

The following program **example-prog** in \mathbb{L} will be implemented using a recurrent neural network:

```

0: dec V0
1: inc V1
2: if V0 != 0 goto 0
3: nop

```

Equations

Let $N(k)$ be the input of the neuron N at iteration k .

- 1) V_i has an input of $V_i(k-1) + \sum_{a \in \text{inc}} N_a(k) - \sum_{b \in \text{dec}} N_b(k)$ at iteration k where **inc** is the set of **inc** instructions referencing V_i and **dec** is the set of **dec** instructions referencing V_i .
- 2) N_j has an input at iteration k of
 - a) $N_{j-1}(k-1) + \sum_{g \in \text{goto}} (N'_g(k-1) - N''_g(k-1))$ if line $j-1$ is an **inc**, **dec**, or **nop**
 - b) $N''_{j-1}(k-1) + \sum_{g \in \text{goto}} (N'_g(k-1) - N''_g(k-1))$ if line $j-1$ is a **goto**, where **goto** is the set of lines that are **gotos** referencing N_j .
- 3) N'_j has an input of $N_j(k-1)$ at iteration k .
- 4) N''_j has an input of $N_j(k-1) - V(k-1)$ at iteration k where V is the variable referenced in the **goto** statement.

States of the Network

Definition 1. A neural network's state is *initial* provided that:

- The input of N_0 is 1.
- The input of all variable neurons is their initial value as given in the program in \mathbb{L} .
- All other neurons have an input of 0.

The inputs at the 0th iteration must be set so that the network is in the *initial* state.

Definition 2. A neural network's state is *legal* provided that:

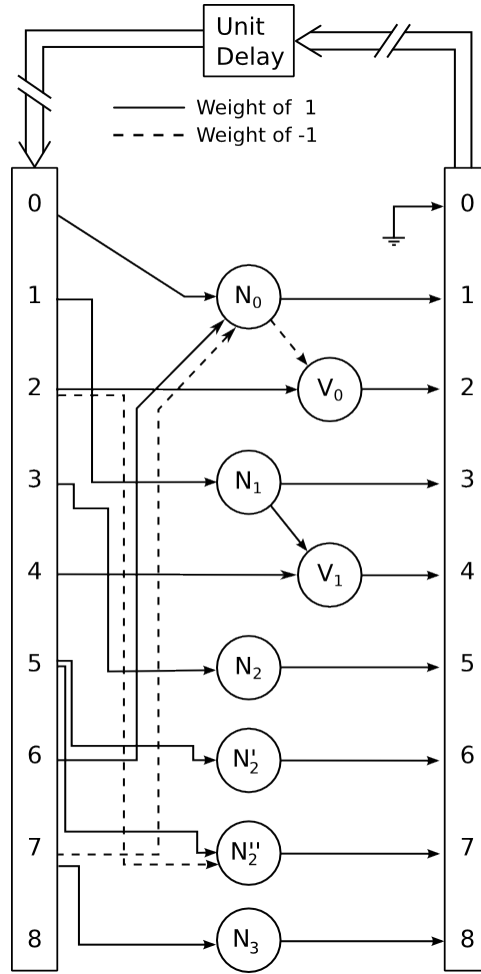
- At most one instruction neuron N_j has an input of 1, the others have an input of 0.
- All transition neuron N'_j and N''_j have an input of 0.

Definition 3. A neural network's state is *transitory* provided that:

- All instruction neurons N_j input 0.
- There exists a unique j such that at least one of the transition neurons N'_j and N''_j input 1.
- All other transition neurons have an input of 0.

Definition 4. A neural network's state is *final* provided that:

- All transition and instruction nodes have an input of 0.

Figure 1: The neural network for `example-prog`.

Position on bus	Iteration										
	0	1	2	3	4	5	6	7	8	9	10
0	1	0	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	1	0	0	0	0	0
2	2	1	1	1	1	0	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0	0	0
4	0	0	1	1	1	1	2	2	2	2	2
5	0	0	0	1	0	0	0	1	0	0	0
6	0	0	0	0	1	0	0	0	1	0	0
7	0	0	0	0	0	0	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0	1	0

Figure 2: Inputs of each position on bus at each iteration.