

# Deep Learning Specialization

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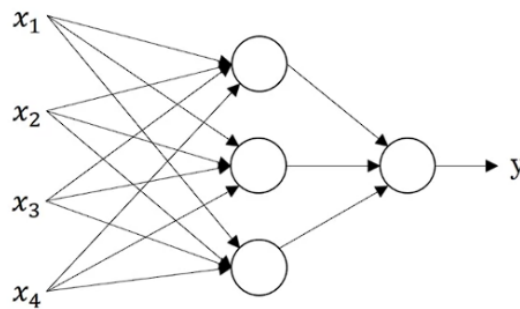
# 1 Neural Networks and Deep Learning

## 1.1 Introduction to Deep Learning

- Takes input  $x$  to a “neuron” and gives some output  $y$



- Simple neural network has a single input, neuron and output
- $x$ : size of the house
- $y$ : price of the house
- Hypothesis (blue line) is a ReLU (Rectified Linear Unit)
- More complex neural networks can be formed by “stacking” neurons



- Every input layer feature is interconnected with every hidden layer feature
  - The neural network will decide what the intermediate features will be
- Most useful in supervised learning settings

### 1.1.1 Supervised Learning

- Aims to learn a function to map an input  $x$  to an output  $y$ 
  - Real estate: predicting house prices from the house features
  - Online advertising: showing ads based on probability of user clicking on ad
  - Photo tagging: tagging images based on objects in the image
  - Speech recognition: generating a text transcript from audio
  - Machine translation: translating from one language to another
  - Autonomous driving: returning the positions of other cars from images and radar info
- Different types of neural network used for different tasks
  - Standard neural network: real estate and online advertising
  - Convolutional neural network (CNN): image data
  - Recurrent neural network (RNN): audio and language data (sequenced data)
  - Hybrid neural network: Autonomous driving (more complex input)



- Supervised learning can be applied to structured and unstructured data
  - Structured data has features with well defined meanings
  - Unstructured data has more abstract features (images, audio, text)
- Deep learning has only recently started to become more widespread
  - Given large amounts of data and a large NN, deep learning will outperform more traditional learning algorithms
  - For small amounts of data, any performance of the algorithm depends on specific implementation
- “Scale drives deep learning progress”
  - Both the scale of the data and the NN
- Recent algorithmic innovations with increase scale of computation



- Idea to switch from sigmoid activation function to ReLu function increased NN performance
- Ends of sigmoid function have close to 0 gradient so and therefore result in small changes in  $\theta$
- ReLu function has gradient of 1 for positive values
- Neural network process is iterative
  - Increasing speed at which a NN can be trained allows different ideas to be tried

## 1.2 Neural Network Basics

### 1.2.1 Logistic Regression as a Neural Network

- Logistic regression used for binary classification
- For a colour image, of  $64 \times 64$  pixels, will have total 12288 input features
  - Image is stored as 3 separate matrices for each colour channel
  - All pixel intensities should be unrolled into a single feature vector

$$n = 12288$$

$$x \in \mathbb{R}^{12288}$$

- For a matrix  $X$  of shape  $(a, b, c, d)$ , want a matrix `X_flatten` of shape  $(b * c * d, 1)$

```
X_flatten = X.reshape(X.shape[0], -1).T
```

### Notation

$$\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})\}$$

- $(x, y)$ : single training example
  - $x \in \mathbb{R}^{n_x}$  ( $n_x$  = number of features)
  - $y \in \{0, 1\}$

- $(x^{(i)}, y^{(i)})$ :  $i^{th}$  training example
- $m = m_{train}$
- $m_{test} = \#$  of test examples
- $X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ | & | & & | \end{bmatrix}$ 
  - $X \in \mathbb{R}^{n_x \times m}$
- $Y = [y^{(1)} \ y^{(2)} \ \dots \ y^{(m)}]$ 
  - $Y \in \mathbb{R}^{1 \times m}$

## Logistic Regression

- Given  $x$ , want  $\hat{y} = P(y = 1|x)$ 
  - Since  $\hat{y}$  is a probability, want  $0 \leq \hat{y} \leq 1$
- Parameters:  $w \in \mathbb{R}^{n_x}, b \in \mathbb{R}$
- Output:  $\hat{y} = \sigma(w^T x + b)$



$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$z = w^T x + b$$

- Aim is to learn parameters  $w$  and  $b$  such that  $\hat{y}$  is a good estimate of the probability
- Previous convention had  $\theta$  vector with an additional  $\theta_0$  parameter
  - Keeping  $\theta_0$  ( $b$ ) separate from the rest of the parameters is easier to implement

## Cost Function

- Given  $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})\}$ , want  $\hat{y}^{(i)} \approx y^{(i)}$
- Squared error function not used for logistic regression loss function

- Optimization problem becomes non convex and will have local optima

$$\mathcal{L}(\hat{y}, y) = -(y \log(\hat{y}) + (1 - y) \log(1 - \hat{y}))$$

- If  $y = 1$ :
  - $\mathcal{L}(\hat{y}, y) = -\log(\hat{y})$
  - Want large  $\log(\hat{y}) \therefore$  want large  $\hat{y}$
  - $\hat{y}$  has a max of 1  $\therefore$  want  $\hat{y} = 1$
- If  $y = 0$ :
  - $\mathcal{L}(\hat{y}, y) = -\log(1 - \hat{y})$
  - Want large  $\log(1 - \hat{y}) \therefore$  want small  $\hat{y}$
  - $\hat{y}$  has a min of 0  $\therefore$  want  $\hat{y} = 0$

- Cost function:

$$\begin{aligned} J(w, b) &= \frac{1}{m} \sum_{i=1}^m \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) \\ &= -\frac{1}{m} \sum_{i=1}^m [y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})] \end{aligned}$$

- Average loss function over all training examples

### ***Gradient Descent***

- Want to find values of  $w$  and  $b$  that minimize the cost function  $J(w, b)$ 
  - For logistic regression,  $w$  and  $b$  usually initialized to 0
- One iteration of gradient descent will take a step in the direction of steepest descent

```
Repeat {
  w := w -  $\alpha \frac{\partial J(w, b)}{\partial w}$ 
  b := b -  $\alpha \frac{\partial J(w, b)}{\partial b}$ 
}
```

- Using the computation graph:

$$\frac{\partial \mathcal{L}(a, y)}{\partial a} = -\frac{y}{a} + \frac{1 - y}{1 - a}$$



$$\begin{aligned}
 \frac{\partial \mathcal{L}(a, y)}{\partial z} &= \frac{\partial \mathcal{L}}{\partial a} \times \frac{\partial a}{\partial z} \\
 &= \left(-\frac{y}{a} + \frac{1-y}{1-a}\right) \times a(1-a) \\
 &= a - y
 \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial w_1} = x_1 \times \frac{\partial \mathcal{L}}{\partial z}$$

$$\frac{\partial \mathcal{L}}{\partial w_2} = x_2 \times \frac{\partial \mathcal{L}}{\partial z}$$

$$\frac{\partial \mathcal{L}}{\partial b} = \frac{\partial \mathcal{L}}{\partial z}$$

- Partial derivative over all training examples calculated by taking the average  $\mathbf{dw1}$

$$\frac{\partial}{\partial w_1} J(w, b) = \frac{1}{m} \sum_{i=1}^m \frac{\partial}{\partial w_1} \mathcal{L}(a^{(i)}, y^{(i)})$$

Initialize  $J = 0$ ,  $\mathbf{dw1} = 0$ ,  $\mathbf{dw2} = 0$ ,  $\mathbf{db} = 0$

For  $i = 1$  to  $m$ :

$$\mathbf{z}^{(i)} = \mathbf{w}^T \mathbf{x}^{(i)} + \mathbf{b}$$

$$\mathbf{a}^{(i)} = \sigma(\mathbf{z}^{(i)})$$

$$J += -[y^{(i)} \log(a^{(i)}) + (1-y^{(i)}) \log(1-a^{(i)})]$$

$$d\mathbf{z}^{(i)} = \mathbf{a}^{(i)} - \mathbf{y}^{(i)}$$

$$d\mathbf{w1} += \mathbf{x}_1^{(i)} d\mathbf{z}^{(i)}$$

$$d\mathbf{w2} += \mathbf{x}_2^{(i)} d\mathbf{z}^{(i)}$$

$$d\mathbf{b} += d\mathbf{z}^{(i)}$$

$J /= m$

$\mathbf{dw1} /= m$

$\mathbf{dw2} /= m$

$\mathbf{db} /= m$

$\mathbf{w1} := \mathbf{w1} - \alpha d\mathbf{w1}$

$\mathbf{w2} := \mathbf{w2} - \alpha d\mathbf{w2}$

$\mathbf{b} := \mathbf{b} - \alpha d\mathbf{b}$

- Above implementation requires **for** loop over all features for all training examples
  - Vectorization can be used to remove explicit **for** loops
  - Vectorization required for deep learning to be efficient

### 1.2.2 Vectorisation in Python

- Deep learning performs best on large data sets
  - Code must be able to run quickly to be effective on large data sets

$$z = w^T x + b$$

$$w \in \mathbb{R}^{n_x} \quad x \in \mathbb{R}^{n_x}$$

- Non vectorized implementation:

```
z = 0
for i in range(n_x):
    z += w[i] * x[i]
z += b
```

- GPUs and CPUs both have parallelization instructions (SIMD: Single Instruction Multiple Data)
  - If built in functions are used, **numpy** will use parallelism to perform computations faster
- For logistic regression, need to calculate  $z$  and  $a$  values for each training example

$$z^{(i)} = w^T x^{(i)} + b$$

$$a^{(i)} = \sigma(z^{(i)})$$

$$X = \begin{bmatrix} | & | & \dots & | \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ | & | & \dots & | \end{bmatrix}$$

$$w \in \mathbb{R}^{n_x} \quad X \in \mathbb{R}^{n_x \times m}$$

$$\begin{aligned} [z^{(1)} \quad z^{(2)} \quad \dots \quad z^{(m)}] &= w^T X + [b \quad b \quad \dots \quad b] \\ &= [w^T x^{(1)} + b \quad w^T x^{(2)} + b \quad \dots \quad w^T x^{(m)} + b] \end{aligned}$$

- In Python:



```
Z = np.dot(w.T, X) + b
```

- Python will broadcast the value **b** so it can be added to the matrix
- Vectorized implementation of sigmoid function can be used on **Z** to calculate **A**

$$A = [a^{(1)} \quad a^{(2)} \quad \dots \quad a^{(m)}]$$

$$dz^{(i)} = a^{(i)} - y^{(i)}$$

$$dz = A - Y$$

$$db = \frac{1}{m} \sum_{i=1}^m dz^{(i)}$$

$$dw = \frac{1}{m} X(dz)^T$$

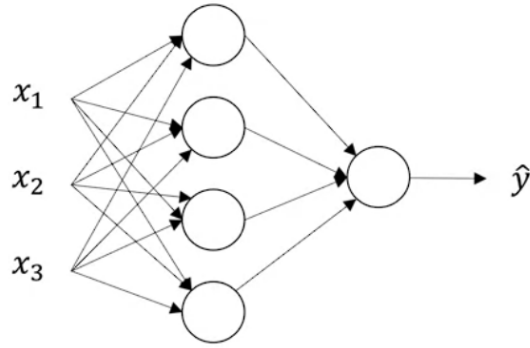
```
Z = np.dot(w.T,X) + b
A = sigmoid(Z)
dz = A - Y
dw = 1/m * np.dot(X, dz.T)
db = 1/m * np.sum(dz)

# Gradient descent update
w = w - alpha * dw
b = b - alpha * db
```

- for loop is required to run multiple iterations of gradient descent

### 1.3 Shallow Neural Networks

- A neural network will have stacked logistic regression units in each layer
  - Logistic regression output from one layer will be fed to another layer



- Input layer of the neural network contains the feature  $x_1, x_2, x_3$ 
  - $a^{[0]} = X$
- Intermediate layers in the network are hidden layers
  - Hidden layers do not have “true” values in the training set
- Final layer in the network is the output layer
  - Generates the predicted value  $\hat{y}$
- Above diagram is a 2 layer NN
  - Input layer is layer 0
- Each layer will have parameters  $w$  and  $b$  associated with them
- Each node in the NN will perform logistic regression with its inputs

$$z_i^{[l]} = w_i^{[l]T} x + b_i^{[l]} \rightarrow a_i^{[l]} = \sigma(z_i^{[l]})$$

$$W^{[1]} = \begin{bmatrix} - & w_1^{[1]T} & - \\ - & w_2^{[1]T} & - \\ - & w_3^{[1]T} & - \\ - & w_4^{[1]T} & - \end{bmatrix}$$

$$a^{[0]} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$b^{[1]} = \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix}$$

$$\begin{aligned}
z^{[1]} &= \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix} \\
&= \begin{bmatrix} w_1^{[1]T} a^{[0]} + b_1^{[1]} \\ w_2^{[1]T} a^{[0]} + b_1^{[1]} \\ w_3^{[1]T} a^{[0]} + b_1^{[1]} \\ w_4^{[1]T} a^{[0]} + b_1^{[1]} \end{bmatrix} \\
&= w^{[1]} a^{[0]} + b^{[1]}
\end{aligned}$$

$$\begin{aligned}
a^{[1]} &= \begin{bmatrix} a_1^{[1]} \\ a_2^{[1]} \\ a_3^{[1]} \\ a_4^{[1]} \end{bmatrix} \\
&= \sigma(z^{[1]})
\end{aligned}$$

$$z^{[2]} = W^{[2]} a^{[1]} + b^{[2]} \rightarrow a^{[2]} = \sigma(z^{[2]})$$

- Vectorized method should be able to work on all training examples at one time
  - Vector for each training example can be stacked horizontally in a matrix
  - Vertical dimension will be the number of units in a layer ( $n_x$  for the input layer)

$$X = \begin{bmatrix} \begin{matrix} | \\ x^{(1)} \\ | \end{matrix} & \begin{matrix} | \\ x^{(2)} \\ | \end{matrix} & \begin{matrix} | \\ x^{(m)} \\ | \end{matrix} \end{bmatrix}$$

$$Z^{[1]} = \begin{bmatrix} \begin{matrix} | \\ z^{[1](1)} \\ | \end{matrix} & \begin{matrix} | \\ z^{[1](2)} \\ | \end{matrix} & \dots & \begin{matrix} | \\ z^{[1](m)} \\ | \end{matrix} \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} \begin{matrix} | \\ a^{[1](1)} \\ | \end{matrix} & \begin{matrix} | \\ a^{[1](2)} \\ | \end{matrix} & \dots & \begin{matrix} | \\ a^{[1](m)} \\ | \end{matrix} \end{bmatrix}$$

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$$A^{[1]} = \sigma(Z^{[1]})$$

$$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

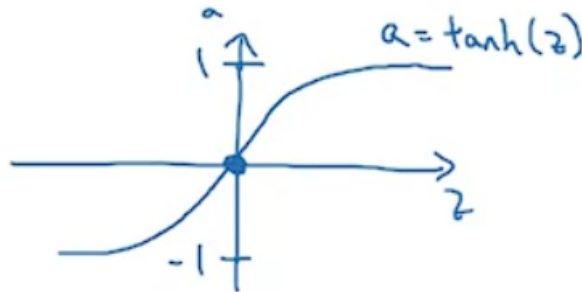
### 1.3.1 Activation Functions

- After  $z$  values are calculated, activation function must be run to get the activation value  $a$

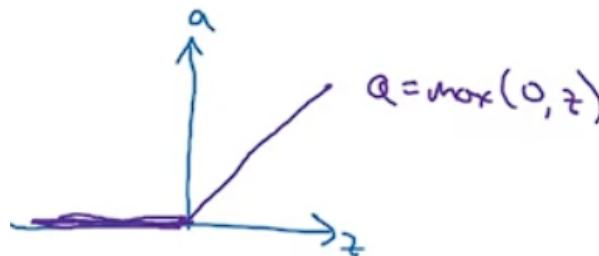
$$a_{sigmoid} = \frac{1}{1 + e^{-z}}$$

- Alternatively  $a^{[1]} = g(z^{[1]})$  where  $g$  is a non linear function
- tanh function almost always performs better than the sigmoid function
  - Equivalent to a transformed version of the sigmoid function
  - tanh function is odd and is “centered” around the origin
  - The mean of the data will be closer to 0 and will help with learning in the next layer

$$a_{tanh} = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$



- For binary classification, the final output layer can use the sigmoid function
  - Want the value of  $\hat{y}$  to be between 0 and 1
- For both the sigmoid and tanh functions, when  $z$  is large, the gradient is very small
  - Results in a slower gradient descent
- ReLU function has a gradient of 1 when  $z$  is positive



- Gradient is 0 when  $z$  is negative
- For majority of the ReLU function, gradient is very different from 0
  - Will typically allow NN to learn much faster than sigmoid or tanh function
- ReLU function should be used as the default activation function
- The leaky ReLU function has a slight positive gradient when  $z$  is negative

$$a_{leakyReLU} = \max(0.01z, z)$$



- For a NN to compute more complex functions, activation function must be non linear
  - If a linear activation function is used, final output of the NN can only be a linear function
  - Multiple linear activation neurons with a sigmoid as the output neuron is equivalent to standard logistic regression
- Linear activation function can be used in the output layer if output is a real number
- Derivative of the activation function must be calculated for backpropagation
  - Sigmoid function

$$g(z) = \frac{1}{1 + e^{-z}}$$

$$\begin{aligned} \frac{d}{dz}g(z) &= \frac{1}{1 + e^{-z}} \left( 1 - \frac{1}{1 + e^{-z}} \right) \\ &= g(z)(1 - g(z)) \end{aligned}$$

- tanh function

$$\begin{aligned} g(z) &= \tanh(z) \\ &= \frac{e^z - e^{-z}}{e^z + e^{-z}} \end{aligned}$$

$$\begin{aligned}\frac{d}{dz}g(z) &= 1 - \left(\frac{e^z - e^{-z}}{e^z + e^{-z}}\right)^2 \\ &= 1 - g(z)^2\end{aligned}$$

– ReLU function

$$\begin{aligned}g(z) &= \max(0, z) \\ \frac{d}{dz}g(z) &= \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}\end{aligned}$$

– Leaky ReLU function

$$\begin{aligned}g(z) &= \max(0.01z, z) \\ \frac{d}{dz}g(z) &= \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}\end{aligned}$$

### 1.3.2 Gradient Descent for Neural Networks

- For a single hidden layer NN, parameters are:  $w^{[1]}, b^{[1]}, w^{[2]}, b^{[2]}$ 
  - $w^{[1]} \in \mathbb{R}^{n_1 \times n_0}$
  - $b^{[1]} \in \mathbb{R}^{n_1 \times 1}$
  - $w^{[2]} \in \mathbb{R}^{n_2 \times n_1}$
  - $b^{[2]} \in \mathbb{R}^{n_2 \times 1}$
- Cost function:  $J(w^{[1]}, b^{[1]}, w^{[2]}, b^{[2]}) = \frac{1}{m} \sum_{i=1}^n \mathcal{L}(\hat{y}, y)$
- For one iteration of gradient descent:

$$w^{[1]} := w^{[1]} - \alpha dw^{[1]}, \quad b^{[1]} := b^{[1]} - \alpha db^{[1]}$$

$$w^{[2]} := w^{[2]} - \alpha dw^{[2]}, \quad b^{[2]} := b^{[2]} - \alpha db^{[2]}$$

– Gradient descent step will take place after backpropagation calculates the derivatives

- Forward propagation:

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = g^{[2]}(Z^{[2]})$$

- Backpropagation:

$$\begin{aligned}
 dz^{[2]} &= A^{[2]} - Y \\
 dw^{[2]} &= \frac{1}{m} dz^{[2]} A^{[1]T} \\
 db^{[2]} &= \frac{1}{m} \text{np.sum}(dz^{[2]}, \text{axis} = 1, \text{keepdims} = \text{True}) \\
 dz^{[1]} &= w^{[2]T} dz^{[2]} \times g^{[1]'}(z^{[1]}) \\
 dw^{[1]} &= \frac{1}{m} dz^{[1]} X^T \\
 db^{[1]} &= \frac{1}{m} \text{np.sum}(dz^{[1]}, \text{axis} = 1, \text{keepdims} = \text{True})
 \end{aligned}$$

### 1.3.3 Random Initialization

- Weights must be initialized randomly for a NN
  - Weights can be initialized to 0 for logistic regression
  - The bias terms  $b$  can be initialized
- If weights are initialized to 0, all neurons in a layer will compute the same hypothesis

```

W1 = np.random.randn((2,2)) * 0.01
b1 = np.zeros((2,1))

```

- Weights should be initialized to small random values
  - If weight is too large, activation value  $z^{[1]}$  will be large
  - If sigmoid or tanh function is used, derivative will be very small and learning will be very slow
- Different constant for `np.random.randn` should be used for deeper neural networks

## 1.4 Deep Neural Networks

- Logistic regression is equivalent to a 1-layer NN
- Deep NN have more hidden layers
  - Number of hidden layers in the network can be a parameter for the ML problem



- Above network has 4 layers,  $L = 4$
- $n^{[l]}$  = number of units in layer  $l$
- $a^{[l]}$  = activations in layer  $l$
- The inputs  $x$  are the activations of the first layer,  $x = a^{[0]}$ 
  - Prediction  $\hat{y}$  will be the activations of the last layer,  $\hat{y} = a^{[L]}$
- Forward propagation for a deep NN will follow the same pattern for all layers

$$z^{[l]} = w^{[l]}a^{[l-1]} + b^{[l]}$$

$$a^{[l]} = g^{[l]}(z^{[l]})$$

- For a vectorized implementation

$$Z^{[l]} = W^{[l]}A^{[l-1]} + b^{[l]}$$

$$A^{[l]} = g^{[l]}(z^{[l]})$$

- Explicit for loop will be used to loop over the layers in the network
- $b$  will still be a column vector but will apply correctly due to broadcasting
- When working with  $W$  and  $A$  matrices,  $A$  will be for the previous layer so the dimensions will fit
- When debugging NN, can look at dimensions of all the matrices
- For a non vectorized implementation:
  - $W^{[l]} : (n^{[l]}, n^{[l-1]})$
  - $b^{[l]} : (n^{[l]}, 1)$
  - Dimensions of  $dw$  and  $db$  should be the same as the dimensions of  $W$  and  $b$
  - $a^{[l]}, z^{[l]} : (n^{[l]}, 1)$
- For a vectorized implementation,  $z$  vectors and  $a$  vectors will be stacked horizontally for all training examples



- $Z^{[l]}, A^{[l]} : (n^{[l]}, m)$
- Deep NN tend to work better as each layer can compute increasingly complex functions
  - Face recognition: edge detection  $\rightarrow$  individual features  $\rightarrow$  large parts of the face
  - Audio: low level waveforms  $\rightarrow$  phonemes  $\rightarrow$  words  $\rightarrow$  sentences
- Functions that can be computed with a “small” deep neural network require exponentially more hidden units in a shallower network
- For each forward propagation step, the value of  $z^{[l]}$  should be cached for backpropagation
  - Values of  $w^{[l]}$  and  $b^{[l]}$  can also be stored in the cache so they can be accessed for backpropagation



- All forward propagation steps will be carried out until the hypothesis,  $\hat{y}$  is found
  - Using cached values, all backpropagation steps will be carried out until  $dz^{[1]}$
  - Parameters  $W^{[l]}$  and  $b^{[l]}$  can be updated accordingly

$$W^{[l]} := W^{[l]} - \alpha dw^{[l]}$$

$$b^{[l]} := b^{[l]} - \alpha db^{[l]}$$

- Backpropagation will also follow a pattern for all layers in the NN
  - $dz^{[l]} = da^{[l]} * g^{[l]'}(z^{[l]})$
  - $dW^{[l]} = dz^{[l]} a^{[l-1]T}$

- $db^{[l]} = dz^{[l]}$
- $da^{[l-1]} = W^{[l]T} dz^{[l]}$
- For a vectorized implementation:
  - $dZ^{[l]} = dA^{[l]} * g^{[l]'}(Z^{[l]})$
  - $dW^{[l]} = \frac{1}{m} dZ^{[l]} A^{[l-1]T}$
  - $db^{[l]} = \frac{1}{m} \text{np.sum}(dZ^{[l]}, \text{axis}=1, \text{keepdims}=\text{True})$
  - $dA^{[l-1]} = W^{[l]T} dZ^{[l]}$

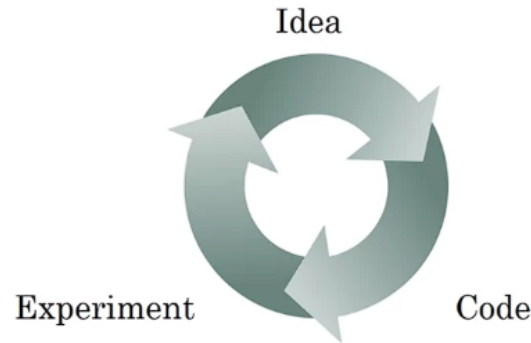
#### 1.4.1 Parameters vs Hyperparameters

- Parameters of the NN are the  $W$  and  $b$  matrices
- NN also has a number of associated hyperparameters:
  - Learning rate  $\alpha$
  - Number of iterations  $z''$
  - Number of layers in the network
  - Number of hidden units
  - Choice of activation function
- Hyperparameters will control the values of  $W$  and  $b$
- Deep learning has many more hyperparameters than earlier eras of machine learning
  - Applying deep learning becomes an empirical process
- Intuitions about hyperparameters may be different across different applications

## 2 Improving Deep Neural Networks: Hyperparameter Tuning, Regularization and Optimization

### 2.1 Practical Aspects of Deep Learning

- Applying ML is a highly iterative process
  - Very hard to choose “correct” values for hyperparameters



- Deep learning used in many different areas
  - NLP
  - Computer vision
  - Speech analysis
  - Structured data
    - \* Advertisement
    - \* Search engines
    - \* Computer security
    - \* Logistics
- Intuitions from one subject area often don't transfer to another application
- Success of deep learning can depend on speed of iteration
  - Choice of split of the data can influence speed of iteration
- Whole dataset should be split into training, development and test set
  - Dev set should be used rate performance of different models
  - Final model should be evaluated on the test set
  - Split will allow better evaluation of bias and variance of the model
- Previous eras of ML had a 60/20/20 split between dataset

- For the big data era, a smaller percentage of data is given to the dev and test sets
  - For 1,000,000 examples, can allocate just 10,000 examples each to dev and test set
  - 10,000 examples is enough to run the algorithm and get a good idea about the algorithm performance
- Recent trends also show mismatched training and test set distributions
  - For images, training set may have very high quality images while test set may have lower quality
  - Dev and test set should come from the same distribution
- Dataset might be split to not include a test set
  - Dev set can be used to get to a “good” model
  - Since data is fit to the dev set, there is no unbiased estimate of performance
  - When data doesn’t include a test set, dev set is usually referred to as “test” set
  - Resulting model may overfit to the dev set

### 2.1.1 Bias and Variance

- In the deep learning era, there tends to be less of a discussion about the bias/variance trade off
- In 2 dimensions, data can be plotted to look for high bias or variance
  - High bias classifiers underfit the data
  - High variance classifiers overfit the data
- For higher dimensions, training set error and dev set error can be used
  - High variance classifier has low training error and high dev set error
  - High bias classifier has high training error and high dev set error
  - Classifier with high bias and high variance will have high training error and even higher dev set error
- Above ideas only work with the assumption that the optimal error is 0%
  - Training and dev set must also come from the same distribution

### 2.1.2 Basic Recipe for Machine Learning

- Train initial algorithm and reduce bias of algorithm to an “acceptable value”
  - Use a larger network
  - Train algorithm for longer

- Reduce variance of the algorithm by getting more data
  - Add regularization terms to the cost function
- Bias and variance can also be reduced by using a more appropriate NN architecture
- In the big data era, bias and variance can be reduced without affecting each other
  - Training a bigger network typically reduce the bias
  - Getting more training data will typically reduce the variance
- Using regularization will have a bias variance trade off

### 2.1.3 Regularization

- Adding regularization will usually help in reducing variance and prevent overfitting
  - Regularization will only affect how the weights change during backpropagation
  - For forward propagation, regularization has no effect
- For logistic regression:

$$J(w, b) = \frac{1}{m} \sum_{i=1}^m \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2m} \|w\|_2^2$$

$$\begin{aligned} \|w\|_2^2 &= \sum_{j=1}^{n_x} w_j^2 \\ &= w^T w \end{aligned}$$

- Above method is  $L_2$  regularization after the  $L_2$  norm (Euclidean norm) of  $w$
  - $b$  values can also be regularized but will have a much smaller effect than  $w$
- $L_1$  regularization adds the term:

$$\frac{\lambda}{m} \sum_{i=1}^{n_x} |w| = \frac{\lambda}{m} \|w\|_1$$

- Using  $L_1$  regularization will result in  $w$  being sparse
  - Can be seen to compressing the model
- $L_2$  regularization is more common for deep learning
- Regularization parameter  $\lambda$  will be set using the cross validation set
  - `lambda` is a reserved keyword in Python

- For a neural network:

$$J(w^{[1]}, b^{[1]}, \dots, w^{[L]}, b^{[L]}) = \frac{1}{m} \sum_{i=1}^m \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2m} \sum_{l=1}^L \|w^{[l]}\|^2$$

$$\|w^{[L]}\|^2 = \sum_{i=1}^{n^{[L]}} \sum_{j=1}^{n^{[L-1]}} (w_{i,j}^{[L]})^2$$

–  $\|W^{[l]}\|_F^2$  known as the Frobenius norm of the matrix

- Since new term added to cost function,  $\frac{\partial J}{\partial W^{[l]}}$  will be different

$$dW^{[l]} = \dots + \frac{\lambda}{m} W^{[l]}$$

$$W^{[l]} = W^{[l]} - \frac{\alpha \lambda}{m} W^{[l]} - \alpha(\dots)$$

– Also known as weight decay as value of  $W$  will decrease on every iteration

$$W^{[l]} - \frac{\alpha \lambda}{m} W^{[l]} = \left(1 - \frac{\alpha \lambda}{m}\right) W^{[l]}$$

– Value of  $\left(1 - \frac{\alpha \lambda}{m}\right)$  will be slightly less than 1

- Adding regularization term will penalize the weight matrix from being too large
  - As the value of  $\lambda$  is increased, the weights in  $w$  will get closer to 0
  - Each hidden unit will have a smaller effect and the resulting NN will be simpler
- When using the tanh function, penalizing  $w$  will make  $z^{[l]}$  smaller
  - For a small  $z^{[l]}$ , tanh function is roughly linear
  - If all hidden units in the network are roughly linear, the result of the NN will also be roughly linear

## Dropout Regularization

- Each layer in the NN has a probability of eliminating a node
  - When a node is eliminated, all outgoing links from the node are also deleted
  - Each example will be trained on a smaller network so will have less chance of overfitting
- For each different training example, the NN is reset and randomly eliminates nodes again
- Inverted dropout:

```
d3 = np.random.rand(a3.shape[0], a3.shape[1]) < keep_prob
a3 = np.multiply(a3, d3)
a3 /= keep_prob
```

- For **keep\_prob** = 0.8 each node has a 0.2 chance of being removed
- Activation values should be scaled by **keep\_prob** so the expected value of  $z$  can stay constant
- On each pass through the training set, a different set of units should be zeroed out
- At test time, dropout should not be used as it will create noise in the predictions
- A single hidden unit cannot rely on a specific feature as it may not be used on each iteration
  - Weights for the unit will be spread out between the units
  - Has the same effect as shrinking the weights like L2 regularization
  - The equivalent L2 penalty on different weights depends on the size of the activations being used for the weight
- **keep\_prob** can be varied between the layers
  - Larger layers may be more prone to overfitting and can have a larger **keep\_prob**
  - For small layers with a very small chance of overfitting, **keep\_prob** can be set to 1
- Many dropout implementations started with computer vision
  - Input size for computer vision is extremely large
- Cost function is not well defined when dropout is used
  - Can set **keep\_prob** to 1 and check for monotonically decreasing  $J$
  - When  $J$  is decreasing, then can reduce the value of **keep\_prob** to use dropout

## Other Regularization Methods

- Getting more training data will almost always help overfitting
  - May not be possible to get more training data or very expensive
- Data augmentation will create new examples and can help reduce overfitting
- For an image dataset:
  - Flipping the image horizontally

- Randomly cropping and distorting the image
- Magnitude of image transformation depends on classifier
  - For a cat dataset, image should not be flipped vertically
  - For OCR, distortions and rotations can be slightly more extreme



- Early stopping can be used to prevent overfitting from happening
  - If the NN is overfitting the data, the dev set error will initially decrease before increasing
  - Training of the NN can be stopped when the dev set error is lowest and the data has not been overfit
- Using early stopping links the task of optimizing  $J$  and not overfitting the data
  - Early stopping will prevent the cost function from being optimized
- L2 regularization is a better method to prevent overfitting
  - Requires a choice for the value of  $\lambda$  and is much more computationally expensive

#### 2.1.4 Setting up the Optimization Problem

- Normalization can be used to speed up the training of a NN
  - Subtract the mean:

$$\mu = \frac{1}{m} \sum_{i=1}^m x^{(i)}$$

$$x := x - \mu$$

- Normalize the variance:

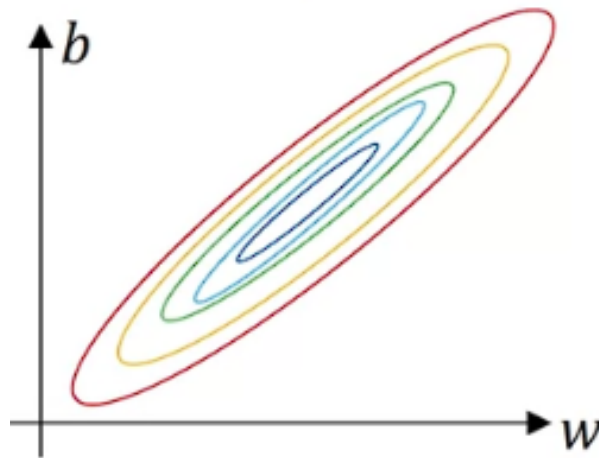
$$\sigma^2 = \frac{1}{m} \sum_{i=1}^m (x^{(i)} - \mu)^2$$

$$x := (x - \mu) / \sigma$$

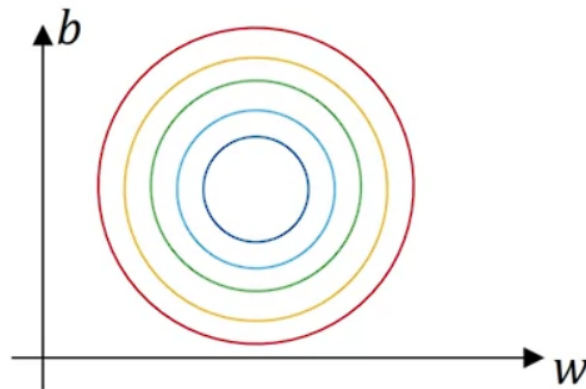
- When normalizing a training set, test set and training set should be processed together



- All the data must go through the same transformation
- For data that is not normalized, the cost function will be very elongated
  - The gradient will be quite shallow and will take longer to converge
  - Algorithm will require a smaller learning rate



- On average, normalized data will have a cost function that is more symmetric
  - Gradient descent will converge faster and can use a larger learning rate



### Vanishing/Exploding Gradients

- For very deep neural networks, the derivatives can get exponentially big or small
- If the weights of a NN are all the same, the prediction  $\hat{y}$  will  $x$  to the  $L$ th power
  - For  $W^{[l]} > I$  the gradient will explode
  - For  $W^{[l]} < I$  the gradient will vanish

- Some modern applications use 152 layer NN
  - Require careful initialization of the weights to ensure correct training
- For a single neuron:
  - The output  $\hat{y}$  will be the sum of all  $w_i x_i$

$$z = w_1 x_1 + w_2 x_2 + \dots + w_n x_n$$

- For a large  $n$ , want a smaller  $w_i$
- Want  $\text{Var}(w_i) = \frac{1}{n}$

$$W^{[l]} = \text{np.random.randn(shape)} * \text{np.sqrt}\left(\frac{1}{n^{[l-1]}}\right)$$

- Variance of Gaussian random variable can be set by multiplying by sqrt tem
- For ReLU activation function, the variance should be set to  $\frac{2}{n}$ 
  - tanh activation uses Xavier initialization  $\frac{1}{n^{[l-1]}}$
  - Yoshua Bengio multiplied random variable by  $\sqrt{\frac{2}{n^{[l-1]} + n^{[l]}}}$
- Initialization of weights aims to set weight matrices close to 1
  - Helps to prevent  $\hat{y}$  from vanishing or exploding too quickly
- Variance parameter can be tuned as another hyperparameter

## Gradient Checking

- Can be used to ensure implementation of backpropagation is correct
- Requires numerical approximations of gradients
  - For a function  $f$  at a point  $\theta$ , gradient can be approximated by looking at  $\theta + \epsilon$  and  $\theta - \epsilon$
  - Approximation is closer when double sided estimate is used
- If  $g$  is the derivative of  $f$ :

$$g(\theta) \approx \frac{f(\theta + \epsilon) - f(\theta - \epsilon)}{2\epsilon}$$

- Using the 2 sided difference will give a much better estimate but is more computationally expensive

- The derivative of a function at a point is the limit of the numerical approximation

$$f'(\theta) = \lim_{\epsilon \rightarrow 0} \frac{f(\theta + \epsilon) - f(\theta - \epsilon)}{2\epsilon}$$

- For a non 0 value of  $\epsilon$ , the error of the approximation is  $O(\epsilon^2)$
- For the single sided numerical approximation, the error is  $O(\epsilon)$
- To perform gradient checking on a NN:
  1. Reshape  $W^{[1]}, b^{[1]}, \dots, W^{[L]}, b^{[L]}$  into a single vector  $\theta$
  2. Reshape  $dW^{[1]}, db^{[1]}, \dots, dW^{[L]}, db^{[L]}$  into a single vector  $d\theta$
  3. For every  $i$  in  $\theta$ , calculate:

$$d\theta_{approx}[i] = \frac{J(\theta_1, \theta_2, \dots, \theta_i + \epsilon) - J(\theta_1, \theta_2, \dots, \theta_i - \epsilon)}{2\epsilon}$$

4. Check if  $d\theta_{approx}$  and  $d\theta$  are reasonably close to each other

For  $\epsilon = 10^{-7}$ :

$$\frac{\|d\theta_{approx} - d\theta\|_2}{\|d\theta_{approx}\|_2 + \|d\theta\|_2} \approx 10^{-7}$$

- Grad check should be only be used when debugging
  - Calculating  $d\theta_{approx}$  is very computationally expensive
- If regularization is used, correct cost function must be used to calculate the gradient
- If dropout is used,  $J$  is not well defined and cannot use grad check
  - Cost function  $J$  that is optimized by dropout is defined by summing over all subsets of nodes that could be eliminated on each iteration
  - Can implement grad check with a `keep_prob` of 1 before turning on dropout
- Implementation of gradient descent may be correct when  $W$  and  $b$  are close to 0
  - Can run grad check just after random initialization
  - After training the network for a number of iterations, can run grad check again

## 2.2 Optimization Algorithms

### 2.2.1 Mini Batch Gradient Descent

- For gradient descent, vectorization will allow computation over all  $m$  training examples
  - If  $m$  is very large, then vectorization will still be very slow
- Gradient descent requires the whole training set to be processed for a single step of gradient descent

- Data from training set can be split into mini batches

$$X^{\{1\}} = [x^{(1)}, x^{(2)}, \dots, x^{(1000)}]$$

$$Y^{\{1\}} = [y^{(1)}, y^{(2)}, \dots, y^{(1000)}]$$

- Mini batch gradient descent looks at one mini batch on each iteration of gradient descent
- For each mini batch in the training set:

- Run forward propagation on  $X^{\{t\}}$

$$Z^{[1]} = W^{[1]}X^{\{t\}} + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

...

$$A^{[l]} = g^{[l]}(Z^{[l]})$$

- Compute cost:  $J^{\{t\}} = \frac{1}{1000} \sum_{i=1}^l \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2 \times 1000} \sum_l \|W^{[l]}\|_F^2$
- Use backpropagation to calculate gradients wrt  $J^{\{t\}}$
- Update weights

$$W^{[l]} := W^{[l]} - \alpha dW^{[l]}$$

$$b^{[l]} := b^{[l]} - \alpha db^{[l]}$$

- A single pass through the training set is known as an epoch
- Algorithm can continue to run for multiple passes through the training set until an optimal solution is found
- For batch gradient descent, the cost should decrease on each iteration
  - If the cost doesn't decrease per iteration, then the algorithm has a bug
- For mini batch gradient descent, the cost will trend downwards but will be more noisy
  - Algorithm is being trained on a different batch of results on each iteration
- When running mini batch gradient descent, must choose the size of the mini batch
  - For mini batch size =  $m$ : Batch gradient descent
  - For mini batch size = 1: Stochastic gradient descent
- For stochastic gradient descent, each example may be good or bad for gradient descent
  - On average the cost function will be minimized for gradient descent
  - Path taken by gradient descent will be very noisy

- Stochastic gradient descent will never converge and just oscillate around the minimum
- Choice of mini batch size should be between 1 and  $m$ 
  - Batch gradient descent will take very long for a single iteration
  - Stochastic gradient descent will lose all the speed from vectorization
- For a small training set ( $m \leq 2000$ ), can just use gradient descent
- Otherwise can try a mini batch size from 64-512
  - Code may run faster if the mini batch size is a power of 2
- A single mini batch should be able to fit in the whole CPU/GPU memory

## Advanced Optimization Algorithms

- Some advanced algorithms require the use of exponentially weighted averages
- Moving averages can be calculated for data such as daily temperature

$$V_0 = 0$$

$$V_t = \beta V_{t-1} + (1 - \beta)\theta_t$$

- $V_t$  is the approximated average temperature over the last  $\frac{1}{1-\beta}$  days
- If  $\beta$  is larger then the average will adapt slower to changes in the data
- Exponentially weighted average can be found by summing the daily temperature with an exponentially decaying function
- If  $\beta = 0.9$ :

$$V_{100} = 0.1\theta_{100} + (0.1)(0.9)\theta_{99} + (0.1)(0.9)^2\theta_{98} + (0.1)(0.9)^3\theta_{97} + \dots$$

- When calculating the exponentially weighted average, the same variable  $v$  should be used and overwritten each time
  - Implementation will be much more efficient than calculating average manually from the past 10 values
- For large values of  $\beta$ , initial average will be much lower than they should be

$$\frac{V_t}{1 - \beta^t}$$

- Bias correction can be used to ensure initial values are correct estimations of the averages
  - As  $t$  becomes larger, denominator becomes closer to 1

## ***Momentum***

- Gradient descent with momentum uses an exponentially weighted average of the gradients to update the weights
    - Almost always performs better than standard gradient descent
- $$V_{dW} = \beta V_{dW} + (1 - \beta)dW$$
- $$V_{db} = \beta V_{db} + (1 - \beta)db$$
- $$W = W - \alpha V_{dW}$$
- $$b = b - \alpha V_{db}$$
- Taking the average of the gradients will slow down any unnecessary oscillations in the algorithm
    - Algorithm may oscillate at first but will start to take more direct steps to the minimum
  - $\beta = 0.9$  is a common choice for most applications of momentum

## ***RMSprop***

- RMSprop takes the weighted average of the squares of the derivatives
  - Derivatives will get divided by the RMS before the weights are updated
- $$S_{dW} = \beta_2 S_{dW} + (1 - \beta_2)dW^2$$
- $$S_{db} = \beta_2 S_{db} + (1 - \beta_2)db^2$$
- $$W = W - \alpha \frac{dW}{\sqrt{S_{dW}}}$$
- $$b = b - \alpha \frac{db}{\sqrt{S_{db}}}$$
- Updates in the direction of oscillation will be divided by a large number
    - Will allow the learning rate to be larger and therefore allows faster training
  - In practice, very small value  $\epsilon$  is added to the denominator for more numerical stability

## ***Adam Optimization Algorithm***

- Adam optimization shown to work well for a range of deep learning architectures
  - Merges Momentum and RMSprop to one algorithm
  - “Adam” stands for adaptive moment estimation
- On iteration  $t$ :
  - Compute  $dW, db$  using the current mini batch

$$\begin{aligned}
V_{dw} &= \beta_1 V_{dw} + (1 - \beta_1) dW, & V_{db} &= \beta_1 V_{db} + (1 - \beta_1) db \\
S_{dw} &= \beta_2 S_{dw} + (1 - \beta_2) dW^2, & S_{db} &= \beta_2 S_{db} + (1 - \beta_2) db^2 \\
V_{dw}^C &= \frac{V_{dw}}{1 - \beta_1^t}, & V_{db}^C &= \frac{V_{db}}{1 - \beta_1^t} \\
S_{dw}^C &= \frac{S_{dw}}{1 - \beta_2^t}, & S_{db}^C &= \frac{S_{db}}{1 - \beta_2^t} \\
W &:= W - \alpha \frac{V_{dw}^C}{\sqrt{S_{dw}^C + \epsilon}} \\
b &:= b - \alpha \frac{V_{db}^C}{\sqrt{S_{db}^C + \epsilon}}
\end{aligned}$$

- Must choose many hyperparameters to run Adam optimization
  - $\alpha$ : needs to be tuned to the specific NN
  - $\beta_1$ : 0.9 (default)
  - $\beta_2$ : 0.999 (default)
  - $\epsilon$ :  $10^{-8}$  (default)

## Learning Rate Decay

- For mini batch gradient descent, the algorithm will oscillate around the minimum point
- If the learning rate is reduced over time, then the oscillations will become smaller
  - During the initial steps of learning, algorithm can afford to take large steps
  - As the algorithm starts to converge, smaller steps are preferred

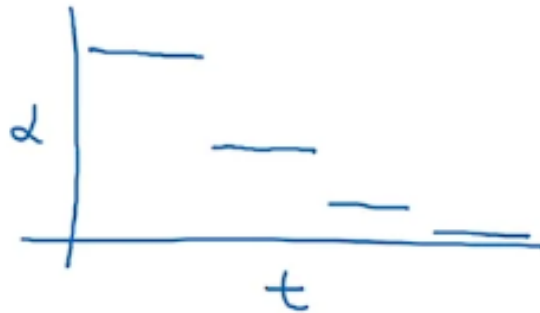
$$\alpha = \frac{1}{1 + \text{decay rate} \times \text{epoch num}} \alpha_0$$

- Other formulas can be used to decay the learning rate

- Exponential decay

$$\alpha = 0.95^{\text{epoch num}} \alpha_0$$

- Discrete staircase



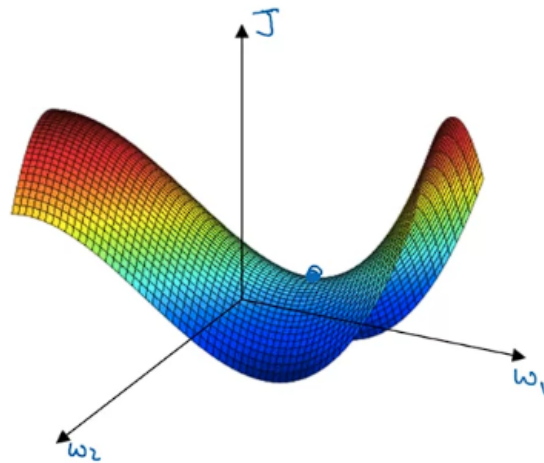
- Square root of epoch number

$$\alpha = \frac{k}{\sqrt{\text{epoch num}}} \alpha_0$$

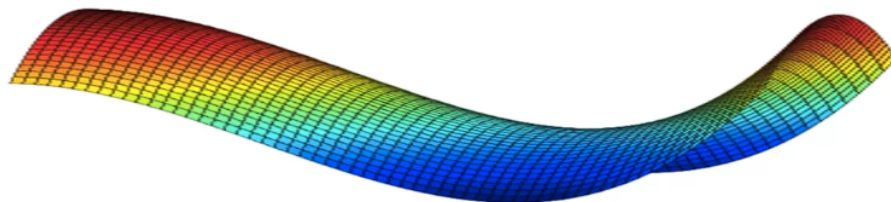
- Manual decay can be used for larger models that take a longer time to train

## Local Optima

- Initial ideas believed that a cost function with many points of 0 gradient would have many local optima
  - When training a NN, most points with 0 gradient are saddle points



- For a point with 0 gradient, Each direction can either be a convex or concave function
  - For a local optima, must have a convex function in all directions
  - In a high dimensional space, chance of all directions being convex functions is very small
- Intuitions about lower dimensional spaces may not transfer to high dimensional spaces
- Plateaus are areas where the gradient is near to 0 for a large area



- Will take a very long time to move down off the plateau



- Learning will be slow but unlikely to get stuck in a local optima
- Optimization algorithms like Adam can help to speed up the training

## 2.3 Hyperparameter Tuning, Batch Normalization and Programming Frameworks

### 2.3.1 Hyperparameter Tuning

- Deep neural networks have many hyperparameters associated with the actual network and the training implementation
  - Numbers of layers and hidden units
  - Learning rate or method for learning rate decay
  - Hyperparameters for momentum or Adam optimization
  - Mini batch size
- Most important hyperparameter is the learning rate
  - Secondary importance can be given to momentum ( $\beta$ ), number of hidden units and the mini batch size
  - Number of layers and learning rate decay can be tuned last
  - Parameters for Adam optimization usually don't need to be tuned
- In practice, random values for the hyperparameters should be sampled and tested
  - If values are arranged in a grid, fewer distinct values can be tested
  - Choosing random values for the hyperparameters gives a higher chance of finding an optimum value for important hyperparameters
- Can use coarse to fine sampling scheme to find optimum values
  - Sample initial values and find which values work the best
  - “Zoom in” to the area and take more samples in the smaller region
- For some hyperparameters (number of layers / hidden units), can sample over a reasonable range
- Some hyperparameters may not have an even distribution (Learning rate between 0.0001 and 1)
  - Can use a log scale to ensure the numbers are better distributed

```
r = -4 * np.random.rand
learning_rate = 10 ** r
```

- Can look for a range  $10^a \dots 10^b$  and take a random sample  $r \in [a, b]$
- For exponentially weighted averages,  $\beta$  will be around 0.9-0.999
  - Equivalent to averaging over the last 10 days or last 1000 days
  - Can sample values for  $1 - \beta$  for  $r \in [-3, -1]$
- For exponentially weighted averages, the sensitivity of the results is very high when  $\beta$  is close to 1
  - A change from 0.999 to 0.9995 will change the average from 1000 to 2000 examples
- Intuitions about the hyperparameters won't always transfer across applications
  - Ideas found in one application can still be applied to other applications
- Hyperparameters can become stale over time with changing data or hardware
  - Hyperparameters should be reevaluated every few months to ensure values are optimal
- Depending on resources, can babysit a single model or train models in parallel
  - For a single model, hyperparameters can be tweaked over time depending on training performance
  - If resources allow, can train the same model with many different hyperparameters and choose the best model

### 2.3.2 Batch Normalization

- Inputs to a NN can be normalized to speed up learning

$$X = \frac{X - \mu}{\sigma}$$

- Batch normalization normalizes the input values  $Z^{[l]}$  to each layer
  - Can instead normalize the values  $A^{[l]}$  after the activation function
- Given intermediate values  $z^{(1)}, \dots, z^{(m)}$ :

$$\mu = \frac{1}{m} \sum_i z^{(i)}$$

$$\sigma^2 = \frac{1}{m} \sum_i (z^{(i)} - \mu)^2$$

$$z_{norm}^{(i)} = \frac{z^{(i)} - \mu}{\sqrt{\sigma^2 + \epsilon}}$$

$$\tilde{z}^{(i)} = \gamma z_{norm}^{(i)} + \beta$$

- $\gamma$  and  $\beta$  are learnable parameters of the model
  - Allows the mean and variance of  $\tilde{z}$  to be set to any value

- If  $\gamma = \sqrt{\sigma^2 + \epsilon}$ ,  $\beta = \mu$ , then  $\tilde{z}^{(i)} = z^{(i)}$
  - May not want mean 0 and standard deviation 1 for the activation function
  - NN will have new parameters  $\beta^{[1]}, \gamma^{[1]}, \dots, \beta^{[L]}, \gamma^{[L]}$ 
    - Will be updated like normal parameters
- $$\beta^{[l]} = \beta^{[l]} - \alpha d\beta^{[l]}$$
- $$\gamma^{[l]} = \gamma^{[l]} - \alpha d\gamma^{[l]}$$
- Batch normalization is typically applied to mini batch gradient descent
    - Mean and variance will be calculated from the mini batch being used
  - When using batch normalization, normalization step removes the need for  $b^{[l]}$  parameters
    - When subtracting the mean from the  $z$  values, the constant will get cancelled out
    - Mean of the  $\tilde{Z}$  values will be decided by the  $\beta^{[l]}$  parameters
  - Batch normalization will make weights deeper in a network more robust to changes earlier in the network
    - Data can have a covariate shift where the distribution changes after a generalization
    - Function mapping from  $X$  to  $Y$  can be the same but model may need to be retrained
    - Batch normalization will reduce the amount of movement of the distribution of the hidden values
  - Even if there is a covariate shift in the data, batch norm will make the  $z$  values have the same mean and variance
    - The individual layers in the network will be more independent of each other
  - Batch norm will also add a slight regularization effect
    - Each mini batch is scaled by the mean/variance of the specific mini batch
    - Normalizing with the mean/variance of the individual mini batch will add noise to the activations
    - Similar to dropout where the algorithm will not rely on any single hidden unit
    - Noise added to the  $z$  values is very small so dropout can be used as well
  - If a larger mini batch size is used, noise is reduced and will have a smaller regularization effect
  - At test time, data will typically be processed one example at a time

- Cannot calculate the mean/variance of a single example
- Mean/variance can be estimated using exponentially weighted averages across the mini batches

### 2.3.3 Multi Class Classification

- Logistic regression can be generalized to apply to multiple classes

$$C = \text{number of classes}$$

- Output layer for the NN will have  $C$  units
  - Each unit will be the probability of each class
  - Sum of all numbers in the vector must be 1
- Softmax layer used in the output layer to output vector of probabilities
  - $Z^{[L]}$  values are calculated as normal:  $Z^{[L]} = W^{[L]}a^{[L-1]} + b^{[L]}$
  - Use the softmax activation function

$$t = e^{(Z^{[L]})}$$

$$a^{[L]} = \frac{t}{\sum_{i=1}^C t_i}$$

- Softmax activation function has a vector for its input and output
  - Other activation functions had a single value for input and output
- Largest input to softmax function will result in the largest output
  - “Hard max” function would return 1 for the largest input and 0 for the other inputs
- If  $C = 2$ , softmax reduces to logistic regression
- Softmax classifier cannot be trained as a normal NN

$$\mathcal{L}(\hat{y}, y) = - \sum_{j=1}^C y_j \log \hat{y}_j$$

- Loss function will only be active for the ground truth class in the training set

$$J(W^{[1]}, b^{[1]}, \dots) = \frac{1}{m} \sum_{i=1}^m \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$$

$$dz^{[L]} = \hat{y} - y$$

### 2.3.4 Deep Learning Frameworks

- For larger NNs, using a framework can save a lot of time
- Can look at the community behind the frameworks and the strengths
  - Ease of programming (development and deployment)
  - Running speed
  - Truly open (open source with good governance)
  - Application of NN

#### Tensorflow

- Assume a simple cost function:

$$J(w) = w^2 + 10w + 25$$

```
import numpy as np
import tensorflow as tf

w = tf.Variable(0, dtype=tf.float32)
optimizer = tf.keras.optimizers.Adam(0.1)

def train_step():
    with tf.GradientTape() as tape():
        cost = w ** 2 - 10 * w + 25
        trainable_variables = [w]
        grads = tape.gradient(cost, trainable_variables)
        optimizer.apply_gradients(zip(grads, trainable_variables))

for i in range(1000):
    train_step()
```

- No need to compute backpropagation steps with tensorflow
- More complex tensorflow program will have cost as a function of variables

```
w = tf.Variable(0, dtype=tf.float32)
x = np.array([1.0, -10.0, 25.0], dtype=np.float32)
optimizer = tf.keras.optmizers.Adam(0.1)
```

```
def training(x, w, optimizer):  
    def cost_fn():  
        return x[0] * w ** 2 + x[1] * w + x[2]  
  
    for i in range(1000):  
        optimizer.minimize(cost_fn, [w])  
  
    return w
```

- Tensorflow will create a computation graph from the defined cost function
  - From the computation graph, tensorflow will compute the backpropagation steps

## 3 Structuring Machine Learning Projects

### 3.1 ML Strategy

#### 3.1.1 Setting up a ML Project

- A machine learning project may have many ideas that can improve performance
  - Collect more data
  - Use a more diverse training set
  - Train the algorithm over a longer period of time
  - Use a different optimization algorithm (Adam instead of gradient descent)
  - Use a bigger/smaller network
  - Add dropout or  $L_2$  regularization
  - Change the network architecture (activation functions or hidden units)
- Some methods may not be useful for the specific scenario
- ML strategy is changing with deep learning
  - Deep learning algorithms have different options when compared with previous generations
- Orthogonalization is where specific functions can be split up into different areas
- For a supervised learning system to perform well, system requires a chain of assumptions
  - Performance of algorithm on the training set must pass some threshold ( $\approx$  human-level performance)
  - Algorithm must be fit well to the dev set
  - Algorithm must be fit well to the test set
  - Algorithm must perform well in the real world
- Each step has specific “knobs” to tune to improve performance in the specific area
  - Training set: bigger network, Adam optimization
  - Dev set: regularization, bigger training set
  - Test set: bigger dev set
  - Real world: change dev set or cost function
- Early stopping can be used but is less orthogonalized
  - Worsens the performance on the training set

- Improves the performance on the dev set
- Single number evaluation metric can be used to test effectiveness of a model
- F1 score combines precision and recall into a single metric
  - Precision is the percentage of positively classified examples that are actually positive
  - Recall is the percentage of positive examples that are correctly classified
  - F1 score takes the harmonic mean of precision and recall

$$F_1 = \frac{2}{\frac{1}{P} + \frac{1}{R}}$$

- Having a well defined test set and single number evaluation metric will speed up iteration
- Scenario may have more than one type of metric that is relevant
  - Classification algorithm may value accuracy as well as running time
  - May not make sense to use a numerical function of some metrics
- Accuracy would be a optimizing metric and running time would be the satisficing metric
  - Goal can be to maximize accuracy subject to running time  $\leq 100\text{ms}$
- For  $N$  different metrics:
  - 1 should be optimizing
  - $N - 1$  should be satisficing
- Dev set and test set should come from the same distribution
  - If different distributions are used, algorithm may perform on the dev set but not on the test set
  - Dev set and test set must have the same target
- Dev set should be used to evaluate the performance of different models
  - Setting up a dev set and an evaluation metric allows teams to iterate quickly

“Choose a dev set and test set to reflect data you expect to get in the future and consider important to do well on”

- Previous eras of machine learning had a 60%, 20%, 20%
- Modern eras of machine learning have much larger datasets
  - For 1000000 examples, can assign 1% each to dev and test set



- Larger amount of data in the training set will help algorithm

“Set your test set to be big enough to give high confidence in the overall performance of your system”

- Some applications may only use a train and dev set
  - Specific scenario may not require high confidence in the overall performance of the algorithm
  - Must be careful to not overfit the dev set too much
- Evaluation metric may not give a full representation of the specific scenario
  - Cat classifier with very low error may allow some pornographic images through the algorithm
  - Algorithm with slightly higher error but no pornographic images would be preferred
- Evaluation metric should be changed if it doesn’t correctly rank the algorithm’s performance
  - Standard error function treats all images equally
  - Weight can be added to the error function to weight unwanted images higher
  - Requires labelling of unwanted images in dev and test set
- Task of changing evaluation metric is separate from changing cost function to achieve good performance
- Metric and/or dev/test set should be changed if performance on the application is not linked

### 3.1.2 Comparing to Human Level Performance

- With advances in deep learning, ML algorithms have much better performance
  - More feasible for algorithms to be competitive with human level performers
- Workflow of designing and building a ML system is more efficient when trying to learn something that humans can do
- For many ML projects, initial learning will be very fast as algorithm approaches human level performance
  - Rate of learning decreases after algorithm surpasses human level performance
  - Algorithm will approach Bayes optimal error
- Bayes optimal error is the best theoretical function for mapping from  $X$  to  $Y$ 
  - For many tasks, human level performance is not very far from Bayes optimal error

- Once human level performance is surpassed, there may not be many areas to improve in
- If algorithm has lower than human level performance:
  - Get labelled data from humans
  - Gain insight from manual error analysis
  - Better analysis of bias/variance
- If human level performance is much lower than the training and dev set error, can focus on the bias of the algorithm
- If human level performance is close to the training error, can focus on the variance of the algorithm
- Human level performance can be used as an estimate for Bayes error
  - Difference between the Bayes error and training error is the avoidable bias
  - Difference between the training and dev set error can measure the variance
- For specialized tasks, different parties may have different errors for human classification
  - For medical image classification, a team of experienced doctors will have much lower error than an average human
  - Bayes error must be less than or equal to the lowest human error
  - Lowest human error can be used as estimate for Bayes error
- For publishing a paper or deploying a system, human error definition may be different
- When algorithm is very close to human level performance, can be hard to see if bias or variance should be trained
- With deep learning, algorithms in some areas can surpass human level performance
  - Online advertising
  - Product recommendations
  - Logistics
  - Loan approvals
- Above areas are not natural perception problems and come from structured data
  - Currently more challenging for computers to surpass humans in natural perception tasks
- ML has also surpassed humans in some natural perception tasks
  - Speech recognition
  - Some image recognition

- Medical tasks
- For supervised learning, must assume that the training set can be fit well (low avoidable bias)
  - The training set performance must also generalize well to the dev/test set (low variance)
- For high bias:
  - Train a bigger model
  - Train for longer or use a better optimization algorithm (momentum, RMSprop, Adam)
  - Change the NN architecture or find better hyperparameters
- For high variance:
  - Use more data
  - Use regularization ( $L_2$ , dropout, data augmentation)
  - Change the NN architecture or find better hyperparameters

### 3.1.3 Error Analysis

- Misclassified examples can be manually examined to look for any patterns
  - Finding patterns can give an upper bound of any increase in performance
- Different ideas for error analysis can be evaluated in parallel with a table
  - For each image, can fill in a checkbox for any patterns
  - Percentage of total for each pattern will give an idea of how to best improve performance
- Manual analysis may show new patterns in the errors
- Some errors may be incorrectly labelled examples in the dev/test set
  - Deep learning algorithms are quite robust to random errors in the training set
  - Algorithms are fairly susceptible to systematic errors in the training set
- Incorrectly labelled examples can be recorded in the error analysis table
  - Percentage of error caused by incorrect labels can be calculated to see if fixing labels is a worthwhile task
- Any processes should be applied to the dev and test set at the same time to ensure they come from the same distribution
  - Training set may end up coming from a different distribution than the dev/test set

- Can also look at examples that the algorithm got right to see if got any errors
- For a new ML system, priority should be to build initial system then iterate
  - Set up a dev/test set and evaluation metric
  - Build initial system quickly
  - Use bias/variance and error analysis to prioritize next steps
- Error analysis will give idea for next steps

### 3.1.4 Mismatched Training and Dev/Test Sets

- Deep learning algorithms perform best with a lot of training data
  - Many teams are putting as much data as possible into training sets
  - Extra data added to the training set will give a different distribution to the training set data
- Other sources of data may have more examples but can come from a slightly different distribution
- Data can be pooled together and randomly split into training, dev and test set
  - All data will come from the same distribution
  - Much of the dev set will come from the additional distribution of images rather than the original distribution
  - Algorithm will optimize to the wrong distribution of images
- Training set can be set to include all images from the additional distribution
  - Examples from the original distribution will be split between the dev and test set
  - Dev and test set will have the correct distribution of images
  - Training set will have a different distribution
- Estimate of bias and variance changes when training set has a different distribution to dev and test set
  - Comparatively high dev set error might mean dev set has more challenging images than training set
  - Data from the dev set will be new to the algorithm and will have a different distribution to the training data
- Portion of the training set can be set as the training-dev set
  - Should not be used for training but will have the same distribution as the training set
- For error analysis, can look at the training set, training-dev set and dev set

- Large difference between the training error and training-dev error indicates a variance problem
- Large difference between the training-dev error and dev error indicates a data mismatch problem
- Large difference between training error and human error indicates high bias problem
- Difference between the dev error and test error indicates degree of overfitting to the dev set
- For each distribution of data, can look at:
  - Human level error
  - Error on examples trained on
  - Error on examples not trained on
- For data mismatch:
  - Use manual data analysis to try understand the difference between training and dev sets
  - Can try to make the training set more similar to the dev set (collect more examples or use artificial data synthesis)
- For some applications, algorithm may overfit during artificial data synthesis
  - For speech recognition, same recording of noise may be added to many examples
  - As much as possible, should aim to get a large range of examples with data synthesis

### 3.1.5 Learning From Multiple Tasks

- For some applications, NN trained for one task can be applied to another task
  - NN trained for cat recognition can be retrained for radiology diagnosis
- After initial NN is trained, output layer should be deleted
  - Weights for the output layer should be randomly initialized
  - Dataset can be switched to new application and NN retrained
- If the new dataset is small, can just retrain the last layer of the NN
  - If there is a lot of data, all layers in the NN can be retrained
  - Pre-training is the training of the NN for the original application
- Learning basic feature of images from a large dataset can help performance of algorithm

- Transfer learning works best when there is comparatively more data for the initial training
  - Initial training will not be useful if there is more data in the fine-tuning dataset
  - Both tasks must have the same input type
  - Low level features should be helpful for learning B
- For multi task learning, a single NN will try to learn multiple things at a time
  - Each task will ideally help the other tasks
- For self driving vehicles, many objects need to be identified from input data
  - Pedestrians
  - Cars
  - Different types of signs
  - Traffic lights
- Output from NN will be a vector for each object

$$\frac{1}{m} \sum_{i=1}^m \sum_{j=1}^4 \mathcal{L}(\hat{y}_j^{(i)}, y_j^{(i)})$$

- Output from NN can have all objects in the same image
- Softmax regression had only one output label for each image
- NN trained to minimize above cost function is using multi task learning
  - Separate NN could have been trained for each object
  - Basic image features for all NN can be shared
- Multi task learning can also be done if the dataset is incomplete
  - Dataset may have missing values for some objects
  - When calculating the cost, missing values can be ignored in the sum
- Multi task learning should have tasks that benefit from having shared lower level features
  - Amount of data for each task tends to be similar
  - Must be able to train a big enough NN to do well on all tasks
- Transfer learning tends to be more common than multi task learning
  - Multi task learning more common in computer vision

### 3.1.6 End to End Deep Learning

- End to end deep learning takes multiple stages of processing and combines it into a single NN
  - For sound recognition:
    - Individual features of the sound (MFCC)
    - Recognizing phonemes
    - Recognizing words
    - Final transcript
  - End to end deep learning requires a lot more data than the standard pipeline
    - A medium sized dataset can use a mixture of end to end learning and the standard pipeline
  - For an identity detection algorithm using a camera, algorithm will first detect the person's face
    - Algorithm will then crop the image to the face and use the image to identify the person
    - Algorithm will compare new image to all existing images of recognized people
  - For each individual step, there is a lot of data for each step
    - Will be a lot harder to find data for both concurrent steps
  - End to end deep learning used for machine translation
  - Estimating a child's age from an x-ray more suited to different tasks
    - Much easier to identify bones from x-ray before estimating age
    - Possible to use end to end method with a lot of data
  - End to end deep learning requires less hand-designing of components
    - Hand-designing components may be constricting the data
  - End to end deep learning requires a large amount of data
    - Hand-designed components could be useful when there is comparatively little data
- “Do you have sufficient data to learn a function of the complexity needed to map  $x$  to  $y$ ”

## 4 Convolutional Neural Networks

### 4.1 Foundations of Convolutional Neural Networks

- Computer vision has benefitted greatly from deep learning
  - Many current applications of computer vision were not possible a few years ago
  - Some ideas in deep learning also transferable across disciplines
- Computer vision can be split into many subareas:
  - Image classification
  - Object detection
  - Neural style transfer
- For computer vision applications, input from an image can be very large
  - $64 \times 64$  color image has 12288 features
  - $1000 \times 1000$  color image (1 megapixel) has 3000000 features
- For a  $1000 \times 1000$  image with 100 hidden units in the first layer,  $W^{[1]}$  will have 3 billion parameters
  - Computational requirements will be very large
  - Also hard to get enough data to prevent the NN from overfitting
- For an object detection problem, can start by detecting vertical and horizontal edges in the image
  - Using a grayscale image, a filter can be convolved with the image
  - Each pixel in the filter takes an element wise product and sum over the whole filter
- $6 \times 6$  grayscale image convolved with a  $3 \times 3$  gives a  $4 \times 4$  image

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

- Above filter used for vertical edge detection
- Filter represents area in image that has a light section on the left section and dark on the right section
- Filter will have better performance on larger images

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$



- Above filter used for horizontal edge detection
- Using the same filter, dark to light and light to dark edges will look different
  - Absolute value can be taken if type of edge detected is not needed
- Different numbers may be used for the edge detection filter
  - Sobel filter
 
$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$
  - Scharr filter
 
$$\begin{bmatrix} 3 & 0 & -3 \\ 10 & 0 & -10 \\ 3 & 0 & -3 \end{bmatrix}$$
- Numbers in the filter can be learned with backpropagation
  - Can define what type of edge the filter should learn

#### 4.1.1 Padding and Strides

- For a  $n \times n$  image with a  $f \times f$  filter, dimensions of the result will be  $n - f + 1$ 
  - Dimensions of the image will shrink with every convolution
  - With the standard convolution operation, corner pixel is only used once
  - Pixels in the center of the image will get used many more times
- Image can be padded with a  $1 \times 1$  border
  - Original image size will be preserved with convolution operation
  - 0s are typically used for padding
- Dimensions of the new image will be  $n + 2p - f + 1$
- Valid convolution has no padding on the input
- Same convolution uses padding such that the output is the same size as the input
  - For same convolution, need  $p = \frac{f-1}{2}$
- Size of filter is usually an odd number
  - For an even number, asymmetrical padding is needed for same convolution
  - Odd filter will always have a central pixel to the filter
- Strided convolutions change the size of the step taken by the filter
  - Standard convolution uses a stride of 1

- For an  $n \times n$  image with an  $f \times f$  filter, size of resultant image is:

$$\left\lfloor \frac{n + 2p - f}{s} + 1 \right\rfloor$$

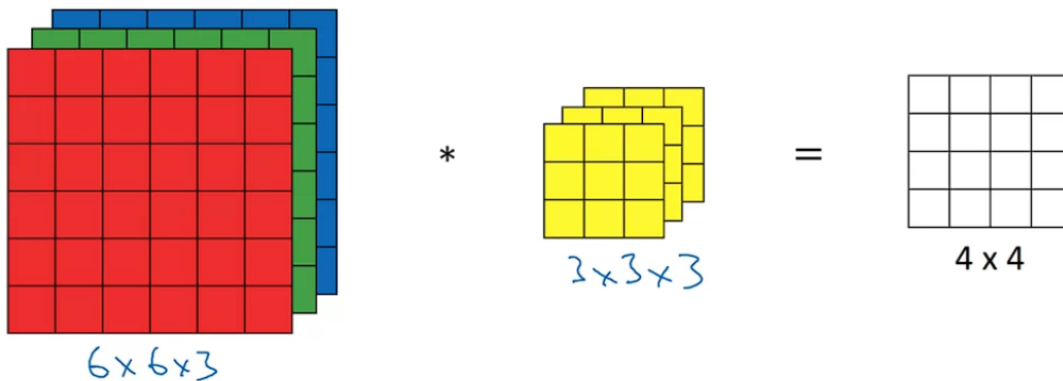
- For a non standard stride length, the filter must be completely within the image for the computation
- Convolution in mathematical literature flips the filter across the horizontal and diagonal before operation

$$\begin{bmatrix} 3 & 4 & 5 \\ 1 & 0 & 2 \\ -1 & 9 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 7 & 9 & -1 \\ 2 & 0 & 1 \\ 5 & 4 & 3 \end{bmatrix}$$

- Convolution operation in deep learning literature known as cross-correlation
- Flipping the filter in convolution gives associativity to the operation

$$(A * B) * C = A * (B * C)$$

- Associativity not required for NN so flipping of filter can be omitted
- For a 3 channel RGB image, filter will also have 3 channels
- Output from the RGB convolution will be a single image



- For RGB image, convolutions in each layer are applied then summed together for each pixel
  - Filter can be set to detect edges in specific colors or all edges
- To detect all edges, vertical filter and horizontal filter can be used
  - Outputs from both filters can be stacked over each other
- Single layer in a convolutional NN will add a bias term and non-linearity to each output
  - Same bias term will be added to all pixels in the image

- If using 10  $3 \times 3 \times 3$  filters, total parameters will be 280
  - Number of parameters is independent of the size of the input
  - Makes CNN less prone to overfitting than standard NN
- For a convolution layer  $l$ :
  - $f^{[l]}$  = filter size
  - $p^{[l]}$  = padding
  - $s^{[l]}$  = stride
  - Input:  $n_H^{[l-1]} \times n_W^{[l-1]} \times n_C^{[l-1]}$
  - Output:  $n_H^{[l]} \times n_W^{[l]} \times n_C^{[l]}$

$$n_H^{[l]} = \left\lfloor \frac{n_H^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

- Filter:  $f^{[l]} \times f^{[l]} \times n_C^{[l-1]}$
  - Activations:  $a^{[l]} \rightarrow n_H^{[l]} \times n_W^{[l]} \times n_C^{[l]}$ 

$$A^{[l]} \rightarrow m \times n_H^{[l]} \times n_W^{[l]} \times n_C^{[l]}$$
  - Weights:  $f^{[l]} \times f^{[l]} \times n_C^{[l-1]} \times n_C^{[l]}$
  - Bias:  $n_C^{[l]}$
- Each layer in a CNN can have different sizes for padding, filters and stride length
  - A lot of the work for CNNs is choosing the hyperparameters for each layer in the network
- Final output from the CNN can be unrolled and fed to a logistic regression unit to make a prediction
- In a CNN, will have convolution layers, pooling layers and fully connected layers
- Pooling layers reduce the size of the representation and can make detected features more robust
  - Max pooling splits the input into sections and takes the maximum value from each section
  - Max pooling will have a filter size and stride length
  - Max pooling will “preserve” any standout features
- For a 3D input to max pooling, output will have the same 3rd dimension
  - Computation will be applied to each channel separately

- Average pooling takes the average from each filter
  - Not as commonly used as max pooling
- Padding size of 0 usually used for pooling layers
- A fully connected layer is the same as a layer in a standard neural network
  - FC layer will have  $W$  and  $b$  parameters
  - Will reduce the dimension of the output of the NN
- Further in the CNN, the height and width of the input will gradually decrease
  - As  $n_W$  and  $n_H$  decrease, the depth of the input will typically increase
- Typical CNN will have one or more conv layers followed by a pool layer
  - CNN will usually finish with some fully connected layers then a softmax layer
- Conv layers help the network with sparsity of connections
  - Using a  $32 \times 32 \times 3$  input image, 6 filters ( $f = 5$ ) will give around 14m parameters
  - Conv layer will have 456 parameters for same calculation
  - In every layer, each output value is depends on only a small number of inputs
- Conv layers use parameter sharing
  - A feature detector (filter) that is useful in one part of an image will likely be useful in another part of the image
- Conv layers and FC layers all have associated parameters
  - Cost function can be defined over the parameters
  - Gradient descent or other optimization algorithm can be used to train the network and reduce  $J$