

Assignment 1: Sets

Declan Murphy Zink

9/10/2020

1 Let A be the set $\{\alpha, \{1, \alpha\}, \{3\}, \{\{1, 3\}\}, 3\}$. Which of the following statements are true and which are false?

1.a $\alpha \in A$.

true

1.b $\{\alpha\} \notin A$.

true

1.c $\{1, \alpha\} \subseteq A$.

false

1.d $\{3, \{3\}\} \subseteq A$.

true

1.e $\{1, 3\} \in A$.

false

1.f $\{\{1, 3\}\} \subseteq A$.

false

1.g $\{\{1, \alpha\}\} \subseteq A$.

true

1.h $\{1, \alpha\} \notin A$.

false

1.i $\emptyset \subseteq A$.

true

2 Let B,C,D,E be the following sets:

$$\begin{aligned} B &= \{x|x \text{ a real number, } x^2 < 4\}, \\ C &= \{x|x \text{ a real number, } 0 \leq x < 2\}, \\ D &= \{x|x \in \mathbb{Z}, x^2 < 1\}, \\ E &= \{1\}. \end{aligned}$$

2.a Which pair of these sets has the property that neither is contained in the other?

Sets D and E

2.b You are given that X is one of the sets B,C,D,E, but you do not know which one. You are also given that $E \subseteq X$ and $X \subseteq B$. What can you deduce about X?

X is either B or E

3 Which of the following arguments are valid? For the valid ones, write down the argument symbolically.

3.a I eat chocolate if I am depressed. I am not depressed. Therefore I am not eating chocolate.

Not valid

3.b I eat chocolate only if I am depressed. I am not depressed. Therefore I am not eating chocolate.

Valid. If C is eating chocolate and D is being depressed, then $P \Leftrightarrow D$, so $\bar{P} \Leftrightarrow \bar{D}$.

3.c If a movie is not worth seeing, then it was not made in England. A movie is worth seeing only if critic Ivor Smallbrain reviews it. The movie *Cat on a Hot Tin Proof* was not reviewed by Ivor Smallbrain. Therefore *Cat on a Hot Tin Proof* was not made in England.

Valid. If W is worth seeing, E is made in England, and I is reviewed by Ivor Smallbrain, then $\bar{W} \Rightarrow \bar{E}$ and $W \Leftrightarrow I$. Therefore $\bar{I} \Leftrightarrow \bar{W}$ and then $\bar{I} \Rightarrow \bar{E}$.