

Non-Interactive Proofs of Proof-of-Work under Velvet Fork

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ABSTRACT

Superlight clients allow decentralized wallets to learn facts about the blockchain without downloading all the block headers. They achieve exponentially faster communication compared to SPV clients. For proof-of-work, they implement the so-called NIPoPoW primitive, for which there exist two variants: Superblock clients and FlyClient. Both of these protocols require consensus changes to existing blockchains and at least a soft fork to implement. In this paper, we discuss how a blockchain can be upgraded to support superblock clients without a soft fork. We show that it is possible to implement the needed changes without modifying the consensus protocol and by requiring only a minority of miners to upgrade, an upgrade termed a “velvet fork” in the literature. While previous work conjectured that NIPoPoW can be safely deployed using velvet forks as-is, we show that previous constructions are insecure, and that using velvet techniques to interlink a blockchain can pose insidious security risks. We describe a novel attack which we term a “chain-sewing” attack which only lurks in velvet situations: An adversary can cut-and-paste portions of various chains from independent forks, sewing them together to form a proof that looks like a chain but is not. We demonstrate that a minority adversary can thwart previous constructions with non-negligible probability. We put forth the first provably secure velvet NIPoPoW construction. Our construction is secure against adversaries that are bounded by 1/4 of the upgraded honest miner population. We prove our construction achieves persistence and liveness and analyze the trade-offs between the upgraded population parameter and the succinctness of the construction. Like non-velvet NIPoPoWs, our approach allows proving generic predicates about chains using infix proofs and as such can be adopted in practice for fast synchronization of transactions and accounts.

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KEYWORDS

blockchain, consensus, lightclient, NIPoPoW

1 INTRODUCTION

Blockchain systems such as Bitcoin [25] and Ethereum [4, 28] have a predetermined expected rate of block production and maintain chains of blocks that are growing linearly with time. A node synchronizing with the rest of the blockchain network for the first time therefore has to download and validate the whole chain, if it does not wish to rely on a trusted third party. While a lightweight node (SPV) can avoid downloading and validating transactions beyond their interest, it must still download the block headers that contain the proof-of-work[7] of each block in order to determine which chain contains the most proof-of-work. Block headers, while smaller by a significant constant factor, still grow linearly with time.

An Ethereum node synchronizing for the first time must download more than 300 MB of block header data for the purpose of proof-of-work verification, even if it elects not to download any transactions. This has become a central problem to the usability of blockchain systems, especially for vendors who are using mobile phones to accept payments or sit behind limited internet bandwidth. They are forced to make a difficult choice between decentralization and the ability to start accepting payments in a timely manner.

Towards the goal of alleviating the burden of this download for SPV clients, a series of *superlight* clients has emerged. These clients are able to choose the best proof-of-work chain by only requesting a small number of *sample* block headers instead of all the block headers. These sample blocks form a Non-Interactive Proof of Proof-of-Work (NIPoPoW), a short string which compresses the proof-of-work of the underlying chain. It has been shown that the block headers in these proofs are secure representatives of the proof-of-work of the underlying chain: A minority adversary can only convince a NIPoPoW client that a certain transaction is confirmed, only if they can convince an SPV client, too.

There are two general directions for superlight client implementations: In the *superblock* [12, 17] approach, the client relies on *superblocks*, blocks that have achieved much better proof-of-work than required for block validity. In the *FlyClient* [2] approach, blocks are sampled at random using a predetermined distribution, akin to a Schnorr protocol [26] modified with the Fiat–Shamir heuristic [8]. The number of block headers that need to be sent then grows only logarithmically with time. The NIPoPoW client, termed a *verifier*, still relies on a connection to full nodes, termed *provers*, which perform the sampling of blocks from the full blockchain. However, no trust assumptions are made for these provers, as the verifier can check the veracity of their claims. As long as the verifier is connected to at least one honest prover (an assumption also made in the SPV protocol [10, 29]), they are able to discern the best chain from the rest.

In both approaches, it is essential for the verifier to check that the blocks sampled one way or another have been generated in the same order as they have been presented by the prover. As such, each block in the proof must contain a pointer to the previous block in the proof. As blocks in these proofs are far apart in the underlying blockchain, the legacy *previous block pointer*, which typically appears within block headers, does not suffice. Both approaches require modifications to the consensus layer of the underlying blockchain to work. In the case of superblock NIPoPoWs, the block header must be modified to include, in addition to a pointer to the previous block, pointers to a small amount of recent high-proof-of-work blocks. In the case of FlyClient, each block must additionally contain pointers to all previous blocks in the chain. Both of these modifications can be made efficiently by organizing these pointers into Merkle Trees [24] or Merkle Mountain Ranges [21, 27] whose root is stored in the block header. The inclusion of extra pointers within blocks is termed *interlinking the chain* [7].

The modified block format, which includes the extra pointers, must be respected and validated by all full nodes and thus requires either a hard fork or at least a soft fork. However, even soft forks require the approval of a supermajority of miners, and new features that are considered non-essential by the community have taken years to receive approval [22]. Towards the goal of implementing superlight clients sooner, we study the question of whether it is possible to deploy superlight clients without a soft fork. We propose a series of modifications to blocks that are *helpful but untrusted*. These modifications mandate that some extra data is included in each block. The extra data is placed inside the block by upgraded miners only, while the rest of the network does not include the additional data into the blocks and does not verify its inclusion, treating them merely as comments. To maintain backwards compatibility, contrary to a soft fork, upgraded miners must accept blocks that do not contain this extra data that have been produced by unupgraded miners, or even blocks that contain invalid or malicious such extra data produced by a mining adversary. This acceptance is necessary in order to avoid causing a chain split with the unupgraded part of the network. Such a modification to the consensus layer is termed a *velvet fork* [31].

A summary of our contributions in this paper is as follows:

- (1) We propose the first *backwards-compatible superlight client*. We put forth an interlinking mechanism implementable through a velvet fork. We then construct a superblock NIPoPoW protocol on top of the velvet forked chain.
- (2) We prove our construction secure in the synchronous static difficulty model against adversaries bounded to $1/4$ of the mining power of the honest upgraded nodes. As such, our protocol works even if a constant minority of miners adopts it.
- (3) We illustrate that, contrary to claims of previous work, superlight clients designed to work in a soft fork cannot be readily plugged into a velvet fork and expected to work. We present a novel and insidious attack termed the *chain-sewing* attack which thwarts the defenses of previous proposals and allows even a minority adversary to cause catastrophic failures.

Previous work. Proofs of Proof-of-Work have been proposed in the context of superlight clients [2, 7, 17], cross-chain communication [13, 18, 30], as well as local data consumption by smart contracts [14]. Superblock NIPoPoWs have been deployed in production using hard forks [6] and have been conjectured to work in velvet fork conditions [17], but we show here that these conjectures are ill-informed in the light of our chain-sewing attack. Velvet forks [31] have been studied for a variety of other applications and have been deployed in practice [11]. In this work, we focus on consensus state compression. Such compression has been explored in the hard-fork setting using zk-SNARKS [23] as well as in the Proof-of-Stake setting [15]. Complementary to consensus state compression (i.e., the compression of block headers and their ancestry) is compression of application state (namely the State Trie, the UTXO, or transaction history). There is a series of works composable with ours that discusses the compression of application state [5, 19].

The structure of the paper is as follows. In Section 2, we give a brief overview of the *backbone model* and its notation; as we heavily leverage the machinery of the model, the section is a necessary prerequisite to follow the analysis. The rest of the paper is structured in the form of a *proof and refutation* [20]. We believe this form is more digestible. In Section 3, we discuss the velvet model and some initial definitions, and we present a first attempt towards a velvet NIPoPoW scheme which was presented in previous work. We discuss the informal argument of why it seems to be secure, which we *refute* with an attack explored in Section 4. In Section 5, we patch the scheme and put forth our more elaborate and novel Velvet NIPoPoW construction. We analyze it and formally prove it secure in Section 7 (this technical section can be omitted without loss of continuity). Our scheme at this point allows verifiers to decide which blocks form a *suffix* of the longest blockchain and thus the protocol supports *suffix proofs*. We extend our scheme to allow any block of interest within the blockchain to be demonstrated to a prover in a straight forward manner in Section 8, giving a full *infix proof* protocol. The latter protocol can be used in practice and can be deployed today in real blockchains, including Bitcoin, to confirm payments achieving both decentralization and timeliness, solving a major outstanding dilemma in contemporary blockchain systems.

2 PRELIMINARIES

We consider a setting where the blockchain network consists of two different types of nodes: The first kind, *full nodes*, are responsible for the maintenance of the chain including verifying it and mining new blocks. The second kind, *verifiers* connect to full nodes and wish to learn facts about the blockchain without downloading it, for example whether a particular transaction is confirmed. The full nodes therefore also function as *provers* for the verifiers. Each verifier connects to multiple provers, at least one of which is assumed to be honest.

We model full nodes according to the Backbone model [9]. There are n full nodes, of which t are adversarial and $n - t$ are honest. All t adversarial parties are controlled by one colluding adversary \mathcal{A} . The parties have access to a hash function H which is modelled as a common Random Oracle [1]. To each novel query, the random oracle outputs κ bits of fresh randomness. Time is split into distinct *rounds* numbered by the integers $1, 2, \dots$. Our treatment is in the *synchronous model*, so we assume messages *diffused* (broadcast) by an honest party at the end of a round are received by all honest parties at the beginning of the next round. This is equivalent to a network connectivity assumption in which the round duration is taken to be the known time needed for a message to cross the diameter of the network. The adversary can inject messages, reorder them, sybil attack by creating multiple messages, but not suppress messages.

Each honest full node locally maintains a *chain* C , a sequence of blocks. In understanding that we are developing an improvement on top of SPV, we use the term *block* to mean what is typically referred to as a *block header*. Each block contains the Merkle Tree root [24] of transaction data \bar{x} , the hash s of the previous block in the chain known as the *previd*, as well as a nonce value ctr . As discussed in the Introduction, the compression of application data \bar{x} is orthogonal to our goals in this paper and has been explored

in independent work [5] which can be composed with ours. Each block $b = s \parallel \bar{x} \parallel ctr$ must satisfy the proof-of-work [7] equation $H(b) \leq T$ where T is a constant *target*, a small value signifying the difficulty of the proof-of-work problem. Our treatment is in the *static difficulty* case, so we assume that T is constant throughout the execution¹. $H(B)$ is known as the *block id*.

Blockchains are finite block sequences obeying the *blockchain property*: that in every block in the chain there exists a pointer to its previous block. A chain is *anchored* if its first block is *genesis*, denoted \mathcal{G} , a special block known to all parties. This is the only node the verifier knows about when it boots up. For chain addressing we use Python brackets $C[\cdot]$. A zero-based positive number in a bracket indicates the indexed block in the chain. A negative index indicates a block from the end, e.g., $C[-1]$ is the tip of the blockchain. A range $C[i:j]$ is a subarray starting from i (inclusive) to j (exclusive). Given chains C_1, C_2 and blocks A, Z we concatenate them as $C_1 C_2$ or $C_1 A$ (if clarity mandates it, we also use the symbol \parallel for concatenation). Here, $C_2[0]$ must point to $C_1[-1]$ and A must point to $C_1[-1]$. We denote $C\{A:Z\}$ the subarray of the chain from block A (inclusive) to block Z (exclusive). We can omit blocks or indices from either side of the range to take the chain to the beginning or end respectively. As long as the blockchain property is maintained, we freely use the set operators \cup, \cap and \subseteq to denote operations between chains, implying that the appropriate blocks are selected and then placed in chronological order.

During every round, every party attempts to *mine* a new block on top of its currently adopted chain. Each party is given q queries to the random oracle which it uses in attempting to mine a new block. Therefore the adversary has tq queries per round while the honest parties have $(n - t)q$ queries per round. When an honest party discovers a new block, they extend their chain with it and broadcast the new chain. Upon receiving a new chain C' from the network, an honest party compares its length $|C'|$ against its currently adopted chain C and adopts the newly received chain if it is longer. It is assumed that the honest parties control the majority of the computational power of the network. This *honest majority assumption* states that there is some δ such that $t < (1 - \delta)(n - t)$. If so, the protocol ensures consensus among the honest parties: There is a constant k , the *Common Prefix* parameter, such that, at any round, all the chains belonging to honest parties share a common prefix of blocks; the chains can deviate only up to k blocks at the end of each chain [9]. Concretely, if at some round r two honest parties have C_1 and C_2 respectively, then either $C_1[-k]$ is a prefix of C_2 or vice versa.

Some valid blocks satisfy the proof-of-work equation better than required. If a block b satisfies $H(b) \leq 2^{-\mu}T$ for some natural number $\mu \in \mathbb{N}$ we say that b is a μ -*superblock* or a block of level μ . The probability of a new valid block achieving level μ is $2^{-\mu}$. The number of levels in the chain will be $\log |C|$ with high probability [16]. Given a chain C , we denote C^\uparrow^μ the subset of μ -superblocks of C .

Non-Interactive Proofs of Proof-of-Work (NIPoPoW) protocols allow verifiers to learn the most recent k blocks of the blockchain adopted by an honest full node without downloading the whole chain. The challenge lies in building a verifier who can find the

suffix of the longest chain between claims of both honest and adversarial provers, while not downloading all block headers. Towards that goal, the *superblock* approach uses superblocks as samples of proof-of-work. The prover sends superblocks to the verifier to convince them that proof-of-work has taken place without actually presenting all this proof-of-work. The protocol is parametrized by a constant security parameter m . The parameter determines how many superblocks will be sent by the prover to the verifier and security is proven with overwhelming probability in m .

The prover selects various levels μ and for each such level sends a carefully chosen portion of its μ -level *superchain* C^\uparrow^μ to the verifier. In standard blockchain protocols such as Bitcoin and Ethereum, each block $C[i + 1]$ in C points to its previous block $C[i]$, but each μ -superblock $C^\uparrow^\mu[i + 1]$ does not point to its previous μ -superblock $C^\uparrow^\mu[i]$. It is imperative that an adversarial prover does not reorder the blocks within a superchain, but the verifier cannot verify this unless each μ -superblock points to its most recently preceding μ -superblock. The proposal is therefore to *interlink* the chain by having each μ -superblock include an extra pointer to its most recently preceding μ -superblock. To ensure integrity, this pointer must be included in the block header and verified by proof-of-work. However, the miner does not know which level a candidate block will attain prior to mining it. For this purpose, each block is proposed to include a pointer to the most recently preceding μ -superblock, for every μ , as illustrated in Figure 1. As these levels are only $\log |C|$, this only adds $\log |C|$ extra pointers to each block header.

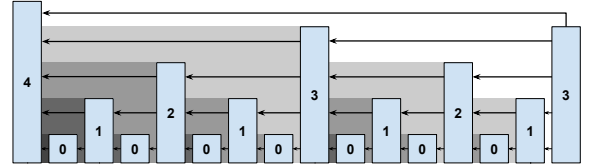


Figure 1: The interlinked blockchain. Each superblock is drawn taller according to its achieved level. Each block links to all the blocks that are not being overshadowed by their descendants. The most recent (right-most) block links to the four blocks it has direct line-of-sight to.

The exact NIPoPoW protocol works like this: The prover holds a full chain C . When the verifier requests a proof, the prover sends the last k blocks of their chain, the suffix $\chi = C[-k:]$, in full. From the larger prefix $C[:-k]$, the prover constructs a proof π by selecting certain superblocks as representative samples of the proof-of-work that took place. The blocks are picked as follows. The prover selects the *highest* level μ^* that has at least m blocks in it and includes all these blocks in their proof (if no such level exists, the chain is small and can be sent in full). The prover then iterates from level $\mu = \mu^* - 1$ down to 0. For every level μ , it includes sufficient μ -superblocks to cover the last m blocks of level $\mu + 1$, as illustrated in Algorithm 1. Because the density of blocks doubles as levels are descended, the proof will contain in expectation $2m$ blocks for each level below μ^* . As such, the total proof size $\pi\chi$ will be $\Theta(m \log |C| + k)$. Such proofs that are polylogarithmic in the chain size constitute an exponential improvement over traditional SPV clients and are called *succinct*.

¹A treatment of variable difficulty NIPoPoWs has been explored in the soft fork case [32], but we leave the treatment of velvet fork NIPoPoWs in the variable difficulty model for future work.

Algorithm 1 The Prove algorithm for the NIPoPoW protocol in a soft fork

```

1: function Provem,k(C)
2:   B ← C[0]                                ▶ Genesis
3:   for μ = |C[-k - 1].interlink| down to 0 do
4:     α ← C[: -k]{B :}↑μ
5:     π ← π ∪ α
6:     if m < |α| then
7:       B ← α[-m]
8:     end if
9:   end for
10:  χ ← C[-k :]
11:  return πχ
12: end function

```

Upon receiving two proofs $\pi_1\chi_1, \pi_2\chi_2$ of this form, the NIPoPoW verifier first checks that $|\chi_1| = |\chi_2| = k$ and that $\pi_1\chi_1$ and $\pi_2\chi_2$ form valid chains. To check that they are valid chains, the verifier ensures every block in the proof contains a pointer to its previous block inside the proof through either the *previd* pointer in the block header, or in the interlink vector. If any of these checks fail, the proof is rejected. It then compares π_1 against π_2 using the \leq_m operator, which works as follows. It finds the lowest common ancestor block $b = (\pi_1 \cap \pi_2)[-1]$; that is, b is the most recent block shared among the two proofs. Subsequently, it chooses the level μ_1 for π_1 such that $|\pi_1\{b:\}\uparrow^{\mu_1}| \geq m$ (i.e., π_1 has at least m superblocks of level μ_1 following block b) and the value $2^{\mu_1}|\pi_1\{b:\}\uparrow^{\mu_1}|$ is maximized. It chooses a level μ_2 for π_2 in the same fashion. The two proofs are compared by checking whether $2^{\mu_1}|\pi_1\{b:\}\uparrow^{\mu_1}| \geq 2^{\mu_2}|\pi_2\{b:\}\uparrow^{\mu_2}|$ and the proof with the largest score is deemed the winner. The comparison is illustrated in Algorithm 2.

Algorithm 2 The implementation of the \geq_m operator to compare two NIPoPoW proofs parameterized with security parameter m . Returns *true* if the underlying chain of party A is deemed longer than the underlying chain of party B .

```

1: function best-argm(π, b)
2:   M ← {μ : |π↑μ{b :}| ≥ m} ∪ {0}          ▶ Valid levels
3:   return maxμ ∈ M{2μ · |π↑μ{b :}|}        ▶ Score for level
4: end function
5: operator πA ≥m πB
6:   b ← (πA ∩ πB)[-1]                      ▶ LCA
7:   return best-argm(πA, b) ≥ best-argm(πB, b)
8: end operator

```

Blockchain protocols can be upgraded using hard or soft forks [3]. In a *hard fork*, blocks produced by upgraded miners are not accepted by unupgraded miners. It is simplest to introduce interlinks using a hard fork by mandating that interlink pointers are included in additional fields in the block header. Unupgraded miners will not recognize these fields and will be unable to parse upgraded blocks. To ensure the block header is of constant size, instead of including all these superblock pointers in the block header individually, they are organized into a Merkle Tree of interlink pointers and only

the root of the Merkle Tree is included in the block header. In this case, the NIPoPoW prover that wishes to show a block b in their proof is connected to its more recently preceding μ -superblock b' , also includes a Merkle Tree proof proving that $H(b')$ is a leaf in the interlink Merkle Tree root included in the block header of b . The verifier has the additional job of verifying the veracity of these Merkle inclusion proofs.

In a *soft forks*, blocks created by unupgraded miners are not accepted by upgraded miners, but blocks created by upgraded miners are accepted by unupgraded miners. As such, any additional data introduced by the upgrade must be included in a field that is treated like a comment by an unupgraded miner. To interlink the chain via a soft fork, the interlink Merkle Tree root is placed in the *coinbase* transaction instead of the block header. Upgraded miners include the correct interlink Merkle Tree root in their coinbase. Upgraded miners receiving a new block validate that the interlink Merkle Tree root is correct before accepting a block as valid. As this root can be calculated in a deterministic manner from the previous blocks in the chain, it can easily be validated. Unupgraded miners ignore this data and accept the block regardless of whether it exists. The success of the fork depends on the majority of the miner population upgrading. Whenever the NIPoPoW prover wishes to show that a block b in the proof contains a pointer to its most recently preceding μ -superblock b' , it must then accompany the block header of $b = s \parallel \bar{x} \parallel ctr$ with the coinbase transaction tx_{cb} of b as well as two Merkle Tree proofs: One proving that the coinbase transaction tx_{cb} is in \bar{x} , and one proving that $H(b')$ is a leaf in the interlink Merkle Tree whose root is included in tx_{cb} .

3 VELVET INTERLINKS

More recently, velvet forks have been introduced [31]. In a velvet fork, blocks created by upgraded miners (called *velvet blocks*) are accepted by unupgraded miners as in a soft fork. Additionally, blocks created by unupgraded miners are also accepted by upgraded miners. This allows the protocol to upgrade even if only a minority of miners chooses to upgrade. To maintain backwards compatibility and to avoid causing forks, the additional data included in a block is *advisory* and must be accepted whether it exists or not. Even if the additional data is invalid or malicious, upgraded nodes (in this context also called *velvet nodes*) are forced to accept the blocks. The simplest approach to velvet fork the chain for interlinking purposes is to have upgraded miners include the interlink pointer in the blocks they produce, but accept blocks with missing or incorrect interlinks. As we show in the next section, this approach is flawed and susceptible to unexpected attacks. A surgical change in the way velvet blocks are produced is necessary to achieve proper security.

In a velvet fork, only a minority of honest parties needs to support the protocol changes. We refer to this percentage as the “velvet parameter”.

Definition 3.1 (Velvet Parameter). The *velvet parameter* g is defined as the percentage of honest parties that have upgraded to the new protocol. The absolute number of honest upgraded parties is denoted n_h and it holds that $n_h = g(n - t)$.

Velvet forks maintain backwards and forwards compatibility. This requires any block produced by upgraded miners to be accepted by unupgraded nodes (as in a soft fork), but also blocks

produced by unupgraded miners to be accepted by upgraded nodes. For the particular case of superblock NIPoPoWs under velvet forks, upgraded miners must include the interlink data structure within their blocks, but must also accept blocks missing the interlink structure or containing an invalid interlink. Unupgraded honest nodes will produce blocks that contain no interlink, while upgraded honest nodes will produce blocks that contain truthful interlinks. Therefore, any block with invalid interlinks will be adversarially generated. However, such blocks cannot be rejected by the upgraded nodes, as that would give the adversary an opportunity to cause a hard fork.

A block generated by the adversary can thus contain arbitrary data in the interlink and yet be adopted by an honest party. Because the honest prover is an upgraded full node, it can determine what the correct interlink pointers are by examining the whole previous chain, and can thus deduce whether a block contains invalid interlink data. In that case, the prover can simply treat such blocks as unupgraded. In the context of the attack that will be presented in the following section, we examine the case where the adversary includes false interlink pointers.

In any velvet protocol, a specific portion within a block, which is treated as a comment by unupgraded nodes, is reused to contain auxiliary data by upgraded miners. Because these auxiliary data can be deterministically calculated, upgraded full nodes can verify the authenticity of the data in a new block they receive. We distinguish blocks based on whether they follow the velvet protocol rules or they deviate from them.

Definition 3.2 (Smooth and Thorny blocks). A block in a velvet protocol upgrade is called *smooth* if it contains auxiliary data and the data corresponds to the honest upgraded protocol. A block is called *thorny* if it contains auxiliary data, but the data differs from the honest upgraded protocol. A block can be neither smooth nor thorny if it does not contain auxiliary data.

In the case of velvet forks for interlink purposes, the auxiliary data consists of the Merkle Tree containing the interlink pointers to the most recent superblock ancestor at every level μ .

A naïve velvet scheme. In previous work [17], it was conjectured that superblock NIPoPoWs remain secure under a velvet fork. We call this scheme the *Naïve Velvet NIPoPoW* protocol, because it is not dissimilar from the NIPoPoW protocol in the soft fork case. In particular, the naïve velvet NIPoPoW protocol that was put forth works as follows. Each upgraded honest miner attempts to mine a block b that includes interlink pointers in the form of a Merkle Tree included in its coinbase transaction. For each level μ , the interlink contains a pointer to the most recent among all the ancestors of b that have achieved at least level μ , regardless of whether the referenced block is upgraded or not and regardless of whether its interlinks are valid. Unupgraded honest nodes will keep mining blocks on the chain as usual; because the status of a block as superblock does not require it to be mined by an upgraded miner, the unupgraded miners contribute mining power to the creation of superblocks as desired.

The prover in the naïve velvet NIPoPoWs then worked as follows. The honest prover constructed the NIPoPoW proof π_χ as usual by selecting certain superblocks from his chain C as representatives in π and by setting $\chi = C[-k:]$. The outstanding issue in this case, however, is that these blocks in π do not form a chain because,

while superblocks, some of them may not be upgraded and they may not contain any pointers (or they may contain invalid pointers). The honest prover needs to provide a connection between two consecutive blocks $\pi[i+1]$ and $\pi[i]$ in the superchain, and suppose $\pi[i]$ is the most recent μ -superblock preceding $\pi[i+1]$. The block $\pi[i+1]$ is a superblock and exists at some position j in the underlying chain C of the prover, i.e., at $\pi[i+1] = C[j]$. If $C[j]$ is a smooth block, then the interlink pointer at level μ within it can be used directly. Otherwise, the prover used the *prevind* pointer of $\pi[i+1] = C[j]$ to repeatedly reach the parents of $C[j]$, namely $C[j-1], C[j-2], \dots$ until a smooth block b between $\pi[i]$ and $\pi[i+1]$ was found in C . The block b then contains a pointer to $\pi[i]$, as $\pi[i]$ is also the most recent μ -superblock ancestor of b . The blocks $C[j-1], C[j-2], \dots, b$ are then included in the proof to illustrate that $\pi[i]$ is an ancestor of $\pi[i+1]$.

The argument for why the above scheme work is as follows. First of all, the scheme does not add many new blocks to the proof. In expectation, if a fully honestly generated chain is processed, after in expectation $\frac{1}{g}$ blocks have been traversed, a smooth block will be found and the connection to $\pi[i]$ will be made. Thus, the number of blocks needed in the proof increases by a factor of $\frac{1}{g}$. Security was argued as follows: An honest party includes in their proof as many blocks as in a soft forked NIPoPoW, albeit by using an indirect connection. The crucial feature is that it is not missing any superblocks. Even if the adversary creates interlinks that skip over some honest superblocks, the honest prover will not utilize these interlinks, but will use the “slow route” of level 0 instead. The adversarial prover, on the other hand, can only use honest interlinks as before, but may also use false interlinks in blocks mined by the adversary. However, these false interlinks cannot point to blocks that are of incorrect level. The reason is that the verifier can look at the hash of each block to verify its level and therefore cannot be lied to. The only problem a fake interlink can cause is that it can point to a μ -superblock which is not *the most recent ancestor*, but some other block. It was then argued that the only other possibility was to point to blocks that are older μ -superblock ancestors in the same chain, as illustrated in Figure 2. However, the adversarial prover can only harm herself by making use of these pointers, as the result will simply by a superchain with fewer blocks.

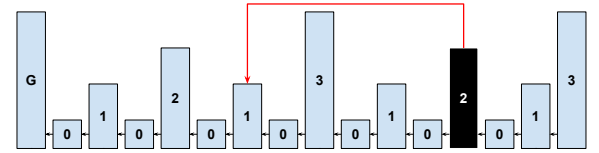


Figure 2: A thorny pointer of an adversarial block, colored black, in an honest party’s chain. The thorny block points to a 1-superboock which is an ancestor 1-superblock, but not *the most recent ancestor* 1-superblock.

As such, we conclude that the honest verifier comparing the honest superchain against the adversarial superchain will reach the same conclusion in the velvet case as he would have reached in the soft fork case: Because the honest superchain in the velvet case contains the same amount of blocks as the honest superchain in

the soft fork case, but the adversarial superchain in the velvet case contains fewer blocks than in the soft fork case, the comparison will remain in favor of the honest party. As we will see in the next section, this conclusion is far from straightforward.

4 THE CHAINSEWING ATTACK

We now make the critical observation that a thorny block can include interlink pointers to blocks that are not its own ancestors in the 0-level chain. Because it must contain a pointer to the hash of the block it points to, they must be blocks that have been generated previously, but they may belong to a different 0-level chain. This is shown in Figure 3.

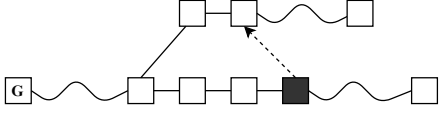


Figure 3: A thorny block, colored black, in an honest party's chain, uses its interlink to point to a fork chain.

In fact, as the interlink vector contains multiple pointers, each pointer may belong to a different fork. This is illustrated in Figure 4. The interlink pointing to arbitrary directions resembles a thorny bush.

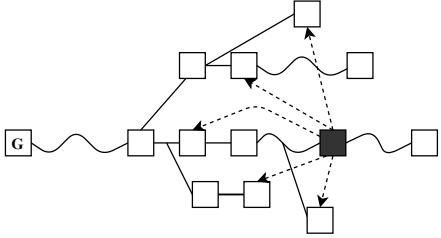


Figure 4: A thorny block appended to an honest party's chain. The dashed arrows are interlink pointers.

We now present the *chain-sewing attack* against the naïve velvet NIPoPoW protocol. The attack leverages thorny blocks in order to enable the adversary to *usurp* blocks belonging to a different chain and claim it as their own. Taking advantage of thorny blocks, the adversary can produce suffix proofs containing an arbitrary number of blocks belonging to several fork chains. The attack works as follows.

Assume chain C_B was adopted by an honest party B and chain C_A , a fork of C_B at some point, is maintained by the adversary A . After the fork point $b = (C_B \cap C_A)[-1]$, the honest party produces a block extending b in C_B containing a transaction tx . The adversary includes a conflicting (double spending) transaction tx' in a block extending b in C_A . The adversary wants to produce a suffix proof convincing a light client that C_A is the longer chain. In order to achieve this, the adversary needs to include a greater amount of total proof-of-work in her suffix proof, π_A , in comparison to that included in the honest party's proof, π_B , so as to achieve $\pi_A \geq_m \pi_B$.

Towards this purpose, she mines intermittently on both C_B and C_A . She produces some thorny blocks in both chains C_A and C_B which will allow her to usurp selected blocks of C_B and present them to the light client as if they belonged to C_A in her suffix proof.

The general form of this attack for an adversary sewing blocks to one forked chain is illustrated in Figure 5. Dashed arrows represent interlink pointers of some level μ_A . Starting from a thorny block in the adversary's forked chain and following the interlink pointers, jumping between C_A and C_B , a chain of blocks crossing forks is formed, which the adversary claims as part of her suffix proof. Blocks of both chains are included in this proof and a verifier cannot distinguish the non-smooth pointers participating in this proof chain and, as a result, considers it a valid proof.

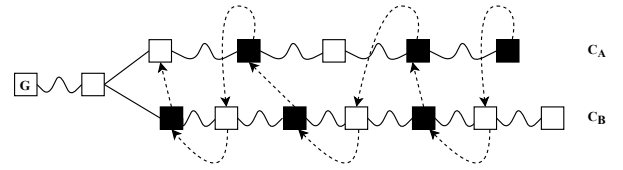


Figure 5: Generic Chainsewing Attack. C_B is the chain of an honest party and C_A the adversary's chain. Adversarially generated blocks are colored black. Dashed arrows represent interlink pointers included in the adversary's suffix proof. Wavy lines imply one or more blocks.

This generic attack can be made concrete as follows. The adversary chooses to attack at some level $\mu_A \in \mathbb{N}$. As shown in Figure 6, she first generates a superblock b' in her forked chain C_A and a thorny block a' in the honest chain C_B which points to b' . Block a' will be accepted as valid in the honest chain C_B despite the invalid interlink pointers. Subsequently, the adversary waits for a period of time until the honest parties have produced a certain number of μ_A -superblocks in C_B . Let b denote the last such honestly generated μ_A -superblock. She then attempts to mine a thorny block a in C_A pointing to b of C_B . Because of the way blocks are generated by upgraded honest miners, there will be successive interlink pointers leading from block b to block a' . Thus following the interlink pointers a chain is formed which connects C_A blocks a and b' and contains an arbitrarily large part of the honest party's chain C_B . At this point, the adversary continues mining on top of block a .

When the time arrives for the adversary to produce the NIPoPoW, she produces a suffix proof for chain C_A containing the subchain $C_A\{b'\} \parallel b' \parallel C\{a':b\} \parallel b \parallel C_A\{a\}$. Note that following the interlink pointers constructed in such a way, a light client perceives this μ_A -superchain as a valid chain, as he cannot detect it is spanning across different forks at level 0.

In this attack the adversary uses thorny blocks to "sew" portions of the honestly adopted chain to her own forked chain. This justifies the name given to the attack.

Note that in order to make this attack successful, the adversary needs only produce few superblocks, but she can arrogate an arbitrarily large number of honestly produced blocks. Thus the attack succeeds with non-negligible probability.

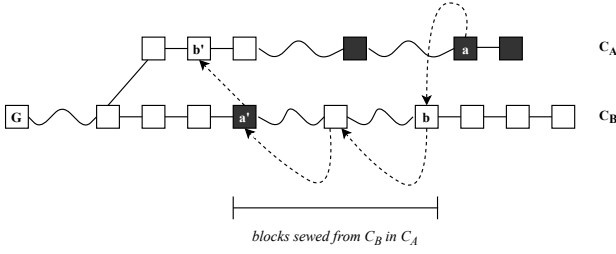


Figure 6: Chainsewing Attack. C_B represents the chain of an honest party. C_A is an adversarial fork. Adversarially generated blocks are colored black. Dashed arrows represent interlink pointers included in the adversary's suffix proof. Wavy lines imply one or more blocks. Firm lines imply the *prevind* relationship between two sequential blocks.

5 VELVET NIPOPOWS

In order to eliminate the Chainsewing Attack we propose an update to the velvet NIPoPoW protocol. The core problem is that, in her suffix proof, the adversary was able to claim not only blocks of shorter forked chains, but also arbitrarily long parts of the chain adopted by an honest party. Since thorny blocks are accepted as valid, the verifier cannot distinguish blocks that actually belong in a chain from blocks that only seem to belong in the same chain because they are pointed to from a thorny block.

Definition 5.1 (Smooth Pointer). A *smooth pointer* of a block b for a specific level μ is the interlink pointer to the most recent μ -level ancestor of b .

We describe a protocol patch that operates as follows. The superbloc NIPoPoW protocol works as usual but each miner constructs smooth blocks. This means that a block's interlink is constructed excluding thorny blocks. In this way, although thorny blocks are accepted in the chain, they are not taken into consideration when updating the interlink structure for the next block to be mined. No honest block could now point to a thorny superbloc that may act as the passing point to the fork chain in an adversarial suffix proof. Thus, after this protocol update the adversary is only able to inject adversarially generated blocks from an honestly adopted chain to her own fork. At the same time, thorny blocks cannot participate in an honestly generated suffix proof except for some blocks in the proof's suffix (χ). This argument holds because thorny blocks do not form a valid chain along with honestly mined blocks anymore. Consequently, as far as the blocks included in a suffix proof is concerned, we can think of thorny blocks as belonging in the adversary's fork chain for the π part of the proof, which is the comparing part between proofs. Figure 7 illustrates this remark.

In order to make NIPoPoWs a secure protocol under velvet fork conditions we suggest that the NIPoPoW protocol under velvet fork works as usual but a miner constructs a block's interlink without pointers to thorny blocks.

In order to prove the security of the updated protocol we need to strengthen the Honest Majority Assumption to 1/4 adversary with respect to upgraded honest miners.

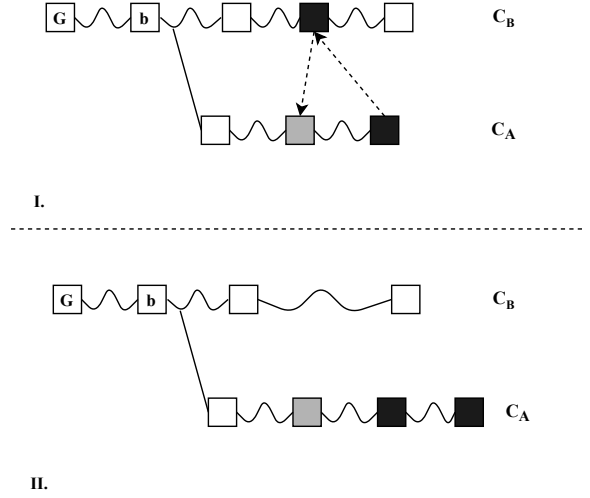


Figure 7: The adversarial fork chain C_A and chain C_B of an honest party. Thorny blocks are colored black. Dashed arrows represent interlink pointers. Wavy lines imply one or more blocks. After the protocol update, when an adversarially generated block is sewed from C_B into the adversary's suffix proof the verifier conceives C_A as longer and C_B as shorter. I: The real picture of the chains. II: Equivalent picture from the verifier's perspective considering the blocks included in the corresponding suffix proof for each chain.

Definition 5.2 (Velvet Honest Majority). Let n_h be the number of upgraded honest miners. Then t out of total n parties are corrupted such that $\frac{t}{n_h} < \frac{1 - \delta_v}{3}$.

The following Lemmas come as immediate results from the suggested protocol update.

LEMMA 5.3. A velvet suffix proof constructed by an honest party cannot contain any thorny block.

LEMMA 5.4. Let $\mathcal{P}_A = (\pi_A, \chi_A)$ be a velvet suffix proof constructed by the adversary and block b_s , generated at round r_s , be the most recent smooth block in the proof. Then $\forall r : r < r_s$ no thorny blocks generated at round r can be included in \mathcal{P}_A .

PROOF. By contradiction. Let b_t be a thorny block generated at some round $r_t < r_s$. Suppose for contradiction that b_t is included in the proof. Then, because \mathcal{P}_A is a valid chain as for interlink pointers, there exist a block path made by interlink pointers starting from b_s and resulting to b_t . Let b' be the most recently generated thorny block after b_t and before b_s included in \mathcal{P}_A . Then b' has been generated at a round r' such that $r_t \leq r' < r_s$. Then the block right after block b' in \mathcal{P}_A must be a thorny block since it points to b' which is thorny. But b' is the most recent thorny block after b_t , thus we have reached a contradiction. \square

LEMMA 5.5. Let $\mathcal{P}_A = (\pi_A, \chi_A)$ be a velvet suffix proof constructed by the adversary. Let b_t be the oldest thorny block included in \mathcal{P}_A which is generated at round r_t . Then any block $b = \{b : b \in \mathcal{P}_A \wedge b \text{ generated at } r \geq r_t\}$ is thorny.

PROOF. By contradiction. Suppose for contradiction that b_s is a smooth block generated at round $r_s > r_t$. Then from Lemma 5.4 any block generated at round $r < r_s$ is smooth. But b_t is generated at round $r_t < r_s$ and is thorny, thus we have reached a contradiction. \square

The following corollary emerges immediately from Lemmas 5.4, 5.5. This result is illustrated in Figure 8.

COROLLARY 5.6. *Any adversarial proof $\mathcal{P}_{\mathcal{A}} = (\pi_{\mathcal{A}}, \chi_{\mathcal{A}})$ containing both smooth and thorny blocks consists of a prefix smooth subchain followed by a suffix thorny subchain.*

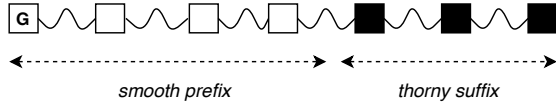


Figure 8: In the general case the adversarial velvet suffix proof $\mathcal{P}_{\mathcal{A}} = (\pi_{\mathcal{A}}, \chi_{\mathcal{A}})$ consists of an initial part of smooth blocks followed by thorny blocks.

We now describe the algorithms needed by the upgraded miner, prover and verifier. The upgraded miner acts as usual except for including the interlink of the newborn block in the coinbase transaction. In order to construct an interlink containing only the smooth blocks, the miner keeps a copy of the “smooth chain” (C_S) which consists of the smooth blocks existing in the original chain C . The algorithm for extracting the smooth chain out of C is given in Algorithm 3. Function *isSmoothBlock*(B) checks whether a block B is a smooth velvet by calling *isSmoothPointer*(B, p) for every pointer p in B ’s interlink. Function *isSmoothPointer*(B, p) returns *true* if p is a valid pointer, in essence a pointer to the most recent *smooth velvet* for the level denoted by the pointer itself. The *updateInterlink* algorithm is given in Algorithm 4, which is essentially the same as in the case of a hard/soft fork, except for working on the smooth chain C_S instead of C .

The construction of the velvet suffix prover is given in Algorithm 5, which is essentially the same to that of a hard/soft fork except for working on smooth chain C_S instead of C .

In conclusion the Verify algorithm for the NIPoPoW suffix protocol remains the same as in the case of hard or soft fork.

Algorithm 3 Smooth chain for suffix proofs

```

1: function smoothChain( $C$ )
2:    $C_S \leftarrow \{\mathcal{G}\}$ 
3:    $k \leftarrow 1$ 
4:   while  $C[-k] \neq \mathcal{G}$  do
5:     if isSmoothBlock( $C[-k]$ ) then
6:        $C_S \leftarrow C_S \cup C[-k]$ 
7:     end if
8:      $k \leftarrow k + 1$ 
9:   end while
10:  return  $C_S$ 
11: end function

```

```

1: function isSmoothBlock( $B$ )
2:   if  $B = \mathcal{G}$  then
3:     return true
4:   end if
5:   for  $p \in B.\text{interlink}$  do
6:     if  $\neg \text{isSmoothPointer}(B, p)$  then
7:       return false
8:     end if
9:   end for
10:  return true
11: end function

```

```

1: function isSmoothPointer( $B, p$ )
2:    $b \leftarrow \text{Block}(B.\text{prevId})$ 
3:   while  $b \neq p$  do
4:     if  $\text{level}(b) \geq \text{level}(p) \wedge \text{isSmoothBlock}(b)$  then
5:       return false
6:     end if
7:     if  $b = \mathcal{G}$  then
8:       return false
9:     end if
10:     $b \leftarrow \text{Block}(b.\text{prevId})$ 
11:  end while
12:  return isSmoothBlock( $b$ )
13: end function

```

Algorithm 4 Velvet updateInterlink

```

1: function updateInterlinkVelvet( $C_S$ )
2:    $B' \leftarrow C_S[-1]$ 
3:    $\text{interlink} \leftarrow B'.\text{interlink}$ 
4:   for  $\mu = 0$  to  $\text{level}(B')$  do
5:      $\text{interlink}[\mu] \leftarrow \text{id}(B')$ 
6:   end for
7:   return  $\text{interlink}$ 
8: end function

```

6 COMBINED ATTACK

After the suggested protocol update the honest prover cannot contain any thorny blocks in his suffix NIPoPow even if these blocks

Algorithm 5 Velvet Suffix Prover

```

function ProveVelvet $_{m,k}(C_S)$ 
   $B \leftarrow C_S[0]$ 
  for  $\mu = |C_S[-k].\text{interlink}|$  down to 0 do
     $\alpha \leftarrow C_S[: -k]\{B;\} \uparrow^\mu$ 
     $\pi \leftarrow \pi \cup \alpha$ 
     $B \leftarrow \alpha[-m]$ 
  end for
   $\chi \leftarrow C_S[-k:]$ 
  return  $\pi\chi$ 
end function

```

are part of C_B . The adversary may exploit this fact and to try to suppress honestly generated blocks in C_B , in order to reduce the blocks that can represent the honest chain in a proof. In parallel, while the adversary mines suppressive thorny blocks on C_B she can still use her blocks in her NIPoPoW proofs, by chainsewing them. Consequently, even if a suppression attempt does not succeed, in case for example that a second honestly generated block is soon enough published, she does not drop the thorny block she generated but include it in her proof.

More in detail, consider that the adversary wishes to attack a specific block level μ_B and generate a NIPoPow proof containing a block b of a fork chain which contains a double spending transaction. Then she acts as follows. She may mine on her fork chain C_A but when she observes a μ_B -level block in C_B she mines a thorny block on C_B which jumps onto her fork chain, in order to suppress this μ_B block. If the suppression succeeds she has managed to damage the μ_B superchain and mine a block that she can afterwards use in her proof. If the suppression does not succeed she can still use the thorny in her proof. The above are illustrated in Figure 9.

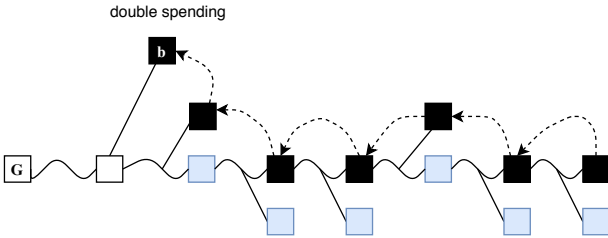


Figure 9: The adversary suppresses honestly generated blocks and chainsews thorny blocks in C_B . Blue blocks are honestly generated blocks of some level of attack. The adversary tries to suppress them. If the suppression is not successful, the adversary can still use the block she mined in her proof.

The described attack consists a combined attack since both suppression and chainsewing are utilized. This combined attack forces us to adopt a stronger Honest Majority Assumption, so as to guarantee that the unsuppressed blocks in C_B suffice for constructing winning NIPoW proofs against the adversarial ones.

7 ANALYSIS

In this section, we prove the security of our scheme. We use the techniques developed in the Backbone line of work [9]. Towards that end, we follow their definitions and call a round *successful* if at least one honest party made a successful random oracle query during the round, i.e., a query b such that $H(b) \leq T$. A round in which exactly one honest party made a successful query is called *uniquely successful* (the adversary could have also made successful queries during a uniquely successful round). Let $X_r \in \{0, 1\}$ and $Y_r \in \{0, 1\}$ denote the indicator random variables signifying that r was a successful or uniquely successful round respectively, and let $Z_r \in \mathbb{N}$ be the random variable counting the number of successful queries of the adversary during round r . For a set of consecutive rounds U , we define $Y(U) = \sum_{r \in U} Y_r$ and similarly define X and Z . We denote $f = \mathbb{E}[X_r] < 0.3$ the probability that a round is successful.

Let λ denote the security parameter (the output size κ of the random oracle is taken to be some polynomial of λ). We make use of the following known [9] results. It holds that $pqt < \frac{t}{n-t} \frac{f}{1-f}$. For the Common Prefix parameter, it holds that $k \geq 2\lambda f$. Additionally, for any set of consecutive rounds U , it holds that $\mathbb{E}[Z(U)] < (1 + \frac{\delta}{2})f \frac{t}{n-t}|U|$, $\mathbb{E}[X(U)] < pq(n-t)|U|$, $\mathbb{E}[Y(U)] > f(1-f)|U|$. An execution is called *typical* if the random variables X, Y, Z do not deviate significantly (more than some error term $\epsilon < 0.3$) from their expectations. It is known that executions are typical with overwhelming probability in λ . Typicality ensures that for any set of consecutive rounds U with $|U| > \lambda$ it holds that $Z(U) < \mathbb{E}[Z(U)] - \epsilon \mathbb{E}[X(U)]$ and $Y(U) > (1 - \epsilon) \mathbb{E}[Y(U)]$.

The following definition and lemma are known [32] results and will allow us to argue that some smooth superblocks will survive in all honestly adopted chains. With foresight, we remark that we will take Q to be the property of a block being both smooth and having attained some superblock level $\mu \in \mathbb{N}$.

Definition 7.1 (Block property, Q-block). A *block property* is a predicate Q defined on a hash output $h \in \{0, 1\}^\kappa$. Given a block property Q , a valid block with hash h is called a Q -block if $Q(h)$ is true.

LEMMA 7.2 (UNSUPPRESSIBILITY). *Consider a collection of polynomially many block properties Q . In a typical execution every set of consecutive rounds U has a subset S of uniquely successful rounds such that*

- $|S| \geq Y(U) - 2Z(U) - 2\lambda f(\frac{t}{n-t} \cdot \frac{1}{1-f} + \epsilon)$
- for any $Q \in \mathcal{Q}$, Q -blocks generated during S follow the distribution as in an unsuppressed chain
- after the last round in S the blocks corresponding to S belong to the chain of any honest party.

We now apply the above lemma to our construction. The following result lies at the heart of our security proof and allows us to argue that an honestly adopted chain will have a better superblock score than an adversarially generated chain.

LEMMA 7.3. Consider Algorithm 4 under velvet fork with parameter g and $(1/4)$ -bounded velvet honest majority. Let U be a set of consecutive rounds $r_1 \cdots r_2$ and C the chain of an honest party at

round r_2 of a typical execution. Let $C_U^S = \{b \in C : b \text{ is smooth} \wedge b \text{ was generated during } U\}$. Let $\mu, \mu' \in \mathbb{N}$. Let C' be a μ' superchain containing only adversarial blocks generated during U and suppose $|C'| > k$. Then it holds that $2^{\mu'} |C'| < 2^\mu |C_U^S|$.

PROOF. From the Unsuppressibility Lemma we have that there is a set of uniquely successful rounds $S \subseteq U$, such that $|S| \geq Y(U) - 2Z(U) - \delta'$, where $\delta' = 2\lambda f(\frac{t}{n-t} \cdot \frac{1}{1-f} + \epsilon)$. We also know that Q -blocks generated during S are distributed as in an unsuppressed chain. Therefore considering the property Q for blocks of level μ that contain smooth interlinks we have that $|C_U^S|^\mu \geq (1 - \epsilon)g2^{-\mu}|S|$. We also know that for the total number of μ' -blocks the adversary generated during U that $|C'| \leq (1 + \epsilon)2^{-\mu'}Z(U)$. Then we have to show that $(1 - \epsilon)g(Y(U) - 2Z(U) - \delta') > (1 + \epsilon)Z(U)$ or $((1 + \epsilon) + 2g(1 - \epsilon))Z(U) < g(1 - \epsilon)(Y(U) + \delta')$. Let $\alpha_z = (1 + \epsilon) + 2g(1 - \epsilon)$. Then we have to show that $\alpha_z Z(U) < g(1 - \epsilon)(Y(U) + \delta')$.

Substituting the bounds of X, Y, Z discussed above, it suffices to show that

$$\alpha_z[(1 + \frac{\delta}{2})f \frac{t}{n-t} |U| + \epsilon pq(n-t)|U|] < (1 - \epsilon)g[(1 - \epsilon)f(1 - f)|U| - \delta']$$

or

$$\alpha_z f |U| \frac{t}{n-t} (1 + \frac{\delta}{2}) + \alpha_z \epsilon pq(n-t)|U| < (1 - \epsilon)g[(1 - \epsilon)f(1 - f)|U| - \delta']$$

or

$$\frac{t}{n-t} < \frac{(1 - \epsilon)g[(1 - \epsilon)f(1 - f)|U| - \delta'] - \alpha_z \epsilon pq(n-t)|U|}{f|U|(\alpha_z(1 + \frac{\delta}{2}))}$$

or

$$\frac{t}{n-t} < \frac{(1 - \epsilon)g[(1 - \epsilon)f(1 - f) - \frac{\delta'}{|U|}] - \alpha_z \epsilon pq(n-t)}{f\alpha_z(1 + \frac{\delta}{2})}$$

But

$$\frac{\epsilon pq(n-t)}{f(1 + \frac{\delta}{2})} = \epsilon' \ll 1$$

thus we have to show that

$$\frac{t}{n-t} < \frac{(1 - \epsilon)g[(1 - \epsilon)f(1 - f) - \frac{\delta'}{|U|}]}{f\alpha_z(1 + \frac{\delta}{2})} - \epsilon'$$

or

$$\frac{t}{n-t} < g \cdot \frac{(1 - \epsilon)^2 f(1 - f) - \frac{(1 - \epsilon)\delta'}{|U|}}{(1 + \frac{\delta}{2})f\alpha_z} - \epsilon' \quad (1)$$

In order to show Equation 1 we use typical bound values for the parameters in our setting. We use $\delta > \frac{2}{3}$ since we assume a (1/4) adversary and $f \leq \frac{1}{10}$. We also need to estimate the value of $\frac{\delta'}{|U|}$. Because all blocks in C' were generated during U and $|C'| > k$, $|U|$ follows negative binomial distribution with probability pqt and number of successes k . Applying a Chernoff bound we have that $|U| > (1 - \epsilon)\frac{k}{pqt}$. Using the inequalities $k \geq 2\lambda f$ and

$pqt < \frac{t}{n-t} \frac{f}{1-f}$, we deduce that $|U| > (1 - \epsilon)[\frac{2\lambda(n-t)(1-f)}{t}]$. So we have that

$$\frac{\delta'}{|U|} < \frac{2\lambda f(\frac{t}{n-t} \frac{1}{1-f} + \epsilon)}{(1 - \epsilon)[\frac{2\lambda(n-t)(1-f)}{t}]}$$

or

$$\frac{\delta'}{|U|} < \left(\frac{t}{n-t}\right)^2 \frac{f}{(1 - \epsilon)(1 - f)^2} + \epsilon < 0.06 + \epsilon \quad (2)$$

By substituting (2) and the typical parameter bounds in Equation (1) we conclude that it suffices to show that

$$\frac{t}{n-t} < \frac{1 - \epsilon''}{3}g \quad (3)$$

But equation (3) is equivalent to $\frac{t}{n-t} < \frac{1 - \delta_v}{3}g$ which is the (1/4) velvet honest majority assumption, so the claim is proven. \square

THEOREM 7.4 (SUFFIX PROOFS SECURITY UNDER VELVET FORK). Assuming honest majority under velvet fork conditions (5.2) such that $t \leq (1 - \delta_v)\frac{n_h}{3}$ where n_h the number of upgraded honest parties, the Non-Interactive Proofs of Proof-of-Work construction for computable k -stable monotonic suffix-sensitive predicates under velvet fork conditions in a typical execution is secure.

PROOF. By contradiction. Let Q be a k -stable monotonic suffix-sensitive chain predicate. Assume for contradiction that NIPoPoWs under velvet fork on Q is insecure. Then, during an execution at some round r_3 , $Q(C)$ is defined and the verifier V disagrees with some honest participant. V communicates with adversary \mathcal{A} and honest prover B . The verifier receives proofs $\pi_{\mathcal{A}}, \pi_B$ which are of valid structure. Because B is honest, π_B is a proof constructed based on underlying blockchain C_B (with $\pi_B \subseteq C_B$), which B has adopted during round r_3 at which π_B was generated. Consider $\tilde{C}_{\mathcal{A}}$ the set of blocks defined as $\tilde{C}_{\mathcal{A}} = \pi_{\mathcal{A}} \cup \{\bigcup\{C_h^r : b_{\mathcal{A}} \in \pi_{\mathcal{A}}, \exists h, r : b_{\mathcal{A}} \in C_h^r\}\}$ where C_h^r the chain that the honest party h has at round r .

Suppose for contradiction that the verifier outputs $\neg Q(C_B)$. Thus it is necessary that $\pi_{\mathcal{A}} \not\geq_m \pi_B$. We show that $\pi_{\mathcal{A}} \geq_m \pi_B$ is a negligible event.

Let the levels of comparison decided by the verifier be $\mu_{\mathcal{A}}$ and μ_B respectively. Let $b_0 = LCA(\pi_{\mathcal{A}}, \pi_B)$. Call $\alpha_{\mathcal{A}} = \pi_{\mathcal{A}} \uparrow^{\mu_{\mathcal{A}}} \{b_0\}$, $\alpha_B = \pi_B \uparrow^{\mu_B} \{b_0\}$.

From Corollary 5.6 we have that the adversarial proof consists of a smooth interlink subchain followed by a thorny interlink subchain. We will refer to the smooth part of $\alpha_{\mathcal{A}}$ as $\alpha_{\mathcal{A}}^S$ and to the thorny part as $\alpha_{\mathcal{A}}^T$.

Our proof construction is based on the following intuition: we consider that $\alpha_{\mathcal{A}}$ consists of three distinct parts $\alpha_{\mathcal{A}}^1, \alpha_{\mathcal{A}}^2, \alpha_{\mathcal{A}}^3$ with the following properties.

Block $b_0 = LCA(\pi_{\mathcal{A}}, \pi_B)$ is the fork point between $\pi_{\mathcal{A}} \uparrow^{\mu_{\mathcal{A}}}, \pi_B \uparrow^{\mu_B}$. Let block $b_1 = LCA(\alpha_{\mathcal{A}}^S, \alpha_B \downarrow)$ be the fork point between $\pi_{\mathcal{A}} \uparrow^{\mu_{\mathcal{A}}}, C_B$ as an honest prover could observe. Part $\alpha_{\mathcal{A}}^1$ contains the blocks between b_0 exclusive and b_1 inclusive generated during the set of consecutive rounds S_1 and $|\alpha_{\mathcal{A}}^1| = k_1$. Consider b_2 the last block in $\alpha_{\mathcal{A}}$ generated by an honest party. Part $\alpha_{\mathcal{A}}^2$ contains

the blocks between b_1 exclusive and b_2 inclusive generated during the set of consecutive rounds S_2 and $|\alpha_{\mathcal{A}}^2| = k_2$. Consider b_3 the next block of b_2 in $\alpha_{\mathcal{A}}$. Then $\alpha_{\mathcal{A}}^3 = \alpha_{\mathcal{A}}[b_3:]$ and $|\alpha_{\mathcal{A}}^3| = k_3$ consisting of adversarial blocks generated during the set of consecutive rounds S_3 . Therefore $|\alpha_{\mathcal{A}}| = k_1 + k_2 + k_3$ and we will show that $|\alpha_{\mathcal{A}}| < |\alpha_B|$.

The above are illustrated, among other, in Parts I, II of Figure 10.

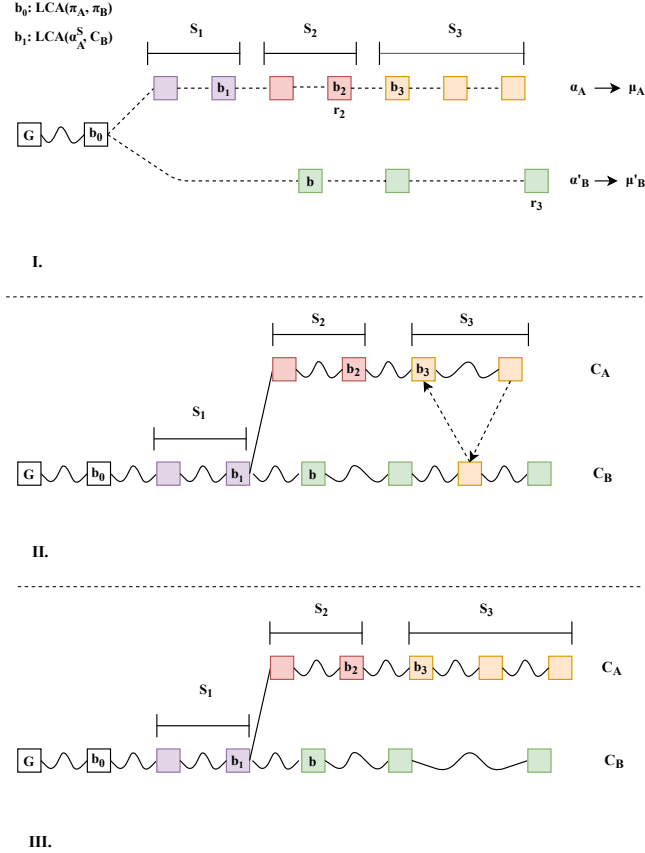


Figure 10: Wavy lines imply one or more blocks. Dashed lines and arrows imply interlink pointers to superblocks. I: the three round sets in two competing proofs at different levels, II: the corresponding 0-level blocks implied by the two proofs, III: blocks participating in chain C_B and block set $\tilde{C}_{\mathcal{A}}$ from the verifier's perspective.

We will now show three successive claims: First that $\alpha_{\mathcal{A}}^1$ contains only a few blocks. Second, $\alpha_{\mathcal{A}}^2$ contains only a few blocks. And third, the adversary is able to produce a winning $\alpha_{\mathcal{A}}$ with negligible probability.

Claim 1: $\alpha_{\mathcal{A}}^1 = \alpha_{\mathcal{A}}(b_0 : b_1]$ contains only a few blocks. We have defined $b_0 = LCA(\pi_{\mathcal{A}}, \pi_B)$ and $b_1 = LCA(\alpha_{\mathcal{A}}^S, \alpha_B \downarrow)$. First observe that there are no thorny blocks in $\alpha_{\mathcal{A}}^1$ since $\alpha_{\mathcal{A}}^1[-1] = b_1$ is a smooth block. This means that if b_1 was generated at round r_{b_1} and $\alpha_{\mathcal{A}}^S[-1]$ in round r then $r \geq r_{b_1}$. So, $\alpha_{\mathcal{A}}^1$ consists a valid chain.

We show the claim by considering the two possible cases for the relation of $\mu_{\mathcal{A}}, \mu_B$.

Claim 1a: If $\mu_B \leq \mu_A$ then they are completely disjoint. In such a case of inequality, every block in α_A would also be of lower level μ_B . Subsequently, any block in $\pi_A \uparrow^{\mu_A} \{b:\}[1:]$ would also be included in proof α_B , but $b = LCA(\pi_A, \pi_B)$ so there can be no succeeding block common in α_A, α_B .

Claim 1b: If $\mu'_B > \mu_A$ then $|\alpha_A[1:] \cap \alpha'_B \downarrow [1:]| = k_1 \leq g(2^{\mu'_B - \mu_A})$. Let's call b the first block in α'_B after block b_0 . Suppose for contradiction that $k_1 > g(2^{\mu'_B - \mu_A})$. Since block b of level μ'_B is also of level μ_A , the adversary could include it in the proof but b cannot exist in both α_A, α'_B since $\alpha_A \cap \alpha'_B = \emptyset$ by definition. In case that the adversary chooses not to include b in the proof then she can include no other blocks of C_B in her proof, since it would not consist a valid chain. Therefore, the adversary can include at most the $\mu_{\mathcal{A}}$ upgraded blocks between b_0, b , which are expected to be equal to $g(2^{\mu'_B - \mu_A})$.

We conclude that $|\alpha_{\mathcal{A}}^S \cap \alpha'_B \downarrow [1:]| = k_1 \leq g(2^{\mu'_B - \mu_{\mathcal{A}}})$, where g the velvet parameter denoting the percentage of upgraded honest parties.

Consequently, there are at least $|\alpha_{\mathcal{A}}| - k_1$ blocks after block b in $\alpha_{\mathcal{A}}$ which are not honestly generated blocks existing in C_B . In other words, there are $|\alpha_{\mathcal{A}}| - k_1$ blocks after block b in $\alpha_{\mathcal{A}}$, which are either thorny blocks existing in C_B either don't belong in C_B .

Claim 2. Part $\alpha_{\mathcal{A}}^2 = \alpha_{\mathcal{A}}(b_1 : b_2]$ consists of only a few blocks. Let $|\alpha_{\mathcal{A}}^2| = k_2$. We have defined $b_2 = \alpha_{\mathcal{A}}^2[-1]$ to be the last block generated by an honest party in $\alpha_{\mathcal{A}}$. Consequently no thorny block exists in $\alpha_{\mathcal{A}}^2$, so all blocks in this part belong in a proper zero-level chain $C_{\mathcal{A}}^2$. Let r_{b_1} be the round at which b_1 was generated. Since b_1 is the last block in $\alpha_{\mathcal{A}}$ which belongs in C_B , then $C_{\mathcal{A}}^2$ is a fork chain to C_B at some block b' generated at round $r' \geq r_{b_1}$.

Let r_2 be the round when b_2 was generated by an honest party. Because an honest party has chain C_B at later round r_3 when the proof π_B is constructed and because of the Common Prefix property on parameter $k_{2\downarrow} = g \cdot 2^{\mu_{\mathcal{A}}} \cdot k_2$, we conclude that $k_2 \leq k$.

Claim 3. The adversary may submit a suffix proof such that $|\alpha_{\mathcal{A}}| \geq |\alpha_B|$ with negligible probability. As explained earlier part $\alpha_{\mathcal{A}}^3$ consists only of adversarially generated blocks. The security parameter m guarantees that $|\alpha_{\mathcal{A}}^3| > k$, as will be later presented in detail. Let U be the set of consecutive rounds $r_2 \dots r_3$. Then all k_3 blocks of this part of the proof are generated during U . Let $\alpha'_B{}^{S_3}$ be the last part of the honest proof containing the interlinked μ_B superblocks generated during U . Then from lemma 7.3 we have that $2^{\mu_{\mathcal{A}}} |\alpha_{\mathcal{A}}^3| < 2^{\mu_B} |\alpha'_B{}^U \uparrow^{\mu_B}|$.

From all the above Claims we have that:

For the first round set S_1 , because of the common underlying chain:

$$2^{\mu_{\mathcal{A}}} |\alpha_{\mathcal{A}}^{S_1}| \leq 2^{\mu'_B} |\alpha'_B{}^{S_1}| \quad (4)$$

For the second round set S_2 because of the adoption by an honest party of chain C_B at a later round r_3 we have:

$$2^{\mu_{\mathcal{A}}} |\alpha_{\mathcal{A}}^{S_2}| \leq 2^{\mu'_B} |\alpha'_B{}^{S_2}| \quad (5)$$

For the third round set S_3 we have:

$$2^{\mu_{\mathcal{A}}} |\alpha_{\mathcal{A}}^{S_3}| < 2^{\mu'_B} |\alpha'_B{}^{S_3}| \quad (6)$$

Consequently we have:

$$2^{\mu_{\mathcal{A}}}(|\alpha_{\mathcal{A}}^{S_1}| + |\alpha_{\mathcal{A}}^{S_2}| + |\alpha_{\mathcal{A}}^{S_3}|) < 2^{\mu'_B}(|\alpha'_{B^{S_1}}| + |\alpha'_{B^{S_2}}| + |\alpha'_{B^{S_3}}|) \Rightarrow$$

$$2^{\mu_{\mathcal{A}}}|\alpha_{\mathcal{A}}| < 2^{\mu'_B}|\alpha'_B| \quad (7)$$

Therefore we have proven that $2^{\mu'_B}|\pi_B \uparrow^{\mu'_B}| > 2^{\mu_{\mathcal{A}}}|\pi_{\mathcal{A}}^{\mu_{\mathcal{A}}}|$. From the definition of μ_B , we know that $2^{\mu_B}|\pi_B \uparrow^{\mu_B}| > 2^{\mu'_B}|\pi_B \uparrow^{\mu'_B}|$ because it was chosen μ_B as level of comparison by the Verifier. So we conclude that $2^{\mu_B}|\pi_B \uparrow^{\mu_B}| > 2^{\mu_{\mathcal{A}}}|\pi_{\mathcal{A}}^{\mu_{\mathcal{A}}}|$. \square

8 INFIX PROOFS

NIPoPoWs infix proofs are proofs that answer any predicate which depends on blocks appearing anywhere in the chain, except for the k suffix for stability reasons. For example, consider the case where a client has received a transaction inclusion proof for a block b and requests an infix proof so as to verify that b is included in the current chain.

Because of the described protocol update for secure NIPoPoW suffix proofs, the infix proofs construction has to be altered as well. In order to construct secure infix proofs under velvet fork conditions, we suggest the following additional protocol patch: each upgraded miner constructs and updates an authenticated data structure for all the blocks in the chain. We suggest Merkle Mountain Ranges (MMR) for this structure. Now an interlinked block's header additionally includes the root of this MMR.

After this additional protocol change the notion of a smooth block changes as well. Smooth blocks are now considered the blocks that contain smooth interlinks but also valid MMR root. A valid MMR root denotes the MMR that contains all the blocks in the chain of an honest full node. Note that a valid MMR contains all the blocks of the longest valid chain, meaning both smooth and thorny. An invalid MMR constructed by the adversary may contain a block of a fork chain. Consequently an upgraded prover has to maintain a local copy of this MMR locally, in order to construct correct proofs. This is crucial for the security of infix proofs, since keeping the notion of a smooth block as before would allow an adversary to produce a block b in an honest party's chain, with b containing a smooth interlink but an invalid MMR, so she could succeed in providing an infix proof about a block of a fork chain.

Considering this additional patch we can now define the final algorithms for the honest miner, infix and suffix prover, as well as for the infix verifier. Because of the new notion of smooth block, the function *isSmoothBlock()* of Algorithm 3 needs to be updated, so that the validity of the included MMR root is also checked. The updated function is given in Algorithm 6. Considering that input C_S is computed using Algorithm 3 with the updated *isSmoothBlock'()* function, *Velvet updateInterlink* and *Velvet Suffix Prover* algorithms remain the same as described in Algorithms 4, 5 respectively. The velvet infix prover and infix verifier algorithms are given in Algorithms 7, 8 respectively. Details about the construction and verification of an MMR and the respective inclusion proofs can be found in [21].

Algorithm 6 Function *isSmoothBlock'()* for infix proof support

```

1: function isSmoothBlock'( $B$ )
2:   if  $B = \mathcal{G}$  then
3:     return true
4:   end if
5:   for  $p \in B.interlink$  do
6:     if  $\neg isSmoothPointer(B, p)$  then
7:       return false
8:     end if
9:   end for
10:  return containsValidMMR( $B$ )
11: end function

```

Algorithm 7 Velvet Infix Prover

```

1: function ProveInfixVelvet( $C_S, b$ )
2:    $(\pi, \chi) \leftarrow ProveVelvet(C_S)$ 
3:    $tip \leftarrow \pi[-1]$ 
4:    $\pi_b \leftarrow MMRinclusionProof(tip, b)$ 
5:   return  $(\pi_b, (\pi, \chi))$ 
6: end function

```

Algorithm 8 Velvet Infix Verifier

```

1: function VerifyInfixVelvet( $b, (\pi_b, (\pi, \chi))$ )
2:    $tip \leftarrow \pi[-1]$ 
3:   return VerifyInclProof( $tip.root_{MMR}, \pi_b, b$ )
4: end function

```

Note that equivalent solution could be formed by using any authenticated data structure that provides inclusion proofs of size logarithmic to the length of the chain. We suggest MMRs because of their efficiency while updating, since they support efficient node addition and deletion algorithms.

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