Permuations 1

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2 Let V, W, and X be the following subsets of S_4:

$$V = \{e, (12)(34), (13)(24), (14)(23)\}$$
$$W = \{e, (12), (34), (12)(34)\}$$
$$X = \{e, (1234), (1432), (13)(24)\}$$

Prove that V, W and X are all subgroups of S_4 and decide which of them are cyclic.

1) $e \in V, W, X$
2) Let $V, W = \{e, a, b, c\}$, then $ab = c, ac = b, bc = a, ba = c, ca = b, cb = a$.
Let $X = \{e, a, b, c\}$, then $ab = e, ba = e, ac = b, ca = b, bc = a, cb = a$.
3) Each element in V and W is its own inverse. Let $X = \{e, a, b, c\}$, then a and b are inverses and c is its own inverse.

X is the cyclic subgroup generated by (1234).

5 Let G be a group with subgroups H and K.

5.a Prove that $H \cap K$ is a subgroup of G.

1) Since $e \in H, K \Rightarrow e \in H \cap K$.
2) If $x \in H, K, H \cap K$ and $y \in H, K, H \cap K$, then $xy \in H, K, H \cap K$.
3) If $x \in H \cap K \Rightarrow x \in H, K \Rightarrow x^{-1} \in H, K \Rightarrow x^{-1} \in H \cap K$.

5.b Show that $H \cup K$ is not a subgroup unless either $H \subseteq K$ or $K \subseteq H$.

If H and K are not subsets of eachother, then $\exists a \in H$ s.t. $a \notin K$ and $\exists b \in K$ s.t. $b \notin H$. Thus $a, b \in H \cup K \Rightarrow ab \in H \cup K$. If this is true, then either $ab \in H$ or $ab \in K$. However if $ab \in H \Rightarrow a^{-1}ab = b \in H$ and if $ab \in K \Rightarrow abb^{-1} = a \in K$, which is a contradiction.

5.c Find an example of a group G with three subgroups H, K, L, none of them equal to G, such that $G = H \cup K \cup L$.

$G = Q_8$. $H = \{1, i, -1, -i\}, K = \{1, j, -1, -j\}, L = \{1, k, -1, -k\}$.