Assignment 1: Sets

Declan Murphy Zink

9/10/2020

1 Let A be the set $\{\alpha, \{1, \alpha\}, \{3\}, \{\{1, 3\}\}, 3\}$. Which of the following statements are true and which are false?

1.a $\alpha \in A$.

true

1.b $\{\alpha\} \notin A$.

true

1.c $\{1, \alpha\} \subseteq A$.

false

1.d $\{3, \{3\}\} \subseteq A$.

true

1.e $\{1, 3\} \in A$.

false

1.f $\{\{1, 3\}\} \subseteq A$.

false

1.g $\{\{1, \alpha\}\} \subseteq A$.

true

1.h $\{1, \alpha\} \notin A$.

false

1.i $\emptyset \subseteq A$.

true

2 Let B,C,D,E be the following sets:

$$B = \{x | x \text{ a real number}, x^2 < 4\},$$
$$C = \{x | x \text{ a real number}, 0 <= x < 2\},$$
$$D = \{x | x \in \mathbb{Z}, x^2 < 1\},$$
$$E = \{1\}.$$

2.a Which pair of these sets has the property that neither is contained in the other?

Sets D and E

2.b You are given that X is one of the sets B,C,D,E, but you do not know which one. You are also given that $E \subseteq X$ and $X \subseteq B$. What can you deduce about X?

X is not the set D

3 Which of the following arguments are valid? For the valid ones, write down the argument symbolically.

3.a I eat chocolate if I am depressed. I am not depressed. Therefore I am not eating chocolate.

Not valid

3.b I eat chocolate only if I am depressed. I am not depressed. Therefore I am not eating chocolate.

Valid. If C is eating chocolate and D is being depressed, then $C \Rightarrow D$, so $\bar{D} \Rightarrow \bar{C}$.

3.c If a movie is not worth seeing, then it was not made in England. A movie is worth seeing only if critic Ivor Smallbrain reviews it. The movie *Cat on a Hot Tin Proof* was not reviewed by Ivor Smallbrain. Therefore *Cat on a Hot Tin Proof* was not made in England.

Valid. If W is worth seeing, E is made in England, and I is reviewed by Ivor Smallbrain, then $\bar{W} \Rightarrow \bar{E}$ and $W \Rightarrow I$. Therefore $\bar{I} \Rightarrow \bar{W}$ and then $\bar{I} \Rightarrow \bar{E}$.