MCMC Produciton Data Analysis with Arps, THM and Jacobi Theta Function Model

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Step 0: Import the basic packages

In [1]:
```
import pandas
import numpy as np
import matplotlib.pyplot as plt
from scipy import optimize
plt.style.use('ggplot')
%matplotlib inline
```

Step 1： Read production data from CSV file （Drillinginfo output）

In [2]:
```
#Extract api, monthly_oil and date from Drillinginfo File
df = pandas.read_csv('Monthly_Production.CSV')
oil = np.array(df['Monthly Oil'].values)
api_well = np.array(df['API/UWI'].values)
pro_month = np.array(df['Monthly Production Date'].values)
unique_ids=len(np.unique(api_well))
# Confirming all 153 wells are read into memory
print('Total number of wells are: {}'.format (unique_ids))
```

```
Total number of wells are: 153
```

Read each individual well's production data into an array

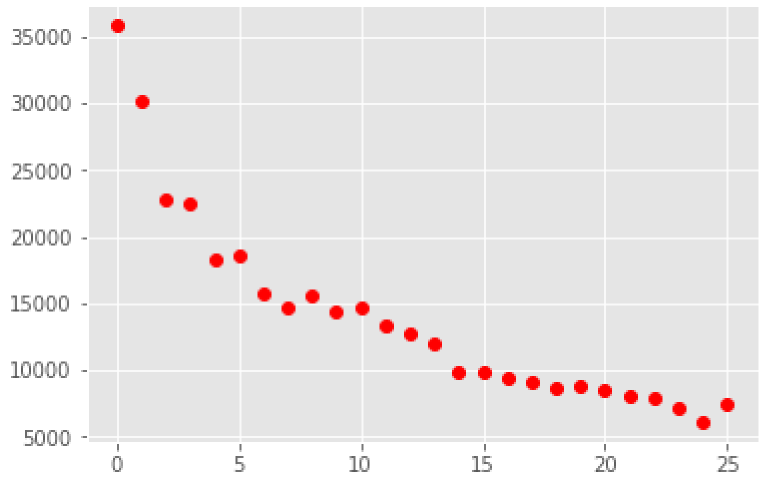
- Chose a well, and plot the original production data
- Take first 12 month of log(oil rate) as training data

In [3]:
```
ID_0 = api_well[0]
i=0
j=0
k=0
oil_array=np.zeros((unique_ids,36),dtype=int)
for ID in api_well:
    if ID==ID_0:
        oil_array[i,j]=oil[k]
        j=j+1
    else:
        ID_0=ID
        i=i+1
        j=0
    k=k+1
```

In this case, we chose well #150, the user can choose any random well

In [4]:
```
cur = oil_array[7]
rec_month = np.count_nonzero(cur)
plt.plot(cur[:rec_month],'ro')
```

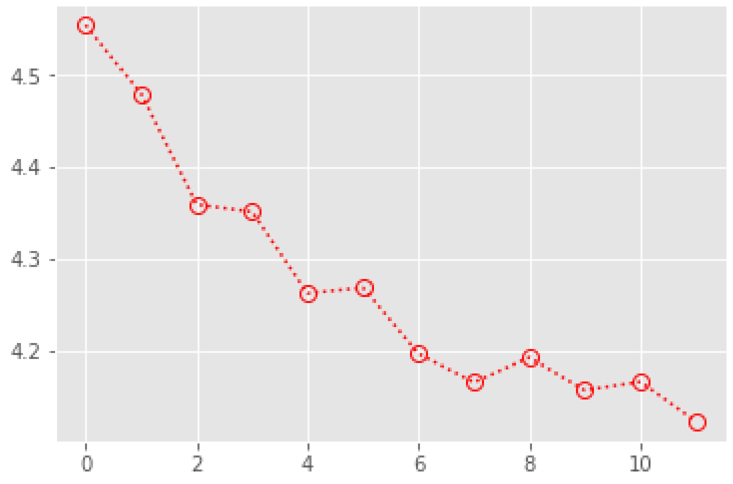
Out[4]:
```
[<matplotlib.lines.Line2D at 0x106113518>]
```



plot the first 12 month as training data

In [5]:
```
data = np.log10(cur[0:12])
marker_style = dict(color='r', linestyle=':', marker='o',
                    markersize=8, markerfacecoloralt='gray')
plt.plot(data,fillstyle='none',**marker_style)
```

Out[5]:
```
[<matplotlib.lines.Line2D at 0x1018eb3f28>]
```



Section 1

In this section, we implement the MCMC Arps model

1. Define Arps equation and get best estimate using least square residual.

- Plot data, initial guess and best estimate

In [6]:
```
def Arps(t,Di,logQi,b):
    Qt = logQi - 1/b*np.log10(1+b*Di*t)
    return Qt
```

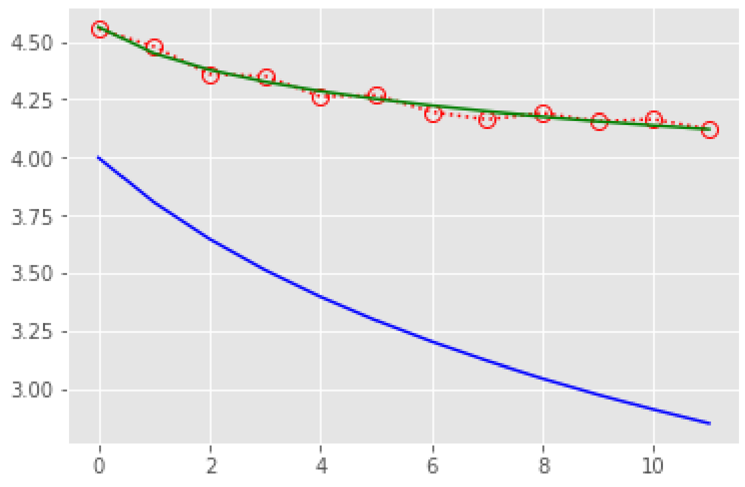
In [7]:
```
Di = 0.5
logQi = 4
b = 0.5
t = np.linspace(0,11,12)
y_init =  Arps(t,Di,logQi,b)
params, params_covariance = optimize.curve_fit(Arps, t, data,
                                               p0=[0.22,4.3,2])
y_hat = Arps(t,params[0],params[1],params[2])
sigma = np.sqrt(np.sum(np.power(data-y_hat,2))/(len(data)-3))

print('The best estimate are: \n Di = {}\n log(Qi) = {}\n b = {}'.format(pa
print('Variance is {}'.format(sigma))
```

```
The best estimate are:
 Di = 0.360589738558525
 log(Qi) = 4.563803782420842
 b = 2.2705705058444425
Variance is 0.02487316937430861
```

The plot below shows the initial guess (blue line) is random, which is far away from best (least square) estimate

In [8]:
```
plt.plot(y_init,'b-') # initial guess
plt.plot(data,fillstyle='none',**marker_style)
plt.plot(y_hat,'g-')
```

Out[8]:
```
[<matplotlib.lines.Line2D at 0x10615b898>]
```



2. MCMC for parameter estimation

- Define posterior function
- Initialize the chain
- Sampling from posterior distribution

In [9]:
```
def post(Di,logQi,b,sigma,t,data):
    like = np.prod(np.exp(-np.power(Arps(t,Di,logQi,b)-data, 2)/sigma**2))
    return like # because constant uniform prior
```

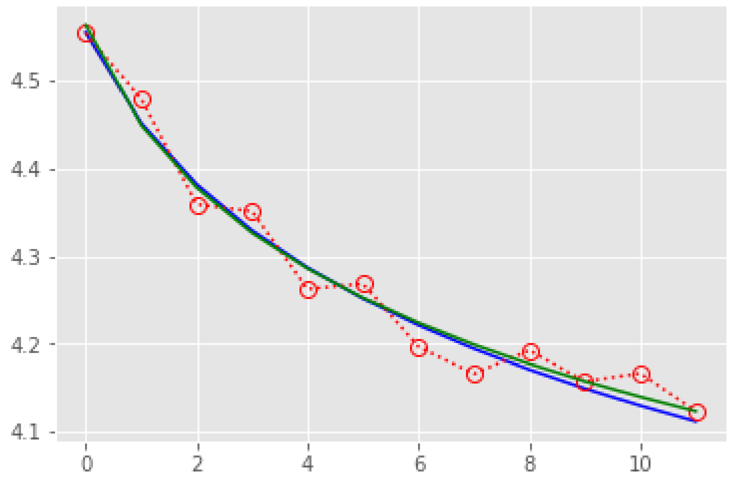
In [10]:
```
logQi = data[0]
Niter = 100000
chain = np.zeros((Niter,3))
chain[0,0] = Di # Initialize the chain with first guess
chain[0,1] = logQi
chain[0,2] = b
```

In [11]:
```
for ii in np.arange(Niter-1):
        if ii%10000 ==0:
            print('iteration {}'.format(ii))
        Di    = chain[ii,0]
        logQi = chain[ii,1]
        b     = chain[ii,2]
        #sigma = chain[ii,3]
        Di_p    = Di + np.random.normal(loc=0.0,scale=0.1,size=1)
        logQi_p = logQi + np.random.normal(loc=0.0,scale=0.1,size=1)
        b_p     = b + np.random.normal(loc=0.0,scale=0.1,size=1)

        if Di_p < 0.1 or Di_p>50 or b_p<0 or b_p>=2 or logQi > 1000000 or l
            chain[ii+1,:] = chain[ii,:]
        elif (post(Di_p,logQi_p,b_p,sigma,t,data) / post(Di,logQi,b,sigma,t
            chain[ii+1,0] = Di_p
            chain[ii+1,1] = logQi_p
            chain[ii+1,2] = b_p
        else:
            chain[ii+1,:] = chain[ii,:]
print('Sampling finished at iteration {}'.format(Niter))
```

```
iteration 0

/Users/weibing/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.p
y:14: RuntimeWarning: invalid value encountered in double_scalars

/Users/weibing/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.p
y:14: RuntimeWarning: divide by zero encountered in double_scalars


iteration 10000
iteration 20000
iteration 30000
iteration 40000
iteration 50000
iteration 60000
iteration 70000
iteration 80000
iteration 90000
Sampling finished at iteration 100000
```

```
In [12]: y_theta =  Arps(t,Di,logQi,b)
         plt.plot(y_theta,'b-')
         plt.plot(data,'ro',fillstyle='none',**marker_style)
         plt.plot(y_hat,'g-')
```

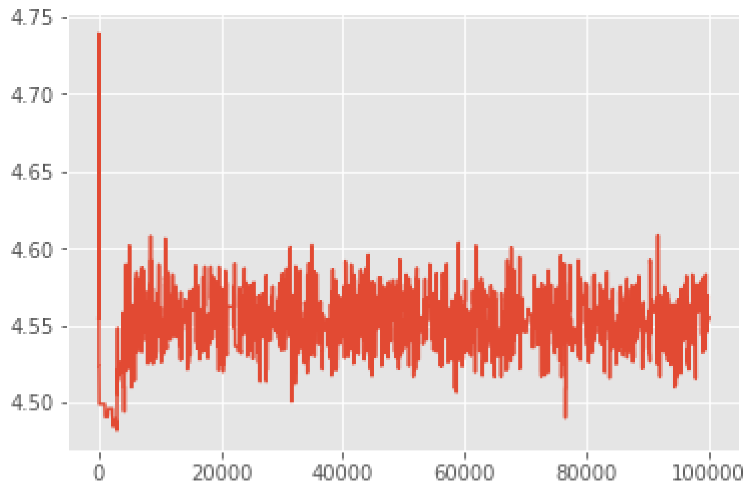
```
Out[12]: [<matplotlib.lines.Line2D at 0x1019088d68>]
```



Check Mixing of the chain, it shows good convergence

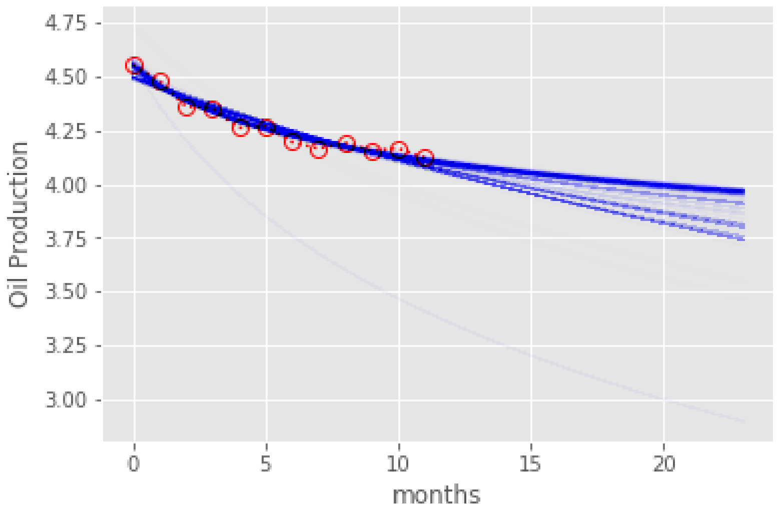
```
In [13]: plt.plot(chain[:,1])
         chain.shape
```

```
Out[13]: (100000, 3)
```



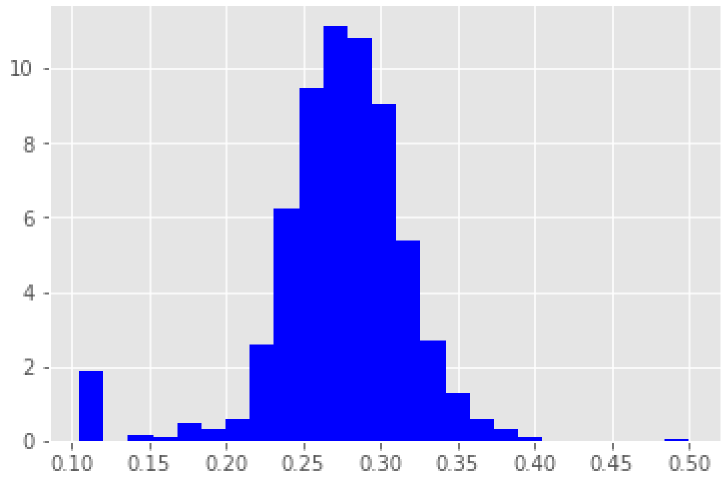
Plot the last 1000 samples from the posterios

In [14]:
```
t_test = np.linspace(0,23,24)
chains_to_plot=np.arange(1000)*9

fig = plt.figure()
aa = plt.axes()
aa.plot(data,'ro',fillstyle='none',**marker_style)
#aa.plot(y_hat,'g-')
kk=0
qq=np.zeros([len(chains_to_plot),24])
for ii in chains_to_plot:
    qq[kk,:]=Arps(t_test,chain[ii,0],chain[ii,1],chain[ii,2])
    aa.plot(qq[kk,:],color='blue',alpha=0.002)
    kk+=1
    aa.set(xlabel='months', ylabel='Oil Production')
```

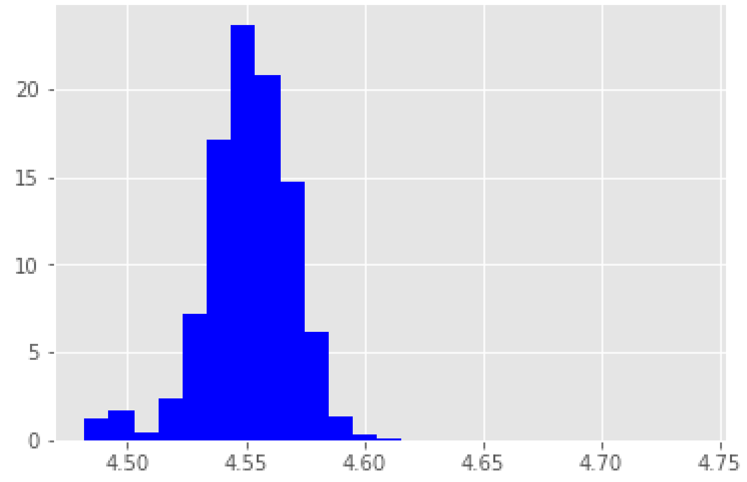


Posterior Summary, the Di, log(Qi), and b parameters

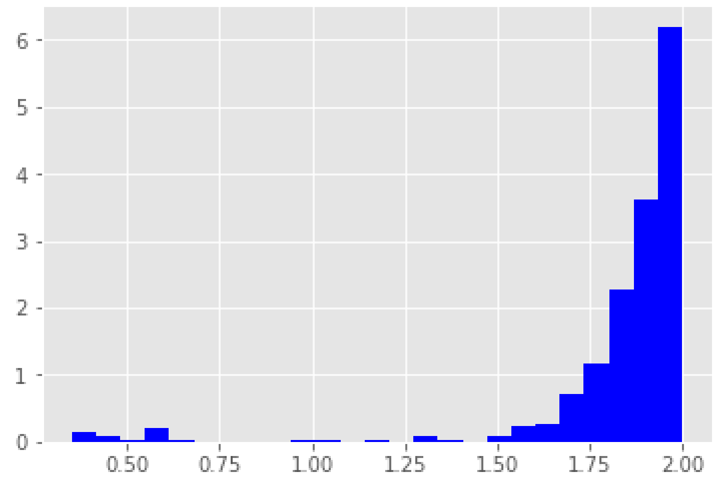
In [15]:
```
fig = plt.figure()
ax = plt.axes()
rectangles = ax.hist(chain[:,0],25,density=True,color='blue')
```



In [16]:
```
fig = plt.figure()
ax = plt.axes()
rectangles = ax.hist(chain[:,1],25,density=True,color='blue')
```



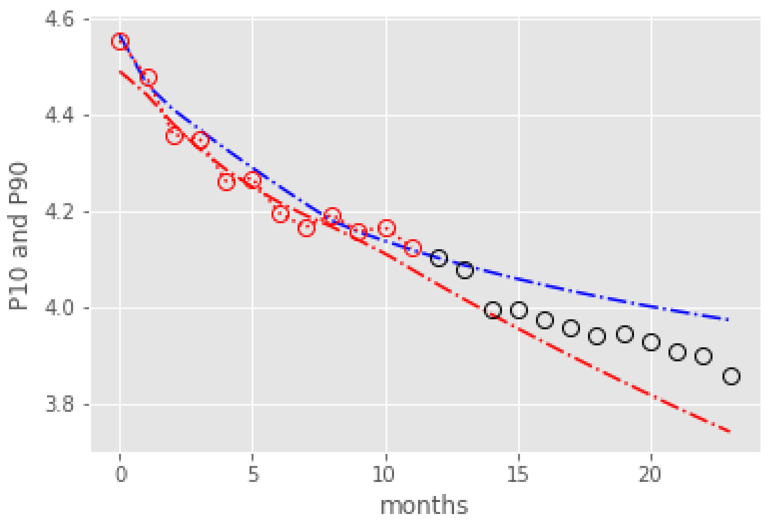
In [17]:
```
fig = plt.figure()
ax = plt.axes()
rectangles = ax.hist(chain[:,2],25,density=True,color='blue')
```



3. Plot of 12month training data, 12-24 month testing data and p10-p90 interval

In [37]:
```
qq_p10=np.zeros(24)
qq_p90=np.zeros(24)
for tt in np.arange(0,24):
    sorted_qq=np.sort(qq[:,tt])
    ixL = np.floor(np.size(sorted_qq)*0.1).astype(int) ## lower index
    ixU = np.floor(np.size(sorted_qq)*0.9).astype(int) ## upper index
    qq_p10[tt]=sorted_qq[ixL]
    qq_p90[tt]=sorted_qq[ixU]

fig = plt.figure()
aa = plt.axes()
aa.plot(qq_p10,'r-.')
aa.plot(qq_p90,'b-.')
aa.set(xlabel='months', ylabel='P10 and P90');
aa.plot(data,'ro',fillstyle='none',**marker_style)
aa.plot(np.linspace(12,23,12),np.log10(cur[12:24]),'ko',fillstyle='none',ma
```

Out[37]:
```
[<matplotlib.lines.Line2D at 0x101acec5f8>]
```

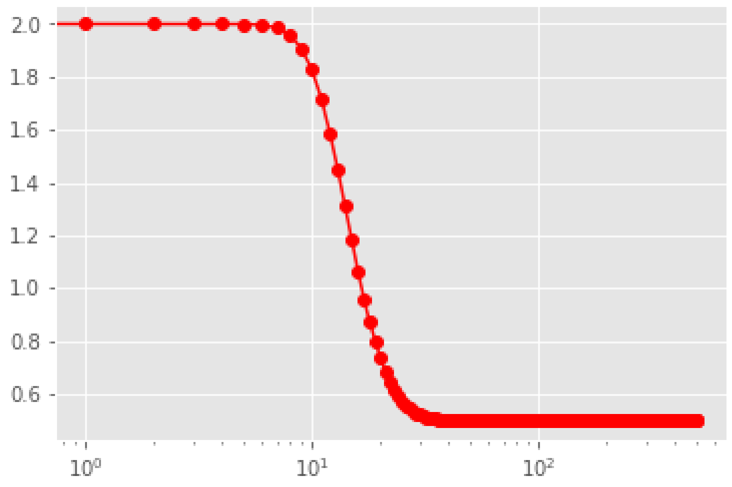


Section 2

In this section, we implement MCMC parameter estimation for the Transient Hyperbolic Model proposed by David Fulfor and Dr. Blasingame

In [38]:
```
# First we plot one example of THM model to see its behavior
t_elf = 12
bf = 0.5
t_test = np.linspace(0,1000,500)
c = np.exp(0.57722)/(1.175*t_elf)
b = 2-(2-bf)*np.exp(-np.exp(-c*(t_test-t_elf)+np.exp(0.57722)))
plt.semilogx(b,'ro-')
```

Out[38]:
```
[<matplotlib.lines.Line2D at 0x101ad4f8d0>]
```



We define the posterior probability function based on changing b parameter

In [39]:
```
def post(Di,logQi,bf,t_elf,sigma,t,data):
    c = np.exp(0.57722)/(1.175*t_elf)
    b = 2-(2-bf)*np.exp(-np.exp(-c*(t-t_elf)+np.exp(0.57722)))
    like = np.prod(np.exp(-np.power(Arps(t,Di,logQi,b)-data, 2)/sigma**2))
    return like # because constant uniform prior
```

In [40]:
```
logQi = data[0]
bf = 0.5
t_elf = 24
Niter = 100000
chain = np.zeros((Niter,4))
chain[0,0] = Di # Initialize the chain with first guess
chain[0,1] = logQi
chain[0,2] = bf
chain[0,3] = t_elf
```

In [41]:
```
for ii in np.arange(Niter-1):
        if ii%10000 ==0:
            print('iteration {}'.format(ii))
        Di     = chain[ii,0]
        logQi = chain[ii,1]
        bf     = chain[ii,2]
        t_elf = chain[ii,3]
        Di_p     = Di + np.random.normal(loc=0.0,scale=0.1,size=1)
        logQi_p = logQi + np.random.normal(loc=0.0,scale=0.1,size=1)
        bf_p     = bf + np.random.normal(loc=0.0,scale=0.1,size=1)
        t_elf_p = t_elf + np.random.normal(loc=0.0,scale=0.1,size=1)

        if Di_p < 0.1 or Di_p>50 or bf_p<0 or bf_p>1 or t_elf_p<0 or logQi
            chain[ii+1,:] = chain[ii,:]
        elif (post(Di_p,logQi_p,bf_p,t_elf_p,sigma,t,data) / post(Di,logQi,
            chain[ii+1,0] = Di_p
            chain[ii+1,1] = logQi_p
            chain[ii+1,2] = bf_p
            chain[ii+1,3] = t_elf_p
        else:
            chain[ii+1,:] = chain[ii,:]
print('Sampling finished at iteration {}'.format(Niter))
```

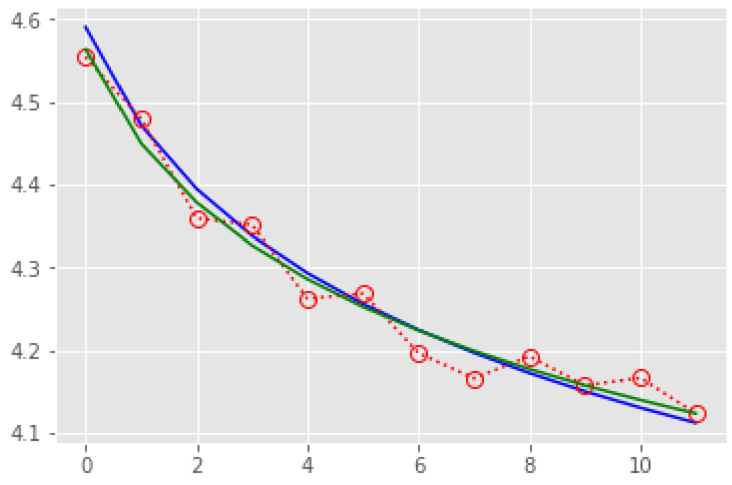
```
iteration 0
iteration 10000
iteration 20000
iteration 30000
iteration 40000
iteration 50000
iteration 60000
iteration 70000
iteration 80000
iteration 90000
Sampling finished at iteration 100000
```

From the converged Markov Chain, we plot one sample THM decline curve vs the least square estimate

- it shows the "end of linear flow time" is about 25 month

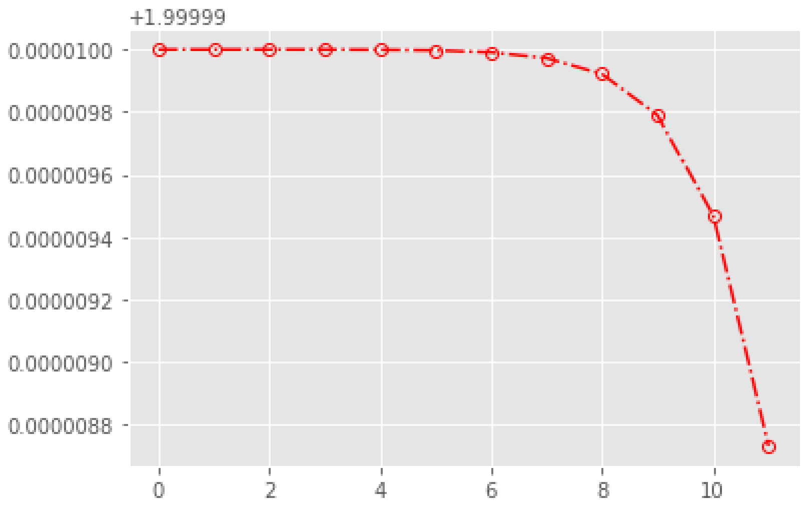
In [42]:
```
c = np.exp(0.57722)/(1.175*t_elf)
b = 2-(2-bf)*np.exp(-np.exp(-c*(t-t_elf)+np.exp(0.57722)))
y_theta =  Arps(t,Di,logQi,b)
plt.plot(y_theta,'b-')
plt.plot(data,'ro',fillstyle='none',**marker_style)
plt.plot(y_hat,'g-');
print('bf is: {}\nt_elf is: {}'.format(bf,t_elf))
```

```
bf is: 0.05149126671846663
t_elf is: 25.26660614137497
```

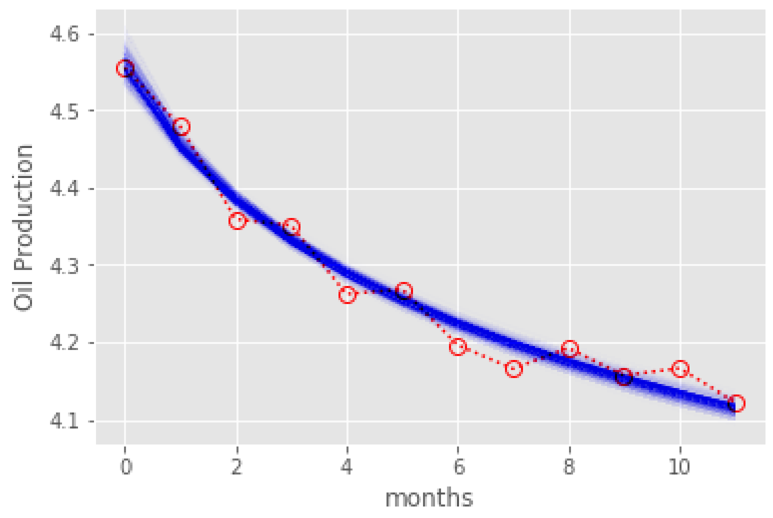


In [55]:
```
# plot the b parameter
plt.plot(b,'ro-.', fillstyle='none')
```

Out[55]:
```
[<matplotlib.lines.Line2D at 0x101a21a438>]
```



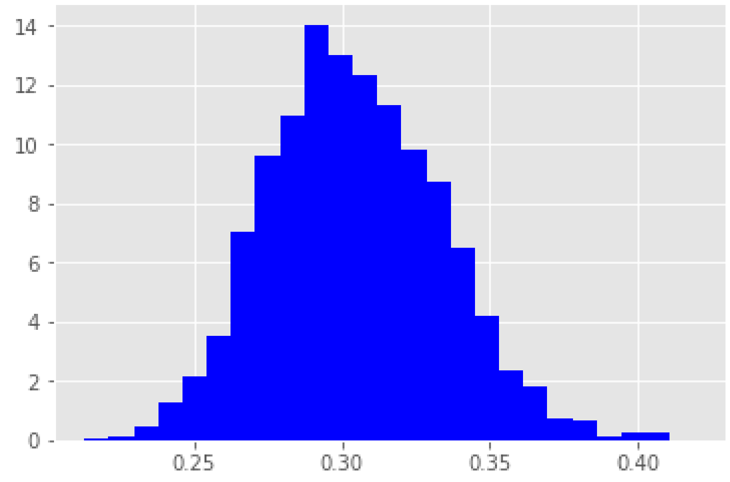
In [44]:
```
chains_to_plot=np.arange(1000)*9

fig = plt.figure()
aa = plt.axes()
aa.plot(data,'ro',fillstyle='none',**marker_style)
#aa.plot(y_hat,'g-')
kk=0
qq=np.zeros([len(chains_to_plot),len(t)])
for ii in chains_to_plot:
    b = 2-(2-chain[ii,2])*np.exp(-np.exp(-c*(t-chain[ii,3])+np.exp(0.57722)
    qq[kk,:]=Arps(t,chain[ii,0],chain[ii,1],b)
    aa.plot(qq[kk,:],color='blue',alpha=0.002)
    kk+=1
    aa.set(xlabel='months', ylabel='Oil Production')
```



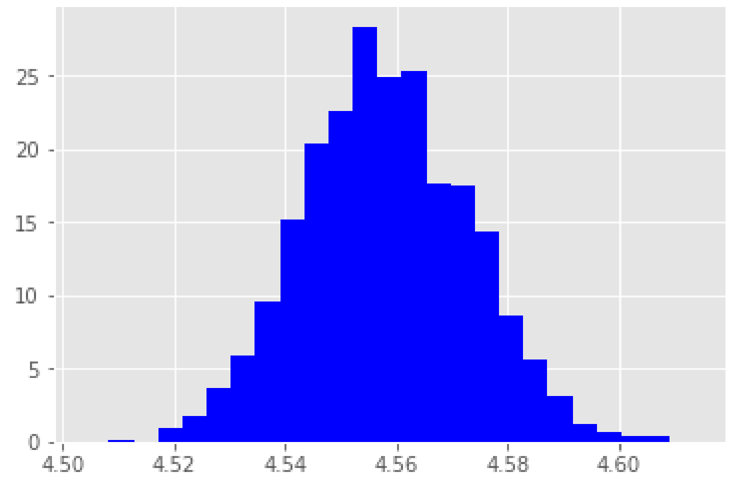
Posterior parameter distribution of

- Di
- log(Qi)
- b_final
- end of linear flow time (t_elf)

In [46]:
```
fig = plt.figure()
ax = plt.axes()
rectangles = ax.hist(chain[:,0],25,density=True,color='blue')
```

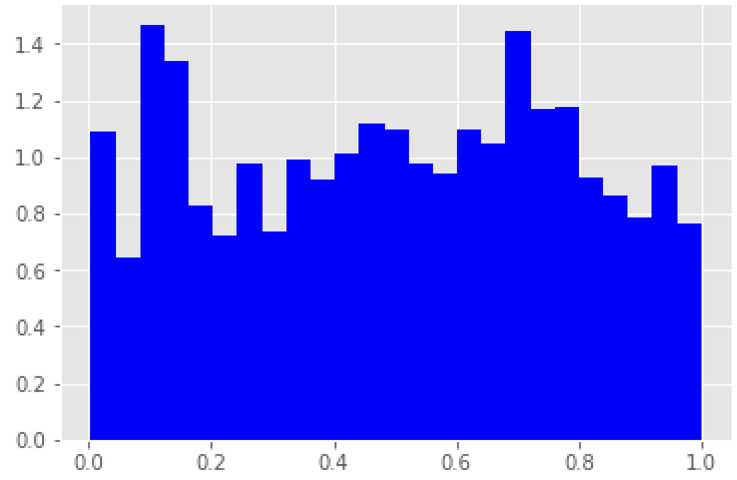


In [48]:
```
fig = plt.figure()
ax = plt.axes()
rectangles = ax.hist(chain[:,1],25,density=True,color='blue')
```

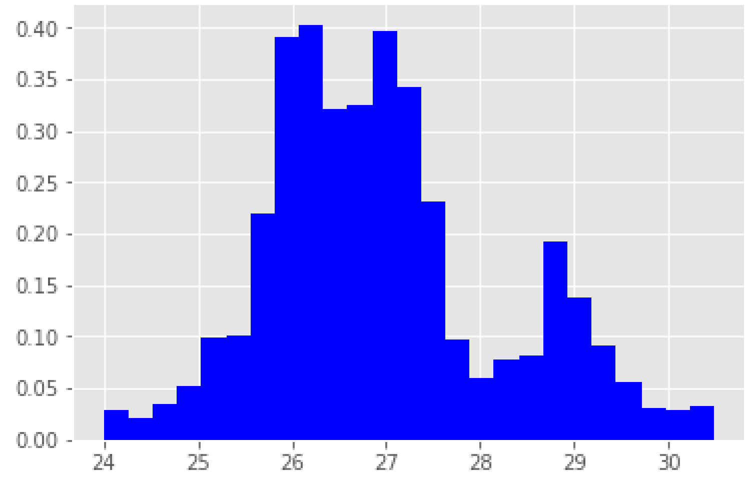


It shows below that b_final is evenly possible for any value between 0-1. This is because the production data is only 12 month, it didn't reach end of linear flow yet. So no estimation on the b_final

In [49]:
```
fig = plt.figure()
ax = plt.axes()
rectangles = ax.hist(chain[:,2],25,density=True,color='blue')
```



In [50]:
```
fig = plt.figure()
ax = plt.axes()
rectangles = ax.hist(chain[:,3],25,density=True,color='blue')
```



Section 3

In this section, we implement the MCMC parameter estimation of Jacobi Theta Function decline curve proposed

In [51]:
```
#import the special function library
import mpmath as mp
```

Define Jacobi Theta Function

In [56]:
```
def Jacobi_theta(t,logQi,chi,eta):
    Qt = np.zeros(len(t))
    for i in range(len(t)):
        Qt[i] = logQi + np.log10(float(mp.jtheta(2,chi,np.exp(-eta*t[i]))))
    return Qt
```

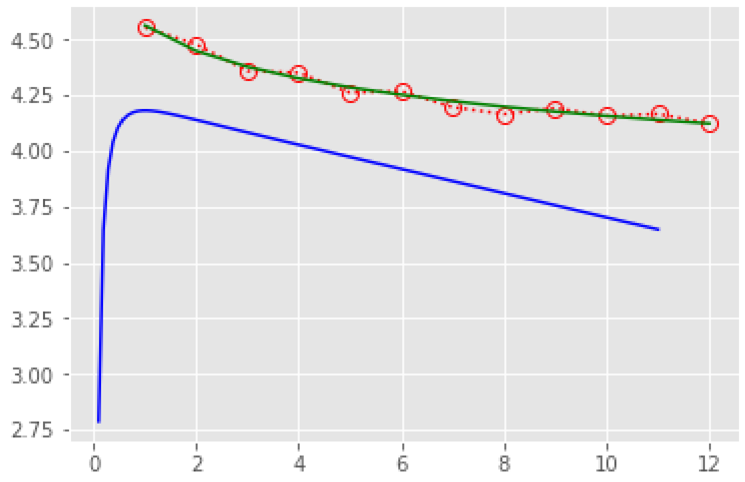
In [105]:
```
logQi = 4
chi = 0.5
eta = 0.5
t_fine = np.linspace(0.01,11,119)
t = np.linspace(1,12,12)
```

Plot the initial guess using data, Jacobi Theta Function and Least Square Estimate

In [106]:
```
y_init =  Jacobi_theta(t_fine,logQi,chi,eta)
plt.plot(t_fine,y_init,'b-') # initial guess
plt.plot(t, data,fillstyle='none',**marker_style)
plt.plot(t, y_hat,'g-')
```

```
/Users/weibing/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.p
y:4: RuntimeWarning: invalid value encountered in log10
  after removing the cwd from sys.path.
```

Out[106]:
```
[<matplotlib.lines.Line2D at 0x101f531588>]
```



In [64]:
```
#### Define posterior probability function, and run MCMC to sample the para
```

In [60]:
```
def post(logQi,chi,eta,sigma,t,data):
    like = np.prod(np.exp(-np.power(Jacobi_theta(t,logQi,chi,eta)-data, 2)/
    return like # because constant uniform prior
```

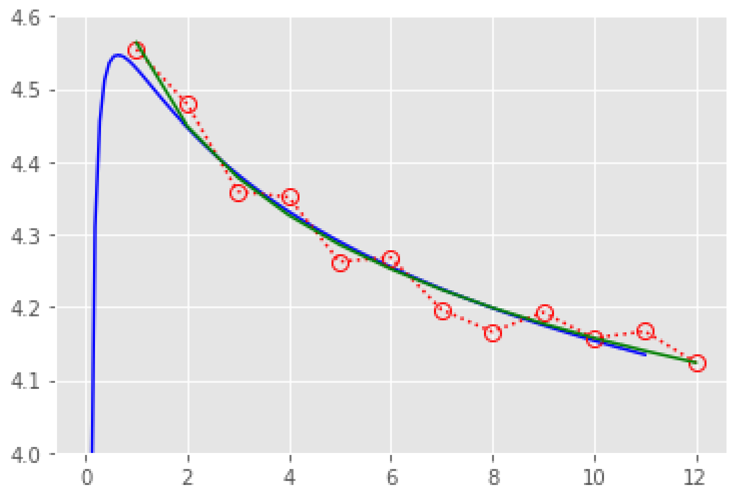
In [62]:
```
logQi = data[0]
Niter = 30000
chain = np.zeros((Niter,3))
chain[0,0] = logQi# Initialize the chain with first guess
chain[0,1] = chi
chain[0,2] = eta
```

In [108]:
```
for ii in np.arange(Niter-1):
        if ii%10000 ==0:
            print('iteration {}'.format(ii))
        logQi    = chain[ii,0]
        chi      = chain[ii,1]
        eta      = chain[ii,2]
        chi_p    = float(chi + np.random.normal(loc=0.0,scale=0.1,size=1))
        logQi_p  = float(logQi + np.random.normal(loc=0.0,scale=0.1,size=1)
        eta_p     = float(eta + np.random.normal(loc=0.0,scale=0.1,size=1))

        if chi_p < 0 or eta_p<0 or logQi_p > 7 or logQi_p< -2:
            chain[ii+1,:] = chain[ii,:]
        elif (post(logQi_p,chi_p,eta_p,sigma,t,data) / post(logQi,chi,eta,s
            chain[ii+1,0] = logQi_p
            chain[ii+1,1] = chi_p
            chain[ii+1,2] = eta_p
        else:
            chain[ii+1,:] = chain[ii,:]
print('Sampling finished at iteration {}'.format(Niter))
```

```
iteration 0

/Users/weibing/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.p
y:4: RuntimeWarning: invalid value encountered in log10
  after removing the cwd from sys.path.

iteration 10000
iteration 20000
Sampling finished at iteration 30000
```

In [109]:
```
y_theta =  Jacobi_theta(t_fine,logQi,chi,eta)
fig = plt.figure()
aa = plt.axes()
plt.plot(t_fine,y_theta,'b-')
plt.plot(t,data,'ro',fillstyle='none',**marker_style)
plt.plot(t,y_hat,'g-')
aa.set(ylim = (4,4.6))
```

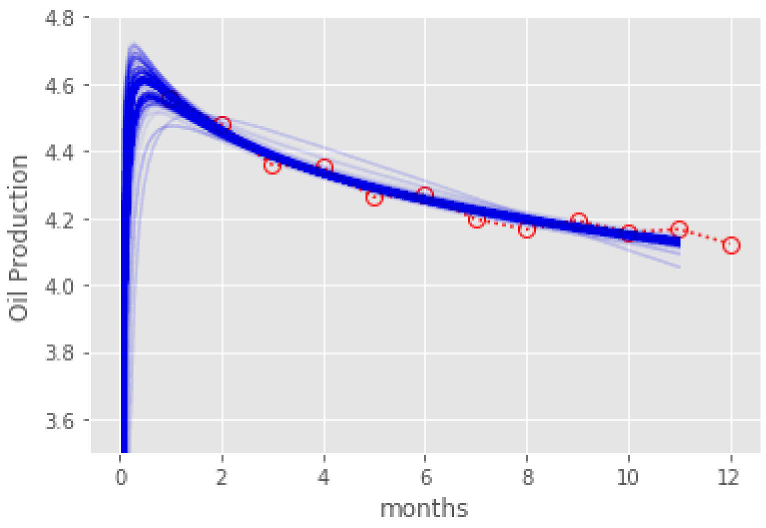
Out[109]:
```
[(4, 4.6)]
```



In [119]:
```
chains_to_plot=np.arange(1000)*9
fig = plt.figure()
aa = plt.axes()
aa.plot(t,data,'ro',fillstyle='none',**marker_style)
#aa.plot(y_hat,'g-')
kk=0
qq=np.zeros([len(chains_to_plot),len(t_fine)])
for ii in chains_to_plot-1:
    qq[kk,:]=Jacobi_theta(t_fine,chain[ii,0] ,chain[ii,1] ,chain[ii,2] )
    aa.plot(t_fine,qq[kk,:],color='blue',alpha=0.02)
    kk+=1
aa.set(ylim= (3.5,4.8), xlabel='months', ylabel='Oil Production')
```

```
/Users/weibing/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.p
y:4: RuntimeWarning: invalid value encountered in log10
  after removing the cwd from sys.path.
```

Out[119]:
```
[(3.5, 4.8), Text(0, 0.5, 'Oil Production'), Text(0.5, 0, 'months')]
```


Oil Production
months