

Engineering Mathematics and Statistics (B39AX) Fall 2023

Tutorial 11

Problem A.

Let X and Y be two discrete RVs (memoryless).

- 1) Prove that the mutual information satisfies $I(X; Y) \geq 0$.
- 2) Give an example where $I(X; Y) = 0$.
- 3) In the case of transmission through a noiseless channel, find the expression of the mutual information $I(X; Y)$ as a function of $H(X)$. If the source X is associated with an alphabet of N symbols, what is the capacity of the channel?

Problem B

Consider a transmission scenario through a standard BSC channel with probability of error given by f . Let p be such that $\mathbb{P}(X = 0) = p$.

- 1) Compute $H(Y|X)$ and the mutual information $I(Y; X)$
- 2) Show that the capacity of the channel is

$$C_s = 1 + f \log_2(f) + (1 - f) \log_2(1 - f)$$

And for which value of p it is obtained.

Problem C

Consider a transmission scenario through a (binary) Z channel with probabilities given by

$$\begin{aligned} \mathbb{P}(Y = 0|X = 0) &= 1, \mathbb{P}(Y = 1|X = 1) = 1 - f \\ \mathbb{P}(Y = 1|X = 0) &= 0, \mathbb{P}(Y = 0|X = 1) = f \end{aligned}$$

Let p be such that $\mathbb{P}(X = 0) = p$.

- 1) Compute $H(X|Y)$ and the mutual information $I(X; Y)$
For which value of p does the transmitted signal reach the channel capacity?
- 2) Compute p for $f = 0.01, 0.1$ and 0.4 .