Non-Interactive Proofs of Proof-of-Work under Velvet Fork

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ABSTRACT

Superlight clients allow decentralized wallets to learn facts about the blockchain without downloading all the block headers. They achieve exponentially faster communication compared to SPV clients. For proof-of-work, they implement the so-called NIPoPoW primitive, for which there exist two variants: Superblock clients and FlyClient. Both of these protocols require consensus changes to existing blockchains and at least a soft fork to implement. In this paper, we discuss how a blockchain can be upgraded to support superblock clients without a soft fork. We show that it is possible to implement the needed changes without modifying the consensus protocol and by requiring only a minority of miners to upgrade, an upgrade termed a "velvet fork" in the literature. While previous work conjectured that NIPoPoW can be safely deployed using velvet forks as-is, we show that previous constructions are insecure, and that using velvet techniques to interlink a blockchain can pose insidious security risks. We describe a novel attack which we term a "chain-sewing" attack which is only possible in velvet situations: An adversary can cut-and-paste portions of various chains from independent forks, sewing them together to form a proof that looks like a chain but is not. We demonstrate that a minority adversary can thwart previous constructions with overwhelming probability. We put forth the first provably secure velvet NIPoPoW construction. Our construction is secure against adversaries that are bounded by 1/4 of the upgraded honest miner population. We prove our construction achieves persistence and liveness and analyze the trade-offs between the upgraded population parameter and the succinctness of the construction. Like non-velvet NIPoPoWs, our approach allows proving generic predicates about chains using infix proofs and as such can be adopted in practice for fast synchronization of transactions and accounts.

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KEYWORDS

blockchain, consensus, lightclient, NIPoPoW

1 INTRODUCTION

Blockchain systems such as Bitcoin [22] and Ethereum [3, 25] have a predetermined expected rate of block production and maintain chains of blocks that are growing linearly with time. A node synchronizing with the rest of the blockchain network for the first time therefore has to download and validate the whole chain, if it does not wish to rely on a trusted third party. While a lightweight node (SPV) can avoid downloading and validating transactions beyond their interest, it must still download the block headers that contain the proof-of-work[5] of each block in order to determine which chain contains the most proof-of-work. Block headers, while smaller by a significant constant factor, still grow linearly with time. An Ethereum node synchronizing for the first time must download more than 300 MB of block header data for the purpose of proof-of-work verification, even if it elects not to download any transactions. This has become a central problem to the usability of blockchain systems, especially for vendors who are using mobile phones to accept payments or sit behind limited internet bandwidth. They are forced to make a difficult choice between decentralization and the ability to start accepting payments in a timely manner.

Towards the goal of alleviating the burden of this download for SPV clients, a series of *superlight* clients has emerged. These clients are able to choose the best proof-of-work chain by only requesting a small number of *sample* block headers instead of all the block headers. These sample blocks form a Non-Interactive Proof of Proof-of-Work (NIPoPoW), a short string which compresses the proof-of-work of the underlying chain. It has been shown that the block headers in these proofs are secure representatives of the proof-of-work of the underlying chain: A minority adversary can only convince a NIPoPoW client that a certain transaction is confirmed, only if they can convince an SPV client, too.

There are two general directions for superlight client implementations: In the *superblock* [10**?**] approach, the client relies on *superblocks*, blocks that have achieved much better proof-of-work than required for block validity. In the *FlyClient* [2] approach, blocks are sampled at random using a predetermined distribution, akin to a Schnorr protocol [23] modified with the Fiat–Shamir heuristic [7]. The number of block headers that need to be sent then grows only logarithmically with time. The NIPoPoW client, termed a *verifier*, still relies on a connection to full nodes, termed *provers*, which perform the sampling of blocks from the full blockchain. However, no trust assumptions are made for these provers, as the verifier can check the veracity of their claims. As long as the verifier is connected to at least one honest prover (an assumption also made in the SPV protocol [9, 26]), they are able to discern the best chain from the rest.

In both approaches, it is essential for the verifier to check that the blocks sampled one way or another have been generated in the same order as they have been presented by the prover. As such, each block in the proof must contain a pointer to the previous block in the proof. As blocks in these proofs are far apart in the underlying blockchain, the legacy *previous block pointer*, which typically appears within block headers, does not suffice. Both approaches require modifications to the consensus layer of the underlying blockchain to work. In the case of superblock NIPoPoWs, the block header must be modified to include, in addition to a pointer to the previous block, pointers to a small amount of recent high-proof-of-work blocks. In the case of FlyClient, each block must additionally contain pointers to all previous blocks in the chain. Both of these modifications can be made efficiently by organizing these pointers into Merkle Trees [21] or Merkle Mountain Ranges [18, 24] whose root is stored in the block header. The inclusion of extra pointers within blocks is termed *interlinking the chain* [5].

The modified block format, which includes the extra pointers, must be respected and validated by all full nodes and thus requires either a hard fork or at least a soft fork. However, even soft forks require the approval of a supermajority of miners, and new features that are considered non-essential by the community have taken years to receive approval [19]. Towards the goal of implementing superlight clients sooner, we study the question of whether it is possible to deploy superlight clients without a soft fork. We propose a series of modifications to blocks that are *helpful but untrusted*. These modifications mandate that some extra data is included in each block. The extra data is placed inside the block by upgraded miners only, while the rest of the network does not include the additional data into the blocks and does not verify its inclusion, treating them merely as comments. To maintain backwards compatibility, contrary to a soft fork, upgraded miners must accept blocks that do not contain this extra data that have been produced by unupgraded miners, or even blocks that contain invalid or malicious such extra data produced by a mining adversary. This acceptance is necessary in order to avoid causing a chain split with the unupgraded part of the network. Such a modification to the consensus layer is termed a *velvet fork* [28].

A summary of our contributions in this paper is as follows:

(1) We propose the first *backwards-compatible superlight client*. We put forth an interlinking mechanism implementable through a velvet fork. We then construct a superblock NIPoPoW protocol on top of the velvet forked chain.
(2) We prove our construction secure in the synchronous static difficulty model against adversaries bounded to $1/4$ of the mining power of the honest upgraded nodes. As such, our protocol works even if a constant minority of miners adopts it.
(3) We illustrate that, contrary to claims of previous work, superlight clients designed to work in a soft fork cannot be readily plugged into a velvet fork and expected to work. We present a novel and insidious attack termed the *chain-sewing* attack which thwarts the defenses of previous proposals and allows even a minority adversary to cause catastrophic failures.

Previous work. Proofs of Proof-of-Work have been proposed in the context of superlight clients [2, 5, 15], cross-chain communication [11, 16, 27], as well as local data consumption by smart contracts [12]. Superblock NIPoPoWs have been conjectured to work in velvet fork conditions [15], but we show here that these conjectures are ill-informed in the light of our chain-sewing attack. Velvet forks [28] have been studied for a variety of other applications. In this work, we focus on consensus state compression. Such compression has been explored in the hard-fork setting using zk-SNARKS [20] as well as in the Proof-of-Stake setting [13]. Complementary to consensus state compression is compression of application state, namely the State Trie, the UTXO, or transaction history. There is a series of works composable with ours that discusses the compression of application state [4, 17].

[8][6]

2 PRELIMINARIES

We consider a setting where the blockchain network consists of two different types of nodes: The first kind, *full nodes*, are responsible for the maintenance of the chain including verifying it and mining new blocks. The second kind, *verifiers* connect to full nodes and wish to learn facts about the blockchain without downloading it, for example whether a particular transaction is confirmed. The full nodes therefore also function as *provers* for the verifiers. Each verifier connects to multiple provers, at least one of which is assumed to be honest.

We model full nodes according to the Backbone model [8]. There are n full nodes, of which t are adversarial and $n - t$ are honest. All t adversarial parties are controlled by one colluding adversary $\mathcal{A}$. The parties have access to a hash function H which is modelled as a common Random Oracle [1]. To each novel query, the random oracle outputs κ bits of fresh randomness. Time is split into distinct *rounds* numbered by the integers $1, 2, \cdots$. Our treatment is in the *synchronous model*, so we assume messages *diffused* (broadcast) by an honest party at the end of a round are received by all honest parties at the beginning of the next round. This is equivalent to a network connectivity assumption in which the round duration is taken to be the known time needed for a message to cross the diameter of the network. The adversary can inject messages, reorder them, sybil attack by creating multiple messages, but not suppress messages.

Each honest full node locally maintains a *chain* $\mathcal{C}$, a sequence of blocks. In understanding that we are developing an improvement on top of SPV, we use the term *block* to mean what is typically referred to as a *block header*. Each block contains the Merkle Tree root [21] of transaction data $\overline{x}$, the hash s of the previous block in the chain known as the *previd*, as well as a nonce value ctr. As discussed in the Introduction, the compression of application data $\overline{x}$ is orthogonal to our goals in this paper and has been explored in independent work [4] which can be composed with ours. Each block $b = s \parallel \overline{x} \parallel ctr$ must satisfy the proof-of-work [5] equation $H(b) \leq T$ where T is a constant *target*, a small value signifying the difficulty of the proof-of-work problem. Our treatment is in the *static difficulty* case, so we assume that T is constant throughout the execution[1]. $H(B)$ is known as the *block id*.

Blockchains are finite block sequences obeying the *blockchain property*: that in every block in the chain there exists a pointer to its previous block. A chain is *anchored* if its first block is *genesis*, denoted $\mathcal{G}$, a special block known to all parties. This is the only node the verifier knows about when it boots up. For chain addressing we use Python brackets $\mathcal{C}[\cdot]$. A zero-based positive number in a bracket indicates the indexed block in the chain. A negative index indicates a block from the end, e.g., $\mathcal{C}[-1]$ is the tip of the blockchain. A range $\mathcal{C}[i{:}j]$ is a subarray starting from i (inclusive) to j (exclusive). Given chains $\mathcal{C}_1, \mathcal{C}_2$ and blocks A, Z we concatenate them as $\mathcal{C}_1\mathcal{C}_2$ or $\mathcal{C}_1 A$ (if clarity mandates it, we also use the symbol $\parallel$ for concatenation). Here, $\mathcal{C}_2[0]$ must point to $\mathcal{C}_1[-1]$ and A must point to $\mathcal{C}_1[-1]$. We denote $\mathcal{C}\{A{:}Z\}$ the subarray of the chain from block A (inclusive) to block Z (exclusive). We can omit blocks or indices from either side

[1]A treatment of variable difficulty NIPoPoWs has been explored in the soft fork case [29], but we leave the treatment of velvet fork NIPoPoWs in the variable difficulty model for future work.

of the range to take the chain to the beginning or end respectively. As long as the blockchain property is maintained, we freely use the set operators $\cup$, $\cap$ and $\subseteq$ to denote operations between chains, implying that the appropriate blocks are selected and then placed in chronological order.

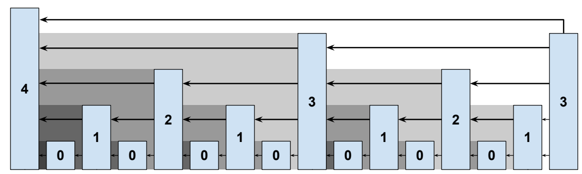
During every round, every party attempts to *mine* a new block on top of its currently adopted chain. Each party is given q queries to the random oracle which it uses in attempting to mine a new block. Therefore the adversary has tq queries per round while the honest parties have $(n-t)q$ queries per round. When an honest party discovers a new block, they extend their chain with it and broadcast the new chain. Upon receiving a new chain C' from the network, an honest party compares its length $|\mathsf{C}'|$ against its currently adopted chain C and adopts the newly received chain if it is longer. It is assumed that the honest parties control the majority of the computational power of the network. This *honest majority assumption* states that there is some δ such that $t < (1-\delta)(n-t)$. If so, the protocol ensures consensus among the honest parties: There is a constant k, the *Common Prefix* parameter, such that, at any round, all the chains belonging to honest parties share a common prefix of blocks; the chains can deviate only up to k blocks at the end of each chain [8]. Concretely, if at some round r two honest parties have C_1 and C_2 respectively, then either $\mathsf{C}_1[:-k]$ is a prefix of C_2 or vice versa.

Some valid blocks satisfy the proof-of-work equation better than required. If a block b satisfies $H(b) \leq 2^{-\mu}T$ for some natural number $\mu \in \mathbb{N}$ we say that b is a *μ-superblock* or a block *of level μ*. The probability of a new valid block achieving level μ is $2^{-\mu}$. The number of levels in the chain will be $\log|\mathsf{C}|$ with high probability [14]. Given a chain C, we denote $\mathsf{C}\uparrow^{\mu}$ the subset of μ-superblocks of C.

Non-Interactive Proofs of Proof-of-Work (NIPoPoW) protocols allow verifiers to learn the most recent k blocks of the blockchain adopted by an honest full node without downloading the whole chain. The challenge lies in building a verifier who can find the suffix of the longest chain between claims of both honest and adversarial provers, while not downloading all block headers. Towards that goal, the *superblock* approach uses superblocks as samples of proof-of-work. The prover sends superblocks to the verifier to convince them that proof-of-work has taken place without actually presenting all this proof-of-work. The protocol is parametrized by a constant security parameter m. The parameter determines how many superblocks will be sent by the prover to the verifier and security is proven with overwhelming probability in m.

The prover selects various levels μ and for each such level sends a carefully chosen portion of its μ-level *superchain* $\mathsf{C}\uparrow^{\mu}$ to the verifier. In standard blockchain protocols such as Bitcoin and Ethereum, each block $\mathsf{C}[i+1]$ in C points to its previous block $\mathsf{C}[i]$, but each μ-superblock $\mathsf{C}\uparrow^{\mu}[i+1]$ does not point to its previous μ-superblock $\mathsf{C}\uparrow^{\mu}[i]$. It is imperative that an adversarial prover does not reorder the blocks within a superchain, but the verifier cannot verify this unless each μ-superblock points to its most recently preceding μ-superblock. The proposal is therefore to *interlink* the chain by having each μ-superblock include an extra pointer to its most recently preceding μ-superblock. To ensure integrity, this pointer must be included in the block header and verified by proof-of-work. However, the miner does not know which level a candidate block will attain prior to mining it. For this purpose, each block is proposed to include a pointer to the most recently preceding μ-superblock, for every μ, as illustrated in Figure 1. As these levels are only $\log|\mathsf{C}|$, this does not add much extra data to the block.

Figure 1: The interlinked blockchain. Each superblock is drawn taller according to its achieved level. Each block links to all the blocks that are not being overshadowed by their descendants. The most recent (right-most) block links to the four blocks it has direct line-of-sight to.



To ensure the block header is of constant size, instead of including all these superblock pointers in the block header individually, they are organized into a Merkle Tree of interlink pointers and only the root of the Merkle Tree is included in the block header. In this manner, a blockchain can be *hard forked* to include interlinks. In this case, the NIPoPoW prover that wishes to show a μ-superblock b in their proof is connected to its more recently preceding μ-superblock b', also includes a Merkle Tree proof proving that $H(b')$ is a leaf in the interlink Merkle Tree root included in the block header of b.

To include interlinks in a *soft fork*, the interlink Merkle Tree root is placed in the *coinbase* transaction instead of the block header. Upgraded miners include the correct interlink Merkle Tree root in their coinbase. Upgraded miners receiving a new block validate that the interlink Merkle Tree root is correct before accepting a block as valid. As this root can be calculated in a deterministic manner from the previous blocks in the chain, it can easily be validated. Whenever the NIPoPoW prover wishes to show that a μ-superblock b contains a pointer to its most recently preceding μ-superblock b', it must then accompany the block header of $b = s \parallel \overline{x} \parallel ctr$ with the coinbase transaction $\mathsf{tx_{cb}}$ of b as well as two Merkle Tree proofs: One proving that the coinbase transaction $\mathsf{tx_{cb}}$ is in $\overline{x}$, and one proving that $H(b')$ is a leaf in the interlink Merkle Tree whose root is included in $\mathsf{tx_{cb}}$.

The exact NIPoPoW protocol works like this: The prover holds a full chain C. When the verifier requests a proof, the prover sends the last k blocks of their chain, the suffix $\chi = \mathsf{C}[-k:]$, in full. From the larger prefix $\mathsf{C}[:-k]$, the prover constructs a proof π by selecting certain superblocks as representative samples of the proof-of-work that took place. The blocks are picked as follows. The prover selects the *highest* level μ^* that has at least m blocks in it and includes all these blocks in their proof (if no such level exists, the chain is small and can be sent in full). The prover then iterates from level $\mu = \mu^* - 1$ down to 0. For every level μ, it includes sufficient μ-superblocks to cover the last m blocks of level $\mu + 1$, as illustrated in Algorithm 1. Because the density of blocks doubles as levels are descended, the proof will contain in expectation $2m$ blocks for each level below μ^*. As such, the total proof size $\pi\chi$ will be $\Theta(m \log|\mathsf{C}| + k)$. Such proofs that are polylogarithmic in the chain size constitute an exponential improvement over traditional SPV clients and are called *succinct*.

Algorithm 1 The Prove algorithm for the NIPoPoW protocol in a soft fork

function $\text{Prove}_{m,k}(\mathsf{C})$
  $B \leftarrow \mathsf{C}[0]$ ▷ Genesis
  for $\mu = |\mathsf{C}[-k-1].\text{interlink}|$ down to 0 **do**
    $\alpha \leftarrow \mathsf{C}[:-k]\{B:\}\uparrow^{\mu}$
    $\pi \leftarrow \pi \cup \alpha$
    if $m < |\alpha|$ **then**
      $B \leftarrow \alpha[-m]$
    end if
  end for
  $\chi \leftarrow \mathsf{C}[-k:]$
  return $\pi\chi$
end function

Upon receiving two proofs $\pi_1\chi_1, \pi_2\chi_2$ of this form, the NIPoPoW verifier first checks that $|\chi_1| = |\chi_2| = k$ and that $\pi_1\chi_1$ and $\pi_2\chi_2$ form valid chains. To check that they are valid chains, the verifier ensures every block in the proof contains a pointer to its previous block inside the proof through either the *previd* pointer in the block header, or a leaf in the interlink Merkle Tree included in the coinbase transaction. In this latter case, Merkle proofs must be provided as discussed previously. If any of these checks fail, the proof is rejected. It then compares π_1 against π_2 using the $\leq_m$ operator, which works as follows. It finds the lowest common ancestor block $b = (\pi_1 \cap \pi_2)[-1]$; that is, b is the most recent block shared among the two proofs. Subsequently, it chooses the level μ_1 for π_1 such that $|\pi_1\{b:\}\uparrow^{\mu_1}| \geq m$ (i.e., π_1 has at least m superblocks of level μ_1 following block b) and the value $2^{\mu_1}|\pi_1\{b:\}\uparrow^{\mu_1}|$ is maximized. It chooses a level μ_2 for π_2 in the same fashion. The two proofs are compared by checking whether $2^{\mu_1}|\pi_1\{b:\}\uparrow^{\mu_1}| \geq 2^{\mu_2}|\pi_2\{b:\}\uparrow^{\mu_2}|$ and the proof with the largest score is deemed the winner. The comparison is illustrated in Algorithm 2.

Algorithm 2 The implementation of the $\geq_m$ operator to compare two NIPoPoW proofs parameterized with security parameter m. Returns *true* if the underlying chain of player A is deemed longer than the underlying chain of player B.

function $\text{best-arg}_m(\pi, b)$
  $M \leftarrow \{\mu : |\pi\uparrow^{\mu} \{b:\}| \geq m\} \cup \{0\}$ ▷ Valid levels
  return $\max_{\mu \in M}\{2^{\mu} \cdot |\pi\uparrow^{\mu} \{b:\}|\}$ ▷ Score for level
end function
operator $\pi_A \geq_m \pi_B$
  $b \leftarrow (\pi_A \cap \pi_B)[-1]$ ▷ LCA
  return $\text{best-arg}_m(\pi_A, b) \geq \text{best-arg}_m(\pi_B, b)$
end operator

3 VELVET INTERLINKS

3.1 Velvet fork parameter

A velvet fork suggest that only a minority of upgraded parties needs to support the protocol changes. Let g express the percentage of honest upgraded parties to the total number of miners. We will refer to g as the "velvet parameter".

Definition 3.1 (Velvet Parameter). Let g be the velvet parameter for NIPoPoW protocols. Then if n_h the upgraded honest miners and n the total number of miners t out of which are corrupted, it holds that $n_h = g(n-t)$.

3.2 Smooth and Thorny blocks

In order to be applied under velvet fork, a protocol has to change in a backwards-compatible manner. In essence, any additional information coming with the protocol upgrade is transparent to the non-upgraded players. This transparency towards the non-upgraded parties requires any block that conforms only to the old protocol rules to be considered a valid one. Considering superblock NIPoPoWs under a velvet fork, any block is to be checked for its validity regardless the validity of the NIPoPoWs protocol's additional information, which is the interlink structure.

A block generated by the adversary could thus contain arbitrary data in the interlink and yet be appended in the chain adopted by an honest party. In case that trash data are stored in the structure this could be of no harm for the protocol routines, since such blocks will be treated as non-upgraded. In the context of the attack that will be presented in the following section, we examine the case where the adversary includes false interlink pointers.

An interlink pointer is the hash of a block. A correct interlink pointer of a block b for a specific level μ is a pointer to the most recent b's ancestor of level μ. From now on we will refer to correct interlink pointers as *smooth pointers*. Pointers of the 0-level *(prevIds)* are always smooth because of the performed proof-of-work.

Definition 3.2 (Smooth Pointer). Smooth pointer of a block b for a specific level μ is the interlink pointer to the most recent μ-level ancestor of b.

A non-smooth pointer may not point to the most recent ancestor of level μ or even point to a superblock of a fork chain, as shown in Figure 2.

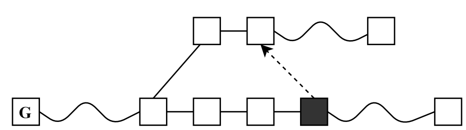


Figure 2: *A non-smooth pointer of an adversarial block, colored black, in an honest player's chain.*

In the same manner it is possible that a false interlink contain arbitrary pointers to blocks of any chain as illustrated in Figure 3. The interlink pointing to arbitrary directions resembles a thorny bush, so we will refer to blocks containing false interlink information as *thorny*.

Definition 3.3 (Thorny Block). Thorny block is a block which contains at least one non-smooth interlink pointer.

Opposite of the thorny are the *smooth* blocks, which may be blocks generated by non-upgraded players or blocks generated by upgraded players and contain only smooth pointers in their interlink.

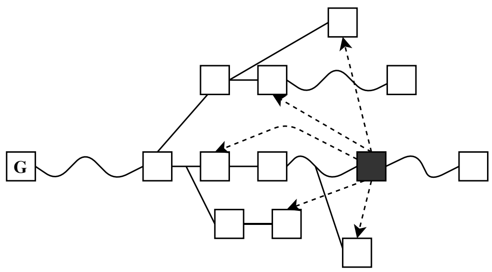


Figure 3: *A thorny block appended in an honest player's chain. The dashed arrows are interlink pointers.*

Definition 3.4 (Smooth Block). Smooth block is any block which is not thorny.

4 THE CHAINSEWING ATTACK

We will now describe an explicit attack against the NIPoPoW suffix proof construction under velvet fork. As already argued, since the protocol is implemented under velvet fork, any thorny block be accepted as valid. Taking advantage of such blocks in the chain, the adversary could produce suffix proofs containing an arbitrary number of blocks belonging in several fork chains. The attack is described in detail in the following.

Assume that chain C_B was adopted by an honest player B and chain $C_{\mathcal{A}}$, a fork of C_B at some point, maintained by adversary $\mathcal{A}$. Assume that the adversary wants to produce a suffix proof in order to attack a light client to have him adopt a chain which contains blocks of $C_{\mathcal{A}}$. In order to achieve this, the adversary needs to include a greater amount of total proof-of-work in her suffix proof, $\pi_{\mathcal{A}}$, in comparison to that included in the honest player's proof, π_B, so as to achieve $\pi_{\mathcal{A}} \geq_m \pi_B$. For this she produces some thorny blocks in chains $C_{\mathcal{A}}$ and C_B which will allow her to claim blocks of chain C_B as if they were of chain $C_{\mathcal{A}}$ in her suffix proof.

The general form of this attack for an adversary sewing blocks to one forked chain is illustrated in Figure 4. Dashed arrows represent interlink pointers of some level $\mu_{\mathcal{A}}$. Starting from a thorny block in the adversary's forked chain and following the interlink pointers, a chain is formed which consists the adversary's suffix proof. Blocks of both chains are included in this proof and a verifier could not distinguish the non-smooth pointers participating in this proof chain and, as a result, would consider it a valid proof.

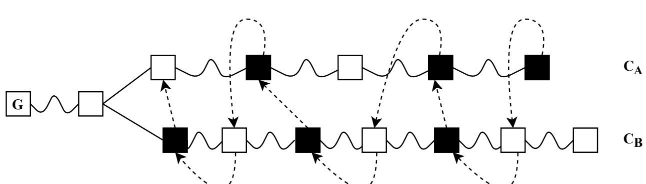


Figure 4: *Generic Chainsewing Attack.* C_B *is the chain of an honest player and* $C_{\mathcal{A}}$ *the adversary's chain. Adversarially generated blocks are colored black. Dashed arrows represent interlink pointers included in the adversary's suffix proof. Wavy lines imply one or more blocks.*

As the generic attack scheme may seem a bit complicated we will now describe a more specific attack case. Consider that the adversary acts as described below. Assume that the adversary chooses to attack at some level $\mu_{\mathcal{A}}$. As shown in Figure 5 she first generates a superblock b' in her forked chain $C_{\mathcal{A}}$ and a thorny block a' in the honest chain C_B which points to b'. As argued earlier, block a' will be accepted as valid in the honest chain C_B despite the invalid interlink pointers. After that, the adversary may mine on chain $C_{\mathcal{A}}$ or C_B, or not mine at all. At some point she produces a thorny block a in $C_{\mathcal{A}}$ pointing to a block b of C_B. Because of the way blocks are generated by updated honest miners there will be successive interlink pointers leading from block b to block a'. Thus following the interlink pointers a chain is formulated which connects $C_{\mathcal{A}}$ blocks a and b' and contains an arbitrarily large part of the honest player's chain C_B.

At this point the adversary will produce a suffix proof for chain $C_{\mathcal{A}}$ containing the subchain $C\{ab\} \cup C\{b{:}a'\} \cup C\{a'{:}b'\}$. Notice that following the interlink pointers constructed in such a way, a light client perceives $C\{ab\} \cup C\{b{:}a'\} \cup C\{a'{:}b'\}$ as a valid chain.

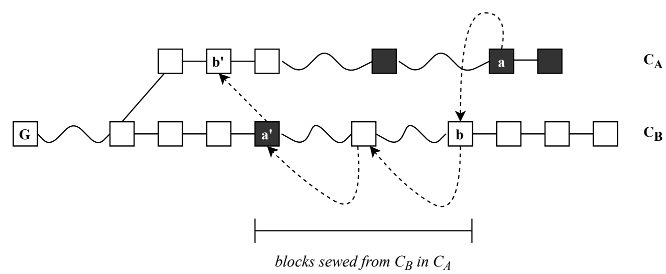


Figure 5: *Chainsewing Attack.* C_B *represents the chain of an honest player.* $C_{\mathcal{A}}$ *is an adversarial fork. Adversarially generated blocks are colored black. Dashed arrows represent interlink pointers included in the adversary's suffix proof. Wavy lines imply one or more blocks. Firm lines imply the previousId relationship between two sequential blocks.*

In this attack the adversary uses thorny blocks to "sew" portions of the chain adopted by an honest player to her own forked chain. This remark justifies the name given to the attack.

Note that in order to make this attack successful, the adversary has to produce only a few superblocks which let her arrogate an arbitrarily large number of blocks. Thus this attack is expected to succeed with overwhelming probability.

5 VELVET NIPOPOWS

In order to eliminate the Chainsewing Attack we propose an update to the velvet NIPoPoW protocol. The core problem is that in her suffix proof the adversary is able to claim not only blocks of forked chains, which are in majority adversarially generated due to the Common Prefix property, but also arbitrarily long parts of the chain adopted by an honest player. Since thorny blocks are accepted as valid, the verifier cannot distinguish blocks that actually belong in a chain from blocks that only seem to belong in the same chain because they are pointed to via a non-smooth pointer.

We describe a protocol patch that operates as follows. The NIPoPoW protocol under velvet fork works as usual but each miner constructs smooth blocks. This means that a block's interlink is constructed excluding thorny blocks. In this way, although thorny blocks are accepted in the chain, they are not taken into consideration when updating the interlink structure for the next block to be mined. No honest block could now point to a thorny superblock that may act as the passing point to the fork chain in an adversarial suffix proof. Thus, after this protocol update the adversary is only able to inject adversarially generated blocks from an honestly adopted chain to her own fork. At the same time, thorny blocks cannot participate in an honestly generated suffix proof except for some blocks in the proof's suffix (χ). This argument holds because thorny blocks do not form a valid chain along with honestly mined blocks anymore. Consequently, as far as the blocks included in a suffix proof is concerned, we can think of thorny blocks as belonging in the adversary's fork chain for the π part of the proof, which is the comparing part between proofs. Figure 6 illustrates this remark.

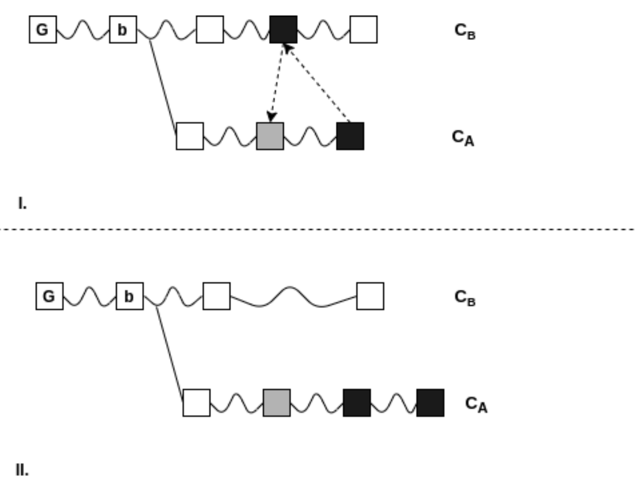


Figure 6: *The adversarial fork chain C_A and chain C_B of an honest player. Thorny blocks are colored black. Dashed arrows represent interlink pointers. Wavy lines imply one or more blocks. After the protocol update, when an adversarially generated block is sewed from C_B into the adversary's suffix proof the verifier conceives C_A as longer and C_B as shorter. I: The real picture of the chains. II: Equivalent picture from the verifier's perspective considering the blocks included in the corresponding suffix proof for each chain.*

5.1 The patch

In order to make NIPoPoWs a secure protocol under velvet fork conditions we suggest that the NIPoPoW protocol under velvet fork works as usual but a miner constructs a block's interlink without pointers to thorny blocks.

In order to prove the security of the updated protocol we need to strengthen the Honest Majority Assumption to 1/4 adversary in respect to upgraded honest miners, as it will be formally stated in the following section.

5.2 Analysis

We now give the formal definition of the Honest Majority Assumption for (1/4)-bounded adversary under velvet fork, some Lemmas that will be used in our security proof as well as the updated algorithms for the honest miner, prover and verifier.

Definition 5.1 (Velvet Honest Majority). Let n_h be the number of upgraded honest miners. Then t out of total n parties are corrupted such that $\frac{t}{n_h} < \frac{1-\delta_v}{3}$.

The following Lemmas come as immediate results from the suggested protocol update.

Lemma 5.2. *A velvet suffix proof constructed by an honest player cannot contain any thorny block.*

Lemma 5.3. *Let $\mathcal{P}_{\mathcal{A}} = (\pi_{\mathcal{A}}, \chi_{\mathcal{A}})$ be a velvet suffix proof constructed by the adversary and block b_s, generated at round r_s, be the most recent smooth block in the proof. Then $\forall r : r < r_s$ no thorny blocks generated at round r can be included in $\mathcal{P}_{\mathcal{A}}$.*

Proof. By contradiction. Let b_t be a thorny block generated at some round $r_t < r_s$. Suppose for contradiction that b_t is included in the proof. Then, because $\mathcal{P}_{\mathcal{A}}$ is a valid chain as for interlink pointers, there exist a block path made by interlink pointers starting from b_s and resulting to b_t. Let b' be the most recently generated thorny block after b_t and before b_s included in $\mathcal{P}_{\mathcal{A}}$. Then b' has been generated at a round r' such that $r_t \leq r' < r_s$. Then the block right after block b' in $\mathcal{P}_{\mathcal{A}}$ must be a thorny block since it points to b' which is thorny. But b' is the most recent thorny block after b_t, thus we have reached a contradiction. □

Lemma 5.4. *Let $\mathcal{P}_{\mathcal{A}} = (\pi_{\mathcal{A}}, \chi_{\mathcal{A}})$ be a velvet suffix proof constructed by the adversary. Let b_t be the oldest thorny bock included in $\mathcal{P}_{\mathcal{A}}$ which is generated at round r_t. Then any block $b = \{b : b \in \mathcal{P}_{\mathcal{A}} \land b \text{ generated at } r \geq r_t\}$ is thorny.*

Proof. By contradiction. Suppose for contradiciton that b_s is a smooth block generated at round $r_s > r_t$. Then from Lemma 5.3 any block generated at round $r < r_s$ is smooth. But b_t is generated at round $r_t < r_s$ and is thorny, thus we have reached a contradiction. □

The following corollary emerges immediately from Lemmas 5.3, 5.4. This result is illustrated in Figure 7.

Corollary 5.5. *Any adversarial proof $\mathcal{P}_{\mathcal{A}} = (\pi_{\mathcal{A}}, \chi_{\mathcal{A}})$ containing both smooth and thorny blocks consists of a prefix smooth subchain followed by a suffix thorny subchain.*

We now describe the algorithms needed by the upgraded miner, prover and verifier. The upgraded miner acts as usual except for including the interlink of the newborn block in the coinbase transaction. In order to construct an interlink containing only the smooth blocks, the miner keeps a copy of the "smooth chain" (C_S) which consists of the smooth blocks existing in the original chain C. The

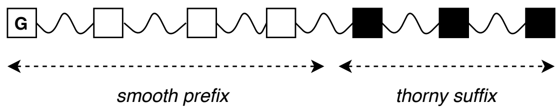


Figure 7: *In the general case the adversarial velvet suffix proof $\mathcal{P}_{\mathcal{A}} = (\pi_{\mathcal{A}}, \chi_{\mathcal{A}})$ consists of an initial part of smooth blocks followed by thorny blocks.*

algorithm for extracting the smooth chain out of C is given in Algorithm 3. Function *isSmoothBlock(B)* checks whether a block B is a smooth velvet by calling *isSmoothPointer(B,p)* for every pointer p in B's interlink. Function *isSmoothPointer(B,p)* returns *true* if p is a valid pointer, in essence a pointer to the most recent *smooth velvet* for the level denoted by the pointer itself. The *updateInterlink* algorithm is given in Algorithm 4, which is essentially the same as in the case of a hard/soft fork, except for working on the smooth chain C_S instead of C.

The construction of the velvet suffix prover is given in Algorithm 5, which is essentially the same to that of a hard/soft fork except for working on smooth chain C_S instead of C.

In conclusion the Verify algorithm for the NIPoPoW suffix protocol remains the same as in the case of hard or soft fork.

Algorithm 3 The computation of a smooth chain

```
function smoothChain(C)
    C_S ← {G}
    k ← 1
    while C[−k] ≠ G do
        if isSmoothBlock(C[−k]) then
            C_S ← C_S ∪ C[−k]
        end if
        k ← k + 1
    end while
    return C_S
end function

function isSmoothBlock(B)
    if B = G then
        return true
    end if
    for p ∈ B.interlink do
        if ¬isSmoothPointer(B, p) then
            return false
        end if
    end for
    return true
end function

function isSmoothPointer(B, p)
    b ← Block(B.prevId)
    while b ≠ p do
        if level(b) ≥ level(p) ∧ isSmoothBlock(b) then
            return false
        end if
        if b = G then
            return false
        end if
        b ← Block(b.prevId)
    end while
    return isSmoothBlock(b)
end function
```

Algorithm 4 Velvet updateInterlink

```
function updateInterlinkVelvet(C_S)
    B′ ← C_S[−1]
    interlink ← B′.interlink
    for μ = 0 to level(B′) do
        interlink[μ] ← id(B′)
    end for
    return interlink
end function
```

6 COMBINED ATTACK

After the suggested protocol update the honest prover cannot contain any thorny blokcs in his suffix NIPoPow even if these blocks

Algorithm 5 Velvet Suffix Prover

```
function ProveVelvet_{m,k}(C_S)
    B ← C_S[0]
    for μ = |C_S[−k].interlink| down to 0 do
        α ← C_S[: − k]{B:}↑^μ
        π ← π ∪ α
        B ← α[−m]
    end for
    χ ← C_S[−k:]
    return πχ
end function
```

are part of C_B. The adversary may exploit this fact and to try to suppress honestly generated blocks in C_B, in order to reduce the blocks that can represent the honest chain in a proof. In parallel, while the adversary mines suppressive thorny blocks on C_B she can still use her blocks in her NIPoPoW proofs, by chainsewing them. Consequently, even if a suppression attempt does not succeed, in case for example that a second honestly generated block is soon enough published, she does not drop the thorny block she generated but include it in her proof.

More in detail, consider that the adversary wishes to attack a specific block level μ_B and generate a NIPoPow proof containing a block b of a fork chain which contains a double spending transaction. Then she acts as follows. She may mine on her fork chain $C_{\mathcal{A}}$ but when she observes a μ_B-level block in C_B she mines a thorny block on C_B which jumps onto her fork chain, in order to suppress this μ_B block. If the suppression succeeds she has managed to damage the μ_B superchain and mine a block that she can afterwards use in her proof. If the suppression does not succeed she can still use the thorny in her proof. The above are illustrated in Figure 8.

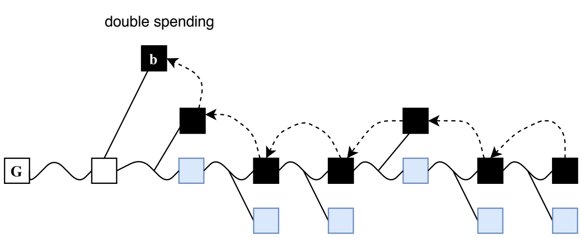


Figure 8: *The adversary suppress honestly generated blocks and chainsew thorny blocks in C_B. Blue blocks are honestly generated blocks of a specific level of attack. The adversary tries to suppress them. If the suppression is not successful, the adversary can still use the block she mined in her proof.*

The described attack consists a combined attack since both suppression and chainsewing are utilized. This combined attack forces us to adopt a stronger Honest Majority Assumption, so as to guarantee that the unsuppressed blocks in C_B suffice for constructing winning NIPoPoW proofs against the adversarial ones.

7 Q-BLOCK SUPPRESSION

Definition 7.1 (block property, Q-block). [29] A block property is a predicate Q defined on a hash output $h \in \{0,1\}^\kappa$. Given a block property Q, a valid block with hash h is called a Q-block if $Q(h)$ is true.

Lemma 7.2 (Unsuppressibility). *[29] Consider a collection of polynomially many block properties $\mathcal{Q}$. In a typical execution every set of consecutive rounds U has a subset S of uniquely successful rounds such that*

- $|S| \geq Y(U) - 2Z(U) - 2\lambda f(\frac{t}{n-t}\cdot\frac{1}{1-f}+\epsilon)$
- *for any $Q \in \mathcal{Q}$, Q-blocks generated during S follow the distribution as in an unsuppressed chain*
- *after the last round in S the blocks corresponding to S belong to the chain of any honest party.*

Lemma 7.3. *Consider Algorithm 4 under velvet fork with parameter g and (1/4)-bounded velvet honest majority. Let U be a set of consecutive rounds $r_1 \cdots r_2$ and C the chain of an honest party at round r_2 of a typical execution. Let $C_U^S = \{b \in C : b \text{ is smooth} \wedge b \text{ was generated during } U\}$. Let $\mu, \mu' \in \mathbb{N}$. Suppose C' a μ' superchain containing only adversarial blocks generated during U and suppose $|C'| > k$. Then it holds that $2^{\mu'}|C'| < 2^{\mu}|C_U^S\uparrow^{\mu}|$.*

Proof. From the Unsuppressibility Lemma we have that there is a set of uniquely successful rounds S, subset of U, such that $|S| \geq Y(U)-2Z(U)-\delta'$, where $\delta' = 2\lambda f(\frac{t}{n-t}\cdot\frac{1}{1-f}+\epsilon)$. We also know that Q-blocks of the property collection $\mathcal{Q} = \{Q(h) = true : h \in \{0,1\}^\kappa, \widetilde{\mu} \leq log(|C|), h \leq 2^{-\widetilde{\mu}}T\}$ that were generated during S are distributed as in an unsuppressed chain. Therefore for the number of interlinked μ-blocks that were generated during S it holds that $|C_U^S\uparrow^{\mu}| \geq (1-\epsilon)g2^{-\mu}|S|$. For the total number of μ'-blocks the adversary generated during U it holds that $|C'| \leq (1+\epsilon)2^{-\mu'}Z(U)$. Then we have to show that $(1-\epsilon)g(Y(U)-2Z(U)-\delta') > (1+\epsilon)Z(U)$ or $((1+\epsilon)+2g(1-\epsilon))Z(U) < g(1-\epsilon)(Y(U)+\delta')$. Let $\alpha_z = (1+\epsilon)+2g(1-\epsilon)$ then we have $\alpha_z Z(U) < g(1-\epsilon)(Y(U)+\delta')$.

Because of the typical exexution we know that $Y(U), Z(U)$ cannot deviate much from their expected values. In particular, we have that[8]: $Z(U) < \mathbb{E}[Z(U)] + \epsilon\mathbb{E}[X(U)]$ and $Y(U) > (1-\epsilon)\mathbb{E}[Y(U)]$. From the Backbone analysis we also know that $\mathbb{E}[Z(U)] < (1+\frac{\delta}{2})f\frac{t}{n-t}|U|$, $\mathbb{E}[X(U)] < pq(n-t)|U|$ and $\mathbb{E}[Y(U)] > f(1-f)|U|$. By substituting the expectation values we have $\alpha_z[(1+\frac{\delta}{2})f\frac{t}{n-t}|U| + \epsilon pq(n-t)|U|] < (1-\epsilon)g[(1-\epsilon)f(1-f)|U|-\delta']$ or $\alpha_z f|U|\frac{t}{n-t}(1+\frac{\delta}{2}) + \alpha_z\epsilon pq(n-t)|U| < (1-\epsilon)g[(1-\epsilon)f(1-f)|U|-\delta']$ or

$$\frac{t}{n-t} < \frac{(1-\epsilon)g[(1-\epsilon)f(1-f)|U|-\delta'] - \alpha_z\epsilon pq(n-t)|U|}{f|U|\alpha_z(1+\frac{\delta}{2})}$$

or

$$\frac{t}{n-t} < \frac{(1-\epsilon)g[(1-\epsilon)f(1-f)-\frac{\delta'}{|U|}] - \alpha_z\epsilon pq(n-t)}{f\alpha_z(1+\frac{\delta}{2})}$$

But

$$\frac{\epsilon pq(n-t)}{f(1+\frac{\delta}{2})} = \epsilon' \ll 1$$

thus

$$\frac{t}{n-t} < \frac{(1-\epsilon)g[(1-\epsilon)f(1-f) - \frac{\delta'}{|U|}]}{f\alpha_z(1+\frac{\delta}{2})} - \epsilon'$$

or

$$\frac{t}{n-t} < \frac{g}{(1+\frac{\delta}{2})f\alpha_z} \cdot \frac{(1-\epsilon)^2 f(1-f) - \frac{(1-\epsilon)\delta'}{|U|}}{(1+\frac{\delta}{2})f\alpha_z} - \epsilon'$$

Consequently we have that $\frac{t}{n-t} < \frac{1-\delta_v}{3}g$ which is the (1/4) velvet honest majority assumption, since

$$\frac{1}{(1+\frac{\delta}{2})f\alpha_z} = \frac{1}{(1+\frac{\delta}{2})f(1+\epsilon) + (1+\frac{\delta}{2})f(1-\epsilon)2g} > \frac{1}{3}$$

and

$$\frac{(1-\epsilon)^2 f(1-f) - \frac{(1-\epsilon)\delta'}{|U|}}{(1+\frac{\delta}{2})f\alpha_z} > (1-\delta_v)$$

. The last holds because we have $|C'| > k$ so $|U| \gg (1-\epsilon)\delta'$. □

8 ANALYSIS

Theorem 8.1 (Suffix Proofs Security under velvet fork). *Assuming honest majority under velvet fork conditions (5.1) such that $t \leq (1-\delta_v)\frac{n_h}{3}$ where n_h the number of upgraded honest players, the non-interactive proofs-of-proof-of-work construction for computable k-stable monotonic suffix-sensitive predicates under velvet fork conditions in a typical execution is secure.*

Proof. By contradiction. Let Q be a k-stable monotonic suffix-sensitive chain predicate. Assume for contradiction that NIPoPoWs under velvet fork on Q is insecure. Then, during an execution at some round r_3, $Q(\mathsf{C})$ is defined and the verifier V disagrees with some honest participant. V communicates with adversary $\mathcal{A}$ and honest prover B. The verifier receives proofs $\pi_{\mathcal{A}}, \pi_B$ which are of valid structure. Because B is honest, π_B is a proof constructed based on underlying blockchain C_B (with $\pi_B \subseteq \mathsf{C}_B$), which B has adopted during round r_3 at which π_B was generated. Consider $\widetilde{\mathsf{C}}_{\mathcal{A}}$ the set of blocks defined as $\widetilde{C}_{\mathcal{A}} = \pi_{\mathcal{A}} \cup \{\bigcup\{\mathsf{C}_h^r\{:b_{\mathcal{A}}\} : b_{\mathcal{A}} \in \pi_{\mathcal{A}}, \exists h, r : b_{\mathcal{A}} \in \mathsf{C}_h^r\}\}$ where C_h^r the chain that the honest player h has at round r.

The verifier outputs $\neg Q(\mathsf{C}_B)$. Thus it is necessary that $\pi_{\mathcal{A}} \geq_m \pi_B$. We show that $\pi_{\mathcal{A}} \geq_m \pi_B$ is a negligible event.

Let the levels of comparison decided by the verifier be $\mu_{\mathcal{A}}$ and μ_B respectively. Let $b_0 = LCA(\pi_{\mathcal{A}}, \pi_B)$. Let μ_B' be the adequate level of proof π_B with respect to block b_0. Call $\alpha_{\mathcal{A}} = \pi_{\mathcal{A}}\uparrow^{\mu_{\mathcal{A}}} \{b_0:\}$, $\alpha_B' = \pi_B\uparrow^{\mu_B'} \{b_0:\}$.

From Corollary 5.5 we have that the adversarial proof consists of a smooth interlink subchain followed by a thorny interlink subchain. We will refer to the smooth part of $\alpha_{\mathcal{A}}$ as $\alpha_{\mathcal{A}}^{\mathcal{S}}$ and to the thorny part as $\alpha_{\mathcal{A}}^{\mathcal{T}}$.

Our proof construction is based on the following intuition: we consider that $\alpha_{\mathcal{A}}$ consists of three distinct parts $\alpha_{\mathcal{A}}^1, \alpha_{\mathcal{A}}^2, \alpha_{\mathcal{A}}^3$ with the following properties.

Consider $b_0 = LCA(\pi_{\mathcal{A}}, \pi_B)$ the fork point between $\pi_{\mathcal{A}}\uparrow^{\mu_{\mathcal{A}}}$, $\pi_B\uparrow^{\mu_B}$ and $b_1 = LCA(\alpha_{\mathcal{A}}^{\mathcal{S}}, \mathsf{C}_B)$ the fork point between $\pi_{\mathcal{A}}^{\mathcal{S}}\uparrow^{\mu_{\mathcal{A}}}$, C_B at the zero level as the honest prover could observe. Part $\alpha_{\mathcal{A}}^1$ contains the blocks between b_0 exclusive and b_1 inclusive generated during the set of consecutive rounds $\mathcal{S}_1$ and $|\alpha_{\mathcal{A}}^1| = k_1$. Consider b_2 the last block in $\alpha_{\mathcal{A}}$ generated by an honest player. Part $\alpha_{\mathcal{A}}^2$ contains the blocks between b_1 exclusive and b_2 inclusive generated during the set of consecutive rounds $\mathcal{S}_2$ and $|\alpha_{\mathcal{A}}^2| = k_2$. Consider b_3 the next block of b_2 in $\alpha_{\mathcal{A}}$. Then $\alpha_{\mathcal{A}}^3 = \alpha_{\mathcal{A}}[b_3:]$ and $|\alpha_{\mathcal{A}}^3| = k_3$ of all adversarially generated blocks generated during the set of rounds $\mathcal{S}_3$. So, $|\alpha_{\mathcal{A}}| = k_1 + k_2 + k_3$ and we will show that $|\alpha_{\mathcal{A}}| < |\alpha_B|$.

The above are illustrated, among other, in Parts I, II of Figure 9.

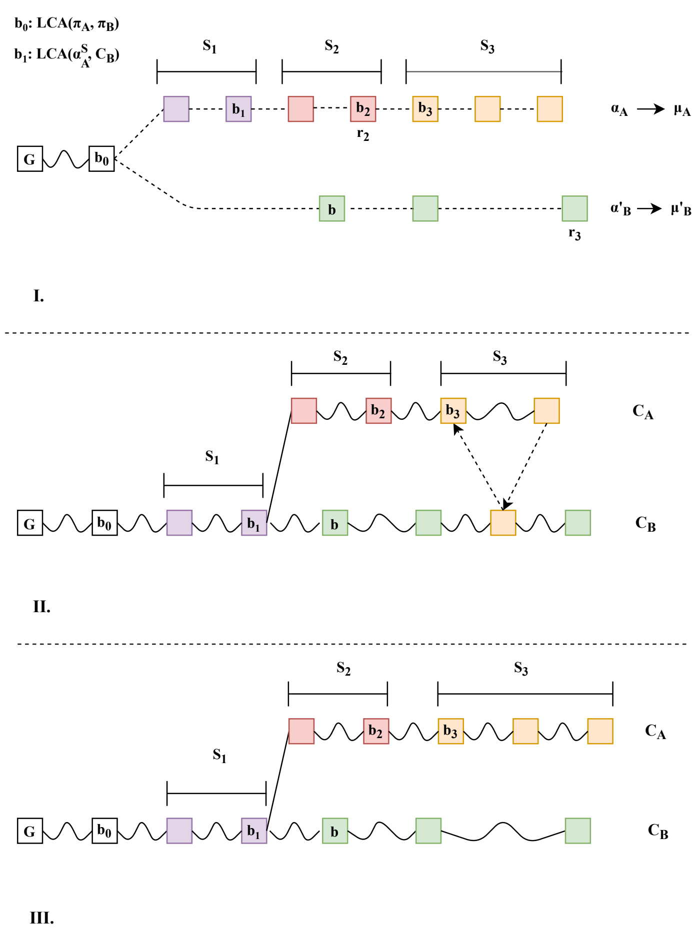


Figure 9: *Wavy lines imply one or more blocks. Dashed lines and arrows imply interlink pointers to superblocks. I: the three round sets in two competing proofs at different levels, II: the corresponding 0-level blocks implied by the two proofs, III: blocks participating in chain* C_B *and block set* $\widetilde{\mathsf{C}}_{\mathcal{A}}$ *from the verifier's perspective.*

We will now show three successive claims under velvet fork conditions: First that $\alpha_{\mathcal{A}}^1$ contains only a few blocks. Second, $\alpha_{\mathcal{A}}^2$

contains only a few blocks. And third, the adversary is able to produce a winning $a_{\mathcal{A}}$ with negligible probability.

Claim 1: $\alpha^1_{\mathcal{A}} = \alpha_{\mathcal{A}}(b_0 : b_1]$ contains only a few blocks. We have defined $b_0 = LCA(\pi_{\mathcal{A}}, \pi_B)$ and $b_1 = LCA(\alpha^S_{\mathcal{A}}, \alpha'_B \downarrow)$. First observe that there are no thorny blocks in $\alpha^1_{\mathcal{A}}$ since $\alpha^1_{\mathcal{A}}[-1] = b_1$ is a smooth block. This means that if b_1 was generated at round r_{b_1} and $\alpha^S_{\mathcal{A}}[-1]$ in round r, then $r \geq r_{b_1}$. So, $\alpha^1_{\mathcal{A}}$ consists a valid chain. We show the statement considering the two possible cases for the relation of $\mu_{\mathcal{A}}, \mu'_B$.

<u>*Claim 1a:*</u> If $\mu'_B \leq \mu_A$ then they are completely disjoint. In such a case of inequality, every block in α_A would also be of lower level μ'_B. Because of the adequate level μ'_B we know that $C\{b{:}\}{\uparrow}^{\mu'_B} = \pi\{b{:}\}{\uparrow}^{\mu'_B}$[15]. Subsequently, any block in $\pi_A{\uparrow}^{\mu_A}$ $\{b{:}\}[1{:}]$ would also be included in proof α'_B, but $b = LCA(\pi_A, \pi_B)$ so there can be no succeeding block common in α_A, α'_B.

<u>*Claim 1b:*</u> If $\mu'_B > \mu_A$ then $|\alpha_A[1{:}] \cap \alpha'_B \downarrow [1{:}]| = k_1 \leq g(2^{\mu'_B - \mu_A})$. Let's call b the first block in α'_B after block b_0. Suppose for contradiction that $k_1 > g(2^{\mu'_B - \mu_A})$. Since block b of level μ'_B is also of level μ_A, the adversary could include it in the proof but b cannot exist in both α_A, α'_B since $\alpha_A \cap \alpha'_B = \emptyset$ by definition. In case that the adversary chooses not to include b in the proof then she can include no other blocks of C_B in her proof, since it would not consist a valid chain. Therefore, the adversary can include at most the $\mu_{\mathcal{A}}$ upgraded blocks between b_0, b, which are expected to be equal to $g(2^{\mu'_B - \mu_A})$.

We conclude that $|\alpha^S_{\mathcal{A}} \cap \alpha'_B \downarrow [1{:}]| = k_1 \leq g(2^{\mu'_B - \mu_{\mathcal{A}}})$, where g the velvet parameter denoting the percentage of upgraded honest parties.

Consequently, there are at least $|\alpha_{\mathcal{A}}| - k_1$ blocks after block b in $\alpha_{\mathcal{A}}$ which are not honestly generated blocks existing in C_B. In other words, there are $|\alpha_{\mathcal{A}}| - k_1$ blocks after block b in $\alpha_{\mathcal{A}}$, which are either thorny blocks existing in C_B either don't belong in C_B.

Claim 2. Part $\alpha^2_{\mathcal{A}} = \alpha_{\mathcal{A}}(b_1 : b_2]$ consists of only a few blocks. Let $|\alpha^2_{\mathcal{A}}| = k_2$. We have defined $b_2 = \alpha^2_{\mathcal{A}}[-1]$ to be the last block generated by an honest player in $\alpha_{\mathcal{A}}$. Consequently no thorny block exists in $\alpha^2_{\mathcal{A}}$, so all blocks in this part belong in a proper zero-level chain $\mathrm{C}^2_{\mathcal{A}}$. Let r_{b_1} be the round at which b_1 was generated. Since b_1 is the last block in $\alpha_{\mathcal{A}}$ which belongs in C_B, then $\mathrm{C}^2_{\mathcal{A}}$ is a fork chain to C_B at some block b' generated at round $r' \geq r_{b_1}$.

Let r_2 be the round when b_2 was generated by an honest party. Because an honest party has chain C_B at later round r_3 when the proof π_B is constructed and because of the Common Prefix property on parameter $k_{2\downarrow} = g \cdot 2^{\mu_{\mathcal{A}}} \cdot k_2$, we conclude that $k_2 \leq k$, where k is the Common Prefix parameter as defined in the Backbone series of papers[8].

Claim 3. The adversary may submit a suffix proof such that $|\alpha_{\mathcal{A}}| \geq |\alpha_B|$ with negligible probability. As explained earlier part $\alpha^3_{\mathcal{A}}$ consists only of adversarially generated blocks. The security parameter m guarantees that $|\alpha^3_{\mathcal{A}}| > k$, as will be later presented in detail. Let U be the set of consecutive rounds $r_2 \ldots r_3$. Then all k_3 blocks of this part of the proof are generated during U. Let $\alpha'^{S_3}_B$ be the last part of the honest proof containing the interlinked μ_B superblocks generated during U. Then from lemma 7.3 we have that $2^{\mu_{\mathcal{A}}}|\alpha^3_{\mathcal{A}}| < 2^{\mu_B}|\alpha'^U_B{\uparrow}^{\mu_B}|$.

From all the above Claims we have that:
For the first round set S_1, because of the common underlying chain:

$$2^{\mu_{\mathcal{A}}}|\alpha^{S_1}_{\mathcal{A}}| \leq 2^{\mu'_B}|\alpha'^{S_1}_B| \quad (1)$$

For the second round set S_2 because of the adoption by an honest party of chain C_B at a later round r_3 we have:

$$2^{\mu_{\mathcal{A}}}|\alpha^{S_2}_{\mathcal{A}}| \leq 2^{\mu'_B}|\alpha'^{S_2}_B| \quad (2)$$

For the third round set S_3 we have:

$$2^{\mu_{\mathcal{A}}}|\alpha^{S_3}_{\mathcal{A}}| < 2^{\mu'_B}|\alpha'^{S_3}_B| \quad (3)$$

Consequently we have:

$$2^{\mu_{\mathcal{A}}}(|\alpha^{S_1}_{\mathcal{A}}| + |\alpha^{S_2}_{\mathcal{A}}| + |\alpha^{S_3}_{\mathcal{A}}|) < 2^{\mu'_B}(|\alpha'^{S_1}_B| + |\alpha'^{S_2}_B| + |\alpha'^{S_3}_B|) \Rightarrow$$

$$2^{\mu_{\mathcal{A}}}|\alpha_{\mathcal{A}}| < 2^{\mu'_B}|\alpha'_B| \quad (4)$$

Therefore we have proven that $2^{\mu'_B}|\pi_B{\uparrow}^{\mu'_B}| > 2^{\mu_{\mathcal{A}}}|\pi^{\mu_{\mathcal{A}}}_{\mathcal{A}}|$. From the definition of μ_B, we know that $2^{\mu_B}|\pi_B{\uparrow}^{\mu_B}| > 2^{\mu'_B}|\pi_B{\uparrow}^{\mu'_B}|$ because it was chosen μ_B as level of comparison by the Verifier. So we conclude that $2^{\mu_B}|\pi_B{\uparrow}^{\mu_B}| > 2^{\mu_{\mathcal{A}}}|\pi_{\mathcal{A}}{\uparrow}^{\mu_{\mathcal{A}}}|$. ◻

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