Permuations 1

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3.a List the numbers that occur as the orders of elements of S_4, and calculate how many elements there are in S_4 of each of these orders.

Possible orders with corresponding cycle shapes and number of elements:
1: (1^4) — 1
2: $(2^2), (2, 1^2)$ — 9
3: $(3, 1)$ — 8
4: (4) — 6

3.b List all the possible cycle-shapes of even permuations in S_6.

$(1^6), (2^2, 1^2), (3, 1^3), (3^2), (5, 1), (4, 2)$

3.c Calculate the largest possible order of any permutation in S_{10}.

Cycle shape (2,3,5) = order 30.

3.d Calculate the largest possible order of any even permutation in S_{10}.

Cycle shape (7,3) = order 21.

3.e Find the value of n such that S_n has an element of order greater than n^2.

Cycle shape to get largest order is increasing primes, so cycle shape (2,3,5,7,11) does it, making $n = 28$, since $2 * 3 * 5 * 7 * 11 = 2310 > 28^2 = 784$.

5 Prove that exactly half of the $n!$ permutations in S_n are even.

Proved by creating a function f that swaps even permutations to odd permutations and vice versa, then prove that f is bijective, thus meaning that there are an equal amount of even and odd permutations for some S_n.

Proof:
$f(p) = p(1, 2)$. Thus if p is even, $f(p)$ is odd, and if p is odd, $f(p)$ is even. $f(p)$ is injective because $f(p_1) = f(p_2) \Rightarrow p_1(1, 2) = p_2(1, 2) \Rightarrow p_1(1, 2)(1, 2) = p_2(1, 2)(1, 2) \Rightarrow p_1 = p_2$. $f(p)$

is surjective because $\forall p \in S_n, f(p) = p(1,2)$ and $f(p(1,2)) = p(1,2)(1,2) = p$, thus for all permutations y in the codomain of f, there exists a permutation x in the domain of f such that $f(x) = y$. Therefore, f is bijective, meaning that for every even permutation there is exactly one odd permutation.