**Engineering Mathematics and Statistics (B39AX) Fall 2023**

**Tutorial 10**

**Problem A.**

Consider the discrete source defined on with probabilities

.

1. Compute the entropy .
2. Use Huffman coding to code the source

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | a | b | c | d | e | f | g | h | i | j | k | l |
|  | 1/48 | 7/48 | 1/24 | 1/8 | 1/16 | 5/48 | 1/48 | 1/48 | 1/24 | 1/16 | 7/48 | 5/24 |
|  | 00000 | 001 | 00010 | 100 | 0100 | 101 | 000010 | 000011 | 00011 | 0101 | 011 | 11 |
|  | 5 | 3 | 5 | 3 | 4 | 3 | 6 | 6 | 5 | 4 | 3 | 2 |

Other solutions possible.

1. What is the expected length of the code obtained?

bits /symbol

1. Is the code obtained via Huffman coding a prefix code?

Yes

1. Is the code the only uniquely decodable code?

No, the Huffman coding procedure can also produce multiple valid codes which are uniquely decodable.

**Problem B**

1. Compute a Huffman code for , where with . First compute and the probability of each word in . Compute and .

With probabilities , and .

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 00 | 01 | 10 | 11 |
|  | 0.81 | 0.09 | 0.09 | 0.01 |
|  | 0 | 10 | 110 | 111 |
|  | 1 | 2 | 3 | 3 |

Other solutions possible.

and

1. Assume that a sequence of symbols from the alphabet {} and compressed using the code below. Imagine picking one bit at random from the binary encoded sequence . What is the probability that this bit is a 0?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
|  | 1/2 | 1/4 | 1/8 | 1/8 |
|  | 0 | 10 | 110 | 111 |
|  | 1 | 2 | 3 | 3 |

Hint: you can define as the fraction of bits equal to 0 in each codeword **.**

Consider a string of codewords from the table above. The number of zeros in each codeword is , thus the expected number of zeros per symbol is

.

Moreover, the expected length of a codeword is

Finally, we obtain that the probability of choosing a 0 is

1. Show that, if and are two independent discrete RVs, the outcome , satisfies

If and are two independent, we have and .