H_0: null hypothesis

H_1:

α

β

vaccine $\begin{cases} H_0: \text{does work} \\ H_1: \text{works} \end{cases}$

· suspect in court $\begin{cases} H_0: \text{non-guilty} \\ H_1: \text{guilty} \end{cases}$

do not reject H_0, but not accept

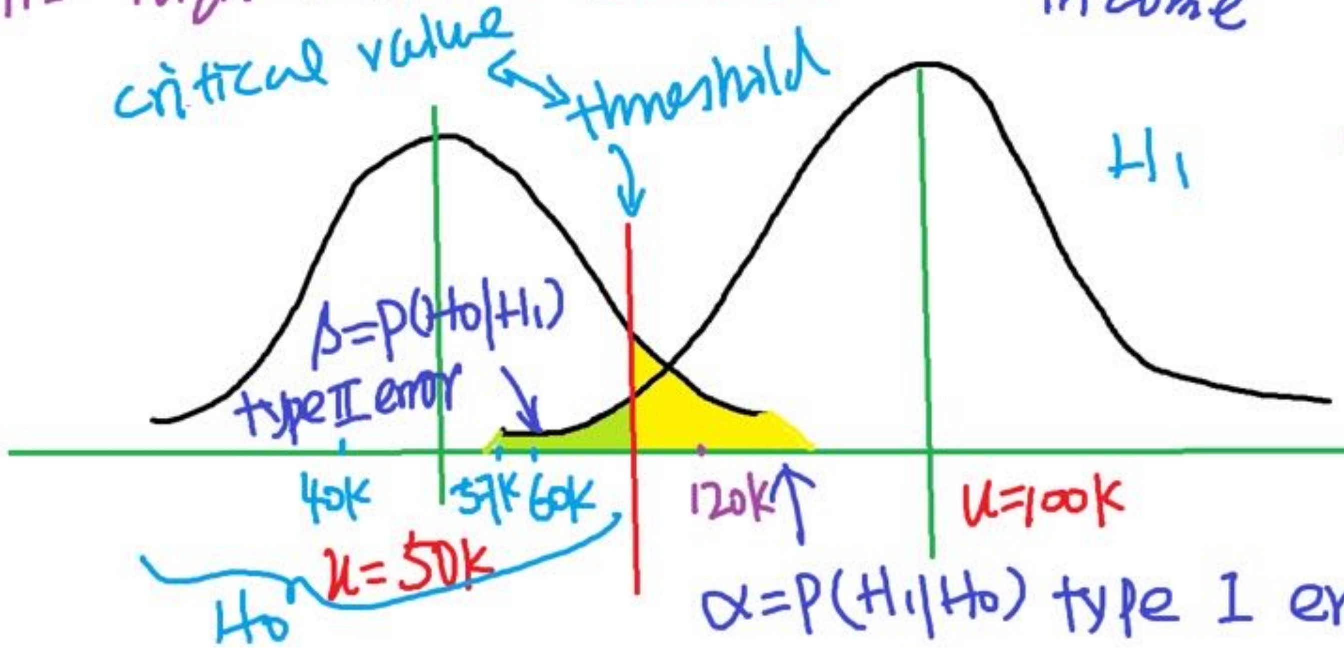
'we do not deny that suspect is non-guilty, but we do not accept it!'

H_0: mid-income $\mu = 50K$

H_1: high-income $\mu = 100K$

30K	80K		50K
x_1	x_2	$\cdots$	x_n

income



classification

1. test statistics
2. threshold
3. compare them

$\beta = P(H_0 | H_1)$ ← probability that classified as H_0 but in fact it is H_1

classified (H_0) truth (H_1)

$\beta = P(H_0 | H_1)$ => in fact vaccine works but it is classified as non-workable.

$\sigma = 0.01 \qquad \bar{x} = 0.26 \qquad n = 10$

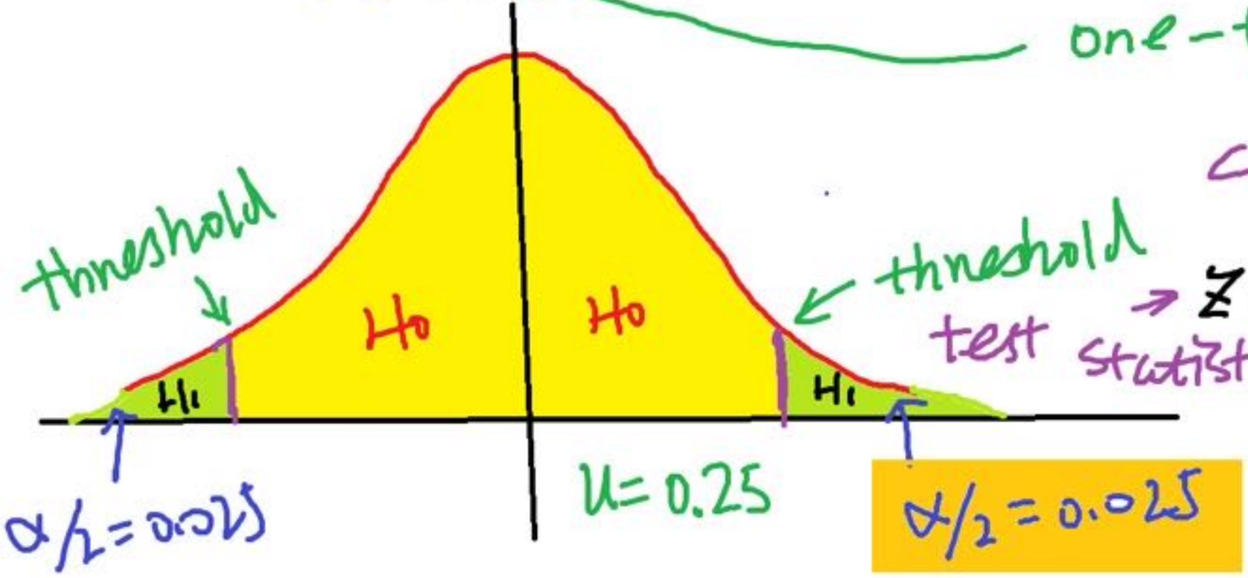
$$\begin{cases} H_0: u = 0.25 \\ H_1: u \neq 0.25 \end{cases}$$

$\alpha = 5\%$ ← type I error

$\alpha = 0.05$ ↖ significant level

one-tail or two tails ?

critical value = 1.96



test statistics → $$Z = \frac{\bar{x} - u}{\sigma/\sqrt{n}} = \frac{0.26 - 0.25}{0.01/\sqrt{10}} = \sqrt{10} \approx 3.16$$

$Z >$ critical value

$\alpha \rightarrow$ critical value (threshold) $\rightarrow$ ① critical value

$\beta \nearrow$

$x_1, x_2 \cdots x_n \rightarrow$ test statistics $\rightarrow$ ② p-value

always needed

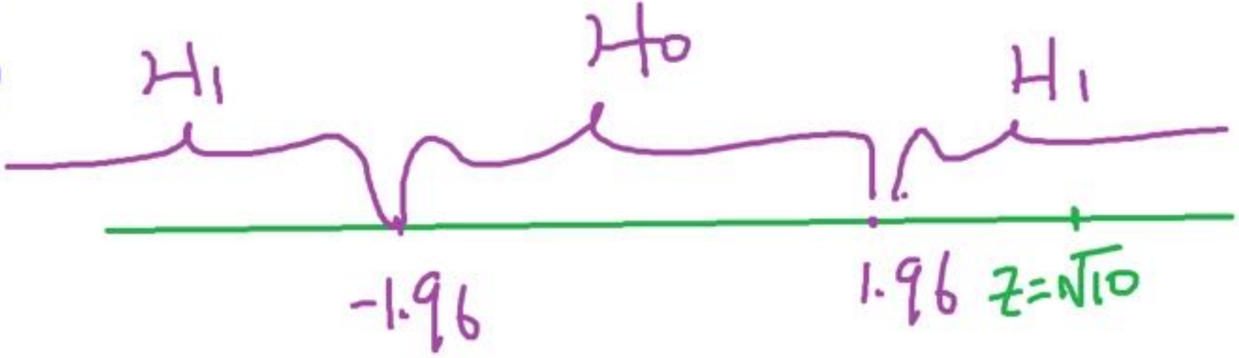
$\downarrow$

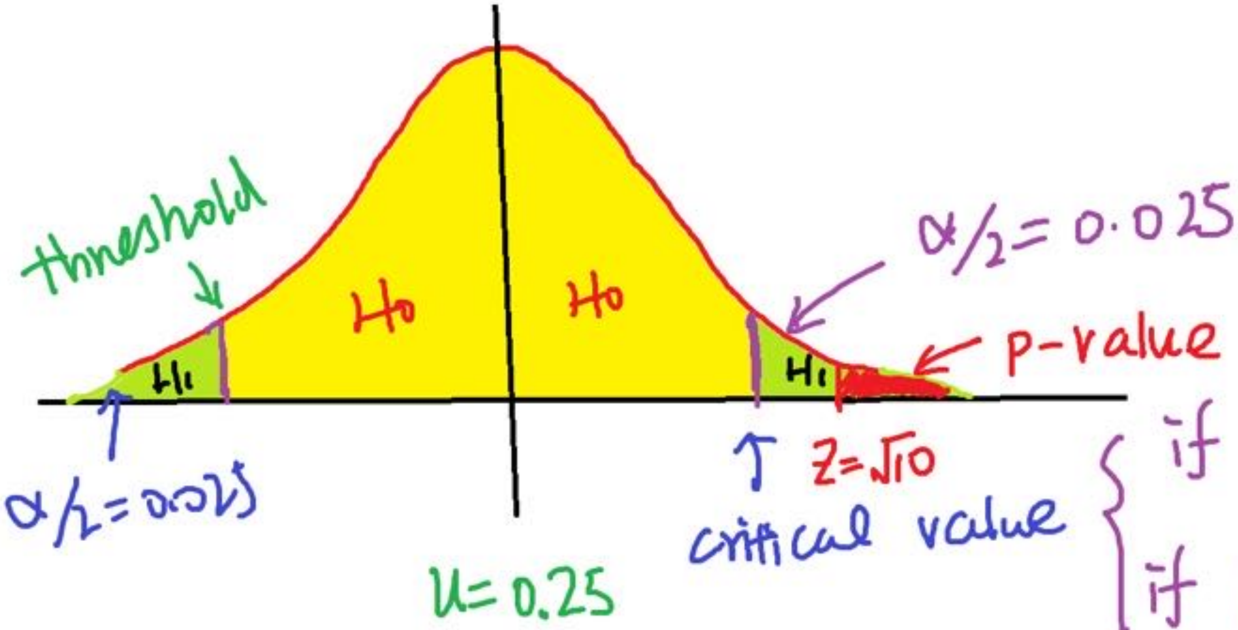
reject or not reject H_0

$$\text{critical value} = 1.96 \rightarrow \begin{cases} \text{if } Z > 1.96 \text{ or } Z < -1.96, \ \text{reject } H_0 \\ \text{if } -1.96 < Z < 1.96, \ \text{do not reject } H_0 \end{cases}$$

rule

what is Z? $Z = \sqrt{10}$





p-value is defined by test statistics.

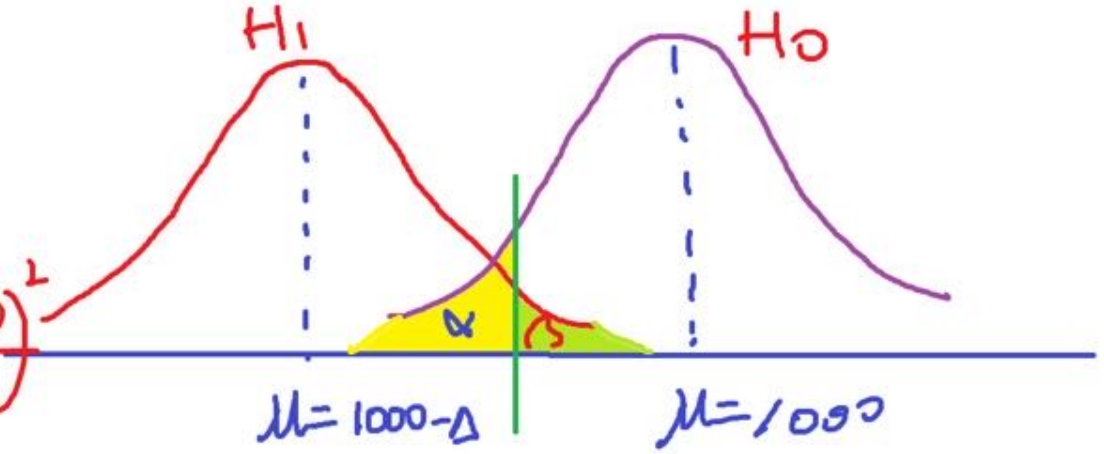
if p-value $< \alpha/2$, reject H_0

if p-value $> \alpha/2$, not reject H_0

p-value $= 0.0008 < \alpha/2 = 0.025 \Rightarrow$ reject H_0

$$\begin{cases} H_0 : \mu = 1000 \\ H_1 : \mu = 1000 - \Delta \end{cases} \qquad \sigma = 10$$

$$n = \left(\frac{(Z_\alpha + Z_\beta)\sigma}{\mu_1 - \mu_0}\right)^2 = \left(\frac{(Z_\alpha + Z_\beta)\sigma}{\Delta}\right)^2$$



$$n = \left(\frac{(1.645 + 1.285)\times 10}{5}\right)^2$$

$\alpha = 0.05 \rightarrow Z_\alpha = 1.645$

$\beta = 0.1 \rightarrow Z_\beta = 1.285$

$\Delta = 5 \rightarrow \mu_1 - \mu_0 = \Delta = 5$

$n \rightarrow \alpha = 0.05 \quad \beta = 0.1 \quad \Delta = 5$

$n \rightarrow \alpha = 0.05 \quad \beta = 0.05 \quad \Delta = 5$

$n \rightarrow \alpha = 0.05 \quad \beta = 0.01 \quad \Delta = 5$

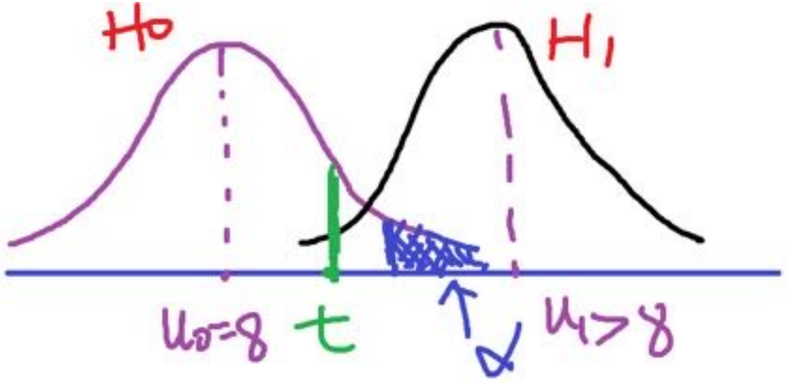
$n \rightarrow \alpha = 0.05 \quad \beta = 0.1 \quad \Delta = 5$

$n \rightarrow \alpha = 0.05 \quad \beta = 0.1 \quad \Delta = 10$

$n \rightarrow \alpha = 0.05 \quad \beta = 0.1 \quad \Delta = 15$

$n \rightarrow \alpha = 0.05 \quad \beta = 0.1 \quad \Delta = 20$

$n=11 \quad \bar{x}=9.7 \quad s^2=14.62$

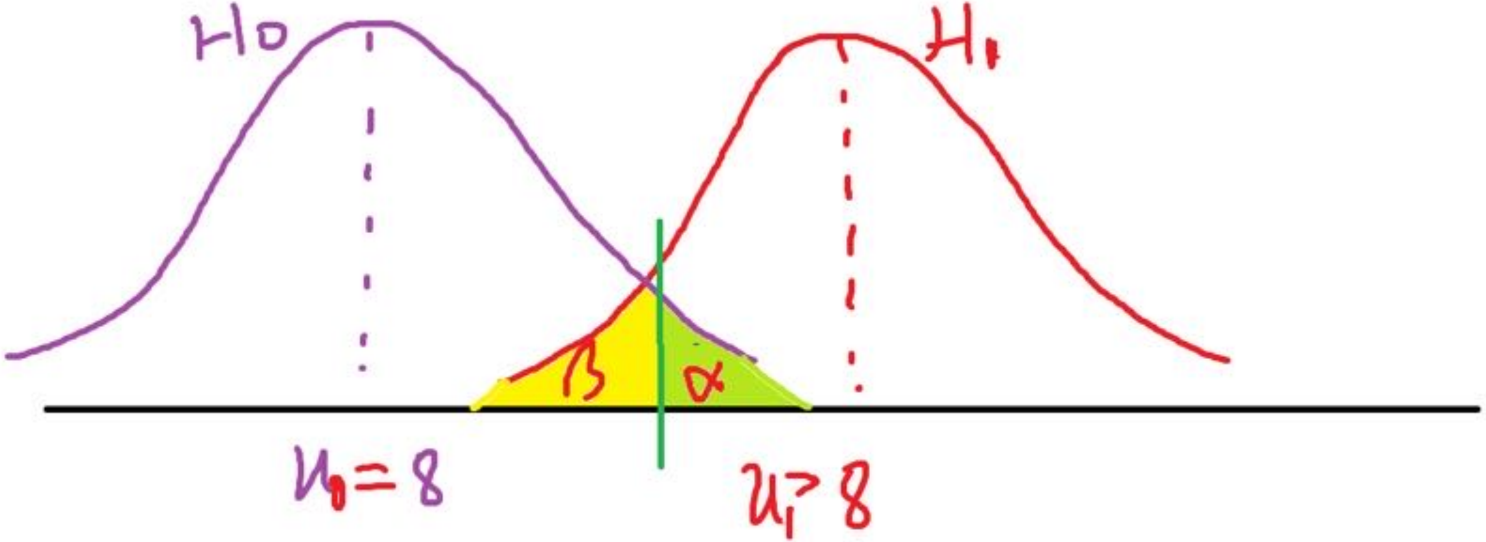


$\begin{cases} H_0: \mu \le 8 \\ H_1: \mu > 8 \end{cases}$ $\quad \alpha = 5\% = 0.05$

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{9.7-8}{\sqrt{14.62}/\sqrt{11}} \approx 1.47$$

$$df = n-1 = 11-1 = 10$$

$\alpha = 0.05 < \text{p-value} < 0.1 \Rightarrow$ do not reject H_0



$\mu_1 \uparrow$ red curve shifts to the right

$\alpha = \beta = 0.1$?

why $\alpha + \beta = 0.1$?

$u_1 - u_0 = 0.01$

$$n = \sqrt{\frac{(Z_\alpha + Z_\beta) \cdot \sigma}{u_1 - u_0}}$$

How to get this ?

A/B test