

COORDINATION IN SOCIAL NETWORKS

COMMUNICATION BY ACTIONS

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- The relevant information in making joint decision is dispersed in the society. (Hayek 1945)
- If so, how people act collectively?
 - Ex.: protest, joint investment, etc.

- In a long-term relationship, people can aggregate such information and coordinate their actions.

WHAT THIS PAPER DOES?

- I model a repeated game with incomplete information and network-monitoring with discount factor.
 - Players can only observe own/neighbors' **types** and own/neighbors' **actions**.
- Look for an equilibrium in which the pay-off relevant information become commonly known in finite time.
 - A strong requirement.
- Such equilibrium can be constructed under some assumptions.

- Strategic learning in infinite repeated game with incomplete information. (See also [Forges 1992]*)

	without discount factor	with discount factor
perfect monitoring	[Aumann and Maschler 1990], etc	[Peski 2014], etc
imperfect monitoring	[Aumann and Maschler 1990], etc	[Fudenberg and Yamamoto 2010,2011] [Wiseman 2012], [Yamamoto 2014], etc.
- network-monitoring	[Renault and Tomala 2004], etc	This paper

- Collective action: [Chwe 2000]*, etc.

Time line

- ① There is a **fixed**, **finite**, **connected**, **undirected**, and **commonly known** network.
- ② Players of two types— S or B —chosen by nature according to a probability distribution.
 - S : Strategic type; B : Behavior type
- ③ Types are then fixed over time.
- ④ Players play a stage game— a collective action —infinitely repeatedly with common discount factor.

What player can/cannot observe

- Players can observe own/neighbors' **types** and **actions**, but not others'.
- Pay-off is hidden.
 - [Aumann and Maschler 1990], [Miyahara and Sekiguchi 2013], [Wolitzky 2013], etc.

- Stage game— k -threshold game: a **protest** ([Chwe 2000])

- S-type's action set= $\{\mathbf{p}, \mathbf{n}\}$
- B-type's action set= $\{\mathbf{n}\}$
- Pay-offs for S-type:

$$u_{S_i}(a_{S_i}, a_{-\theta_i}) = 1 \quad \text{if } a_{S_i} = \mathbf{p} \text{ and } \#\{j : a_{\theta_j} = \mathbf{p}\} \geq k$$

$$u_{S_i}(a_{S_i}, a_{-\theta_i}) = -1 \quad \text{if } a_{S_i} = \mathbf{p} \text{ and } \#\{j : a_{\theta_j} = \mathbf{p}\} < k$$

$$u_{S_i}(a_{S_i}, a_{-\theta_i}) = 0 \quad \text{if } a_{S_i} = \mathbf{n}$$

STATIC EX-POST PARETO EFFICIENT OUTCOME

Type profile	Static ex-post efficient outcome
At least k S-types exist	All S-types play p
Otherwise	All S-types play n

- WPBE (weak perfect Bayesian equilibrium)
- Sequential equilibrium

APEX (*approaching ex-post efficient*) equilibrium

DEFINITION (APEX STRATEGY)

An equilibrium is APEX \Leftrightarrow

$\forall \theta$, there is a finite time T^θ

such that the actions in the equilibrium path repeats the static ex-post efficient outcome after T^θ .

RESULT 1: APEX FOR $k = n$

THEOREM ($k = n$)

If $k = n$, then an APEX sequential equilibrium exists whenever discount factor is sufficiently high.

DEFINITION FOR APEX FOR $k < n$

DEFINITION

θ has **strong connectedness** \Leftrightarrow for every pair of S-types, there is a path consisting of S-types to connect them.

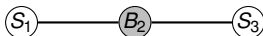
DEFINITION

π has **full support on strong connectedness** \Leftrightarrow

$\pi(\theta) > 0$ if and only if θ has strong connectedness.

WITHOUT STRONG CONNECTEDNESS

Let $k=2$ and $n=3$



- A B-type will not reveal information.
- **Without** full support on strong connectedness, in general, an APEX equilibrium does not exist when pay-off is hidden.

RESULT 2: APEX FOR $k < n$

THEOREM ($k < n$)

If $k < n$, then if network is a *tree*, if prior π has *full support on strong connectedness*, then an APEX WPBE exists whenever discount factor is sufficiently high.

OUTLINE FOR EQUILIBRIUM CONSTRUCTION

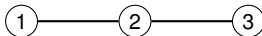
① APEX sequential equilibrium for $k = n$.

- An example.
- Sketch of proof.

② APEX WPBE for $k < n$.

- ① Consider cheap talk.
- ② Consider “costly” talk.
- ③ Sketch of proof.

AN EXAMPLE FOR $k = n$



Let $k = n = 3$, when discount factor is high enough, an APEX sequential equilibrium can be constructed by

- Period 1

- S-type 2: choose **n** if $\theta \neq (S, S, S)$, and then choose **n forever**;
- S-type 2: choose **p** if $\theta = (S, S, S)$.
- S-type 1 (or S-type 3): **p**.

- Period 2

- If S-type 2 chooses **n** in the last period \Rightarrow S-type 1 (or S-type 3) chooses **n forever**.
- If S-type 2 chooses **p** in the last period \Rightarrow S-type 1 (or S-type 3) chooses **p forever**;
- Any deviation \Rightarrow Choosing **n forever**.

AN EXAMPLE FOR $k = n$

Main features in equilibrium construction in this example

- The **1st-period** actions serve as “**messages**” to reveal the relevant information.
- The **2nd-period** is a commonly known “**timing**” to coordinate (i.e. a part of equilibrium strategy).
- **Playing n forever** serves as a “**grim trigger**”.

Sketch of proof:

- 1 “messages” to reveal the relevant information.
 - Some B-types neighbors \Rightarrow play **n** forever.
 - No B-type neighbor \Rightarrow play **p** unless **n** is observed, and then play **n** forever.

Sketch of proof:

- ① “messages” to reveal the relevant information.
 - Some B-types neighbors \Rightarrow play **n** forever.
 - No B-type neighbor \Rightarrow play **p** unless **n** is observed, and then play **n** forever.
- ② “Timing” to coordinate.
 - Finite network \Rightarrow there is a finite time $T(= n)$ such that players coordinate to the static ex-post efficient outcome.

Sketch of proof:

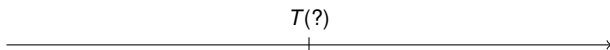
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- ④ Let discount factor be sufficiently high to impede deviation.

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 - Finite network \Rightarrow there is a finite time $T(= n)$ such that players coordinate to the static ex-post efficient outcome.
- ③ Any deviation \Rightarrow play “**n** forever”.
- ④ Let discount factor be sufficiently high to impede deviation.
- ⑤ A belief system for sequential equilibrium can be chosen.

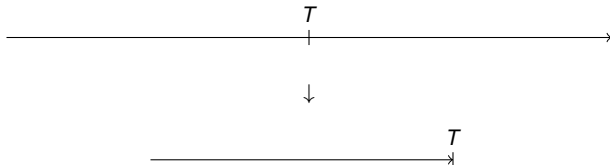


- Challenges:

- Only two actions— $\{\mathbf{n}, \mathbf{p}\}$ — used for transmit relevant information.
- How to find that finite time “ T ” for every state?
- Group punishment is hard to be made. (Network-monitoring)

EQUILIBRIUM CONSTRUCTION FOR $k < n$

For simplicity, assume T is fixed, commonly known, and independent from states.



- By definition of APEX,
 - After T , actions are infinitely repeated and thus information can not be updated.
- Idea:
 - Suppose players can transmit information by “talking” within T rounds and then play a one-shot game.
 - 1 Consider an augmented T -round “cheap talk” phase.
 - 2 Consider an augmented T -round “costly talk” phase.

k -THRESHOLD GAME AUGMENTED BY T -ROUND CHEAP TALK

Time line

- Nature choose θ according to π .
- Types are then fixed over time.
- At the first T rounds, players play T -round cheap talk.
- At $T + 1$ round, players play a one-shot k -Threshold game.
- Game ends.

k -THRESHOLD GAME AUGMENTED BY T -ROUND CHEAP TALK

- T is a big number.
- A “letter-writing technology” for player i :
 - A set of sentences: $W = \{\mathbf{n}, \mathbf{p}\}^L$, where L is a big number.
 - A fixed grammar M for each round:

$$M_i^1 = \{f | f : \Theta_{G_i} \rightarrow W\} \cup \{\emptyset\}$$

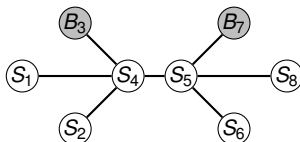
$$\text{for } 2 \leq t \leq T, M_i^t = \{f | f : \prod_{j \in G_i} M_j^{t-1} \rightarrow W\} \cup \{\emptyset\}$$

- i 's neighbors can observe what i write for each round.

k -THRESHOLD GAME AUGMENTED BY T -ROUND CHEAP TALK

Example of a WPBE construction:

- $k = 5$, $n = 8$ and $T = 2$.
- G and $\theta =$



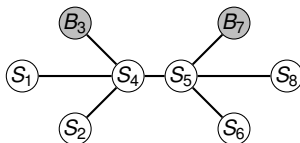
- Equilibrium path
 - At $t = 1$,

	$L=8$
S-type 4	$(\mathbf{p}, \mathbf{p}, \mathbf{n}, \mathbf{p}, \mathbf{p}, \mathbf{n}, \mathbf{n}, \mathbf{n})$
	$L=8$
S-type 5	$(\mathbf{n}, \mathbf{n}, \mathbf{n}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{n}, \mathbf{p})$
S-type 1,2,6,8	\emptyset

k -THRESHOLD GAME AUGMENTED BY T -ROUND CHEAP TALK

Example of a WPBE construction:

- $k = 5$, $n = 8$ and $T = 2$.
- G and $\theta =$



- Equilibrium path
 - At $t = 2$,

	$L=8$
S-type 4	$(\mathbf{p}, \mathbf{p}, \mathbf{n}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{n}, \mathbf{p})$
	$L=8$
S-type 5	$(\mathbf{p}, \mathbf{p}, \mathbf{n}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{n}, \mathbf{p})$
S-type 1,2,6,8	\emptyset

- At $t = 3$, all S-types play \mathbf{p} , then game ends.

- Off-path strategy
 - If S-type 4 (or 5) make detectable deviation (e.g. wrong sentence)
 \Rightarrow others play **n** and then **n**.
 - If S-type 4 (or 5) make undetectable deviation \Rightarrow he is facing a possibility of failure to coordinate.
- Off-path belief
 - If a player observes a detectable deviation \Rightarrow he believes that all players outside neighborhood are B-types.

k -THRESHOLD GAME AUGMENTED BY T -ROUND COSTLY TALK

If there is a fixed cost ϵ to send the letter...

- Off-path strategy
 - If S-type 4 (or 5) make detectable deviation (e.g. wrong sentence, playing \emptyset)
 \Rightarrow others play \emptyset and then **n**.
 - If S-type 4 (or 5) make undetectable deviation \Rightarrow he is facing a possibility of failure to coordinate.
- Off-path belief
 - If a player observes a detectable deviation \Rightarrow he believes that all players outside neighborhood are B-types.

k -THRESHOLD GAME AUGMENTED BY T -ROUND COSTLY TALK

If there is a fixed cost ϵ to send the letter...

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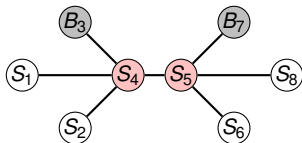
So, when ϵ is small enough and T is large enough, a WPBE can be constructed when ϵ is independent from messages.

k -THRESHOLD GAME AUGMENTED BY T -ROUND COSTLY TALK

FREE RIDER PROBLEM

However, if ϵ is **not independent from messages**, then a **Free Rider Problem** may occur.

- Suppose $\epsilon \downarrow$ when announce **more** S-types in the **1st** round.
- $k = 5$, $n = 8$ and $T = 2$.
- G and $\theta =$



- 1 S-type **4** and S-type **5** will deviate from truthfully announcement.
- 2 Why? They will report more S-types to save costs in the 1st round and “wait for” each others’ truthfully announcement (Free Rider Problem).

k -THRESHOLD GAME AUGMENTED BY T -ROUND COSTLY TALK

FREE RIDER PROBLEM

Free rider problem occurs when

- Multiple players can assure that they can learn the relevant information, given others' truthful announcement.

How to solve the Free Rider Problem? Main idea:

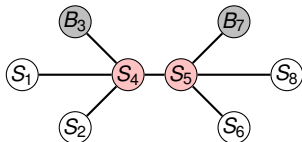
- Let some of them be free rider, while letting others report truthfully.

k -THRESHOLD GAME AUGMENTED BY T -ROUND COSTLY TALK

FREE RIDER PROBLEM

Example of solving Free Rider Problem (Suppose $\epsilon \downarrow$ when announce **more** S-types in the **1st** round; After 1st round, ϵ is fixed):

- $k = 5$, $n = 8$ and let $T = 3$.
- G and $\theta =$



- Equilibrium path
 - At $t = 1$, let S-type 4 be free rider.

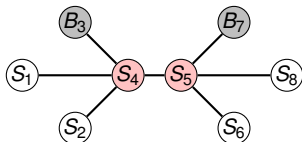
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S-type 1,2,6,8	\emptyset

k -THRESHOLD GAME AUGMENTED BY T -ROUND COSTLY TALK

FREE RIDER PROBLEM

Example of solving Free Rider Problem (Suppose $\epsilon \downarrow$ when announce **more** S-types in the **1st** round; After 1st round, ϵ is fixed):

- $k = 5$, $n = 8$ and let $T = 3$.
- G and $\theta =$



- Equilibrium path
 - At $t = 2$,

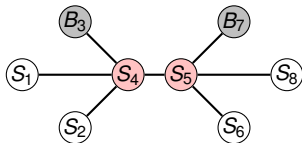
	$L=8$
S-type 4	$(\mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{p})$
S-type 5	\emptyset
S-type 1,2,6,8	\emptyset

k -THRESHOLD GAME AUGMENTED BY T -ROUND COSTLY TALK

FREE RIDER PROBLEM

Example of solving Free Rider Problem (Suppose $\epsilon \downarrow$ when announce **more** S-types in the **1st** round; After 1st round, ϵ is fixed):

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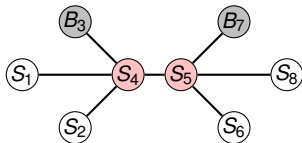
S-type 4	\emptyset
S-type 5	$\overbrace{(\mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{p})}^{L=8}$
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FREE RIDER PROBLEM

Example of solving Free Rider Problem (Suppose $\epsilon \downarrow$ when announce **more** S-types in the **1st** round; After 1st round, ϵ is fixed):

- $k = 5$, $n = 8$ and let $T = 3$.
- G and $\theta =$



- Equilibrium path
 - At $t = 3$,

S-type 4	\emptyset
S-type 5	$\overbrace{(\mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{p}, \mathbf{p})}^{L=8}$
S-type 1,2,6,8	\emptyset

- At $t = 4$, all S-types play \mathbf{p} , then game ends.

RESULT 2: APEX FOR $k < n$

THEOREM ($k < n$)

If $k < n$, then if network is a *tree*, if prior π has *full support on strong connectedness*, then an APEX WPBE exists whenever discount factor is sufficiently high.

Sketch of proof:

- ① The Free Rider Problem may exist in tree networks, but it can be solved.
- ② Detectable deviation \Rightarrow playing \mathbf{n} forever (by off-path belief).
- ③ Undetectable deviation \Rightarrow facing a possibility of coordination failure.
- ④ Any deviation will let APEX fail with positive probability.
- ⑤ APEX outcome gives maximum ex-post continuation pay-off after T .
- ⑥ Sufficiently high discount factor will impede deviation.

- 1 Tackle cyclic networks.
- 2 Look for a general model such that a finite-time communication protocol exists and this protocol can be extended to an equilibrium.