Coordination in Social Networks

Communication by Actions

Chun-Ting Chen

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INTRODUCTION

Motivation

- How people act collectively under uncertainty?
    - Ex.: protest, joint investment, etc.

This paper shows

- In a long-term relationship, people can aggregate such information and coordinate their actions.

What this paper does?

- I model a repeated game with incomplete information and network-monitoring with discount factor.
  - Players can only observe own/neighbors' types and own/neighbors' actions.
- Look for an equilibrium in which the pay-off relevant information become commonly known in finite time.
- Such equilibrium can be constructed under some assumptions.

Time line

1. There is a fixed, finite, connected, undirected, and commonly known network.
2. Players of two types— S or B —chosen by nature according to a probability distribution.
   - S: Strategic type; B: Behavior type
3. Types are then fixed over time.
4. Players play a stage game— a collective action —infinitely repeatedly with common discount factor.

Model

What player can/cannot observe

- Players can observe own/neighbors' types and actions, but not others'.
- Pay-off is hidden.
    - [Aumann and Maschiler 1990], [Miyahara and Sekiguchi 2013], [Wolitzky 2013], etc.

Model

- Stage game—k-threshold game: a protest ([Chwe 2000])

MODEL

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- Stage game—k-threshold game: a protest ([Chwe 2000])
  - There are n players, and $k \leq n$
  - S-type's action set$= \{\mathbf{p}, \mathbf{n}\}$
  - B-type's action set$= \{\mathbf{n}\}$
  - Pay-offs for S-type:

$$
\begin{aligned}
u_{S_i}(a_{S_i}, a_{-\theta_i}) &= 1 && \text{if } a_{S_i} = \mathbf{p} \text{ and } \#\{j : a_{\theta_j} = \mathbf{p}\} \geq k \\
u_{S_i}(a_{S_i}, a_{-\theta_i}) &= \text{-1} && \text{if } a_{S_i} = \mathbf{p} \text{ and } \#\{j : a_{\theta_j} = \mathbf{p}\} < k \\
\\
u_{S_i}(a_{S_i}, a_{-\theta_i}) &= 0 && \text{if } a_{S_i} = \mathbf{n}
\end{aligned}
$$

Static ex-post Pareto efficient outcome

Type profile	Static ex-post efficient outcome
At least k S-types exist	All S-types play **p**
Otherwise	All S-types play **n**

EQUILIBRIUM CONCEPT

- WPBE (weak perfect Bayesian equilibrium)
- Sequential equilibrium

APEX (*approaching ex-post efficient*) equilibrium

Definition (APEX strategy)

An equilibrium is APEX $\Leftrightarrow$

$$\forall\theta, \text{ there is a finite time } T^{\theta}$$

such that the actions in the equilibrium path repeats the static ex-post efficient outcome after T^{θ}.

Result 1: APEX for $k = n$

Theorem ($k = n$)

If $k = n$, then an APEX sequential equilibrium exists whenever discount factor is sufficiently high.

Definition for APEX for $k < n$

Definition

θ has **strong connectedness** $\Leftrightarrow$ for every pair of S-types, there is a path consisting of S-types to connect them.

Definition

π has **full support on strong connectedness** $\Leftrightarrow$

$\pi(\theta) > 0$ **if and only if** θ has strong connectedness.

Without strong connectedness

Let k=2 and n=3

- A B-type will not reveal information.
- **Without** full support on strong connectedness, in general, an APEX equilibrium does not exist when pay-off is hidden.

Result 2: APEX for $k < n$

Theorem ($k < n$)

If $k < n$, then if network is a tree, if prior π has full support on strong connectedness, then an APEX WPBE exists whenever discount factor is sufficiently high.

Outline for Equilibrium Construction

1. APEX sequential equilibrium for $k = n$.
   - Sketch of proof.
2. APEX WPBE for $k < n$.
   1. Consider cheap talk.
   2. Consider "costly" talk.
   3. Sketch of proof.

EQUILIBRIUM CONSTRUCTION FOR $k = n$

Sketch of proof:

1. "messages" to reveal the relevant information.
   - Some B-types neighbors $\Rightarrow$ play **n** forever.
   - No B-type neighbor $\Rightarrow$ play **p** unless **n** is observed, and then play **n** forever.

Equilibrium construction for $k = n$

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   - Finite network $\Rightarrow$ there is a finite time $T(= n)$ such that players coordinate to the static ex-post efficient outcome.

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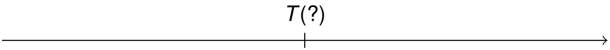
4. Let discount factor be sufficiently high to impede deviation.
5. A belief system for sequential equilibrium can be chosen.

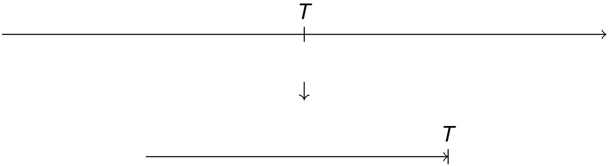
Equilibrium construction for $k < n$



- Challenges:
  - Only two actions—$\{\mathbf{n}, \mathbf{p}\}$— used for transmit relevant information.
  - How to find that finite time "T" for every state?
  - Group punishment is hard to be made. (Network-monitoring)

EQUILIBRIUM CONSTRUCTION FOR $k < n$

For simplicity, assume T is fixed, commonly known, and independent from states.



- Idea:
  - Suppose players can transmit information by “talking” within $\hat{T}$ rounds, where there are multiple periods in each round, and then play a one-shot game.

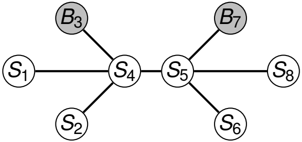
k-Threshold game augmented by $\hat{T}$-round cheap talk

Time line

- Nature choose θ according to π.
- Types are then fixed over time.
- At the first $\hat{T}$ rounds, players play $\hat{T}$-rounds of cheap talk (or costly talk).
- At $\hat{T}+1$ round, players play a one-shot k-Threshold game.
- Game ends.

Example of a WPBE construction:

- $k = 5$, $n = 8$ and $\hat{T} = 2$.
- G and θ=



- Equilibrium path
  - At $\hat{t} = 1$,

S-type 4	$(\overbrace{\mathbf{p}, \mathbf{p}, \mathbf{n}, \mathbf{p}, \mathbf{p}, \mathbf{n}, \mathbf{n}, \mathbf{n}}^{L=8})$
S-type 5	$(\overbrace{\mathbf{n}, \mathbf{n}, \mathbf{n}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{n}, \mathbf{p}}^{L=8})$
S-type 1,2,6,8	$\emptyset$

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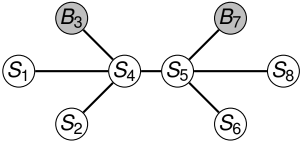

- Equilibrium path
  - At $\hat{t} = 2$,

$$\begin{array}{ll} \text{S-type 4} & (\overbrace{\mathbf{p}, \mathbf{p}, \mathbf{n}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{n}, \mathbf{p}}^{L=8}) \\ \text{S-type 5} & (\overbrace{\mathbf{p}, \mathbf{p}, \mathbf{n}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{n}, \mathbf{p}}^{L=8}) \\ \text{S-type 1,2,6,8} & \emptyset \end{array}$$

  - At $\hat{t} = 3$, all S-types play **p**, then game ends.

k-Threshold game augmented by $\hat{T}$-round cheap talk

- Off-path strategy can be constructed
- Off-path belief can be constructed

- Off-path strategy can be constructed
  - If S-type 4 (or 5) make detectable deviation (e.g. wrong sentence) $\Rightarrow$ others play **n** and then **n**.
  - If S-type 4 (or 5) make undetectable deviation $\Rightarrow$ he is facing a possibility of failure to coordinate.
- Off-path belief can be constructed
  - If a player observes a detectable deviation $\Rightarrow$ he believes that all players outside neighborhood are B-types.

If there is a fixed cost ϵ to send the letter...

- Off-path strategy can be constructed
  - If S-type 4 (or 5) make detectable deviation (e.g. wrong sentence, playing $\emptyset$) $\Rightarrow$ others play $\emptyset$ and then **n**.
  - If S-type 4 (or 5) make undetectable deviation $\Rightarrow$ he is facing a possibility of failure to coordinate.
- Off-path belief can be constructed
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If there is a fixed cost ϵ to send the letter...

- Off-path strategy can be constructed
  - If S-type 4 (or 5) make detectable deviation (e.g. wrong sentence, playing $\emptyset$) $\Rightarrow$ others play $\emptyset$ and then **n**.
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- Off-path belief can be constructed
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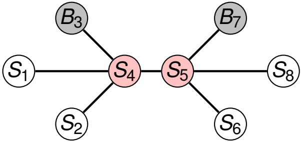
So, when ϵ is small enough and $\hat{T}$ is large enough, a WPBE can be constructed when ϵ is independent from messages.

However, if ϵ is not independent from messages, then a Free Rider Problem may occur.

- Suppose $\epsilon \downarrow$ when announce more S-types in the 1^{st} round.
- $k = 5$, $n = 8$ and $T = 2$.
- G and θ=



1. S-type 4 and S-type 5 will deviate from truthfully announcement.
2. Why? They will report more S-types to save costs in the 1^{st} round and "wait for" each others' truthfully announcement (Free Rider Problem).

How to solve the Free Rider Problem? Main idea:

- Let some of them be free rider, while letting others report truthfully.

Result 2: APEX for $k < n$

Theorem ($k < n$)

If $k < n$, then if network is a tree, if prior π has full support on strong connectedness, then an APEX WPBE exists whenever discount factor is sufficiently high.

Sketch of proof:

1. The Free Rider Problem may exist in tree networks, but it can be solved.
2. Detectable deviation $\Rightarrow$ playing **n** forever (by off-path belief).
3. Undetectable deviation $\Rightarrow$ facing a possibility of coordination failure.
4. Any deviation will let APEX fail with positive probability.
5. APEX outcome gives maximum ex-post continuation pay-off after T.
6. Sufficiently high discount factor will impede deviation.

Conclusion

In the repeated k-threshold game played in the finite networks, if the network is a tree, then players can act collectively after finite time under an assumption on connectedness.

Thank you.