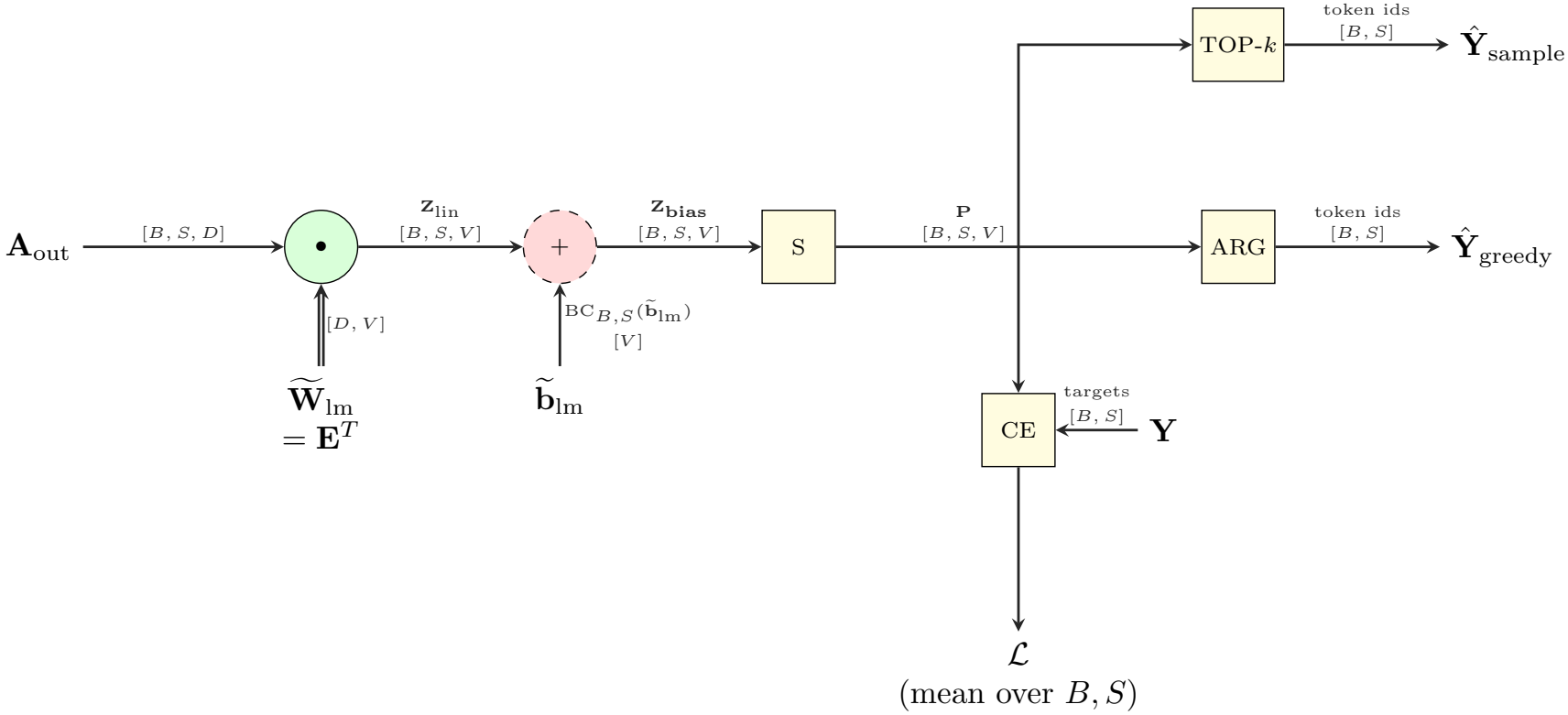
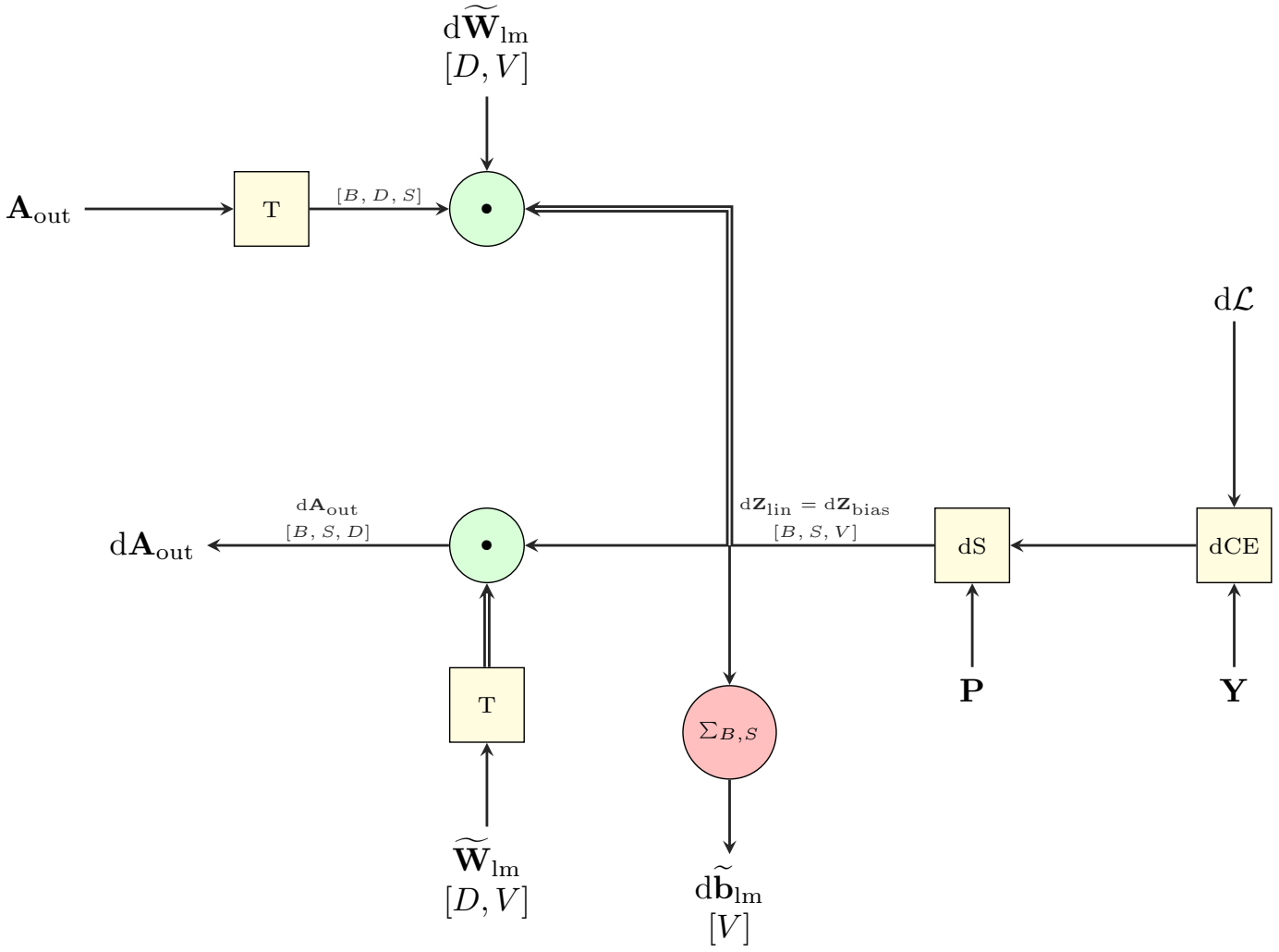
Token Generation & Loss (Forward)



Token Generation & Loss — Backward (Corrected)



Operations (Ops)

Abbrev	Name	Type / Shape	Notes
S	Softmax	op	Over vocab axis V; outputs probabilities $\mathbf{P}$.
CE	Cross-Entropy	op	Usually *sparse* CE consuming label indices $\mathbf{Y}$.
ARG	Argmax (greedy)	op	argmax_V to get token ids (no gradient).
TOP-k	Top-k / sampling	op	Optional decoding path (or nucleus sampling); no gradient.
T	Transpose	op	E.g., $\widetilde{\mathbf{W}}_{\text{lm}}^T \in \mathbb{R}^{V\times D}$.
$\text{BC}_{B,S}(\cdot)$	Broadcast	op	Expand $[V]\to[B,S,V]$ for bias add.
dS	Softmax backward	op	The output is $\text{d}\mathbf{Z}_{\text{bias}} = \mathbf{P} - \text{onehot}(\mathbf{Y})$ (with CE).
dAddB	Addition (Bias) backward	op	Passes $\text{d}\mathbf{Z}_{\text{bias}}$ to $\text{d}\mathbf{Z}_{\text{lin}}$ and $\sum_{B,S}$.
$\sum_{B,S}$	Summation	op	Sums $\text{d}\mathbf{Z}_{\text{bias}}$ over axes B and S to get $\text{d}\widetilde{\mathbf{b}}_{\text{lm}}$.

Data Tensors (Values)

Symbol	Name	Shape	Notes
$\mathbf{A}_{\text{out}}$	Transformer output (hidden)	$[B,S,D]$	Final hidden from the Transformer block(s).
$\widetilde{\mathbf{W}}_{\text{lm}}$	LM head weight (tied)	$[D,V]$	Typically tied to $\mathbf{E}^T$.
$\widetilde{\mathbf{b}}_{\text{lm}}$	LM head bias	$[V]$	Broadcast-added over $[B,S,V]$.
$\mathbf{Z}_{\text{lin}}$	Logits (linear output)	$[B,S,V]$	$\mathbf{Z}_{\text{lin}} = \mathbf{A}_{\text{out}}\widetilde{\mathbf{W}}_{\text{lm}}$.
$\mathbf{Z}_{\text{bias}}$	Logits (final/Softmax input)	$[B,S,V]$	$\mathbf{Z}_{\text{bias}} = \mathbf{Z}_{\text{lin}} + \widetilde{\mathbf{b}}_{\text{lm}}$.
$\mathbf{P}$	Probabilities	$[B,S,V]$	$\mathbf{P} = \text{softmax}(\mathbf{Z}_{\text{bias}})$.
$\mathbf{Y}$	Target token ids	$[B,S]$	Ground-truth indices (sparse labels).
$\mathcal{L}$	Loss	scalar or $[B,S]$	Typically mean over B,S.
$\text{d}\mathcal{L}$	Loss gradient	scalar-grad	Starting signal for backward pass.
$\text{d}\mathbf{Z}_{\text{bias}}$	Final Logits gradient	$[B,S,V]$	From CE+Softmax: $\mathbf{P} - \text{onehot}(\mathbf{Y})$.
$\text{d}\mathbf{Z}_{\text{lin}}$	Linear output grad	$[B,S,V]$	Same as $\text{d}\mathbf{Z}_{\text{bias}}$ (input to $\mathbf{Z}_{\text{lin}}$ matmul).
$\text{d}\widetilde{\mathbf{W}}_{\text{lm}}$	LM weight grad	$[D,V]$	$= \mathbf{A}_{\text{out}}^T \text{d}\mathbf{Z}_{\text{lin}}$.
$\text{d}\widetilde{\mathbf{b}}_{\text{lm}}$	LM bias grad	$[V]$	$= \sum_{B,S}(\text{d}\mathbf{Z}_{\text{bias}})$.
$\text{d}\mathbf{A}_{\text{out}}$	Hidden grad	$[B,S,D]$	$= \text{d}\mathbf{Z}_{\text{lin}}\,\widetilde{\mathbf{W}}_{\text{lm}}^T$.

Shapes: B=batch, S=sequence length, D=hidden dim, V=vocab size.