Engineering Mathematics and Statistics (B39AX)
Fall 2023

Tutorial 4

Problem A. The Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time and/or space. It is parameterized by $\lambda \geq 0$, and its probability mass function is

$$p(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k\,!},$$

for $k = 0, 1, 2, \ldots$.

(a) Show that the expected value of a random variable with Poisson distribution is λ.

(b) The number of people that arrive at a shop within a 10 minute period is Poisson distributed with mean $\lambda = 2$. What is the probability nobody arrives within a 15 minute period?

Problem B. Markov's inequality states that any nonnegative random variable $X \geq 0$ satisfies[1]

$$\mathbb{P}(X \geq c) \leq \frac{\mathbb{E}[X]}{c},$$

for any $c > 0$. Use it to show Chebyshev's inequality: for any random variable X and any $c > 0$,

$$\mathbb{P}\Big(\big|X - \mathbb{E}[X]\big| \geq c\Big) \leq \frac{\mathrm{Var}(X)}{c^2}.$$

Hint: Apply Markov's inequality to the random variable $Y = \big(X - \mathbb{E}[X]\big)^2$.

Problem C. We toss a fair coin 40 times and want to compute the probability that we get at least 30 heads. Use two methods:

a) The central limit theorem.

b) Chebyshev's inequality (from the previous problem).

[1]This can be shown by the following argument (for continuous random variables)

$$\mathbb{E}[X] = \int_0^{+\infty} x \cdot f_X(x)\,dx \geq \int_c^{+\infty} x \cdot f_X(x)\,dx \geq c \int_c^{+\infty} f_X(x)\,dx = c \cdot \mathbb{P}(X \geq c),$$

where the first inequality uses the fact that $x \cdot f_X(x) \geq 0$ for $x \geq 0$, and the second one the fact that $x \cdot f_X(x) \geq c \cdot f_X(x)$ for all $x \geq c$.