Logic programming semantics

Cheat sheet

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1 Stable Models Semantics[3]

Gelfond-Lifschitz operator Let P be a normal logic program and I a 2-valued interpretation. The GL-transformation of P modulo I is the program $\frac{P}{I}$ obtained from P by performing the following operations:

- remove from P all rules which contain a default literal *not* A such that $A \in I$;
- remove from the remaining rules alll default literals.

Fixed points of the Gelfond-Lifschitz operator Γ for a program P are always models of P.

Stable Model Semantics A 2-valued interpretation I of a normal logic program P is a stable model of P if and only if $\Gamma(I) = I$. An atom A of P is true under the stable model semantics if and only if A belongs to all stable models of P.

2 Well-Founded Semantics[2]

Γ^* **operator** Let P be a normal logic program and I a 3-valued interpretation. The extended GL-transformation of P modulo I is the program $\frac{P}{I}$ obtained from P by performing the following operations:

- remove from P all rules which contain a default literal *not* A such that $I(A) = 1$;
- replace in the remaining rules of P those default rules *not* A such that $I(A) = \frac{1}{2}$ by the reserved literal u;
- remove from the remaining rules alll default literals.

Since the resulting program is non-negative, it has a unique 3-valued interpretation least model J. We define $\Gamma^*(I) = J$.

Well-founded semantics A 3-valued interpretation I of a logic program P is a partial stable model of P if and only if $\Gamma^*(I) = I$. The well-founded semantics of P is determined by the unique $F - least$ partial stable model of P, and can be obtained by the (bottom-up) iteration of Γ^* starting from the empty interpretation.

3 Revised Stable Models Semantics[4]

Gelfond-Lifschitz operator Let P be a normal logic program and I a 2-valued interpretation. The GL-transformation of P modulo I is the program $\frac{P}{I}$ obtained from P by performing the following operations:

- remove from P all rules which contain a default literal *not* A such that $A \in I$;
- remove from the remaining rules alll default literals.

Fixed points of the Gelfond-Lifschitz operator Γ for a program P are always models of P.

As a shorthand notation, let $WFM(P)$ denote the positive atoms of the Well-Founded Model of P, that is WFM(P) is the least fixpoint of operator Γ_p^2[5], i.e., Γ_p applied twice.

Sustainable Set Intuitively, we say a set S is sustainable in P if and only if any atom A in S does no go against the well-founded consequences of the remaining atoms in S whenever $S\{A\}$ itself is a sustainable set. The empty set by definition is sustainable. "Not going against" means that atom A cannot be false (A is true or undefined) in the well-founded model of $P \cup S \setminus \{A\}$, i.e., it belongs to set $\Gamma_{P \cup S \setminus \{A\}}(WFM(P \cup S \setminus \{A\}))$

Formally we say S is sustainable if and only if:

$$\forall_{a \in s} S \setminus A \textit{ is sustainable} \Rightarrow a \in \Gamma_{P \cup S \setminus \{A\}} \left(WFM \left(P \cup S \setminus \{A\}\right)\right)$$

If S is empty the condition is trivially true.

Revised Stable Models and Semantics Let $RAA_P(M) \equiv M - \Gamma_P(M)$. M is a Revised Stable Model of an Normal Logic Program P, if and only if:

- M is a minimal classical model, with "$\sim$" interpreted as classical negation;

- $\exists_{\alpha \geq 2}$ such that $\Gamma_P^{\alpha}(M) \supseteq RAA_P(M)$
- $RAA_P(M)$ is sustainable.

The Revised Stable Models is the intersection of its models, just as in the Stable Model semantics.

4 Disjunctive Well-Founded Semantics[1]

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