**Engineering Mathematics and Statistics (B39AX) Fall 2023**

**Tutorial 11**

**Problem A.**

Let and be two discrete RVs (memoryless).

1. Prove that the mutual information satisfies .

measures the amount of surprised contained in . Knowing can reduce the surprise contained in but the amount of surprised contained in cannot be larger than the amount of surprise in , i.e., . This leads to .

1. Give an example where .

when , i.e., when knowing does not provide additional information about . This is for instance the case if and are independent. In that case and .

Rq: Another (more specific) example is when one of the two variables (say ) is no longer random, but takes a specific value with a probability equal to 1. If , .

1. In the case of transmission through a noiseless channel, find the expression of the mutual information as a function of . If the source is associated with an alphabet of symbols, what is the capacity of the channel?

If the channel is noiseless (and thus lossless), b and thus , which leads to .

The capacity of the channel is obtained by maximizing , i.e., . The entropy of the source is maximizing when the symbols are uniformly distributed and in that case, b/symbols.

**Problem B**

Consider a transmission scenario through a standard BSC channel with probability of error given by . Let be such that .

1. Compute and the mutual information

which does not depend on .

1. Show that the capacity of the channel is

And for which value of it is obtained.

By differentiating with respect to we find , for which and

**Problem C**

Consider a transmission scenario through a (binary) Z channel with probabilities given by

Let be such that .

1. Compute and the mutual information
2. For which value of does the transmitted signal reach the channel capacity?

By differentiating with respect to , we find

With

1. Compute for and .