Course code and name:	B38EM Introduction to Electricity and Magnetism
Type of assessment:	**Individual**
Coursework Title:	**Take home Assignment 1**
Student Name:	MA XUNCHI
Student ID Number:	H00392669 2101210015

Declaration of authorship. By signing this form:

- **I declare** that the work I have submitted for individual assessment OR the work I have contributed to a group assessment, is entirely my own. I have NOT taken the ideas, writings or inventions of another person and used these as if they were my own. My submission or my contribution to a group submission is expressed in my own words. Any uses made within this work of the ideas, writings or inventions of others, or of any existing sources of information (books, journals, websites, etc.) are properly acknowledged and listed in the references and/or acknowledgements section.

- I confirm that I have read, understood and followed the University's Regulations on plagiarism as published on the University's website, and that I am aware of the penalties that I will face should I not adhere to the University Regulations.

- I confirm that I have read, understood and avoided the different types of plagiarism explained in the University guidance on Academic Integrity and Plagiarism

Student Signature: MA XUNCHI

Date: 03-18-2023

Copy this page and insert it into your coursework file in front of your title page.
For group assessment each group member must sign a separate form and all forms must be included with the group submission.

Your work will not be marked if a signed copy of this form is not included with your submission.

Introduction to Electricity and Magnetism B38EM

Assignment 1

Name: MA XUNCHI HWU ID: H00392669 .

$\varepsilon_0 = 8.85\text{x}10^{-12}\ \text{Fm}^{-1}$, $e = 1.6\text{x}10^{-19}\ \text{C}$, $1\ \text{nC} = 10^{-9}\ \text{C}$

1) Gradient of scalar function: Find the gradient of the following scalar function and then evaluate it at the given point.

$V_1 = 24V_0 \cos(\pi y/3) \sin(2\pi z/3)$ at (3, 2, 1) in Cartesian coordinates.

Solution:

$\nabla V_1 = (0, -8\pi V_0 \sin(\frac{4\pi}{3}) \sin(\frac{2\pi}{3}), 16\pi V_0 \cos(\frac{4\pi}{3}) \cos(\frac{2\pi}{3}))$

(3 marks)

$= (0, -6\pi V_0, 4\pi V_0)$

2) Calculating the Divergence: Determine the divergence of the following vector field and then evaluate it at the indicated point:

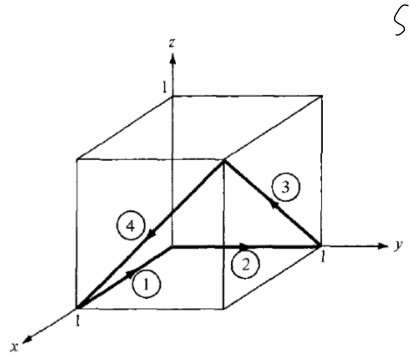
$\mathbf{E} = \hat{\mathbf{x}}\, 3x^2 + \hat{\mathbf{y}}\, 2z + \hat{\mathbf{z}}\, x^2 z$ at (2, −2, 0) in Cartesian coordinates.

Solution:

$\text{div}\, E = \frac{\partial E}{\partial x} + \frac{\partial E}{\partial y} + \frac{\partial E}{\partial z} = 6x + 0 + x^2 = 12 + 4 = 16$

(3 marks)

3) Circulation: Given that $\mathbf{F} = x^2\mathbf{a}_x - xz\mathbf{a}_y - y^2\mathbf{a}_z$, calculate the circulation of **F** around the (closed) path shown in the Figure.



Solution: $\oint F.dl = (\int_{①} + \int_{②} + \int_{③} + \int_{④}) F.dl$

$= \int_0^1 x^2 dx + 0 + \int_0^1 (x^2 - 1) dx + \int_0^1 (-y - y^2) dy$

$= -\frac{1}{3} + 0 - \frac{2}{3} + \frac{5}{6}$

$= -\frac{1}{6}$

(3 marks)

4) In a certain region, the electric flux density is given by:

$\mathbf{D} = 2\rho(z+1)\cos\varphi\ \mathbf{a}_\rho - \rho(z+1)\sin\varphi\ \mathbf{a}_\varphi + \rho^2 \cos\varphi\ \mathbf{a}_z$ $\mu\text{C/m}^2$.

a) Show that the charge density is equal to $\rho_v = 3(z+1)\cos\varphi$ $\mu\text{C/m}^2$

b) Calculate the total charge enclosed by the volume $0<\rho<2$, $0<\varphi<\pi/2$, $0<z<4$.

Solution: a) $\rho_v = \nabla \cdot D = \nabla \left(\frac{\partial(2\rho^2(z+1)\cos\varphi)}{\rho\,\partial\rho} + \frac{\partial(-\rho(z+1)\sin\varphi)}{\rho\,\partial\varphi} + \frac{\partial(\rho^2\cos\varphi)}{\partial z} \right)$

(5 marks)

$= 4(z+1)\cos\varphi - (z+1)\cos\varphi + 0 = 3(z+1)\cos\varphi\ \mu C/m^2$

b) $Q = \int_0^4 \int_0^{\frac{\pi}{2}} \int_0^2 3(z+1)\cos\varphi\, d\rho\, d\varphi\, dz$

$= \int_0^4 \int_0^{\frac{\pi}{2}} 6(z+1)\cos\varphi\, d\varphi\, dz = \int_0^4 6(z+1)\, dz = 72\mu C$

5) A spherical shell with outer radius *b* surrounds a charge-free cavity of radius $a < b$ (Fig. 2). If the shell contains a charge density given by

$$\rho_V = -\frac{\rho_{V0}}{R^2}, \quad a \le R \le b,$$

where ρ_{V0} is a positive constant, determine the electric flux density D in all regions.

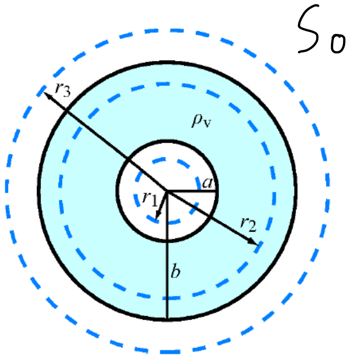


Fig. 2 for problem 5.

Solution: Since it is a sphrical shell according to Gauss's law

$$DS = Q \Rightarrow D = \frac{Q}{4\pi r^2}$$

1) $r < a$, $Q = 0 \Rightarrow D = 0$

2) $a < r < b$, $Q = \int_{V_a}^{V_{r(r<b)}} \rho_V dv = \int_a^{r(r<b)} -\frac{\rho_{V0}}{R^2} 4\pi R^2 dR = -4\pi \rho_{V0}(r-a)$

$$D = -\frac{\rho_{V0}(r-a)}{R^2}$$

3) $r > b$, all q is within, Let $r = b$

$$D = -\frac{\rho_{V0}(b-a)}{R^2}$$

(5 marks)

6) Spherical Capacitor: Show that the capacitance of two concentric spherical metal shells with radii *a* and *b* is equal to $C = 4\pi\varepsilon_0[ab/(b-a)]$. Assume the inner shell has charge +Q and the outer -Q.

(3 marks)

Solution: $C = \frac{Q}{\Delta V}$ $\left(\Delta V = \int_a^b \vec{E} dr = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2} \hat{r}\cdot\hat{r} dr\right.$

$\left.\oint_S \vec{E}\cdot\hat{n} dA = E(4\pi r^2) = \frac{Q}{\varepsilon_0} \Rightarrow \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}\right)$ $= \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

7) Consider a straight non-magnetic conductor of circular cross-section and radius *a* carrying a current with uniform current density **J** (A/m^2) in the vertical direction. Using Ampere's law find the magnetic field inside and outside the conductor.

(3 marks)

Solution: 1) inside the conductor:

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 I$$

$$\oint_C \vec{B}\, d\vec{s} = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$I = \pi r^2 J$$

$$B = \frac{\mu_0 \pi r^2 J}{2\pi r} = \frac{\mu_0 r J}{2}$$

2) outside the conductor

$$I = \pi a^2 J$$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\pi a^2 J}{2\pi r} = \frac{a^2 J}{2r}$$