Coordination in Social Networks

Chun-ting Chen

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Motivation

- Consider a rigid regime. An powerful discontent against the regime may exist, but due to the communication barrier, it is hard to be put together.
- Communication barrier
    1. Taking actions is risky: being exiled, being eavesdropped, being threatened by suppression.
    2. Information is private: No (fair) voting system, No (fair) mass media, No (uncensored) discussion forum.
- *How did Rebels made decisive collective action in this regime?*

Motivation

History tells us:

- Although taking action is risky, consecutive contributions may trigger later decisive action.
  - Dr. Sun Yat-Sen, founding father of Republic of China, initiated failed uprising ten times before 1911 Revolution.
  - Monday Demonstrations are consecutively held in Leipzig, Germany in 1989/9-1989/12 .
  - Prof. Benny Tai, a leader of Occupy Central, has said "It (Umbrella Protest) is beyond what I imagined", while Occupy Central trigger the Umbrella Protest in Hong Kong, China 2014.
- Although information is private, social networks serve as routes for communication.
  - Ex., Gangster networks (1911 Revolution); Church networks (1989 Berlin Uprising, 2014 Umbrella Protest); Friend networks, etc.

Motivation

Question

- *If rational rebels know that "tiny" contributions can trigger later events, how did they conduct a decisive collective action in the social networks under communication barrier?*

Objective

- What kinds of networks can conduct a decisive collective action?
- Use a game-theoretic model to capture
  1. Incomplete information about social discontent.
  2. Information cascades in social networks.
  3. Information generation is not free but risky.
  4. No "signals" other than actions can be generated.

Motivation

Modeling

- An incomplete information k-threshold game repeatedly played in networks
  - Time line
    - Players locate in a fixed network.
    - Players' types, *Rebel* or *Inert*, chosen by nature before a game is played.
    - Players play a game infinitely repeatedly afterwards.
  - Assumption
    - Players can perfectly only observe their neighbors' types and actions.
    - Common prior π. Common discount factor δ. Payoff is hidden.
- k-threshold game
  - $A_{Rebel} = \{\textbf{revolt}, \textbf{stay}\}$. $A_{Inert} = \{\textbf{inert}\}$.
  - Payoff for Rebels:
    1. play **revolt** and more than k **revolts**, get 1.
    2. play **revolt** and less than k **revolts**, get -1.
    3. play **stay**, get 0.
  - Payoff for Inert:
    1. play **inert**, get 1.
- Remark: **revolt** is a risky arm; **stay** is a safe arm

Motivation

Looking for the networks in which

- An equilibrium, where the ex-post efficient outcome in static game played repeatedly after a finite time T in the path, exists when δ is high enough.
  - I.e., If there are more than k Rebels, all Rebels play **revolt** afterwards; otherwise, all Rebels play **stay** afterwards.
  - I.e., After T, Rebels learn if there are more than k Rebels.
  - I.e., After T, Rebels share with a collective action; Before T, Rebels contribute their private information about "how many Rebels they know" .

Motivation

Example, $k = n = 3$ and let the network be

If $\pi(Rebel, Rebel, Rebel) > 0$, we can construct such equilibrium

- After nature moves, Rebel 2 chooses **revolt** if he observes $\theta = (Rebel, Rebel, Rebel)$, and plays **revolt** in this period. Otherwise, he chooses **stay** and keeps playing **stay** afterwards.
- After nature moves, Rebel 1 and Rebel 3 play **stay**.
- If Rebel 2 chooses **revolt** in the last period, then Rebel 1 (or Rebel 3) plays **revolt** in this period; if Rebel 2 chooses **stay** in the last period, then Rebel 1 (or Rebel 3) keeps playing **stay** afterwards.

Related Literature

- Public good provision.
  - One strand: costly information generation. [Lohmann, 1993,1994], [Bolton and Harris, 1999], [Bramoulléfh and Kranton, 2007]
  - Here, add network-monitoring
- Social learning.
  - One strand: learning in network. [Goyal, 2012], [Acemoglu et al., 2011], [Chatterjee and Dutta, 2011].
  - Here, farsighted-learning in the game
- Repeated game.
  - One strand: repeated game in network. [Laclau, 2012], [Wolitzky, 2013], [Wolitzky, 2014]
  - Here, incomplete information imperfect monitoring
  - One strand: folk theorem with incomplete information imperfect monitoring. [Fudenberg and Yamamoto, 2010] [Fudenberg and Yamamoto, 2011] [Wiseman, 2012]
  - Here, not a folk theorem. No assumptions on public or private signal generated by single-period actions.

Model

Network

- n players. Let $N = \{1, \ldots, n\}$ be the set of players.
- G_i is a subset of N, where $i \in G_i$
- G_i is i's neighborhood. $\bar{G}_i = G_i \backslash \{i\}$ is i's neighborhood excluding i.
- $G = \{G_i\}_i$ is the network.
- G is fixed if G is not random; finite if N is finite; undirected if $j \in G_i \Rightarrow i \in G_j$.

Definition

1. A *path* from i to j, $i \neq j$ in an undirected network G is a finite sequence $l_1, \ldots, l_q$ such that $l_1 = i, l_2 \in \bar{G}_{l_1}, l_3 \in \bar{G}_{l_2}, \ldots, l_q = j$ and $l_1, \ldots, l_q$ are all distinct.
2. *connectedness*: An undirected network is connected if and only if for all i, j, $i \neq j$ there is a path from i to j.

Model

Static k-threshold game [Chwe 2000]

- Each player i's type $\theta_i \in \Theta_i = \{Rebel, Inert\}$
- Denote $\Theta = \times_{i \in N} \Theta_i$
- Prior π over Θ.
- $A_{Rebel_i} = \{\textbf{revolt}, \textbf{stay}\}$; $A_{Inert_i} = \{\textbf{inert}\}$
- A parameter k with $1 \leq k \leq n$
- Static game payoff for player i: $u_{\theta_i}(a_{\theta_i}, a_{-\theta_i})$

$$
\begin{array}{lcll}
u_{Rebel_i}(a_{Rebel_i}, a_{-\theta_i}) & = & 1 & \text{if } a_{Rebel_i} = \textbf{revolt} \text{ and } \#\{j : a_{\theta_j} = \textbf{revolt}\} \geq k \\
u_{Rebel_i}(a_{Rebel_i}, a_{-\theta_i}) & = & -1 & \text{if } a_{Rebel_i} = \textbf{revolt} \text{ and } \#\{j : a_{\theta_j} = \textbf{revolt}\} < k \\
u_{Rebel_i}(a_{Rebel_i}, a_{-\theta_i}) & = & 0 & \text{if } a_{Rebel_i} = \textbf{stay} \\
u_{Inert_i}(a_{Inert_i}, a_{-\theta_i}) & = & 1 & \text{if } a_{Inert_i} = \textbf{inert}
\end{array}
$$

- Players only know their neighbor's types.

Model

- Denote $[Rebels](\theta) = \{j : \theta_j = Rebel\}$ for $\theta \in \Theta$.
- Denote $\#A$ as the cardinality of a finite set A.

Model

Repeated k-threshold game

- Common δ. Time is infinite, discrete.
- Nature choose θ at 0 period; players play the static k-threshold game infinitely repeatedly.
- Players perfectly only observe his neighbors' actions.
- $h^m_{G_i}$: the history i can observe up to period m
- $\beta_i(\theta|h^m_{G_i})$: i's belief for θ at period m.
- Payoff is hidden.
- Equilibrium concept: (weak) sequential Equilibrium.

Model

Assumption

G is commonly known

Definition

π has full support if and only if $\pi(\theta) > 0$ for all $\theta \in \Theta$.

Definition

A sequential equilibrium is *approaching efficient* (APEX) if and only if for all θ there is a finite time T^θ such that the tails of actions after T^θ in equilibrium path repeats the ex-post efficient outcome.

Equilibrium: $k = n$

Theorem

In *any fixed, finite, connected, commonly known, undirected (FFCCU) network,* ***if*** *the prior has full support ,* ***then*** *for n-person repeated k-Threshold game with parameter $k = n$ played in such networks,* ***there is*** *a δ such that a sequential equilibrium which is APEX* ***exists****.*

Proof:

- If there is an Inert neighbor, then play **stay** forever.
- If there is no Inert neighbor, then play **revolt** until he observe some neighbors play **stay**, and then play **stay** forever.
- If he deviates, then play **revolt** forever.
- Since networks are FFCCU, there is a finite time T^{θ} such that ex-post efficient outcome repeats afterwards.

Equilibrium: $k = n$

$k = n$ is a trivial case.

- Rebels will never play **revolt** if one of his neighbor is Inert.
- $\{$**revolt**, **stay**$\}$ reveals "no Inert", "some Inert".
- When someone deviates, group punishment is not needed.

Equilibrium: $k < n$

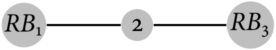
$k < n$ is not a trivial case

- Rebels **may** play **revolt** if one of his neighbor is Inert.
- $\{$**revolt**, **stay**$\}$ has to reveal more information.
- When someone deviates, group punishment may be needed.

Equilibrium: $k < n$

Rebels **may** play **revolt** even if one of his neighbor is Inert.

- Let $k = 2$
- Assume $\theta = (Rebel_1, Inert_2, Rebel_3)$.
- Let G be



1. Inert 2 block the information transmission.
2. This is an incomplete information game without communication
3. Rebel 1 still has incentive to play **revolt**.
   - $\pi(\{\theta : \theta_3 = Rebel\})$ is high
   - Rebel 3 will play revolt.
4. Generally, achieving APEX is impossible.

Equilibrium: $k < n$

Given G,

Definition

Strong connectedness $\Leftrightarrow$ for every pair of Rebels, there is a path consisting of Rebels to connect them.

Definition

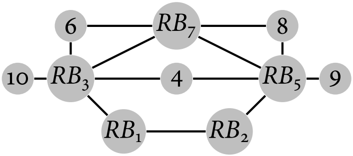
Full support on strong connectedness$\Leftrightarrow$

1. $1 > \pi(\theta) > 0$ whenever θ has strong connectedness.
2. $\pi(\theta) = 0$ whenever θ did not satisfy strong connectedness.

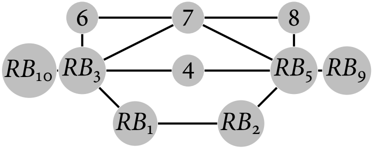
Equilibrium: $k < n$

{**revolt**, **stay**} has to reveal more information. Suppose Rebel 3, 5 "talk to" Rebel 1, 2.

- Let $k = 5$



- Let $k = 6$

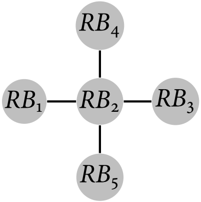


- "Talking about how many nearby Rebels" is not enough.
- "Talking about how many nearby Rebels" and "Talking about nearby Rebels' locations"

Equilibrium: $k < n$

When someone deviates, group punishment may be needed.

- Let $k = 4$



1. Rebel 1 can only be monitored by Rebel 2.
2. Given some strategies, suppose Rebel 2,3,4,5 can coordinate at period T and play **revolt** forever.
3. If Rebel 1 deviate at period $T - 1$, Rebel 2 has no incentive to punish him.
   - Different from the case $k = n = 5$

Equilibrium: $k < n$

Theorem

In any *FFCCU network without cycles,* ***if*** *θ has strong connectedness and π has full support on strong connectedness,* ***then*** *for n-person repeated k-Threshold game with parameter $1 \leq k < n$ played in networks,* ***there is*** *a δ such that a weak sequential equilibrium which is APEX* ***exists****.*

Definition

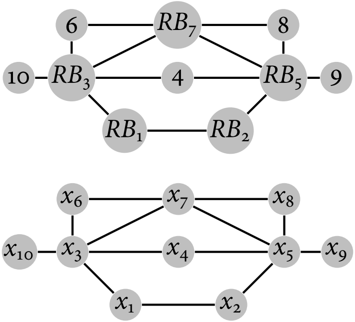
Cycles: A FFCCU network is without cycles $\Leftrightarrow$ the path from i to j, for $i \neq j$, is unique.

Equilibrium construction

{**revolt**, **stay**} has to reveal more information

- Indexing each node i as a distinct prime number x_i.
- Rebel 3 report $x_1 \times x_7 \times x_1$ to Rebel 1 by sending a finite sequence

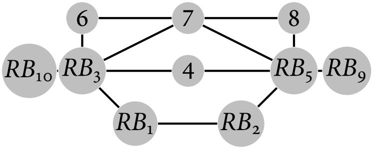
$$\textbf{stay}, \ldots, \textbf{stay}, \underbrace{\textbf{revolt}, \textbf{stay}, \ldots, \textbf{stay}}_{x_1 \times x_7 \times x_3}$$



Equilibrium construction

Using $\{$**revolt**, **stay**$\}$-sequences to transmit information

- Naively, we may let Rebel 3 report → Rebel 5 report → Rebel 1 report →...



- → since network is finite, then some Rebels will know the true state → some more Rebels will know the true state. →...→ all Rebels will know the true state →...→ all Rebels will know all Rebels ... know the true state, etc.
- APEX requires that there is a timing to play **revolt** or **stay** forever.
  1. However, calculating such timing given all possible θ is tedious.
  2. Higher-order belief is apparently a giant object.

Equilibrium construction

Using $\{$**revolt**, **stay**$\}$-sequences to transmit information

1. about θ (**Reporting messages**)
2. about "Have some Rebels known $\#[Rebels](\theta) \geq k$ or $\#[Rebels](\theta) < k$?" (**Coordination messages**)
    - To bypass the tracking of higher-order belief when network is finite.

Specifically,

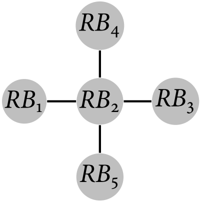
$$\underbrace{\langle\text{coordination period}\rangle}_{0-block}\underbrace{\langle\text{reporting period}\rangle\langle\text{coordination period}\rangle}_{1-block}\dots$$

- If some Rebels know the relevant information, then send coordination messages to other Rebels.
- And then Rebels play **revolt** or **stay** after coordination period.

Equilibrium construction

When someone deviates, group punishment may be needed.

- Recall



- Rebel 1 has less incentive to send information.
- Rebel 2 have strictly more information than all of his neighbors.
- Characterize those Rebels who have strictly more information about θ than any of their neighbors - **Information hierarchy**.

Equilibrium construction

$$\underbrace{\langle\text{coordination period}\rangle}_{0-block}\underbrace{\langle\text{reporting period}\rangle\langle\text{coordination period}\rangle}_{1-block}\ldots$$

1. Step 1: Characterize information hierarchy for each t-block.
2. Step 2: Build reporting and coordination messages in the path, and show the in-path belief updating.
3. Step 3: Set up off-path belief.

Information Hierarchy

At 0-block, let

- $$R^0 = [Rebels](\theta)$$

Information Hierarchy

At 1-block, let

$$\begin{aligned} N_i^0 &\equiv G_i \\ I_i^0 &\equiv G_i \cap R^0 \end{aligned}$$

Define $\leq^0$ by

$$i \in \leq^0 \Leftrightarrow \exists j \in \bar{G}_i (I_i^0 \subseteq N_j^0 \cap R^0)$$

- $\leq^0$ is the set of players whose Rebel neighbors are covered by his neighbors' Rebel neighbors.

Let

$$R^1 \equiv \{ i \in R^0 | i \notin \leq^0 \}$$

- R^1 are those Rebels whose Rebel neighbors can not covered by all of his neighbors' Rebel neighbors.

Information Hierarchy

Ex., Rebel 1 is a non-R^1 node. Rebel 2 is a R^1 node.

Information Hierarchy

In $t+1$-block, denote

$$N_i^t \equiv \bigcup_{k \in I_i^{t-1}} G_k$$
$$I_i^t \equiv \bigcup_{k \in G_i \cap R^t} I_k^{t-1}$$

- N_i^t is i's *extended* neighborhood given i's information I_i^{t-1}
- I_i^t is i's *extended* Rebel neighbors given j's information I_j^{t-1}, where j is a R^t Rebel.

Define $\leq^t$ by

$$i \in \leq^t \Leftrightarrow \exists j \in \bar{G}_i (I_i^t \subseteq N_j^t \cap R^0)$$

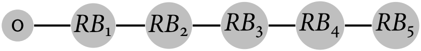
- $\leq^t$ is the set of players whose extended Rebel neighbors are covered by his neighbors' extended Rebel neighbors.

Let

$$R^{t+1} \equiv \{ i \in R^t | i \notin \leq^t \}$$

Information Hierarchy

1. Rebel 1 is a non-R^1 node. Rebel 2 is a R^1 node. Rebel 3 is a R^1 node.
2. Rebel 1 is a non-R^2 node. Rebel 2 is a non-R^2 node. Rebel 3 is a R^2 node.



Theorem

If the network is FFCCU without cycle and if the state has strong connectedness, then

$$R^0 \neq \varnothing \Rightarrow \exists t \geq 0 (\exists i \in R^t (I_i^t = R^0))$$

Equilibrium path

At t-block,

- RP^t: the reporting period
- CD^t: the coordination period
  - There are some sub-periods in divisions in a coordination period.

$$\overbrace{\langle\underbrace{\langle\cdot\rangle\cdots\langle\cdot\rangle}_{\text{sub-blocks}}\rangle\langle\underbrace{\langle\cdot\rangle\cdots\langle\cdot\rangle}_{\text{sub-blocks}}\rangle\langle\underbrace{\langle\cdot\rangle\cdots\langle\cdot\rangle}_{\text{sub-blocks}}\rangle}^{\text{coordination period}}$$

  - $CD^t_{p,q}$: the p sub-blocks in q division in t-block coordination period.
- $\langle RP^t\rangle$: the reporting messages
- $\langle CD^t\rangle$: the coordination messages
- Players use messages, which length is the same as the length of corresponding period.

Equilibrium path

At t-block ($t > 0$), denote

- $\langle I_i^{t-1} \rangle$=**s**, ..., **s**, $\underbrace{\textbf{r}, \textbf{s}, \ldots, \textbf{s}}_{\times_{j \in I_i^{t-1}} x_j}$
- $\langle$**stay**$\rangle$=**s**, ..., **s**

At t-block ($t > 0$), ideally,

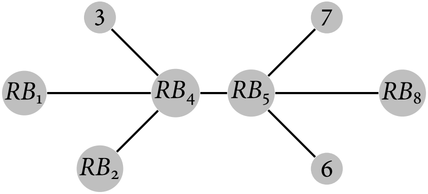
- Let R^t report $\langle I^{t-1} \rangle$ truthfully in reporting period.
- Let R^t send "coordination messages" to coordinate in coordination period.
- Let non-R^t report $\langle$**stay**$\rangle$ truthfully in reporting period.

Not so obvious. Sending message incurs expected cost and therefore players may want to deviate from truthful reporting.

Equilibrium path

A free rider problem. Pivotal player case 1.

- $k = 5$
- Assume players' strategies are starting with a *RP* and then a *CD* follows.
- Assume there is a action-irrelevant message $\langle M \rangle$.
- When Rebel observe $\langle M \rangle$ immediately after *PR*, they play **revolt** forever; Otherwise, play **stay** forever.

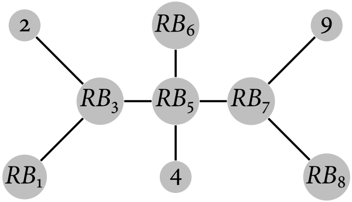


- **Problem**: Both Rebel 4 and Rebel 5 are pivotal; they will shift to play $\langle$**stay**$\rangle$ if others report truthfully.

Equilibrium path

Pivotal Player Case 2

- $k = 6$
- Assume players' strategies are starting with a *RP* and then a *CD* follows.
- Assume there is a action-irrelevant message $\langle M \rangle$.
- When Rebel observe $\langle M \rangle$ immediately after *PR*, they play **revolt** forever; Otherwise, play **stay** forever.

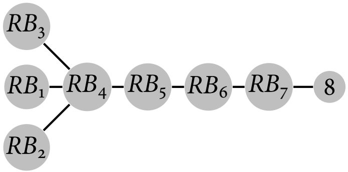


- **Problem**: Rebel 5 is pivotal; he shifts to play $\langle$**stay**$\rangle$. Why?

Equilibrium path

Pivotal Player Case 3

- $k = 6$
- Assume players' strategies are starting with a RP and then a CD follows.
- Assume there is a action-irrelevant message $\langle M \rangle$.
- When Rebel observe $\langle M \rangle$ immediately after PR, they play **revolt** forever; Otherwise, play **stay** forever.



- **Problem**: Rebel 4 is pivotal; he shifts to play $\langle$**stay**$\rangle$. Why?

Equilibrium path

- Thus, some Rebels have incentives to deviate from truthfully reporting $\langle I^{t-1} \rangle$ to $\langle \mathbf{stay} \rangle$.
  - Free rider problems occurs.
  - The "meaning" of $\langle \mathbf{stay} \rangle$ is vague.
- Remedy: Create another message, $\langle 1 \rangle = \mathbf{s}, \ldots, \mathbf{s}, \mathbf{r}$, as the message used by pivotal player.
- Remedy: If there is a free rider problem, choose some players as free rider and choose some to contribute.
- Remedy: Let Rebels' continuation behavior be not only contingent on coordination message M but also on reporting message.
- Good news: The pivotal problems can be identified as the above three cases.
- Good news: The free rider problem can be identified as the above case.
  - Two nearby Rebels.

Equilibrium path

- Good news: With suitable coordination messages and continuation behavior
  1. Pivotal players will not deviate from playing $\langle 1 \rangle$.
  2. Only pivotal players will play $\langle 1 \rangle$
- Good news: the belief updating after CD^t, $t > 0$ in the equilibrium path will be

Table : Belief updating after $CD^t_{1,2}$, $t > 0$

In RP^t	In $CD^t_{1,1}$	In $CD^t_{1,2}$	
i plays	i plays	i plays	The events j believe with probability one
$\langle \mathbf{stay} \rangle$	$\langle \mathbf{1}_i \rangle$	$\langle \mathbf{stay} \rangle$	$i \notin R^t$
$\langle I_i^{t-1} \rangle$	$\langle \mathbf{stay} \rangle$	$\langle \mathbf{stay} \rangle$	$\#[Rebels](\theta) < k$
$\langle I_i^{t-1} \rangle$	$\langle \mathbf{1}_i \rangle$	$\langle \mathbf{stay} \rangle$	$\#[Rebels](\theta) \geq k$
$\langle I_i^{t-1} \rangle$	$\langle \mathbf{1}_i \rangle$	$\langle \mathbf{1}_i \rangle$	$i \in R^t$
$\langle 1 \rangle$	$\langle \mathbf{stay} \rangle$	$\langle \mathbf{stay} \rangle$	$\#[Rebels](\theta) < k$
$\langle 1 \rangle$	$\langle \mathbf{1}_i \rangle$	$\langle \mathbf{stay} \rangle$	$\#[Rebels](\theta) \geq k$

, where, $\langle \mathbf{1}_i \rangle = \mathbf{s}, \ldots, \mathbf{s}, \underbrace{\mathbf{r}, \mathbf{s}, \ldots, \mathbf{s}}_{x_i}$

Off-path Belief

Whenever Rebel i detects a deviation at s period, he forms the belief

$$\sum_{\theta \in \{\theta : \theta_j = Inert, j \notin G_i\}} \beta^{\pi,\tau}_{G_i}(\theta | h^{s'}_{G_i}) = 1 \tag{1}$$

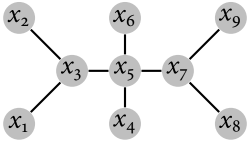
for all $s' \geq s$.

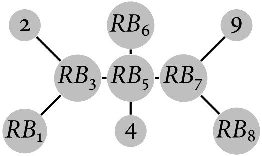
1. If $\#I^{o}_{i} < k$, he will play **stay** forever.
2. This off-path belief then serve as a grim trigger.

Off-path Belief

Without ⟨1⟩, using this grim-trigger-like belief may not sustain APEX

- $k = 5$





- **Problem**: Without ⟨1⟩ being considered as an in-path strategies;
- Rebel 4 is pivotal; He shifts to report $x_3 \times x_5 \times x_7$ instead of $x_3 \times x_5 \times x_7 \times x_6$.
- Coordination can be made, but Rebel 6 is out of coordination since he detects a deviation.

Equilibrium: $k < n$: Discussion

- From the above steps, an APEX equilibrium is constructed.
- We can relaxed the assumption that payoff is hidden.
  - payoff is perfectly observed: easy to construct an APEX equilibrium.
  - payoff is noisy: with full support assumption, the existing equilibrium is APEX
- This proof is still open for FFCCU network with cycles.
- Off-path belief did not satisfy full consistency property for FFCCU network without cycles.
  - Belief updating is had to track
  - Imperfect monitoring impedes the group punishments.
- Prime number indexing also works for other discreet and finite state space.

Conclusion

1. I show that, without cheap talk, in this repeated k-threshold game played in FFCCU networks without cycles, coordination still can happen.
    - Using sequence of actions to communicate.
2. The equilibrium is constructive and does not rely on public or private signals other than actions.
3. We can use prime number to index the states given that states are discrete and finite.
4. For the network with circle, it is still remaining to tackle with.