1 Model

Given a finite set X, denote $\#X$ as its cardinality .

There is a set of players $N = \{1, 2, ..., n\}$. They constitute a network $G = (V, E)$ so that the vertices are players ($V = N$) and an edge is a pair of them (E is a subset of the set containing all two-element subsets of N). Throughout this paper, G is assumed to be finite, commonly known, fixed, undirected, and connected.[1] For the sake of convenience, $G_i \equiv \{j : \{i, j\} \in E\}$.

Time is discrete with index $t \in \{0, 1, ...\}$. At $t = 0$, the nature chooses a state $\theta \in \Theta = \{R, I\}^n$ once and for all according to a common prior π. R and I represent as Rebel and Inert respectively. For the sake of convenience, let $[R](\theta)$ be the set of Rebels given θ and the notion *relevant information* indicate to the information about whether or not $\#[R](\theta) \geq k$.

Definition 1.1 (Strong connectedness). *Given G, a state θ has strong connectedness if, for every pair of Rebels, there is a path consisting of Rebels to connect them.*

In the language of graph theory, this definition is equivalent to: "Given G, θ has strong connectedness if the induced graph by $[R](\theta)$ is connected."

Definition 1.2 (Full support on strong connectedness). *Given G, π has full support on strong connectedness if*

$$\pi(\theta) > 0 \Leftrightarrow \text{ } \theta \textit{ has strong connectedness}$$

1.0.1 Information hierarchy

The information hierarchy across Rebels at t in G is a tuple

$$(\{G_i^t\}_{i \in N}, \{I_i^t\}_{i \in N}, R^t, \theta).$$

[1]A path in G from i to j is a finite sequence $(l_1, l_2, ..., l_L)$ without repetition so that $l_1 = i$, $l_L = j$, and $\{l_q, l_{q+1}\} \in E$ for all $1 \leq q < L$. G is fixed if G is not random, and G is undirected if there is no order relation over each edge. G is connected if, for all $i, j \in N$, $i \neq j$, there is a path from i to j.

G_i^t is meant to represent *the extended neighbors* to represent the following. $j \in G_i^t$ if j can be reached by t consecutive edges from i such that the endpoints of $t-1$ edges are both Rebels but the endpoints of the remaining one are j and a Rebel; I_i^t is interpreted as *the extended Rebel neighbors*—the set of Rebels in G_i^t; R^t is interpreted as *the active Rebels*—those Rebels who are *active* in the sense that their extended Rebel neighbors are not a subset their direct neighbors' extended Rebel neighbors. They are defined recursively:

At $t = 0$,

$$\text{if } \theta_i = I,\ G_i^0 \equiv \emptyset,\ I_i^0 \equiv \emptyset.$$

$$\text{if } \theta_i = R,\ G_i^0 \equiv \{i\},\ I_i^0 \equiv \{i\}.$$

$$R^0 \equiv [R](\theta).$$

At $t = 1$,

$$\text{if } \theta_i = I,\ G_i^1 \equiv \emptyset,\ I_i^1 \equiv \emptyset.$$

$$\text{if } \theta_i = R,\ G_i^1 \equiv G_i,\ I_i^0 \equiv G_i \cap R^0.$$

$$R^1 \equiv \{i \in R^0 : \nexists j \in G_i \text{ such that } I_i^1 \subseteq G_j^1\}.$$

At $t > 1$,

$$\text{if } \theta_i = I,\ G_i^t \equiv \emptyset,\ I_i^t \equiv \emptyset.$$

$$\text{if } \theta_i = R,\ G_i^t \equiv \bigcup_{j \in G_i} G_j^{t-1},\ I_i^t \equiv \bigcup_{j \in G_i} I_j^{t-1}.$$

$$R^t \equiv \{i \in R^{t-1} : \nexists j \in G_i \text{ such that } I_i^t \subseteq G_j^t\}.$$

For convince, also denote $I_{ij}^{t+1} = I_i^t \cap I_j^t$ if $j \in G_i \cap R^t$. It can be shown that $R^t \subset R^{t-1}$ by the following lemma.

Lemma 1.1. *If the network is acyclic and if the θ has strong connectedness, then*

$$R^t \subset R^{t-1}$$

for all $t \geq 1$.