**Engineering Mathematics and Statistics (B39AX) Fall 2023**

**Tutorial 9**

**Problem A.**

Consider a binary source defined on the alphabet , such that , with .

We want to transmit messages from this source through a binary symmetric channel with probability of error using a code with repetitions. Let denote the sum of the binary values measured at the receiver, where is an odd number.

1. What is the expression of the PMFs (likelihoods) and ?

If , corresponds to the number of times where the wrong symbol (i.e, 1) has been received, out of repetitions.

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If , corresponds to the number of times where the correct symbol (i.e, 1) has been received, out of repetitions.

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1. What it the PMF of the marginal distribution (evidence) ?

1. What is (in the Bayesian sense) the best decision criterion to estimate from the binary values measured at the receiver? Compute the value of for the different values of .

Using the Bayes rule, we have

The optimal decision criterion is the MAP decision criterion given by

Thus, we decide if

Or

, i.e.,

Since , it means or

Thus we decide if and otherwise.

1. Using this criterion, compute the probability of error of the coding/decoding strategy.

The probability of error is given by

**Problem B.**

Consider the discrete random variable (or source) defined on with probabilities , , , , ,

1. Compute the information content of the outcome .
2. Compute the entropy for ,
3. Compute the value of for which is maximized?

By differentiating with respect to , and finding the value of for which the gradient is 0, we obtain

1. For and assuming that the source as a rate of 120 symbol/s, what is the information rate of the source?

For , we obtain b/symbol so the information rate is b/s