Experiment : 01

1.1 Title : Introduction to MATLAB/Simulink: Plot the step response of first order and second order systems.

1.2 Apparatus :

MATLAB/Simulink Software

1.3 Theory :

let us consider first order system of which has transfer function is given below

$$G_1(s) = \frac{1}{1+s} \tag{1.1}$$

and second order system which transfer function is

$$G_2(s) = \frac{10}{s(s+15)} \tag{1.2}$$

Now evaluate the closed loop transfer function of above systems in which step response is given as input and negetive unity feedback is used. general closed loop transfer function is defined as below

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

where $G(s)$ is feed forward gain and $H(s)$ is feedback gain, where $C(s)$ referred as controlled ouput variable and $R(s)$ as reference set point or input variable

as per above equation, we get $Y_1(s)$ and $Y_2(s)$ as first order and second order closed loop transfer function respectively

$$Y_1(s) = \frac{1}{s+2} \tag{1.3}$$

$$Y_2(s) = \frac{10}{s^2+15s+10} \tag{1.4}$$

1.4 Observation :

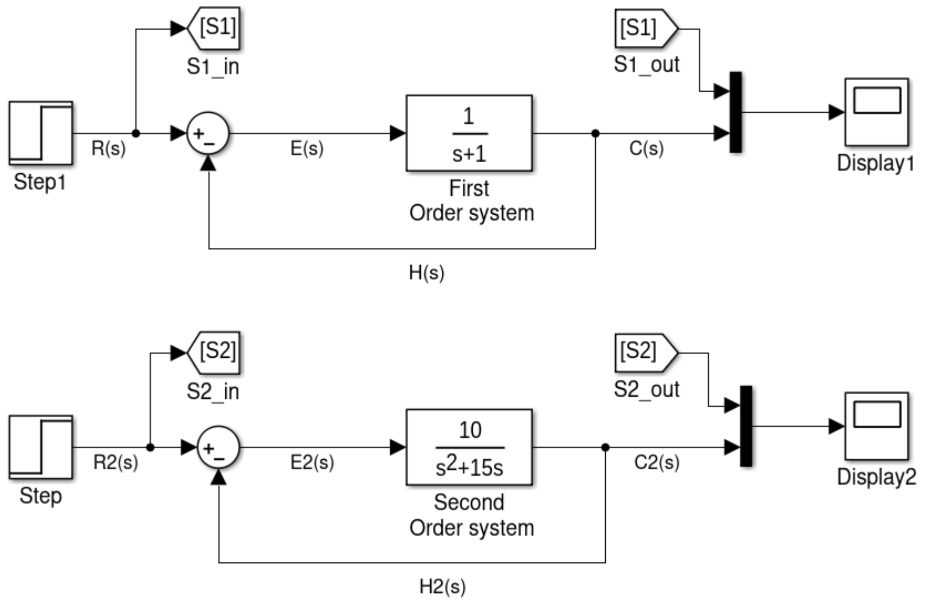


Figure 1.1 : Simulink model of system under consideration

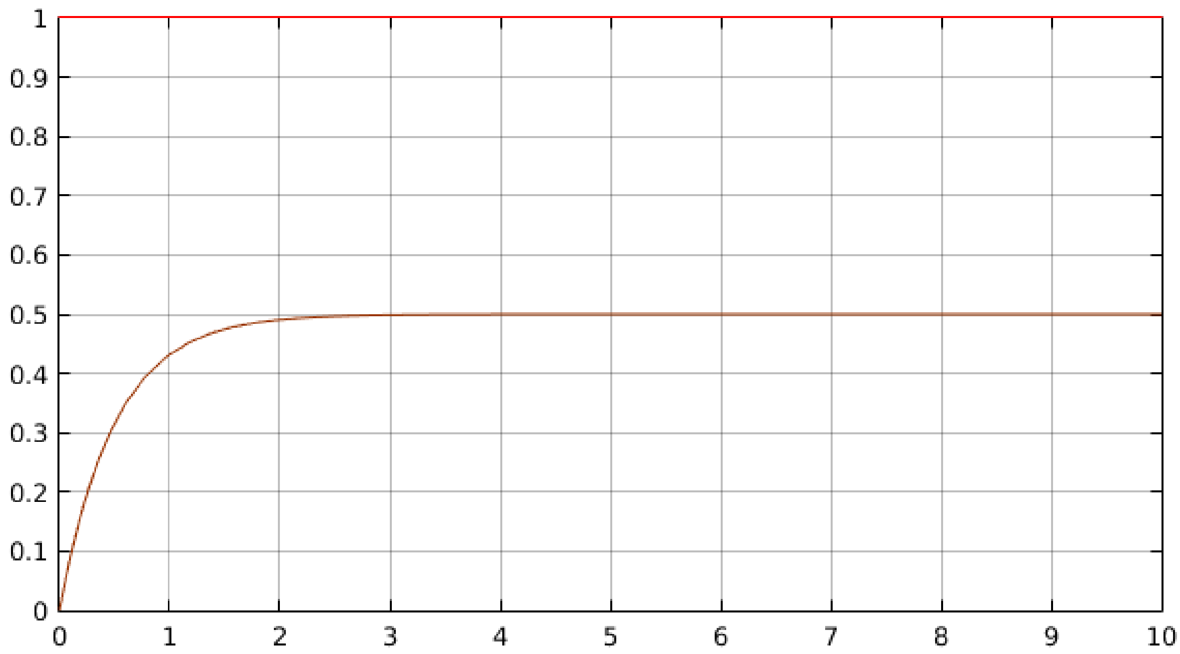


Figure 1.2 : Step response of first order system

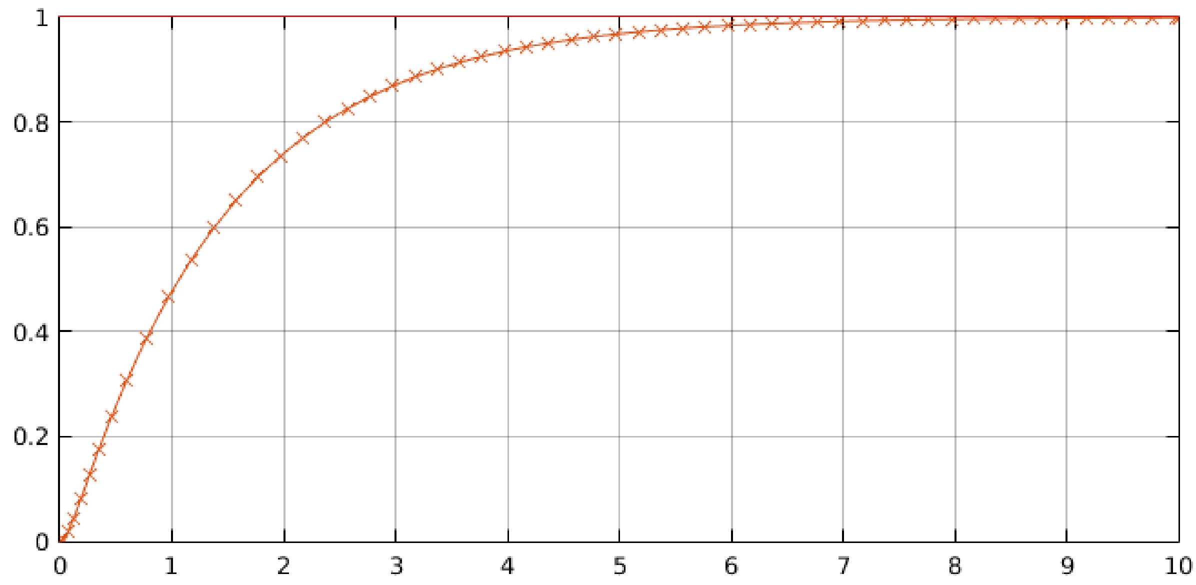


Figure 1.3 : Step response of second order system

1.5 Conclusion :

- by giving step response or bounded input, we get an bounded output in both the systems, so they are behave as stable.
- first order system will settled down to final value at approx 2.2 seconds when step input is given
- in second order system final value achieve at approx 7 seconds due to its Overdamped response.

Experiment : 02

2.1 Title : Study the effect of varying system gain for a second order overdamped system, and verifying the results using Root locus.

2.2 Aim :

1. **Analyse the step responses of Second order system at different values of K_P.**

2. **Analyse the step response of Second order system with transportation delay with and without disturbances.**

2.3 Apparatus :

MATLAB/Simulink Software

2.4 Theory :

In a first part we have $G_1(s)$ of second order system and varying the proportional gain(K_P) to 50, 100, 1500 values and evaluating its step responses.

$$G_1(s) = \frac{8}{s^2 + 50s} \tag{2.1}$$

In second part we have second order system with dead time or transportation delay $G_2(s)$ and disturbance with delay as $G_3(s)$ defined below

$$G_2(s) = \frac{2e^{-5s}}{50s^2 + 15s + 1} \tag{2.2}$$

$$\tag{2.3}$$

$$G_3(s) = \frac{0.3e^{-5s}}{15s + 1} \tag{2.4}$$

2.5 Observation :

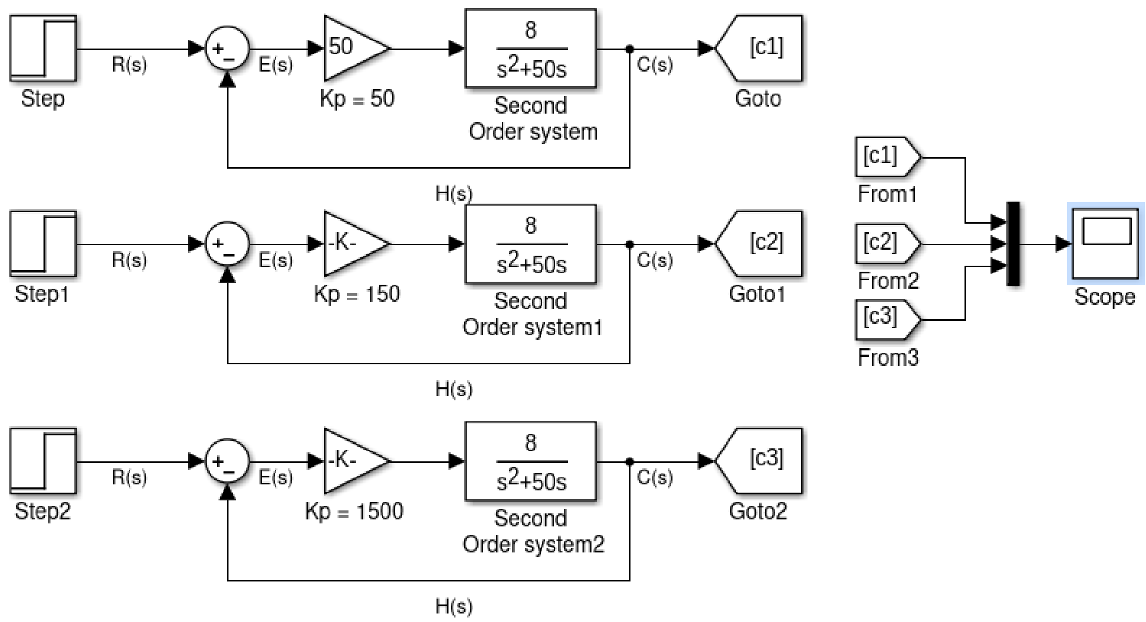


Figure 2.1: Simulink model of above system

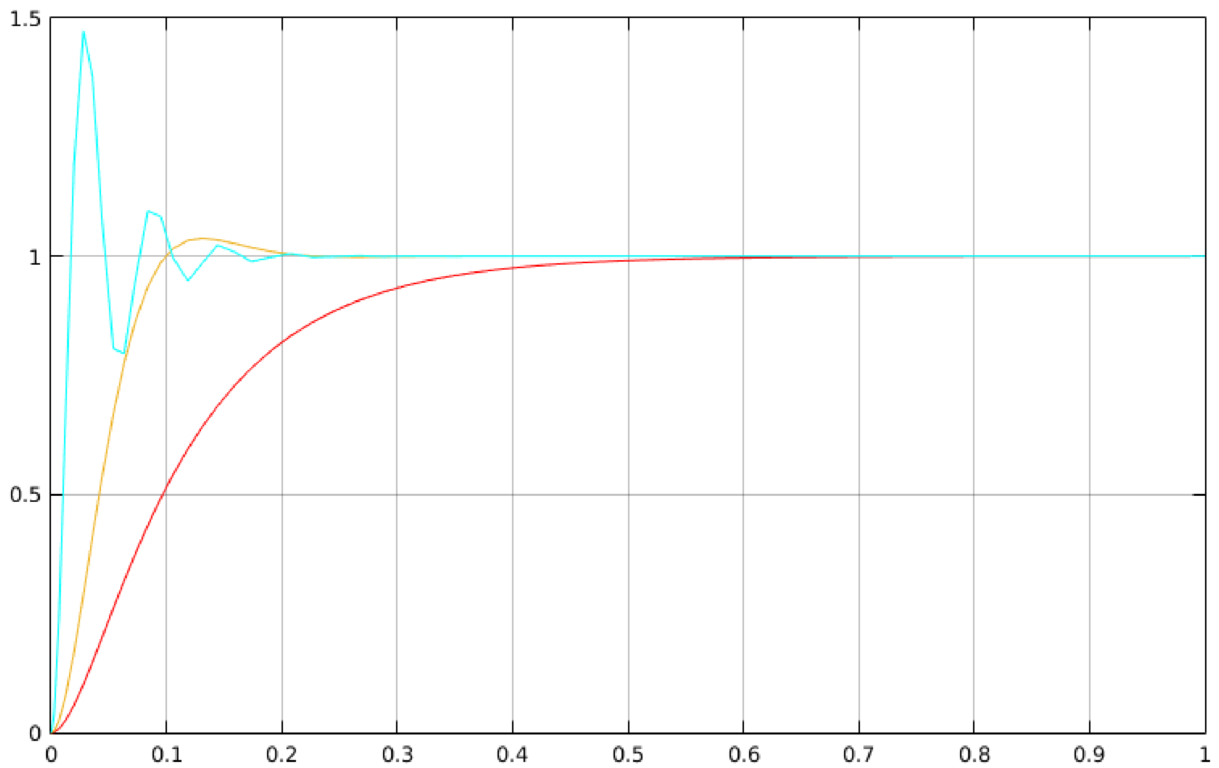
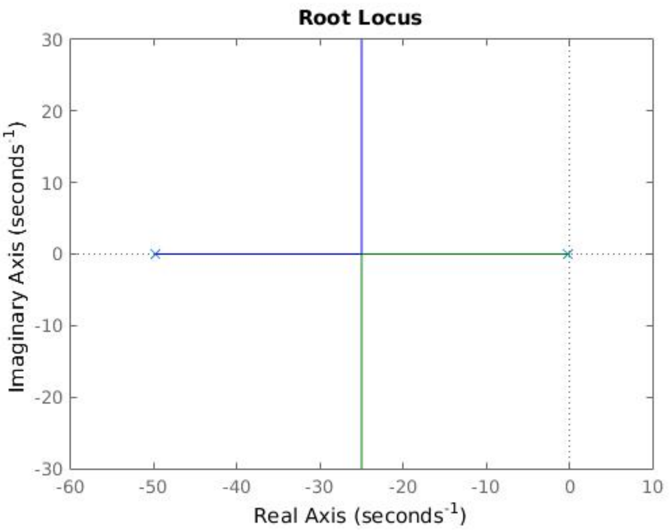


Figure 2.2: Figure 2.2 : Step responses of second order system with different value of K_p




```
#Code for plot Root locus
# in MATLAB  programme
num = [1];
den = [1 50 8];
G = tf(num,den);
rlocus(G)
```

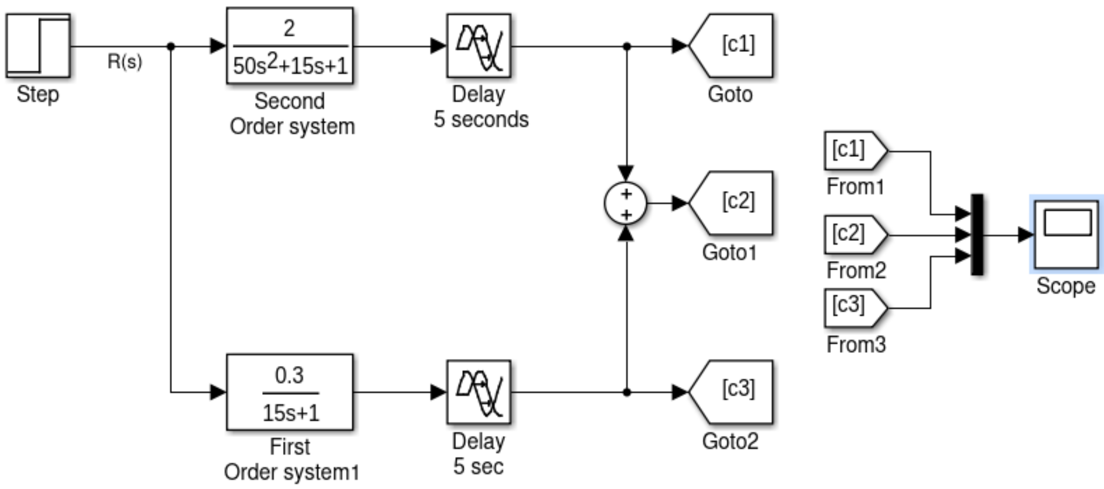


Figure 2.3: Simulink Model of second order system with time delay

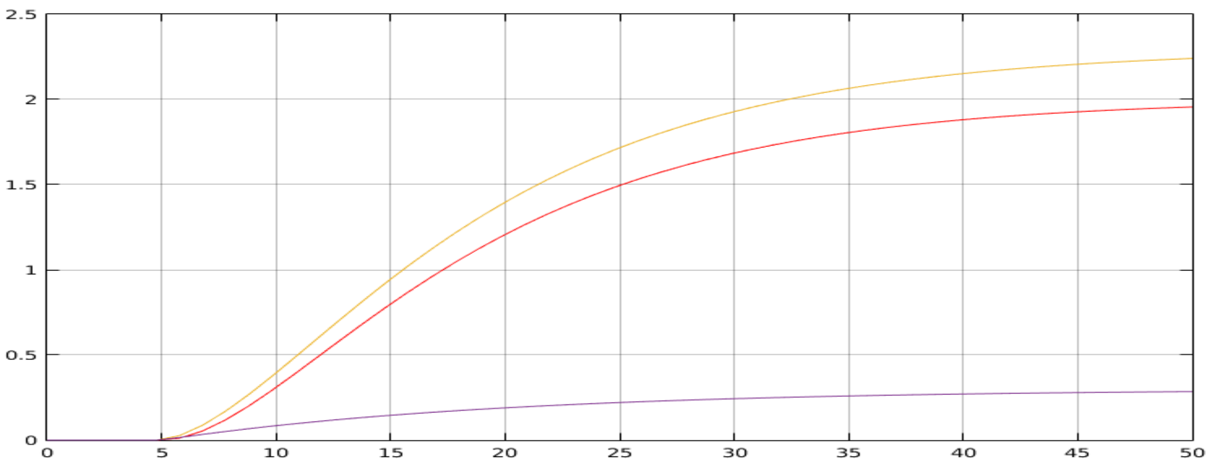


Figure 2.4: Step responses of Second order system with dead time

2.6 Conclusion :

- In first part we have observed that on increasing the value of K_P output response is reaching at final value very fast. It is verified from Root locus that at $K_P = 50$ we get critical damped response while on $K_P = 150$, 1500 we get underdamped response i.e. having overshoot.
- In second part due to disturbance, output response is getting deviated from its reference value and reaching at peak which is greater than input.

Experiment : 03

3.1 Title : Dynamic response of First order system with time delay

3.2 Aim :

1. Plot the responses of firsr order system with given condition:
   (a) Exact response
   (b) **I order Padé** Approximation
   (c) **II order Padé** Approximation
2. Proportional only control of First order system with dead time
3. Intergral only control of Second order system with dead time

3.3 Apparatus :

MATLAB/Simulink Software

3.4 Theory :

In a first part we have plant transfer function with transportation delay as $G_1(s)$

$$G_1(s) = \frac{Ke^{-sT_d}}{sT_p + 1} \tag{3.1}$$

This is exact or actual transfer function, which we have to approximate using Taylor series approximation by expanding transportation delay term into ints factors into poles and zeros.Now First order Padé approximation for a time delay consist of half magnitude of right hand side s plane zero and half magnitude of left hand side s plane pole, or ratio of two polynomials in 's' with with coefficient term calculating by taylor series expansion of $e^{-s\theta}$.

First Order Padé Approximation is,

$$e^{-s\theta} = \left(\frac{1-\frac{s\theta}{2}}{1+\frac{s\theta}{2}}\right)$$

second Order Padé Aprroximation is,

$$e^{-s\theta} = \frac{1-\frac{s\theta}{2}+\frac{s^2(\theta)^2}{12}}{1+\frac{s\theta}{2}+\frac{s^2(\theta)^2}{12}}$$

For a first case we have $T_d = 5$ second, $T_p = 10$ second and K = 1, so we have to evaluate the responses, so that

$$G(s) = \frac{e^{-5s}}{10s+1} \tag{3.2}$$

I Order Padé Approximation

$$G_1(s) = \frac{1-2.5s}{25s^2+12.5s+1} \tag{3.3}$$

II Order Padé Approximation

$$G_2(s) = \frac{2.083s^2-2.5s+1}{20.83s^3+27.083s^2+12.5s+1} \tag{3.4}$$

In second part firstly we analyse the response of G(s) at different values of K=1.05, 1.15, 2, 4.5, 8.5 and selecting value of $T_d = 5$ and T_p=1.

Now in second part we change value of k_i= 0.2,0.5,0.8,1 respectively and add disturbance in system with time delay of 25 seconds.

3.5 Observation :

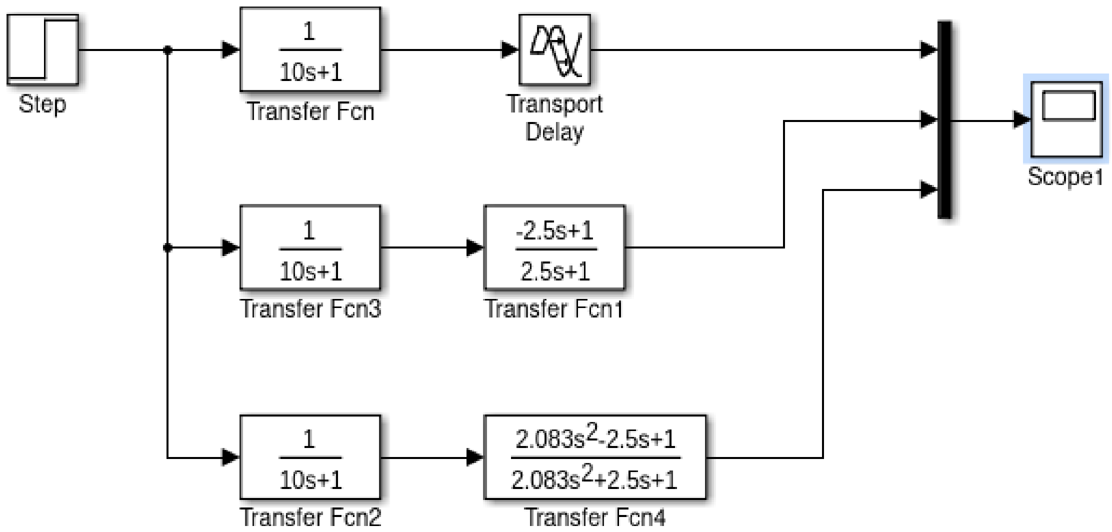


Figure 3.1: Simulink model of system of part 1

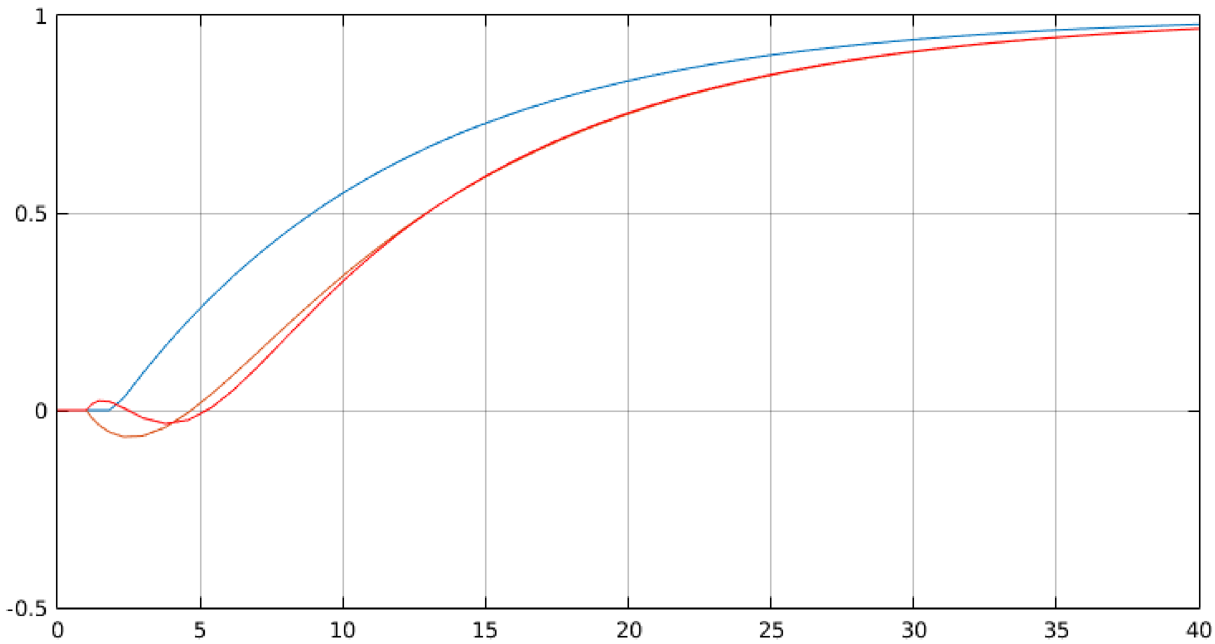


Figure 3.2: step response of All system in combined axis

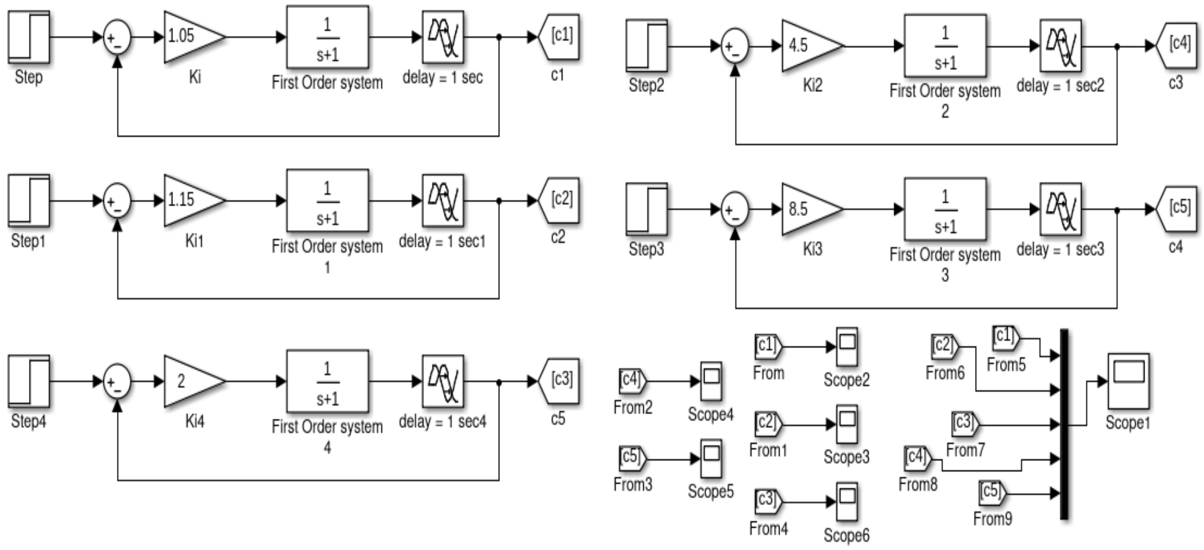


Figure 3.3: simulink model of part 2

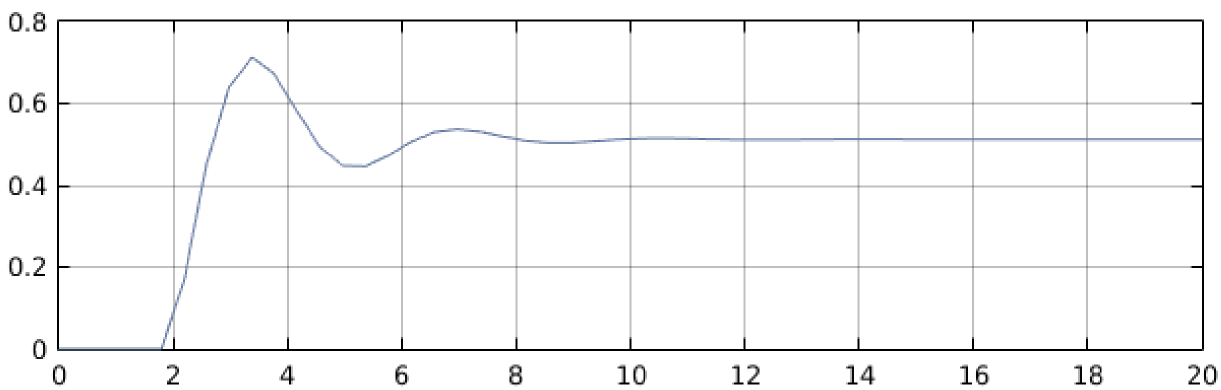


Figure 3.4: step response with $K = 1.05$

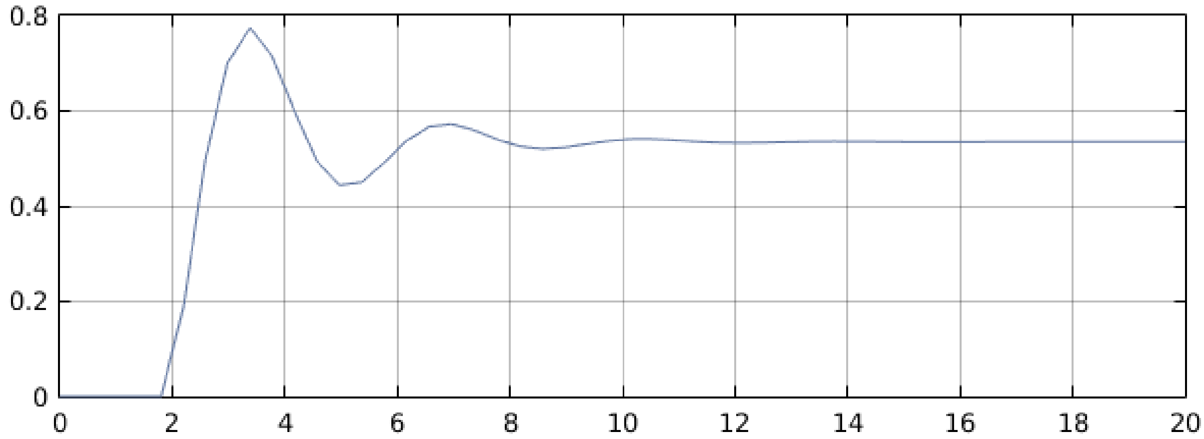


Figure 3.5: step response with $K = 1.15$

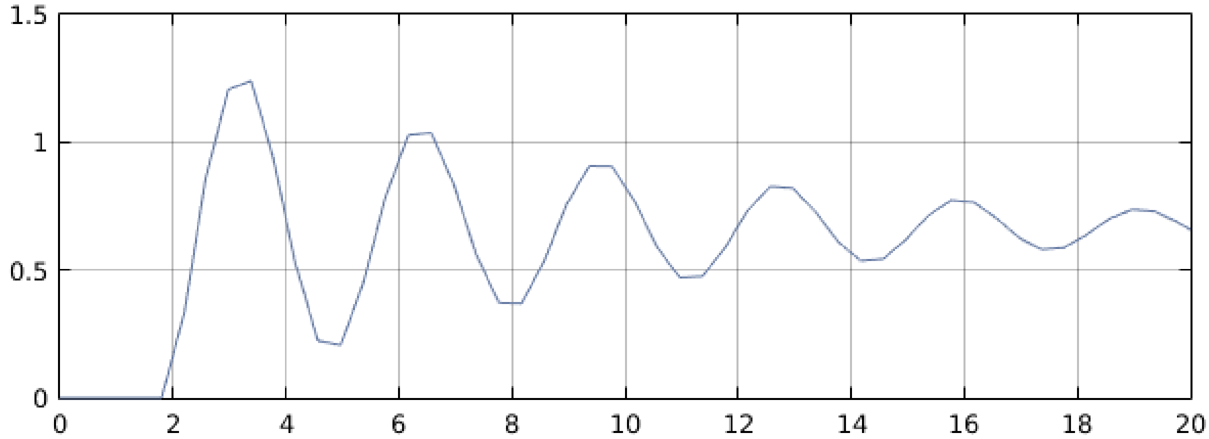


Figure 3.6: step response with $K = 2$

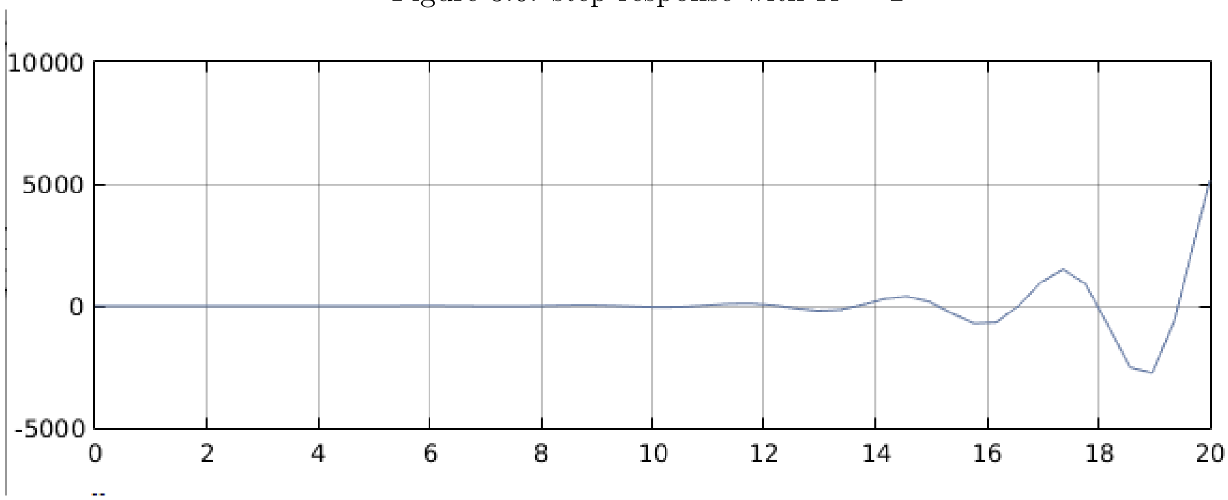


Figure 3.7: step response with $K = 4.5$

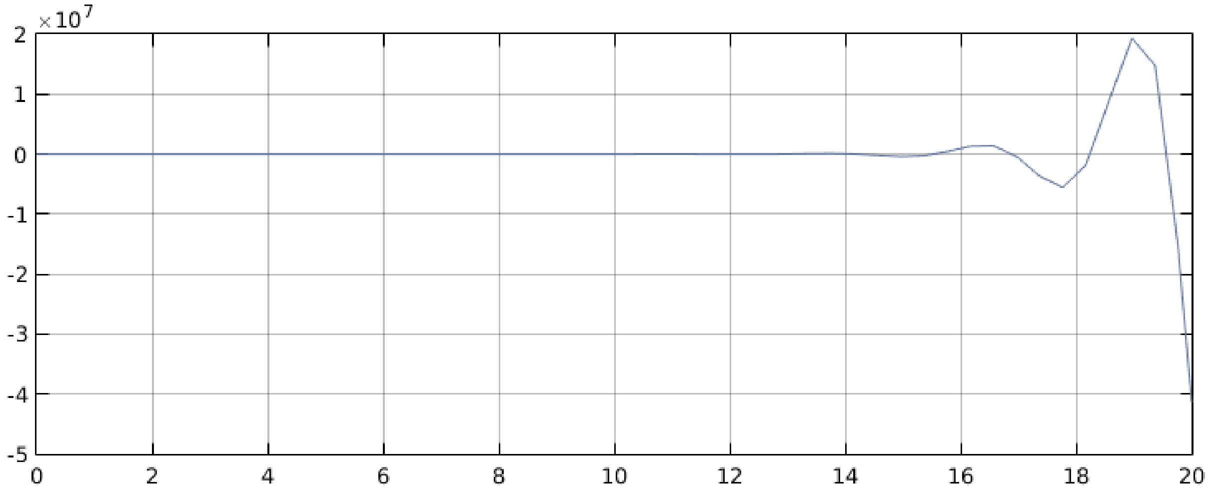


Figure 3.8: step response with $K = 8.5$

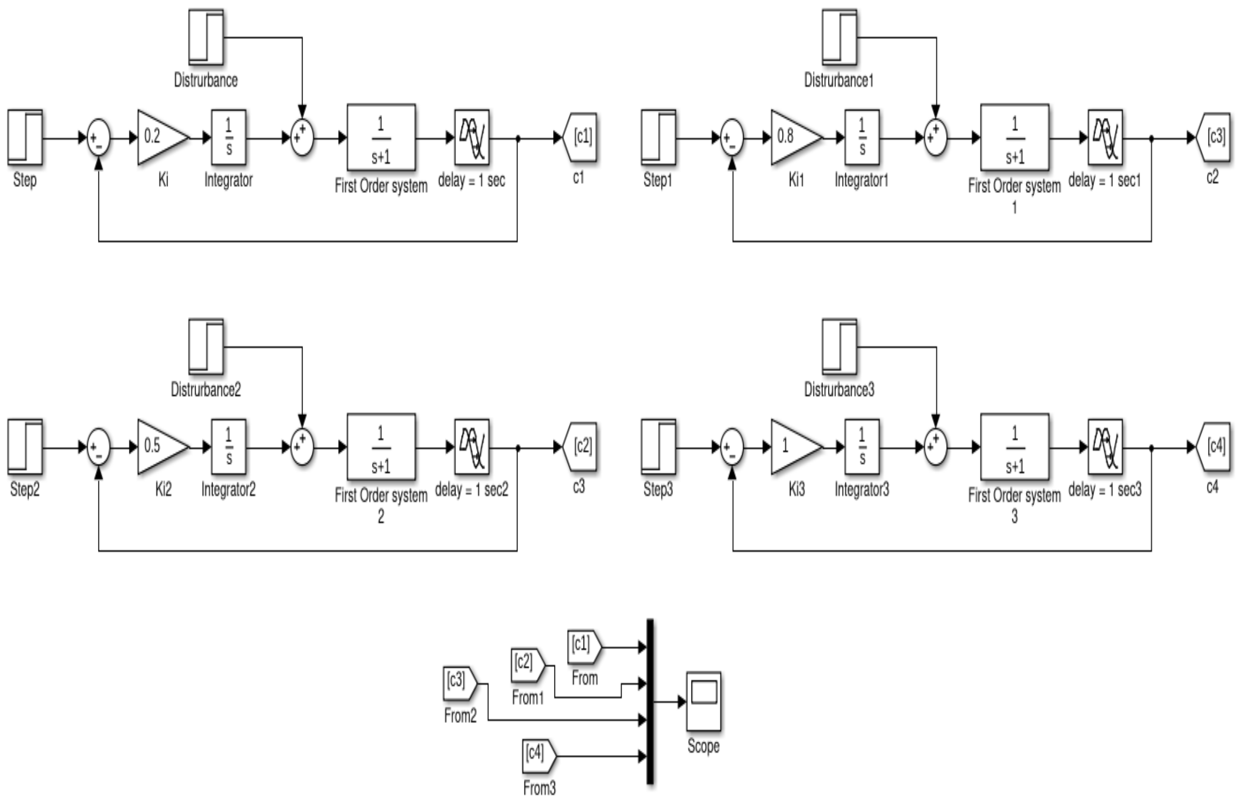


Figure 3.9: simulink model of system of part 3

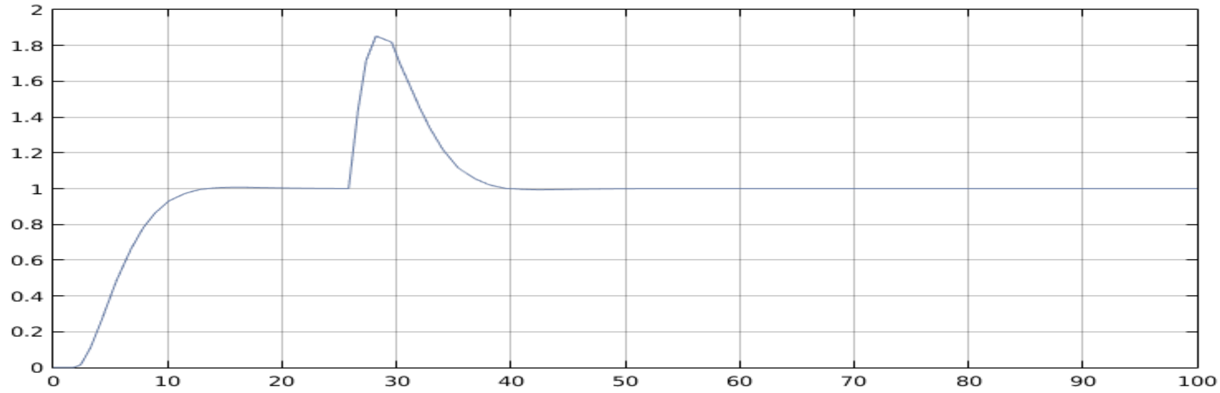


Figure 3.10: step response with $K_i = 0.2$

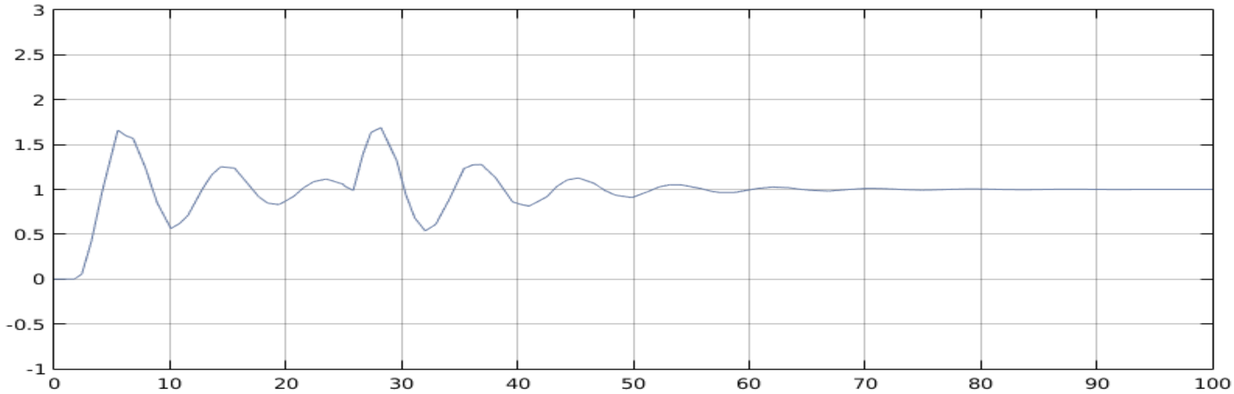


Figure 3.11: step response with $K_i = 0.5$

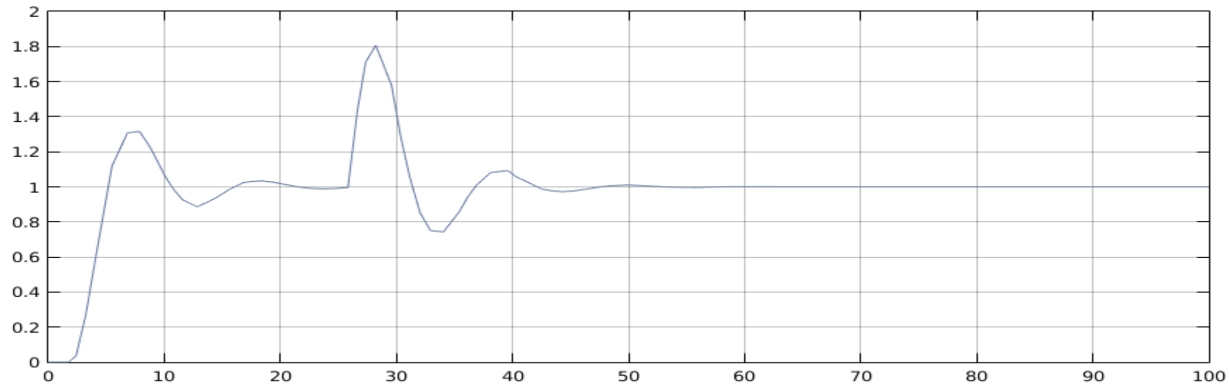


Figure 3.12: step response with $K_i = 0.8$

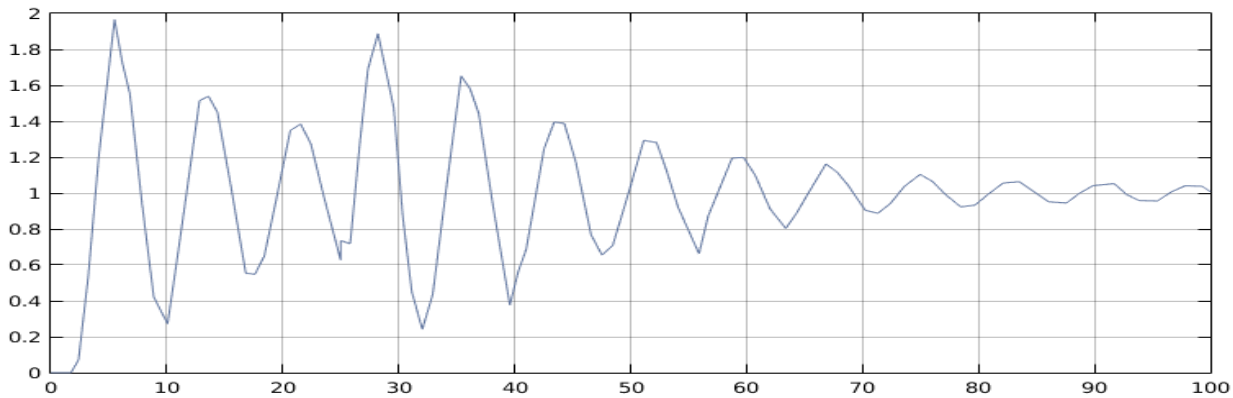


Figure 3.13: step response with $K_i = 1$

3.6 Conclusion

- In First Part of system we can verify that system incorporate inverse response when padé approximation is used, in which initial slope is negetive and response goes to reverse direction to the reference value.
- Also if we increase order of padé approximation then inverse response increase or become double.
- In second part as value of K (gain) is increased then its step response has increasing time delay and oscillation respectively.
- Now in Third part, for analysis of integral control only we had inserted additional disturbance of step input after 25 sec and check that system will become oscillatory in first some cycle of operation as value of K_i is increasing.

Experiment : 04

4.1 Title : Approximation of Higher order system into first and second order systems with time delay using Taylor series approximation and Skogestad's Half rule.

4.2 Apparatus :

MATLAB Software

4.3 Theory :

For Approximation of system, we need to linearized higher order nonlinear model into its equivalent first order or second order model so we can deal easily with its dynamic and steady and state characteristics. So that we are using methods like Taylor series and SKOGESTAD's Half rule.

Tailor series approximation is expressed up to second term,so that

$$e^{-\theta s} \approx 1 - \theta s \tag{4.1}$$

$$e^{-\theta s} = \frac{1}{e^{\theta s}} \approx \frac{1}{1+\theta s} \tag{4.2}$$

Now from SKOGESTAD's methods, there are some rules to transform higher order system into first order or second order equivalent system is given below

- if model has multiple time constants than manipulation takes place with largest neglected Time constant
- in denominator one half of neglected T is added to the existing time delay and other half of neglected T is added to the time constant that is retained
- Third largest time constants that are smaller than largest neglected T will be considered & added as time delay term

Part : A

$$G(s) = \frac{K(-0.1s+1)}{(5s+1)(3s+1)(0.5s+1)} \tag{4.3}$$

Derive an approximation First order plus time delay model(FOPTD) using:

- Taylor series approximation
- SKOGESTAD's Half rule

we get our transform or linearized plant transfer function as

$$G_T = \frac{Ke^{-3.6s}}{5s+1}, G_H = \frac{Ke^{-2.1s}}{6.5s+1} \tag{4.4}$$

where G_T = approximated transfer function using taylor series
G_H = approximated transfer function using SKOGESTAD's half rule

Part : B

$$H(s) = \frac{K(1-s)e^{-s}}{(12s+1)(3s+1)(0.2s+1)(0.05s+1)} \tag{4.5}$$

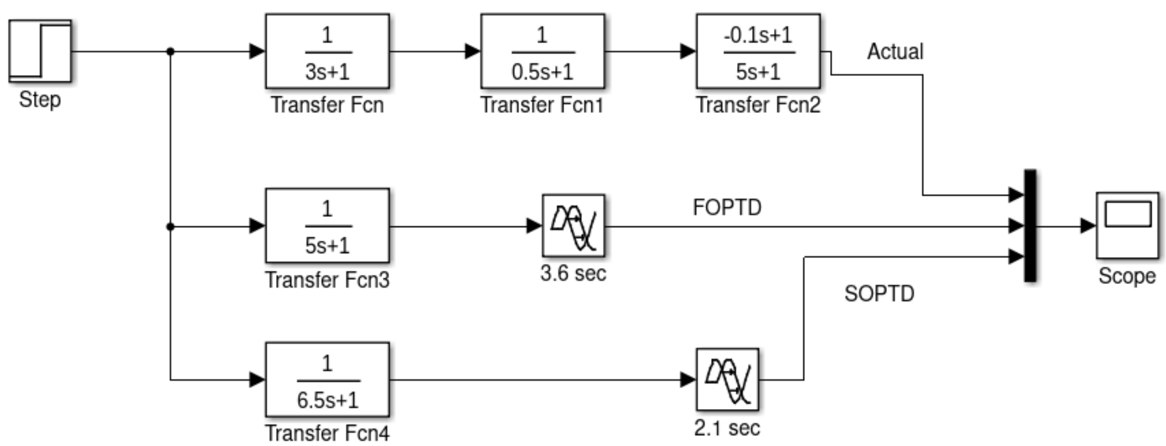
Derive models using Skogestad's rule:

- First order plus time delay
- Second order plus time delay take K = 1

we get transformed approximated plant transfer function as

$$H_{T1}(s) = \frac{e^{-3.75s}}{13.5s+1}, H_{H2}(s) = \frac{Ke^{-2.15s}}{(12s+1)(3.1s+1)} \tag{4.6}$$

4.4 Observation :



Simulink model of systems given is Part A

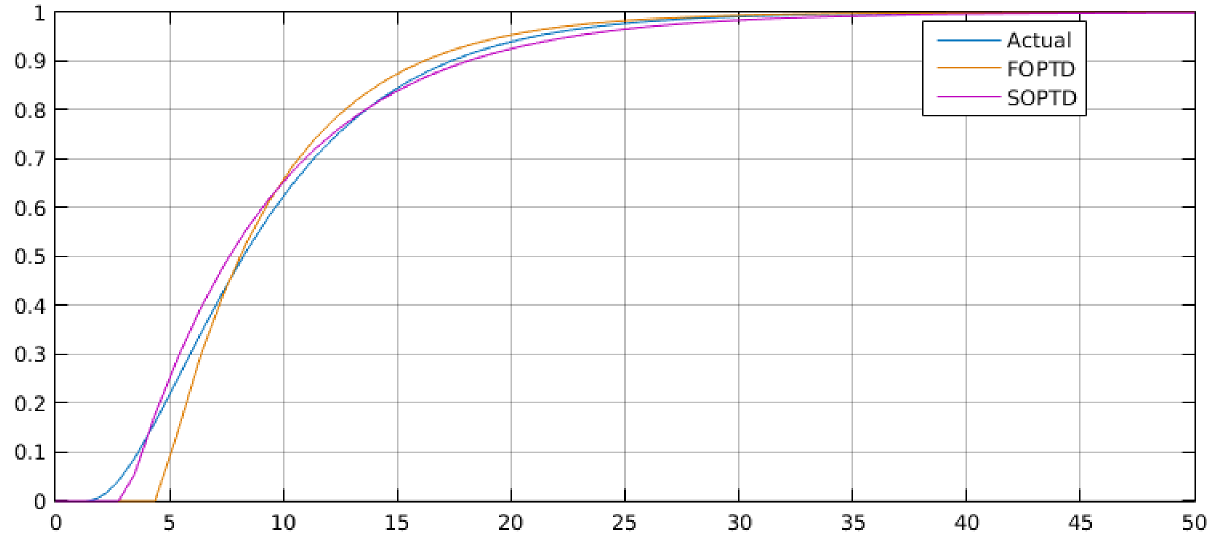


Figure 4.1: Step response of system with time delay

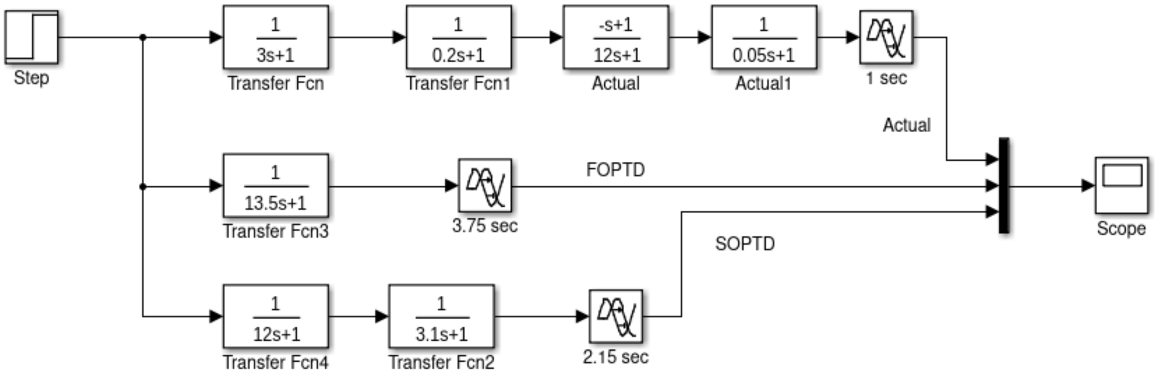


Figure 4.2: Simulink model of systems given in Part B

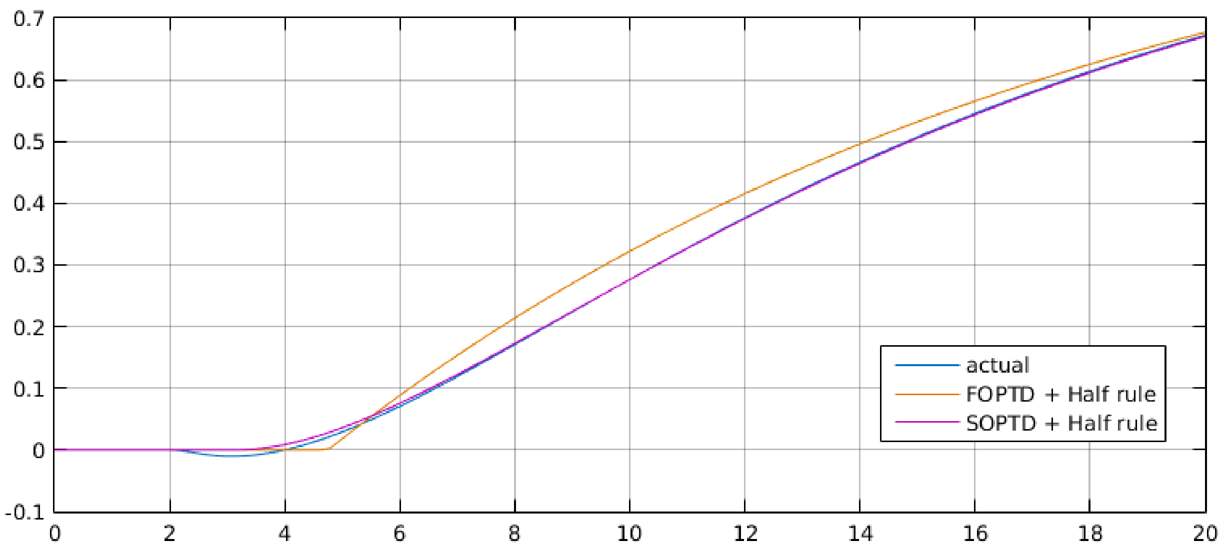


Figure 4.3: Step response of system approximated by Half rule

4.5 Conclusion :

- In part A we get the more time delay in Half rule approximation, then in Taylor series approximation in output response but settling time is almost same.
- In part B we get little spike in actual response. But in approximation we do not get any spike. Here FOPTD has more delay than SOPTD, and no changes in other characteristics.

Experiment : 05

5.1 Title : Study of offset in a response of First order system with proportional controller, and eliminating it using PI controller.

5.2 Apparatus :

MATLAB/Simulink software

5.3 Theory :

there is a plant $G_p(s)$ has first order transfer function and controller transfer function is termed as $G_c(s)$, we are giving step response to the closed loop unity feedback system in which,

$$G_p(s) = \frac{K_p}{1 + s\tau_p}, G_c(s) = K_c(\frac{1 + s\tau_i}{s\tau_i}) \tag{5.1}$$

Part A

first we apply proportional controller, i.e. K_c to first order system and analyse offset which is difference between actual value and desired value of response. Now Closed loop transfer function in this case is,

$$H_{cl}(s) = \frac{C(s)}{R(s)} = \frac{K_c K_p}{1 + K_c K_p + sT_p} \tag{5.2}$$

here $K_p = 1$, $T_p = 5$, Applying Routh's stability criterion, we get stability condition as

$$T_p > 0 \tag{5.3}$$

$$(1 + K_p K_c) > 0 \tag{5.4}$$

$$K_p K_c > -1 \tag{5.5}$$

here we need to change the value of K_c = -2,-0.5,1,5,10 and analyse the response of c(t), u(t),and its offset.

Part B

In this part we are using PI controller to cotrol the behavious of plant tansfer function and eliminate offset of proportional controller, so that closed transfer function of entire system is

$$Y_{cl}(s) = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)} \tag{5.6}$$

After substituting values

$$Y_{CL} = \frac{1+s\tau_i}{\frac{\tau_p\tau_i}{K_cK_p}s^2+\frac{\tau_i(1+K_pK_c)}{K_pK_c}s+1}$$

Now applying Routh Hurwitz's stability criteria to Y_{CL} and $\tau_i\tau_p > 0$ so that

$$\frac{\tau_i\tau_p}{K_cK_p} > 0$$
$$\frac{\tau_i(1+K_pK_c)}{K_cK_p} > 0$$
$$K_pK_c > 0$$

here $\tau_p = 5$, $K_i = 0.2$, 4,($\tau_i = 0.25$,5) and $K_c = 5$ and obeserve that whether offset is eliminated by PI only control or not.

5.4 Observation :

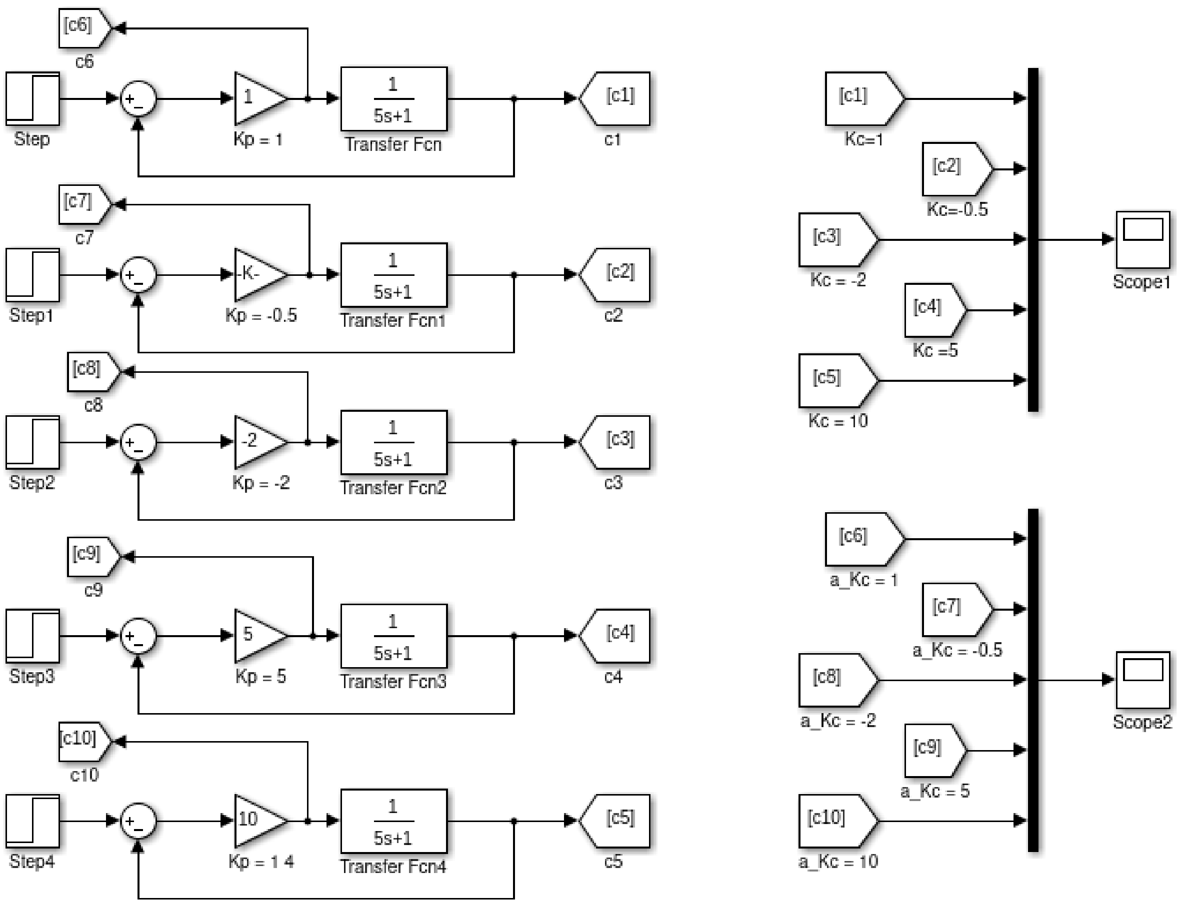


Figure 5.1: Simulink model of system for proportional control only

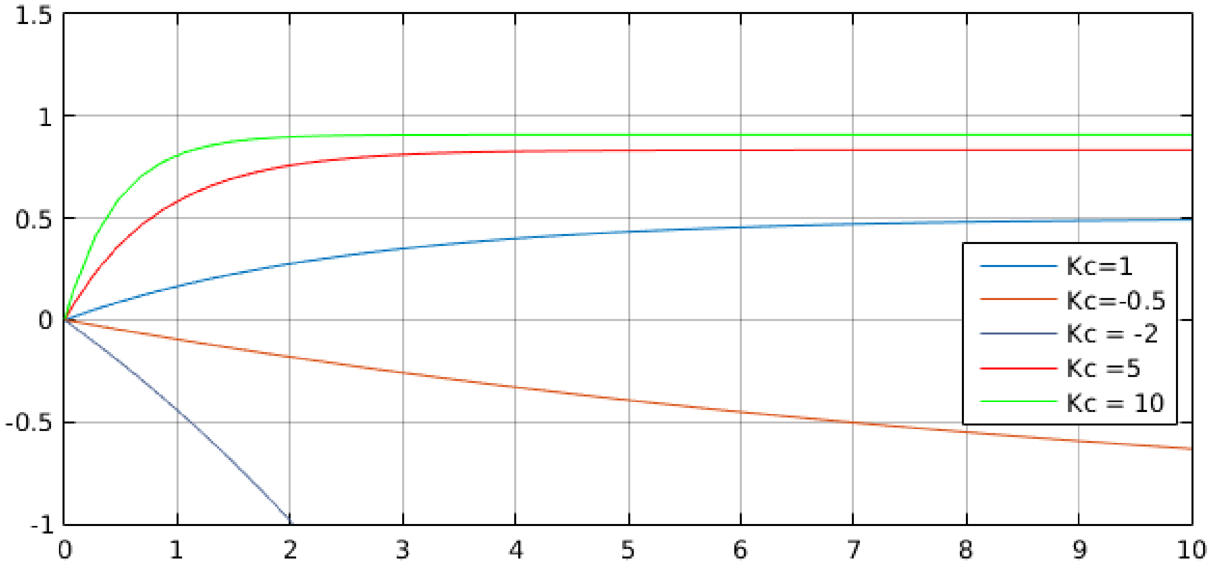


Figure 5.2: step response of systems with different value of K_c

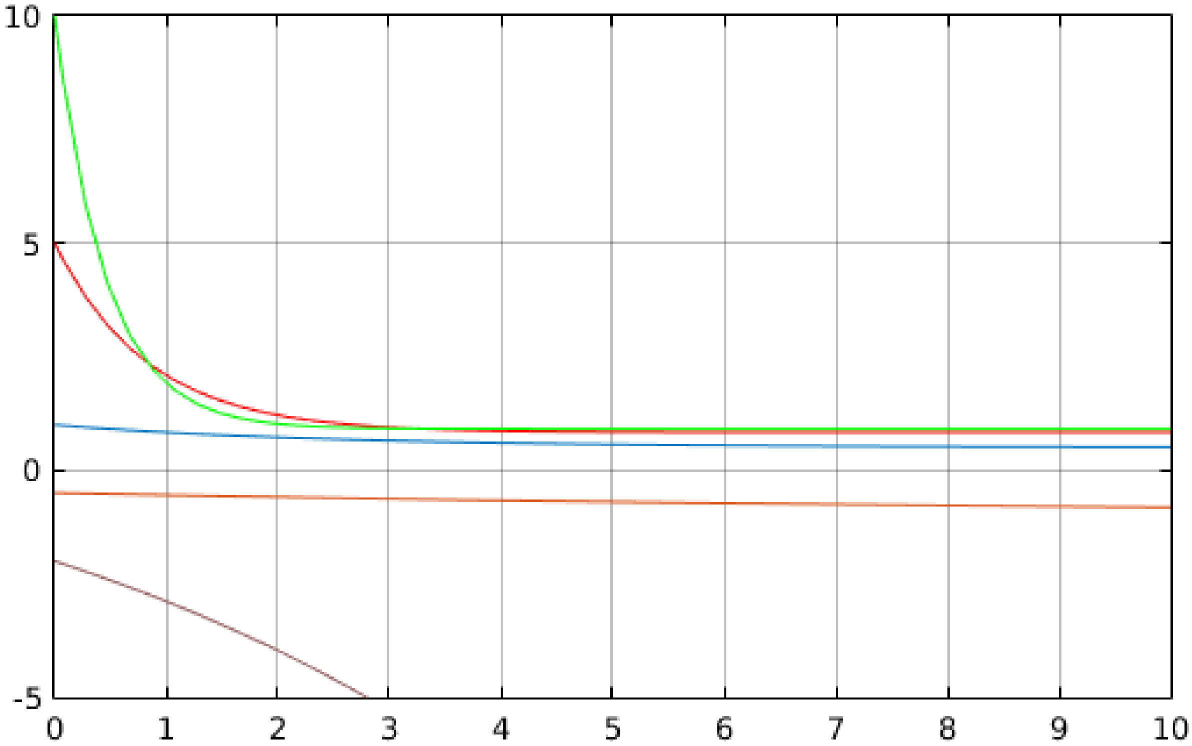


Figure 5.3: controller response of systems with different value of K_c

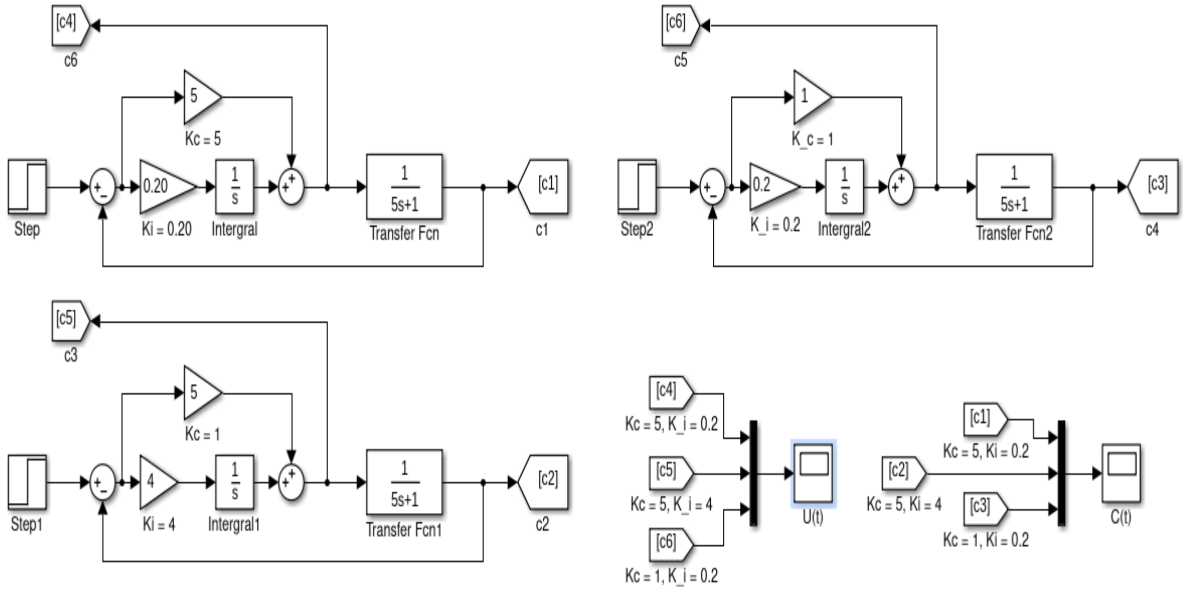


Figure 5.4: simulink model of systems given in Part B

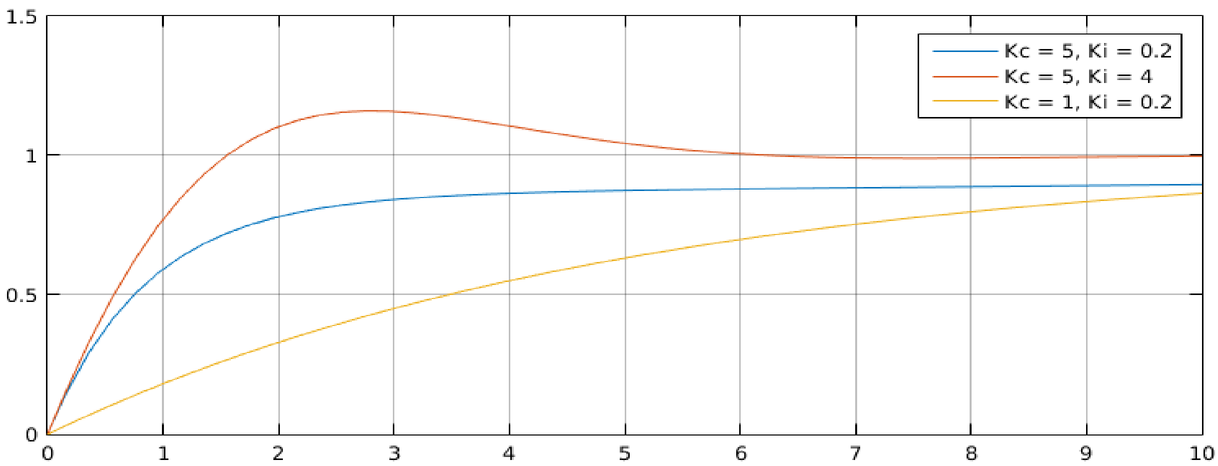


Figure 5.5: step response of systems with different value of K_i

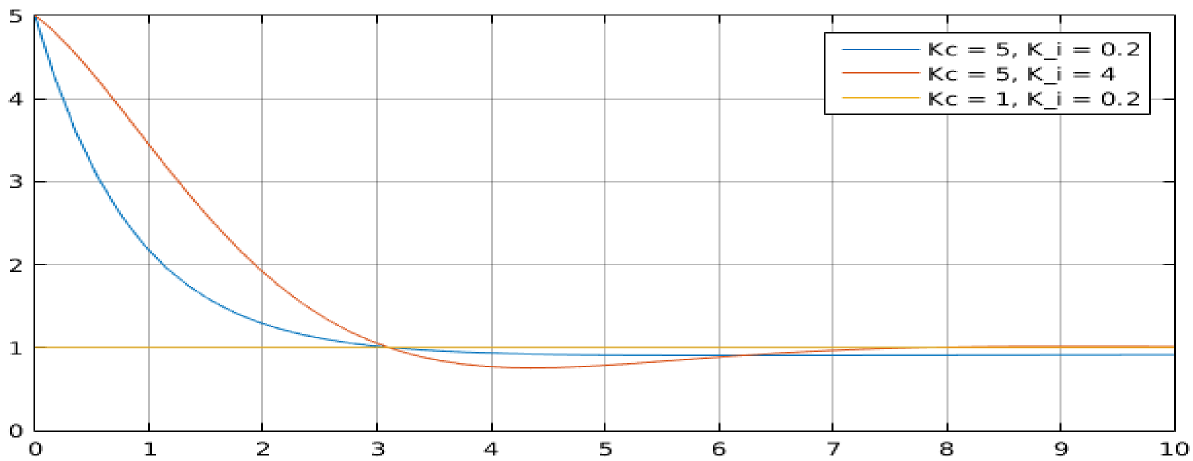


Figure 5.6: controller response of systems with different value of K_i

5.5 Result :

Value of Kc	Nature of response C(t)	Offset E(s) = 1/(1+KpKc)	Condition of Kc & Kp
10	Stable and Overdamped	0.091	Same sign Kp Kc >0
5	Stable and Overdamped	0.168	Same sign Kp Kc >0
1	Stable and Overdamped	0.5	Same sign Kp Kc >0
-0.5	system is conditiionally stable	2	not same sign -1<KpKc <0
-2	Unstable	–	not same sign Kp Kc <-1

Table 5.1: analysis of step response of system with different value of K_C

In part A we analyzes various step response and get details about how closed loop system responded at different value of K_c and details of offset and stability details given in table

Proportional Gain Kc	Integral gain Ki $(1\backslash\tau_i)$	Output Response C(t)	speed of controller
5	0.2	Stable and No offset	medium
5	4	Stable and No offset	faster
1	0.2	Stable and Offset	slow

Table 5.2: step response with varying K_c and K_i values

In part B we are using PI only control to achieve errorless steady state response

5.6 Conclusion :

- by increasing value of proportional gain K_c we get smaller offset but it has never becomes zero, it means some amount of band of error remains always in system in case of proportional only control.
- Due to presense of integrator, past value of system responses takes into consideration and by using those value error becomes zero and also becomes negetive because of continuosly increasing of sum at zero error also.
- to reduce steady state error we can use PI only control that will results satisfatorily, and achieves set point after 4 or 5 time contants but it will increase oscillation and system response becomes slower.

Experiment : 06

6.1 Title : Stability analysis of a first order open loop unstable process with proportional control.

6.2 Apparatus :

MATLAB/Simulink Software

6.3 Theory :

take a unstable first order plant transfer function $G_p(s)$ and Controller $G_c(s)$

$$G_p(s) = \frac{1}{1-5s} \tag{6.1}$$

$$\tag{6.2}$$

$$G_c(s) = K_c \tag{6.3}$$

closed loop unity feedback transfer function is Y_{CL}, with step input

$$Y_{CL} = \frac{C(s)}{R(s)} = \frac{K_c}{(1+K_c)-5s} \tag{6.4}$$

Now applying RH's stability criteria to find condition for stability

$$if : -5 < 0 \tag{6.5}$$

$$then : Kc + 1 < 0 \tag{6.6}$$

then system is stable if $K_c <$ -1, Now observe C(t)=controlled output and U(t)=controller output at value of different proportional gain of

$$K_c = 1, -1, -10, -30$$

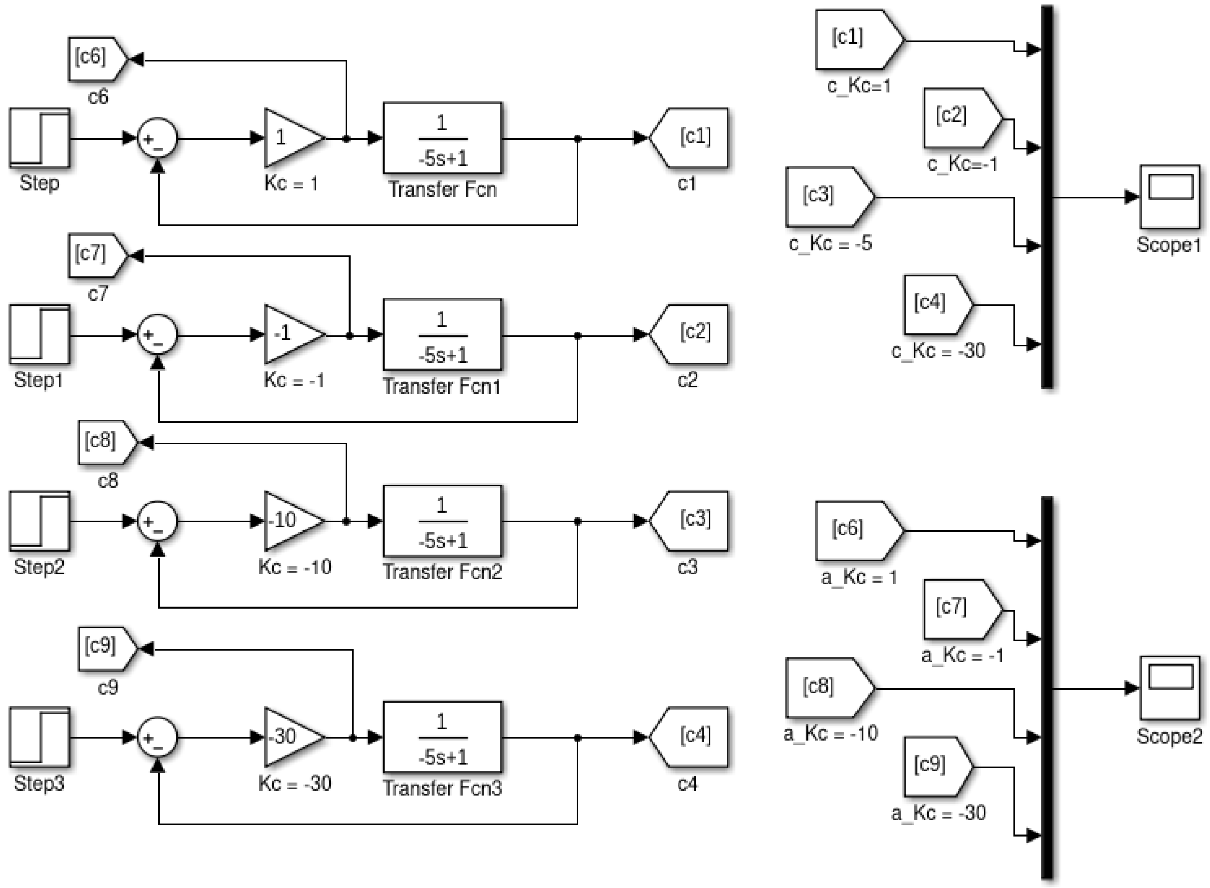


Figure 6.1: Simulink model of system for proportional control only

6.4 Observation :

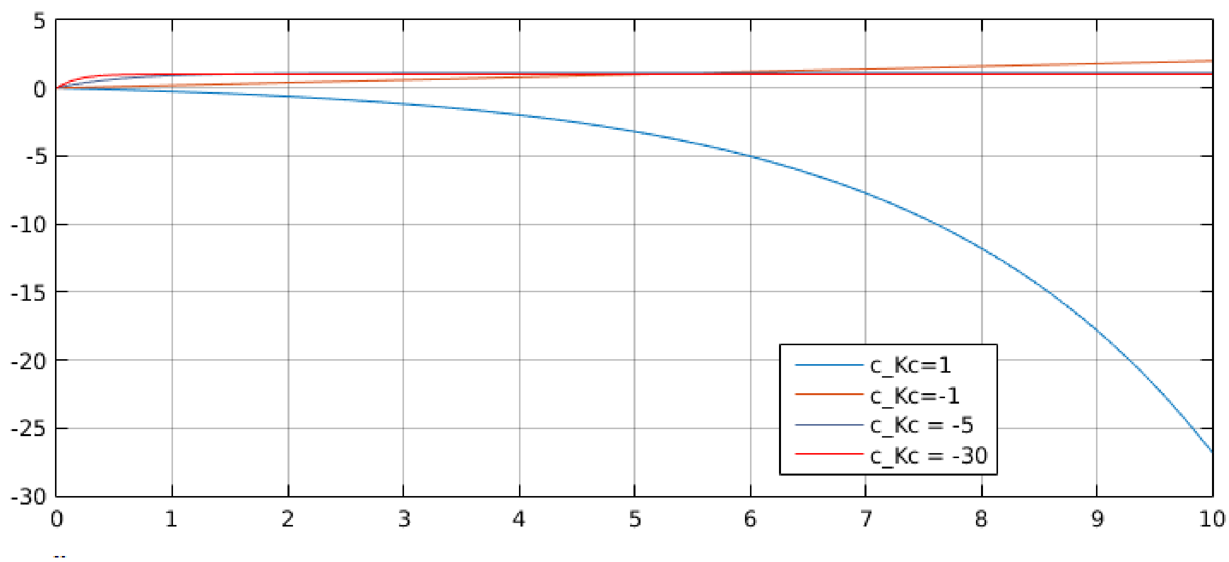


Figure 6.2: step response of systems with different value of K_c

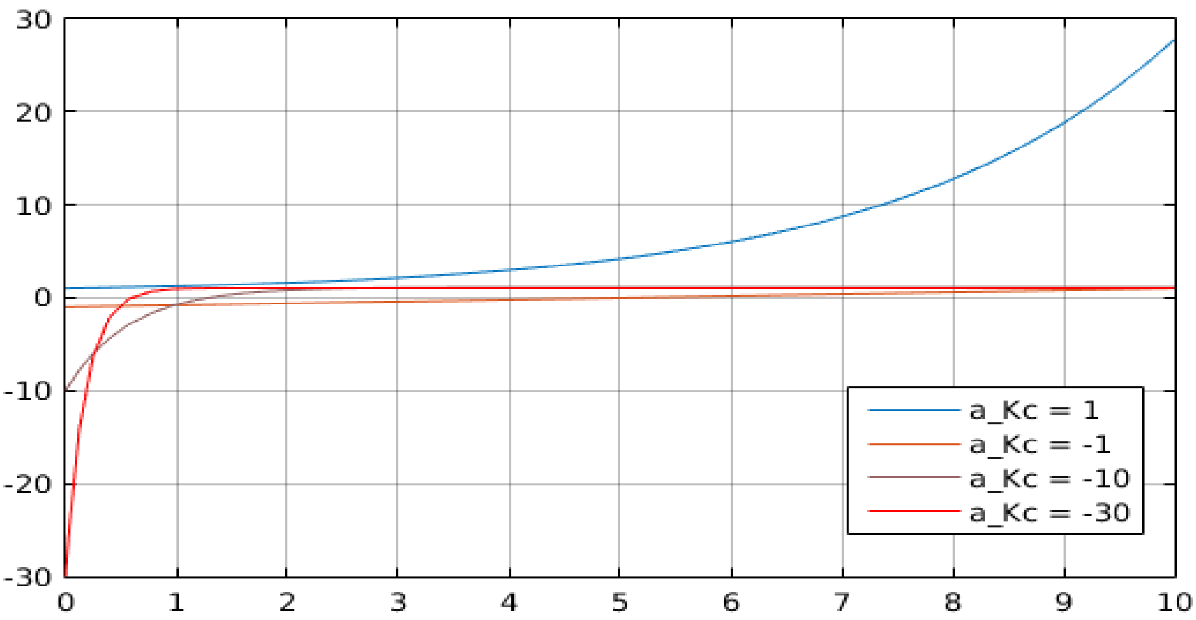


Figure 6.3: controller response of systems with different value of K_c

6.5 Result :

Proportional Gain Kc	Output Response	Offset	condition of Kp & Kc
1	Unstable and Unbounded	infinite	Kp Kc >-1
-1	Unstable and Unbounded	infinite	Kp Kc = -1
-10	Stable and Bounded	-0.1	Kp Kc <-1
-30	Stable and Bounded	-0.03	Kp Kc <<-1

Table 6.1: step response with varying K_c value

6.6 Conclusion :

- step response given to the unstable system gives unbounded output and approaches to infinite value
- from RH's stability criteria we can say that to operate system in stable region we need to make $K_c <$ -1, or in more general way it is $K_c < 0$
- from varying value K_c to negetive magnitude side we can observe that system stabilizes as closed loop right hand side pole effect is dilute huge and negetive gain value of K_c, in other sense we can get bounded output with some amount of offset.

Experiment : 07

7.1 Title : Application of DS method for PID controller settings for a second order plus time delay system

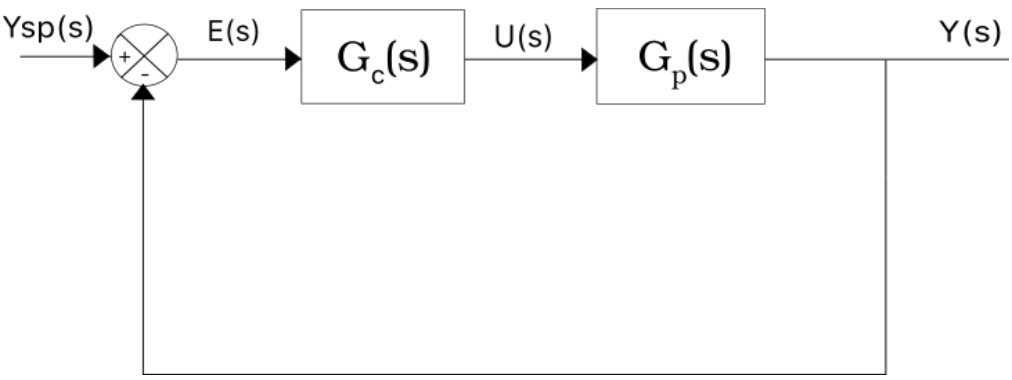
7.2 Aim : Design a PID controller for desired closed loop time constants

7.3 Apparatus :

Matlab/Simulink Software

7.4 Theory :

In the Direct Synthesis (DS) method, the controller design is based on a process model and a desired closed-loop transfer function. The latter is usually specified for set-point changes, but responses to disturbances can also be utilized. Although these feedback controllers do not always have a PID structure, the DS method does produce PI or PID controllers for common process models. As a starting point for the analysis, consider the block diagramof a feedback control system in Figure 7.1 The closed-loop transfer function for set-point changes was derived



$$\frac{Y}{Y_{sp}} = \frac{G_c G_p}{1 + G_c G_p}$$

Rearranging and sloving it we can get,

$$G_c = \frac{1}{G_p} \frac{Y/Y_{sp}}{1 - Y/Y_{sp}}$$

Also, it is useful to distinguish between the actual process G_p and the model,$\tilde{G}_p$, that provides an approximation of the process behavior. A practical design equation can be derived by replacing the unknown G_c by$\tilde{G}_p$, and Y/Ysp by a desired closed-loop transfer function, $(Y/Y_{sp})_d$.

$$G_c(s) = \frac{1}{\tilde{G}_p(s)} \frac{(Y/Y_{sp})_d}{1 - (Y/Y_{sp})_d} \tag{7.1}$$

hence desired closed loop transfer function for process without time delays, the
first order model :

$$(\frac{Y}{Y_{sp}})_d = \frac{1}{\tau_c s + 1}$$

and now substituting above values in 7.1 and solving for $G_C(s)$,

$$G_c(s) = \frac{1}{\tilde{G}_p} \frac{1}{\tau_c s} \tag{7.2}$$

the $\frac{1}{\tau_s}s$term provides integral control action and thus eliminate offset.
first order plus time delay model :
If the process transfer function contains a known time delay ,a reasonable choice for the desired closed-loop transfer

$$(\frac{Y}{Y_{sp}})_d = \frac{e^{-\theta s}}{\tau_c s + 1}$$

Combining above equation gives:

$$G_c(s) = \frac{1}{\tilde{G}_p \frac{e^{-\theta s}}{\tau_c s + 1 - e^{-\theta s}} (7.3)}$$

now approximate time delay term as Taylor series expansion:

$$e^{-\theta s} \approx 1 - \theta s$$

and then re arranging above eq7.3 gives,

$$G_c(s) = \frac{(1 - \theta s)}{\tilde{G}_p s(\tau_c + \theta)} \tag{7.4}$$

Now consider standard FOPTD model as

$$\tilde{G}_p(s) = \frac{K_p e^{-\theta s}}{\tau_p s + 1}$$

Now standard PI controller equation as

$$G_c = K_c(1 + \frac{1}{\tau_i s})$$

now comparing it with eq 7.4 we can get values of

$$k_c = \frac{\tau_p}{k_p(\theta + \tau_c)}$$

and also

$$\tau_i = \tau_p$$

Second order plus time delay model :

$$\tilde{G}_p(s) = \frac{k_p e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{7.5}$$

Now substituting into eq 7.4 and comparing with standard PID controller equation as shown below

$$G_c = K_c(1 + \frac{1}{\tau_i s} + \tau_d s)$$

we can get value of

$$k_c = \frac{1}{k_p}\frac{\tau_1 + \tau_2}{\tau_c + \theta}$$

and also

$$\tau_i = \tau_1 + \tau_2$$

$$\tau_d = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$$

7.5 Procedure :

Now we have process transfer function for analysis as

$$G_p(s) = \frac{2e^{-s}}{(10s+1)(5s+1)}$$

$$\tilde{G}_p(s) = \frac{0.9e^{-s}}{(10s+1)(5s+1)}$$

- Analyze step response when process is perfect($G_p = \tilde{G}_p$)
- Analyse with model gain $k_p = 0.9$ instead of 2(not perfect)
- when unit step change occured in both set point and disturbance, in disturbance it is occured at 80 sec
- simulate for 160 sec time and disturbance is introduced at 80 sec.

7.6 Observation :

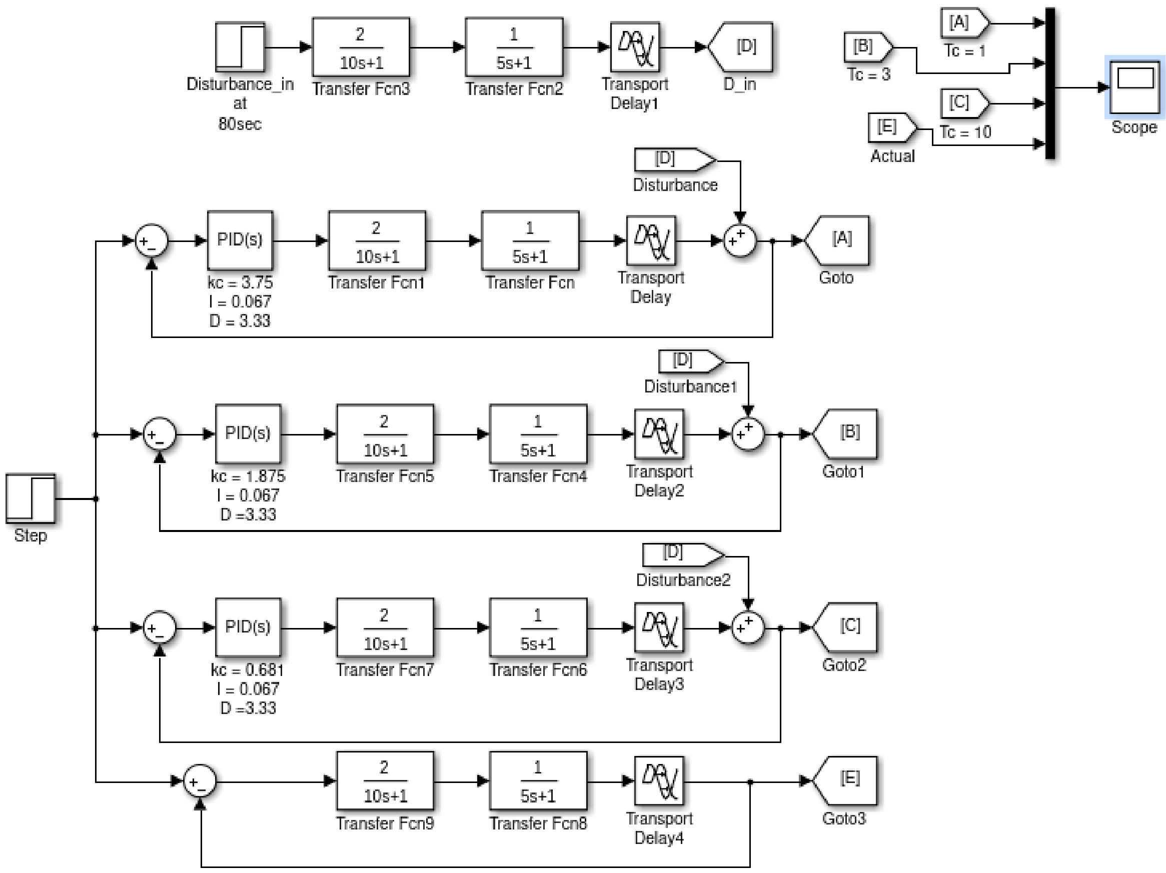


Figure 7.1: for K = 2, and τ_c= 1,3, and 10

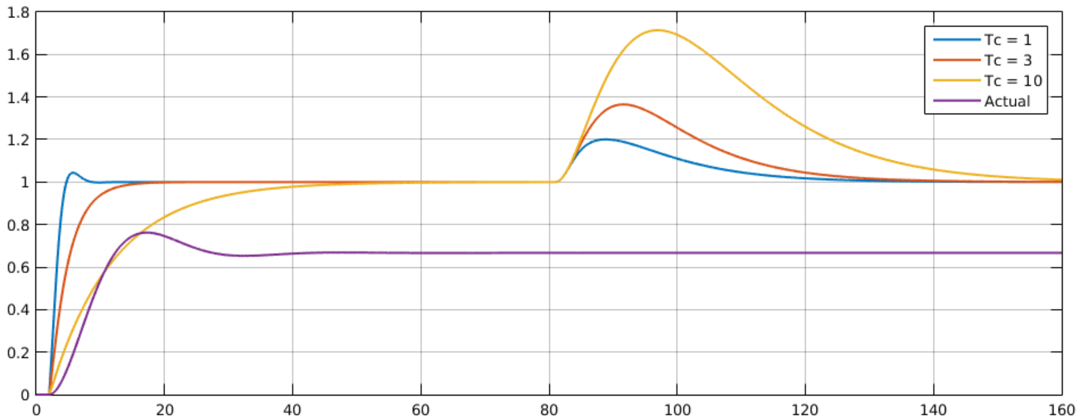


Figure 7.2: step response with disturbance at 80 sec of second order system

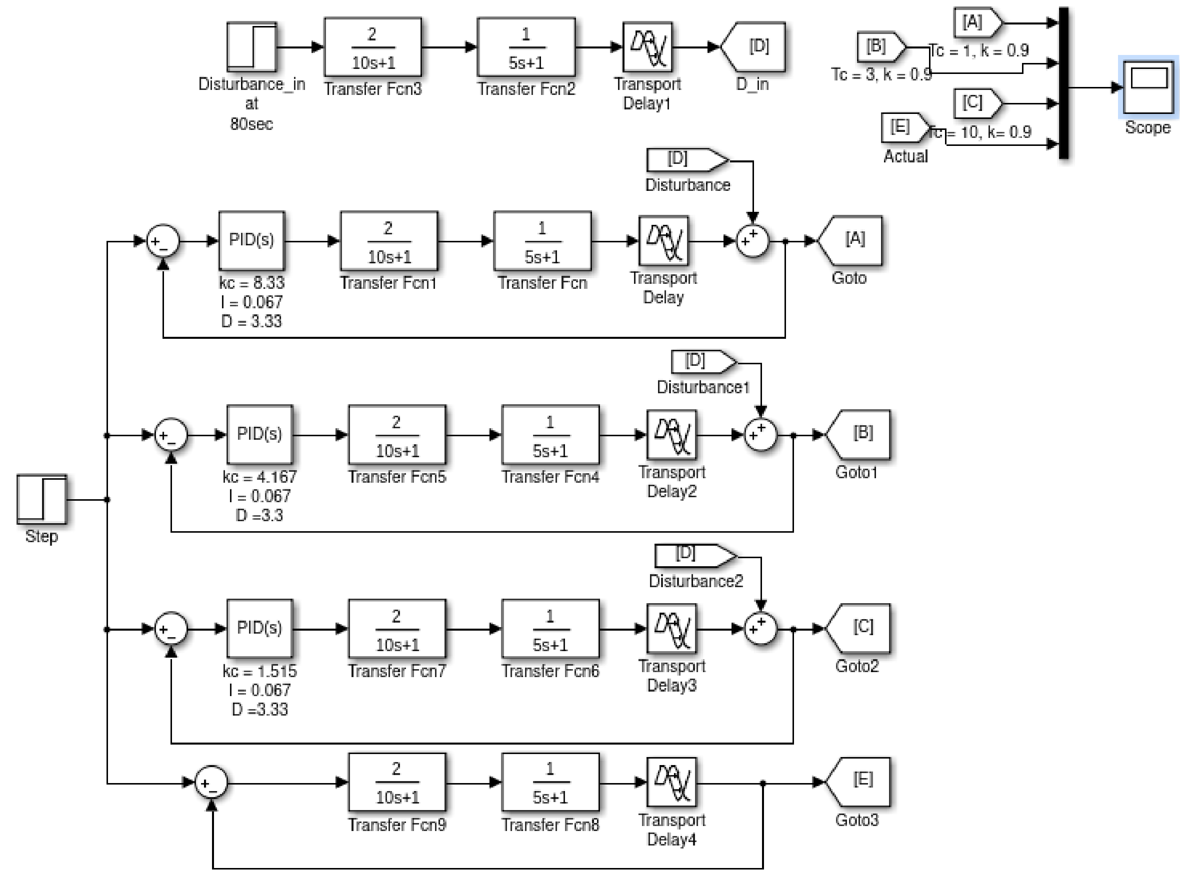


Figure 7.3: for K = 0.9, and τ_c= 1,3, and 10

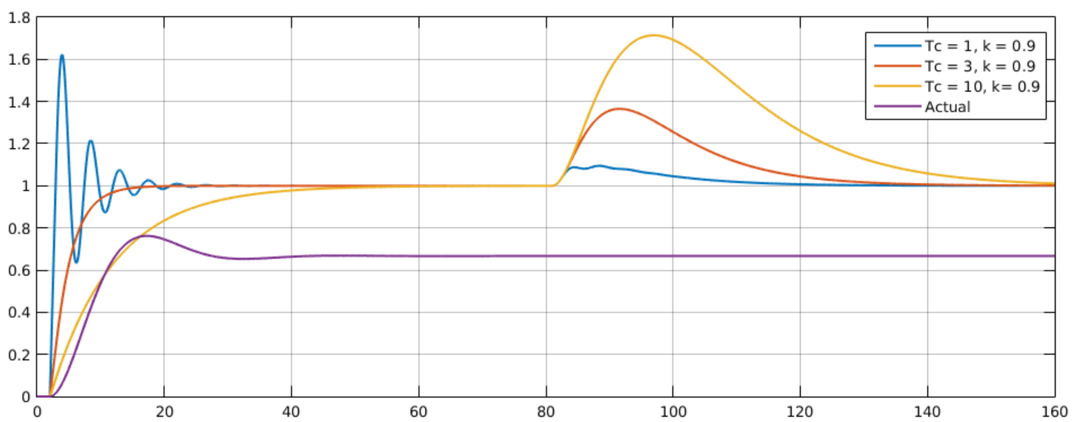


Figure 7.4: step response with disturbance at 80 sec of second order system

7.7 Results :

	$\tau_c = 1$	$\tau_c = 3$	$\tau_c = 10$
$k_c(k_p = 2)$	3.75	1.88	0.68
$k_c(k_p = 0.9)$	8.33	4.17	1.51
τ_i	15	15	15
τ_d	3.33	3.33	3.33

Table 7.1: Values of k_c and τ_i and τ_d

7.8 Conclusion :

- value of τ_c decreases as k_c increases or vice versa, but τ_i τ_d values fixed, deviation is large after disturbance occures.
- as τ_c increase response become sluggish
- in imperfact model oscillation starts quickly than perfact assumption
- decreasing k_p values cause oscillatory response or make system unstable at instant hence less value than 0.9 make it to unstable operating point.

Experiment : 08

8.1 Title : Internal Model Control (IMC) based PID controller design for I^{st} Order & II^{nd} Order systems

8.2 Apparatus

MATLAB/Simulink Software

8.3 Theory

The most common type of industrial controller is still the PID controller, and need of tuning and stabile performance is always desire. so for that we are using here IMC based controller that can be formulated in the standard feedback control structure and will result in equivalent PID controller.

PID tuning parameters are tweaked on tranfer function model, but it is not always clear how the process model effects the tuning decision. In the IMC formulation the controller $G_c(s)$, is based directly on the "good" part of the process transfer function.

we derive the feedback equivalence to IMC by using block diagram manipulation, begin with IMC structure shown in fig8.1 #A is converted to fig8.2 #B and then finally inner loop of the rearranged IMC structure shown in fig8.3 #C, internal loop with positive has transfer function like this.
let

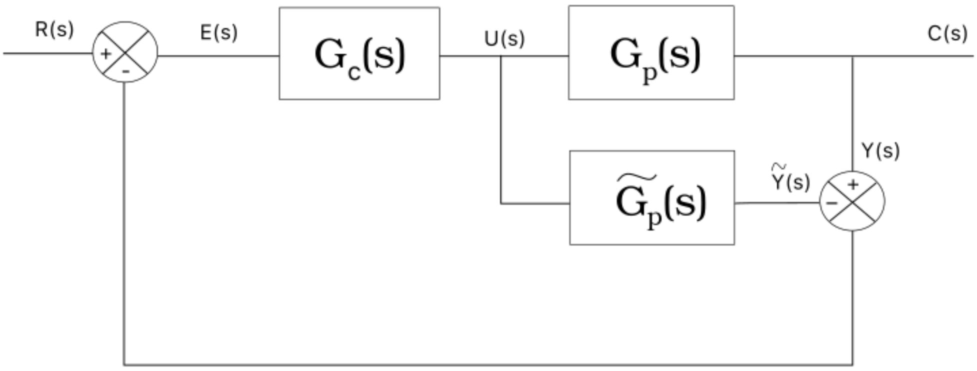
$$\begin{aligned}
C(s) = Y(s) &= G_p(s)U(s) \\
&= G_p(s)[G_c(s)e(s)] \\
&= G_p(s)[G_c(s)(R(s) - Y(s) + \widetilde{Y}(s))] \\
&= G_p(s)G_c(s)R(s) - G_p(s)G_c(s)Y(s) + G_p(s)G_c(s)U(s)\widetilde{G_p}(s) \\
&= G_c(s)G_p(s)R(s) - Y(s)G_c(s)(G_p(s) - \widetilde{G_p}(s))
\end{aligned}$$

$$\frac{Y(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 - G_c(s)(\widetilde{G_p}(s) - G_p(s))}$$

Or also we can write for Internal loop,

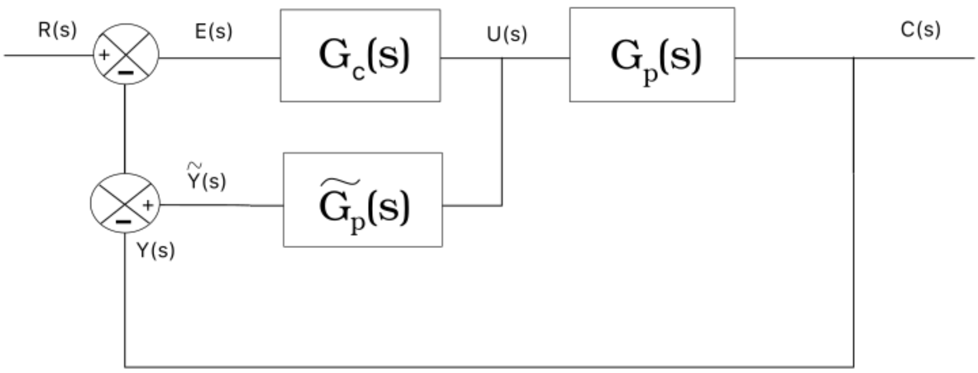
$$G_c^{\star} = \frac{U(s)}{R(s) - Y(s)} = \frac{G_c(s)}{1 - \widetilde{G_p}(s)G_c(s)} \tag{8.1}$$

where R(s)-Y(s) is error term used by a standard feedback controller.



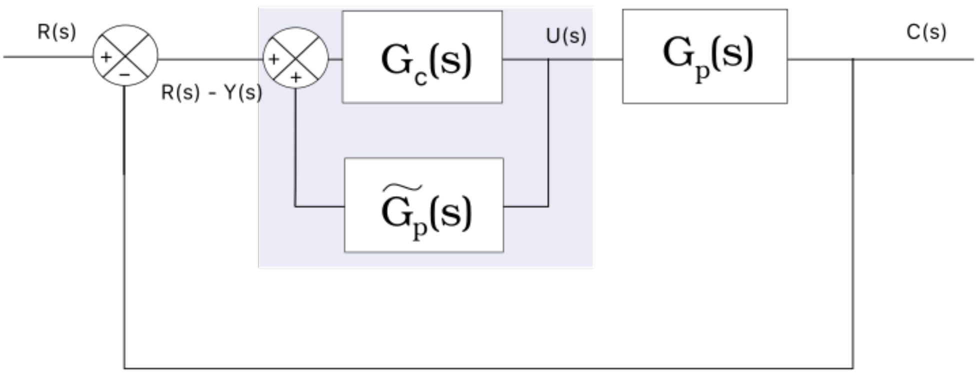
#[A]

Figure 8.1: Basic Block diagram of feedback conntrol system model using IMC



#[B]

Figure 8.2:



#[C]

Figure 8.3: Final FBC system model using IMC

8.3.1 Part 1: IMC Based PID Design for a First Order Process

Find the PID equivalent to IMC for a first-order process

$$\widetilde{G_p}(s) = \frac{k_p}{\tau_p s + 1} \tag{8.2}$$

Step 1: Develop a process model $\widetilde{G_p}$(s) and factor it into invertible(Good) and Non-invertible (Bad) portions.

$$\widetilde{G_p}(s) = \widetilde{G_{p-}}(s)\widetilde{G_{p+}}(s)$$

Step 2: Ideal IMC controller will be inverse of invertible part of $\widetilde{G_c}$(s)

$$\widetilde{G_c}(s) = (\widetilde{G_p}(s))^{-1}$$

Step 3: Find the IMC controller transfer function, $G_c(s)$, which includes a filter to make $G_c(s)$ semi proper

$$\mathbf{G_c(s) = \widetilde{G_c}(s)f(s) = G_{p-}^{-1}(s)f(s) = \frac{\tau_p s + 1}{k_p}\frac{1}{\lambda s + 1}} \tag{8.3}$$

so that

$$G_c(s) = \frac{\tau_p s + 1}{k_p(\lambda s + 1)}$$

Step 4: Find the equivalent standard feedback controller using the transformation

$$G_c^{\star}(s) = \frac{G_c(s)}{1 - \widetilde{G_p}(s)G_c(s)}$$

$$= \frac{\frac{\tau_p s+1}{k_p(\lambda s+1)}}{1-\frac{k_p(\tau_p s+1)}{k_p(\tau_p s+1)(\lambda s+1)}}$$

$$G_c^{\star}(s) = \frac{\tau_p s + 1}{k_p \lambda s} \tag{8.4}$$

Step 6: Rearrange above equation as per time constant form of PI controller as shown below

$$= k_c(1 + \frac{1}{\tau_i s})$$

$$G_c^{\star}(s) = \frac{\tau_p}{k_p\lambda} + \frac{1}{s k_p \lambda}$$

$$k_c = \frac{\tau_p}{k_p\lambda}$$

$$\tau_i = \tau_p$$

For analysis of that we have taken plant transfer function

$$G_p(s) = \frac{2}{1 + 10s}$$

so here we have value of $k_p = 2, \tau_p = 10$ and $\lambda = 2\&5$, and then we get the value of k_c = 2.5 & 1, and $\tau_i = 10$. we have analyse step response of IMC based PI controlled first order system with different value of λ.

8.3.2 Part 2: IMC based PID design for Second Order Process

Find the PID equivalent to IMC for a Second-order process

$$\widetilde{G_p}(s) = \frac{k_p}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{8.5}$$

Step 1: Develop a process model $\widetilde{G_p}$(s) and factor it into invertible(Good) and Non-invertible (Bad) portions.

$$\widetilde{G_p}(s) = \widetilde{G_{p-}}(s)\widetilde{G_{p+}(s)}$$

Step 2: Ideal IMC controller will be inverse of invertible part of $\widetilde{G_c}$(s)

$$\widetilde{G_c}(s) = (\widetilde{G_p}(s))^{-1}$$

Step 3: Find the IMC controller transfer function, $G_c(s)$, which includes a filter to make $G_c(s)$ semi proper

$$\mathbf{G_c(s) = \widetilde{G_c}(s)f(s) = G_{p-}^{-1}(s)f(s) = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k_p}\frac{1}{1 + s\lambda}} \tag{8.6}$$

Step 4: Find the equivalent standard feedback controller using the transformation

$$G_c^\star(s) = \frac{G_c(s)}{1 - \widetilde{G_p}(s)G_c(s)}$$

$$G_c^\star(s) = \frac{\tau_1\tau_2 s^2 + (\tau_1 + \tau_2)s + 1}{k_p \lambda s}$$

$$G_c^\star(s) = \frac{(\tau_1 + \tau_2)}{k_p\lambda} + \frac{1}{k_p\lambda s} + \frac{\tau_1\tau_2 s}{k_p\lambda} \tag{8.7}$$

Step 6: Rearrange above equation as per time constant form of PID controller as shown below

$$= k_c(1 + \frac{1}{\tau_i s} + s\tau_d)$$

$$k_c = \frac{\tau_1 + \tau_2}{k_p\lambda}$$

$$\tau_i = \tau_1 + \tau_2$$

$$\tau_d = \frac{\tau_1\tau_2}{\tau_1 + \tau_2}$$

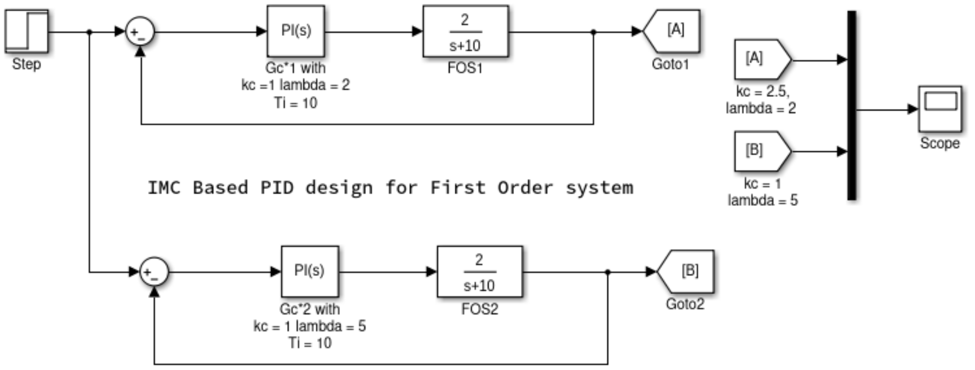
For analysis of that we have taken plant transfer function

$$G_p(s) = \frac{2}{(1 + 10s)(1 + 5s)}$$

so here we have value of $k_p = 2, \tau_1 = 10, \tau_2 = 5$ and $\lambda = 2\&5$, and then we get the value of $k_c = 3.75$ & 1.5, and $\tau_i = 15, \&\tau_d = 3.33$. we have analyse step response of IMC based PID controlled second order system with different value of λ.

8.4 Observation :

Part 1



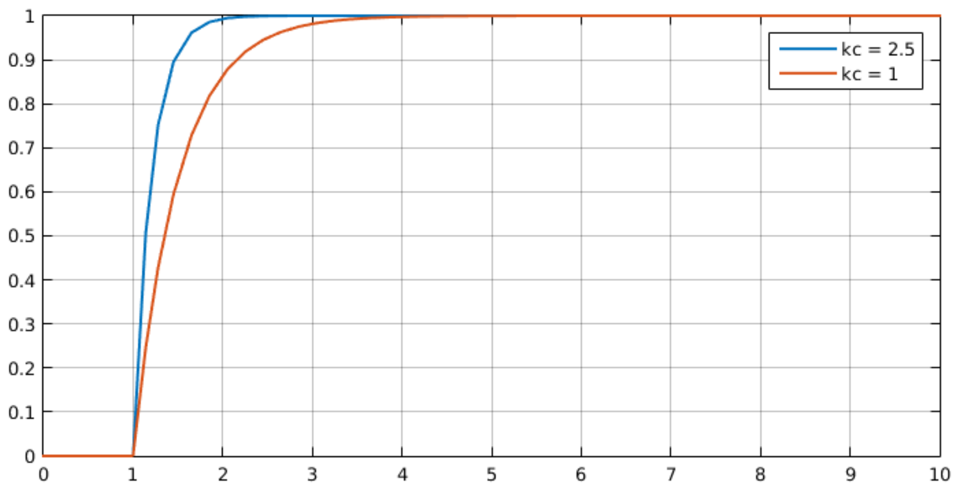
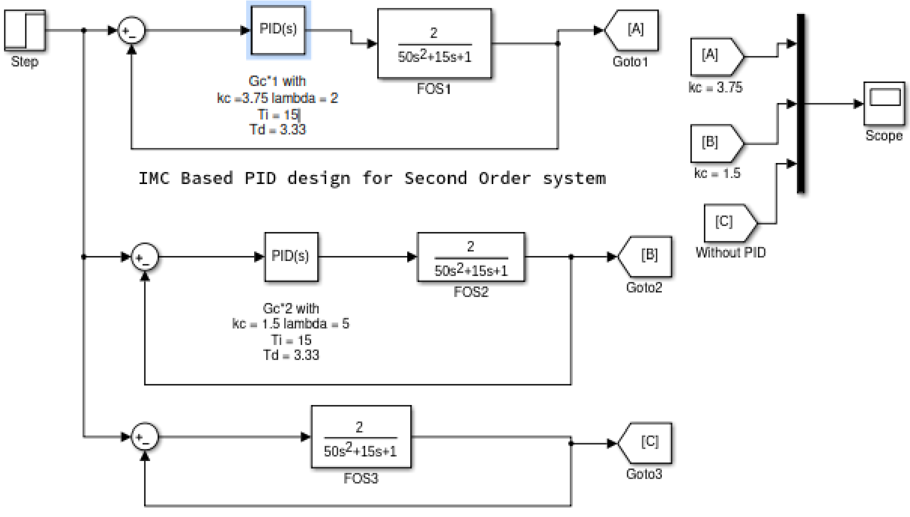


Figure 8.4: Step response of IMC based PI control of First order system

Part 2



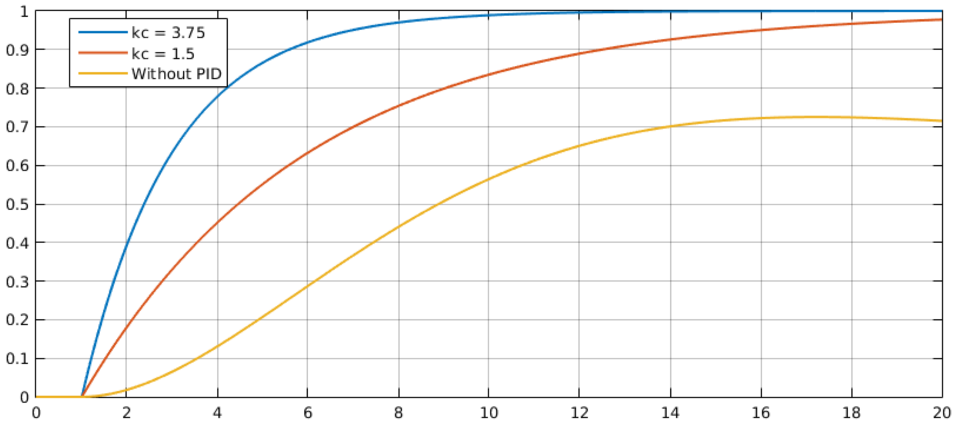


Figure 8.5: Step response of IMC based PID control of Second Order System

8.5 Results :

	k_c	τ_i
$\lambda = 2$	2.5	10
$\lambda = 5$	1	10

Table 8.1: Values of k_c and τ_i

	k_c	τ_i	τ_d
$\lambda = 2$	3.75	15	3.33
$\lambda = 5$	1.5	15	3.33

Table 8.2: Values of k_c and τ_i and τ_d

8.6 Conclusion :

- in part 1 : we get offset in output without controller so using IMC based controller we get better response (IMC based using PI only)
- as value of λ is increasing settling time increasing
- in part 2 of second order system PID controller removes offset efficiently and output response is nicely tracking set point
- on changing value of λ only k_c changes other have no effect and settling time also increases in case of second order system also as λ increases

Experiment : 09

9.1 IMC based PID Controller design for Systems with dead time, and Unstable system

9.2 Apparatus :

MATLAB/Simulink

9.3 Theory :

In order to arrive at a PID equivalent form for processes with a time-delay, we must make some approximation to the deadtime,so We will use a first-order Padé approximation for deadtime. I^{st} order padé approx as

$$e^{-\theta s} = \frac{1 - 0.5\theta s}{1 + 0.5\theta s}$$

9.3.1 Part 1 : FOPTD system

Now lets take, First Order Plus Delay Time FOPTD system:

$$\widetilde{G_p}(s) = \frac{k_p e^{-\theta s}}{1 + s\tau_p}$$

Step : 1

$$\widetilde{G_p}(s) = \frac{k_p(1 - 0.5\theta s)}{(1 + s\tau_p)(1 + 0.5\theta s)}$$

Step : 2 Factor out non invertible terms(stable part or LHS part)

$$\widetilde{G_{p-}}(s) = \frac{k_p}{(1 + s\tau_p)(1 + 0.5\theta s)}$$

Step : 3 Ideal IMC controller as

$$\widetilde{G_c}(s) = \frac{(1 + s\tau_p)(1 + 0.05\theta s)}{k_p}$$

Step : 4 IMC controller with filter

$$G_c(s) = \frac{(1 + s\tau_p)(1 + 0.05\theta s)}{k_p} \frac{1}{1 + \lambda s}$$

Step : 5

$$G_c^{\star}(s) = \frac{G_c(s)}{1 - \widetilde{G_p}(s) G_c(s)}$$

$$G_c^{\star} = \frac{\frac{(1+s\tau_p)(1+0.5\theta s)}{k_p(1+\lambda s)}}{1-\frac{(k_p(1-0.5\theta s))(1+s\tau_p)(1+0.5\theta s)}{(1+s\tau_p)(1+s\theta 0.5)k_p(1+\lambda s)}}$$

$$= \frac{(1+s\tau_p)(1+0.5\theta s)}{k_p(1+\lambda s) - k_p(1-0.5\theta s)}$$

$$= \frac{s^2\tau_p 0.5\theta s + s(\tau_p + 0.5\theta) + 1}{sk_p(\lambda + 0.5\theta)}$$

$$= \frac{(\tau_p + 0.5\theta)}{k_p(\lambda + 0.5\theta)} + \frac{1}{s(k_p(\lambda + 0.5\theta))} + \frac{s(\tau_p 0.5\theta)}{k_p(\lambda + 0.5\theta)}$$

Step : 6 Comparing it with Ideal PID controller equation we can get values of k_c,τ_i, τ_d,

$$k_c = \frac{\tau_p + 0.5\theta}{k_p(\lambda + 0.5\theta)}$$

$$\tau_i = \tau_p + 0.5\theta)$$

$$\tau_d = \frac{\theta\tau_p}{2\tau_p + \theta}$$

now we have plant transfer function with time dalay as

$$\widetilde{G_p}(s) = \frac{2e^{-5s}}{1+10s}$$

so from that we can get $k_p = 2$, $\theta = 5$, and $\tau_p = 10$, Now substituting above values in eq() we can get $k_c = 0.833$, $\tau_i = 12.5$, and $\tau_d = 2$. here we have assume value of $\lambda = \theta$ for study of performance but there is value of $\lambda > 0.8\theta$ is conclude by other researchers experiment that will results in more accurate response of systems.

9.3.2 Part 2 : Unstable System

For unstable process more complicated filter transfer function required, so here we have first order unstable process is

$$\widetilde{G_p}(s) = \frac{k_p}{1 - \tau_u s}$$

Step 1 : Find the IMC controller transfer function, $G_c(s)$

here we also need to make controller transfer function to semi proper so that, $f(s)\mid_{s=p_u} = 1$

$$f(s) = \frac{1 + s\gamma}{(1+s\lambda)^n}$$

value of γ that satisfies

$$f(s = p_u) = 1$$

$$P_u = \frac{1}{\tau_u}$$

$$G_c(s) = \frac{(1-\tau_u s)(\gamma s + 1)}{k_p(\lambda s + 1)^2} \mid_{n=2}$$

$$f(s) = \frac{\gamma s + 1}{k_p(\lambda s + 1)^2} = 1$$

after solving, we can get

$$\gamma = \lambda[\frac{\lambda}{\tau_u} + 2]$$

Step 2: Find the equivalent standard feedback controller using the transformation

$$G_c^\star = \frac{G_c(s)}{1 - \widetilde{G_p}(s)G_c(s)}$$

$$= \frac{\frac{1-\tau_u s}{k_p}\frac{\gamma s+1}{(\lambda s+1)^2}}{1-\frac{k_p}{1-\tau_u s}\frac{1-\tau_u s}{k_p}\frac{\gamma s+1}{(\lambda s+1)^2}}$$

$$= \frac{(1 - \tau_u s)(\gamma s + 1)}{k_p(\lambda^2 s^2 + 2\lambda s - \gamma s)}$$

multiply and divide with γs

$$= \frac{\frac{1}{k_p}(1-\tau_u s+1)(1+\gamma s)}{\lambda^2 s^2+s(2\lambda-\gamma)}\frac{\gamma s}{\gamma s}$$

by simplifying it

$$= \frac{\frac{1}{k_p}(1-\tau_u s)\frac{\gamma s+1}{\gamma s}}{\frac{\lambda^2 s}{\gamma}+\frac{(2\lambda-\gamma)}{\gamma}}$$

and then we can get,

$$= \frac{\frac{\gamma(1-\tau_u s)(\gamma s+1)}{k_p\gamma s(2\lambda-\gamma)}}{\frac{\lambda^2 s}{2\lambda-\gamma}+1}$$

Now substituting value of γ as

$$\gamma = \lambda(\frac{\lambda}{\tau_u} + 2)$$

$$G_c^\star = \frac{\frac{\gamma(1-\tau_u s)(1+\gamma s)}{k_p\gamma s(2\lambda-\gamma)}}{\frac{\lambda^2}{2\lambda-(\frac{\lambda^2}{\tau_u}+2\lambda)}s+1}$$

$$G_c^{\star} = \frac{\gamma}{k_p(2\lambda - \gamma)} \frac{\gamma s + 1}{\gamma s}$$

Step : 3 This is in form of PI controller, in which

$$k_c = \frac{\gamma}{k_p(2\lambda - \gamma)} \tag{9.1}$$

$$\tau_i = \gamma \tag{9.2}$$

here we have $k_p = 1$, $\tau_p = 1$, and we have to analyse all responses with different γ values, like $\gamma = 0.5, 1, 2$, by substituting above values in above equation we can get values for $\lambda = 0.5$ as $k_c = -1$, $\gamma = \tau_i = 1.25$ for $\lambda = 1$ we get $k_c = -1$, $\tau_i = 3$ for $\lambda = 2$ we get $k_c = -2$, $\tau_i = 8$

9.4 Observation :

Part 1

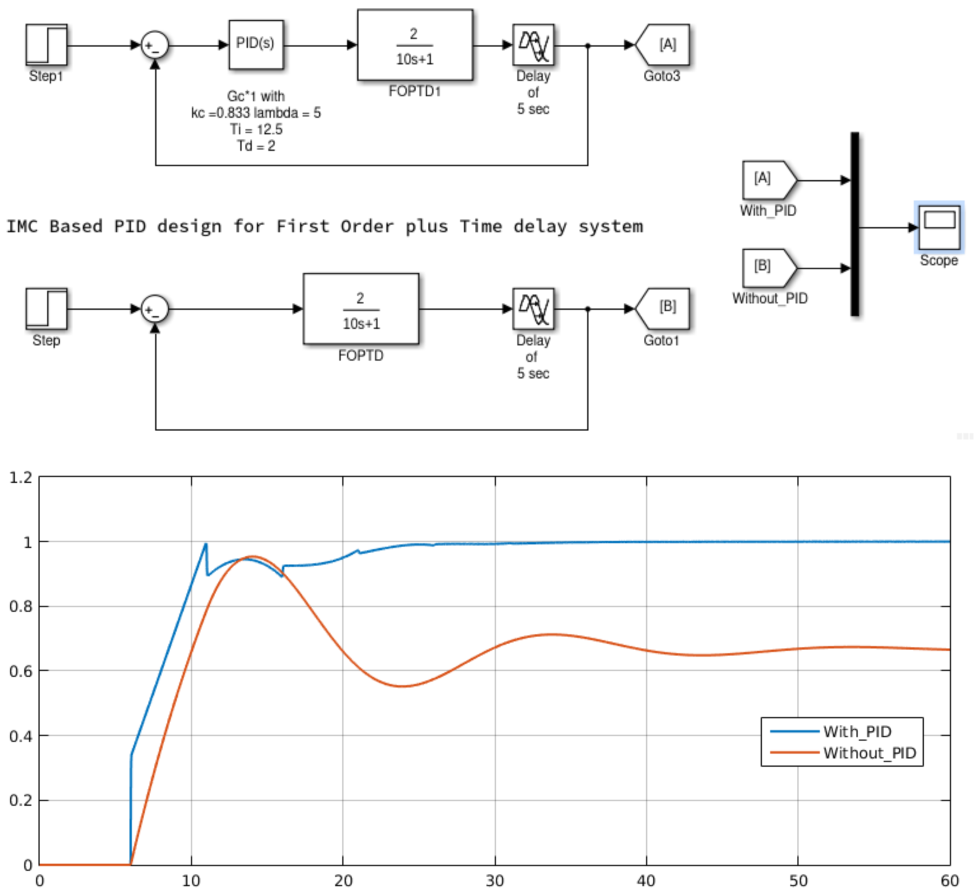


Figure 9.1: Step response of IMC based PID control of First order plus time delay system

Part 2

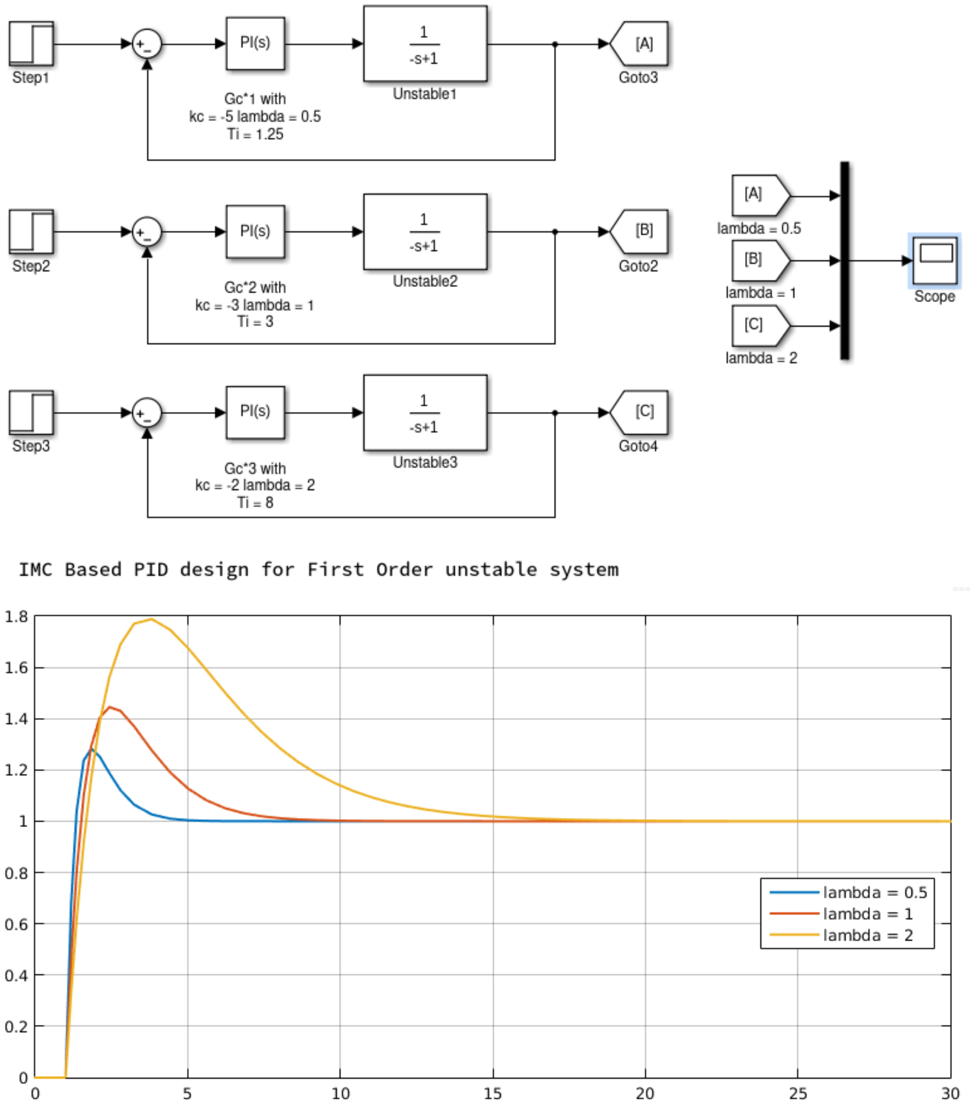


Figure 9.2: Step response of IMC based PI control of Unstalbe System

9.5 Results :

	k_c	τ_i	τ_d
$\lambda = 5$	0.833	12.5	2
$\lambda = 10$	0.5	12.5	2
$\lambda = 15$	0.35	12.5	2

Table 9.1: Values of k_c and τ_i and τ_d

	k_c	τ_i
$\lambda = 0.5$	-5	1.25
$\lambda = 1$	-3	3
$\lambda = 2$	-2	8

Table 9.2: Values of k_c and τ_i

9.6 Conclusion :

- in part 1 : oscillatory response and offset both can be eliminated at desired level using IMC based PID controller in FOPTD as λ values are increased response smooth out or good set point tracking achieved
- in part 2 : in unstable process we need to choose the desired value for λ as a trade-off between performance and robustness and it is achieved better response as λ uncreases peak value increases.

Experiment : 10

10.1 Design of PID Controller based on Zeigler Nichols closed loop oscillation & Tyreus Luyben method

10.2 Aim:

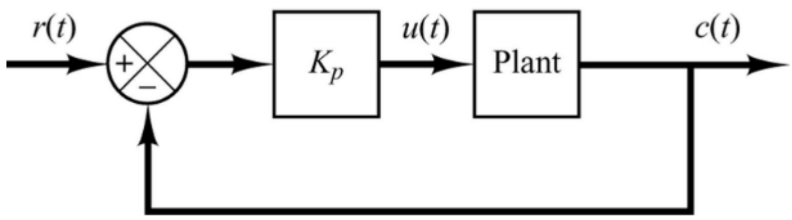
- Design PID controller based on Ziegler Nichols & Tyreus Luyben method
- Evaluate performance criteria using IAE, ISE, & ITAE

10.3 Apparatus

Matlab Simulink Software

10.4 Theory:

Let's start with closed loop system as shown in figure



So here we have plant with proportional controller **K** in unity feedback closed loop configuration in which we have to increase value of K upto K_u where ,K_u is value at firstly sustained oscillation occures.

Now find value of τ_u from steady state sustained oscillation condition, where τ_u is time period in seconds for oscillating waveform

For slightly larger values of controller gain, the closed-loop system is unstable, while for slightly lower values the system is stable. so we can choose the values of controller as per table shown below for which we first need to calculate K_u and τ_u for particular system by using routh-array stability criteria or by any means.

based on Ziegler Nichols method

Type of Controller	K_c	τ_i	τ_d
P	$0.5K_u$	∞	0
PI	$0.45K_u$	$\frac{1}{1.2\tau_u}$	0
PID	$0.6K_u$	$\frac{\tau_u}{2}$	$\frac{\tau_u}{8}$

Tyreus and Luyben have suggested tuning parameter rules that result in less oscillatory responses and that are less sensitive to changes in the process condition. so we can tune PI , and PID controller by putting values of K_u and τ_u in below table.

Based on Tyreus Luyben method

Type of Controller	K_c	τ_i	τ_d
P	$0.5K_u$	∞	0
PI	$\frac{K_u}{3.2}$	$2.2\tau_u$	0
PID	$\frac{K_u}{2.2}$	$2.2\tau_u$	$\frac{\tau_u}{6.3}$

So to compare effectiveness and evaluate its performance we need to use some criteria IAE, ISE, & ITAE as shown below

- IAE = Integral Absolute Error $\int_0^\infty |e(t)|\,dt$
- ISE = Integral Square Error $\int_0^\infty e^2(t)\,dt$
- ITAE = Integral Time Absolute Error $\int_0^\infty t|e(t)|\,dt$

10.5 Procedure

we have plant transfer function as

$$G(s) = \frac{1}{6s^3 + 11s^2 + 6s + 1}$$

and by using routh array stability criteria we can get value of $K_u = 10$ and $\tau_u = 6.28$ and then evaluate above system using Ziegler Nichols and Tyreus Luyben method and find the value of IAE, ISE, and ITAE for each controller tuning methods.

10.6 Observation

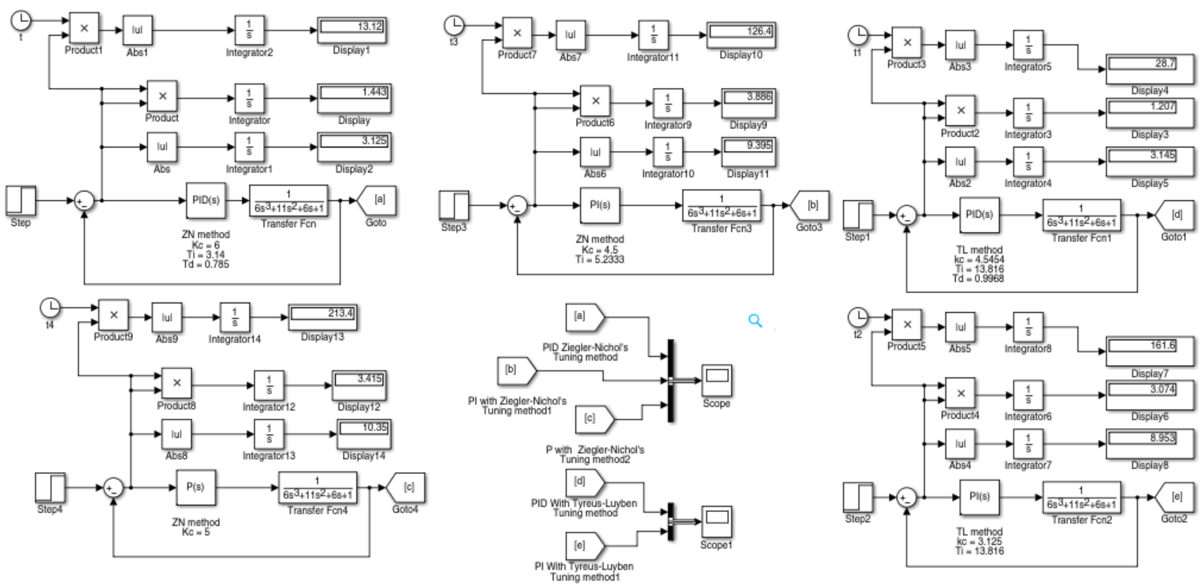


Fig 1: System under Observation

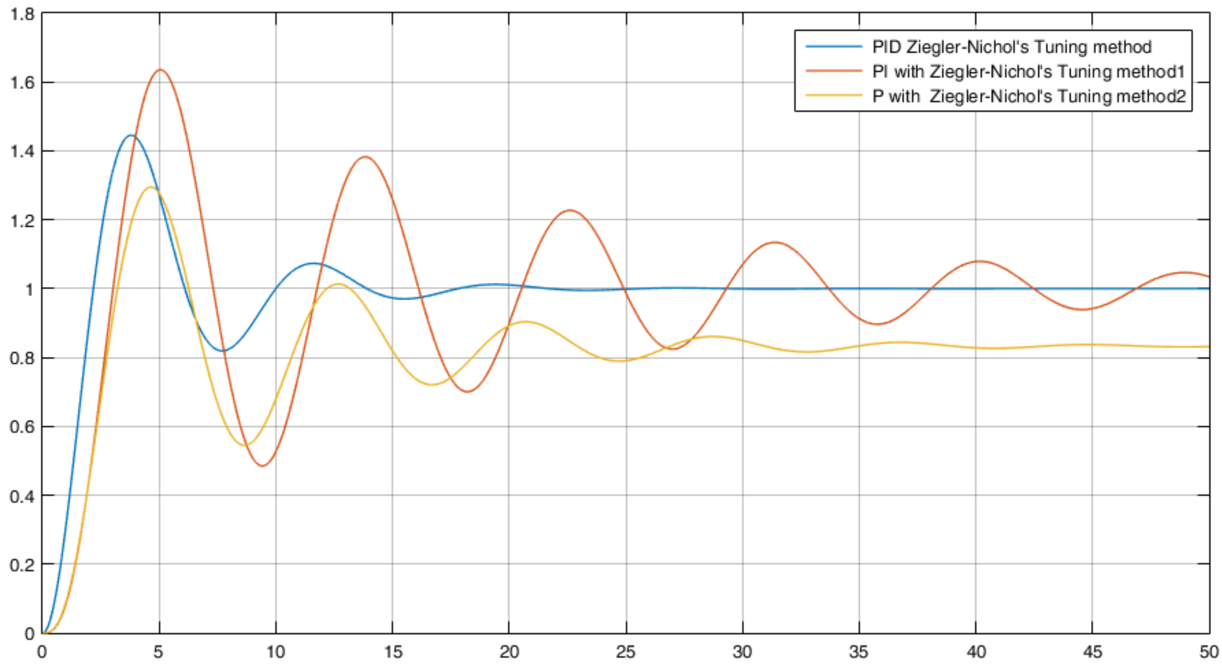


Fig 2: Responses with the ZN method

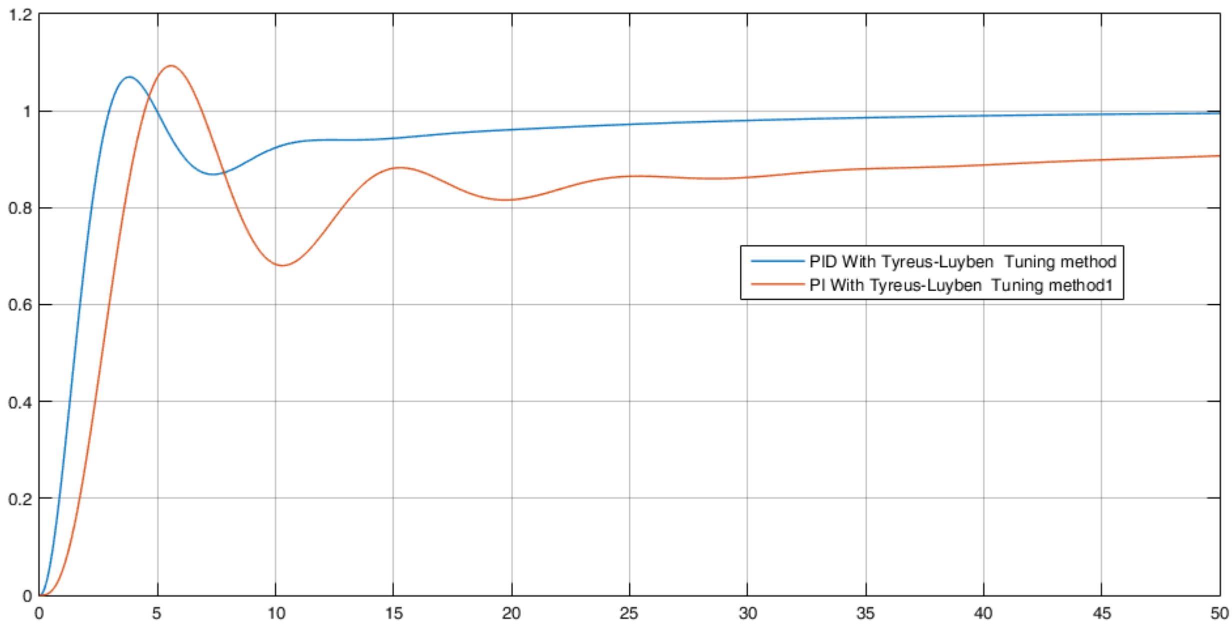


Fig 3: Responses with TL method

10.7 Results

Evalution of IAE, ISE, & ITAE of system tuned by ZN method

Type of Controller	IAE	ISE	ITAE
P	10.35	3.415	213.4
PI	9.395	3.886	126.4
PID	3.125	1.443	13.12

Evalution of IAE, ISE, & ITAE of system tuned by TL method

Type of Controller	IAE	ISE	ITAE
PI	8.593	3.074	161.6
PID	3.145	1.207	28.7

10.8 Conclusion

- closed-loop behavior tends to be oscillatory and sensitive to uncertainty with P and PI only control because for finding tuning parameter we need to drive system at point where system have sustained oscillation
- The tuning parameters are also not very robust, that is, they are very sensitive to process uncertainty. If the process conditions change, then the control system may become unstable.
- Tyreus-Luyben parameters result in less oscillatory responses and will be less sensitive to uncertainty in compare to ZN tuning method.