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Deep Learning Specialization

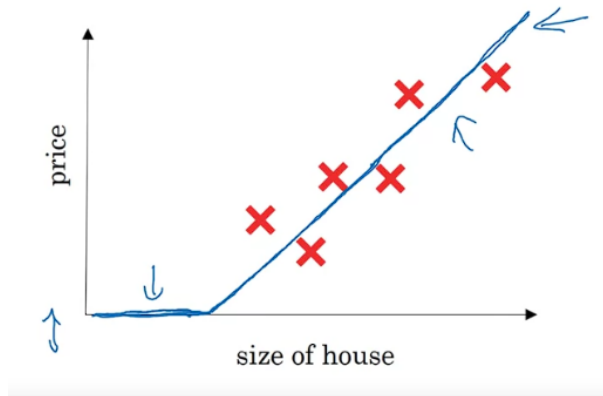
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July 14, 2022

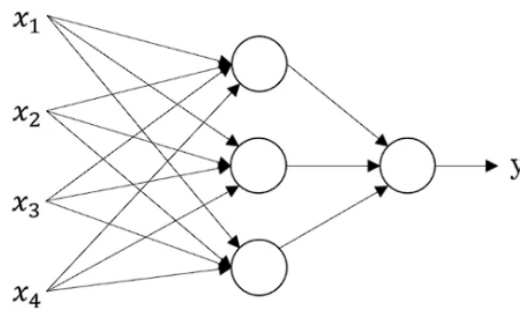
1 Neural Networks and Deep Learning

1.1 Introduction to Deep Learning

- Takes input x to a “neuron” and gives some output y



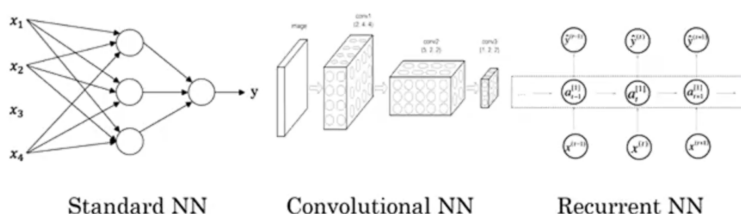
- Simple neural network has a single input, neuron and output
- x : size of the house
- y : price of the house
- Hypothesis (blue line) is a ReLU (Rectified Linear Unit)
- More complex neural networks can be formed by “stacking” neurons



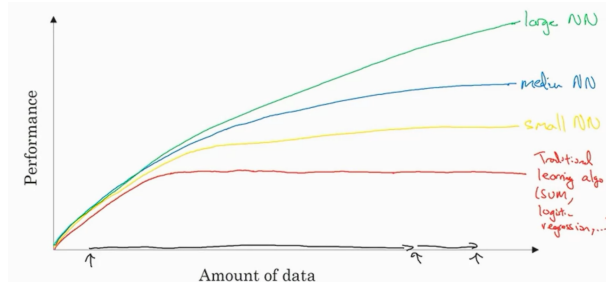
- Every input layer feature is interconnected with every hidden layer feature
 - The neural network will decide what the intermediate features will be
- Most useful in supervised learning settings

1.1.1 Supervised Learning

- Aims to learn a function to map an input x to an output y
 - Real estate: predicting house prices from the house features
 - Online advertising: showing ads based on probability of user clicking on ad
 - Photo tagging: tagging images based on objects in the image
 - Speech recognition: generating a text transcript from audio
 - Machine translation: translating from one language to another
 - Autonomous driving: returning the positions of other cars from images and radar info
- Different types of neural network used for different tasks
 - Standard neural network: real estate and online advertising
 - Convolutional neural network (CNN): image data
 - Recurrent neural network (RNN): audio and language data (sequenced data)
 - Hybrid neural network: Autonomous driving (more complex input)



- Supervised learning can be applied to structured and unstructured data
 - Structured data has features with well defined meanings
 - Unstructured data has more abstract features (images, audio, text)
- Deep learning has only recently started to become more widespread
 - Given large amounts of data and a large NN, deep learning will outperform more traditional learning algorithms
 - For small amounts of data, any performance of the algorithm depends on specific implementation
- “Scale drives deep learning progress”
 - Both the scale of the data and the NN
- Recent algorithmic innovations with increase scale of computation



- Idea to switch from sigmoid activation function to ReLu function increased NN performance
- Ends of sigmoid function have close to 0 gradient so and therefore result in small changes in θ
- ReLu function has gradient of 1 for positive values
- Neural network process is iterative
 - Increasing speed at which a NN can be trained allows different ideas to be tried

1.2 Neural Network Basics

1.2.1 Logistic Regression as a Neural Network

- Logistic regression used for binary classification
- For a colour image, of 64×64 pixels, will have total 12288 input features
 - Image is stored as 3 separate matrices for each colour channel
 - All pixel intensities should be unrolled into a single feature vector

$$n = 12288$$

$$x \in \mathbb{R}^{12288}$$

- For a matrix X of shape (a, b, c, d) , want a matrix `X_flatten` of shape $(b * c * d, 1)$

```
X_flatten = X.reshape(X.shape[0], -1).T
```

Notation

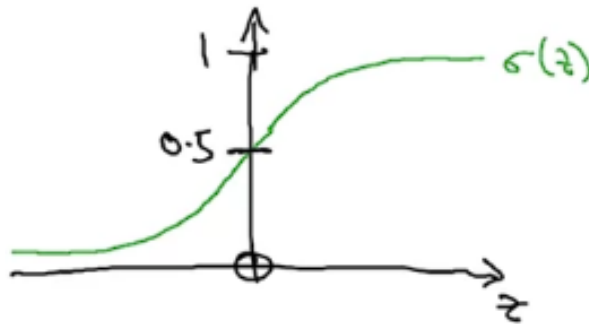
$$\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})\}$$

- (x, y) : single training example
 - $x \in \mathbb{R}^{n_x}$ (n_x = number of features)
 - $y \in \{0, 1\}$

- $(x^{(i)}, y^{(i)})$: i^{th} training example
- $m = m_{train}$
- $m_{test} = \#$ of test examples
- $X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ | & | & & | \end{bmatrix}$
 - $X \in \mathbb{R}^{n_x \times m}$
- $Y = [y^{(1)} \ y^{(2)} \ \dots \ y^{(m)}]$
 - $Y \in \mathbb{R}^{1 \times m}$

Logistic Regression

- Given x , want $\hat{y} = P(y = 1|x)$
 - Since \hat{y} is a probability, want $0 \leq \hat{y} \leq 1$
- Parameters: $w \in \mathbb{R}^{n_x}, b \in \mathbb{R}$
- Output: $\hat{y} = \sigma(w^T x + b)$



$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$z = w^T x + b$$

- Aim is to learn parameters w and b such that \hat{y} is a good estimate of the probability
- Previous convention had θ vector with an additional θ_0 parameter
 - Keeping θ_0 (b) separate from the rest of the parameters is easier to implement

Cost Function

- Given $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})\}$, want $\hat{y}^{(i)} \approx y^{(i)}$
- Squared error function not used for logistic regression loss function

- Optimization problem becomes non convex and will have local optima

$$\mathcal{L}(\hat{y}, y) = -(y \log(\hat{y}) + (1 - y) \log(1 - \hat{y}))$$

- If $y = 1$:
 - $\mathcal{L}(\hat{y}, y) = -\log(\hat{y})$
 - Want large $\log(\hat{y}) \therefore$ want large \hat{y}
 - \hat{y} has a max of 1 \therefore want $\hat{y} = 1$
- If $y = 0$:
 - $\mathcal{L}(\hat{y}, y) = -\log(1 - \hat{y})$
 - Want large $\log(1 - \hat{y}) \therefore$ want small \hat{y}
 - \hat{y} has a min of 0 \therefore want $\hat{y} = 0$
- Cost function:

$$\begin{aligned} J(w, b) &= \frac{1}{m} \sum_{i=1}^m \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) \\ &= -\frac{1}{m} \sum_{i=1}^m [y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})] \end{aligned}$$

- Average loss function over all training examples

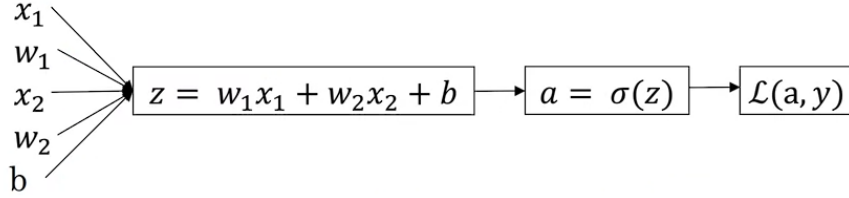
Gradient Descent

- Want to find values of w and b that minimize the cost function $J(w, b)$
 - For logistic regression, w and b usually initialized to 0
- One iteration of gradient descent will take a step in the direction of steepest descent

```
Repeat {
    w := w -  $\alpha \frac{\partial J(w, b)}{\partial w}$ 
    b := b -  $\alpha \frac{\partial J(w, b)}{\partial b}$ 
}
```

- Using the computation graph:

$$\frac{\partial \mathcal{L}(a, y)}{\partial a} = -\frac{y}{a} + \frac{1 - y}{1 - a}$$



$$\begin{aligned}
 \frac{\partial \mathcal{L}(a, y)}{\partial z} &= \frac{\partial \mathcal{L}}{\partial a} \times \frac{\partial a}{\partial z} \\
 &= \left(-\frac{y}{a} + \frac{1-y}{1-a}\right) \times a(1-a) \\
 &= a - y
 \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial w_1} = x_1 \times \frac{\partial \mathcal{L}}{\partial z}$$

$$\frac{\partial \mathcal{L}}{\partial w_2} = x_2 \times \frac{\partial \mathcal{L}}{\partial z}$$

$$\frac{\partial \mathcal{L}}{\partial b} = \frac{\partial \mathcal{L}}{\partial z}$$

- Partial derivative over all training examples calculated by taking the average $\mathbf{dw1}$

$$\frac{\partial}{\partial w_1} J(w, b) = \frac{1}{m} \sum_{i=1}^m \frac{\partial}{\partial w_1} \mathcal{L}(a^{(i)}, y^{(i)})$$

Initialize $J = 0$, $\mathbf{dw1} = 0$, $\mathbf{dw2} = 0$, $\mathbf{db} = 0$

For $i = 1$ to m :

$$\mathbf{z}^{(i)} = \mathbf{w}^T \mathbf{x}^{(i)} + \mathbf{b}$$

$$\mathbf{a}^{(i)} = \sigma(\mathbf{z}^{(i)})$$

$$J += -[y^{(i)} \log(a^{(i)}) + (1-y^{(i)}) \log(1-a^{(i)})]$$

$$d\mathbf{z}^{(i)} = \mathbf{a}^{(i)} - \mathbf{y}^{(i)}$$

$$d\mathbf{w1} += \mathbf{x}_1^{(i)} d\mathbf{z}^{(i)}$$

$$d\mathbf{w2} += \mathbf{x}_2^{(i)} d\mathbf{z}^{(i)}$$

$$d\mathbf{b} += d\mathbf{z}^{(i)}$$

$J /= m$

$\mathbf{dw1} /= m$

$\mathbf{dw2} /= m$

$\mathbf{db} /= m$

$$\mathbf{w1} := \mathbf{w1} - \alpha d\mathbf{w1}$$

$$\mathbf{w2} := \mathbf{w2} - \alpha d\mathbf{w2}$$

$$\mathbf{b} := \mathbf{b} - \alpha d\mathbf{b}$$

- Above implementation requires **for** loop over all features for all training examples
 - Vectorization can be used to remove explicit **for** loops
 - Vectorization required for deep learning to be efficient

1.2.2 Vectorisation in Python

- Deep learning performs best on large data sets
 - Code must be able to run quickly to be effective on large data sets

$$z = w^T x + b$$

$$w \in \mathbb{R}^{n_x} \quad x \in \mathbb{R}^{n_x}$$

- Non vectorized implementation:

```
z = 0
for i in range(n_x):
    z += w[i] * x[i]
z += b
```

- GPUs and CPUs both have parallelization instructions (SIMD: Single Instruction Multiple Data)
 - If built in functions are used, **numpy** will use parallelism to perform computations faster
- For logistic regression, need to calculate z and a values for each training example

$$z^{(i)} = w^T x^{(i)} + b$$

$$a^{(i)} = \sigma(z^{(i)})$$

$$X = \begin{bmatrix} | & | & \dots & | \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$w \in \mathbb{R}^{n_x} \quad X \in \mathbb{R}^{n_x \times m}$$

$$\begin{aligned} [z^{(1)} \quad z^{(2)} \quad \dots \quad z^{(m)}] &= w^T X + [b \quad b \quad \dots \quad b] \\ &= [w^T x^{(1)} + b \quad w^T x^{(2)} + b \quad \dots \quad w^T x^{(m)} + b] \end{aligned}$$

- In Python:

```
Z = np.dot(w.T, X) + b
```

- Python will broadcast the value **b** so it can be added to the matrix
- Vectorized implementation of sigmoid function can be used on **Z** to calculate **A**

$$A = [a^{(1)} \quad a^{(2)} \quad \dots \quad a^{(m)}]$$

$$dz^{(i)} = a^{(i)} - y^{(i)}$$

$$dz = A - Y$$

$$db = \frac{1}{m} \sum_{i=1}^m dz^{(i)}$$

$$dw = \frac{1}{m} X(dz)^T$$

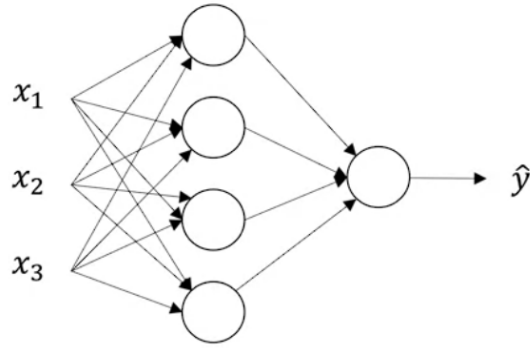
```
Z = np.dot(w.T,X) + b
A = sigmoid(Z)
dz = A - Y
dw = 1/m * np.dot(X, dz.T)
db = 1/m * np.sum(dz)

# Gradient descent update
w = w - alpha * dw
b = b - alpha * db
```

- for loop is required to run multiple iterations of gradient descent

1.3 Shallow Neural Networks

- A neural network will have stacked logistic regression units in each layer
 - Logistic regression output from one layer will be fed to another layer



- Input layer of the neural network contains the feature x_1, x_2, x_3
 - $a^{[0]} = X$
- Intermediate layers in the network are hidden layers
 - Hidden layers do not have “true” values in the training set
- Final layer in the network is the output layer
 - Generates the predicted value \hat{y}
- Above diagram is a 2 layer NN
 - Input layer is layer 0
- Each layer will have parameters w and b associated with them
- Each node in the NN will perform logistic regression with its inputs

$$z_i^{[l]} = w_i^{[l]T} x + b_i^{[l]} \rightarrow a_i^{[l]} = \sigma(z_i^{[l]})$$

$$W^{[1]} = \begin{bmatrix} - & w_1^{[1]T} & - \\ - & w_2^{[1]T} & - \\ - & w_3^{[1]T} & - \\ - & w_4^{[1]T} & - \end{bmatrix}$$

$$a^{[0]} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$b^{[1]} = \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix}$$

$$\begin{aligned}
z^{[1]} &= \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix} \\
&= \begin{bmatrix} w_1^{[1]T} a^{[0]} + b_1^{[1]} \\ w_2^{[1]T} a^{[0]} + b_1^{[1]} \\ w_3^{[1]T} a^{[0]} + b_1^{[1]} \\ w_4^{[1]T} a^{[0]} + b_1^{[1]} \end{bmatrix} \\
&= w^{[1]} a^{[0]} + b^{[1]}
\end{aligned}$$

$$\begin{aligned}
a^{[1]} &= \begin{bmatrix} a_1^{[1]} \\ a_2^{[1]} \\ a_3^{[1]} \\ a_4^{[1]} \end{bmatrix} \\
&= \sigma(z^{[1]})
\end{aligned}$$

$$z^{[2]} = W^{[2]} a^{[1]} + b^{[2]} \rightarrow a^{[2]} = \sigma(z^{[2]})$$

- Vectorized method should be able to work on all training examples at one time
 - Vector for each training example can be stacked horizontally in a matrix
 - Vertical dimension will be the number of units in a layer (n_x for the input layer)

$$X = \begin{bmatrix} \begin{matrix} | \\ x^{(1)} \\ | \end{matrix} & \begin{matrix} | \\ x^{(2)} \\ | \end{matrix} & \begin{matrix} | \\ x^{(m)} \\ | \end{matrix} \end{bmatrix}$$

$$Z^{[1]} = \begin{bmatrix} \begin{matrix} | \\ z^{1} \\ | \end{matrix} & \begin{matrix} | \\ z^{[1](2)} \\ | \end{matrix} & \dots & \begin{matrix} | \\ z^{[1](m)} \\ | \end{matrix} \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} \begin{matrix} | \\ a^{1} \\ | \end{matrix} & \begin{matrix} | \\ a^{[1](2)} \\ | \end{matrix} & \dots & \begin{matrix} | \\ a^{[1](m)} \\ | \end{matrix} \end{bmatrix}$$

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$$A^{[1]} = \sigma(Z^{[1]})$$

$$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

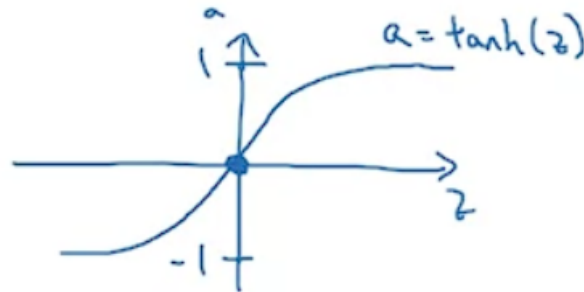
1.3.1 Activation Functions

- After z values are calculated, activation function must be run to get the activation value a

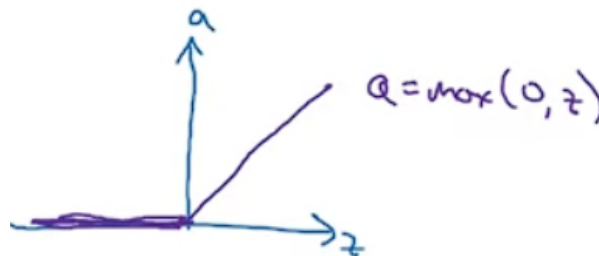
$$a_{sigmoid} = \frac{1}{1 + e^{-z}}$$

- Alternatively $a^{[1]} = g(z^{[1]})$ where g is a non linear function
- tanh function almost always performs better than the sigmoid function
 - Equivalent to a transformed version of the sigmoid function
 - tanh function is odd and is “centered” around the origin
 - The mean of the data will be closer to 0 and will help with learning in the next layer

$$a_{tanh} = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

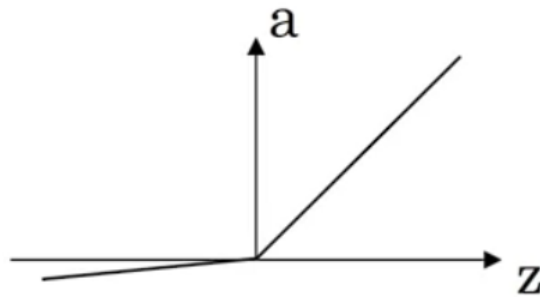


- For binary classification, the final output layer can use the sigmoid function
 - Want the value of \hat{y} to be between 0 and 1
- For both the sigmoid and tanh functions, when z is large, the gradient is very small
 - Results in a slower gradient descent
- ReLU function has a gradient of 1 when z is positive



- Gradient is 0 when z is negative
- For majority of the ReLU function, gradient is very different from 0
 - Will typically allow NN to learn much faster than sigmoid or tanh function
- ReLU function should be used as the default activation function
- The leaky ReLU function has a slight positive gradient when z is negative

$$a_{leakyReLU} = \max(0.01z, z)$$



- For a NN to compute more complex functions, activation function must be non linear
 - If a linear activation function is used, final output of the NN can only be a linear function
 - Multiple linear activation neurons with a sigmoid as the output neuron is equivalent to standard logistic regression
- Linear activation function can be used in the output layer if output is a real number
- Derivative of the activation function must be calculated for backpropagation
 - Sigmoid function

$$g(z) = \frac{1}{1 + e^{-z}}$$

$$\begin{aligned} \frac{d}{dz}g(z) &= \frac{1}{1 + e^{-z}} \left(1 - \frac{1}{1 + e^{-z}} \right) \\ &= g(z)(1 - g(z)) \end{aligned}$$

- tanh function

$$\begin{aligned} g(z) &= \tanh(z) \\ &= \frac{e^z - e^{-z}}{e^z + e^{-z}} \end{aligned}$$

$$\begin{aligned}\frac{d}{dz}g(z) &= 1 - \left(\frac{e^z - e^{-z}}{e^z + e^{-z}}\right)^2 \\ &= 1 - g(z)^2\end{aligned}$$

– ReLU function

$$\begin{aligned}g(z) &= \max(0, z) \\ \frac{d}{dz}g(z) &= \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}\end{aligned}$$

– Leaky ReLU function

$$\begin{aligned}g(z) &= \max(0.01z, z) \\ \frac{d}{dz}g(z) &= \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}\end{aligned}$$

1.3.2 Gradient Descent for Neural Networks

- For a single hidden layer NN, parameters are: $w^{[1]}, b^{[1]}, w^{[2]}, b^{[2]}$
 - $w^{[1]} \in \mathbb{R}^{n_1 \times n_0}$
 - $b^{[1]} \in \mathbb{R}^{n_1 \times 1}$
 - $w^{[2]} \in \mathbb{R}^{n_2 \times n_1}$
 - $b^{[2]} \in \mathbb{R}^{n_2 \times 1}$
- Cost function: $J(w^{[1]}, b^{[1]}, w^{[2]}, b^{[2]}) = \frac{1}{m} \sum_{i=1}^n \mathcal{L}(\hat{y}, y)$
- For one iteration of gradient descent:

$$w^{[1]} := w^{[1]} - \alpha dw^{[1]}, \quad b^{[1]} := b^{[1]} - \alpha db^{[1]}$$

$$w^{[2]} := w^{[2]} - \alpha dw^{[2]}, \quad b^{[2]} := b^{[2]} - \alpha db^{[2]}$$

– Gradient descent step will take place after backpropagation calculates the derivatives

- Forward propagation:

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = g^{[2]}(Z^{[2]})$$

- Backpropagation:

$$\begin{aligned}
 dz^{[2]} &= A^{[2]} - Y \\
 dw^{[2]} &= \frac{1}{m} dz^{[2]} A^{[1]T} \\
 db^{[2]} &= \frac{1}{m} \text{np.sum}(dz^{[2]}, \text{axis} = 1, \text{keepdims} = \text{True}) \\
 dz^{[1]} &= w^{[2]T} dz^{[2]} \times g^{[1]'}(z^{[1]}) \\
 dw^{[1]} &= \frac{1}{m} dz^{[1]} X^T \\
 db^{[1]} &= \frac{1}{m} \text{np.sum}(dz^{[1]}, \text{axis} = 1, \text{keepdims} = \text{True})
 \end{aligned}$$

1.3.3 Random Initialization

- Weights must be initialized randomly for a NN
 - Weights can be initialized to 0 for logistic regression
 - The bias terms b can be initialized
- If weights are initialized to 0, all neurons in a layer will compute the same hypothesis

```

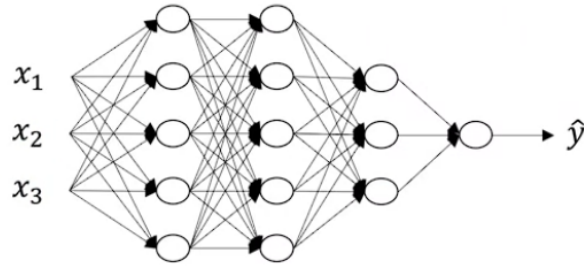
W1 = np.random.randn((2,2)) * 0.01
b1 = np.zeros((2,1))

```

- Weights should be initialized to small random values
 - If weight is too large, activation value $z^{[1]}$ will be large
 - If sigmoid or tanh function is used, derivative will be very small and learning will be very slow
- Different constant for `np.random.randn` should be used for deeper neural networks

1.4 Deep Neural Networks

- Logistic regression is equivalent to a 1-layer NN
- Deep NN have more hidden layers
 - Number of hidden layers in the network can be a parameter for the ML problem



- Above network has 4 layers, $L = 4$
- $n^{[l]}$ = number of units in layer l
- $a^{[l]}$ = activations in layer l
- The inputs x are the activations of the first layer, $x = a^{[0]}$
 - Prediction \hat{y} will be the activations of the last layer, $\hat{y} = a^{[L]}$
- Forward propagation for a deep NN will follow the same pattern for all layers

$$z^{[l]} = w^{[l]}a^{[l-1]} + b^{[l]}$$

$$a^{[l]} = g^{[l]}(z^{[l]})$$

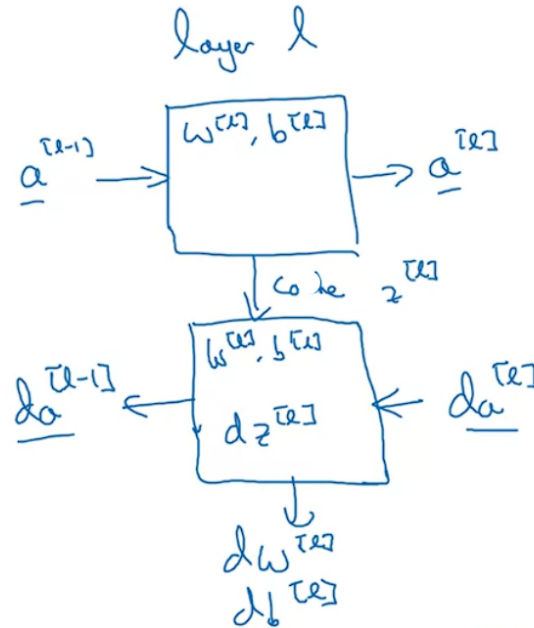
- For a vectorized implementation

$$Z^{[l]} = W^{[l]}A^{[l-1]} + b^{[l]}$$

$$A^{[l]} = g^{[l]}(z^{[l]})$$

- Explicit for loop will be used to loop over the layers in the network
- When debugging NN, can look at dimensions of all the matrices
- For a non vectorized implementation:
 - $W^{[l]} : (n^{[l]}, n^{[l-1]})$
 - $b^{[l]} : (n^{[l]}, 1)$
 - Dimensions of dw and db should be the same as the dimensions of W and b
 - $a^{[l]}, z^{[l]} : (n^{[l]}, 1)$
- For a vectorized implementation, z vectors and a vectors will be stacked horizontally for all training examples
 - $Z^{[l]}, A^{[l]} : (n^{[l]}, m)$
- Deep NN tend to work better as each layer can compute increasingly complex functions
 - Face recognition: edge detection \rightarrow individual features \rightarrow large parts of the face

- Audio: low level waveforms \rightarrow phonemes \rightarrow words \rightarrow sentences
- Functions that can be computed with a “small” deep neural network require exponentially more hidden units in a shallower network
- For each forward propagation step, the value of $z^{[l]}$ should be cached for backpropagation
 - Values of $w^{[l]}$ and $b^{[l]}$ can also be stored in the cache so they can be accessed for backpropagation



- All forward propagation steps will be carried out until the hypothesis, \hat{y} is found
 - Using cached values, all backpropagation steps will be carried out until $dz^{[1]}$
 - Parameters $W^{[l]}$ and $b^{[l]}$ can be updated accordingly

$$W^{[l]} := W^{[l]} - \alpha dw^{[l]}$$

$$b^{[l]} := b^{[l]} - \alpha db^{[l]}$$

1.4.1 Gradient Descent Algorithm