

The second-order geometry of short-and-sparse blind deconvolution

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Abstract

These notes describe a set of experiments for interrogating the curvature of nonconvex objective functions for short-and-sparse blind deconvolution (SaS-BD) over the course of optimization.

1 Introduction

In SaS-BD, we observe a length- m signal

$$\mathbf{y} = \iota \mathbf{a}_0 \circledast \mathbf{x}_0 + \mathbf{w},$$

the cyclic convolution between a *short* kernel $\mathbf{a}_0 \in \mathbb{S}^{p_0-1}$, zero-padded to length m , and a *sparse* activation map $\mathbf{x}_0 \in \mathbb{R}^m$, with possible additive noise \mathbf{w} . Our goal is to recover \mathbf{a}_0 and \mathbf{x}_0 up to some scaling or cyclic shift, as

$$\iota \mathbf{a}_0 \circledast \mathbf{x}_0 \equiv \alpha s_\ell[\iota \mathbf{a}_0] \circledast \alpha^{-1} s_{-\ell}[\mathbf{x}_0]$$

for any $\alpha \in \{\mathbb{R} \setminus 0\}$ and $\ell \in \mathbb{Z}$. We refer to these as the scaling and shift symmetries respectively.

A natural approach is to begin by formulating SaS-BD as a nonconvex optimization problem,

$$\min_{\mathbf{a} \in \mathbb{S}^{p-1}, \mathbf{x}} \left[\Psi_\lambda(\mathbf{a}, \mathbf{x}) \doteq \psi(\mathbf{a}, \mathbf{x}; \mathbf{y}) + \lambda \rho(\mathbf{x}) \right], \quad (1.1)$$

which minimizes a reconstruction error between \mathbf{y} and $\mathbf{a} \circledast \mathbf{x}$, plus a sparse penalty ρ on \mathbf{x} . For example, combining a squared error loss with an ℓ_1 -norm penalty yields the *bilinear-lasso* (BL) formulation

$$\Psi_\lambda(\mathbf{a}, \mathbf{x}) = \frac{1}{2} \|\mathbf{a} \circledast \mathbf{x} - \mathbf{y}\|_2^2 + \lambda \|\mathbf{x}\|_1. \quad (\text{BL})$$

A simplified problem, known as the *dropped-quadratic* formulation (DQ), can be obtained when the circulant matrix of \mathbf{a} , $\mathbf{C}_\mathbf{a}$, satisfies $\mathbf{C}_\mathbf{a}^T \mathbf{C}_\mathbf{a} \simeq \mathbf{I}$ over the course of optimization,

$$\Psi_\lambda(\mathbf{a}, \mathbf{x}) = \frac{1}{2} \|\mathbf{x}\|_2^2 - \langle \mathbf{a} \circledast \mathbf{x}, \mathbf{y} \rangle + \frac{1}{2} \|\mathbf{y}\|_2^2 + \lambda \rho(\mathbf{x}), \quad (\text{DQ})$$

$$\rho(\mathbf{x}) = \sum_i (x_i^2 + \delta^2)^{\frac{1}{2}}. \quad (1.2)$$

1.1 Regional landscape geometry

When \mathbf{x}_0 is generic, the objective function becomes regular on \mathbf{a} after marginalization on \mathbf{x} ,

$$(1.1) \equiv \min_{\mathbf{a} \in \mathbb{S}^{p-1}} \left[\varphi_\lambda(\mathbf{a}) \doteq \min_{\mathbf{x}} \Psi_\lambda(\mathbf{a}, \mathbf{x}) \right]. \quad (1.3)$$

This phenomenon appears to be *largely independent* of the specific formulation used for SaS-BD, but finds a relatively simple expression in the (DQ) formulation, for which it is summarized as follows [KLZW19]:

1.2 Sparsity-coherence tradeoff

2 Experimental outline

References

[KLZW19] Han-Wen Kuo, Yenson Lau, Yuqian Zhang, and John Wright. Geometry and symmetry in short-and-sparse deconvolution. *arXiv preprint arXiv:1901.00256*, 63(7):4497–4520, 2019.