1.Given $X_1, X_2, \ldots, X_n$ a random sample from a population of average μ and of variance σ^2

a) Is either of the two estimators of p defined below unbiased?

$$\hat{\mu}_1 = \frac{X_1 + X_2 + \cdots + X_7}{7}$$

$$\hat{\mu}_2 = \frac{2X_1 - X_6 + X_4}{2}$$

b) Which of these estimators has the smallest mean squared error?

2. Let Z be a geometric random variable with parameter p. Find an estimator of to from a random sample of size n,

a) With the Maximum likelihood estimation.

b) With the method of moments

3. Given 75 W bulbs whose lifespan, in hours, is known to obey approximately to a normal distribution, with a standard deviation σ of 25 hours. The 20 ampoules from a random sample lasted an average of $\bar{x}$ = 1014 hours.

a) Establish a 95% confidence interval for the average bulb life.

b) Suppose that from this sample, we have constructed, for the average bulb life, a confidence interval of a total length corresponding to 8 hours. What is the confidence level of this interval?

c) For a future experiment, we want to make 95% sure that the mistake made by estimating the average life of the bulbs will be less than 5 hours. What should be the size of the sample used?

4. A calculator manufacturer wants to estimate the proportion of defective calculators in his production. However, 18 of the 8,000 calculators in a random sample were found to be defective.

a) Establish, for the proportion of defective calculator, an interval of 99% confidence.

b) If we had constructed a confidence interval for the proportion of non-defective calculators from the same data, would we have obtained a greater margin of error?

c) We consider acceptable a proportion of defective calculators of 3/1000. Are we sure to respect the standard in this factory?