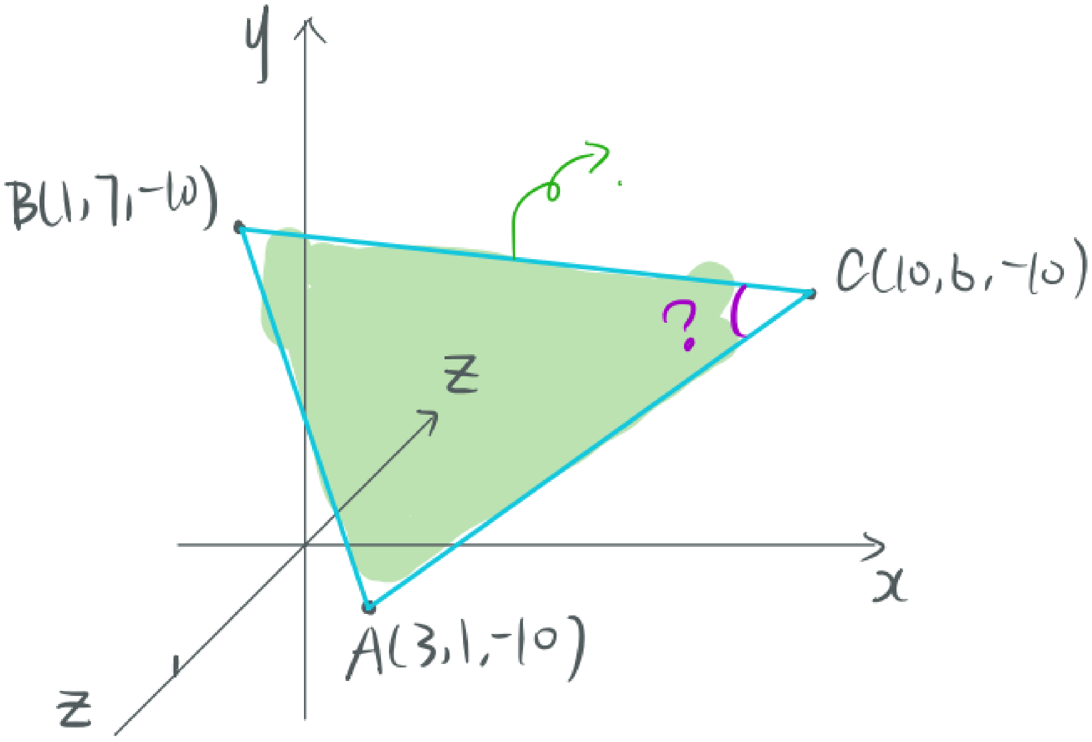
1. Field

场的建模与分析

1. 标定每个点，清楚接下来要算什么



2. 向量
   从A指到B的向量为r_{AB}，它等于终点坐标减去起点坐标

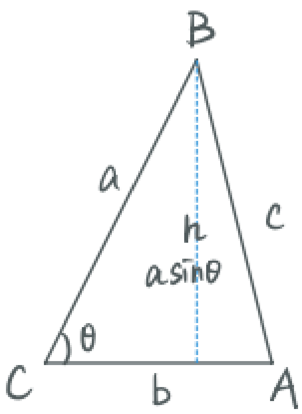
$$r_{AB} = r_B - r_A$$

3. 角度计算
   由矢量的点乘推出角度的计算公式：

$$\overrightarrow{r_{CB}} \cdot \overrightarrow{r_{CA}} = \left|\overrightarrow{r_{CB}}\right| \cdot \left|\overrightarrow{r_{CA}}\right| \cdot \cos\theta_C$$

$$\theta_C = \cos^{-1}\left(\frac{\overrightarrow{r_{CB}} \cdot \overrightarrow{r_{CA}}}{\left|\overrightarrow{r_{CB}}\right| \cdot \left|\overrightarrow{r_{CA}}\right|}\right)$$

4. 面积(area)计算
   由矢量的叉乘推导出面积计算公式



$$area = \frac{1}{2} h \cdot b = \frac{1}{2} ab\sin\theta = \frac{1}{2}|\overline{r_{BC}} \times \overline{r_{CA}}|$$

5. 周长(perimeter)计算
   三个向量模长(magnitude)的和。

2. Coodinate System 坐标系

2. 拉梅系数

各个坐标系的微分算子：

笛卡尔坐标系	$dl_x = dx$	$dl_y = dy$	$dl_z = dz$
柱坐标系	$dl_\rho = d\rho$	$dl_\phi = \rho d\phi$	$dl_z = dz$
球坐标系	$dl_r = dr$	$dl_\theta = rd\theta$	$dl_\phi = rsin\theta d\phi$

Del operator ▽

定义	三维空间笛卡尔系	三维空间柱坐标系
$\nabla = \frac{d}{dl}$	$(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$	$(\frac{\partial}{\partial \rho}, \frac{\partial}{\rho\partial \phi}, \frac{\partial}{\partial z})$

球坐标系的Del算子类似，可以自行写出。

3. 笛卡尔坐标和柱坐标的转换

Table 3-2: Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2+y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2+y^2+z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2+y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2+z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

partial ∂为取偏导数

4. Gradient 梯度

标量场的梯度表示标量场在某一点沿着最快增长方向的变化率，其方向与等值面垂直。

$$\nabla f = \frac{\partial f}{\partial x}\mathbf{a_x} + \frac{\partial f}{\partial y}\mathbf{a_y} + \frac{\partial f}{\partial z}\mathbf{a_z}$$

5. Divergence 散度

矢量场的散度表示矢量场在某一点从该点发散或汇聚的程度，其正负与流体流出或流入有关。

$$\nabla \cdot \mathbf{P} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (A_x, A_y, A_z)$$

$$\nabla \cdot \mathbf{P} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

6. Curl 旋度

矢量场的旋度表示矢量场在某一点绕该点旋转的强度和方向，其大小与旋转速率成正比，其方向与右手定则一致。

$$\nabla \times \mathbf{P} = \begin{vmatrix} \hat{a_x} & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & Az \end{vmatrix}$$

$$\nabla \times \mathbf{P} = (\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z})\mathbf{a_x} + (\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x})\mathbf{a_y} + (\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y})\mathbf{a_z}$$

以上