A QUICK GUIDE FOR INTEGRALS.

IN OUR COURSE.

I : INDEFINITE INTEGRALS

An INDEFINITE integral is written:

$$\int f(x)\,dx = g(x). \qquad \ldots\ldots(1)$$

This simply means

$$\frac{dg}{dx} = f(x) \quad . \qquad \ldots\ldots(1a)$$

(i.e. the derivative of the (indef.) integral equals $f(x)$, the "integrand".

- For this reason, indefinite integrals are often called "ANTI-DERIVATIVES".

Examples :

(a) $\displaystyle\int x\,dx = \frac{x^2}{2} + \underset{\text{constant}}{\underset{\uparrow}{A}}$.

Check : $\dfrac{d}{dx}\left(\dfrac{x^2}{2} + A\right) = \dfrac{1}{2}\cdot 2x + 0 = x$ ✓

Using (1a)

(b) $\displaystyle\int (x + x^4)\,dx = \frac{x^2}{2} + \frac{x^5}{5} + \underset{\text{cons}}{\underset{\uparrow}{B}}$.

(c) $\displaystyle\int (\sin x)\,dx = -\cos x + \underset{\text{const}}{\underset{\uparrow}{C}}$.

(d) $\displaystyle\int (\cos x)\,dx = \sin x + \underset{\text{const}}{\underset{\uparrow}{E}}$.

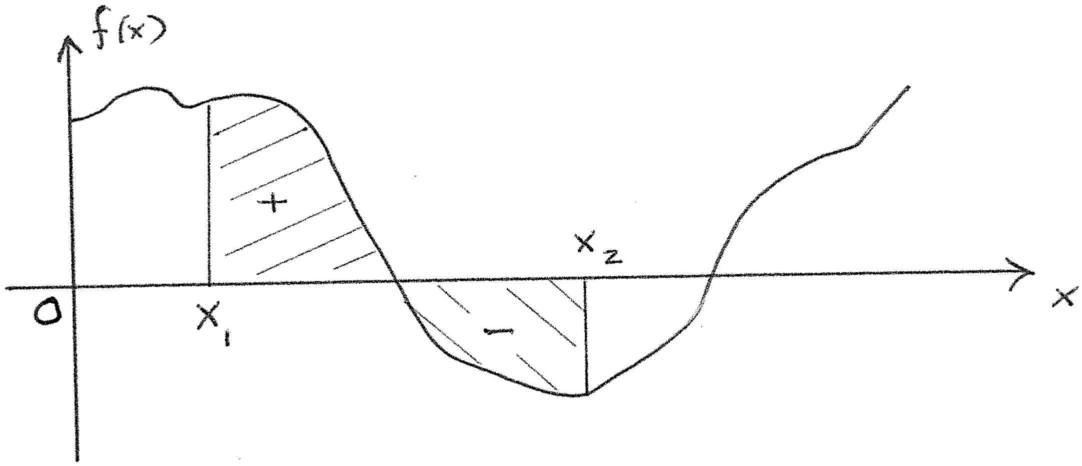
Notes :

- Values of additive constants depend on the problem.
- You can ALWAYS check indefinite integrals using eqn (1a).

II DEFINITE INTEGRALS

Part A : What does a definite integral represent ?

Imagine a curve of $f(x)$ vs x :



Suppose we want to find the AREA below this curve between $x = x_1$ (Lower limit) and $x = x_1$ (Upper limit). As shown, area is + or − depending whether the curve is above or below the X axis.

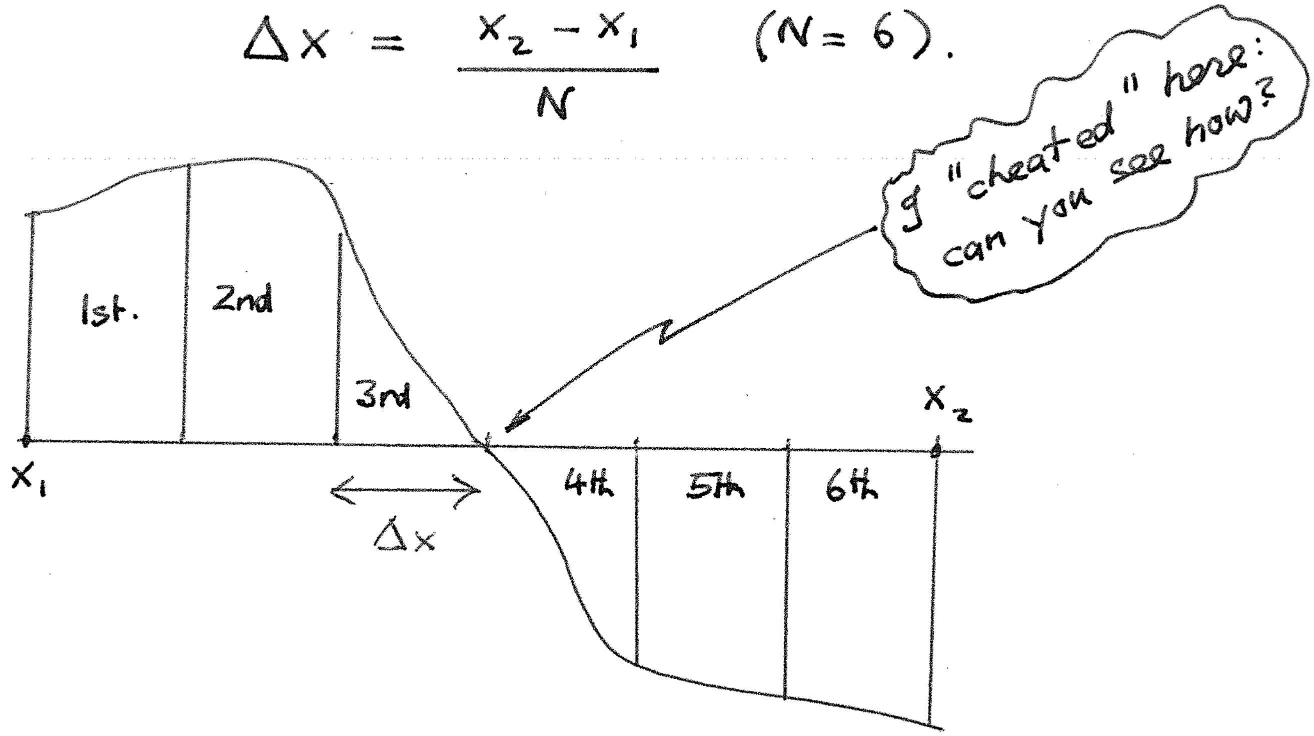
Call the required area I .

We start by APPROXIMATING I.

Below, I divide the interval $x_2 - x_1$ into

$N = 6$ equally wide strips; each width being

$$\Delta x = \frac{x_2 - x_1}{N} \qquad (N = 6).$$



The approximation to I is of the form

$$I_{\substack{(\text{approx,}) \\ \text{for } N=6}} = f(x_1)\Delta x + f(x_1 + \Delta x)\Delta x + \dots + f(x_1 + 5\Delta x)\Delta x.$$

$$= \sum_{i=1}^{6} f(x_i)\Delta x \quad .$$

Now, IT CAN BE SHOWN that I is EXACTLY the "Riemann Sum":

$$I = \lim_{\Delta x \to 0} \sum_{i=1}^{\infty} f(x_i)\Delta x \quad \text{with}$$

$$\Delta x = \frac{x_2 - x_1}{N} \text{ as above, i.e. } N \to \infty \text{ here.}$$

..... (2)

Now, the AMAZING thing is that this sum in (2) is also connected to indefinite integrals:

$$\boxed{\begin{aligned} I &= \int_{x_1}^{x_2} f(x)\,dx \\ &= g(x)\Big|_{x=x_1}^{x_2} \overset{\text{means}}{=} g(x_2) - g(x_1), \\ &\text{with } f(x), g(x) \text{ satisfying (1), (1a) above.} \end{aligned}}$$

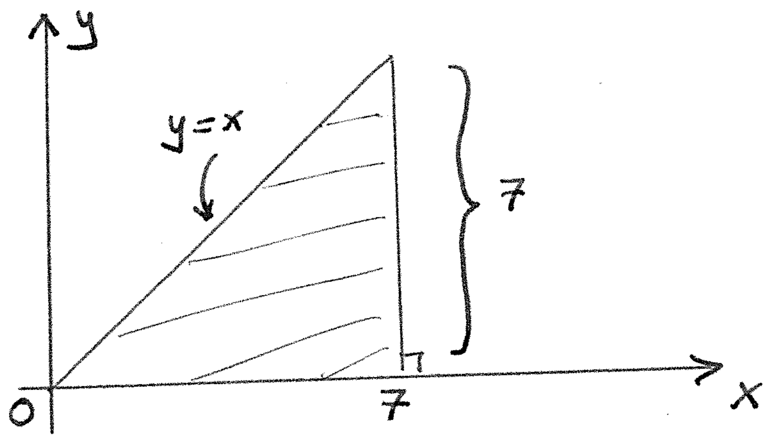
Examples:

(a) $$\int_0^7 x\,dx = \left(\frac{x^2}{2} + A\right)\Bigg|_{x=0}^{7} = \frac{7^2}{2} + A - A = \underline{\underline{\frac{49}{2}}}.$$

Note 2 things:

(i) Any additive constants in $g(x)$ in (1) are cancelled out in definite integrals; we can ignore them!

(ii) $\frac{49}{2}$ is this area:



check:

ie area $= \frac{1}{2}(7)(7) = \frac{49}{2}$ ✓

(b) $$\int_{-1}^{+1} (x + x^4)\,dx = \left(\frac{x^2}{2} + \frac{x^5}{5}\right)\Bigg|_{x=-1}^{+1}$$

$$= \frac{1}{2} + \frac{1}{5} - \left(\frac{1}{2} - \frac{1}{5}\right) = \frac{2}{5}.$$

(c) $$\int_0^{\pi/2} (\sin x)\,dx = -\cos x\Big|_{x=0}^{\pi/2} = -\cos\frac{\pi}{2} + \cos(0)$$

$$= 0 + 1 = 1$$

(d) $$\int_{-\pi/2}^{\pi/2} (\cos x)\, dx = \sin x \Big|_{x=-\pi/2}^{\pi/2}$$

$$= \sin \frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right) = 1 - (-1) = \underline{\underline{2}}.$$