QEA

Quantitative Engineering Analysis

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Background

This is a project where we'll be modeling a passive solar house. A passive solar house is a house that heats solely from radiation from the sun [2]. The radiation passes through south facing windows and hits a thermal mass inside the house. The thermal mass stores energy and slowly releases it as heat to keep the house warm [1]. Fully passive solar heated houses, such as the one we modeled, reduce energy usage in the home while maintaining a comfortable living environment. Through strategic design, these dwellings are also functional through the summer months [2].

We will be modeling a passive solar house in winter temperatures in Massachusetts. Our model consists of a 256 sq ft house with one big south facing window and a floor heat storage unit that absorbs the solar radiation passing through the window. Our house needs to stay at a reasonable temperature for living (17-25 degrees C) in Massachusetts winters (which average a daily low of 0 C in January, 2020) [3]. We will use a constant outdoor air temperature of -3C for our modeling. Our design will also incorporate an overhang that will serve to block the summer sun while allowing the winter sun lower in the sky to still heat the house.

Modeling:

We can model the energy flux of the heat storage unit of a passive solar house as:

$$Qnet = mc\frac{dT}{dt} = Qin - Qout$$

Where $Qnet$ is the energy of the tile heat storage. m is the mass of the heat storage unit. c is the heat capacity of the heat storage, and $\frac{dT}{dt}$ is the change in temperature of the unit over time.

Qin is radiation from the sun through the window and absorbed by the tile floor. We're assuming that all the sun energy that hits the window is being absorbed by the floor. We can model our system this way because the solar energy absorbed by the window and air is negligible in comparison with the energy that is being absorbed by the heat storage unit due to its much higher heat capacity.

$$Qin = qsun * Awin = -361cos(\pi t/(12 * 3600)) + 224cos(\pi t/(6 * 3600)) + 210$$
[4]

Where $qsun$ is the normal component of solar flux through south-facing window in the winter, and $Awin$ is the area of the window.

We are modeling $Qout$ as energy from the floor to the outside air:

$$Qout = (Tf - Ta)/Rtot$$

Where Tf is the temperature of the tile heat storage unit, Ta is the temperature of the outside air. For us Ta will be -3 degrees C because we are just modeling this house for a Massachusetts winter day and we are assuming the temperature will remain constant at -3. $Rtot$ is the total thermal resistance between the heat storage unit and the outside air. Figures 2 and 3 show the components of $Rtot$.

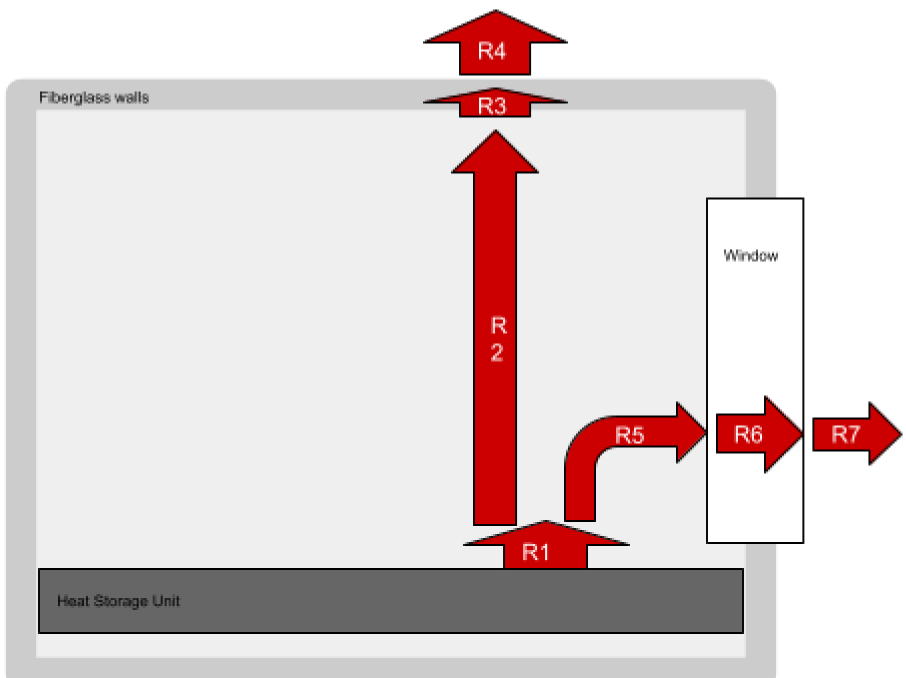

Figure 1: Model of the thermal resistance of the house where R1 is the resistance from the heat storage to the inside air, R2 is from the inside air to the walls, R3 is through the fiberglass walls, R4 is from the walls to the outside air, R5 is from the inside air to the window, R6 is resistance through the window, and R7 is from the outside of the window to the outside air

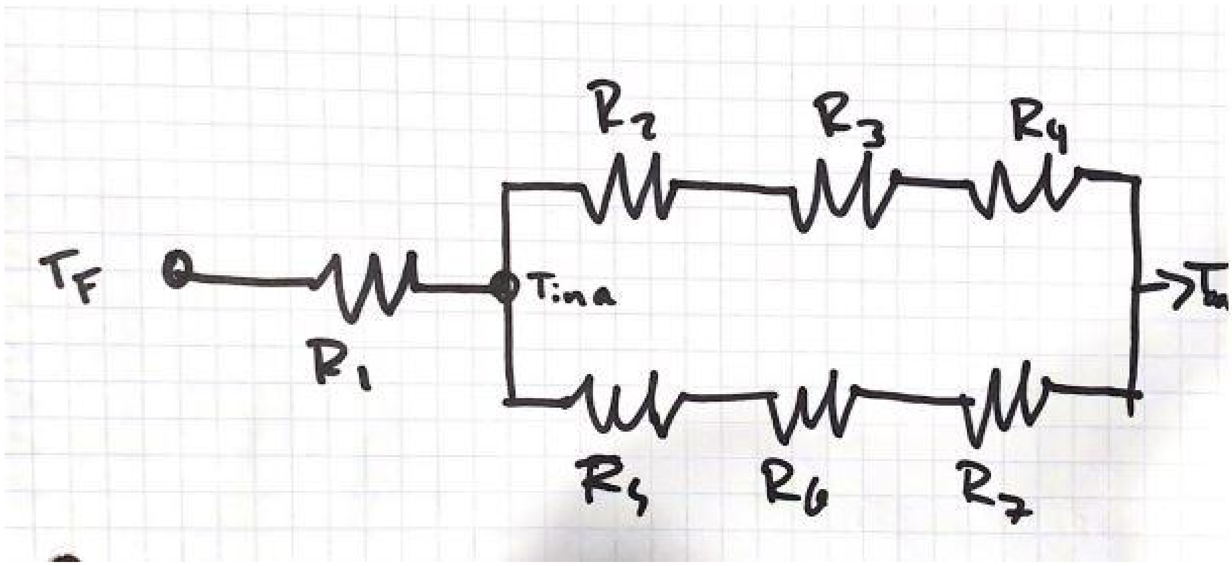

Figure 2: Thermal resistance diagram showing that the thermal resistance through the walls and through the window are in parallel to each other.

If h is the heat transfer coefficient and A is the area of something, then the thermal resistance to convection of that object is:

$$Rconv = \frac{1}{hA}$$

The resistance to conduction is:

$$Rcond = \frac{L}{kA}$$

Where L is the objects width, and $k$is the substance's thermal conductivity.

In our model R1 is convection between the tile and inside air. R2 is convection from inside air to the fiber glass insulation. R3 is conduction through the fiber glass. R4 is convection from the fiberglass to the outside air. R5 is convection from the inside air to the window. R6 is the resistance across the window, which can be calculated using our convection equation with an h value that represents the heat transfer coefficient across the whole window. R7 is conduction from the window to the outside air.

$$Rtot = R1 + \frac{1}{\frac{1}{R2 + R3 + R4} + \frac{1}{R5 + R6 + R7}}$$

$$R1 = \frac{1}{hi^*Af} \qquad R2 = \frac{1}{hi^*Awa}$$

$$R3 = \frac{Lw}{k^*Awa} \qquad R4 = \frac{1}{ho^*Awa}$$

$$R5 = \frac{1}{hi^*Awi} \qquad R6 = \frac{1}{heq^*Awin}$$

$$R7 = \frac{1}{ho^*Awin}$$

Where *hi* is heat transfer coefficient of indoor surfaces. *ho* is the heat transfer coefficient of outdoor surfaces. *heq* is heat transfer coefficient of the entire window. *Af* is area of the heat storage unit, *Awa* is surface area of the house (including walls, floors and ceiling). *Awi* is area of the window. *k* is the conductivity of the walls.

With all of this information we can solve the ODE stated above for the temperature of the heat storage unit. From there we can find the temperature inside our house because we know that the energy flow through our system remains constant. This means that the heat flow from the heat storage unit to the inside air will be the same as the heat flow of the whole system (from the heat storage unit to the outside air. Therefore:

$$U = \frac{Tf - Toa}{Rtot} = \frac{Tf - Tia}{R1}$$

and

$$Tia = Tf - \frac{R1(Tf - Toa)}{Rtot}$$

Where *U* is the heat flow. Tf is the temperature of the floor. Toa is the temperature of indoor air (which is what we are looking for).

In the following section we will further discuss our design decisions and plug in specific values for our variables in these equations. In our results sections you can see the temperature inside our passive solar house over a winter day.

Design and optimization:

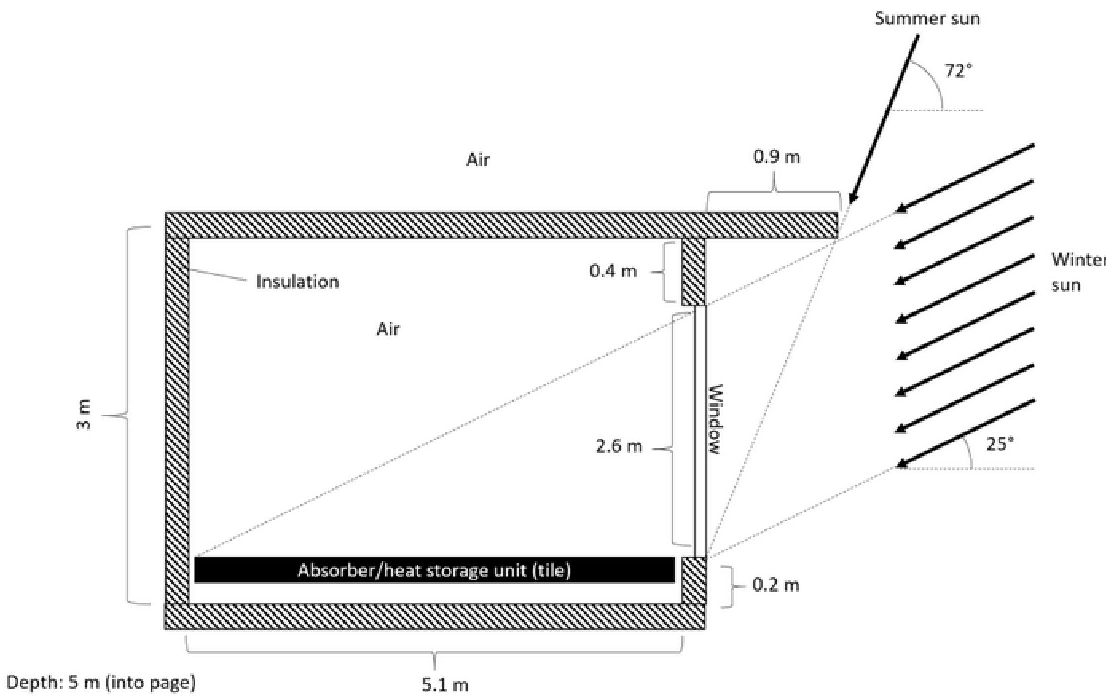

Figure 3: Model of our passive solar house with representation of how the sun energy goes through the window. [4]

Our passive solar house is 5 by 5 by 3 meter rectangular prism with a 2.6 by 2.6 meter north facing window to let in sunlight in the winter, and a 0.9 meter overhang to shade the window from the sun in the summer. We are using 7 cm thick fiberglass insulation for our floor, walls, and ceiling. Our window is a double paned piece of glass. Our heat storage unit is made of 20 cm thick tile suspended above the floor of our house. Our house sits on stilts, to ensure a nice view, meaning there is air on all sides of the house. Some useful properties of the material making up our house are listed in Table 1. The heat transfer coefficient of the indoor surfaces is 15 W/m^2*K; the heat transfer coefficient of the outdoor surfaces is 30 W/m^2*K

Table 1:

Material	Conductivity (W/m K)	Heat capacity (J/kg K)	density (kg/m^3)	Our Area (m^2)	Our thickness (m)
Fiberglass	0.04			143.24	.07
Tile (heat storage unit)		800	3000	25	.2

Window				6.76	.05

Plot 3 shows how changing insulation thickness while maintaining the same value for thermal mass only changes the values for max and min temp over the course of the day but not the range of these values. With our simple model, the only parameter that we would optimize for governing the range of temperatures at equilibrium is the insulation thickness. We want our house to stay at a comfortable temperature which would be around the .08 m (red) and the .05 (blue) meter plots. We ended up with .07 meter thick insulation.

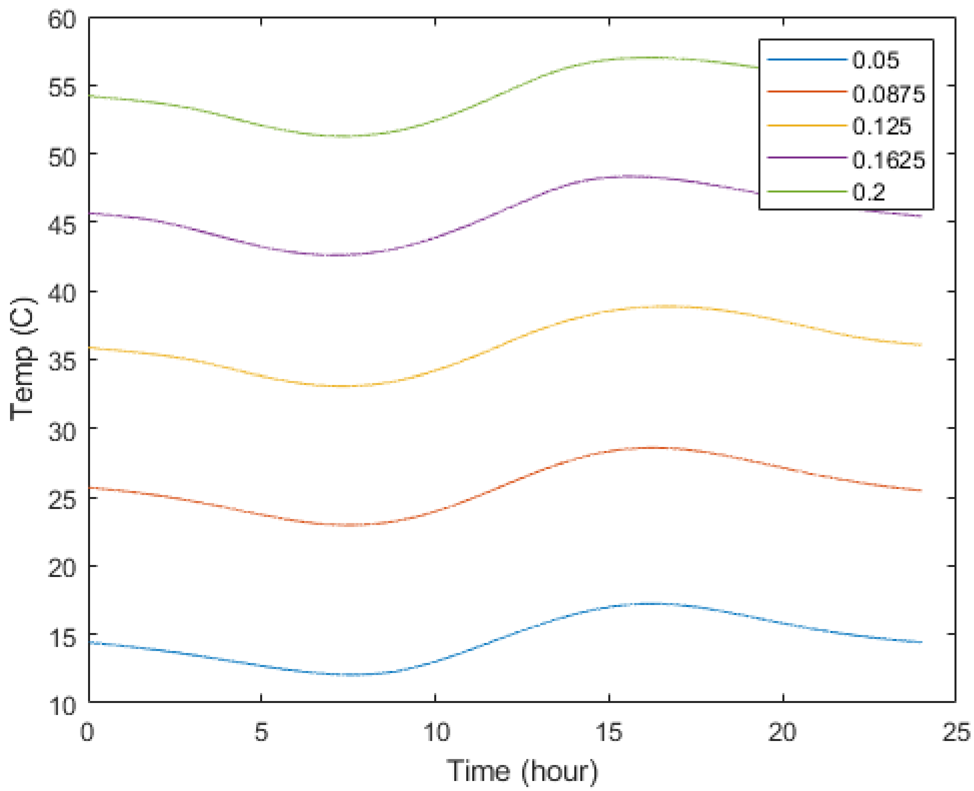

Figure 4: The temperature of the indoor air with varying values for the thickness of insulation over the course of day 45 once our model reaches equilibrium. Insulation thickness ranges from 5cm to 20cm

Figure 5 shows how changing the thickness of the thermal mass, and thus its overall heat capacity, impacts the indoor air temperature. A higher thermal mass causes the indoor air temperature to be moderated over the course of the day night cycle. Changing the thermal mass does not however change the equilibrium temperature around which the indoor air temperature fluctuates. Furthermore, the time of day when the indoor air temperature reaches a maximum and minimum shifts slightly due to the fact that a larger thermal mass has a larger time delay to change temperature. For living in a house it is probably preferable to have less dramatic changes in temperature throughout the day so we chose to model our house with a thicker heat storage unit of .2 meters, which would be close to the purple and yellow plots on figure 5.

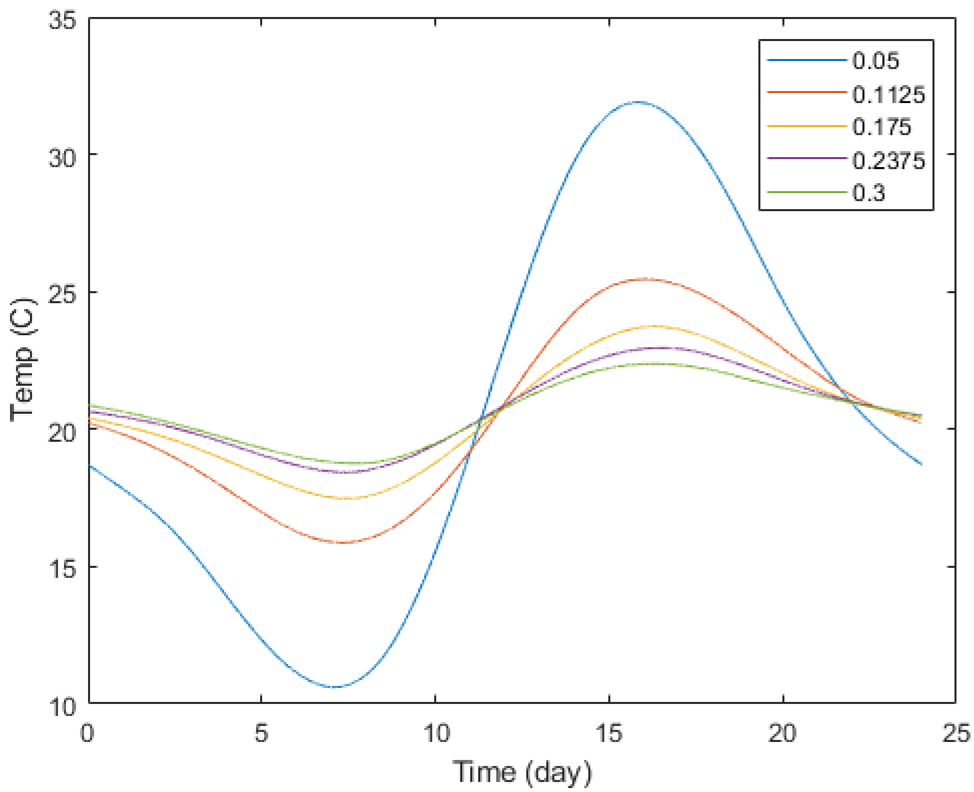

Figure 5: The temperature of the indoor air with varying values for the thickness of thermal mass over the course of day 45 once our model reaches equilibrium. With a range thickness of 5cm to 30cm

Results and Discussion

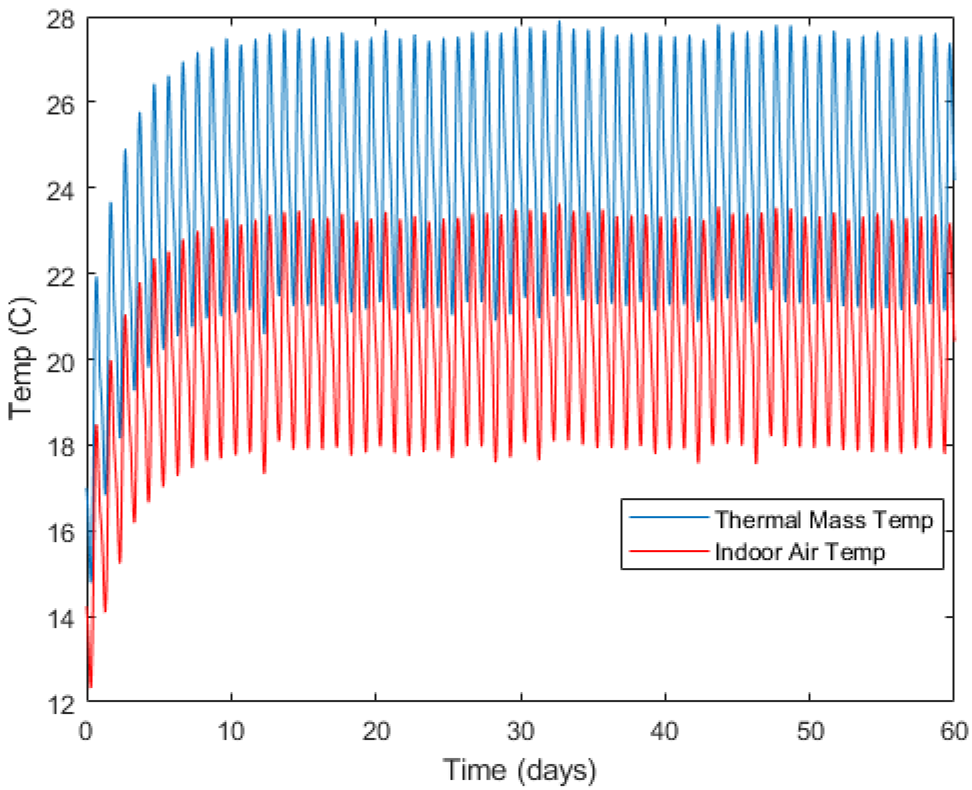

Figure 6: The temperature of the indoor air and thermal mass from our final model over the course of 60 days.

Figure 6 shows the behavior of our model over the course of 60 days. After 10 days or so the temperature of the thermal mass and the indoor air become periodic over the course of 24 hours. Also, the temperature of the thermal mass always remains higher than the temperature of the indoor air due to the thermal resistance the energy has to overcome in order to warm up the air.

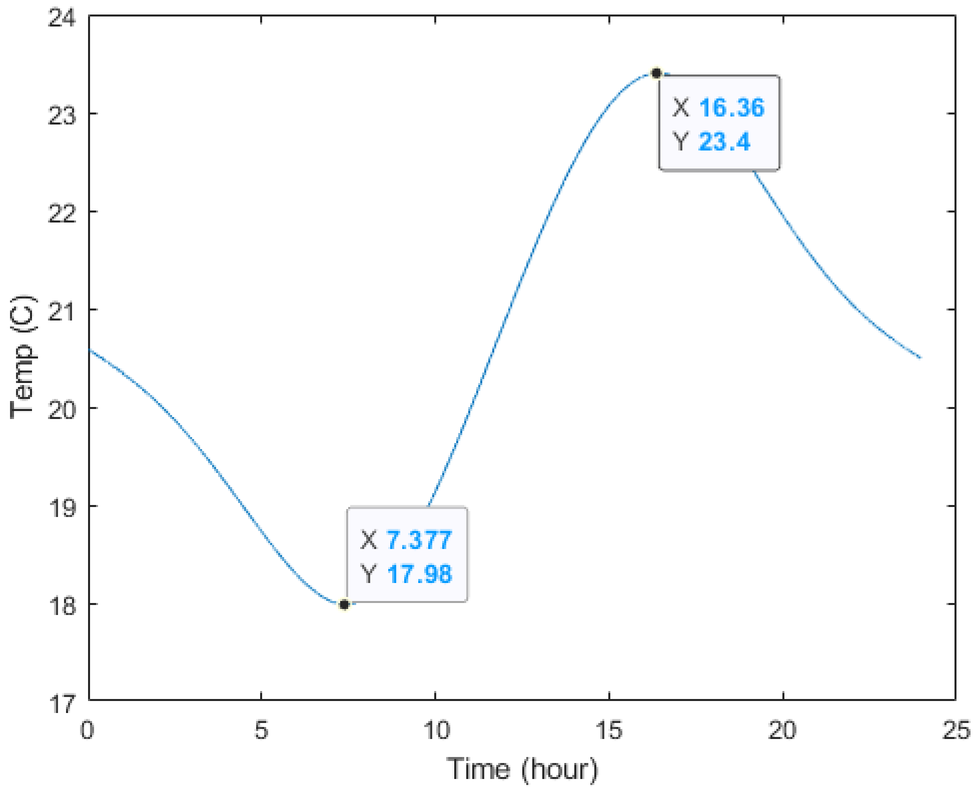

Figure 7: The temperature of the indoor air over the course of day 45 once our model reaches equilibrium

Figure 7 shows the indoor air temperature over the course of one day after our model reaches equilibrium. The minimum indoor air temperature is about 18 degrees celsius which occurs around 7:20am and the maximum indoor air temperature is about 23 degrees celsius at 4:20pm. This would be a noticeable swing in temperature over the day, but would be comfortable throughout.

Discussion

A house that is fully passively heated is difficult to achieve in the northeast without major compromises in the design that impacts comfort and function. After doing our analysis and changing some of the parameters, it is clear that each parameter is quite sensitive to changes that impact the comfort of the house as a whole. The best method for new construction would be to use some of the principles of passive solar design incorporated in the design, but not the main method for indoor air temperature regulation. Furthermore, as the ambient temperatures continue to rise in Massachusetts, it is projected that the heating degree-days will be 11-24% lower, but cooling degree-days will be 57-150% higher by the middle of the century [6], the need for passive cooling will far exceed the need for passive heating. Thus using a relatively high thermal mass is a poor design choice.

Overall, our house as modeled would be habitable, but certainly not the most practical from a livability point of view. Some people don't want to live in square houses with no rooms and only one window even if it is the ideal temperature at all times. Though our model doesn't address this, summer temperatures in our well insulated house with a high thermal mass would likely be unlivable even with the strategically located overhang blocking overhead sunlight. The model we constructed served the purpose of displaying the relative effects of changing the thermal mass and the insulation thickness, but our house is simplified so significantly from a realistic dwelling that it is difficult to draw any real design insight from the model.

References

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Source Code