Engineering Mathematics and Statistics (B39AX) Fall 2023

Tutorial 10

Problem A.

Consider the discrete source X defined on $\mathcal{A}_X = \{a, b, c, d, e, f, g, h, i, j, k, l\}$ with probabilities

$$\begin{cases} \mathbb{p}(X = a) = 1/48 \\ \mathbb{p}(X = b) = 7/48 \\ \mathbb{p}(X = c) = 1/24 \\ \mathbb{p}(X = d) = 1/8 \\ \mathbb{p}(X = e) = 1/16 \\ \mathbb{p}(X = f) = 5/48 \\ \mathbb{p}(X = g) = 1/48 \\ \mathbb{p}(X = h) = 1/48 \\ \mathbb{p}(X = i) = 1/24 \\ \mathbb{p}(X = j) = 1/16 \\ \mathbb{p}(X = k) = 7/48 \\ \mathbb{p}(X = l) = 5/24 \end{cases}.$$

1) Compute the entropy $H(X)$.

2) Use Huffman coding to code the source X

3) What is the expected length of the code obtained?

4) Is the code obtained via Huffman coding a prefix code?

5) Is the code the only uniquely decodable code?

Problem B

1) Compute a Huffman code for X^2, where $\mathcal{A}_X = \{0,1\}$ with $\mathbb{p}(X = 0) = 0.9$. First compute $\mathcal{A}_{X^2}$ and the probability of each word in $\mathcal{A}_{X^2}$. Compute $H(X^2)$ and $L(X^2, C)$.

2) Assume that a sequence of symbols from the alphabet $\{1,2,3,4\}$ and compressed using the code below. Imagine picking one bit at random from the binary encoded sequence $c^{+} = c(x_1)c(x_2)$ .... What is the probability that this bit is a 0?

$\boldsymbol{a_i}$	1	2	3	4
$\boldsymbol{p_i}$	1/2	1/4	1/8	1/8
$\boldsymbol{c(a_i)}$	0	10	110	111
$\boldsymbol{l_i}$	1	2	3	3

Hint: you can define f_i as the fraction of bits equal to 0 in each codeword $\boldsymbol{c(a_i)}$**.**

3) Show that, if X and Y are two independent discrete RVs, the outcome x, y satisfies

$$H(X,Y) = H(X) + H(Y).$$