Engineering Mathematics and Statistics (B39AX)
Fall 2023

Tutorial 3

Problem A. A factory produces resistors of $1000\,\Omega$, and the quality control department specifies a tolerance of 10%. Assume that the resistance of each resistor is modeled as a Gaussian random variable with mean $\mu = 1000\,\Omega$ and standard deviation $\sigma = 40\,\Omega$.

(a) What fraction of resistors do you expect to be rejected?

(b) Suppose that a faulty machine produces resistors with $\mu = 1030\,\Omega$, but with the previous standard deviation. What is now the fraction to be rejected?

Problem B. The lifetime of a high performance electrical motor is expressed in weeks as a Rayleigh random variable, which has the following pdf

$$f_X(x) = \begin{cases} \dfrac{x}{\beta} \exp\Big(-\dfrac{x^2}{2\beta}\Big) & x \geq 0 \\ 0 & x < 0\,, \end{cases}$$

with $\beta = 200$. What is the probability that the motor will fail within the first week?

Problem C. A binary message is transmitted as a signal s, which is either -1 or $+1$. The communication channel corrupts the transmission with additive Gaussian noise with mean $\mu = 0$ and variance $\sigma^2 = 4$. The receiver concludes that the signal -1 (or $+1$) was transmitted if the value received is < 0 (or > 0, respectively). If the probability of transmitting a $+1$ is 0.7, and the probability of transmitting a -1 is 0.3, what is the probability of error?