Engineering Mathematics and Statistics (B39AX)
Fall 2023

Tutorial 1

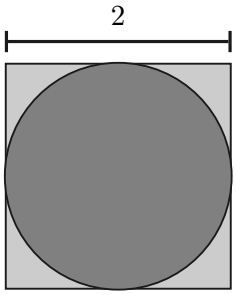
Problem A. A die is weighted so that the probability of each face is proportional to the number that it contains. For example, 6 is twice as likely to occur as 3.

(a) Describe the sample space and find the probability of each outcome.

(b) What is the probability of obtaining an even number? And what is the probability of obtaining a prime number?

(c) What is the probability of obtaining a number larger than or equal to 3?

(d) What is the probability of obtaining 1? Is there an alternative way to obtain this result using the previous answers?



Problem B. A square of side 2 has a circle perfectly inscribed, as shown in the figure. We throw a dart, which lands at any point inside the square with equal probability. What is the probability that it lands outside the circle?

Problem C. Let A and B be events with probabilities 3/4 and 1/3, respectively.

(a) Show that the probability of $A \cap B$ is smaller than or equal to 1/3. Describe the situation in which the probability is equal to 1/3.

(b) Show that the probability of $A \cap B$ is larger than or equal to 1/12. Describe the situation in which the probability is equal to 1/12.

Problem D. Suppose I toss a fair coin three times. In each toss, let H denote heads and T denote tails.

(a) Describe the sample space and determine the size of the set of possible events.

(b) Let A be the event "obtain exactly two heads." Compute $\mathbb{P}(A)$.

(c) Let B be the event "obtain heads in the first toss." Is B independent from A?

Problem E. An urn contains 7 red and 3 white marbles. The marbles are taken out from the urn one at a time. What is the probability that the first two marbles are red and the third one white?