Coordination in Social Networks

Communication by Actions

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Motivation

- The relevant information in making joint decision is dispersed in the society. (Hayek 1945)
- If so, how people act collectively? How do they form an uprising, a currency attack,...,etc.

I WANT TO SHOW

- In a long-term relationship, people can aggregate such information and coordinate their actions.

Time line:

1. There is a fixed, finite, connected, undirected, and commonly known network.
2. Players of two types— S or B —chosen by nature according to a probability distribution.
   - S: Strategic type; B: Behavior type
3. Types are then fixed over time.
4. Players play a collective action—a protest—many many many...many periods with discount factor.

Model

- There are L players.
- The game played in each period —k-threshold game: a protest
- S-type's action set$= \{\mathbf{p}, \mathbf{n}\}$
- B-type's action set$= \{\mathbf{n}\}$
- Pay-offs for S-type:

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  If he chooses **n**, he gets payoff 0.

Model

What player can/cannot observe

- Players can observe own/neighbors' types and actions, but not others'.
- In each day, the pay-off is hidden.

STATIC EFFICIENT OUTCOME

Type profile	Static efficient outcome
At least *k* S-types exist	All S-types play **p**
Otherwise	All S-types play **n**

An APEX (*approaching ex-post efficient*) equilibrium

DEFINITION (APEX STRATEGY)

An equilibrium is APEX $\Leftrightarrow$ for all type distribution, there is a period, T, such that the actions in the equilibrium path repeats the static ex-post efficient outcome after T.

Outline

1. APEX equilibrium for $k = n$.
   - An example.
   - Sketch of proof.
2. APEX equilibrium for $k < n$.
   1. Where the main challenge is.

Result 1: APEX for $k = L$

Theorem ($k = L$)

If $k = L$, then an APEX equilibrium exists whenever discount factor is sufficiently high.

AN EXAMPLE FOR $k = L$

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- Period 2
  - If S-type 2 chooses **n** in the last period $\Rightarrow$ S-type 1 (or S-type 3) chooses **n** forever.
  - If S-type 2 chooses **p** in the last period $\Rightarrow$ S-type 1 (or S-type 3) chooses **p** forever;
- Any deviation $\Rightarrow$ Choosing **n** forever.

An example for $k = L$

Main features in equilibrium construction in the above example

- The 1st-period actions serve as "messages" to reveal the relevant information.
- The 2nd-period is a commonly known "timing" to coordinate (i.e. a part of equilibrium strategy).
- Playing **n** forever serves as a "grim trigger".

And, by very similar argument as this example, the APEX equilibrium when $k = L$ can be constructed.

DEFINITION FOR APEX FOR $k < L$

To find an APEX for $k < L$ is more challenging, and therefore some assumptions and definitions are introduced.

DEFINITION

The type distribution has **strong connectedness**$\Leftrightarrow$ for every pair of S-types, there is a path consisting of S-types to connect them.

DEFINITION

The prior of type distribution has **full support on strong connectedness**$\Leftrightarrow$ The prior puts positive weight only on those type distributions which has strong connectedness.

WITHOUT STRONG CONNECTEDNESS

Let k=2 and l=3

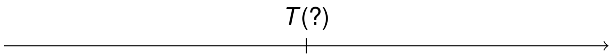
- Note that B-type will not reveal information.
- **Without** full support on strong connectedness, in general, an APEX equilibrium does not exist when pay-off is hidden.

RESULT 2: APEX FOR $k < L$

THEOREM ($k < L$)

If $k < L$, then if network is a tree, if prior has full support on strong connectedness, then an APEX WPBE exists whenever discount factor is sufficiently high.

EQUILIBRIUM CONSTRUCTION FOR $k < L$



- Challenges:
  - Only two actions—$\{\mathbf{n}, \mathbf{p}\}$— used for transmit relevant information.
  - How to find that finite time "T" for every state?
  - Group punishment is hard to be made. (Network-monitoring)

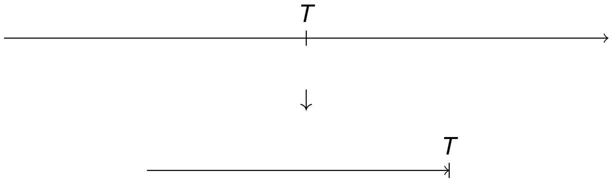
EQUILIBRIUM CONSTRUCTION FOR $k < L$

To solve the challenge:

1. We first consider fixed T.
2. Then allow indeterministic T.

Equilibrium construction for $k < L$

Assume T is fixed, commonly known, and independent from states.



- By definition of APEX,
  - After T, actions are infinitely repeated and thus information can not be updated.
- Idea:
  - Suppose players can transmit information by “talking” within T rounds and then play a one-shot game.
    1. Consider an augmented T-round “cheap talk” phase.
    2. Consider an augmented T-round “costly talk” phase.

Further works

1. Look for a general model such that a finite-time communication protocol exists and this protocol can be extended to an equilibrium.