Coordination in Network

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Motivation

What kind of social network guarantee the successfulness or revolution? - Chwe[2000]

- The revolutionary may require certain amount of people to revolve against the government. Otherwise, those “rebels” may go to jail if revolution failed.
- However, due to the given communication barrier (social structural), they may only know their “neighbors”’ tendency in revolving. Therefore, those rebel may not be willing to revolve if the risk to jail is too high.

In Chwe[2000], the answer is: The social network which provide common knowledge about people’s willing (type) guarantee such successfulness.

- I.e. No matter what the prior is, if certain amount of rebels is in the society, then all the rebels revolve is an equilibrium.

Motivation

In Chwe[2000],

- Providing a model to investigate the coordination problem within a social network, where communication barriers are specified by network structure.
- The game is modelled as an incomplete information, static, coordination game

However, in reality, the success of revolution is rarely made in one night

- Revolutionary is dynamic. The rebels may exchange current failure to future success, since their failure may reveal their tendency ("they are rebels") to let other potential rebels know their types!

Motivation

In this paper, we then model a repeated coordination game with communication barriers specified by social network, and investigate what kinds of social network can guarantee the successfulness of revolution.

- Two kinds of communication barrier:
  - No cheap talk.
    - Players "communicate" only through their actions.
  - The underlying incomplete information and imperfect monitoring is given and specified by social network.

Model

The social network is modelled as a communication barrier

- Different types of players. Players can only observe their neighbours' type.
- Players can only observe their neighbours' past actions.
- Network is finite, fixed, undirected, and commonly known.

The game

- A coordination game, Threshold game Chwe[2000], is repeated played.
- Players simultaneously play against all other players.
- Common δ. Time is infinite, discrete.
- Payoff is hidden.
- Consider sequential equilibrium.

Goal: What kinds of social network ensure an equilibrium in which the future successfulness of revolution can repeated always?

Related Literature

- Learning in network
  - Bayesian learning: Bala and Goyal[1998], Gale and Kariv[2003], Acemoglu[2011], Muller[2013]
  - naive learning: Golub and Jackson[2010]
- Game in network
  - One-shot game: Chwe[2000]
  - Sequential game: Acemoglu[2011], Chatterjee and Dutta[2012]
- Repeated game in incomplete information: Aumann and Maschler[1995].
- Folk Theorem
  - A common assumption in incomplete information repeated game: players got some additional "signals" (such as payoffs) in every periods.
  - Different (but sufficient) assumptions in imperfect monitoring repeated game.

Static Game (Threshold game [Chwe 2000]) in network

- A parameter s with $1 \leq s \leq n$
- n players.
- Each player i's type $t_i \in T_i = \{H, L\}$.
- A Prior π over type space.
- Action set: player i's action is chosen from $A_i = \{h, l\}$ (High input or Low input).
- Static game payoff for player i: $u_{t_i}(a_i, a_{-i})$
  - For L-type player: always playing l.
  - For H-type player:
    - $u_H(h, a_{-i}) = -1$, if $\#\{j | a_j = h\} < s$
    - $u_H(h, a_{-i}) = 1$, if $\#\{j | a_j = h\} \geq s$
    - $u_H(H, l, a_{-i}) = 0$

Static Game (Threshold game [Chwe 2000]) in network

- Ex-post efficiency: if $\#H \geq s$ then all H-type players should play h; otherwise, playing l.

Repeated Threshold game in network

Definition

Approaching efficient: the tails of actions in the equilibrium path repeats the ex-post efficient outcome in the static Threshold game.

- So, what kind of network ensure approaching efficiency when δ is high enough?
  - $s = 1$: all network does.
  - $s = n$:

Theorem

$s = n$: if $\pi(H) < \pi(L)$, then all finite connected undirected network ensure approaching efficiency.

  - $1 < s < n$: Later on

Repeated Threshold game in network

$1 < s < n$:

- If $\pi(H)$ is i.i.d. across nodes: **generally impossible to achieve approaching efficiency**.

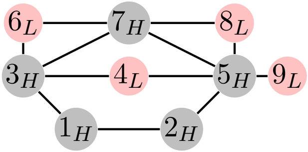
$$1_H — 2_L — 3_L — 4_L — 5_L — 6_L — 7_L — 8_L — 9_H — 10_L$$

  - There is no chance to learn if L-type separate H-nodes.

Repeated Threshold game in network

$1 < s < n$:

- Even there is a chance to learn



- We want to use $\{h, l\}$ to report the correct number of H-nodes.
- We also want to use $\{h, l\}$ to report the location of H-nodes.
- There is a free-rider problem: although players' incentives are aligned in the future successfulness,
  - , however they have to take the "risk" to report others' types.
  - , however there is a discount if the players can take the risks later.

Repeated Threshold game in network

Definition

Strong connectivity: For every pair of H-type nodes, there is a path consisting H-type nodes to connect them.

Assumption

1. *T has strong connectivity*
2. *The prior:* $0 < \pi(H) < 1$

Repeated Threshold game in network

Main theorem: There is a class of type spaces in which the revolution can success in the tree networks.

Theorem

If T *has strong connectivity, then for all* $0 < \pi(H) < 1$*, for all n-person repeated Threshold game with parameter* $1 \leq s \leq n$ *played in any finite connected undirected network without circle, there is a* δ *such that there is an equilibrium which is approaching efficient.*

Conjecture

Further goal:

Conjecture

If T has strong connectivity, then for all $0 < \pi(H) < 1$, for all n-person repeated Threshold game with parameter $1 \leq s \leq n$ played in any finite connected undirected network, there is a δ such that there is an equilibrium which is approaching efficient.