HERIOT-WATT UNIVERSITY

B38EM – Introduction to Electricity and Magnetism

Laboratory 1: Electrostatics

Instructor: Dr. Dimitris Anagnostou & Dr. Yuan Ding

1. Note

Please read the introduction and complete the pre-lab prior to entering the laboratory. You present the completed pre-lab to the instructor and/or demonstrator upon arrival.

2. Objectives

The main aim of this experiment is to further understand the electrostatic concept of electric field around dipoles/charged bodies. Different configurations will be investigated, and we will see how the field lines are produced by different distributions of charges.

3. Background

Any distribution of static charges produces an electric field surrounding them in their vicinity. Coulomb's Law and the principle of superposition makes it possible to theoretically calculate the field produced by these set of charges. In practice, however, it is very difficult to mathematically represent all but the simplest charge configurations as they are vector quantities that are a function of position, this implies that it can be different at every point in space.

One way to visualise the electric field near an arrangement of charges is by using **electric flux lines** introduced by Michael Faraday in his experimental investigation as a way of visualising the electric field**.** An electric flux line is an imaginary path or line drawn in such a way that its direction at any point is the direction of the electric field at that point. They are the lines to which the electric field density, *D*, is tangential at every point.

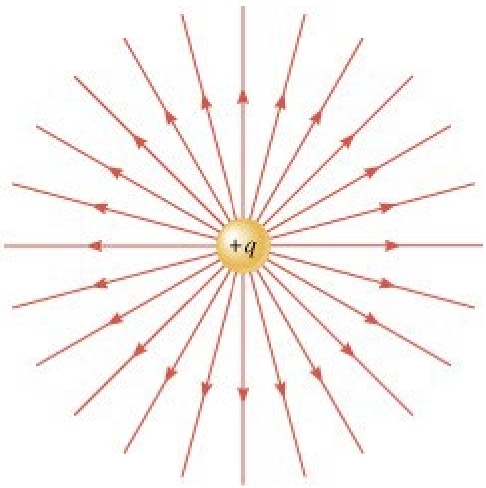


Figure 1: Flux distribution around a point charge

The situation gets a little complex when another charge is added. This changes the field lines around the vicinity make a more complicated charge distributions resulting from an **electric dipole**. An **electric dipole** is formed when two-point charges of equal magnitude, but opposite sign are separated by a small distance.

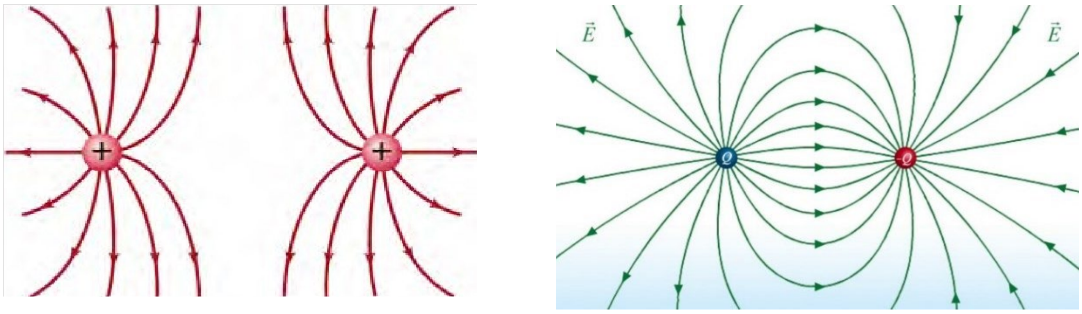


Figure 2: Electrostatic distributions of (a) Double charge (b) Dipole Charge.

From the lectures, you know electric field intensity (or electric field strength) ***E*** is the force per unit charge when placed in the electric field. This is given by

$$\boldsymbol{E} = k \cdot \frac{Q}{|\boldsymbol{R}|^2}\boldsymbol{a_R} = k \cdot \frac{Q(\boldsymbol{r} - \boldsymbol{r}')}{(\boldsymbol{r} - \boldsymbol{r}')^3}$$

$$\text{Where } k = \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 (Nm^2/C^2)$$

And for *N* point charges $Q_1, Q_2, \ldots, Q_N$ located at $\boldsymbol{r_1}, \boldsymbol{r_2}, \ldots, \boldsymbol{r_N}$, the electric field intensity at point ***r*** can be obtained from

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N} Q_i \frac{(\boldsymbol{r} - \boldsymbol{r}_i)}{|\boldsymbol{r} - \boldsymbol{r}_i|^3} = \frac{Q_1(\boldsymbol{r} - \boldsymbol{r}_1)}{4\pi\varepsilon_0|\boldsymbol{r} - \boldsymbol{r}_1|^3} + \frac{Q_2(\boldsymbol{r} - \boldsymbol{r}_2)}{4\pi\varepsilon_0|\boldsymbol{r} - \boldsymbol{r}_2|^3} + \cdots + \frac{Q_N(\boldsymbol{r} - \boldsymbol{r_N})}{4\pi\varepsilon_0|\boldsymbol{r} - \boldsymbol{r_N}|^3}$$

Assume there are two positive charged spheres, Sphere A with charge Q_A and Sphere B with charge Q_B in each region of space with a distance *d* between their midpoints. Each sphere creates an **E**-field at its surrounding area. The strength of **E**-field can be calculated using:

$$E = \frac{k \cdot Q}{|\boldsymbol{d}|^2}$$

where $Q = Q_A = Q_B$. Since there are two charges, the electric field strength calculation would have to be performed twice to obtain relative strength values between spheres A and sphere B from where they are located. The electric field relative to sphere A (E_A) and sphere B (E_B) is given by

$$\boldsymbol{E_A} = \frac{k \cdot Q_A}{|\Delta \boldsymbol{d_A}|^2}, \boldsymbol{E_B} = \frac{k \cdot Q_B}{|\Delta \boldsymbol{d_B}|^2}$$

$\Delta \boldsymbol{d_A}$*: the displacement vector relative to the centre of the sphere A;*
$\Delta \boldsymbol{d_B}$*: the displacement vector relative to the centre of the sphere B.*

The electric field strength and direction at a multitude of locations can be known by the sum of $\boldsymbol{E}_A$ and $\boldsymbol{E}_B$ to give $\boldsymbol{E}_{sum}$. As shown below:

$$\boldsymbol{E}_{sum} = \boldsymbol{E}_A + \boldsymbol{E}_B = \frac{1}{4\pi\varepsilon_0}\left[Q_A \frac{\boldsymbol{d} - \boldsymbol{d}_A}{|\boldsymbol{d} - \boldsymbol{d}_A|^3} + Q_B \frac{\boldsymbol{d} - \boldsymbol{d}_B}{|\boldsymbol{d} - \boldsymbol{d}_B|^3}\right]$$

$\mathbf{d}$: the position vector

$\mathbf{d}_A$: the position vector of the centre of the sphere A;
$\mathbf{d}_B$: the position vector of the centre of the sphere B.

4. Pre-Lab Assignment: *(To be used as self-assessment)*

1. An electric dipole consisting of charges of magnitude 10 nC are separated by a distance from their centres. What is the magnitude and direction of the net electric field created halfway between the two charges? If the positive charge is located at (1, 0, 0) mm and the negative charge is located at (–6, 0, 0) mm.

2. Repeat Question 1 above, using the parameters in Table shown in **Lab Activity 2**. Find the maximum electric field, Emax and the electric field located at the mid-point (0,0,0).
   (Note: When calculating distance, assume charges are located at the centre of sphere)

3. Point charges 1 mC and –2 mC are located at (3, 2, –1) m and (–1, –1, 4) m, respectively. Calculate the electric force on a 10 nC charge located at (0, 3, 1) m and the electric field intensity at that point.

5. Introduction

Electrostatics is a fascinating subject that has grown up in diverse areas of application. Electric power transmission, X-ray machines, and lightning protection are associated with strong electric fields and will require a knowledge of electrostatics to understand and design suitable equipment. The devices used in solid-state electronics such as resistors, capacitors, and active devices such as bipolar and field effect transistors, are based on electrostatics. Transistors in computer chips work based on control of electron motion by electrostatic fields. In fact, almost all computer peripheral devices, except for magnetic memory, are based on electrostatic fields. Touch pads, capacitance keyboards, cathode-ray tubes, liquid crystal displays, and electrostatic printers are typical examples. In medical work, diagnosis is often carried out with the aid of electrostatics, as incorporated in electrocardiograms, electroencephalograms, and other recordings of organs with electrical activity including eyes, ears, and stomachs.

The experimental setup consists of two perfectly conducting, charged spheres with radius R_0 = 1 mm in free space. The midpoints of the sphere are separated by D = 6 mm and the charges are given by q = 8 nC. The highest electric field strength **Emax** can be found at the points on the spheres which are closest to each other (P_1 and P_2 in Fig. 3).

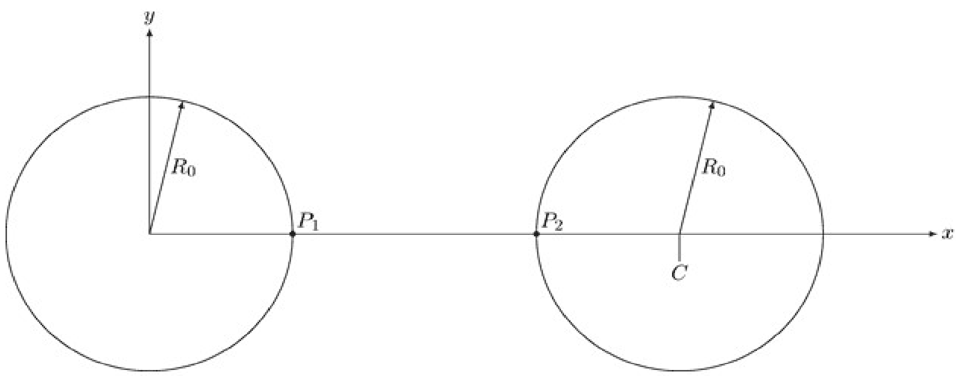
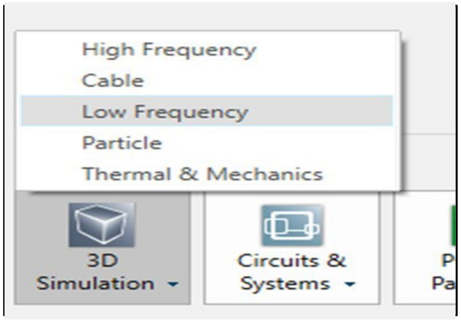
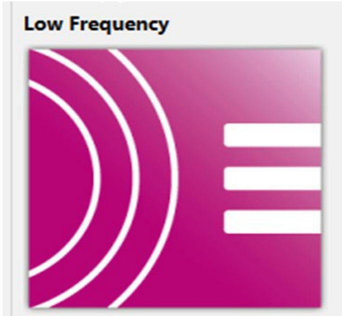


Figure 3: Diagram of the problem with all relevant variables.

6. Lab Activity:

1. **Launch CST**: From the File page launch **CST EM STUDIO** which is an electrostatic solver. We will not need to create a project template for this, simply select **DO NOT USE A PROJECT TEMPLATE.**



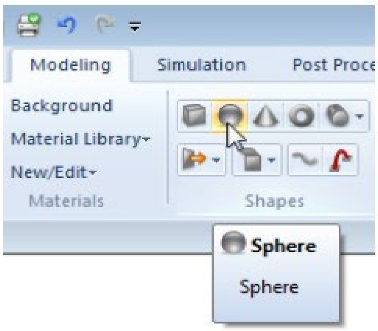


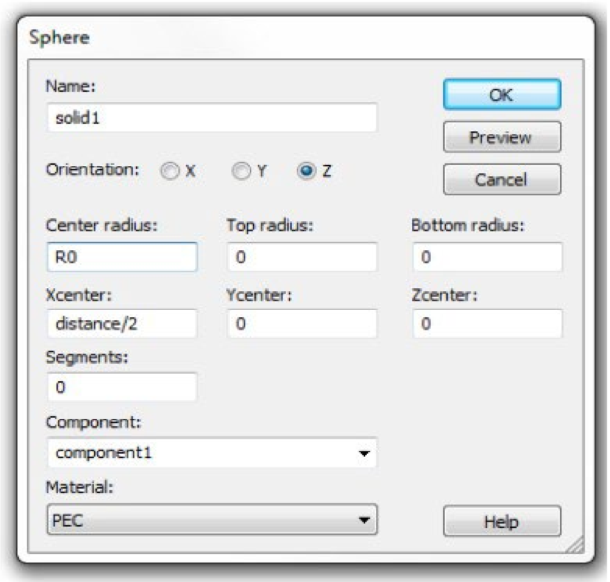
2. **Model Sphere**: We can model the sphere parameters on the parameter list, this is useful when evaluating a simulation as we will see later. Below is a guide to help you fill the parameter list**.**

Save a screenshot of your parameter table with description [1 mark]

Parameter	Value	Description
R_0	1 mm	PEC sphere radius
D	6 mm	Separation of spheres from their midpoints
q_1	8nC	Charge of sphere 1
q_2	$-$8nC	Charge of sphere 2
P_1	$(D/2) - R_0$	Point of highest E-field on sphere 1
P_2	$(-D/2)+R_0$	Point of highest E-field on sphere 2
P_3	$(P_1+P_2)/2$	Mid-point between spheres

Model the first sphere to have radius R_0 centred at (D/2, 0, 0), with **PEC material.**

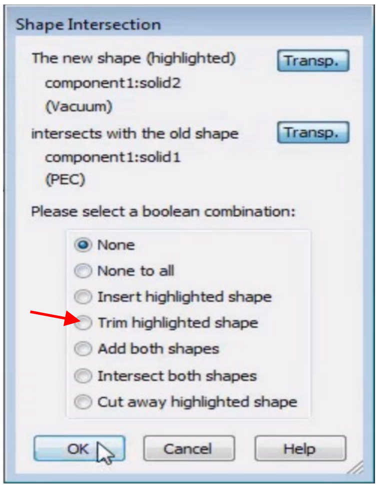


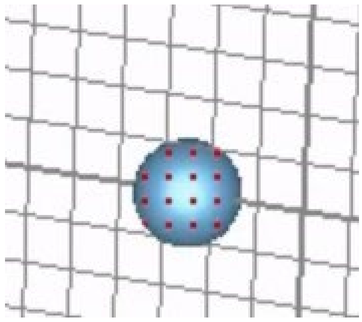


Note: ‘distance’ in the screenshot should be replaced with ‘D’.

Around this sphere create a **vacuum** by defining a second sphere, with radius (R_0+0.2), also centred at point (D/2, 0, 0) but this time with **vacuum/Air material**.

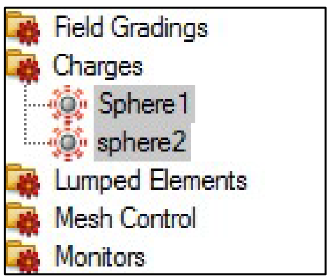
A dialog will appear because these objects intersect, asking how this intersection should be handled. Ensure the vacuum sphere is highlighted and select **TRIM HIGHLIGHTED SHAPE**.

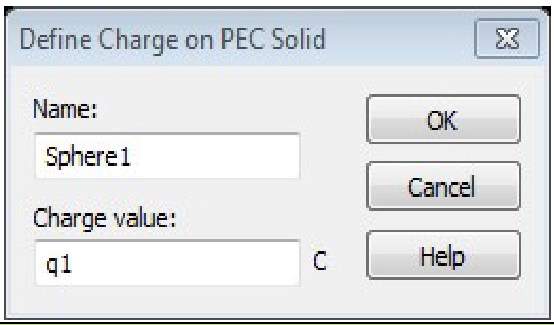


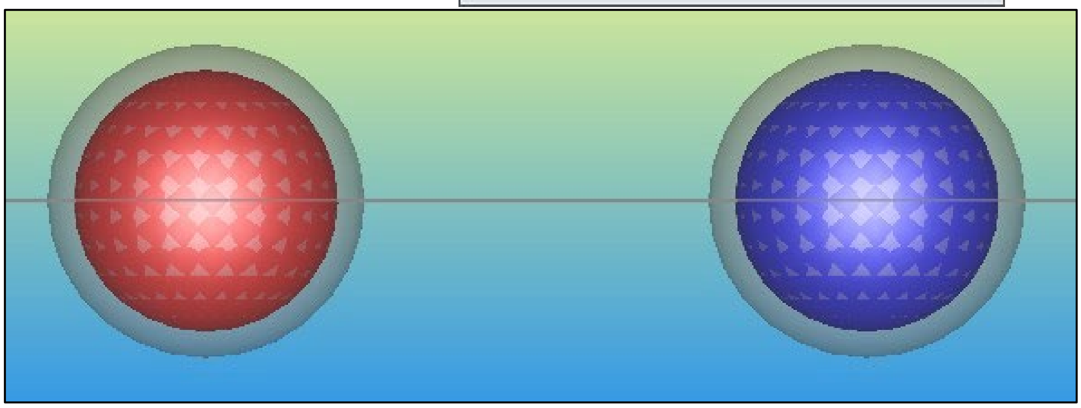


Repeat this process for the second sphere, which should be centred at the point (–D/2, 0, 0). The model should appear as shown in Figure.

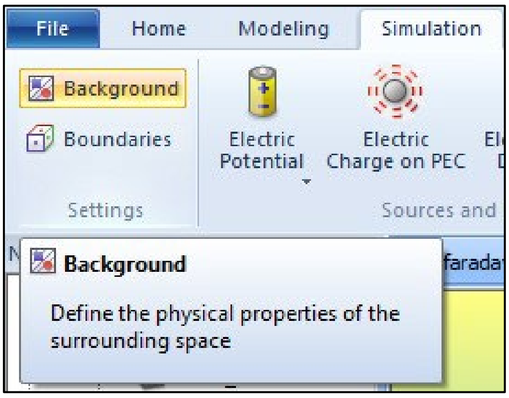
3. **Define charges of sphere**: Right-click on the **Charges** folder and select the **define Charge** tool. The first sphere should have a positive charge of **q**1 C, and the other a negative **q**2 C as below:

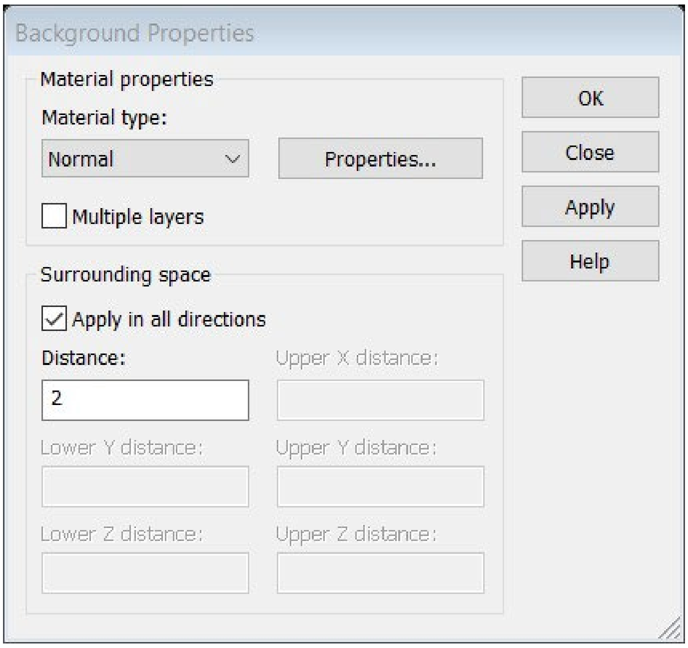




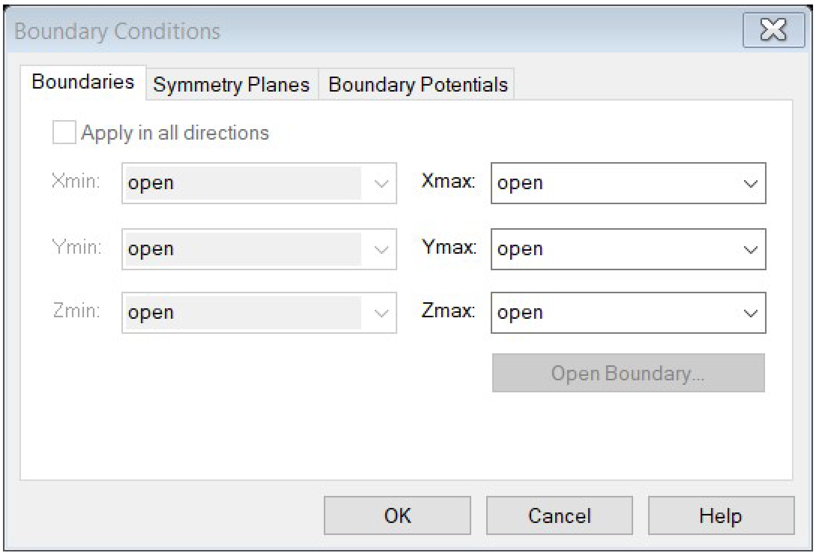


4. **Setting Background**: From the **simulation menu** select the **Background tool**, set material to **normal** and **add 2 mm of space**. The background is the area around the components to be analysed.

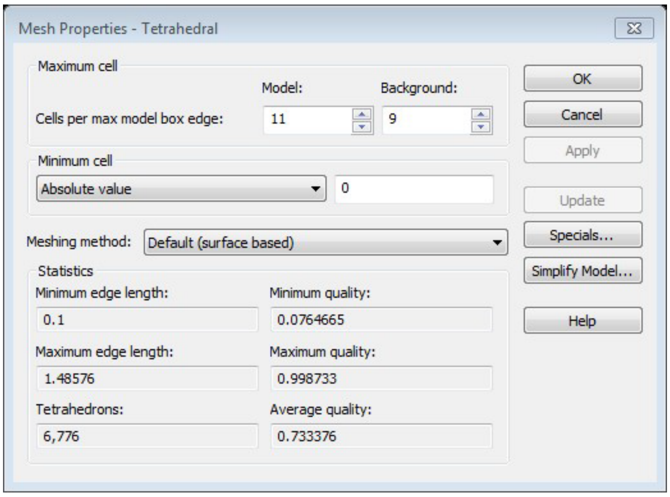




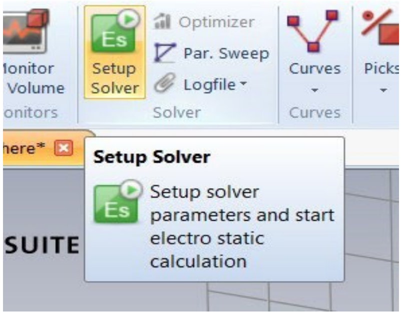
5. **Set boundary conditions**: The boundary properties can be set from the **Boundaries** tool below the **Background** tool. They should be all **open** in all directions.

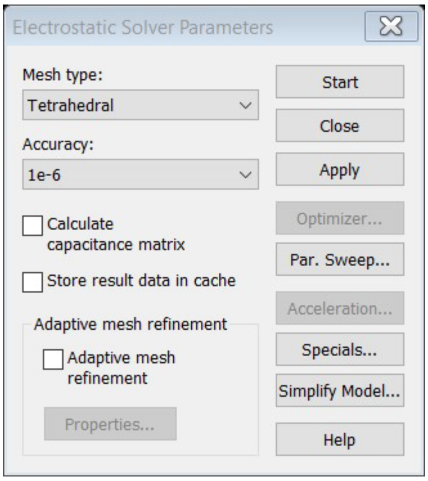


For improved accuracy of the field gradients, we must refine the mesh.
Select **Mesh Properties** from the Simulation ribbon and set the **number of cells** per max model box edge to: **80 in the model** and **12 in the background** for *commercial license* or **11 in the model and 9 in the background** for *student version*.



6. **Simulation**: Use the electrostatic solver to simulate the setup.



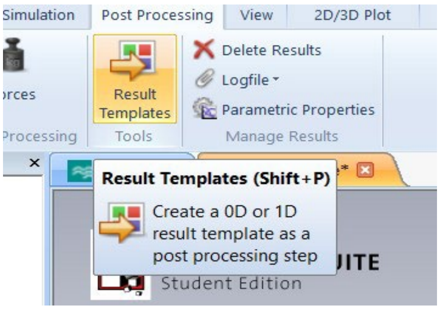


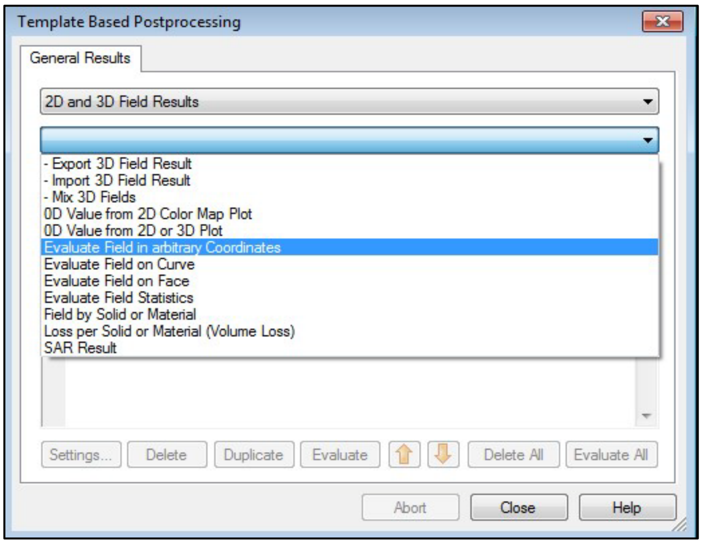
Click on **Potential** on the **2D and 3D Field Results** folder to show the distribution of potential around the sphere. Select **Fields on Plane** and change normal to **Y**.

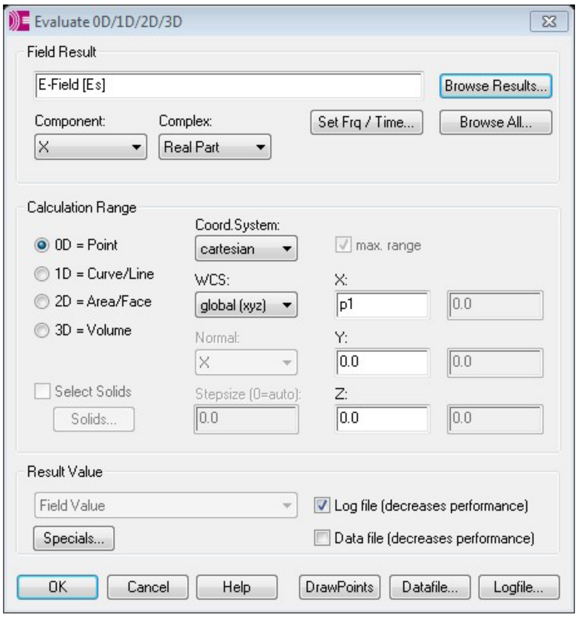
Record a screenshot of your results. **[1 mark]**

7. Exercise 1

Compare the simulation to theory by calculating the **E**-field (Emax) at the edge of the sphere at the point of the negatively charged sphere nearest to the opposite sphere. To do this, open the **Template Based Post Processing** dialog and select **Evaluate Field in arbitrary Coordinates** from Add new post processing. Select the **E-Field Result** and select **0D type** calculation. This should correspond to the **P_1** and **P_2** on the **x-axis as we just calculate** the x-component of the E-field.







Once the template has been created, click **Evaluate All** to calculate the field at the point.

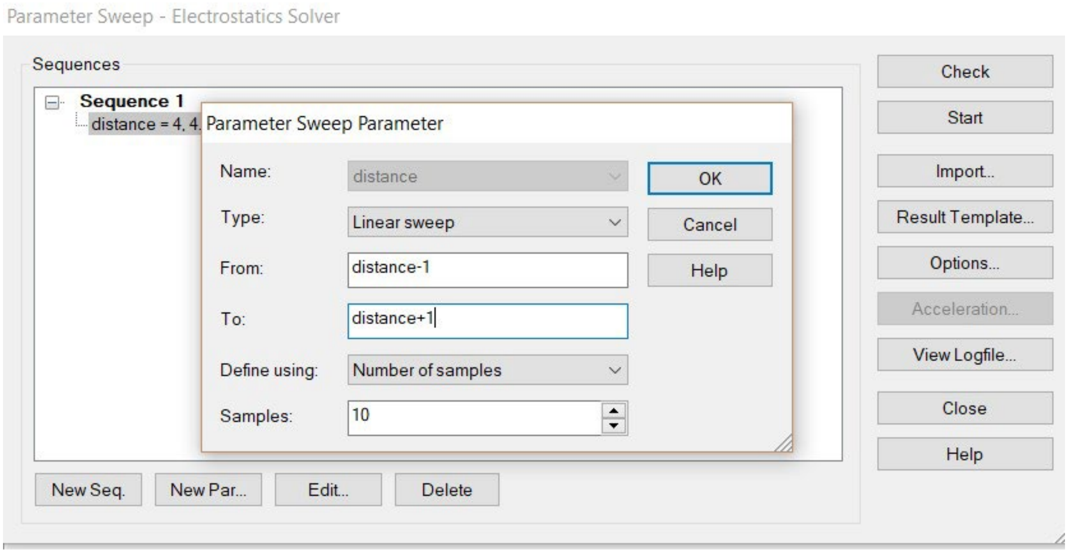
Record and comment on the value of Emax. [2 marks]

Repeat this step for a distance located halfway between the spheres and record your results. Is it the same? Why? [4 marks]

Discuss observations of these results with your solutions from Pre-Lab assignment. [2 marks]

8. Exercise 2

By Varying the distance between the spheres, we should observe a change in **Emax,** this can be done by a **Parameter Sweep Function.** Select **Parameter Sweep** from the home menu and then **New Sequence-New Parameter**, sweep the distance between **distance–1 (or D–1 depending on the parameters defined) to distance+1** with 10 samples

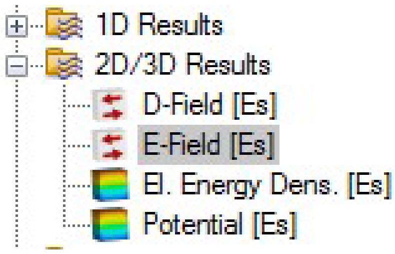


Then press **Start.** In the **0D Results** folder located under the Table folder, we can view the **Emax** calculated at the edge of the sphere for each run.

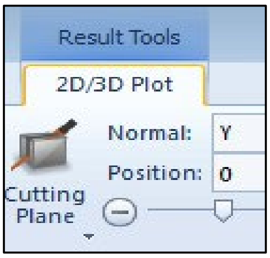
Record and Comment on results. **[2 marks]**

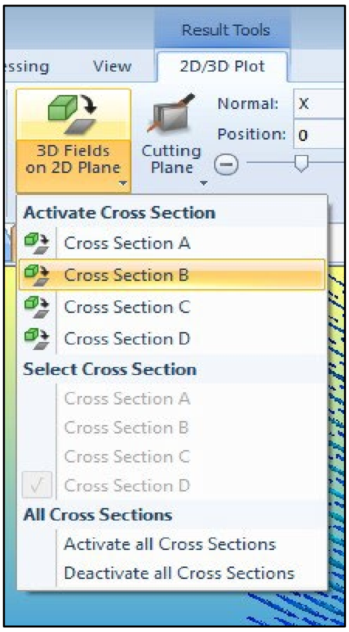
9. Exercise 3

We can see a visual representation of the electric field orientation of the charged spheres. Under the **2D/3D Result** select **E-Field [ES].**

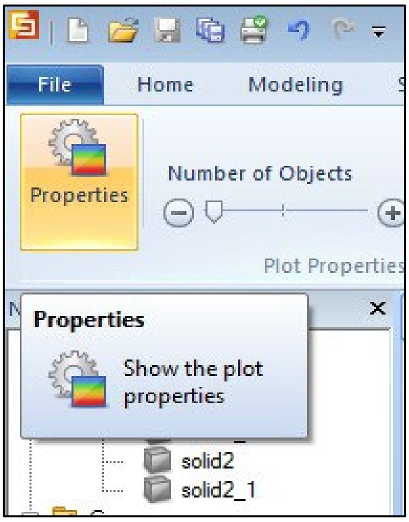


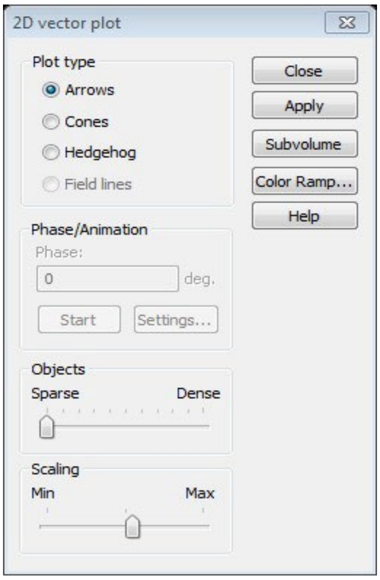
After simulating as previously done**,** view the results under the **2D/3D plot.** To better understand the results, select a single cross-section from **3D fields on 2D plane.**





We can adjust the orientation view from properties by sliding the **Object** and **Scaling bars** as shown below**.**





Record the electric field orientation for the electric dipole and discuss your observations. [2 marks]

Repeat the same process for a configuration with two positive charges, is it the same? Why? [2 marks]

10. Lab Report

The report should be typed and not more than 5 pages including figures (this excludes, an appendix, cover page, etc.). Please note that screen shots from the CST simulator are encouraged in your report. The report should not re-state the goals and the procedures as contained within this document. All your observations must be clearly indicated, along with comparisons to theoretical values. Comparisons should also be thoroughly discussed where needed. Your conclusion should be a general assessment of the verification of the theory and should indicate what you have learned while completing the laboratory activity. Sparsely commented reports will be sparsely rewarded.

Lab reports are to be completed and submitted 2 weeks from the laboratory activity to EPSSO by days end. No excuses, No exceptions. One typed lab report is required for each two-person lab group team. The report should be typed and not more than 5 pages including figures (this excludes, an appendix, cover page, etc.). Please note that screen shots from the CST simulator are encouraged in your report.

A marking scheme will also be provided in advance of the deadline. The report should not re-state the goals and the procedures as contained within this document. All your observations must be clearly indicated, along with comparisons to theoretical values. Comparisons should also be thoroughly discussed where needed. Your conclusion should be a general assessment of the verification of the theory and should indicate what you have learned while completing the laboratory activity. Sparsely commented reports will be sparsely rewarded.