# Group Theory 3

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**1   If aH and bH are distinct left cosets of H in G, are Ha and Hb distinct right cosets of H in G? Prove that this is true or give a counterexample.**

This is false. Counterexample:

In $D_6$, let $H = \{e, a\}$. Thus $bH = \{b, ab^2\}$ and $abH = \{ab, b^2\}$, which are distinct left cosets. However $Hb = \{b, ab\}$ and $Hab = \{ab, b\}$, thus $Hb = Hab$, so this statement is false.

**2   If in G $a^5 = e$ and $aba^{-1} = b^2$, find the order of b if $b \neq e$.**

$$aba^{-1} = b^2$$
$$ab = b^2a$$

And...

$$a^5 = e$$
$$a^4 = a^{-1}$$

Then...

$$aba^{-1} = b^2$$
$$ba^{-1} = a^{-1}b^2$$
$$ba^4 = a^4b^2$$
$$ba^4 = aaaabb$$
$$ba^4 = aaab^2ab$$
$$ba^4 = aab^4aab$$
$$\cdots$$
$$ba^4 = b^{32}a^4$$
$$b = b^{32}$$
$$e = b^{31}$$

Since 31 is a prime number, the order of b must be 31.