1. A procurement officer bought 25 components of provider 1 and 30 of provider 2. Suppose the values $X_{1,1}$,$X_{1,2}$,…,$X_{1,25}$ the electric resistance obtained of the components from provider 1 are independent and follows a normal distribution with an average of 100 ohms and a standard deviation of 1.5 ohm suppose that the values $X_{2,1}$,$X_{2,2}$,…,$X_{2,30}$ the electric resistance obtained of the components from provider 2 are independent and follow a normal distribution with an average of 105 ohms and a standard deviation of 2.0 ohms.

a) What is the sampled distribution of $\bar{X}_1$-$\bar{X}_2$

b) What is the error type of $\bar{X}_1$-$\bar{X}_2$

c) If it was impossible to suppose that the electric resistance follows a normal distribution, what could we have affirmed about the sampled distribution of $\bar{X}_1$-$\bar{X}_2$

d) What is the sampled distribution of the statistic $\frac{s1^2/2.25}{s2^2/4.00}$?

e) What is the expectation of the statistic in d)?

2. We randomly choose a sample of 100 semiconductor wafers and check if they are defective or not. They come from a manufacturing process that generates a proportion θ of defective platelets with 0 <θ <1. We set Xi = 1 if the i-th platelet is defective (and X_i else) with $i \in \{1,2, \dots ,100\}$, $Y = \sum_{i=1}^{100} Xi$ the number of defective pads in the sample and X = Y / 100 the proportion of defective platelets in the sample.

a) Determine the exact law of Y and its parameters.

b) Calculate P (1≤Y ≤5) if θ = 0.03.

c) Determine an approximate distribution of Y based on the central limit theorem.

d) Determine an approximate distribution of X.

e) Use this approximation to calculate P (0.01 ≤ X ≤ 0.05) if θ = 0.03, without the correction for continuity.

f) Compare the answers obtained in b) and in e) (give the errors). This time use the correction for continuity and give the new error.

3. Let the independent random variables $Z_i \sim N(0, 1)$ and $U_i \sim \chi_1^2$ be with $i \in \{1, 2, 3, 4\}$ Determine the probability law of each of the following random variables as well as their parameters.

a) $T_1 = \sum_{i=1}^{4} Z_i.$

b) $T_2 = \sum_{i=1}^{3} U_i.$

c) $T_3 = \dfrac{2U_1}{U_2 + U_3}.$

d) $T_4 = \dfrac{\sum_{i=1}^{4} Z_i}{\sqrt{\sum_{i=1}^{4} U_i}}.$

e) $T_5 = (U_1 + U_2)/2.$

4. With the same variables as in the previous exercise:

a) Find a such that $P(U1 / U2 \geq a) = 0.1$.

b) Find b such that $P\left(\frac{U1}{1+U1} \geq b\right) = 0.01$

c) Find c such that $P\left(\frac{(Z1+Z2)^2}{Z3^2+Z4^2} \geq c\right) = 0.05$

5. Let X be a population which follows a uniform law $U(0, \theta)$ with $\theta > 0$ and let $X1, X2, \ldots, Xn$ a random sample of X. We set $M = \max\{X1, X2, \ldots, Xn\}$ the maximum value observed in the sample.

a) Use the fact that $P(M \leq x) = P(X1 \leq x, X2 \leq x, \ldots, Xn \leq x)$ to determine the sampling distribution of M.

b) Check that the sampling distribution of $Y = M / \theta$ does not depend on θ.

c) Determine the value $C_{0.05}$ which satisfies $P(Y > C_{0.05}) = 0.05$ and calculate it for a sample of size $n = 20$.