

Course code and name:	B38EM Introduction to Electricity and Magnetism
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Coursework Title:	Take home Assignment 2
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Introduction to Electricity and Magnetism B38EM

Assignment #2

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1}, \quad e = 1.6 \times 10^{-19} \text{ C}, \quad \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$$

1. Consider the system shown in **Figure Q1** where a metal bar of mass m slides frictionless on two parallel conducting rails separated by a distance l . A resistor R is connected across the rails. If a uniform magnetic field B , pointing into the page, fills the entire region, find the following:

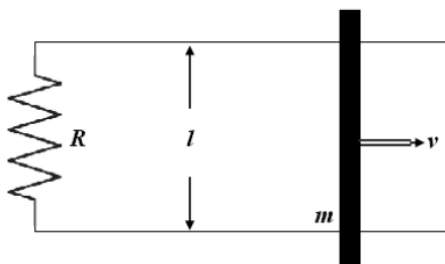


Figure Q1

- (a) If the bar moves to the right at speed v , what is the current in the resistor? In which direction does it flow? (3 marks)
- (b) Determine the magnetic force. What is the direction of the force? (3 marks)
- (c) If the bar starts out with speed v_0 at $t=0$, and it is left to slide, what is its speed at a later t ? (4 marks)
- (d) If the initial kinetic energy of the bar was $\frac{1}{2}mv_0^2$. Check that the energy delivered to the resistor is exactly that. (Hint: First find the power delivered to the resistor) (3 marks)

1. (a) $I = \frac{U}{R} = \frac{Bvl}{R}$

(b) $F = BIl = \frac{B^2 l^2 v}{R}$

The current in the conductor goes upwards,
So the direction of magnetic force is left

(c) $F = -\frac{B^2 l^2 v}{R}$

$$m \frac{dv}{dt} = -\frac{B^2 l^2 v}{R}$$

$$\int_{v_0}^v \frac{dv}{v} = -\int_0^t \frac{B^2 l^2}{Rm} dt \Rightarrow v = v_0 e^{-\frac{B^2 l^2}{Rm} t}$$

(d) $W = \int_0^\infty \left(\frac{B^2 l^2 v}{R} \right) dt$
Since $v = v_0 e^{-\frac{B^2 l^2}{Rm} t}$

$$W = \left[-\frac{(B^2 l^2 v_0)^2}{R} \times \frac{Rm}{2B^2 l^2} e^{-\frac{2B^2 l^2}{Rm} t} \right]_0^\infty$$

$$= v_0^2 \times \frac{m}{2} = \frac{1}{2} m v_0^2$$

Proved

2. The transformer shown in **Figure Q2** consists of a long wire coincident with the z -axis carrying a current $I = I_0 \cos \omega t$, coupling magnetic energy to a toroidal coil situated in the x - y plane and centred at the origin. The toroidal core uses iron material with relative permeability, μ_r , around which 100 turns of a tightly wound coil serves to induce a voltage V_{emf} , as shown in the figure.

(a) Develop an expression for V_{emf} (3 marks)

(b) Calculate V_{emf} , for the parameters in **Table Q2** (2 marks)

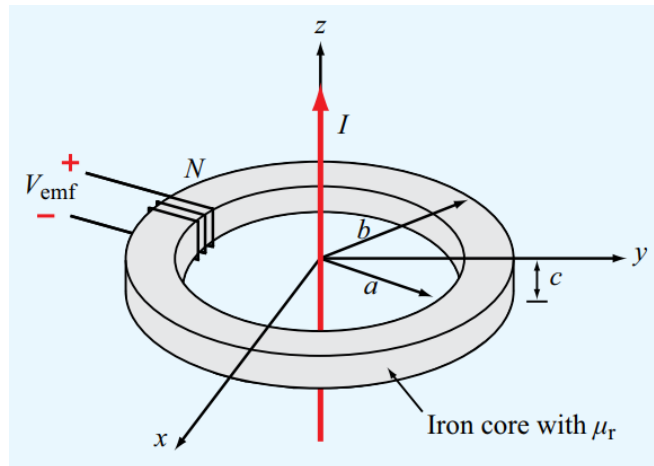


Figure Q2

Table Q2

Parameter	Value
Frequency, f	60 Hz
Relative permeability, μ_r	4000
a	5 cm
b	6 cm
c	2 cm
Current amplitude, I_0	50 A

$$\Phi = \oint \vec{B} \cdot d\vec{s}$$

$$= \int_a^b \frac{\mu I}{2\pi r} \cdot c dr$$

$$= \frac{\mu c I}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$V_{emf} = -N \frac{d\Phi}{dt}$$

$$= \frac{\mu c N^2 I_0}{2\pi} \ln\left(\frac{b}{a}\right) \sin \omega t$$

(b) according to (a)

$$V_{emf} \approx 5.5 \sin 377t$$

3. In a Cartesian coordinate, a plane wave is polarized with its electric vector along z . The wave propagates along the y -axis. The electric field is given by $E_z(y,t) = E_0 e^{i(ky - \omega t)}$ Volts/metre. This wave is propagating in vacuum; its amplitude is $E_0 = 7$ V/m and its wavelength is $\lambda = 0.3$ meters.

- a) What is the frequency of the wave? (2 marks)
 b) How large is the magnetic field associated with this wave (2 marks) and in what direction is it oriented (2 marks)?
 c) What is the average rate at which energy is transported by this wave (per square metres)? (2 marks)

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{0.3 \text{ m}}$$

$$= 10^9 \text{ Hz}$$

$$b) B = \frac{E}{c} = \frac{1}{3} \times 10^{-8} E_0 = \frac{7}{3} \times 10^{-8} \text{ T}$$

it orients x -axis

$$c) I = \frac{1}{2} c \epsilon_0 E_0^2 \approx 0.065 \text{ W/m}^2$$

4. A coaxial cable that connects a radar receiver to its antenna is 30-m-long and it is lossless. The cable has a characteristic impedance $Z_0 = 50 \Omega$ and operates at 2 MHz. The cable is terminated at an unmatched antenna that has an impedance of $Z_L = 60 + j40 \Omega$. If the phase velocity on this transmission line is $u = 0.6c$ (where c is the speed of light in vacuum), find:
- d) The complex reflection coefficient Γ at the cable and antenna interface; **(2 marks)**
 - e) The voltage standing wave ratio; **(2 marks)**
 - f) The input impedance of this transmission line. **(1 marks)**

$$\text{d) } \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{10 + j40}{110 + j40} \approx 0.352 \angle 55.98^\circ$$

$$\text{e) } S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.352}{1 - 0.352} \approx 2.086$$

$$\text{f) } \beta = \frac{2\pi f}{u}$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \approx (23.973 + j1.352) \Omega$$