Course code and name:	B38EM Introduction to Electricity and Magnetism
Type of assessment:	**Individual**
Coursework Title:	**Take home Assignment 2**
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Introduction to Electricity and Magnetism B38EM
Assignment #2

$\varepsilon_0 = 8.85\text{x}10^{-12}\ \text{Fm}^{-1}$, $e = 1.6\text{x}10^{-19}\ \text{C}$, $\mu_0 = 4\pi\ 10^{-7}\ \text{N/A}^2$

1. Consider the system shown in **Figure Q1** where a metal bar of mass *m* slides frictionless on two parallel conducting rails separated by a distance *l*. A resistor *R* is connected across the rails. If a uniform magnetic field B, pointing into the page, fills the entire region, find the following:

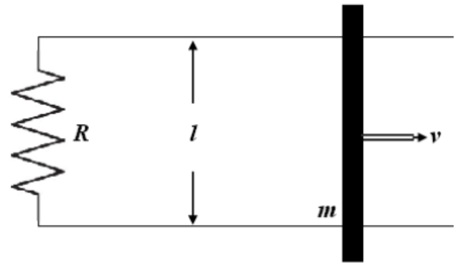

Figure Q1

(a) If the bar moves to the right at speed *v*, what is the current in the resistor? In which direction does it flow? *(3 marks)*

(b) Determine the magnetic force. What is the direction of the force? *(3 marks)*

(c) If the bar starts out with speed v_0 at t=0, and it is left to slide, what is its speed at a later t? *(4 marks)*

(d) If the initial kinetic energy of the bar was $\frac{1}{2}mv_0^2$. Check that the energy delivered to the resistor is exactly that. (Hint: First find the power delivered to the resistor) *(3 marks)*

1. (a) $I = \frac{U}{R} = \frac{Bvl}{R}$

(b) $F = BIl = \frac{B^2l^2v}{R}$

The current in the conductor goes upwards, So the direction of magnetic force is left

(c) $F = -\frac{B^2l^2v}{R}$

$m\frac{dv}{dt} = -\frac{B^2l^2v}{R}$

$\int_{v_0}^{v}\frac{dv}{v} = -\int_0^t \frac{B^2I^2}{Rm}dt \Rightarrow v = v_0 e^{-\frac{B^2I^2}{Rm}t}$

(d) $W = \int_0^\infty \left(\frac{BIv}{R}\right)^2 R\,dt$

Since $v = v_0 e^{-\frac{B^2l^2v}{Rm}t}$

$W = \left[-\frac{(BIv_0)^2}{R} \times \frac{Rm}{2B^2I^2} e^{-\frac{2B^2I^2v_t}{Rm}t}\right]_0^\infty$

$= v_0^2 \times \frac{m}{2} = \frac{1}{2}mv_0^2$

Proved

2. The transformer shown in **Figure Q2** consists of a long wire coincident with the z-axis carrying a current $I = I_0 \cos \omega t$, coupling magnetic energy to a toroidal coil situated in the x–y plane and centred at the origin. The toroidal core uses iron material with relative permeability, μ_r, around which 100 turns of a tightly wound coil serves to induce a voltage V_{emf}, as shown in the figure.
   (a) Develop an expression for V_{emf} *(3 marks)*
   (b) Calculate V_{emf}, for the parameters in **Table Q2** *(2 marks)*

2.
(a)

$$\Phi = \oint \vec{B}\cdot ds$$
$$= \int_a^b \frac{\mu I}{2\pi r}\cdot c\,dr$$
$$= \frac{\mu c I}{2\pi}\ln\left(\frac{b}{a}\right)$$
$$V_{emf} = -N\frac{d\Phi}{dt}$$
$$= \frac{\mu c N w I_0}{2\pi}\ln\left(\frac{b}{a}\right)\sin wt$$

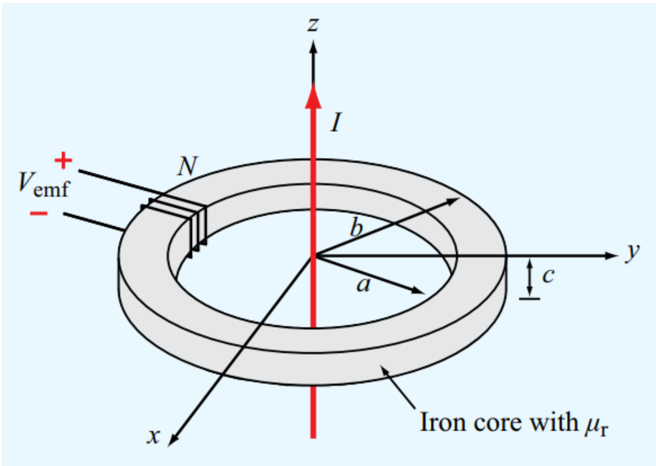

Figure Q2

Table Q2

Parameter	Value
Frequency, f	60 Hz
Relative permeability, μ_r	4000
a	5 cm
b	6 cm
c	2 cm
Current amplitude, I_0	50 A

(b) according to (a)

$$V_{emf} \approx 5.5 \sin 377t$$

3. In a Cartesian coordinate, a plane wave is polarized with its electric vector along z. The wave propagates along the y-axis. The electric field is given by $E_z(y,t) = E_0 e^{j(ky-\omega t)}$ Volts/metre. This wave is propagating in vacuum; its amplitude is $E_0 = 7$ V/m and its wavelength is $\lambda = 0.3$ meters.
a) What is the frequency of the wave? **(*2 marks*)**
b) How large is the magnetic field associated with this wave **(*2 marks*)** and in what direction is it oriented **(*2 marks*)**?
c) What is the average rate at which energy is transported by this wave (per square metres)? **(*2 marks*)**

3.
(a) $f = \frac{c}{\lambda} = \frac{3\times10^8 m/s}{0.3m} = 10^9$ Hz

(b) $B = \frac{E}{c} = \frac{1}{3}\times10^{-8}E_0 = \frac{7}{3}\times10^{-8}$ T
$H = \frac{B}{\mu_0} \approx 0.0186$ A/m
it orients x-axis

(c) $I = \frac{1}{2}c\varepsilon_0 E_0^2 \approx 0.065$ W/m^3

4. A coaxial cable that connects a radar receiver to its antenna is 30-m-long and it is lossless. The cable has a characteristic impedance $Z_0 = 50\ \Omega$ and operates at 2 MHz. The cable is terminated at an unmatched antenna that has an impedance of $Z_L = 60 + j40\ \Omega$. If the phase velocity on this transmission line is $u = 0.6c$ (where c is the speed of light in vacuum), find:

d) The complex reflection coefficient Γ at the cable and antenna interface; **(*2 marks*)**

e) The voltage standing wave ratio; **(*2 marks*)**

f) The input impedance of this transmission line. **(*1 marks*)**

4.(d) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{10+j40}{110+j40} \approx 0.352\ \angle 55.98^\circ$

(e) $S = \dfrac{1+|\Gamma|}{1-|\Gamma|} = \dfrac{1+0.352}{1-0.352} \approx 2.086$

$\beta = \dfrac{2\pi f}{u}$

(f) $Z_{in} = Z_0 \dfrac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \approx (23.973 + j1.352)\ \Omega$