Engineering Mathematics and Statistics (B39AX) Fall 2023

Tutorial 9

Problem A.

Consider a binary source X defined on the alphabet $\mathcal{A} = \{0,1\}$, such that $\mathbb{p}(X = 0) = p$, $\mathbb{p}(X = 1) = 1 - p$ with $p = 0.5$.

We want to transmit messages from this source through a binary symmetric channel with probability of error $f = 0.1$ using a code with N repetitions. Let Y denote the sum of the N binary values measured at the receiver, where N is an odd number.

1) What is the expression of the PMFs (likelihoods) $\mathbb{p}(Y|X = 0)$ and $\mathbb{p}(Y|X = 1)$?

2) What it the PMF of the marginal distribution (evidence) $\mathbb{p}(Y)$?

3) What is (in the Bayesian sense) the best decision criterion to estimate X from the N binary values measured at the receiver? Compute the value of $\hat{X}_{MAP}$ for the different values of Y.

4) Using this criterion, compute the probability of error of the R_N coding/decoding strategy.

Problem B.

Consider the discrete random variable (or source) X defined on $\mathcal{A}_X = \{1,2,3,4,5,6\}$ with probabilities $\mathbb{p}(X = 1) = 1/6$, $\mathbb{p}(X = 2) = 1/12$, $\mathbb{p}(X = 3) = 1/12$, $\mathbb{p}(X = 4) = 2p/3$, $\mathbb{p}(X = 5) = (1 - p)/3$,$\mathbb{p}(X = 6) = (1 - p)/3$.

1) Compute the information content of the outcome $X = 2$.

2) Compute the entropy $H(X)$ for $p = 2/3$.

3) Compute the value of p for which $H(X)$ is maximized?

4) For $p = 1/3$ and assuming that the source X as a rate of 120 symbol/s, what is the information rate of the source?