Coordination in Social Networks

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Motivation

- The relevant information in making joint decision is dispersed in the society. (Hayek 1945)
- If so, how people act collectively?
  - Ex.: protest, currency attack, joint investment, etc.

THIS PAPER SHOWS

- In a long-term relationship, people can aggregate such information and coordinate their actions.

What this paper does?

- I model a repeated game with incomplete information and network-monitoring.
  - Players can only observe own/neighbors' types and own/neighbors' actions.
- Look for an equilibrium in which the pay-off relevant information become commonly known in finite time.
  - A strong requirement.
- Such equilibrium can be constructed under some assumptions.

Related literature

- Strategic learning in repeated game with incomplete information

	without discount factor	with discount factor
perfect monitoring	[Forges 1992]*, etc	[Jordan 1995],[Peski 2014]*, etc
imperfect monitoring	[Aumann and Maschiler 1990], etc	
network-monitoring	[Renault and Tomala 1998], etc	This paper

- Collective action: [Chwe 2000]*, [Morris and Shin 1998], etc.

Time line

1. There is a fixed, finite, connected, undirected, and commonly known network.
2. Players of two types— S or B —chosen by nature according to a probability distribution.
3. Types are then fixed over time.
4. Players play a stage game— a collective action —infinitely repeatedly with common discount factor.

What player can/cannot observe

- Players can observe own/neighbors' types and actions, but not others'.
- Pay-off is hidden.
    - Viewing the pay-off as the expected pay-off: [Aumann and Maschiler 1990], [Miyahara and Sekiguchi 2013], [Wolitzky 2013], etc.

- Stage game—k-threshold game: a protest ([Chwe 2000])
  - S-type's action set$= \{\mathbf{p}, \mathbf{n}\}$
  - B-type's action set$= \{\mathbf{n}\}$
  - (Expected) pay-offs for S-type:

$$
\begin{aligned}
u_{S_i}(a_{S_i}, a_{-\theta_i}) &= 1 && \text{if } a_{S_i} = \mathbf{p} \text{ and } \#\{j : a_{\theta_j} = \mathbf{p}\} \geq k \\
u_{S_i}(a_{S_i}, a_{-\theta_i}) &= \text{-1} && \text{if } a_{S_i} = \mathbf{p} \text{ and } \#\{j : a_{\theta_j} = \mathbf{p}\} < k \\
\\
u_{S_i}(a_{S_i}, a_{-\theta_i}) &= 0 && \text{if } a_{S_i} = \mathbf{n}
\end{aligned}
$$

Static ex-post Pareto efficient outcome

Type profile	Static ex-post efficient outcome
At least k S-types exist	All S-types play **p**
Otherwise	All S-types play **n**

- WPBE (weak perfect Bayesian equilibrium)
- Sequential equilibrium

APEX (*approaching ex-post efficient*) equilibrium

DEFINITION (APEX STRATEGY)

An equilibrium is APEX $\Leftrightarrow$

$$\forall\theta, \text{ there is a finite time } T^{\theta}$$

such that the actions in the equilibrium path repeats the static ex-post efficient outcome after T^{θ}.

Result 1: APEX for $k = n$

Theorem ($k = n$)

If $k = n$, then an APEX sequential equilibrium exists whenever discount factor is sufficiently high.

DEFINITION FOR APEX FOR $k < n$

DEFINITION

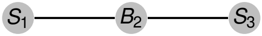
θ has **strong connectedness**$\Leftrightarrow$ for every pair of S-types, there is a path consisting of S-types to connect them.

DEFINITION

π has **full support on strong connectedness**$\Leftrightarrow$

$\pi(\theta) > 0$ **if and only if** θ has strong connectedness.

Let k=2 and n=3



- A B-type will not reveal information.
- **Without** full support on strong connectedness, in general, an Apex equilibrium does not exist when pay-off is hidden.

RESULT 2: APEX FOR $k < n$

THEOREM ($k < n$)

If $k < n$, then if network is a tree, if prior π has full support on strong connectedness, then an APEX WPBE exists whenever discount factor is sufficiently high.

Outline for Equilibrium Construction

1. APEX sequential equilibrium for $k = n$.
    - An example.
    - Sketch of proof.
2. APEX WPBE for $k < n$.
    1. Consider cheap talk.
    2. Consider "costly" talk.
    3. Sketch of proof.

AN EXAMPLE FOR $k = n$

Let $k = n = 3$, when discount factor is high enough, an APEX sequential equilibrium can be constructed by

- Period 1
  - S-type 2: choose **p** if $\theta = (S, S, S)$;
  - S-type 2: choose **n** if $\theta \neq (S, S, S)$, and then choose **n** forever.
  - S-type 1 (or S-type 3): **p**.
- Period 2
  - If S-type 2 chooses **p** in the last period $\Rightarrow$ S-type 1 (or S-type 3) chooses **p** forever;
  - If S-type 2 chooses **n** in the last period $\Rightarrow$ S-type 1 (or S-type 3) chooses **n** forever.
- Any deviation $\Rightarrow$ Choosing **n** forever.

Main features in equilibrium construction in this example

- The 1st-period actions serve as “messages” to reveal the relevant information.
- The 2nd-period is a commonly known “timing” to coordinate (part of equilibrium strategy).
- Playing **n** forever serves as a “grim trigger”.

EQUILIBRIUM CONSTRUCTION FOR $k = n$

Sketch of proof:

1. "messages" to reveal the relevant information.
    - Some B-types neighbors $\Rightarrow$ play **n** forever.
    - No B-type neighbor $\Rightarrow$ play **p** unless **n** is observed, and then play **n** forever.

Sketch of proof:

1. "messages" to reveal the relevant information.
   - Some B-types neighbors $\Rightarrow$ play **n** forever.
   - No B-type neighbor $\Rightarrow$ play **p** unless **n** is observed, and then play **n** forever.
2. "Timing" to coordinate.
   - Finite network $\Rightarrow$ there is a finite time T such that players coordinate to the static ex-post efficient outcome.

Equilibrium construction for $k = n$

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2. "Timing" to coordinate.
   - Finite network $\Rightarrow$ there is a finite time T such that players coordinate to the static ex-post efficient outcome.
3. Any deviation $\Rightarrow$ play "**n** forever".

Equilibrium construction for $k = n$

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   - Some B-types neighbors $\Rightarrow$ play **n** forever.
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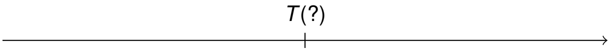
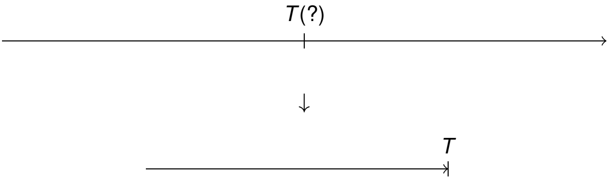
2. "Timing" to coordinate.
   - Finite network $\Rightarrow$ there is a finite time T such that players coordinate to the static ex-post efficient outcome.
3. Any deviation $\Rightarrow$ play "**n** forever".
4. A belief system for sequential equilibrium can be chosen.

EQUILIBRIUM CONSTRUCTION FOR $k < n$



- Challenges:
    - How to find that finite time "T" for every state?
    - Only two actions used for transmit relevant information.
    - Group punishment is hard to be made. (Network-monitoring)

Equilibrium construction for $k < n$



- By definition of APEX,
  - After T, actions are infinitely repeated and thus information can not be updated.
- Idea:
  1. Assume T is fixed, commonly known, and independent from states.
  2. Players can transmit information by "talking" only within T rounds.
     - Consider an augmented T-round "cheap talk" phase.
     - Consider an augmented T-round "costly talk" phase.

Time line

- Nature choose θ according to π.
- Types are then fixed over time.
- At the first T rounds, players play T-round cheap talk.
- At $T + 1$ round, players play a one-shot k-Threshold game.
- Game ends.

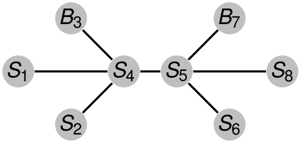
- T is a big number.
- A "letter-writing technology" for player i:
  - A set of sentences: $W = \{n, p\}^L$, where L is a big number.
  - A fixed grammar:

$$M_i^1 = \{f | f : \Theta_{G_i} \to W\} \cup \{\emptyset\} \ ; M_i^{t+1} = \{f | f : \prod_{j \in G_i} M_j^t \to W\} \cup \{\emptyset\} \text{ for } T \geq t \geq 1$$

k-THRESHOLD GAME AUGMENTED BY T-ROUND CHEAP TALK

Example of a WPBE construction:

- $k = 5$, $n = 8$ and $T = 2$.
- G and θ=

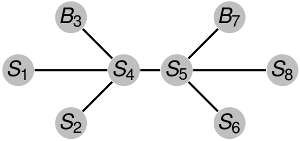


- Equilibrium path
  - At $t = 1$,

S-type 4	$\overbrace{(n, n, n, p, p, n, p, p)}^{8}$
S-type 5	$\overbrace{(p, n, p, p, p, n, n, n)}^{8}$
S-type 1,2,6,8	$\emptyset$

Example of a WPBE construction:

- $k = 5$, $n = 8$ and $T = 2$.
- G and θ=



- Equilibrium path
  - At $t = 2$,

S-type 4	$(\overbrace{p, n, p, p, p, n, p, p}^{8})$
S-type 5	$(\overbrace{p, n, p, p, p, n, p, p}^{8})$
S-type 1,2,6,8	$\emptyset$

  - At $t = 3$, all S-types play **p**, then game ends.

k-THRESHOLD GAME AUGMENTED BY T-ROUND CHEAP TALK

- Off-path strategy
  - If S-type 4 (or 5) make detectable deviation (e.g. wrong sentence)
    $\Rightarrow$ others play **n** and then **n**.
  - If S-type 4 (or 5) make undetectable deviation $\Rightarrow$ he is facing a possibility of failure to coordinate.
- Off-path belief
  - Detectable deviation $\Rightarrow$ believing that all players outside neighborhood are B-types.

k-THRESHOLD GAME AUGMENTED BY T-ROUND COSTLY TALK

If there is a fixed cost ϵ to send the letter...

- Off-path strategy
  - If S-type 4 (or 5) make detectable deviation (e.g. wrong sentence, playing $\emptyset$)

    $\Rightarrow$ others play $\emptyset$ and then **n**.
  - If S-type 4 (or 5) make undetectable deviation $\Rightarrow$ he is facing a possibility of failure to coordinate.
- Off-path belief
  - Detectable deviation $\Rightarrow$ believing that all players outside neighborhood are B-types.

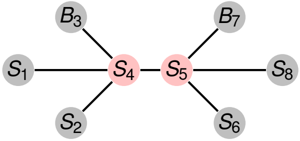
If there is a fixed cost ϵ to send the letter...

- Off-path strategy
  - If S-type 4 (or 5) make detectable deviation (e.g. wrong sentence, playing $\emptyset$) $\Rightarrow$ others play $\emptyset$ and then **n**.
  - If S-type 4 (or 5) make undetectable deviation $\Rightarrow$ he is facing a possibility of failure to coordinate.
- Off-path belief
  - Detectable deviation $\Rightarrow$ believing that all players outside neighborhood are B-types.

So, when ϵ is small enough and T is large enough, an weak equilibrium can be constructed when ϵ is independent from messages.

However, if ϵ is not independent from messages, then a Free Rider Problem may occur.

- Suppose $\epsilon \downarrow$ when announce more S-types in the 1^{st} round.
- $k = 5$, $n = 8$ and $T = 2$.
- G and θ=



1. S-type 4 and S-type 5 will deviate from truthfully announcement.
2. Why? They will report more S-types to save costs in the 1st round and “wait for” each others' truthfully announcement (Free Rider Problem).

Result 2: APEX for $k < n$

Theorem ($k < n$)

If $k < n$, then if network is a tree, if prior π has full support on strong connectedness, then an APEX WPBE exists whenever discount factor is sufficiently high.

Sketch of proof:

1. The Free Rider Problem may exist in tree networks, but it can be solved.
2. Detectable deviation $\Rightarrow$ playing **n** forever (by off-path belief).
3. Undetectable deviation $\Rightarrow$ facing a possibility of coordination failure.
4. Any deviation will let APEX fail with positive probability.
5. APEX outcome gives maximum ex-post continuation pay-off after T.
6. Sufficiently high discount factor will impede deviation.

Further works

1. Tackle cyclic networks.
2. Look for a general model such that a finite-time communication protocol exists and this protocol can be extended to an equilibrium.