¿

Deep Learning Specialization

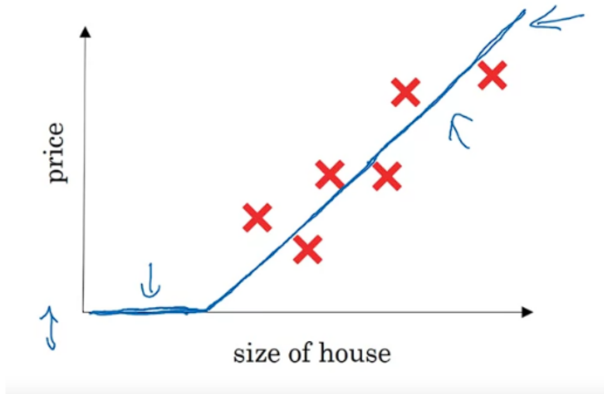
Declan Lim

July 15, 2022

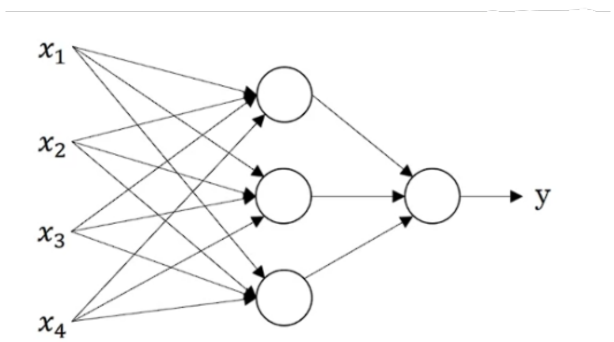
1 Neural Networks and Deep Learning

1.1 Introduction to Deep Learning

- Takes input x to a "neuron" and gives some output y



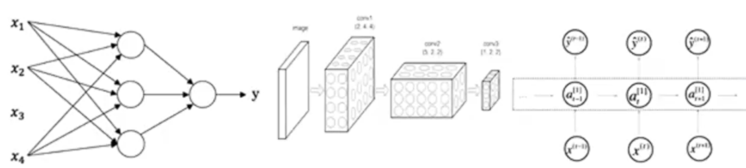
  - Simple neural network has a single input, neuron and output
  - x: size of the house
  - y: price of the house
  - Hypothesis (blue line) is a ReLU (Rectified Linear Unit)
- More complex neural networks can be formed by "stacking" neurons



- Every input layer feature is interconnected with every hidden layer feature
  - The neural network will decide what the intermediate features will be
- Most useful in supervised learning settings

1.1.1 Supervised Learning

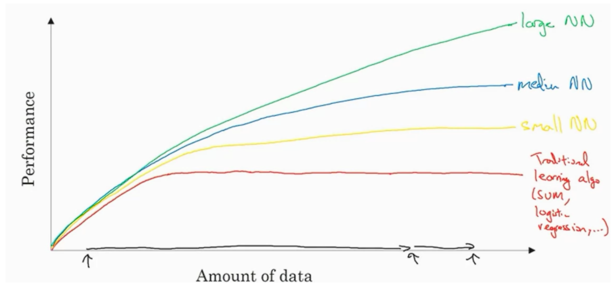
- Aims to learn a function to map an input x to an output y
  - Real estate: predicting house prices from the house features
  - Online advertising: showing ads based on probability of user clicking on ad
  - Photo tagging: tagging images based on objects in the image
  - Speech recognition: generating a text transcript from audio
  - Machine translation: translating from one language to another
  - Autonomous driving: returning the positions of other cars from images and radar info
- Different types of neural network used for different tasks
  - Standard neural network: real estate and online advertising
  - Convolutional neural network (CNN): image data
  - Recurrent neural network (RNN): audio and language data (sequenced data)
  - Hybrid neural network: Autonomous driving (more complex input)



Standard NN   Convolutional NN   Recurrent NN

- Supervised learning can be applied to structured and unstructured data
  - Structured data has features with well defined meanings
  - Unstructured data has more abstract features (images, audio, text)

- Deep learning has only recently started to become more widespread
  - Given large amounts of data and a large NN, deep learning will outperform more traditional learning algorithms
  - For small amounts of data, any performance of the algorithm depends on specific implementation
- "Scale drives deep learning progress"
  - Both the scale of the data and the NN
- Recent algorithmic innovations with increase scale of computation



- Idea to switch from sigmoid activation function to ReLu function increased NN performance
- Ends of sigmoid function have close to 0 gradient so and therefore result in small changes in θ
- ReLu function has gradient of 1 for positive values

- Neural network process is iterative
  - Increasing speed at which a NN can be trained allows different ideas to be tried

1.2 Neural Network Basics

1.2.1 Logistic Regression as a Neural Network

- Logistic regression used for binary classification
- For a colour image, of 64×64 pixels, will have total 12288 input features
  - Image is stored as 3 separate matrices for each colour channel
  - All pixel intensities should be unrolled into a single feature vector

$$n = 12288$$

$$x \in \mathbb{R}^{12288}$$

  - For a matrix `X` of shape (a, b, c, d), want a matrix `X_flatten` of shape $(b * c * d, 1)$

```
X_flatten = X.reshape(X.shape[0], -1).T
```

Notation

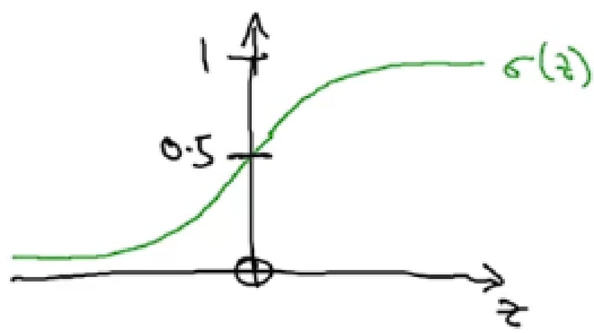
$$\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), ..., (x^{(m)}, y^{(m)})\}$$

- (x, y): single training example
  - $x \in \mathbb{R}^{n_x}$ (n_x = number of features)
  - $y \in \{0, 1\}$

- $(x^{(i)}, y^{(i)})$: i^{th} training example
- $m = m_{train}$
- $m_{test} = \#$ of test examples
- $X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & ... & x^{(3)} \\ | & | & & | \end{bmatrix}$
    - $X \in \mathbb{R}^{n_x \times n}$
- $Y = \begin{bmatrix} y^{(1)} & y^{(2)} & ... & y^{(m)} \end{bmatrix}$
    - $Y \in \mathbb{R}^{1 \times m}$

Logistic Regression

- Given x, want $\hat{y} = P(y = 1|x)$
    - Since $\hat{y}$ is a probability, want $0 \leq \hat{y} \leq 1$
- Parameters: $w \in \mathbb{R}^{n_x}, b \in \mathbb{R}$
- Output: $\hat{y} = \sigma(w^T x + b)$



$$\sigma(z) = \frac{1}{1 + e^{-z}}$$
$$z = w^T x + b$$

- Aim is to learn parameters w and b such that $\hat{y}$ is a good estimate of the probability
- Previous convention had θ vector with an additional θ_0 parameter
    - Keeping θ_0 (b) separate from the rest of the parameters is easier to implement

Cost Function

- Given $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), ..., (x^{(m)}, y^{(m)})\}$, want $\hat{y}^{(i)} \approx y^{(i)}$
- Squared error function not used for logistic regression loss function

- Optimization problem becomes non convex and will have local optima

$$\mathcal{L}(\hat{y}, y) = -(y\log(\hat{y}) + (1-y)\log(1-\hat{y}))$$

- If $y = 1$:
  - $\mathcal{L}(\hat{y}, y) = -\log(\hat{y})$
  - Want large $\log(\hat{y})$ $\therefore$ want large $\hat{y}$
  - $\hat{y}$ has a max of 1 $\therefore$ want $\hat{y} = 1$
- If $y = 0$:
  - $\mathcal{L}(\hat{y}, y) = -log(1-\hat{y})$
  - Want large $\log(1-\hat{y})$ $\therefore$ want small $\hat{y}$
  - $\hat{y}$ has a min of 0 $\therefore$ want $\hat{y} = 0$
- Cost function:

$$\begin{aligned} J(w,b) &= \frac{1}{m}\sum_{i=1}^{m}\mathcal{L}(\hat{y}^{(i)}, y^{(i)}) \\ &= -\frac{1}{m}\sum_{i=1}^{m}[y^{(i)}\log(\hat{y}^{(i)}) + (1-y^{(i)})\log(1-\hat{y}^{(i)})] \end{aligned}$$

  - Average loss function over all training examples

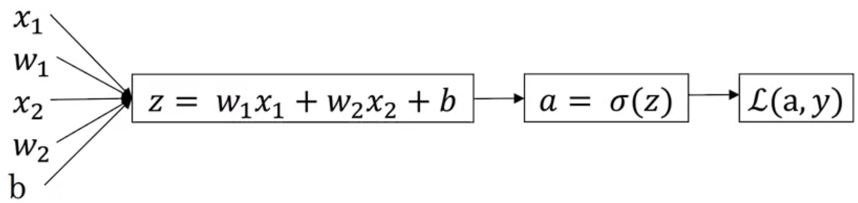
Gradient Descent

- Want to find values of w and b that minimize the cost function $J(w,b)$
  - For logistic regression, w and b usually initialized to 0
- One iteration of gradient descent will take a step in the direction of steepest descent

```
Repeat {
    w := w - α ∂J(w,b)/∂w
    b := b - α ∂J(w,b)/∂b
}
```

- Using the computation graph:

$$\frac{\partial\mathcal{L}(a,y)}{\partial a} = -\frac{y}{a} + \frac{1-y}{1-a}$$



$$
\begin{aligned}
\frac{\partial \mathcal{L}(a,y)}{\partial z} &= \frac{\partial \mathcal{L}}{\partial a} \times \frac{\partial a}{\partial z} \\
&= (-\frac{y}{a} + \frac{1-y}{1-a}) \times a(1-a) \\
&= a - y
\end{aligned}
$$

$$
\frac{\partial \mathcal{L}}{\partial w_1} = x_1 \times \frac{\partial \mathcal{L}}{\partial z}
$$

$$
\frac{\partial \mathcal{L}}{\partial w_2} = x_2 \times \frac{\partial \mathcal{L}}{\partial z}
$$

$$
\frac{\partial \mathcal{L}}{\partial b} = \frac{\partial \mathcal{L}}{\partial z}
$$

- Partial derivative over all training examples calculated by taking the average `dw1`

$$
\frac{\partial}{\partial w_1} J(w,b) = \frac{1}{m} \sum_{i=1}^{m} \frac{\partial}{\partial w_1} \mathcal{L}(a^{(i)}, y^{(i)})
$$

Initialize `J = 0, dw1 = 0, dw2 = 0, db = 0`

```
For i = 1 to m:
    z^(i) = w^T x^(i) + b
    a^(i) = σ(z^(i))

    J += -[y^(i) log(a^(i)) + (1-y^(i))log(1-a^(i))]
    dz^(i) = a^(i) - y^(i)
    dw1 += x_1^(i) dz^(i)
    dw2 += x_2^(i) dz^(i)
    db += dz^(i)

J /= m
dw1 /= m
dw2 /= m
db /= m

w1 := w1 - α dw1
w2 := w2 - α dw2
b := b - α db
```

- Above implementation requires `for` loop over all features for all training examples
  - Vectorization can be used to remove explicit `for` loops
  - Vectorization required for deep learning to be efficient

1.2.2 Vectorisation in Python

- Deep learning performs best on large data sets
  - Code must be able to run quickly to be effective on large data sets

$$z = w^T x + b$$

$$w \in \mathbb{R}^{n_x} \quad x \in \mathbb{R}^{n_x}$$

- Non vectorized implementation:

```python
z = 0
for i in range(n_x):
    z += w[i] * x[i]
z += b
```

- GPUs and CPUs both have parallelization instructions (SIMD: Single Instruction Multiple Data)
  - If built in functions are used, `numpy` will use parallelism to perform computations faster
- For logistic regression, need to calculate z and a values for each training example

$$z^{(i)} = w^T x^{(i)} + b$$

$$a^{(i)} = \sigma(z^{(i)})$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$w \in \mathbb{R}^{n_x} \quad X \in \mathbb{R}^{n_x \times m}$$

$$\begin{aligned} \begin{bmatrix} z^{(1)} & z^{(2)} & \dots & z^{(m)} \end{bmatrix} &= w^T X + \begin{bmatrix} b & b & \dots & b \end{bmatrix} \\ &= \begin{bmatrix} w^T x^{(1)} + b & w^T x^{(2)} + b & \dots & w^T x^{(m)} + b \end{bmatrix} \end{aligned}$$

- In Python:

```python
Z = np.dot(w.T, X) + b
```

– Python will broadcast the value `b` so it can be added to the matrix

- Vectorized implementation of sigmoid function can be used on `Z` to calculate `A`

$$A = \begin{bmatrix} a^{(1)} & a^{(2)} & ... & a^{(m)} \end{bmatrix}$$

$$dz^{(i)} = a^{(i)} - y^{(i)}$$
$$dz = A - Y$$
$$db = \frac{1}{m}\sum_{i=1}^{m} dz^{(i)}$$
$$dw = \frac{1}{m} X(dz)^T$$

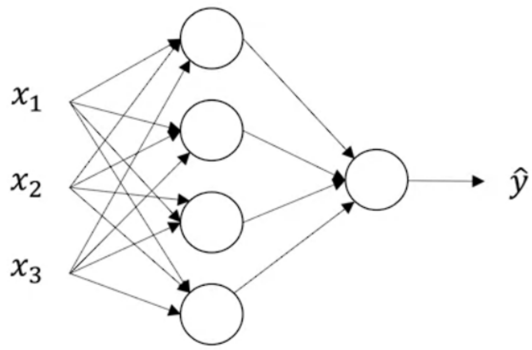
```python
Z = np.dot(w.T,X) + b
A = sigmoid(Z)
dz = A - Y
dw = 1/m * np.dot(X, dz.T)
db = 1/m * np.sum(dz)

# Gradient descent update
w = w - alpha * dw
b = b - alpha * db
```

- `for` loop is required to run multiple iterations of gradient descent

1.3 Shallow Neural Networks

- A neural network will have stacked logistic regression units in each layer
  – Logistic regression output from one layer will be fed to another layer



- Input layer of the neural network contains the feature x_1, x_2, x_3
  - $a^{[0]} = X$
- Intermediate layers in the network are hidden layers
  - Hidden layers do not have "true" values in the training set
- Final layer in the network is the output layer
  - Generates the predicted value $\hat{y}$
- Above diagram is a 2 layer NN
  - Input layer is layer 0
- Each layer will have parameters w and b associated with them
- Each node in the NN will perform logistic regression with its inputs

$$z_i^{[l]} = w_i^{[l]T} x + b_i^{[l]} \quad \rightarrow \quad a_i^{[l]} = \sigma(z_i^{[l]})$$

$$W^{[1]} = \begin{bmatrix} - & w_1^{[1]T} & - \\ - & w_2^{[1]T} & - \\ - & w_3^{[1]T} & - \\ - & w_4^{[1]T} & - \end{bmatrix}$$

$$a^{[0]} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$b^{[1]} = \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix}$$

$$
\begin{aligned}
z^{[1]} &= \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix} \\
&= \begin{bmatrix} w_1^{[1]T} a^{[0]} + b_1^{[1]} \\ w_2^{[1]T} a^{[0]} + b_1^{[1]} \\ w_3^{[1]T} a^{[0]} + b_1^{[1]} \\ w_4^{[1]T} a^{[0]} + b_1^{[1]} \end{bmatrix} \\
&= w^{[1]} a^{[0]} + b^{[1]}
\end{aligned}
$$

$$
\begin{aligned}
a^{[1]} &= \begin{bmatrix} a_1^{[1]} \\ a_2^{[1]} \\ a_3^{[1]} \\ a_4^{[1]} \end{bmatrix} \\
&= \sigma(z^{[1]})
\end{aligned}
$$

$$
z^{[2]} = W^{[2]} a^{[1]} + b^{[2]} \quad \rightarrow \quad a^{[2]} = \sigma(z^{[2]})
$$

- Vectorized method should be able to work on all training examples at one time
  - Vector for each training example can be stacked horizontally in a matrix
  - Vertical dimension will be the number of units in a layer (n_x for the input layer)

$$
X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & & x^{(m)} \\ | & | & & | \end{bmatrix}
$$

$$
Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & ... & z^{[1](m)} \\ | & | & & | \end{bmatrix}
$$

$$
A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & ... & a^{[1](m)} \\ | & | & & | \end{bmatrix}
$$

$$
\begin{aligned}
Z^{[1]} &= W^{[1]} X + b^{[1]} \\
A^{[1]} &= \sigma(Z^{[1]}) \\
Z^{[2]} &= W^{[2]} A^{[1]} + b^{[2]} \\
A^{[2]} &= \sigma(Z^{[2]})
\end{aligned}
$$

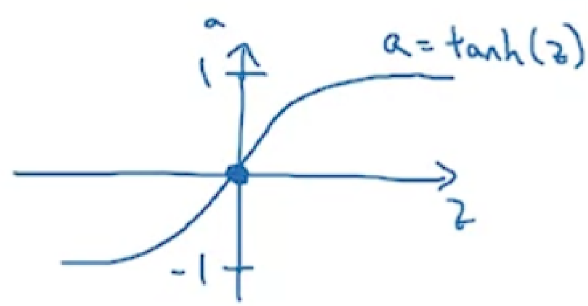
1.3.1 Activation Functions

- After z values are calculated, activation function must be run to get the activation value a

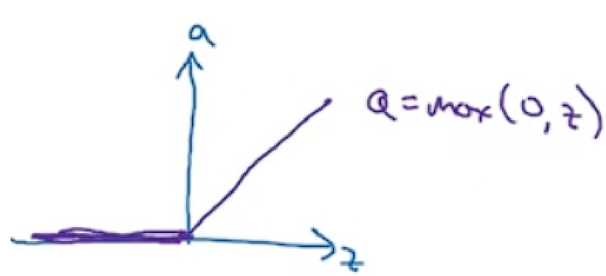
$$a_{sigmoid} = \frac{1}{1+e^{-z}}$$

- Alternatively $a^{[1]} = g(z^{[1]})$ where g is a non linear function
- tanh function almost always performs better than the sigmoid function
  - Equivalent to a transformed version of the sigmoid function
  - tanh function is odd and is "centered" around the origin
  - The mean of the data will be closer to 0 and will help with learning in the next layer

$$a_{\tanh} = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

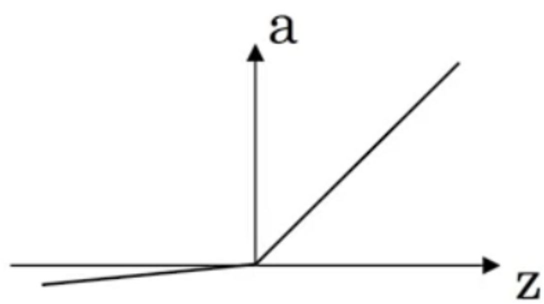


- For binary classification, the final output layer can use the sigmoid function
  - Want the value of $\hat{y}$ to be between 0 and 1
- For both the sigmoid and tanh functions, when z is large, the gradient is very small
  - Results in a slower gradient descent
- ReLU function has a gradient of 1 when z is positive



  - Gradient is 0 when z is negative
- For majority of the ReLU function, gradient is very different from 0
  - Will typically allow NN to learn much faster than sigmoid or tanh function
- ReLU function should be used as the default activation function
- The leaky ReLu function has a slight positive gradient when z is negative

$$a_{leakyReLU} = \max(0.01z, z)$$



- For a NN to compute more complex functions, activation function must be non linear
  - If a linear activation function is used, final output of the NN can only be a linear function
  - Multiple linear activation neurons with a sigmoid as the output neuron is equivalent to standard logistic regression
- Linear activation function can be used in the output layer if output is a real number
- Derivative of the activation function must be calculated for backpropagation
  - Sigmoid function

$$g(z) = \frac{1}{1+e^{-z}}$$

$$\begin{aligned}\frac{d}{dz}g(z) &= \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right)\\ &= g(z)(1-g(z))\end{aligned}$$

  - tanh function

$$\begin{aligned}g(z) &= \tanh(z)\\ &= \frac{e^{z}-e^{-z}}{e^{z}+e^{-z}}\end{aligned}$$

$$\frac{d}{dz}g(z) = 1 - \left(\frac{e^z - e^{-z}}{e^z + e^{-z}}\right)^2$$
$$= 1 - g(z)^2$$

- ReLU function

$$g(z) = \max(0, z)$$
$$\frac{d}{dz}g(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$

- Leaky ReLU function

$$g(z) = \max(0.01z, z)$$
$$\frac{d}{dz}g(z) = \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$

1.3.2 Gradient Descent for Neural Networks

- For a single hidden layer NN, parameters are: $w^{[1]}, b^{[1]}, w^{[2]}, b^{[2]}$
  - $w^{[1]} \in \mathbb{R}^{n_1 \times n_0}$
  - $b^{[1]} \in \mathbb{R}^{n_1 \times 1}$
  - $w^{[2]} \in \mathbb{R}^{n_2 \times n_1}$
  - $b^{[2]} \in \mathbb{R}^{n_2 \times 1}$
- Cost function: $J(w^{[1]}, b^{[1]}, w^{[2]}, b^{[2]}) = \frac{1}{m}\sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$
- For one iteration of gradient descent:

$$w^{[1]} := w^{[1]} - \alpha dw^{[1]},\ b^{[1]} := b^{[1]} - \alpha db^{[1]}$$

$$w^{[2]} := w^{[2]} - \alpha dw^{[2]},\ b^{[2]} := b^{[2]} - \alpha db^{[2]}$$

  - Gradient descent step will take place after backpropagation calculates the derivatives
- Forward propagation:

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$
$$A^{[1]} = g^{[1]}(Z^{[1]})$$
$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$
$$A^{[2]} = g^{[2]}(Z^{[2]})$$

- Backpropagation:

$$
\begin{aligned}
dz^{[2]} &= A^{[2]} - Y \\
dw^{[2]} &= \frac{1}{m} dz^{[2]} A^{[1]T} \\
db^{[2]} &= \frac{1}{m} \texttt{np.sum}(dz^{[2]}, \texttt{axis = 1, keepdims = True}) \\
dz^{[1]} &= w^{[2]T} dz^{[2]} \times g^{[1]\prime}(z^{[1]}) \\
dw^{[1]} &= \frac{1}{m} dz^{[1]} X^T \\
db^{[1]} &= \frac{1}{m} \texttt{np.sum}(dz^{[1]}, \texttt{axis = 1, keepdims = True})
\end{aligned}
$$

1.3.3 Random Initialization

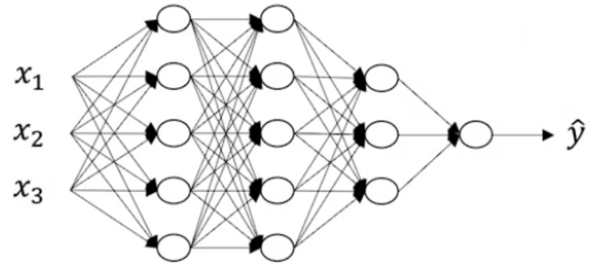
- Weights must be initialized randomly for a NN
  - Weights can be initialized to 0 for logistic regression
  - The bias terms b can be initialized
- If weights are initialized to 0, all neurons in a layer will compute the same hypothesis

```
W1 = np.random.randn((2,2)) * 0.01
b1 = np.zero((2,1))
```

- Weights should be initialized to small random values
  - If weight is too large, activation value $z^{[1]}$ will be large
  - If sigmoid or tanh function is used, derivative will be very small and learning will be very slow
- Different constant for `np.random.randn` should be used for deeper neural networks

1.4 Deep Neural Networks

- Logistic regression is equivalent to a 1-layer NN
- Deep NN have more hidden layers
  - Number of hidden layers in the network can be a parameter for the ML problem



  - Above network has 4 layers, $L = 4$
  - $n^{[l]}$ = number of units in layer l
  - $a^{[l]}$ = activations in layer l
- The inputs x are the activations of the first layer, $x = a^{[0]}$
  - Prediction $\hat{y}$ will be the activations of the last layer, $\hat{y} = a^{[L]}$
- Forward propagation for a deep NN will follow the same pattern for all layers

$$z^{[l]} = w^{[l]}a^{[l-1]} + b^{[l]}$$

$$a^{[l]} = g^{[l]}(z^{[l]})$$

- For a vectorized implementation

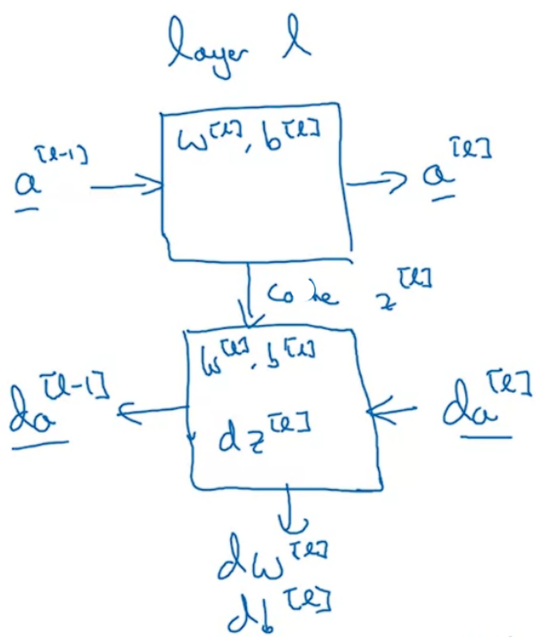
$$Z^{[l]} = W^{[l]}A^{[l-1]} + b^{[l]}$$

$$A^{[l]} = g^{[l]}(z^{[l]})$$

  - Explicit for loop will be used to loop over the layers in the network
  - b will still be a column vector but will apply correctly due to broadcasting
  - When working with W and A matrices, A will be for the previous layer so the dimensions will fit
- When debugging NN, can look at dimensions of all the matrices
- For a non vectorized implementation:
  - $W^{[l]} : (n^{[l]}, n^{[l-1]})$
  - $b^{[l]} : (n^{[l]}, 1)$
  - Dimensions of dw and db should be the same as the dimensions of W and b
  - $a^{[l]}, z^{[l]} : (n^{[l]}, 1)$
- For a vectorized implementation, z vectors and a vectors will be stacked horizontally for all training examples

– $Z^{[l]}, A^{[l]} : (n^{[l]}, m)$

- Deep NN tend to work better as each layer can compute increasingly complex functions
  – Face recognition: edge detection → individual features → large parts of the face
  – Audio: low level waveforms → phonemes → words → sentences
- Functions that can be computed with a "small" deep neural network require exponentially more hidden units in a shallower network
- For each forward propagation step, the value of $z^{[l]}$ should be cached for backpropagation
  – Values of $w^{[l]}$ and $b^{[l]}$ can also be stored in the cache so they can be accessed for backpropagation



- All forward propagation steps will carried out until the hypothesis, $\hat{y}$ is found
  – Using cached values, all backpropagation steps will be carried out until $dz^{[1]}$
  – Parameters $W^{[l]}$ and $b^{[l]}$ can be updated accordingly

$$W^{[l]} := W^{[l]} - \alpha dw^{[l]}$$

$$b^{[l]} := b^{[l]} - \alpha db^{[l]}$$

- Backpropagation will also follow a pattern for all layers in the NN
  – $dz^{[l]} = da^{[l]} * g^{[l]\prime}(z^{[l]})$
  – $dW^{[l]} = dz^{[l]} a^{[l-1]T}$

- $db^{[l]} = dz^{[l]}$
- $da^{[l-1]} = W^{[l]T} dz^{[l]}$

- For a vectorized implementation:
  - $dZ^{[l]} = dA^{[l]} * g^{[l]\prime}(Z^{[l]})$
  - $dW^{[l]} = \frac{1}{m} dZ^{[l]} A^{[l-1]T}$
  - $db^{[l]} = \frac{1}{m}\texttt{np.sum}(dZ^{[l]}\texttt{, axis=1, keepdims=True})$
  - $dA^{[l-1]} = W^{[l]T} dZ^{[l]}$

1.4.1 Parameters vs Hyperparameters

- Parameters of the NN are the W and b matrices
- NN also has a number of associated hyperparameters:
  - Learning rate α
  - Number of iterations z‘’
  - Number of layers in the network
  - Number of hidden units
  - Choice of activation function
- Hyperparameters will control the values of W and b
- Deep learning has many more hyperparameters than earlier eras of machine learning
  - Applying deep learning becomes an empirical process
- Intuitions about hyperparameters may be different across different applications