Comparing Math Performance to National Average

April 30, 2018

1 Comparing Math Performance to National Average

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This report will compare a certain high school's performance on the math SAT to the national average of 513.

The data are held in `sat_scores.csv`, telling the gender, verbal SAT score, and math SAT score for the students in this high school who took the SAT. To load it, we will use pandas, a python library built for working with and analyzing data.

```
In [1]: import pandas as pd
        sat_df = pd.read_csv("sat_scores.csv")
        print(sat_df)
```

```
gender  verbal  math
0        f      630   660
1        f      590   580
2        m      750   800
3        m      600   690
4        m      610   550
5        f      490   800
6        f      680   610
7        m      520   540
8        f      680   660
9        m      650   700
10       m      600   560
11       f      550   560
12       m      490   390
13       f      530   530
14       m      560   560
15       f      630   590
16       f      510   520
17       m      710   740
18       f      550   560
19       m      690   620
20       m      700   700
21       m      540   620
22       f      280   500
23       m      710   760
24       f      640   710
25       m      600   590
26       m      610   670
27       m      680   670
28       f      520   470
```

```
29       f     730   740
..     ...     ...   ...
273      f     570   530
274      f     560   540
275      f     670   520
276      m     650   710
277      f     690   700
278      m     610   740
279      f     500   650
280      m     560   700
281      m     640   650
282      m     430   490
283      f     700   570
284      m     620   670
285      f     610   640
286      m     580   640
287      f     730   570
288      f     520   530
289      m     540   580
290      m     640   610
291      m     680   720
292      m     580   490
293      f     640   630
294      f     700   650
295      m     600   630
296      f     540   510
297      f     480   540
298      f     710   700
299      m     650   780
300      f     640   570
301      f     370   410
302      m     710   700

[303 rows x 3 columns]
```

We don't need the gender or verbal SAT score columns, so let's create a new dataframe that only has a math SAT score column.

```
In [2]: math_sat_df = pd.DataFrame(sat_df["math"])
        print(math_sat_df)
```

```
math
0     660
1     580
2     800
3     690
4     550
5     800
6     610
7     540
8     660
9     700
10    560
11    560
12    390
```

```
13     530
14     560
15     590
16     520
17     740
18     560
19     620
20     700
21     620
22     500
23     760
24     710
25     590
26     670
27     670
28     470
29     740
..     ...
273    530
274    540
275    520
276    710
277    700
278    740
279    650
280    700
281    650
282    490
283    570
284    670
285    640
286    640
287    570
288    530
289    580
290    610
291    720
292    490
293    630
294    650
295    630
296    510
297    540
298    700
299    780
300    570
301    410
302    700

[303 rows x 1 columns]
```

Although these data come from a census, and population parameters can be calculated, we will take a random sample and run a test upon that to compare this high school's math performance to the national average. To generate a sample, we will use `pandas.DataFrame.sample`. For the test that will be done later, it is important that size of the sample not exceed 10% of the size of the population. We will take a sample with a size 9% of the population.

```
In [3]: math_sat_sample_df = math_sat_df.sample(frac=0.09)
        print(math_sat_sample_df)
```

```
math
128   630
274   540
188   710
136   470
280   700
78    660
270   640
281   650
160   520
273   530
277   700
272   630
202   540
94    570
162   570
99    550
265   570
230   530
154   570
268   700
51    530
302   700
125   670
43    560
17    740
38    580
256   640
```

Now that we have our sample, we need to calculate it's summary statistics. For this, we will use numpy, a python library designed for scientific computing.

```
In [4]: import numpy as np

        n                       = len(math_sat_sample_df)
        degrees_of_freedom      = n-1
        mean                    = float(np.mean(math_sat_sample_df))
        standard_deviation      = float(np.std(math_sat_sample_df))
        mean_standard_deviation = float(standard_deviation / np.sqrt(n))

        print("n                       =", n)
        print("degrees of freedom      =", degrees_of_freedom)
        print("mean                    =", mean)
        print("standard deviation      =", standard_deviation)
        print("mean standard deviation =", mean_standard_deviation)
```

```
n                       = 27
degrees of freedom      = 26
mean                    = 607.4074074074074
```

```
standard deviation      = 71.88429966079455
mean standard deviation = 13.834139919889154
```

With these statistics calculated, we can run a **one-sample two-tail t-test** to compare this high school's math performance to the national average. Before we do, though, we have to check certain conditions.

1.0.1 1. Random sample of math SAT scores from this high school

We used `pandas.DataFrame.sample`, which generates a pseudo-random sample. It may not be perfect, but it's random enough.

1.0.2 2. n is less than 10% of the total population of math SAT scores from this high school

We kept this in mind when generating the sample, using only 9% of the total population of math SAT scores from this high school. This condition checks out.

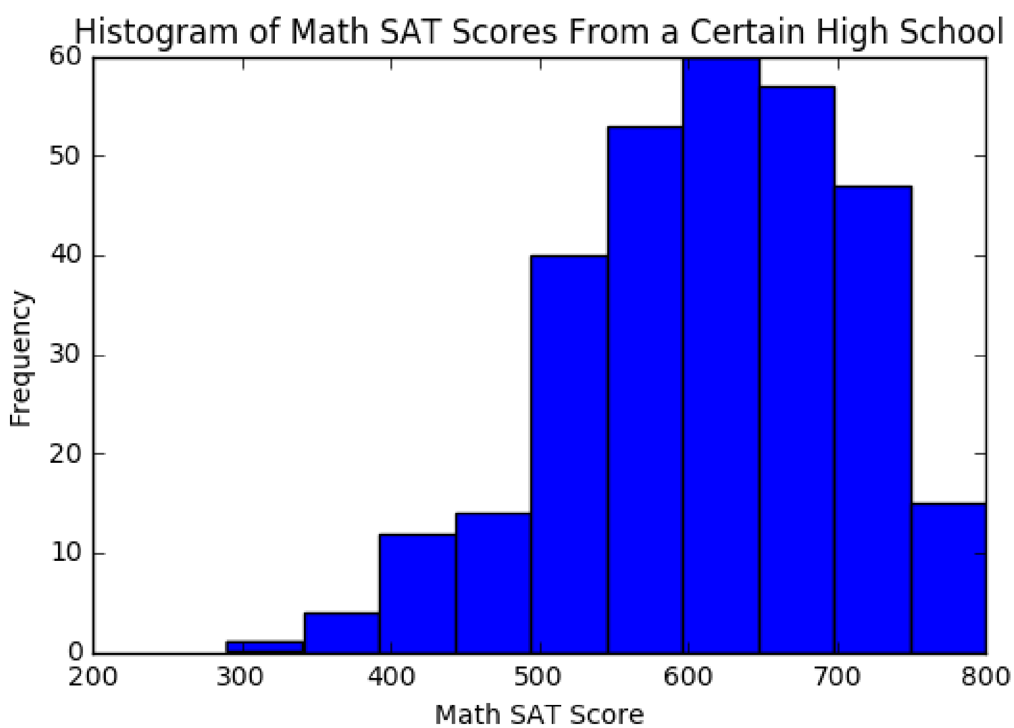
1.0.3 3. Sample comes from a distribution that is unimodal and symmetric

Let's generate a histogram of all the data. We'll use matplotlib, a python library for plotting data.

```
In [5]: %matplotlib inline
        from matplotlib import pyplot as plt

        plt.xlabel('Math SAT Score')
        plt.ylabel('Frequency')
        plt.title('Histogram of Math SAT Scores From a Certain High School')
        plt.hist(sat_df["math"])
```

```
Out[5]: (array([  1.,   4.,  12.,  14.,  40.,  53.,  60.,  57.,  47.,  15.]),
         array([ 290.,  341.,  392.,  443.,  494.,  545.,  596.,  647.,  698.,
                 749.,  800.]),
         <a list of 10 Patch objects>)
```



It could be said that the data are unimodal skewed left, but they will be "normal enough" to work with.

With these conditions met, we may run the **one-sample two-tail t-test**. Let's define the null and alternative hypotheses.

$$H_0 : \mu = 513$$

$$H_A : \mu \neq 513$$

The null hypothesis (H_0) states that the true mean of math SAT scores is equal to 513, the national average.

The alternative hypothesis (H_A) states that the true mean of math SAT scores is not equal to 513, the national average.

We can determine whether the null hypothesis should be accepted or not by calculating the t-statistic and its corresponding p-value.

$$t - statistic = t_{df} = \frac{\bar{X} - \mu_0}{SE(\bar{X})}$$

df represents the degrees of freedom, $\bar{X}$ represents the mean of our sample, μ_0 represents the national average, and $SE(\bar{X})$ represents the mean standard deviation of our sample.

```
In [6]: t_stat = (mean - 513) / mean_standard_deviation
        print("t-statistic =", t_stat)
```

```
t-statistic = 6.824233957015221
```

To find the p-value of a two-tailed t-test, we take the probability that any t-statistic from a student's t-distribution with the same degrees of freedom would be greater than or equal to this one, and then multiply it by two.

$$p - value = 2P(t_{26} \geq t - statistic)$$

scipy has a convenient function for calculating such probabilities: `scipy.stats.t.sf`

```
In [7]: import scipy.stats

        p_val = 2 * scipy.stats.t.sf(t_stat, degrees_of_freedom)
        alpha = 0.05

        if p_val < alpha:
            print("With a p-value of %s%%, this test rejects the null hypothesis in favor of the alterna
        else:
            print("With a p-value of %s%%, this test fails to reject the null hypothesis" % (round(p_va
```

```
With a p-value of 0.0%, this test rejects the null hypothesis in favor of the alternative
```

The test has rejected that the true mean of math SAT scores is equal to 513, the national average. Now we should create a confidence interval to estimate what the true mean really is. We will use a 90% confidence interval. It is calculated with the following expression.

$$\bar{X} \pm t^*_{df} * SE(\bar{X})$$

We can use `scipy.stats.t.isf` to determine t^*_{26} based on our 90% confidence interval.

```
In [8]: confidence = 0.9
        upper_probability = (1 - confidence) / 2  # Parameter for isf
        t_star = scipy.stats.t.isf(upper_probability, df=26)

        print(t_star)
```

```
1.70561791976
```

We know the other values in the expression, so it is easy to calculate.

```
In [9]: interval = [mean - (t_star * mean_standard_deviation), mean + (t_star * mean_standard_deviation)
        print(interval)
```

```
[583.81165045558737, 631.00316435922741]
```

It can be said with 90% confidence that the true mean value of math SAT scores for this particular high school lies on the above interval.