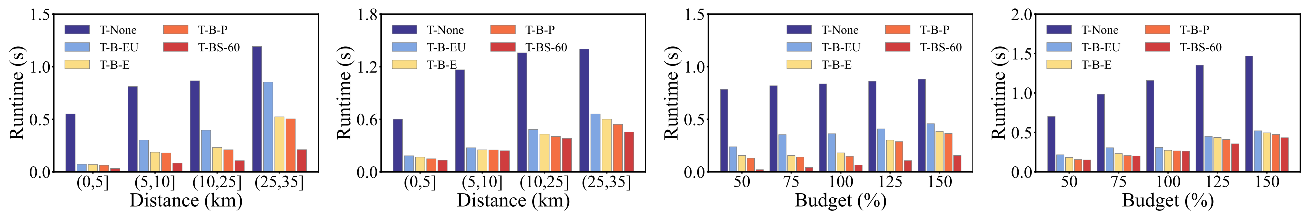
Data Set D_1								
δ	Off-Peak Hours				Peak Hours			
	30	60	120	240	30	60	120	240
Run time (H)	43.2	33.4	20.4	16.8	56	28.5	17.4	13.2
Storage (GB)	5.77	5.13	4.67	4.29	5.10	4.75	4.46	4.17
Data Set D_2								
δ	Off-Peak Hours				Peak Hours			
	30	60	120	240	30	60	120	240
Run time (H)	48.5	23.4	13.4	10.9	40.2	28.4	20.1	16.7
Storage (GB)	3.49	3.16	2.84	2.56	3.23	2.99	2.76	2.53

Table 2: Budget-Specific Heuristics Pre-computation



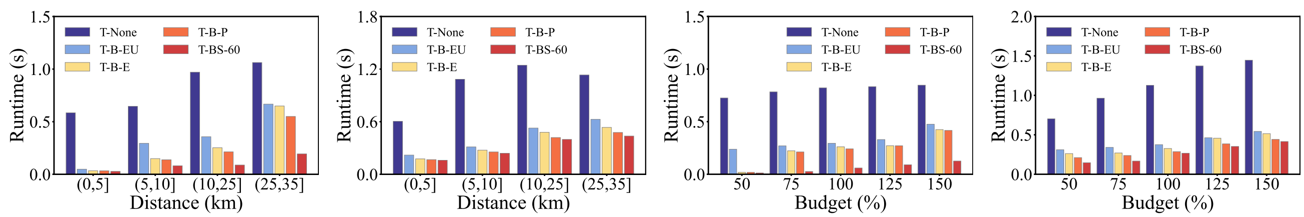
(a) Binary Heuristics, D_1 (b) Binary Heuristics, D_2 (c) Budget-Specific Heuristics, D_1 (d) Budget-Specific Heuristics, D_2

Figure 1: Offline Building Binary Heuristics and Budget-Specific Heuristics



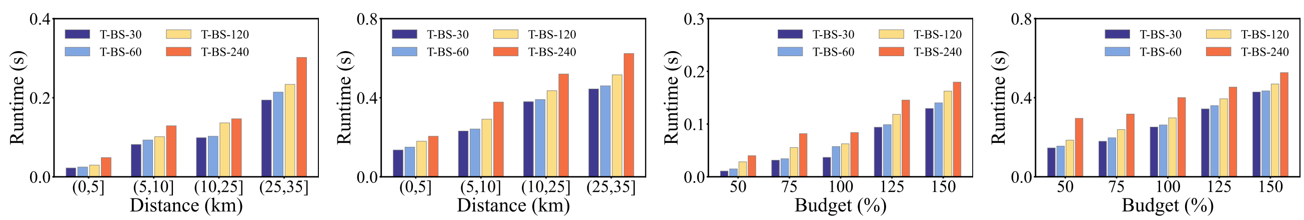
(a) By distances, D_1 (b) By distances, D_2 (c) By budget values, D_1 (d) By budget values, D_2

Figure 2: Stochastic Routing with Binary Heuristics at Off-Peak Hours



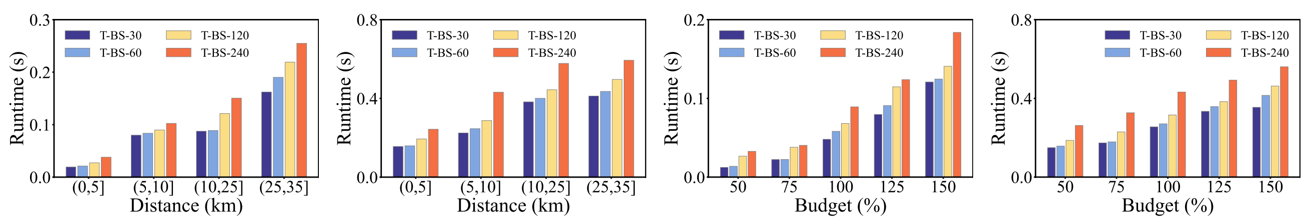
(a) By distances, D_1 (b) By distances, D_2 (c) By budget values, D_1 (d) By budget values, D_2

Figure 3: Stochastic Routing with Binary Heuristics at Peak Hours



(a) By distances, D_1 (b) By distances, D_2 (c) By budget values, D_1 (d) By budget values, D_2

Figure 4: Stochastic Routing with Budget-Specific Heuristics at Off-Peak Hours



(a) By distances, D_1 (b) By distances, D_2 (c) By budget values, D_1 (d) By budget values, D_2

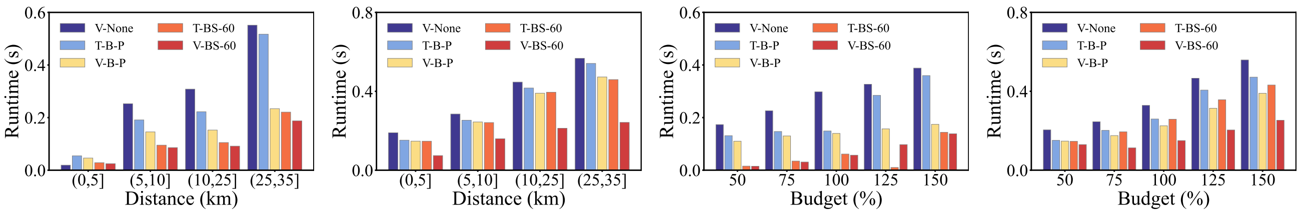
Figure 5: Stochastic Routing with Budget-Specific Heuristics at Peak Hours

4 EXPERIMENTAL RESULTS

We provide experimental results for stochastic routing with binary heuristics at off-peak and peak hours for data sets D_1 and D_2 in Figure 2 and 3, respectively.

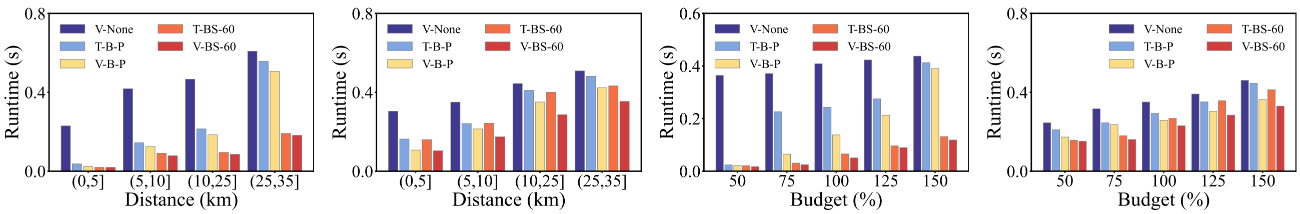
We provide experimental results for stochastic routing with budget-specific heuristics at off-peak and peak hours for data sets D_1 and D_2 in Figure 4 and 5, respectively.

We provide experimental results for V-path based stochastic routing at off-peak and peak hours for data sets D_1 and D_2 in Figure 6 and 7, respectively.



(a) By distances, D_1 (b) By distances, D_2 (c) By budget values, D_1 (d) By budget values, D_2

Figure 6: V-Path based Stochastic Routing at Off-Peak Hours



(a) By distances, D_1 (b) By distances, D_2 (c) By budget values, D_1 (d) By budget values, D_2

Figure 7: V-Path based Stochastic Routing at Peak Hours