Introduction to Electric and Magnetic Fields B38EM
Tutorial #7

1. A plane wave traveling along the x-axis in a polystyrene-filled region with $\varepsilon_r = 2.54$ has an electric field given by $E_y = E_0 \cos(\omega t - kx)$. The frequency is 2.4 GHz, and $E_0 = 5.0$ V/m. Find the following:

(a) the amplitude and direction of the magnetic field,
(b) the phase velocity,
(c) the wavelength,
(d) the phase shift between the positions $x_1 = 0.1$ m and $x_2 = 0.15$ m.

(a)

$$H = \frac{1}{\eta}\hat{k} \times \vec{E}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \eta = \sqrt{\frac{\mu_o \mu_r}{\varepsilon_o \varepsilon_r}}$$

$$\eta = \sqrt{\frac{\mu_o(1)}{\varepsilon_o(2.54)}}$$

$$\eta = 236.38$$

$$\hat{a}_x \times \hat{a}_y = \hat{a}_z$$

$$H_z = \frac{E_o}{\eta}\cos(\omega t - kx)$$

$$H_z = \frac{5}{236.38}\cos(\omega t - kx)$$

$$H_z = 0.0211\cos(\omega t - kx)\, A/m$$

(b)

$$v = \frac{c}{\sqrt{\varepsilon_r}}$$

$$v = \frac{3 \times 10^8}{\sqrt{2.54}}$$

$$v = 1.88 \times 10^8 m/s$$

(c)

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{v}{f}$$

$$\lambda = \frac{1.88 \times 10^8}{2.4 \times 10^9}$$

$$\lambda = 0.0784m$$

$$k = \beta = \frac{2\pi}{\lambda} = \frac{2\pi}{0.0784}$$

$$k = 80.11m^{-1}$$

(d)

$$\Delta\phi = k(x_2 - x_1)$$
$$\Delta\phi = 80.11(0.15 - 0.10)$$
$$\Delta\phi = 4.0055 rad = 229.5°$$

2. A 75 Ω coaxial line has a current $i(t, z) = 1.8 \cos(3.77 \times 10^9 t - 18.13z)$ mA. Determine:

(a) the frequency,
(b) the phase velocity,
(c) the wavelength,
(d) the relative permittivity of the line,
(e) the phasor form of the current, and
(f) the time domain voltage on the line.

(a)

$$\omega = 2\pi f$$
$$f = \frac{\omega}{2\pi}$$
$$f = \frac{3.77 \times 10^9}{2\pi}$$
$$f = 600 MHz$$

(b)

$$v = \frac{\omega}{\beta}$$
$$v = \frac{3.77 \times 10^9}{18.13}$$
$$v = 2.08 \times 10^8 m/s$$

(c)

$$\beta = \frac{2\pi}{\lambda}$$
$$\lambda = \frac{2\pi}{\beta}$$
$$\lambda = \frac{2\pi}{18.13}$$
$$\lambda = 0.346 m$$

(d)

$$v = \frac{c}{\sqrt{\varepsilon_r}}$$
$$\sqrt{\varepsilon_r} = \frac{c}{v}$$
$$\varepsilon_r = \left(\frac{c}{v}\right)^2$$
$$\varepsilon_r = \left(\frac{3 \times 10^8}{2.08 \times 10^8}\right)^2$$
$$\varepsilon_r = 2.08$$

(e)

$$i(z,t) = 1.8\,cos(\,3.77 \times 10^9 t - 18.13z)$$
$$I(z) = 1.8e - j^{\beta z}$$
$$I(z) = 1.8e - j^{18.13z} mA$$

(f)

$$v(z,t) = i(z,t)Z_o$$
$$v(z,t) = 1.8\,cos(\,3.77 \times 10^9 t - 18.13z)75$$
$$v(z,t) = 0.135\,cos(\,3.77 \times 10^9 t - 18.13z)V$$

3. A transmission line has the following per-unit-length parameters: L = 0.5 μ H/m, C = 200 pF/m, R = 4.0 Ω/m, and G = 0.02 S/m. Calculate the propagation constant and characteristic impedance of this line at 800 MHz. If the line is 30 cm long, what is the attenuation in dB? Recalculate these quantities in the absence of loss (R = G = 0).

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{\big(4 + j(2\pi(800)0.5\mu)\big)\big(0.02 + j(2\pi(800)200p)\big)}$$
$$\gamma = \alpha + j\beta = 0.54 + j50.268\ /m$$

$$Z_o = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} = \sqrt{\frac{\big(4 + j(2\pi(800)0.5\mu)\big)}{\big(0.02 + j(2\pi(800)200)\big)}} = 49.99 + j0.46\ \Omega$$

$$R = 0, G = 0$$
$$\beta = \omega\sqrt{LC} = 2\pi(800)\sqrt{(0.5\mu)(200p)} = 50.265\ rad/m$$

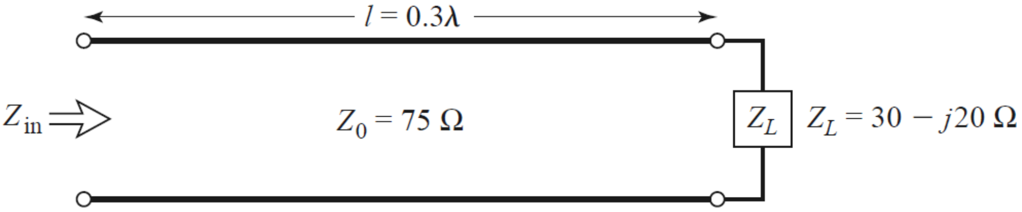
$$Z_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{(0.5\mu)}{(200p)}} = 50\ \Omega$$

$$l = 30cm$$

$$\alpha = 0.54\ Np/m$$

Attenuation, $\alpha = 20\ log_{10} e^{-\alpha l} = 20\ log_{10} e^{-(0.54)(30\times 10^{-2})} = -1.4\ dB$

4. A lossless transmission line of electrical length $\ell = 0.3\lambda$ is terminated with a complex load impedance as shown in the accompanying figure. Find the reflection coefficient at the load, the SWR on the line, the reflection coefficient at the input of the line, and the input impedance to the line.



$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}$$
$$\Gamma = \frac{30 - j20 - 75}{30 - j20 + 75}$$
$$\Gamma = \frac{-45 - j20}{105 - j20}$$
$$\Gamma = \frac{-45 - j20(105 + j20)}{105 - j20(105 + j20)}$$
$$\Gamma = \frac{(-45)(105) + (-45)(j20) + (-j20)(105) + (-j20)(j20)}{(105)(105) + (105)(j20) + (-j20)(105) + (-j20)(j20)} \frac{-4725 - j900 - j2100 + 400}{11025 + 400}$$
$$\Gamma = \frac{-4725 - j900 - j2100 - j^2 400}{11025 - j^2 400} = \frac{-4725 - j900 - j2100 + 400}{11025 + 400}$$
$$\Gamma = \frac{-4325 - j3000}{11425}$$
$$\Gamma = -0.379 - j0.263$$
$$\Gamma = |\Gamma| e^{j\theta}$$
$$|\Gamma| = \sqrt{(-0.379)^2 + (-0.263)^2}$$
$$|\Gamma| = 0.46$$
$$\theta = tan^{-1}\left(\frac{0.263}{0.379}\right)$$
$$\theta = 35^\circ$$
$\Gamma = -0.379 - j0.263$ third quadrant
$$\theta = 35^\circ + 180^\circ$$
$$\theta = 215^\circ$$
$$\Gamma = |\Gamma| e^{j\theta} = 0.46 e^{j215^\circ}$$

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$
$$SWR = \frac{1 + 0.46}{1 - 0.46}$$
$$SWR = 2.7$$

$$\Gamma_{in} = \Gamma e^{j2\beta\ell}$$
$$\Gamma_{in} = (-0.379 - j0.263) e^{j2\frac{2\pi}{\lambda}0.3\lambda}$$
$$\Gamma_{in} = (-0.379 - j0.263) e^{j1.2\pi}$$

$$Z_{in} = Z_o\left(\frac{Z_L + jZ_o\, tan\, \beta\, \ell}{Z_o + jZ_L\, tan\, \beta\, \ell}\right)$$
$$Z_{in} = 75\left(\frac{(30 - j20) + j75\, tan\left(\frac{2\pi}{\lambda}\right)(0.3\lambda)}{75 + j(30 - j20)\, tan\left(\frac{2\pi}{\lambda}\right)(0.3\lambda)}\right)$$
$$Z_{in} = 75\left(\frac{(30 - j20) + j75\, tan\, 0.6\pi}{75 + j(30 - j20)\, tan\, 0.6\pi}\right)$$
$$Z_{in} = 75\left(\frac{(30 - j20) + j75(-3.077)}{75 + j(30 - j20)(-3.077)}\right)$$
$$Z_{in} = 75\left(\frac{(30 - j20) - j230.775}{75 + j(-92.31 + j61.54)}\right)$$
$$Z_{in} = 75\left(\frac{30 - j250.775}{75 - j92.31 + j^2 61.54}\right)$$
$$Z_{in} = 75\left(\frac{30 - j250.775}{13.46 - j92.31}\right)$$
$$Z_{in} = \frac{2250 - j18808.125}{13.46 - j92.31}$$
$$Z_{in} = \frac{2250 - j18808.125(13.46 + j92.31)}{13.46 - j92.31(13.46 + j92.31)}$$
$$Z_{in} = \frac{30285 + j207697.5 - j253157.3625 + 1736178.019}{8702.3077}$$
$$Z_{in} = \frac{30285 + j207697.5 - j253157.3625 + 1736178.019}{8702.3077}$$
$$Z_{in} = (202.99 - j5.224)\Omega$$

5. A 75 Ω coaxial transmission line has a length of 2.0 cm and is terminated with a load impedance of 37.5 + *j*75 Ω. If the relative permittivity of the line is 2.56 and the frequency is 3.0 GHz, find the input impedance to the line, Z_{in}, the reflection coefficient, Γ and the SWR on the line.

$Z_o = 75\Omega$
$Z_L = 37.5 + j75\Omega$
$\varepsilon_r = 2.56$
$f = 3GHz$

$$\lambda = \frac{\lambda_o}{\sqrt{\varepsilon_r}} = \frac{c}{f}\frac{1}{\sqrt{2.56}} = \frac{3 \times 10^8}{3 \times 10^9}\frac{1}{\sqrt{2.56}} = 6.25cm$$
$$\beta l = \frac{2\pi}{\lambda} l = \frac{2\pi}{6.25 \times 10^{-2}} 2 \times 10^{-2} = 2.0106\, rad = 115.2°$$
$$Z_{in} = Z_o\left(\frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}\right) = 75\left(\frac{(37.5 + j75) + j75 \tan 115.2°}{75 + j(37.5 + j75)\tan 115.2°}\right) = 18.98 - j20.55\Omega$$
$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}$$
$$\Gamma = \frac{(37.5 + j75) - 75}{(37.5 + j75) + 75}$$
$$\Gamma = 0.077 + j0.615 = 0.62\angle 83°$$

$$SWR = \frac{1 + |0.62|}{1 - |0.62|}$$
$$SWR = \frac{1 + 0.62}{1 - 0.62}$$
$$SWR = 4.26$$

6. A lossless transmission line is terminated with a 100 Ω load. If the SWR on the line is 1.5, find the two possible values for the characteristic impedance of the line.

$$|\Gamma| = \frac{S - 1}{S + 1} = \frac{1.5 - 1}{1.5 + 1} = 0.2$$
$$|\Gamma| = \left|\frac{Z_L - Z_o}{Z_L + Z_o}\right| = \left|\frac{100 - Z_o}{100 + Z_o}\right|$$
$$\left|\frac{100 - Z_o}{100 + Z_o}\right| = 0.2$$
$$\left|\frac{100 - Z_o}{100 + Z_o}\right| = -0.2$$
$$Z_o = Z_L \frac{1 - \Gamma}{1 + \Gamma} = 100 \frac{1 - 0.2}{1 + 0.2} = 66.7\Omega$$
$$Z_o = Z_L \frac{1 - \Gamma}{1 + \Gamma} = 100 \frac{1 - (-0.2)}{1 + (-0.2)} = 100 \frac{1 + 0.2}{1 - 0.2} = 150\Omega$$

7. A radio transmitter is connected to an antenna having an impedance 80 + *j*40 Ω with a 50 Ω coaxial cable. If the 50 Ω transmitter can deliver 30 W when connected to a 50 Ω load, how much power is delivered to the antenna?

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{(80 + j40) - 50}{(80 + j40) + 50} = 0.297 + j0.216 = 0.367\angle 36°$$
$$P_{load} = P_{inc} - P_{ref} = P_{inc}\left(1 - |\Gamma|^2\right) = 30\left(1 - 0.367^2\right) = 25.9W$$