The Velvet Path to Superlight Blockchain Clients

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ABSTRACT

Superlight blockchain clients learn facts about the blockchain state while requiring merely polylogarithmic communication in the total number of blocks. For proof-of-work blockchains two known constructions exist: Superblock and FlyClient. Unfortunately, none of them can be deployed to existing blockchains as they require consensus changes and at least a soft fork to implement.

In this paper, we investigate how a blockchain can be upgraded to support superblock clients without a soft fork. We show that it is possible to implement the needed changes without modifying the consensus protocol and by requiring only a minority of miners to upgrade, a process termed a "velvet fork" in the literature. While previous work conjectured that superblock clients can be safely deployed using velvet forks as-is, we show that previous constructions are insecure, and that using velvet techniques to interlink a blockchain can pose insidious security risks. We describe a novel class of attacks, called "chain-sewing", which arise in the velvet fork setting: an adversary can cut-and-paste portions of various chains from independent forks, sewing them together to fool a superlight client into accepting a false claim. We show how previous velvet fork constructions can be attacked via chain-sewing. Next we put forth the first provably secure velvet superblock client construction which we show secure against adversaries that are bounded by 1/4 of the upgraded honest miner population. Like non-velvet superlight clients, our approach allows proving generic predicates about chains using infix proofs and as such can be adopted in practice for fast synchronization of transactions and accounts.

CCS CONCEPTS

• **Security and privacy** → *Cryptography*;

KEYWORDS

blockchain, consensus, lightclients, NIPoPoW

1 INTRODUCTION

Blockchain systems such as Bitcoin [26] and Ethereum [4, 30] have a predetermined expected rate of block production and maintain chains of blocks that are growing linearly with time. A node synchronizing with the rest of the blockchain network for the first time therefore has to download and validate the whole chain, if it does not wish to rely on a trusted third party. While a lightweight node (SPV) can avoid downloading and validating transactions beyond their interest, it must still download the block headers that contain the proof-of-work [8] of each block in order to determine which chain contains the most proof-of-work. The block header data, while smaller by a significant constant factor, still grow linearly with time. An Ethereum node synchronizing for the first time must download more than 300 MB of block header data for the purpose of proof-of-work verification, even if it elects not to download any transactions. This has become a central problem to the usability of blockchain systems, especially for vendors who are using mobile phones to accept payments or sit behind limited internet bandwidth. They are forced to make a difficult choice between decentralization and the ability to start accepting payments in a timely manner.

Towards the goal of alleviating the burden of this download for SPV clients, a number of *superlight* clients has emerged. These protocols give rise to Non-Interactive Proofs of Proof-of-Work (NIPoPoW) [18], which are short strings that "compress" the proof-of-work information of the underlying chain. The necessary security property of such proofs is that a minority adversary can only convince a NIPoPoW client that a certain transaction is confirmed, only if they can convince an SPV client, too.

There are two general directions for superlight client implementations: In the *superblock* [13, 18] approach, the client relies on *superblocks*, blocks that have achieved much better proof-of-work than required for block validity. In the *FlyClient* [2] approach, blocks are sampled and committed at random as in a Σ-protocol (e.g. Schnorr's discrete-log protocol [28]) and then using the Fiat–Shamir heuristic [9] a non-interactive proof is calculated. The number of block headers that need to be sent then grows only logarithmically with time. The NIPoPoW client, which is the proof *verifier* in this context, still relies on a connection to full nodes, who, acting as *provers*, perform the sampling of blocks from the full blockchain. No trust assumptions are made for these provers, as the verifier can check the veracity of their claims. As long as the verifier is connected to at least one honest prover (an assumption also made in the SPV protocol [11, 31]), they are able to arrive at the correct claim.

In both approaches, it is essential for the verifier to check that the blocks sampled one way or another have been generated in the same order as they have been presented by the prover. As such, each block in the proof must contain a pointer to the previous block in the proof. As blocks in these proofs are far apart in the underlying blockchain, the legacy *previous block pointer*, which typically appears within block headers, does not suffice. Both approaches require modifications to the consensus layer of the underlying blockchain to work. In the case of superblock NIPoPoWs, the block header must be modified to include, in addition to a pointer to the previous block, pointers to a small amount of recent high-proof-of-work blocks. In the case of FlyClient, each block must additionally contain pointers to all previous blocks in the chain. Both of these modifications can be made efficiently by organizing these pointers into Merkle Trees [25] or Merkle Mountain Ranges [22, 29] whose root is stored in the block header. The inclusion of extra pointers within blocks is termed *interlinking the chain* [17].

The modified block format, which includes the extra pointers, must be respected and validated by all full nodes and thus requires either a hard fork or at least a soft fork. However, even soft forks require the approval of a supermajority of miners, and new features that are considered non-essential by the community have taken years to receive approval [23]. Towards the goal of implementing superlight clients sooner, we study the question of whether it is possible to deploy superlight clients without a soft fork. We propose

a series of modifications to blocks that are *helpful but untrusted*. These modifications mandate that some extra data is included in each block. The extra data is placed inside the block by upgraded miners only, while the rest of the network does not include the additional data into the blocks and does not verify its inclusion, treating them merely as comments. To maintain backwards compatibility, contrary to a soft fork, upgraded miners must accept blocks that do not contain this extra data that have been produced by unupgraded miners, or even blocks that contain invalid or malicious such extra data produced by a mining adversary. This acceptance is necessary in order to avoid causing a chain split with the unupgraded part of the network. Such a modification to the consensus layer is termed a *velvet fork* [33]. A summary of our contributions in this paper is as follows:

(1) We illustrate that, contrary to claims of previous work, superlight clients designed to work in a soft fork cannot be readily plugged into a velvet fork and expected to work. We present a novel and insidious attack termed the *chain-sewing* attack which thwarts the defenses of previous proposals and allows even a minority adversary to cause catastrophic failures.
(2) We propose the first *backwards-compatible superlight client*. We put forth an interlinking mechanism implementable through a velvet fork. We then construct a superblock NIPoPoW protocol on top of the velvet forked chain and show it allows to build superlight clients for various statements regarding the blockchain state via both "suffix" and "infix" proofs.
(3) We prove our construction secure in the synchronous static difficulty model against adversaries bounded to 1/4 of the mining power of the honest upgraded nodes. As such, our protocol works even if a constant minority of miners adopts it.

Previous work. Proofs of Proof-of-Work have been proposed in the context of superlight clients [2, 8, 18], cross-chain communication [14, 19, 32], as well as local data consumption by smart contracts [15]. Superblock NIPoPoWs have been deployed in production using hard forks [7] and have been conjectured to work in velvet fork conditions [18] (we show here that these conjectures are ill-informed in the light of our chain-sewing attack). Velvet forks [33] have been studied for a variety of other applications and have been deployed in practice, e.g., see [12]. In this work, we focus on consensus state compression. Such compression has been explored in the hard-fork setting using zk-SNARKS [24] as well as in the Proof-of-Stake setting [16]. Complementary to consensus state compression (i.e., the compression of block headers and their ancestry) is compression of application state (namely the State Trie, the UTXO, or transaction history). There is a series of works complementary and composable with ours that discusses the compression of application state [5, 20].

Organization. The structure of the paper is as follows. In Section 2, we give a brief overview of the *backbone model* and its notation; as we heavily leverage the machinery of the model, the section is a necessary prerequisite to follow the analysis. The rest of the paper is structured in the form of *a proof and refutation* [21]. We believe this form is more digestible. In Section 3, we discuss the velvet model and some initial definitions, and we present a first attempt towards a velvet NIPoPoW scheme which was presented in previous work. We discuss the informal argument of why it seems to be secure, which we *refute* with an attack explored in Section 4. In Section 5, we patch the scheme and put forth our more elaborate and novel Velvet NIPoPoW construction. We analyze it and formally prove it secure in Section **??** (this technical section can be omitted without loss of continuity). Our scheme at this point allows verifiers to decide which blocks form a *suffix* of the longest blockchain and thus the protocol supports *suffix proofs*. We extend our scheme to allow any block of interest within the blockchain to be demonstrated to a prover in a straight forward manner in Section 7, giving a full *infix proof* protocol. The latter protocol can be used in practice and can be deployed today in real blockchains, including Bitcoin, to confirm payments achieving both decentralization and timeliness, solving a major outstanding dilemma in contemporary blockchain systems.

2 PRELIMINARIES

We consider a setting where the blockchain network consists of two different types of nodes: The first kind, *full nodes*, are responsible for the maintenance of the chain including verifying it and mining new blocks. The second kind, *verifiers* connect to full nodes and wish to learn facts about the blockchain without downloading it, for example whether a particular transaction is confirmed. The full nodes therefore also function as *provers* for the verifiers. Each verifier connects to multiple provers, at least one of which is assumed to be honest.

We model full nodes according to the Backbone model [10]. There are n full nodes, of which t are adversarial and $n - t$ are honest. All t adversarial parties are controlled by one colluding adversary $\mathcal{A}$. The parties have access to a hash function H which is modelled as a common Random Oracle [1]. To each novel query, the random oracle outputs κ bits of fresh randomness. Time is split into distinct *rounds* numbered by the integers $1, 2, \cdots$. Our treatment is in the *synchronous model*, so we assume messages *diffused* (broadcast) by an honest party at the end of a round are received by all honest parties at the beginning of the next round. This is equivalent to a network connectivity assumption in which the round duration is taken to be the known time needed for a message to cross the diameter of the network. The adversary can inject messages, reorder them, sybil attack by creating multiple messages, but not suppress messages.

Each honest full node locally maintains a *chain* C, a sequence of blocks. In understanding that we are developing an improvement on top of SPV, we use the term *block* to mean what is typically referred to as a *block header*. Each block contains the Merkle Tree root [25] of transaction data $\overline{x}$, the hash s of the previous block in the chain known as the *previd*, as well as a nonce value ctr. As discussed in the Introduction, the compression of application data $\overline{x}$ is orthogonal to our goals in this paper and has been explored in independent work [5] which can be composed with ours. Each block $b = s \parallel \overline{x} \parallel ctr$ must satisfy the proof-of-work [8] equation $H(b) \leq T$ where T is a constant *target*, a small value signifying the difficulty of the proof-of-work problem. Our treatment is in the

static difficulty case, so we assume that T is constant throughout the execution[1]. $H(B)$ is known as the *block id*.

Blockchains are finite block sequences obeying the *blockchain property*: that in every block in the chain there exists a pointer to its previous block. A chain is *anchored* if its first block is *genesis*, denoted $\mathcal{G}$, a special block known to all parties. This is the only node the verifier knows about when it boots up. For chain addressing we use Python brackets $\mathsf{C}[\cdot]$. A zero-based positive number in a bracket indicates the indexed block in the chain. A negative index indicates a block from the end, e.g., $\mathsf{C}[-1]$ is the tip of the blockchain. A range $\mathsf{C}[i{:}j]$ is a subarray starting from i (inclusive) to j (exclusive). Given chains $\mathsf{C}_1, \mathsf{C}_2$ and blocks A, Z we concatenate them as $\mathsf{C}_1\mathsf{C}_2$ or $\mathsf{C}_1 A$ (if clarity mandates it, we also use the symbol $\|$ for concatenation). Here, $\mathsf{C}_2[0]$ must point to $\mathsf{C}_1[-1]$ and A must point to $\mathsf{C}_1[-1]$. We denote $\mathsf{C}\{A{:}Z\}$ the subarray of the chain from block A (inclusive) to block Z (exclusive). We can omit blocks or indices from either side of the range to take the chain to the beginning or end respectively. As long as the blockchain property is maintained, we freely use the set operators $\cup$, $\cap$ and $\subseteq$ to denote operations between chains, implying that the appropriate blocks are selected and then placed in chronological order.

During every round, every party attempts to *mine* a new block on top of its currently adopted chain. Each party is given q queries to the random oracle which it uses in attempting to mine a new block. Therefore the adversary has tq queries per round while the honest parties have $(n-t)q$ queries per round. When an honest party discovers a new block, they extend their chain with it and broadcast the new chain. Upon receiving a new chain C' from the network, an honest party compares its length $|\mathsf{C}'|$ against its currently adopted chain C and adopts the newly received chain if it is longer. It is assumed that the honest parties control the majority of the computational power of the network. This *honest majority assumption* states that there is some δ such that $t < (1-\delta)(n-t)$. If so, the protocol ensures consensus among the honest parties: There is a constant k, the *Common Prefix* parameter, such that, at any round, all the chains belonging to honest parties share a common prefix of blocks; the chains can deviate only up to k blocks at the end of each chain [10]. Concretely, if at some round r two honest parties have C_1 and C_2 respectively, then either $\mathsf{C}_1[:-k]$ is a prefix of C_2 or vice versa.

Some valid blocks satisfy the proof-of-work equation better than required. If a block b satisfies $H(b) \leq 2^{-\mu}T$ for some natural number $\mu \in \mathbb{N}$ we say that b is a *μ-superblock* or a block *of level μ*. The probability of a new valid block achieving level μ is $2^{-\mu}$. The number of levels in the chain will be $\log|\mathsf{C}|$ with high probability [17]. Given a chain C, we denote $\mathsf{C}{\uparrow}^{\mu}$ the subset of μ-superblocks of C.

Non-Interactive Proofs of Proof-of-Work (NIPoPoW) protocols allow verifiers to learn the most recent k blocks of the blockchain adopted by an honest full node without downloading the whole chain. The challenge lies in building a verifier who can find the suffix of the longest chain between claims of both honest and adversarial provers, while not downloading all block headers. Towards that goal, the *superblock* approach uses superblocks as samples of proof-of-work. The prover sends superblocks to the verifier to convince them that proof-of-work has taken place without actually presenting all this proof-of-work. The protocol is parametrized by a constant security parameter m. The parameter determines how many superblocks will be sent by the prover to the verifier and security is proven with overwhelming probability in m.

The prover selects various levels μ and for each such level sends a carefully chosen portion of its μ-level *superchain* $\mathsf{C}{\uparrow}^{\mu}$ to the verifier. In standard blockchain protocols such as Bitcoin and Ethereum, each block $\mathsf{C}[i+1]$ in C points to its previous block $\mathsf{C}[i]$, but each μ-superblock $\mathsf{C}{\uparrow}^{\mu}[i+1]$ does not point to its previous μ-superblock $\mathsf{C}{\uparrow}^{\mu}[i]$. It is imperative that an adversarial prover does not reorder the blocks within a superchain, but the verifier cannot verify this unless each μ-superblock points to its most recently preceding μ-superblock. The proposal is therefore to *interlink* the chain by having each μ-superblock include an extra pointer to its most recently preceding μ-superblock. To ensure integrity, this pointer must be included in the block header and verified by proof-of-work. However, the miner does not know which level a candidate block will attain prior to mining it. For this purpose, each block is proposed to include a pointer to the most recently preceding μ-superblock, for every μ, as illustrated in Figure 1. As these levels are only $\log|\mathsf{C}|$, this only adds $\log|\mathsf{C}|$ extra pointers to each block header.

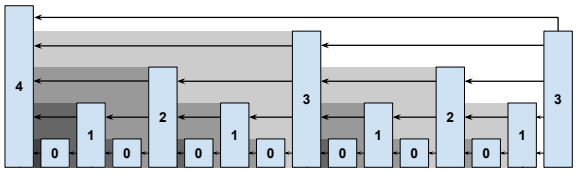

Figure 1: The interlinked blockchain. Each superblock is drawn taller according to its achieved level. Each block links to all the blocks that are not being overshadowed by their descendants. The most recent (right-most) block links to the four blocks it has direct line-of-sight to.

The exact NIPoPoW protocol works like this: The prover holds a full chain C. When the verifier requests a proof, the prover sends the last k blocks of their chain, the suffix $\chi = \mathsf{C}[-k{:}]$, in full. From the larger prefix $\mathsf{C}[:-k]$, the prover constructs a proof π by selecting certain superblocks as representative samples of the proof-of-work that took place. The blocks are picked as follows. The prover selects the *highest* level μ^* that has at least m blocks in it and includes all these blocks in their proof (if no such level exists, the chain is small and can be sent in full). The prover then iterates from level $\mu = \mu^* - 1$ down to 0. For every level μ, it includes sufficient μ-superblocks to cover the last m blocks of level $\mu+1$, as illustrated in Algorithm 1. Because the density of blocks doubles as levels are descended, the proof will contain in expectation $2m$ blocks for each level below μ^*. As such, the total proof size $\pi\chi$ will be $\Theta(m\log|\mathsf{C}|+k)$. Such proofs that are polylogarithmic in the chain size constitute an exponential improvement over traditional SPV clients and are called *succinct*.

[1] A treatment of variable difficulty NIPoPoWs has been explored in the soft fork case [34], but we leave the treatment of velvet fork NIPoPoWs in the variable difficulty model for future work.

Algorithm 1 The Prove algorithm for the NIPoPoW protocol in a soft fork

```
function Prove_{m,k}(C)
    B ← C[0]                                   ▷ Genesis
    for μ = |C[−k − 1].interlink| down to 0 do
        α ← C[: −k]{B :}↑^μ
        π ← π ∪ α
        if m < |α| then
            B ← α[−m]
        end if
    end for
    χ ← C[−k :]
    return πχ
end function
```

Upon receiving two proofs $\pi_1\chi_1, \pi_2\chi_2$ of this form, the NIPoPoW verifier first checks that $|\chi_1| = |\chi_2| = k$ and that $\pi_1\chi_1$ and $\pi_2\chi_2$ form valid chains. To check that they are valid chains, the verifier ensures every block in the proof contains a pointer to its previous block inside the proof through either the *previd* pointer in the block header, or in the interlink vector. If any of these checks fail, the proof is rejected. It then compares π_1 against π_2 using the $\leq_m$ operator, which works as follows. It finds the lowest common ancestor block $b = (\pi_1 \cap \pi_2)[-1]$; that is, b is the most recent block shared among the two proofs. Subsequently, it chooses the level μ_1 for π_1 such that $|\pi_1\{b:\}\uparrow^{\mu_1}| \geq m$ (i.e., π_1 has at least m superblocks of level μ_1 following block b) and the value $2^{\mu_1}|\pi_1\{b:\}\uparrow^{\mu_1}|$ is maximized. It chooses a level μ_2 for π_2 in the same fashion. The two proofs are compared by checking whether $2^{\mu_1}|\pi_1\{b:\}\uparrow^{\mu_1}| \geq 2^{\mu_2}|\pi_2\{b:\}\uparrow^{\mu_2}|$ and the proof with the largest score is deemed the winner. The comparison is illustrated in Algorithm 2.

Algorithm 2 The implementation of the $\geq_m$ operator to compare two NIPoPoW proofs parameterized with security parameter m. Returns *true* if the underlying chain of party A is deemed longer than the underlying chain of party B.

```
function best-arg_m(π, b)
    M ← {μ : |π↑^μ {b :}| ≥ m} ∪ {0}           ▷ Valid levels
    return max_{μ∈M}{2^μ · |π↑^μ {b :}|}       ▷ Score for level
end function
operator π_A ≥_m π_B
    b ← (π_A ∩ π_B)[−1]                        ▷ LCA
    return best-arg_m(π_A, b) ≥ best-arg_m(π_B, b)
end operator
```

Blockchain protocols can be upgraded using hard or soft forks [3]. In a *hard fork*, blocks produced by upgraded miners are not accepted by unupgraded miners. It is simplest to introduce interlinks using a hard fork by mandating that interlink pointers are included in additional fields in the block header. Unupgraded miners will not recognize these fields and will be unable to parse upgraded blocks. To ensure the block header is of constant size, instead of including all these superblock pointers in the block header individually, they are organized into a Merkle Tree of interlink pointers and only the root of the Merkle Tree is included in the block header. In this case, the NIPoPoW prover that wishes to show a block b in their proof is connected to its more recently preceding μ-superblock b', also includes a Merkle Tree proof proving that $H(b')$ is a leaf in the interlink Merkle Tree root included in the block header of b. The verifier has the additional job of verifying the veracity of these Merkle inclusion proofs.

In a *soft forks*, blocks created by unupgraded miners are not accepted by upgraded miners, but blocks created by upgraded miners are accepted by unupgraded miners. As such, any additional data introduced by the upgrade must be included in a field that is treated like a comment by an unupgraded miner. To interlink the chain via a soft fork, the interlink Merkle Tree root is placed in the *coinbase* transaction instead of the block header. Upgraded miners include the correct interlink Merkle Tree root in their coinbase. Upgraded miners receiving a new block validate that the interlink Merkle Tree root is correct before accepting a block as valid. As this root can be calculated in a deterministic manner from the previous blocks in the chain, it can easily be validated. Unupgraded miners ignore this data and accept the block regardless of whether it exists. The success of the fork depends on the majority of the miner population upgrading. Whenever the NIPoPoW prover wishes to show that a block b in the proof contains a pointer to its most recently preceding μ-superblock b', it must then accompany the block header of $b = s \,\|\, \overline{x} \,\|\, ctr$ with the coinbase transaction $\mathsf{tx}_{\mathsf{cb}}$ of b as well as two Merkle Tree proofs: One proving that the coinbase transaction $\mathsf{tx}_{\mathsf{cb}}$ is in $\overline{x}$, and one proving that $H(b')$ is a leaf in the interlink Merkle Tree whose root is included in $\mathsf{tx}_{\mathsf{cb}}$.

3 VELVET INTERLINKS

More recently, velvet forks have been introduced [33]. In a velvet fork, blocks created by upgraded miners (called *velvet blocks*) are accepted by unupgraded miners as in a soft fork. Additionally, blocks created by unupgraded miners are also accepted by upgraded miners. This allows the protocol to upgrade even if only a minority of miners chooses to upgrade. To maintain backwards compatibility and to avoid causing forks, the additional data included in a block is *advisory* and must be accepted whether it exists or not. Even if the additional data is invalid or malicious, upgraded nodes (in this context also called *velvet nodes*) are forced to accept the blocks. The simplest approach to velvet fork the chain for interlinking purposes is to have upgraded miners include the interlink pointer in the blocks they produce, but accept blocks with missing or incorrect interlinks. As we show in the next section, this approach is flawed and susceptible to unexpected attacks. A surgical change in the way velvet blocks are produced is necessary to achieve proper security.

In a velvet fork, only a minority of honest parties needs to support the protocol changes. We refer to this percentage as the "velvet parameter".

Definition 3.1 (Velvet Parameter). The *velvet parameter* g is defined as the percentage of honest parties that have upgraded to the new protocol. The absolute number of honest upgraded parties is denoted n_h and it holds that $n_h = g(n - t)$.

Velvet forks maintain backwards and forwards compatibility. This requires any block produced by upgraded miners to be accepted by unupgraded nodes (as in a soft fork), but also blocks

produced by unupgraded miners to be accepted by upgraded nodes. For the particular case of superblock NIPoPoWs under velvet forks, upgraded miners must include the interlink data structure within their blocks, but must also accept blocks missing the interlink structure or containing an invalid interlink. Unupgraded honest nodes will produce blocks that contain no interlink, while upgraded honest nodes will produce blocks that contain truthful interlinks. Therefore, any block with invalid interlinks will be adversarially generated. However, such blocks cannot be rejected by the upgraded nodes, as that would give the adversary an opportunity to cause a hard fork.

A block generated by the adversary can thus contain arbitrary data in the interlink and yet be adopted by an honest party. Because the honest prover is an upgraded full node, it can determine what the correct interlink pointers are by examining the whole previous chain, and can thus deduce whether a block contains invalid interlink data. In that case, the prover can simply treat such blocks as unupgraded. In the context of the attack that will be presented in the following section, we examine the case where the adversary includes false interlink pointers.

In any velvet protocol, a specific portion within a block, which is treated as a comment by unupgraded nodes, is reused to contain auxiliary data by upgraded miners. Because these auxiliary data can be deterministically calculated, upgraded full nodes can verify the authenticity of the data in a new block they receive. We distinguish blocks based on whether they follow the velvet protocol rules or they deviate from them.

Definition 3.2 (Smooth and Thorny blocks). A block in a velvet protocol upgrade is called *smooth* if it contains auxiliary data and the data corresponds to the honest upgraded protocol. A block is called *thorny* if it contains auxiliary data, but the data differs from the honest upgraded protocol. A block can be neither smooth nor thorny if it does not contain auxiliary data.

In the case of velvet forks for interlink purposes, the auxiliary data consists of the Merkle Tree containing the interlink pointers to the most recent superblock ancestor at every level μ.

A naïve velvet scheme. In previous work [18], it was conjectured that superblock NIPoPoWs remain secure under a velvet fork. We call this scheme the *Naïve Velvet NIPoPoW* protocol, because it is not dissimilar from the NIPoPoW protocol in the soft fork case. In particular, the naïve velvet NIPoPoW protocol that was put forth works as follows. Each upgraded honest miner attempts to mine a block b that includes interlink pointers in the form of a Merkle Tree included in its coinbase transaction. For each level μ, the interlink contains a pointer to the most recent among all the ancestors of b that have achieved at least level μ, regardless of whether the referenced block is upgraded or not and regardless of whether its interlinks are valid. Unupgraded honest nodes will keep mining blocks on the chain as usual; because the status of a block as superblock does not require it to be mined by an upgraded miner, the unupgraded miners contribute mining power to the creation of superblocks as desired.

The prover in the naïve velvet NIPoPoWs then worked as follows. The honest prover constructed the NIPoPoW proof $\pi\chi$ as usual by selecting certain superblocks from his chain C as representatives in π and by setting $\chi = \mathrm{C}[-k:]$. The outstanding issue in this case, however, is that these blocks in π do not form a chain because, while superblocks, some of them may not be upgraded and they may not contain any pointers (or they may contain invalid pointers). The honest prover needs to provide a connection between two consecutive blocks $\pi[i+1]$ and $\pi[i]$ in the superchain, and suppose $\pi[i]$ is the most recent μ-superblock preceding $\pi[i+1]$. The block $\pi[i+1]$ is a superblock and exists at some position j in the underlying chain C of the prover, i.e., at $\pi[i+1] = \mathrm{C}[j]$. If $\mathrm{C}[j]$ is a smooth block, then the interlink pointer at level μ within it can be used directly. Otherwise, the prover used the *previd* pointer of $\pi[i+1] = \mathrm{C}[j]$ to repeatedly reach the parents of $\mathrm{C}[j]$, namely $\mathrm{C}[j-1], \mathrm{C}[j-2], \cdots$ until a smooth block b between $\pi[i]$ and $\pi[i+1]$ was found in C. The block b then contains a pointer to $\pi[i]$, as $\pi[i]$ is also the most recent μ-superblock ancestor of b. The blocks $\mathrm{C}[j-1], \mathrm{C}[j-2], \cdots, b$ are then included in the proof to illustrate that $\pi[i]$ is an ancestor of $\pi[i+1]$.

The argument for why the above scheme work is as follows. First of all, the scheme does not add many new blocks to the proof. In expectation, if a fully honestly generated chain is processed, after in expectation $\frac{1}{g}$ blocks have been traversed, a smooth block will be found and the connection to $\pi[i]$ will be made. Thus, the number of blocks needed in the proof increases by a factor of $\frac{1}{g}$. Security was argued as follows: An honest party includes in their proof as many blocks as in a soft forked NIPoPoW, albeit by using an indirect connection. The crucial feature is that it is not missing any superblocks. Even if the adversary creates interlinks that skip over some honest superblocks, the honest prover will not utilize these interlinks, but will use the "slow route" of level 0 instead. The adversarial prover, on the other hand, can only use honest interlinks as before, but may also use false interlinks in blocks mined by the adversary. However, these false interlinks cannot point to blocks that are of incorrect level. The reason is that the verifier can look at the hash of each block to verify its level and therefore cannot be lied to. The only problem a fake interlink can cause is that it can point to a μ-superblock which is not *the most recent ancestor*, but some other block. It was then argued that the only other possibility was to point to blocks that are older μ-superblock ancestors in the same chain, as illustrated in Figure 2. However, the adversarial prover can only harm herself by making use of these pointers, as the result will simply by a superchain with fewer blocks.

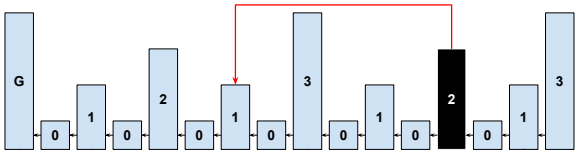

Figure 2: A thorny pointer of an adversarial block, colored black, in an honest party's chain. The thorny block points to a 1-superboock which is an ancestor 1-superblock, but not *the most recent ancestor* 1-superblock.

As such, we conclude that the honest verifier comparing the honest superchain against the adversarial superchain will reach the same conclusion in the velvet case as he would have reached in the soft fork case: Because the honest superchain in the velvet case contains the same amount of blocks as the honest superchain in

the soft fork case, but the adversarial superchain in the velvet case contains fewer blocks than in the soft fork case, the comparison will remain in favor of the honest party. As we will see in the next section, this conclusion is far from straightforward.

4 THE CHAINSEWING ATTACK

We now make the critical observation that a thorny block can include interlink pointers to blocks that are not its own ancestors in the 0-level chain. Because it must contain a pointer to the hash of the block it points to, they must be blocks that have been generated previously, but they may belong to a different 0-level chain. This is shown in Figure 3.

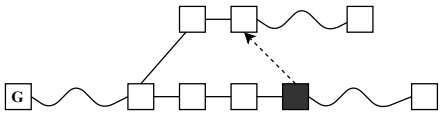

Figure 3: A thorny block, colored black, in an honest party's chain, uses its interlink to point to a fork chain.

In fact, as the interlink vector contains multiple pointers, each pointer may belong to a different fork. This is illustrated in Figure 4. The interlink pointing to arbitrary directions resembles a thorny bush.

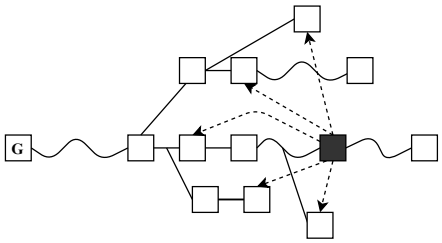

Figure 4: A thorny block appended to an honest party's chain. The dashed arrows are interlink pointers.

We now present the *chain-sewing attack* against the naïve velvet NIPoPoW protocol. The attack leverages thorny blocks in order to enable the adversary to *usurp* blocks belonging to a different chain and claim it as their own. Taking advantage of thorny blocks, the adversary can produce suffix proofs containing an arbitrary number of blocks belonging to several fork chains. The attack works as follows.

Assume chain C_B was adopted by an honest party B and chain $\mathsf{C}_{\mathcal{A}}$, a fork of C_B at some point, is maintained by the adversary $\mathcal{A}$. After the fork point $b = (\mathsf{C}_B \cap \mathsf{C}_{\mathcal{A}})[-1]$, the honest party produces a block extending b in C_B containing a transaction tx. The adversary includes a conflicting (double spending) transaction tx' in a block extending b in $\mathsf{C}_{\mathcal{A}}$. The adversary wants to produce a suffix proof convincing a light client that $\mathsf{C}_{\mathcal{A}}$ is the longer chain. In order to achieve this, the adversary needs to include a greater amount of total proof-of-work in her suffix proof, $\pi_{\mathcal{A}}$, in comparison to that included in the honest party's proof, π_B, so as to achieve $\pi_{\mathcal{A}} \geq_m \pi_B$. Towards this purpose, she miners intermittently on both C_B and $\mathsf{C}_{\mathcal{A}}$. She produces some thorny blocks in both chains $\mathsf{C}_{\mathcal{A}}$ and C_B which will allow her to usurp selected blocks of C_B and present them to the light client as if they belonged to $\mathsf{C}_{\mathcal{A}}$ in her suffix proof.

The general form of this attack for an adversary sewing blocks to one forked chain is illustrated in Figure 5. Dashed arrows represent interlink pointers of some level $\mu_{\mathcal{A}}$. Starting from a thorny block in the adversary's forked chain and following the interlink pointers, jumping between $\mathsf{C}_{\mathcal{A}}$ and C_B, a chain of blocks crossing forks is formed, which the adversary claims as part of her suffix proof. Blocks of both chains are included in this proof and a verifier cannot distinguish the non-smooth pointers participating in this proof chain and, as a result, considers it a valid proof. Importantly, the adversary must ensure that any blocks usurped from the honest chain are not included in the honest NIPoPoW to force the NIPoPoW verifier to consider an earlier LCA block b; otherwise, the adversary will compete after a later fork point, negating any sewing benefits.

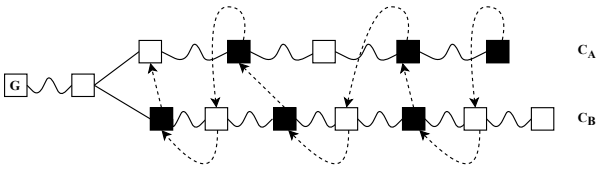

Figure 5: Generic Chainsewing Attack. C_B is the chain of an honest party and $\mathsf{C}_{\mathcal{A}}$ the adversary's chain. Adversarially generated blocks are colored black. Dashed arrows represent interlink pointers included in the adversary's suffix proof. Wavy lines imply one or more blocks.

This generic attack can be made concrete as follows. The adversary chooses to attack at some level $\mu_{\mathcal{A}} \in \mathbb{N}$ (ideally, if the honest verifier does not impose any succinctness limits, the adversary sets $\mu_A = 0$). As shown in Figure 6, she first generates a block b' in her forked chain $\mathsf{C}_{\mathcal{A}}$ containing the double spend, and a block a' in the honest chain C_B which thorny-points to b'. Block a' will be accepted as valid in the honest chain C_B despite the invalid interlink pointers. The adversary also chooses a desired superblock level $\mu_B \in \mathbb{N}$ that she wishes the honest party to attain. Subsequently, the adversary waits for the honest party to mine and sews any blocks mined on the honest chain that are of level below μ_B. However, she must bypass blocks that she thinks the honest party will include in their final NIPoPoW, which are of level μ_B (the blue block designated c in Figure 6). To bypass a block, the adversary mines her own thorny block d on top of the current honest tip (which could be equal to the block to be bypassed, or have progressed further), containing a thorny pointer to the block preceding the block to be bypassed and hoping that it will not exceed level μ_B (if it exceeds that level, she discards her d block). Once m blocks of level μ_B have been bypassed in this manner, the adversary starts bypassing blocks of level $\mu_B - 1$, because the honest NIPoPoW will start including lower-level blocks. The adversary continues descending in levels until a sufficiently low level min μ_B has been reached at which point it becomes uneconomical for the adversary to continue bypassing blocks (typically for a 1/4 adversary, min $\mu_B = 2$). At this point, the adversary forks off of the last sewed honest block. This last honest

block will be used as the last portion of the adversarial π part of the NIPoPoW proof. She then independently mines a k-long suffix for the χ portion and creates her NIPoPoW $\pi\chi$. Lastly, she waits for enough time to pass so that the honest party's chain progresses sufficiently to make the previous bypassing guesses correct and so that no blocks in the honest NIPoPoWs coincide with blocks that have not been bypassed. This requires to wait for the following blocks to appear in the honest chain: $2m$ blocks of level μ_B; after the m^{th} first μ_B-level block, a further $2m$ blocks of level $\mu_B - 1$; after the m^{th} such block, a further $2m$ blocks of the preceding level, and so on until level 0 is reached.

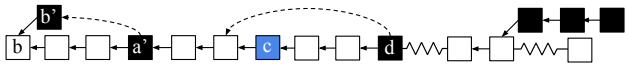

Figure 6: A portion of the concrete Chainsewing Attack. The adversary's blocks are shown in black, while the honestly generated blocks are shown in white. Block b' contains a double spend, while block a' sews it in place. The blue block c is a block included in the honest NIPoPoW, but it is bypassed by the adversary by introducing block d which, while part of the honest chain, points to c's parent. After a point, the adversary forks off and creates $k = 3$ of their own blocks.

In this attack the adversary uses thorny blocks to "sew" portions of the honestly adopted chain to her own forked chain. This justifies the name given to the attack. Note that in order to make this attack successful, the adversary needs only produce few superblocks, but she can arrogate an arbitrarily large number of honestly produced blocks. Thus the attack succeeds with non-negligible probability.

We illustrate simulation results[2] for the success rate of our attack in Appendix A. Our experiments find the attack with parameters $\mu_B = 10, \mu_A = 0, t = 1, n = 5, k = 15$ succeeds with a constant rate of success of approximately 0.26, when the security parameter m ranges from 3 to 15. This is in contrast to the best previously known attack (which does not make us of thorny blocks), which succeeds with probability less than 0.01. Previous work recommends $m = 15$ for a 1/3 adversary for a probability of failure bounded by 0.001.

5 VELVET NIPOPOWS

In order to eliminate the Chainsewing Attack we propose an update to the velvet NIPoPoW protocol. The core problem is that, in her suffix proof, the adversary was able to claim not only blocks of shorter forked chains, but also arbitrarily long parts of the chain generated by an honest party. Since thorny blocks are accepted as valid, the verifier cannot distinguish blocks that actually belong in a chain from blocks that only seem to belong in the same chain because they are pointed to from a thorny block.

The idea for a secure protocol is to distinguish the smooth from the thorny blocks, so that smooth blocks can never point to thorny blocks. In this way we can make sure that thorny blocks acting as passing points to fork chains, as block a' does in Figure 6, cannot be pointed to by honestly generated blocks. Therefore, the adversary cannot utilize honest mining power to construct a stronger suffix proof for her fork chain. Our velvet construction mandates that honest miners create blocks that contain interlink pointers pointing only to previous smooth blocks. As such, newly created smooth blocks can only point to previously created smooth blocks and not thorny blocks. Following the terminology of Section 3, the smoothness of a blocks in this new construction is a stricter notion than smoothness in the naïve construction.

In order to formally describe the suggested protocol patch, redefine the notion of a smooth block recursively by introducing the notion of a smooth interlink pointer.

Definition 5.1 (Smooth Pointer). A *smooth pointer* of a block b for a specific level μ is the interlink pointer to the most recent μ-level smooth ancestor of b.

We describe a protocol patch that operates as follows. The superblock NIPoPoW protocol works as usual but each honest miner constructs smooth blocks whose interlink contains only smooth pointers; thus it is constructed excluding thorny blocks. In this way, although thorny blocks are accepted in the chain, they are not taken into consideration when updating the interlink structure for the next block to be mined. No honest block could now point to a thorny superblock that may act as the passing point to the fork chain in an adversarial suffix proof. Thus, after this protocol update the adversary is only able to inject adversarially generated blocks from an honestly adopted chain to her own fork. At the same time, thorny blocks cannot participate in an honestly generated suffix proof except for some blocks in the proof's suffix (χ). This argument holds because thorny blocks do not form a valid chain along with honestly mined blocks anymore. Consequently, as far as the blocks included in a suffix proof are concerned, we can think of thorny blocks as belonging in the adversary's fork chain for the π part of the proof, which is the comparing part between proofs. Figure 7 illustrates this remark. The velvet NIPoPoW verifier is also modified to only follow interlink pointers, and never previd pointers (which could be pointing to thorny blocks, even if honestly generated).

With this protocol patch we conclude that the adversary cannot usurp honest mining power to her fork chain. This comes with the cost that the honest prover cannot utilize thorny blocks despite belonging in the honest chain. Due to this fact we define the Velvet Honest Majority Assumption for (1/4)-bounded adversary under which the security of the protocol is guaranteed.

Definition 5.2 (Velvet Honest Majority). Let n_h be the number of upgraded honest miners. Then t out of total n parties are corrupted such that $\frac{t}{n_h} < \frac{1-\delta_v}{3}$.

The following Lemmas come as immediate results from the suggested protocol update.

Lemma 5.3. *A velvet suffix proof constructed by an honest party cannot contain any thorny block.*

Lemma 5.4. *Let $\mathcal{P}_{\mathcal{A}} = (\pi_{\mathcal{A}}, \chi_{\mathcal{A}})$ be a velvet suffix proof constructed by the adversary and block b_s, generated at round r_s, be the most recent smooth block in the proof. Then $\forall r : r < r_s$ no thorny blocks generated at round r can be included in $\mathcal{P}_{\mathcal{A}}$.*

[2] A link to the open source implementation of the attack simulation has been removed for anonymization purposes

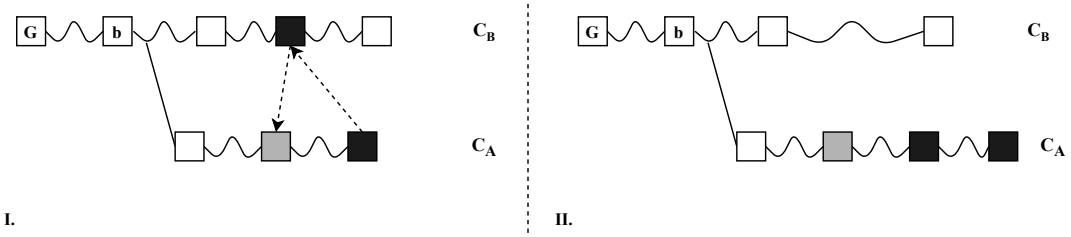

Figure 7: The adversarial fork chain C_A and chain C_B of an honest party. Thorny blocks are colored black. Dashed arrows represent interlink pointers. Wavy lines imply one or more blocks. After the protocol update, when an adversarially generated block is sewed from C_B into the adversary's suffix proof the verifier conceives C_A as longer and C_B as shorter. I: The real picture of the chains. II: Equivalent picture from the verifier's perspective considering the blocks included in the corresponding suffix proof for each chain.

Proof. By contradiction. Let b_t be a thorny block generated at some round $r_t < r_s$. Suppose for contradiction that b_t is included in the proof. Then, because $\mathcal{P}_{\mathcal{A}}$ is a valid chain as for interlink pointers, there exist a block path made by interlink pointers starting from b_s and resulting to b_t. Let b' be the most recently generated thorny block after b_t and before b_s included in $\mathcal{P}_{\mathcal{A}}$. Then b' has been generated at a round r' such that $r_t \leq r' < r_s$. Then the block right after block b' in $\mathcal{P}_{\mathcal{A}}$ must be a thorny block since it points to b' which is thorny. But b' is the most recent thorny block after b_t, thus we have reached a contradiction. □

Lemma 5.5. *Let $\mathcal{P}_{\mathcal{A}} = (\pi_{\mathcal{A}}, \chi_{\mathcal{A}})$ be a velvet suffix proof constructed by the adversary. Let b_t be the oldest thorny bock included in $\mathcal{P}_{\mathcal{A}}$ which is generated at round r_t. Then any block $b = \{b : b \in \mathcal{P}_{\mathcal{A}} \wedge b$ generated at $r \geq r_t\}$ is thorny.*

Proof. By contradiction. Suppose for contradiciton that b_s is a smooth block generated at round $r_s > r_t$. Then from Lemma 5.4 any block generated at round $r < r_s$ is smooth. But b_t is generated at round $r_t < r_s$ and is thorny, thus we have reached a contradiction. □

The following corollary emerges immediately from Lemmas 5.4, 5.5. This result is illustrated in Figure 8.

Corollary 5.6. *Any adversarial proof $\mathcal{P}_{\mathcal{A}} = (\pi_{\mathcal{A}}, \chi_{\mathcal{A}})$ containing both smooth and thorny blocks consists of a prefix smooth subchain followed by a suffix thorny subchain.*

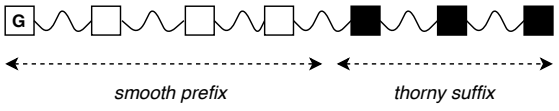

Figure 8: General case of the adversarial velvet suffix proof $\mathcal{P}_{\mathcal{A}} = (\pi_{\mathcal{A}}, \chi_{\mathcal{A}})$ consisting of an initial part of smooth blocks followed by thorny blocks.

We now describe the algorithms needed by the upgraded miner, prover and verifier. The upgraded miner acts as usual except for including the interlink of the newborn block in the coinbase transaction. In order to construct an interlink containing only the smooth blocks, the miner keeps a copy of the "smooth chain" (C_S) which consists of the smooth blocks existing in the original chain C. The algorithm for extracting the smooth chain out of C is given in Algorithm 3. Function *isSmoothBlock(B)* checks whether a block B is smooth by calling *isSmoothPointer(B,p)* for every pointer p in B's interlink. Function *isSmoothPointer(B,p)* returns *true* if p is a valid pointer, in essence a pointer to the most recent *smooth velvet* for the level denoted by the pointer itself. The *updateInterlink* algorithm is given in Algorithm 4. It is the same as in the case of a soft fork, but works on the smooth chain C_S instead of C.

The construction of the velvet suffix prover is given in Algorithm 5. Again it deviates from the soft fork case by working on the smooth chain C_S instead of C. Lastly, the Verify algorithm for the NIPoPoW suffix protocol remains the same as in the case of hard or soft fork, keeping in mind that no *previd* links can be followed when verifying the ancestry of the chain to avoid hitting any thorny blocks.

6 ANALYSIS

In this section, we prove the security of our scheme. Before we delve in detail into the formal details of the proof, let us first observe why the 1/4 bound is necessary through a combined attack on our construction.

After the suggested protocol update the honest prover cannot include any thorny blocks in his suffix NIPoPoW even if these blocks are part of his chain C_B. The adversary may exploit this fact as follows. She tries to suppress high-level honestly generated blocks in C_B, in order to reduce the blocks that can represent the honest chain in a proof. This can be done by mining a *suppressive block* on the parent of an honest superblock on the honest chain and hoping that she will be faster than the honest parties. In parallel, while she mines suppressive thorny blocks on C_B she can still use her blocks in her NIPoPoW proofs, by chainsewing them. Consequently, even if a suppression attempt does not succeed, in case for example a second honestly generated block is published soon enough, she does not lose the mining power spent but can still utilize it by including the block in her proof.

In more detail, consider the adversary who wishes to attack a specific block level μ_B and generates a NIPoPoW proof containing a block b of a fork chain which contains a double spending transaction.

Algorithm 3 Smooth chain for suffix proofs

```
function smoothChain(C)
    C_S ← {G}
    k ← 1
    while C[−k] ≠ G do
        if isSmoothBlock(C[−k]) then
            C_S ← C_S ∪ C[−k]
        end if
        k ← k + 1
    end while
    return C_S
end function
function isSmoothBlock(B)
    if B = G then
        return true
    end if
    for p ∈ B.interlink do
        if ¬isSmoothPointer(B, p) then
            return false
        end if
    end for
    return true
end function
function isSmoothPointer(B, p)
    b ← Block(B.prevId)
    while b ≠ p do
        if level(b) ≥ level(p) ∧ isSmoothBlock(b) then
            return false
        end if
        if b = G then
            return false
        end if
        b ← Block(b.prevId)
    end while
    return isSmoothBlock(b)
end function
```

Algorithm 4 Velvet updateInterlink

```
function updateInterlinkVelvet(C_S)
    B′ ← C_S[−1]
    interlink ← B′.interlink
    for μ = 0 to level(B′) do
        interlink[μ] ← id(B′)
    end for
    return interlink
end function
```

Then she acts as follows. She mines on her fork chain $\mathsf{C}_{\mathcal{A}}$ but when she observes a μ_B-level block in C_B she tries to mine a thorny block on C_B in order to suppress this μ_B block. This thorny block contains an interlink pointer which jumps onto her fork chain, but a previd pointer to the honest chain. If the suppression succeeds she has managed to damage the distribution of μ_B-superblocks within the honest chain, at the same time, to mine a block that she can afterwards use in her proof. If the suppression does not succeed she

Algorithm 5 Velvet Suffix Prover

```
function ProveVelvet_{m,k}(C_S)
    B ← C_S[0]
    for μ = |C_S[−k].interlink| down to 0 do
        α ← C_S[: − k]{B:}↑^μ
        π ← π ∪ α
        B ← α[−m]
    end for
    χ ← C_S[−k:]
    return πχ
end function
```

can still use the thorny block in her proof. The above are illustrated in Figure 9.

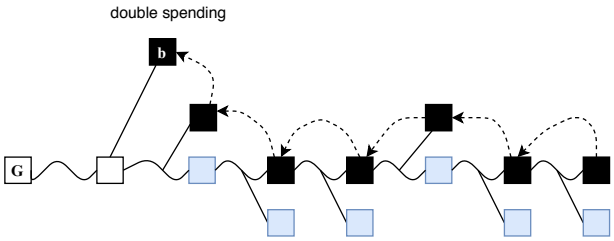

Figure 9: The adversary suppresses honestly generated blocks and chainsews thorny blocks in C_B. Blue blocks are honestly generated blocks of some level of attack. The adversary tries to suppress them. If the suppression is not successful, the adversary can still use the block she mined in her proof.

The described attack is a combined attack which combines both superblock suppression (initially described in [18]) and chainsewing (introduced in this work). This combined attack forces us to consider the Velvet Honest Majority Assumption of (1/4)-bounded adversary, so as to guarantee that the unsuppressed blocks in C_B suffice for constructing winning NIPoPoW proofs against the adversarial ones.

For the analysis, we use the techniques developed in the Backbone line of work [10]. Towards that end, we follow their definitions and call a round *successful* if at least one honest party made a successful random oracle query during the round, i.e., a query b such that $H(b) \leq T$. A round in which exactly one honest party made a successful query is called *uniquely successful* (the adversary could have also made successful queries during a uniquely successful round). Let $X_r \in \{0,1\}$ and $Y_r \in \{0,1\}$ denote the indicator random variables signifying that r was a successful or uniquely successful round respectively, and let $Z_r \in \mathbb{N}$ be the random variable counting the number of successful queries of the adversary during round r. For a set of consecutive rounds U, we define $Y(U) = \sum_{r \in U} Y_r$ and similarly define X and Z. We denote $f = \mathbb{E}[X_r] < 0.3$ the probability that a round is successful.

Let λ denote the security parameter (the output size κ of the random oracle is taken to be some polynomial of λ). We make use of the following known [10] results. It holds that $pq(n-t) < \frac{f}{1-f}$. For the Common Prefix parameter, it holds that $k \geq 2\lambda f$.

Additionally, for any set of consecutive rounds U, it holds that $\mathbb{E}[Z(U)] < \frac{t}{n-t} \cdot \frac{f}{1-f}|U|$, $\mathbb{E}[X(U)] < pq(n-t)|U|$, $\mathbb{E}[Y(U)] > f(1-f)|U|$. An execution is called *typical* if the random variables X, Y, Z do not deviate significantly (more than some error term $\epsilon < 0.3$) from their expectations. It is known that executions are typical with overwhelming probability in λ. Typicality ensures that for any set of consecutive rounds U with $|U| > \lambda$ it holds that $Z(U) < \mathbb{E}[Z(U)] - \epsilon\,\mathbb{E}[X(U)]$ and $Y(U) > (1-\epsilon)\,\mathbb{E}[Y(U)]$. From the above we can conclude to $Y(U) > (1-\epsilon)f(1-f)|U|$ and $Z(U) < \frac{t}{n-t} \cdot \frac{f}{1-f}|U| + \epsilon f|U|$ which will be used in our proofs. We consider $f < \frac{1}{20}$ a typical bound for parameter f. This is because in our (1/4)-bounded adversary assumption we need to reach about 75% of the network, which requires about 20 seconds [6]. Considering also that in Bitcoin the block generation time is in expectation 600 seconds, we conclude to an estimate $f = \frac{18}{600}$ or $f = 0.03$.

The following definition and lemma are known [34] results and will allow us to argue that some smooth superblocks will survive in all honestly adopted chains. With foresight, we remark that we will take Q to be the property of a block being both smooth and having attained some superblock level $\mu \in \mathbb{N}$.

Definition 6.1 (Q-block). A *block property* is a predicate Q defined on a hash output $h \in \{0,1\}^\kappa$. Given a block property Q, a valid block with hash h is called a Q-block if $Q(h)$ holds.

Lemma 6.2 (Unsuppressibility). *Consider a collection of polynomially many block properties $\boldsymbol{Q}$. In a typical execution every set of consecutive rounds U has a subset S of uniquely successful rounds such that*

- $|S| \geq Y(U) - 2Z(U) - 2\lambda f(\frac{t}{n-t} \cdot \frac{1}{1-f} + \epsilon)$
- *for any $Q \in \boldsymbol{Q}$, Q-blocks generated during S follow the distribution as in an unsuppressed chain*
- *after the last round in S the blocks corresponding to S belong to the chain of any honest party.*

We now apply the above lemma to our construction. The following result lies at the heart of our security proof and allows us to argue that an honestly adopted chain will have a better superblock score than an adversarially generated chain.

Lemma 6.3. *Consider Algorithm 4 under velvet fork with parameter g and (1/4)-bounded velvet honest majority. Let U be a set of consecutive rounds $r_1 \cdots r_2$ and C the chain of an honest party at round r_2 of a typical execution. Let $\mathsf{C}_U^S = \{b \in \mathsf{C} : b \text{ is smooth} \wedge b \text{ was generated during } U\}$. Let $\mu, \mu' \in \mathbb{N}$. Let C' be a μ' superchain containing only adversarial blocks generated during U and suppose $|\mathsf{C}_U^S\uparrow^\mu| > k$. Then for any $\delta_3 \leq \frac{3\lambda f}{5}$ it holds that $2^{\mu'}|\mathsf{C}'| < 2^\mu(|\mathsf{C}_U^S\uparrow^\mu| + \delta_3)$.*

Proof. From the Unsuppressibility Lemma we have that there is a set of uniquely successful rounds $S \subseteq U$, such that $|S| \geq Y(U) - 2Z(U) - \delta'$, where $\delta' = 2\lambda f(\frac{t}{n-t} \cdot \frac{1}{1-f} + \epsilon)$. We also know that Q-blocks generated during S are distributed as in an unsuppressed chain. Therefore considering the property Q for blocks of level μ that contain smooth interlinks we have that $|\mathsf{C}_U^S\uparrow^\mu| \geq (1-\epsilon)g2^{-\mu}|S|$. We also know that for the total number of μ'-blocks the adversary generated during U that $|\mathsf{C}'| \leq (1+\epsilon)2^{-\mu'}Z(U)$. Then we have to show that $(1-\epsilon)g(Y(U) - 2Z(U) - \delta') > (1+\epsilon)Z(U)$ or $((1+\epsilon) + 2g(1-\epsilon))Z(U) < g(1-\epsilon)(Y(U) + \delta')$. But it holds that $(1+\epsilon) + 2g(1-\epsilon) < 3$, therefore it suffices to show that $3Z(U) < g(1-\epsilon)(Y(U) + \delta') - 2^\mu\delta_3$.

Substituting the bounds of X, Y, Z discussed above, it suffices to show that

$$3[\frac{t}{n-t} \cdot \frac{f}{1-f}|U| + \epsilon f|U|] < (1-\epsilon)g[(1-\epsilon)f(1-f)|U| - \delta'] - 2^\mu\delta_3$$

$$\text{or } \frac{t}{n-t} < \frac{(1-\epsilon)g[(1-\epsilon)f(1-f) - \frac{\delta'}{|U|}] - 3\epsilon f - \frac{2^\mu\delta_3}{|U|}}{3\frac{f}{1-f}}.$$

But $\epsilon(1-f) \ll 1$ thus we have to show that

$$\frac{t}{n-t} < \frac{g}{3} \cdot \frac{(1-\epsilon)^2 f(1-f) - \frac{(1-\epsilon)\delta'}{|U|} - \frac{2^\mu\delta_3}{|U|}}{\frac{f}{1-f}} - \epsilon' \quad (1)$$

In order to show Equation 1 we use $f \leq \frac{1}{20}$ which is a typical bound for our setting as discussed above. Because all blocks in C were generated during U and $|\mathsf{C}| > k$, $|U|$ follows negative binomial distribution with probability $2^{-\mu}pq(n-t)$ and number of successes k. Applying a Chernoff bound we have that $|U| > (1-\epsilon)\frac{k}{2^{-\mu}pq(n-t)}$. Using the inequalities $k \geq 2\lambda f$ and $pq(n-t) < \frac{f}{1-f}$, we deduce that $|U| > (1-\epsilon)2^\mu 2\lambda(1-f)$. So we have that $\frac{\delta'}{|U|} < \frac{2\lambda f(\frac{t}{n-t}\frac{1}{1-f} + \epsilon)}{(1-\epsilon)2^\mu 2\lambda(1-f)}$ or $\frac{\delta'}{|U|} < \frac{t}{n-t} \cdot \frac{f}{(1-\epsilon)(1-f)^2} + \epsilon < 0.01 + \epsilon$. We also know that $\delta_3 \leq \frac{3\lambda f}{5}$, so $\frac{2^\mu\delta_3}{|U|} < \frac{2^\mu\frac{3\lambda f}{5}}{2^\mu 2\lambda(1-f)}$ or $\frac{2^\mu\delta_3}{|U|} < \frac{3f}{10(1-f)} < 0.01 + \epsilon$. By substituting the above and the typical f parameter bound in Equation (1) we conclude that it suffices to show that $\frac{t}{n-t} < \frac{1-\epsilon''}{3}g$ which is equivalent to $\frac{t}{n-t} < \frac{1-\delta_v}{3}g$ for $\epsilon'' = \delta_v$, which is the (1/4) velvet honest majority assumption, so the claim is proven. □

Lemma 6.4. *Consider Algorithm 4 under velvet fork with parameter g and (1/4)-bounded velvet honest majority. Consider the property Q for blocks of level μ. Let U be a set of consecutive rounds and C the chain of an honest party at the end of U of a typical execution and $\mathsf{C}_U = \{b \in \mathsf{C} : b \text{ was generated during } U\}$. Suppose that no block in C_U is of level μ. Then $|U| \leq \delta_1$ where $\delta_1 = \frac{(2+\epsilon)2^\mu + \delta'}{(1-\epsilon)f(1-f) - 2\frac{t}{n-t}\frac{f}{1-f} - 3\epsilon f}$.*

Proof. The statement results immediately form the Unsuppressibility Lemma. Suppose for contradiciton that $|U| > \delta_1$. Then from the Unsuppressibility Lemma we have that there is a subset of consecutive rounds S of U for which it holds that $|S| \geq Y(U) - 2Z(U) - \delta'$ where $\delta' = 2\lambda f(\frac{t}{n-t} \cdot \frac{1}{1-f} + \epsilon)$. By substituting $Y(U) > (1-\epsilon)f(1-f)|U|$ and $Z(U) < \frac{t}{n-t}\frac{f}{1-f} + \epsilon f|U|$ we have that $|S| > (2+\epsilon)2^{\mu}$ but Q-blocks generated during S follow the distribution as in a chain where no suppression attacks occur. Therefore at least one block of level μ would appear in C_U, thus we have reached a contradiction and the statement is proven. □

Theorem 6.5 (Suffix Proofs Security under velvet fork). *Assuming honest majority under velvet fork conditions (5.2) such that $t \leq (1-\delta_v)\frac{n_h}{3}$ where n_h the number of upgraded honest parties, the Non-Interactive Proofs of Proof-of-Work construction for computable k-stable monotonic suffix-sensitive predicates under velvet fork conditions in a typical execution is secure.*

Proof. By contradiction. Let Q be a k-stable monotonic suffix-sensitive chain predicate. Assume for contradiction that NIPoPoWs under velvet fork on Q is insecure. Then, during an execution at some round r_3, $Q(\mathsf{C})$ is defined and the verifier V disagrees with some honest participant. V communicates with adversary $\mathcal{A}$ and honest prover B. The verifier receives proofs $\pi_{\mathcal{A}}, \pi_B$ which are of valid structure. Because B is honest, π_B is a proof constructed based on underlying blockchain C_B (with $\pi_B \subseteq \mathsf{C}_B$), which B has adopted during round r_3 at which π_B was generated. Consider $\widetilde{\mathsf{C}}_{\mathcal{A}}$ the set of blocks defined as $\widetilde{C}_{\mathcal{A}} = \pi_{\mathcal{A}} \cup \{\bigcup\{\mathsf{C}_h^r\{:b_{\mathcal{A}}\} : b_{\mathcal{A}} \in \pi_{\mathcal{A}}, \exists h, r : b_{\mathcal{A}} \in \mathsf{C}_h^r\}\}$ where C_h^r the chain that the honest party h has at round r. Consider also C_B^S the set of smooth blocks of honest chain C_B. We apply security parameter

$$m = 2k + \frac{2+\epsilon+\delta'}{\frac{t}{n-t}\frac{f}{1-f}[f(1-f) - \frac{2}{3}\frac{f}{1-f}]}$$

Suppose for contradiction that the verifier outputs $\neg Q(\mathsf{C}_B)$. Thus it is necessary that $\pi_{\mathcal{A}} \geq_m \pi_B$. We show that $\pi_{\mathcal{A}} \geq_m \pi_B$ is a negligible event. Let the levels of comparison decided by the verifier be $\mu_{\mathcal{A}}$ and μ_B respectively. Let $b_0 = LCA(\pi_{\mathcal{A}}, \pi_B)$. Call $\alpha_{\mathcal{A}} = \pi_{\mathcal{A}}\uparrow^{\mu_{\mathcal{A}}}\{b_0:\}$, $\alpha_B = \pi_B\uparrow^{\mu_B}\{b_0:\}$.

From Corollary 5.6 we have that the adversarial proof consists of a smooth interlink subchain followed by a thorny interlink subchain. We refer to the smooth part of $\alpha_{\mathcal{A}}$ as $\alpha_{\mathcal{A}}^{\mathcal{S}}$ and to the thorny part as $\alpha_{\mathcal{A}}^{\mathcal{T}}$.

Our proof construction is based on the following intuition: we consider that $\alpha_{\mathcal{A}}$ consists of three distinct parts $\alpha_{\mathcal{A}}^1, \alpha_{\mathcal{A}}^2, \alpha_{\mathcal{A}}^3$ with the following properties. Block $b_0 = LCA(\pi_{\mathcal{A}}, \pi_B)$ is the fork point between $\pi_{\mathcal{A}}\uparrow^{\mu_{\mathcal{A}}}, \pi_B\uparrow^{\mu_B}$. Let block $b_1 = LCA(\alpha_{\mathcal{A}}^{\mathcal{S}}, \mathsf{C}_B^S)$ be the fork point between $\pi_{\mathcal{A}}^{\mathcal{S}}\uparrow^{\mu_{\mathcal{A}}}, \mathsf{C}_B$ as an honest prover could observe. Part $\alpha_{\mathcal{A}}^1$ contains the blocks between b_0 exclusive and b_1 inclusive generated during the set of consecutive rounds $\mathcal{S}_1$ and $|\alpha_{\mathcal{A}}^1| = k_1$. Consider b_2 the last block in $\alpha_{\mathcal{A}}$ generated by an honest party. Part $\alpha_{\mathcal{A}}^2$ contains the blocks between b_1 exclusive and b_2 inclusive generated during the set of consecutive rounds $\mathcal{S}_2$ and $|\alpha_{\mathcal{A}}^2| = k_2$. Consider b_3 the next block of b_2 in $\alpha_{\mathcal{A}}$. Then $\alpha_{\mathcal{A}}^3 = \alpha_{\mathcal{A}}[b_3:]$ and $|\alpha_{\mathcal{A}}^3| = k_3$ consisting of adversarial blocks generated during the set of consecutive rounds $\mathcal{S}_3$. Therefore $|\alpha_{\mathcal{A}}| = k_1 + k_2 + k_3$ and we will show that $|\alpha_{\mathcal{A}}| < |\alpha_B|$.

The above are illustrated, among other, in Parts I, II of Figure 10.

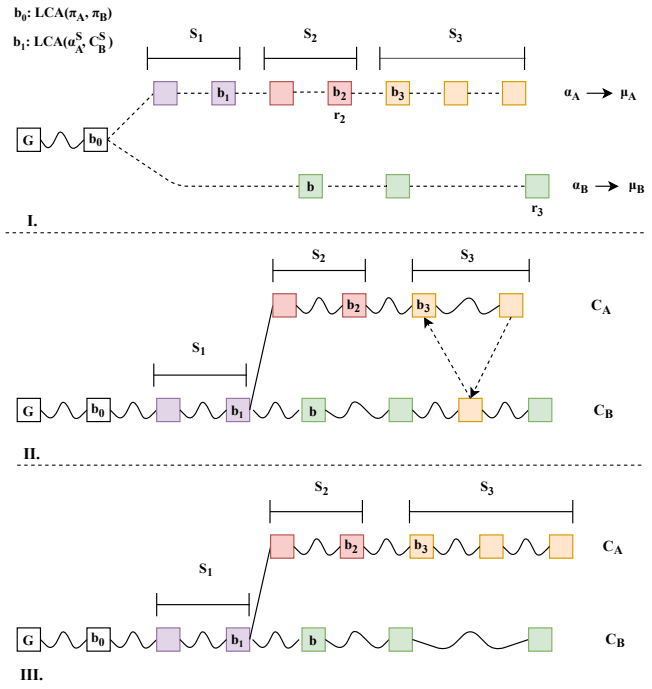

Figure 10: I. the three round sets in two competing proofs at different levels, II. the corresponding 0-level blocks implied by the two proofs, III: blocks in C_B and block set $\widetilde{\mathsf{C}}_{\mathcal{A}}$ from the verifier's perspective.

We now show three successive claims: First that $\alpha_{\mathcal{A}}^1$ contains few blocks. Second, $\alpha_{\mathcal{A}}^2$ contains few blocks. And third, the adversary can produce a winning $a_{\mathcal{A}}$ with negligible probability.

Claim 1: $\alpha_{\mathcal{A}}^1 = (\alpha_{\mathcal{A}}\{b_0 : b_1\} \cup b_1)$ *contains only a few blocks.* Let $|\alpha_{\mathcal{A}}^1| = k_1$. We have defined the blocks $b_0 = LCA(\pi_{\mathcal{A}}, \pi_B)$ and $b_1 = LCA(\alpha_{\mathcal{A}}^{\mathcal{S}}, \mathsf{C}_B^S)$. First observe that because of the Corollary 5.6 there are no thorny blocks in $\alpha_{\mathcal{A}}^1$ since $\alpha_{\mathcal{A}}^1[-1] = b_1$ is a smooth block. This means that if b_1 was generated at round r_{b_1} and $\alpha_{\mathcal{A}}^{\mathcal{S}}[-1]$ in round r then $r \geq r_{b_1}$. Therefore, $\alpha_{\mathcal{A}}^1$ contains smooth blocks of C_B. We show the claim by considering the two possible cases for the relation of $\mu_{\mathcal{A}}, \mu_B$.

Claim 1a: If $\mu_B \leq \mu_{\mathcal{A}}$ then $k_1 = 0$. In order to see this, first observe that every block in $\alpha_{\mathcal{A}}$ would also be of lower level μ_B. Subsequently, any block in $\alpha_{\mathcal{A}}\{b_0:\}$ would also be included in proof α_B but this contradicts the minimality of block b_0.

Claim 1b: If $\mu_B > \mu_{\mathcal{A}}$ then $k_1 \leq \frac{\delta_1 2^{-\mu_{\mathcal{A}}}}{(1+\epsilon)\frac{t}{n-t}\frac{f}{1-f}}$. In order to show this we consider block b the first block in α_B. Now suppose

for contradiction that $k_1 > \frac{\delta_1 2^{-\mu_{\mathcal{A}}}}{(1+\epsilon)\frac{t}{n-t}\frac{f}{1-f}}$. Then from lemma 6.4 we have that block b is generated during S_1. But b is of lower level $\mu_{\mathcal{A}}$ and $\alpha^1_{\mathcal{A}}$ contains smooth blocks of C_B. Therefore b is also included in $\alpha^1_{\mathcal{A}}$, which contradicts the minimality of block b_0.

Consequently, there are at least $|\alpha_{\mathcal{A}}| - k_1$ blocks in $\alpha_{\mathcal{A}}$ which are not honestly generated blocks existing in C_B. In other words, these are blocks which are either thorny blocks existing in C_B either don't belong in C_B.

Claim 2. *Part $\alpha^2_{\mathcal{A}} = (\alpha_{\mathcal{A}}\{b_1 : b_2\} \cup b_2)$ consists of only a few blocks.* Let $|\alpha^2_{\mathcal{A}}| = k_2$. We have defined $b_2 = \alpha^2_{\mathcal{A}}[-1]$ to be the last block generated by an honest party in $\alpha_{\mathcal{A}}$. Consequently no thorny block exists in $\alpha^2_{\mathcal{A}}$, so all blocks in this part belong in a proper zero-level chain $\mathsf{C}^2_{\mathcal{A}}$. Let r_{b_1} be the round at which b_1 was generated. Since b_1 is the last block in $\alpha_{\mathcal{A}}$ which belongs in C_B, then $\mathsf{C}^2_{\mathcal{A}}$ is a fork chain to C_B at some block b' generated at round $r' \geq r_{b_1}$. Let r_2 be the round when b_2 was generated by an honest party. Because an honest party has adopted chain C_B at a later round r_3 when the proof π_B is constructed and because of the Common Prefix property on parameter k_2, we conclude that $k_2 \leq 2^{-\mu_{\mathcal{A}}}k$.

Claim 3. *The adversary may submit a suffix proof such that $|\alpha_{\mathcal{A}}| \geq |\alpha_B|$ with negligible probability.* Let $|\alpha^3_{\mathcal{A}}| = k_3$. As explained earlier part $\alpha^3_{\mathcal{A}}$ consists only of adversarially generated blocks. Let S_3 be the set of consecutive rounds $r_2 ... r_3$. Then all k_3 blocks of this part of the proof are generated during S_3. Let α^3_B be the last part of the honest proof containing the interlinked μ_B superblocks generated during S_3. Then by applying lemma 6.3 $\frac{m}{k}$ times we have that $2^{\mu_{\mathcal{A}}}|\alpha^3_{\mathcal{A}}| < 2^{\mu_B}(|\alpha^{S_3}_B\uparrow^{\mu_B}| + \frac{m\delta_3}{k})$. By substituting the values from all the above Claims and because every block of level μ_B in $a\alpha_B$ is of equal hashing power to $2^{\mu_B - \mu_{\mathcal{A}}}$ blocks of level $\mu_{\mathcal{A}}$ in the adversary's proof we have that: $2^{\mu_B}|\alpha^3_B| - 2^{\mu_{\mathcal{A}}}|\alpha^3_{\mathcal{A}}| > 2^{\mu_{\mathcal{A}}}(k_1 + k_2)$ or $2^{\mu_B}|\alpha^3_B| > 2^{\mu_{\mathcal{A}}}|\alpha^1_{\mathcal{A}} + \alpha^2_{\mathcal{A}} + \alpha^3_{\mathcal{A}}|$ or $2^{\mu_B}|\alpha_B| > 2^{\mu_{\mathcal{A}}}|\alpha_{\mathcal{A}}|$ Therefore we have proven that $2^{\mu_B}|\pi_B\uparrow^{\mu_B}| > 2^{\mu_{\mathcal{A}}}|\pi^{\mu_{\mathcal{A}}}_{\mathcal{A}}|$. □

7 INFIX PROOFS

NIPoPoW infix proofs answer any predicate which depends on blocks appearing anywhere in the chain, except for the k suffix for stability reasons. For example, consider the case where a client has received a transaction inclusion proof for a block b and requests an infix proof so as to verify that b is included in the current chain. Because of the described protocol update for secure NIPoPoW suffix proofs, the infix proofs construction has to be altered as well. In order to construct secure infix proofs under velvet fork conditions, we suggest the following additional protocol patch: each upgraded miner constructs and updates an authenticated data structure for all the blocks in the chain. We suggest Merkle Mountain Ranges *(MMR)* for this structure. Now a velvet block's header additionally includes the root of this MMR.

After this additional protocol change the notion of a smooth block changes as well. Smooth blocks are now considered the blocks that contain truthful interlinks and valid MMR root too. A valid MMR root denotes the MMR that contains all the blocks in the chain of an honest full node. Note that a valid MMR contains all the blocks of the longest valid chain, meaning both smooth and thorny. An invalid MMR constructed by the adversary may contain a block of a fork chain. Consequently an upgraded prover has to maintain a local copy of this MMR locally, in order to construct correct proofs. This is crucial for the security of infix proofs, since keeping the notion of a smooth block as before would allow an adversary to produce a block b in an honest party's chain, with b containing a smooth interlink but invalid MMR, so she could succeed in providing an infix proof about a block of a fork chain.

Considering this addtional patch we can now define the final algorithms for the honest miner, infix and suffix prover, as well as for the infix verifier. Because of the new notion of smooth block, the function *isSmoothBlock()* of Algorithm 3 needs to be updated, so that the validity of the included MMR root is also checked. The updated function is given in Algorithm 6. Considering that input C_S is computed using Algorithm 3 with the updated *isSmoothBlock'()* function, *Velvet updateInterlink* and *Velvet Suffix Prover* algorithms remain the same as described in Algorithms 4, 5 repsectively. The velvet infix prover and infix verifier algorithms are given in Algorithms 7, 8 respectively. Details about the construction and verification of an MMR and the respective inclusion proofs can be found in [22]. Note that equivalent solution could be formed by using any authenticated data structure that provides inclusion proofs of size logarithmic to the length of the chain. We suggest MMRs because of they come with efficient update operations.

Algorithm 6 Function isSmoothBlock'() for infix proof support

```
1:  function isSmoothBlock'(B)
2:      if B = G then
3:          return true
4:      end if
5:      for p ∈ B.interlink do
6:          if ¬isSmoothPointer(B, p) then
7:              return false
8:          end if
9:      end for
10:     return containsValidMMR(B)
11: end function
```

Algorithm 7 Velvet Infix Prover

```
1: function ProveInfixVelvet(C_S, b)
2:     (π, χ) ← ProveVelvet(C_S)
3:     tip ← π[−1]
4:     π_b ← MMRinclusionProof(tip, b)
5:     return (π_b, (π, χ))
6: end function
```

Algorithm 8 Velvet Infix Verifier

```
1: function VerifyInfixVelvet(b, (π_b, (π, χ)))
2:     tip ← π[−1]
3:     return VerifyInclProof(tip.root_MMR, π_b, b)
4: end function
```

A CHAINSEWING ATTACK SIMULATION

To measure the success rate of the chainsewing attack against the naïve NIPoPoW construction described in Section 4, we implemented a simulation to estimate the probability of the adversary generating a winning NIPoPoW against the honest party. Our experimental setting is as follows. We fix $\mu_A = 0$ and $\mu_B = 10$ as well as the required length of the suffix $k = 15$. We fix the adversarial mining power to $t = 1$ and $n = 5$ which gives a 20% adversary. We then vary the NIPoPoW security parameter for the π portion from $m = 3$ to $m = 15$. We then run 100 Monte Carlo simulations and measure whether the adversary was successful in generating a competing NIPoPoW which compares favourably against the adversarial NIPoPoW.

For performance reasons, our model for the simulation slightly deviates from the Backbone model on which the theoretical analysis of Section **??** is based and instead follows the simpler model of Ren [27]. This model favours the honest parties, and so provides a lower bound for probability of adversarial success, strengthening our results. Here, block arrival is modelled as a Poisson process and blocks are deemed to belong to the adversary with probability t/n, while they are deemed to belong to the honest parties with probability $(n - t)/n$. Block propagation is assumed instant and every party learns about a block as soon as it is mined. As such, the honest parties are assumed to work on one common chain and the problem of non-uniquely successful rounds does not occur.

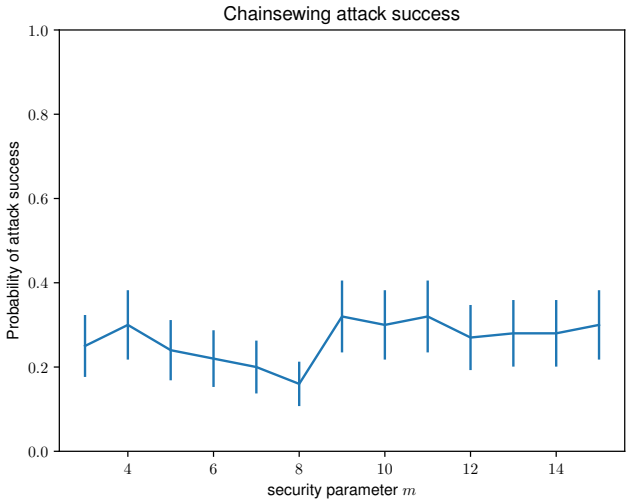

Figure 11: The measured probability of success of the Chainsewing attack mounted under our parameters for varying values of the security parameter m. Confidence intervals at 95%.

We consistently find a success rate of approximately 0.26 which remains more or less constant independent of the security parameter, as expected. We plot our results with 95% confidence intervals in Figure 11. This is in contrast with the best previously known attack in which, for all examined values of the security parameter, the probability of success remains below 1%.

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