SFG conversion efficiency measurement

Here are several different ways to calculate the SFG conversion efficiency in the waveguide. We assume that λ_1 is the pump wavelength (e.g. 1550 nm), λ_2 is the input signal wavelength (e.g. 920 nm), and λ_{SFG} is the output SFG wavelength (e.g. 580 nm).

1. Low power limit (the standard way)

$$\eta_{SFG} = P_{SFG} / (P_1 \times P_2),$$

where the powers are the output powers and P_{SFG} is much less than P_1 and P_2 (less than 10% for both for ~10% accuracy).

2. 100% photon conversion

$$\eta_{SFG} = (\lambda_2/\lambda_{SFG}) \times \pi^2 / (4\, P_1),$$

Valid when the pump power P_1 is increased to the point at which P_{SFG} is maximized, which is also the same as the point at which P_2 is minimized to zero. It is required that the pump power P_1 has less than 10% depletion.

3. Re-emergent 100% photon conversion

$$\eta_{SFG} = m^2 \times (\lambda_2/\lambda_{SFG}) \times \pi^2 / (4\, P_1),$$

where m is an odd number. Pump power P_1 is increased past the point at which P_{SFG} is maximized until it is zeroed and then maximized again at 9× the original power, then this formula can be used with $m = 3$. (Similarly, $m = 5$ for 3rd maximum, $m = 7$ for 4th maximum, etc.) Must have less than 10% depletion in P_1. Also works at the points where P_{SFG} is zero for even m, for instance, $m = 2$ for 2nd zero (1st zero after 1st maximum), $m = 4$ for 3rd zero, etc. Since there is no depletion of P_1 at the zero point, the even m measurements are valid even for high P_1 depletion conditions.

4. Small P_2 depletion

$$\eta_{SFG} = (\lambda_2/\lambda_{SFG}) \times \gamma / P_1,$$

where γ is the amount of depletion at λ_2,

$$\gamma = 1 - (P_2 \text{ in the presence of } P_1) \div (P_2 \text{ in the absence of } P_1).$$

Instead of turning P_1 on and off to measure P_2, it is even better to tune λ_1 on and off of peak phase matching. Must have less than 10% depletion in P_2 and even less in P_1.

5. Arbitrary P_2 depletion

$$\eta_{SFG} = (\lambda_2/\lambda_{SFG}) \times (\sin^{-1} \sqrt{[(\lambda_{SFG}/\lambda_2) \times (P_{SFG}/P_2)]})^2 / P_1.$$

This is exact for any amount of P_2 depletion up to 100%, however this formula still requires that the pump depletion P_1 is small. This formula can be used to derive the alternative forms above. This equation can be rewritten as:

$$P_{SFG} = P_2 \times (\lambda_2/\lambda_{SFG}) \times \sin^2 \sqrt{[(\lambda_{SFG}/\lambda_2) \times P_1 \times \eta_{SFG}]}.$$

6. SFG in the presence of loss

If there is loss in the waveguide, the output SFG power is reduced. Instead of $P_{SFG} = P_1 \times P_2 \times \eta_{SFG}$ in the low power limit, we have:

$$P_{SFG} = P_1 \times P_2 \times \eta_{SFG} \times 10^{-0.1(\alpha_1 + \alpha_2 + \alpha_{SFG})L/2},$$

where α_1 , α_2, α_{SFG} are the losses in dB/cm at each of the wavelengths λ_1, λ_2, λ_{SFG}. For instance, if the loss is 0.1 dB/cm at each wavelength and the length is 4 cm, then the total loss in conversion is 3×0.1 dB/cm $\times$ 4 cm $\div$ 2 = 0.6 dB and the effective conversion efficiency is $10^{-0.06}$ = 87% of the theoretical conversion efficiency.

Note that there are other ways that upconversion efficiency can be defined. It is defined here so that it is obvious and useful in the low power limit ($\eta_{SFG} = P_{SFG} / (P_1 \times P_2)$). Another way that even some textbooks use is the SHG-equivalent conversion efficiency:

$$\eta_{SHG\text{-}equivalent} = \eta_{SFG} \div 4.$$

Another way it can be defined is as the photon conversion efficiency:

$$\eta_{photon} = (\lambda_{SFG}/\lambda_2) \times \eta_{SFG},$$

which can be thought of as the percent of the number of photons converted per unit of pump power, and is seen alternatively to Equation 2 as $\eta_{photon} = \pi^2 / (4\ P_1)$.

SHG conversion efficiency measurement

λ is the pump wavelength (e.g. 1064 nm) with output power P and λ_{SHG} is the SHG wavelength (e.g. 532 nm) with output power P_{SHG}.

1. Low power limit (the standard way)

$$\eta_{SHG} = P_{SHG} / P^2,$$

where the powers are the output powers and P_{SHG} is much less than P (less than 10% for ~10% accuracy).

2. Arbitrary pump depletion

$$\eta_{SHG} = (\tanh^{-1} \sqrt{[P_{SHG}/P]})^2 / P.$$

This is exact for any amount of depletion. This formula is equivalent to Equation 1 at low power. This equation can be rewritten as:

$$P_{SHG} = P \times \tanh^2 \sqrt{[P \times \eta_{SHG}]}$$

3. Multimode pump

$$\eta_{SHG} = \tfrac{1}{2} P_{SHG} / P^2,$$

where the factor of two is due to the fact that for a multi-longitudinal-mode pump there is both SFG and SHG occurring among multiple lines. (The figure is from Helmfrid and Arvidsson, Second-harmonic generation in quasi-phase-matching waveguides with a multimode pump (1991).)

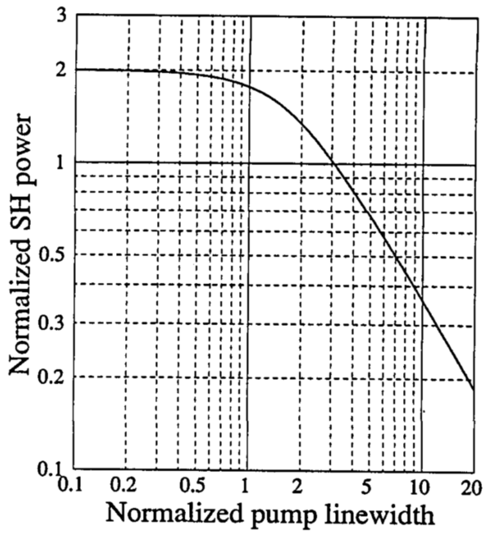

Fig. 2. Second-harmonic (SH) power generated with a multi-mode pump as a function of pump linewidth. The output power is normalized to be 1 for a corresponding case with a single-mode pump, and the linewidth is normalized by the acceptance bandwidth of the process.

4. SHG in the presence of loss

Let's consider a few different cases for calculating the internal (η_I) and external (η_E) conversion efficiency in the presence of loss, based on where we assume the loss to be.

T = pump transmission
P = measured output pump power
P_{SHG} = measured output SHG power
$\eta = P_{SHG} / P^2$ = conversion efficiency as normally measured using output powers

External conversion efficiency, based on pump power at input and SHG power at output, is what really matters for a device and will always be equal to $T^2\eta$ no matter where the losses are coming from.

$$\eta_E = T^2 \times \eta$$

Internal conversion efficiency is the theoretically expected conversion efficiency that you would get if all the losses were eliminated, which we calculate differently from the measurements depending on where the loss is coming from. Here are six different scenarios:

A.) Assume all loss is at input (no loss at output, no propagation loss). This is what we normally assume, which seems pretty fair for short chips considering that we expect mode match loss at the input but not output:

$$\eta_I = \eta$$

B.) Assume all loss is at output (no loss at input, no propagation loss), and that SHG loss is the same as pump loss:

$$\eta_I = T \times \eta$$

C.) Assume input and output losses are equal, $T_{IN} = T_{OUT} = \sqrt{T}$, and are the same for SHG and pump (no propagation loss).

$$\eta_I = \sqrt{T} \times \eta$$

D.) Assume all loss is propagation loss (no loss at input or output), and is the same for SHG and pump. Then $T = 10^{-0.1\alpha L}$ where α is the propagation loss in dB/cm and L is the chip length.

$$\eta_I = \sqrt{T} \times \eta$$

E.) Assume all loss is at input except for Fresnel loss, T_F, at output (no propagation loss). $T = T_{IN} \times T_F$. For free-space coupling, $T_F = (n-1)^2/(n+1)^2$ where n is the index. (T_F = 92% for KTP. T_F = 86% for LN.)

$$\eta_I = T_F \times \eta$$

F.) General case, $T = T_{IN} \times T_P \times T_{OUT}$, assuming pump and SHG losses are the same. The propagation loss is $T_P = 10^{-0.1\alpha L}$ where α is the propagation loss in dB/cm and L is the chip length.

$$\eta_I = \sqrt{T_P} \times T_{OUT} \times \eta$$

DFG conversion efficiency

Rough notes for now:

Assume P'_1 at 100% photon conversion point. Then for $\lambda_2 \rightarrow \lambda_3$ in presence of P'_1 (from Eq. 2 above):

$\eta = (\lambda_2/\lambda_3) \times \pi^2 / (4 P'_1)$ where $P_3 = P_1 \times P_2 \times \eta$ for low power SFG

Similarly, for $\lambda_3 \rightarrow \lambda_2$ in presence of P_1:

$\eta = (\lambda_3/\lambda_2) \times \pi^2 / (4 P'_1)$ where $P_2 = P_1 \times P_3 \times \eta$ for low power DFG

First one is SFG, second one is DFG, therefore:

$$(\lambda_3/\lambda_2) \times \eta_{SFG} = (\lambda_2/\lambda_3) \times \eta_{DFG}$$

$$\eta_{DFG} = (\lambda_3/\lambda_2)^2 \times \eta_{SFG}$$

Quantum frequency conversion

$QE = \sin^2\sqrt{[\, \eta_{SFG} L^2 P_1 (\lambda_3/\lambda_2) \,]}$

$P_3 = P_2 (\lambda_3/\lambda_2) \sin^2\sqrt{[\, \eta_{SFG} L^2 P_1 (\lambda_3/\lambda_2) \,]}$

Estimate QE with output loss of λ_3:

QE = $\sin^2\sqrt{(\eta L^2 P_1(\lambda_3/\lambda_2))}$ * 10^(-(loss/10)L/2)

Coupled wave equations

We can define the coupled wave equations in convenient units as follows:

$dE_1/dz = -i \sqrt{\eta} \exp(-i \Delta k z) E_3 E_2^* (\lambda_3/\lambda_1)$
$dE_2/dz = -i \sqrt{\eta} \exp(-i \Delta k z) E_3 E_1^* (\lambda_3/\lambda_2)$
$dE_3/dz = -i \sqrt{\eta} \exp(+i \Delta k z) E_1 E_2$

where $P_i(z) = E_i^*(z)E_i(z)$ is the power vs distance for each wavelength in watts, the E_i are in $\sqrt{W}$, z is in cm, Δk is in cm^{-1}, and η is the SFG conversion efficiency in %/W/cm².

Optical parametric amplification

Solving the coupled wave equations for E_3 = constant (undepleted pump assumption), $E_1(0) = 0$, $E_2(0) = \sqrt{P_0}$, and $\Delta k=0$, we find

$d^2E_1/dz^2 = \gamma^2 E_1$
$d^2E_2/dz^2 = \gamma^2 E_2$

$\gamma^2 = \eta E_3^*E_3 (\lambda_3/\lambda_1) (\lambda_3/\lambda_2) = \eta P_3 (\lambda_3/\lambda_1) (\lambda_3/\lambda_2)$

The general solution is $E_i(z) = a_i \exp(+\gamma z) + b_i \exp(-\gamma z)$, and given the initial conditions the solution for power vs distance is

$P_1(z) = P_0 (\lambda_2/\lambda_1) \sinh^2(\gamma z)$
$P_2(z) = P_0 \cosh^2(\gamma z) = P_0 (1+\sinh^2(\gamma z))$

Let's consider a pulsed pump input at λ_3 with pulse energy ρ and pulse duration τ, and a cw seed at λ_2 with power P_0. To get to pump depletion levels, i.e. in which a significant fraction of the pump pulse energy is converted to λ_1 and λ_2, requires that the seed power is amplified to the scale of the pulse peak power, $P_3 \approx \rho/\tau$:

$P_0 \cosh^2(\gamma L) \approx P_3$

Solving for γL we have $\gamma L = \cosh^{-1}\sqrt{(P_3/P_0)}$, or

$\eta L^2 = (1/P_3) [(\lambda_1/\lambda_3) (\lambda_2/\lambda_3) \cosh^{-1}\sqrt{(P_3/P_0)}]^2$

As specific example, assume P_0 = 1mW, ρ = 1nJ, τ = 1ps, and $(\lambda_1/\lambda_3)(\lambda_2/\lambda_3) \approx 4$. Then P_3 = 1kW, $P_3/P_0 \approx 10^6$, and $\eta L^2 \approx (1/P_3) [4 \cosh^{-1}(10^3)]^2$ = 92%/W. Keep in mind that L must be less than the walkoff distance.