B39HF High Frequency Circuits

Chapter 2 Transmission Lines

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Chapter 2 Outline

- Transmission lines
  - General Consideration
  - Lumped-Element Model
  - Transmission Lines Equations
  - Wave Propagation of Transmission Lines
- Lossless Transmission lines
  - General Consideration
  - Wave Impedance of the Lossless Line
  - Special Cases of the Lossless Line
  - Power Flow on a Lossless Transmission Line

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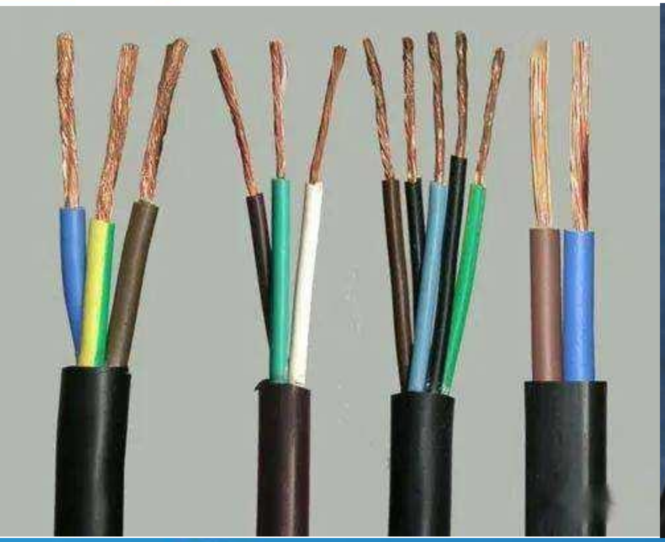
  - Wave Propagation of Transmission Lines

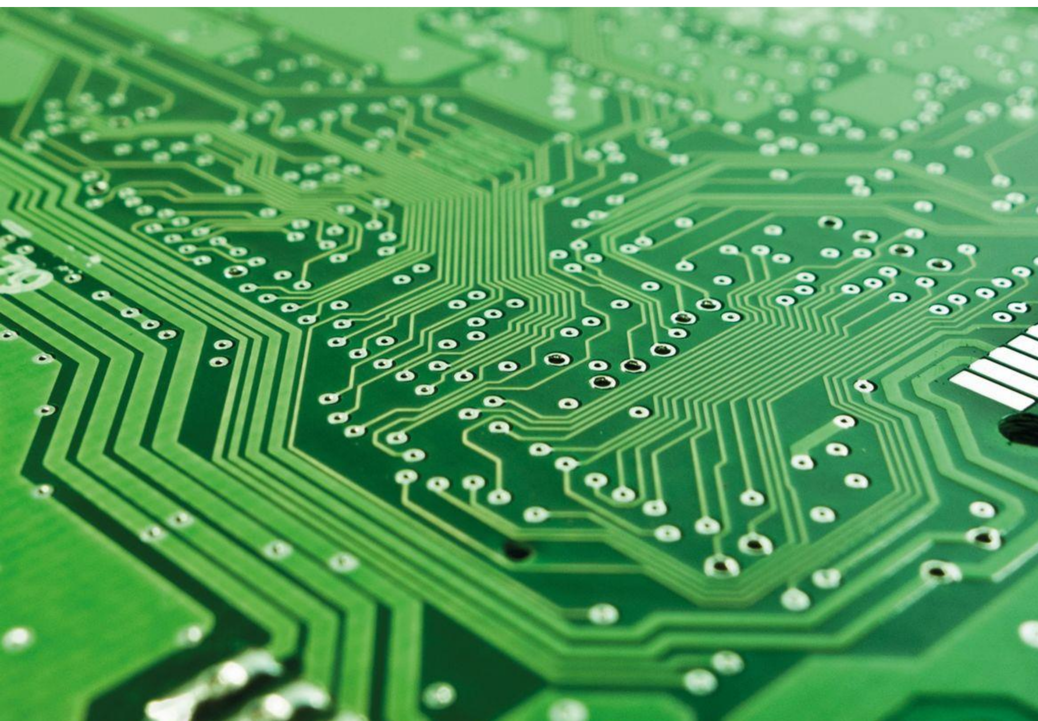
Transmission Line Motivation

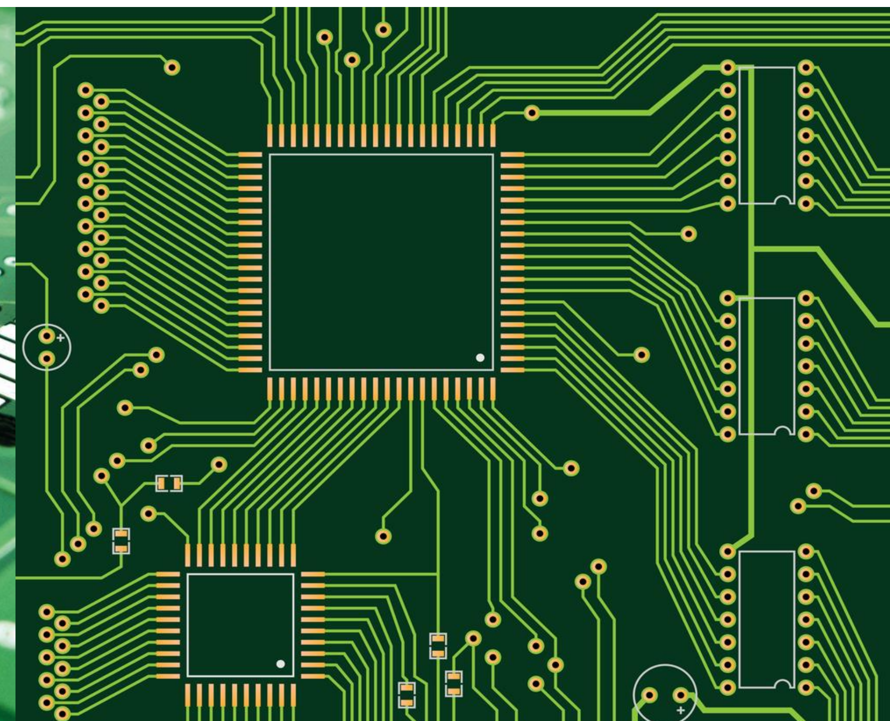


Transmission Line Motivation

When the length of the line is much smaller than the wavelength, the effects caused by the line in the electric/magnetic fields is negligible.

- Circuit theory can be applied (KCL, KVL,…)





Transmission Line Motivation

Why Transmission Line Theory？

Transmission Line Motivation

Circuit Theory: theory of lumped elements (R, C, L), conducting wires play no role (space-independent v, i), simple, approximate.

EM Theory : A full vector analysis based on Maxwell's equations, is most complete, accurate, however, too complex .

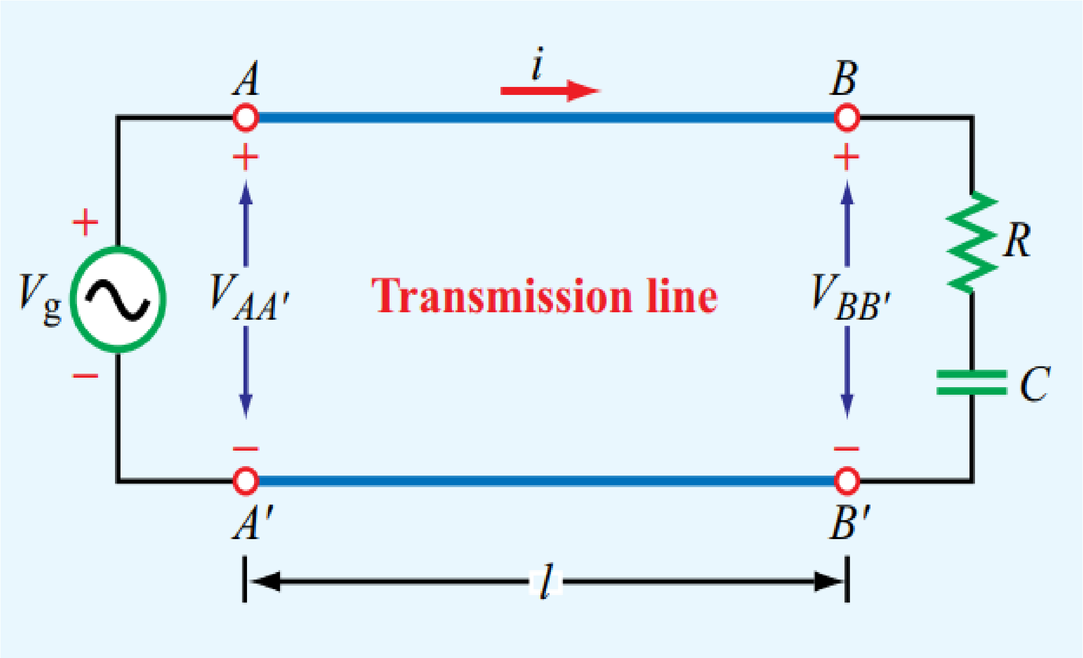
Circuit Theory → Transmission Line Theory → Electromagnetic Theory

$i(t)v(t)$ $\qquad$ $i(z,t)v(z,t)$ $\qquad$ $E(z,t)H(z,t)$

Transmission line Theory : bridge circuit theory and EM theory, theory of distributed circuits, using the EM theory to derive the distributed parameters, dealing with them by using circuit theory.

The Role of Wavelength

A simple circuit, elements usually are interconnected using a pair of wires. Generator is connected to a RC load via a pair of wires.



Is the pair of wires between AA' and BB' a transmission line?

The factors that determine whether or not we should treat the wires as a transmission line, are governed by the length of the line l and the frequency f of the signal provided by the generator.

The Role of Wavelength

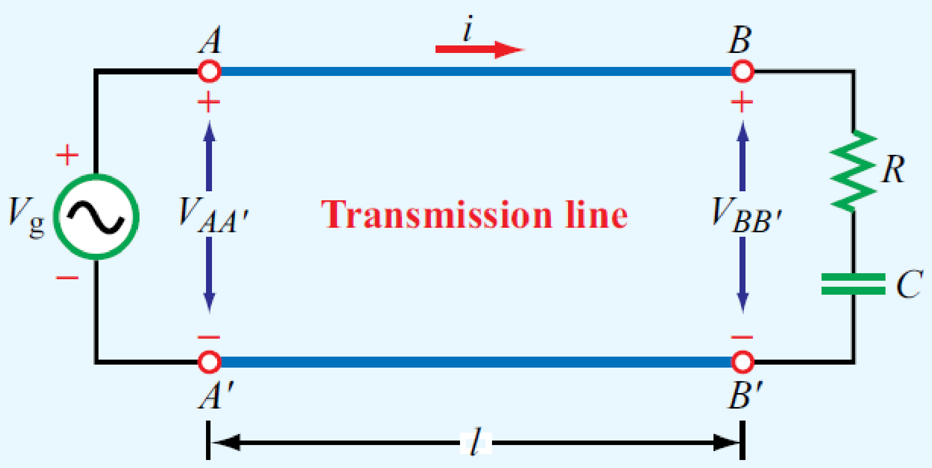


Figure 2-2 Generator connected to an RC circuit through a transmission line of length l.

When l/λ is very small, line effects may be ignored, but when ***$l/\lambda \geq 0.01$***, it may be necessary to account not only for the phase shift due to the time delay, but also for the presence of **reflected** signals that may have been bounced back by the load toward the generator.

$$V_{AA'} = V_g(t) = V_0 \cos \omega t \;\; (\text{V})$$

Voltage across BB' is delayed in time relative to that across AA' ,by the travel delay-time l/c ,

$$V_{BB'} = V_{AA'}(t - l/c) = V_0 \cos[\omega(t - l/c)]$$
$$= V_0 \cos(\omega t - \phi_0) \;\; (\text{V})$$

Where $\phi_0 = \dfrac{\omega l}{c}$

Due to $f\lambda = c \; (m/s)$

Hence,

$$\phi_0 = \frac{\omega l}{c} = \frac{2\pi f l}{c} = 2\pi \frac{l}{\lambda}$$

The Role of Wavelength

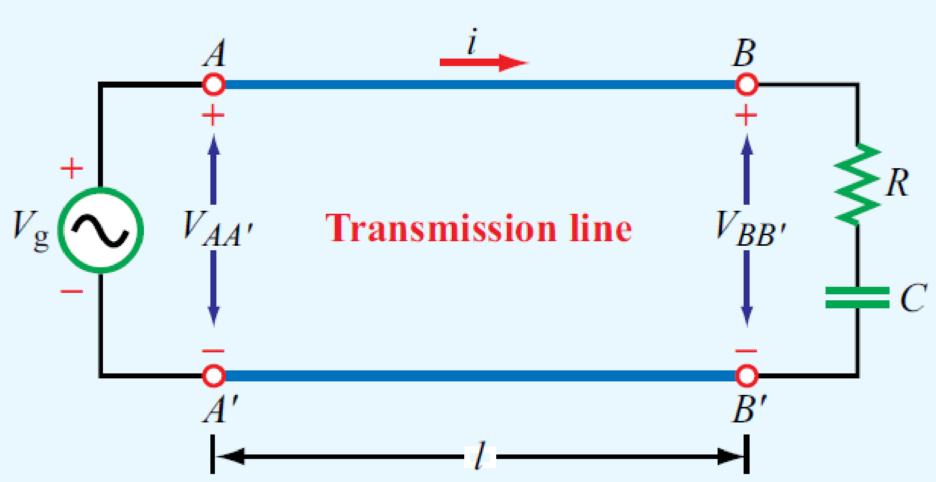


Figure 2-2 Generator connected to an RC circuit through a transmission line of length l.

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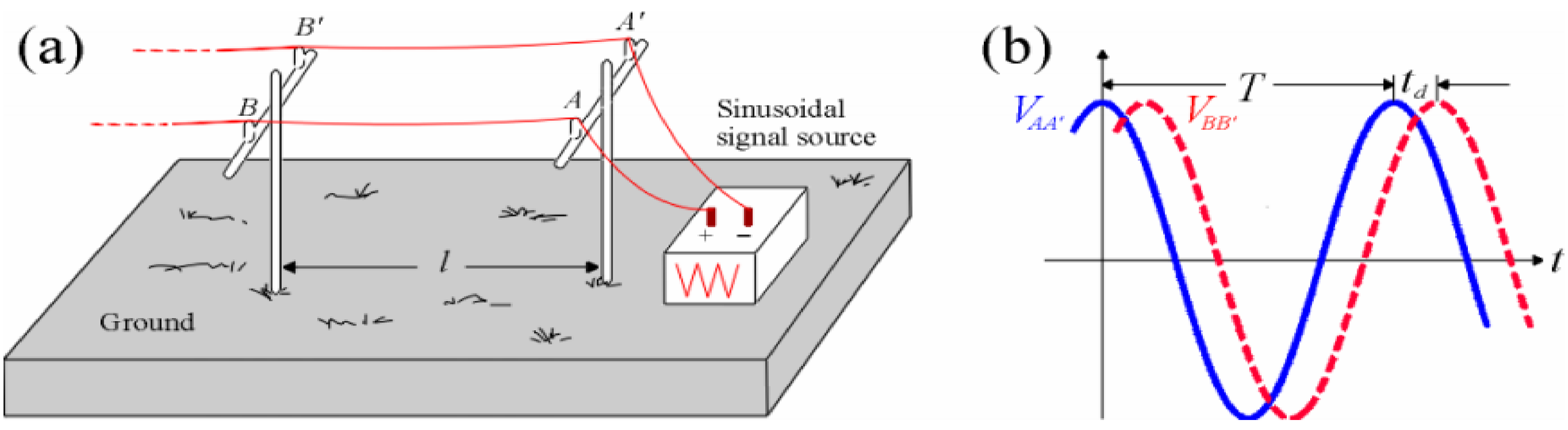


Fig. 2-1. (a) Schematic of the power line. (b) Definitions of oscillating period and delay.

Transmission Line Modeling

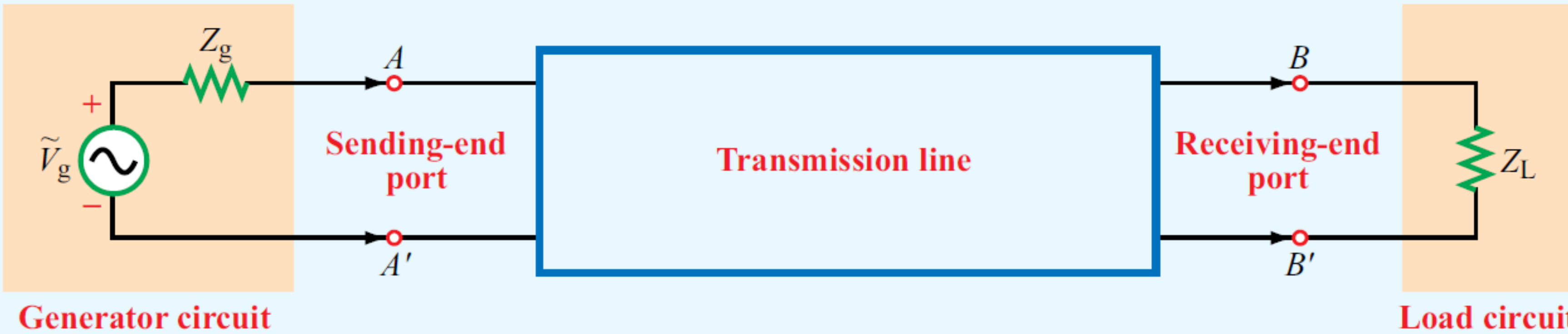


Figure 2-1 A transmission line is a two-port network connecting a generator circuit at the sending end to a load at the receiving end.

- Transmission lines (TLs) use two-port circuit theory for representation, the operating wavelength is proportional to the circuit size.

- By modelling TLs in the form of equivalent circuits at high frequencies (HFs), Kirchhoff’s voltage and current laws to develop wave equations.

- Allows characterization of standing waves, power transfer, and voltage and current along the TL.

The Role of Wavelength

- At HF, since the wavelength compared to the circuit, power loss and dispersion effects have to be considered
- TL with dispersion: the wave velocity is not constant as a function of frequency.
- The shape of a rectangular pulse is distorted, as it travels down the line
- Differ. frequency components not propagate at the same frequency.

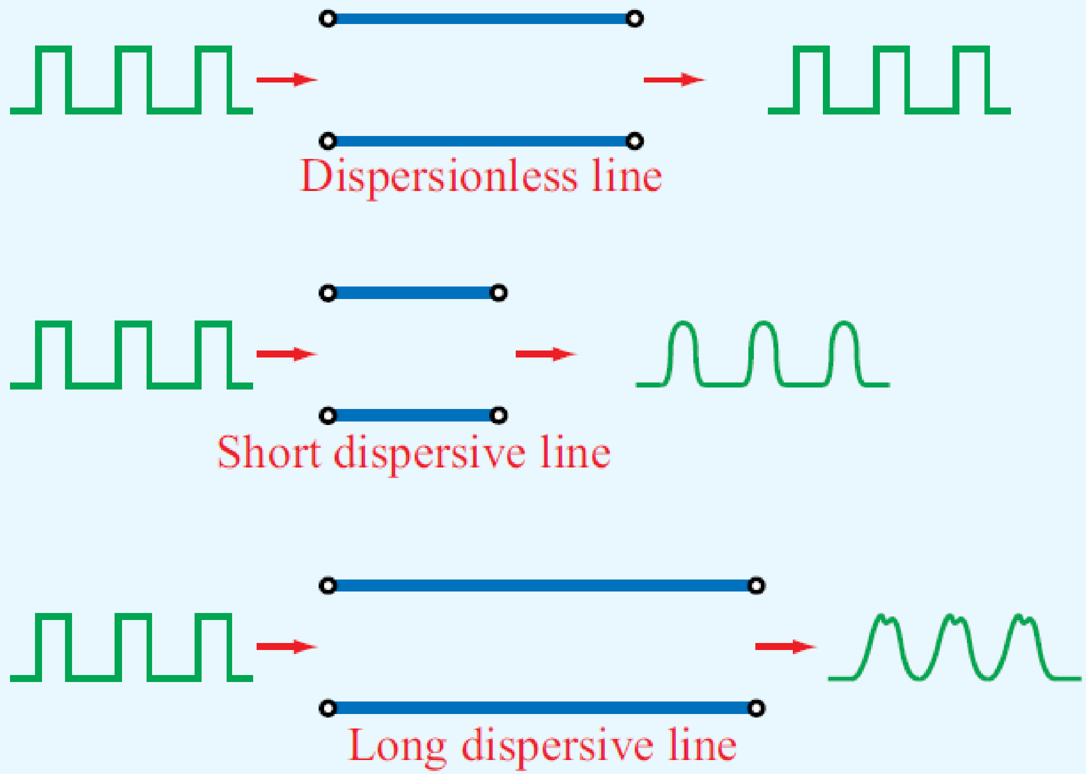


Figure 2-3 A dispersionless line does not distort signals passing through it regardless of its length, whereas a dispersive line distorts the shape of the input pulses because the different frequency components propagate at different velocities. The degree of distortion is proportional to the length of the dispersive line.

■ 傅里叶级数的复数表达形式:

利用欧拉公式:

$$e^{\pm jnw_0t} = \cos nw_0t \pm j\sin nw_0t$$

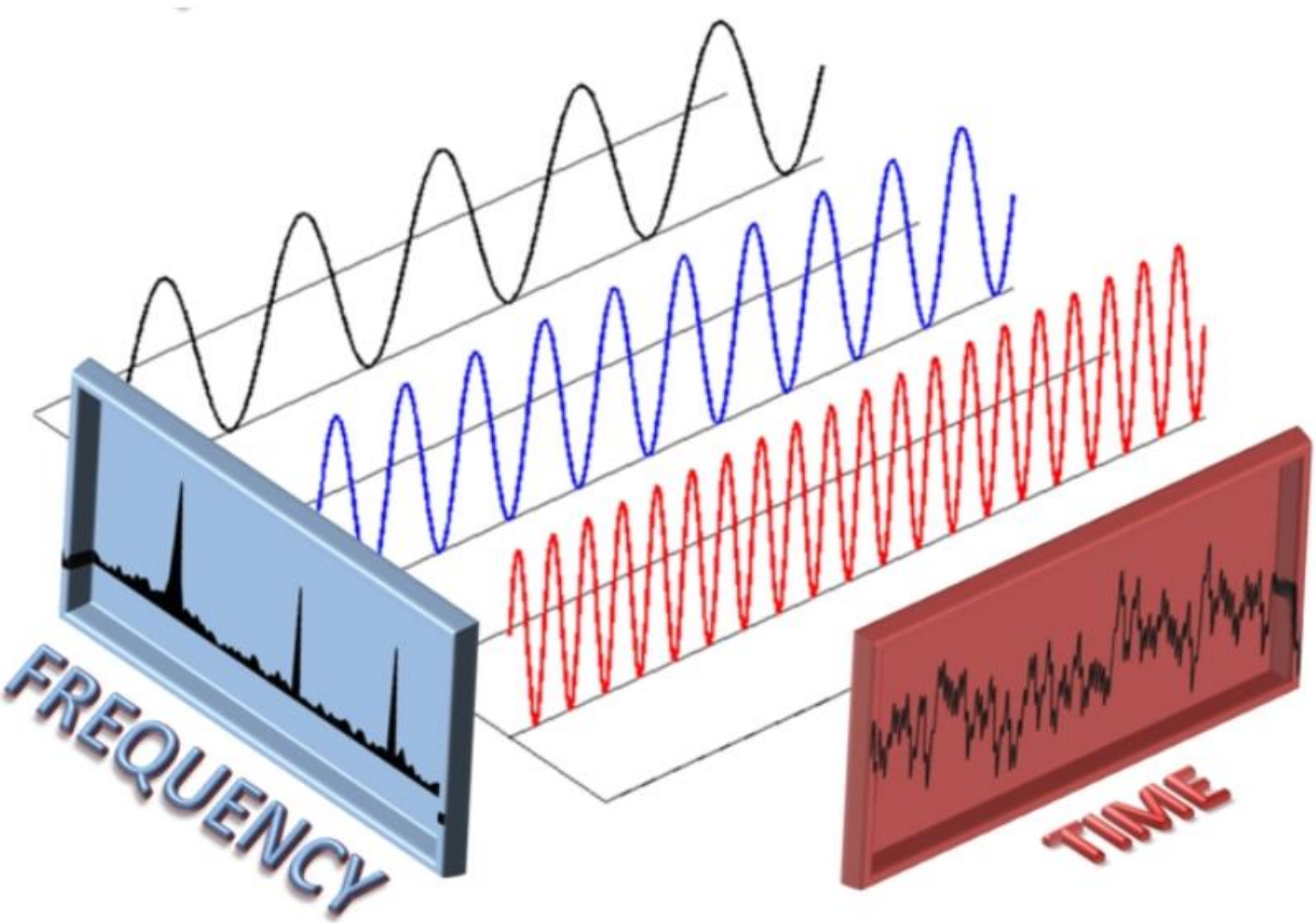
$$\cos nw_0t = \frac{1}{2}(e^{-jnw_0t} + e^{jnw_0t})$$

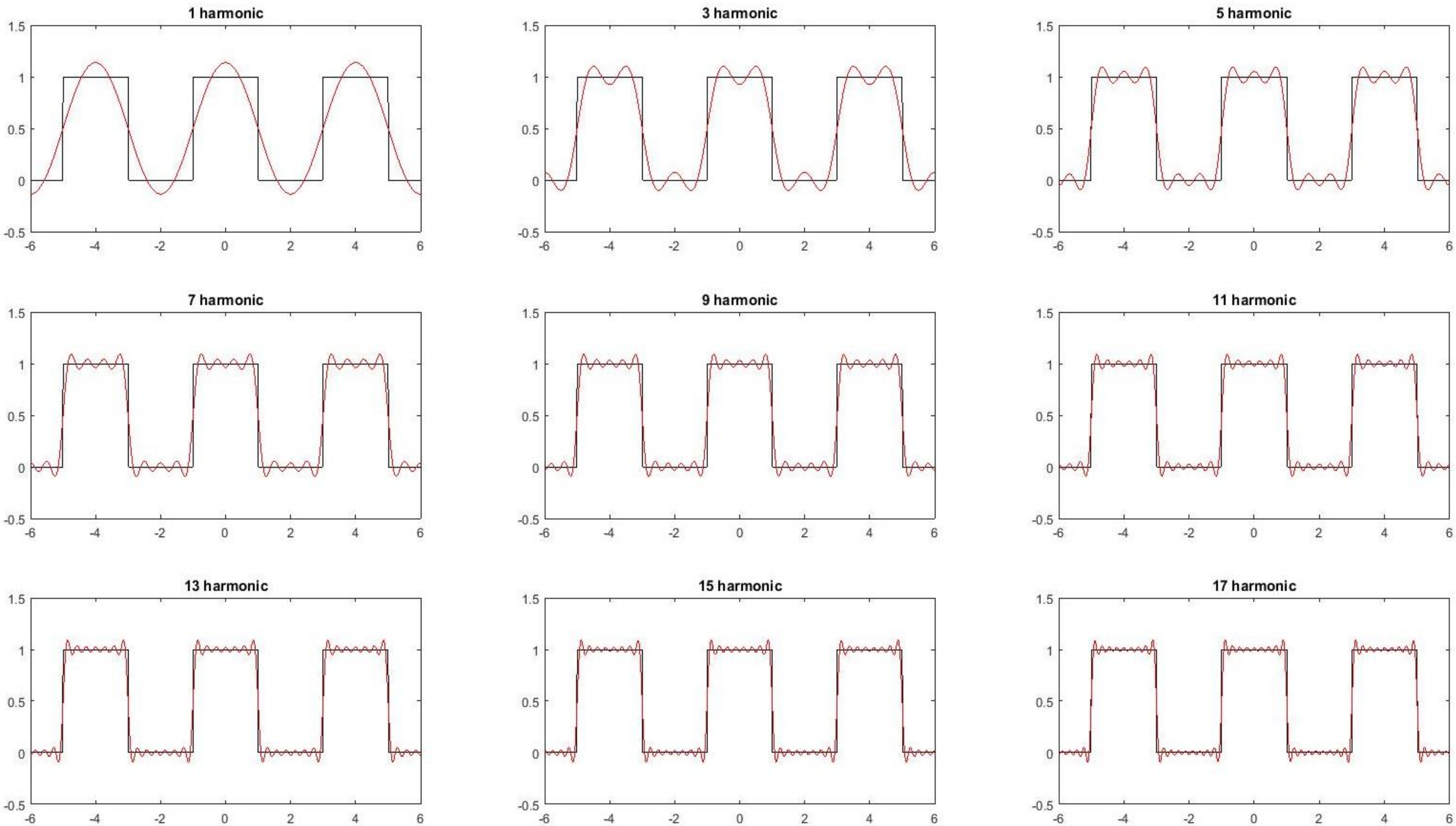
$$\sin nw_0t = \frac{j}{2}(e^{-jnw_0t} - e^{jnw_0t})$$

将:

$$x(t) = a_0 + \sum_{n=1}^{\infty}(a_n\cos n\omega_0 t + b_n\sin n\omega_0 t) \qquad (n = 1,2,,3,\ldots)$$

正余弦用欧拉公式代换　　带入并合并同类项


FREQUENCY
TIME


1 harmonic
3 harmonic
5 harmonic
7 harmonic
9 harmonic
11 harmonic
13 harmonic
15 harmonic
17 harmonic

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Propagation Modes

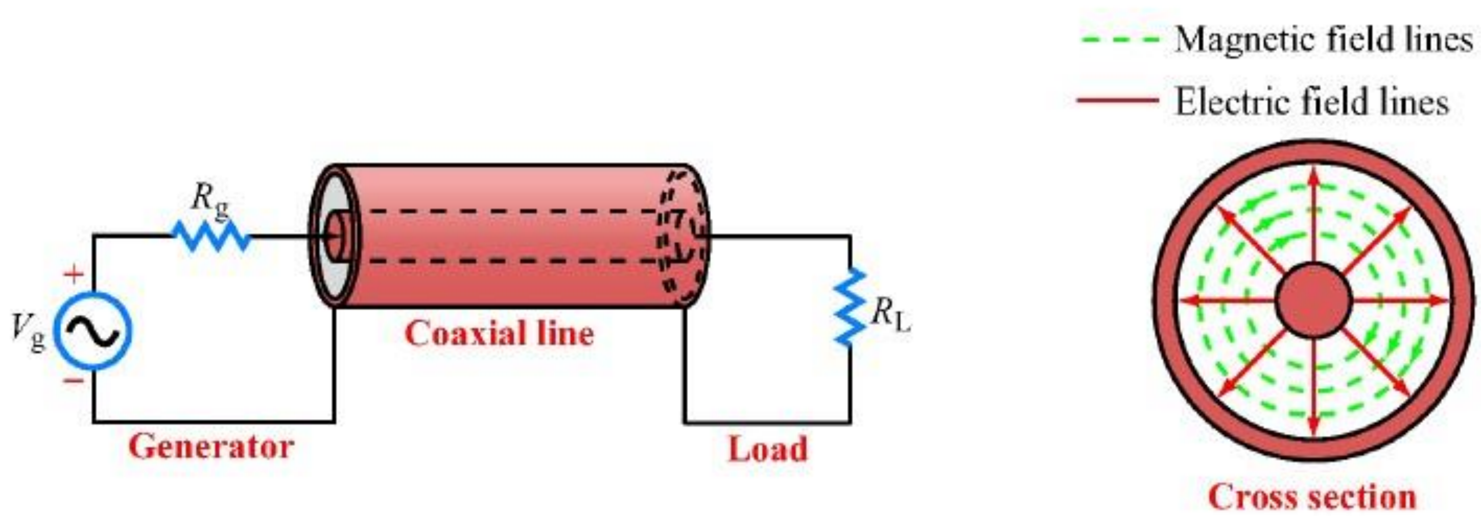
Propagation Modes

Transmission lines may be classified into two basic types:

- Transverse electromagnetic (TEM) transmission lines
- Higher-order transmission lines

Propagation Modes

Transverse electromagnetic (TEM) transmission lines:

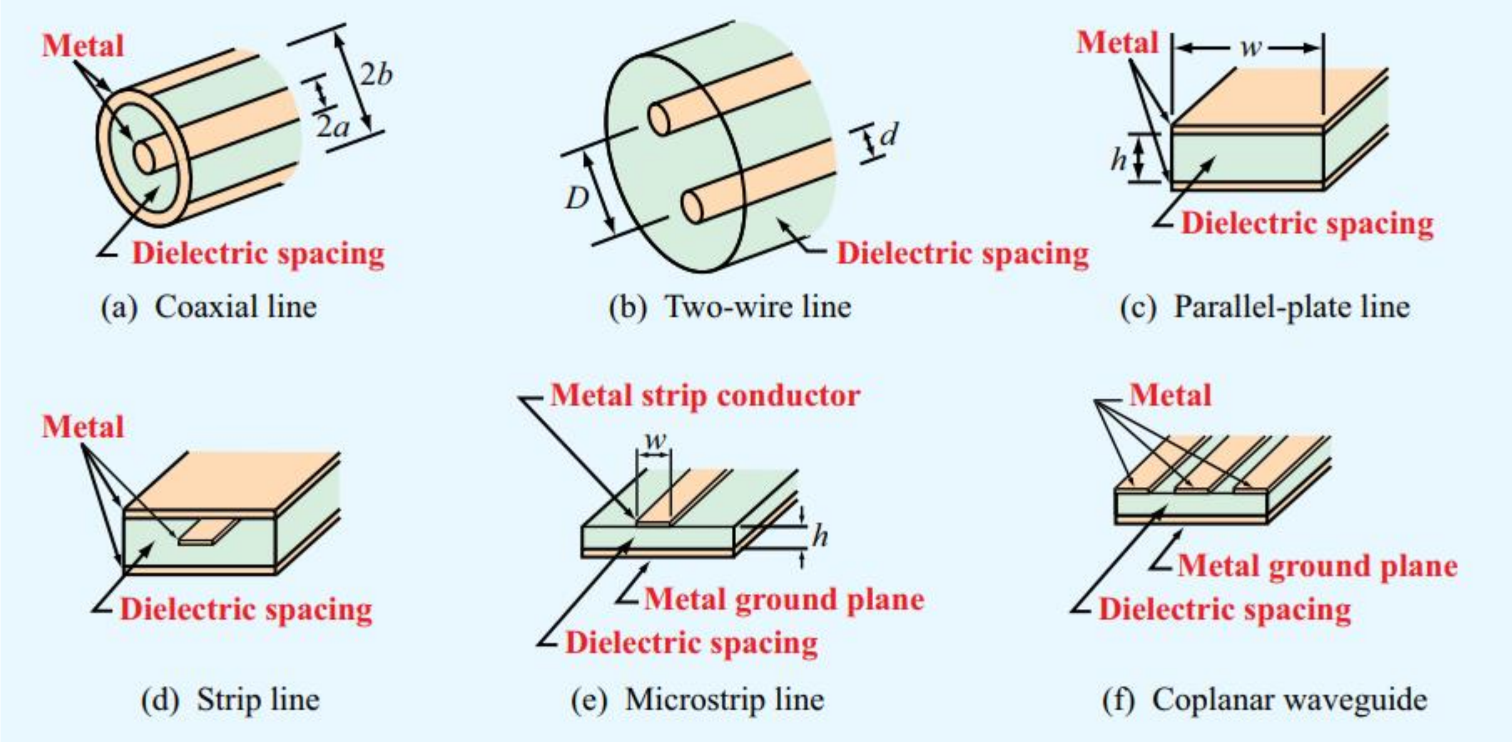


- Wave propagation on TEM TLs are defined by electric and magnetic fields which are entirely transverse to the direction of propagation.
- Coaxial line example: the electric field is in the radial direction from signal line to ground. The magnetic field circles the inner conductor.
- Common features of TEM TLs is that have two parallel conducting surfaces; thus the used circuit representation.
- Only TEM TLs will be addressed in this course.

Propagation Modes

- **Transverse electromagnetic (TEM) transmission lines:**

Waves propagating along these lines are characterized by electric and magnetic fields that are entirely transverse to the direction of propagation. Such an orthogonal configuration is called a TEM mode.



TEM Transmission Lines

Propagation Modes

- **Higher-order transmission lines:**

Waves propagating along these lines have at least one significant field component in the direction of propagation. Hollow conducting waveguides, dielectric rods, and optical fibers belong to this class of lines.

(g) Rectangular waveguide

(h) Optical fiber

Higher-Order Transmission Lines

Note: Only TEM-mode transmission lines are treated in this chapter. This is because they are more commonly used in practice.

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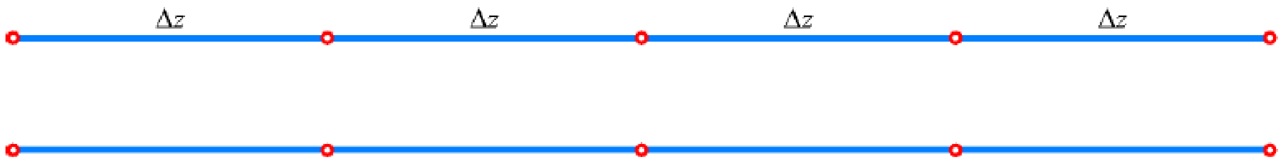
Lumped-Element Model

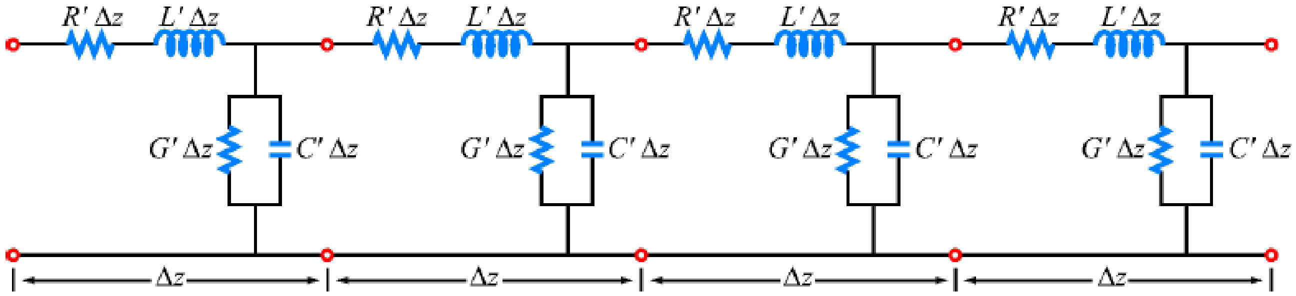
Lumped-Element Model

When we draw a schematic of an electronic circuit, we use specific symbols to represent resistors, capacitors, inductors, diodes, and the like. In each case, the symbol represents the functionality of the device, rather than its shape, size, or other attributes. We shall do the same for transmission lines.

(a) Parallel-wire representation



(b) Differential sections each Δz long



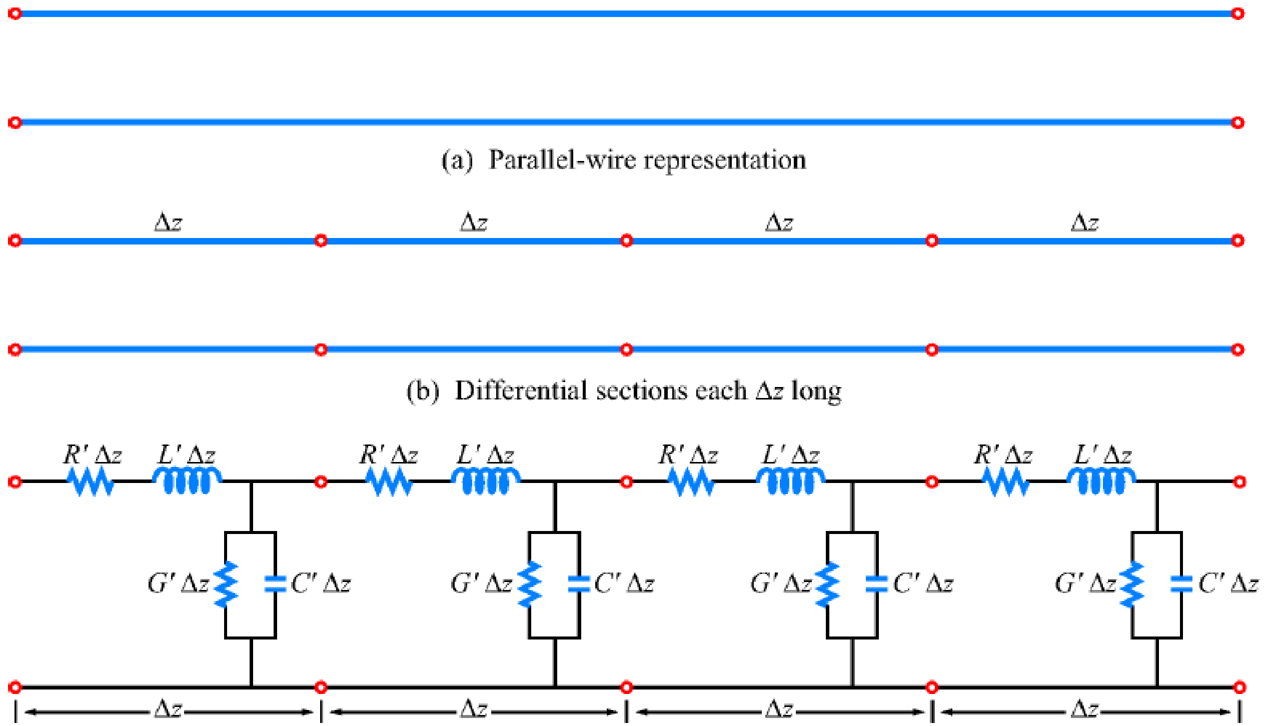
(c) Each section is represented by an equivalent circuit

Lumped-Element Model

(a)

(b)

(c)



(a) Parallel-wire representation

(b) Differential sections each Δz long

(c) Each section is represented by an equivalent circuit

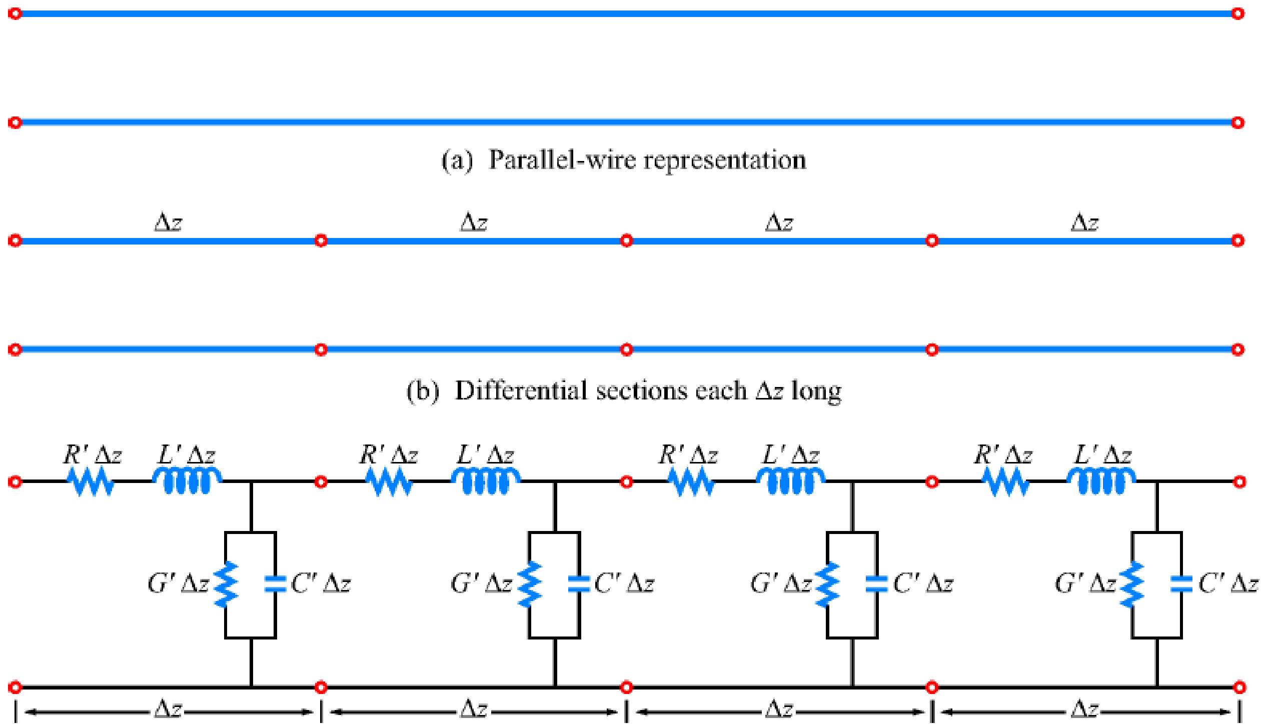
- Regardless of its cross-sectional shape, a TEM transmission line is represented by the parallel-wire configuration shown in (a).
- Figure(a) may represent a coaxial line, a two-wire line, or any other TEM line.

Lumped-Element Model

(a)

(b)

(c)



(a) Parallel-wire representation

(b) Differential sections each Δz long

(c) Each section is represented by an equivalent circuit

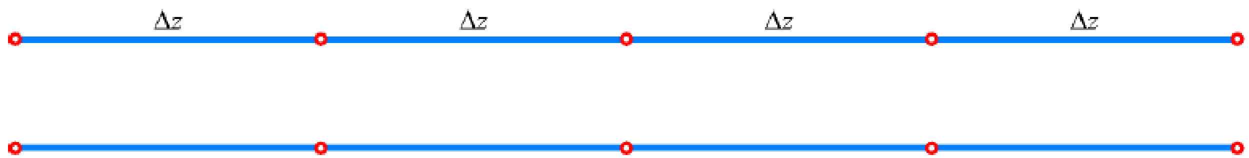
- Regardless of its cross-sectional shape, a TEM transmission line is represented by the parallel-wire configuration shown in (a).
- To obtain equations relating voltages and currents, the line is subdivided into small differential sections (b), each of which is then represented by an equivalent circuit or unit cell (c).

Lumped-Element Model

(a)

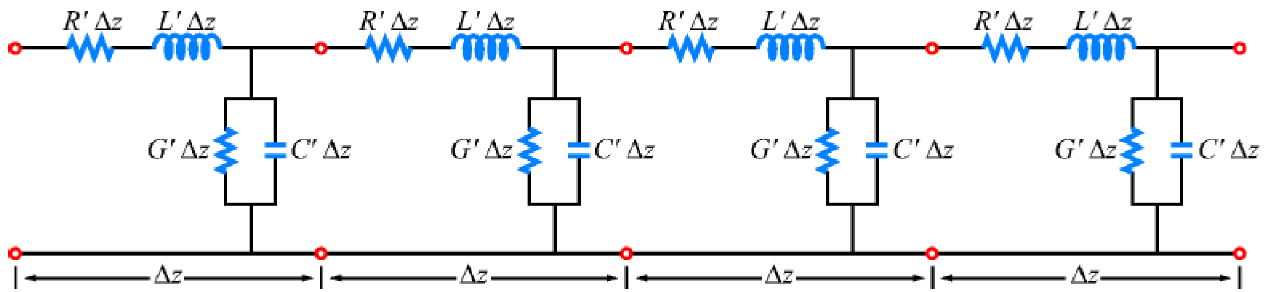
(a) Parallel-wire representation

(b)



(b) Differential sections each Δz long

(c)

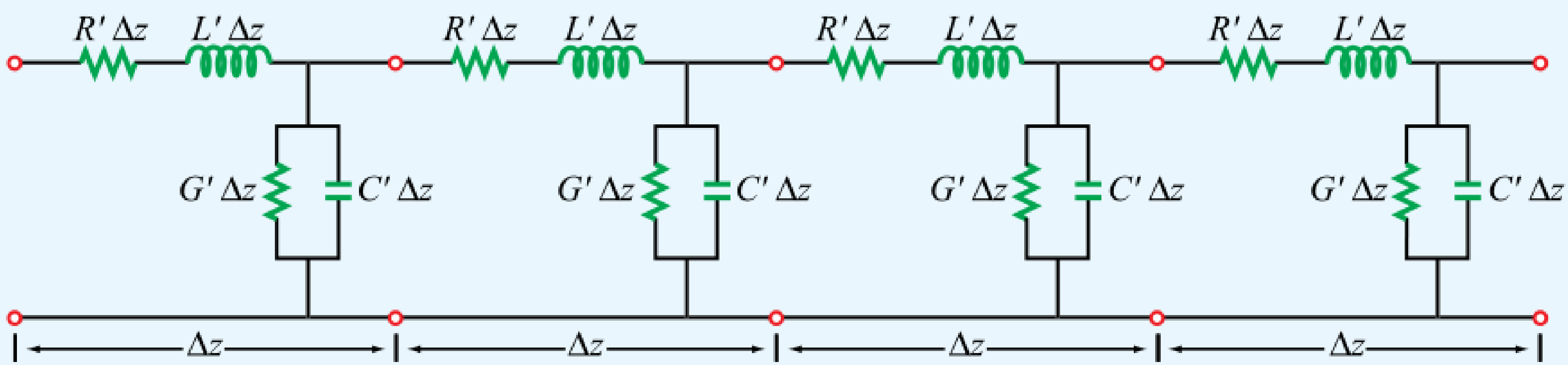


(c) Each section is represented by an equivalent circuit

- when we analyze a circuit containing a transistor, we mimic the functionality of the transistor by an equivalent circuit composed of sources, resistors, and capacitors.
- We apply the same approach to the transmission line by orienting the line along the z direction, subdividing it into differential sections each of length Δz.

Lumped-Element Model

Representing each section by an equivalent circuit, as illustrated in Figure(c)



(c) Each section is represented by an equivalent circuit.

This representation, often called the lumped-element circuit model, consists of four basic elements, with values that henceforth will be called the transmission line parameters R’ , L’, G’, C' .

Whereas the four line parameters are characterized by different formulas for different types of transmission lines, the equivalent model represented by Figure(c) is equally applicable to all TEM transmission lines.

Lumped-Element Model

The lumped element circuit model consists of four basic circuit elements, called transmission line parameters:

R' : The combined resistance of both conductors per unit length, in Ω/m.

L' : The combined inductance of both conductors per unit length, in H/m.

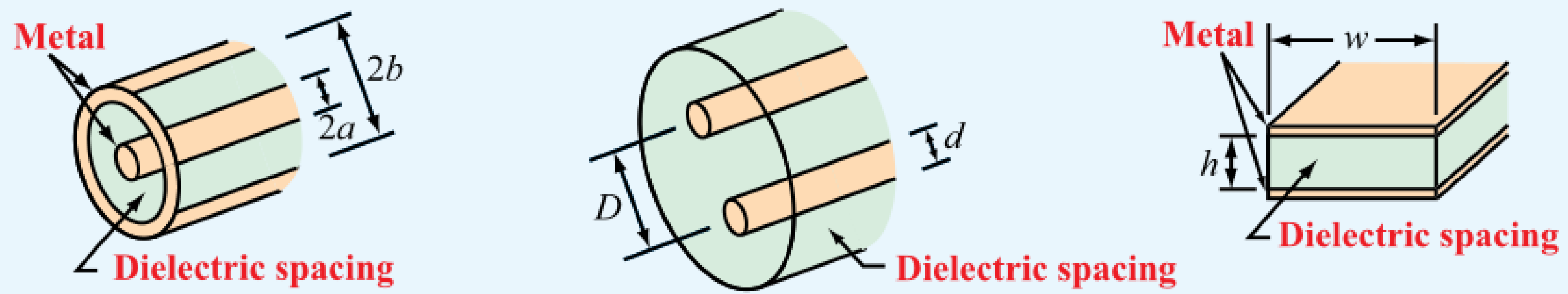
G' : The conductance of the insulating medium between the two conductors per unit length, in S/m.

C' : The capacitance of the two conductors per unit length, in F/m.

The prime superscript is used as a reminder that the line parameters are differential quantities whose units are per unit length.

Lumped-Element Model

Expressions for the line parameters R', L', G', and C' are given in Table for the three types of TEM transmission lines diagrammed in parts (a) through (c).

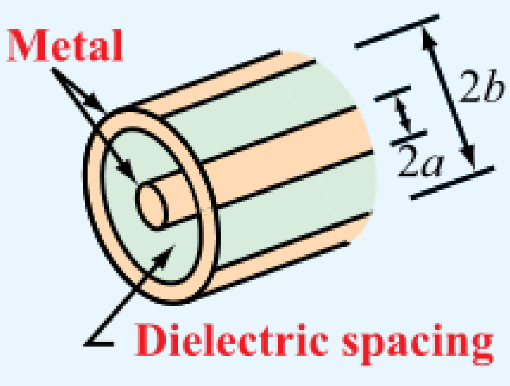


(a) Coaxial line (b) Two-wire line (c) Parallel-plate line

Lumped-Element Model

For each of these lines, the expressions are functions of two sets of parameters: (1) geometric parameters defining the cross-sectional dimensions of the given line and (2) the electromagnetic constitutive parameters of the conducting and insulating materials. The pertinent geometric parameters are:

(a) Coaxial line



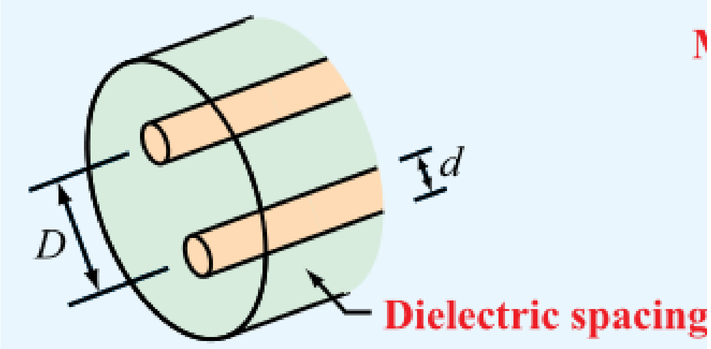
(a) Coaxial line

$a =$ outer radius of inner conductor, *m*

$b =$ inner radius of outer conductor, *m*

Lumped-Element Model

(b) Two-wire line

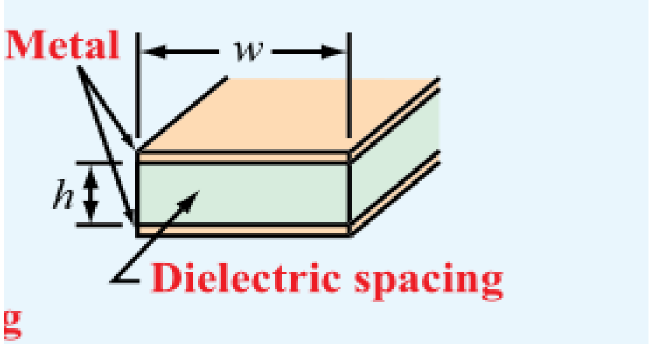


(b) Two-wire line

d = diameter of each wire, *m*

D = spacing between wires'centers, *m*

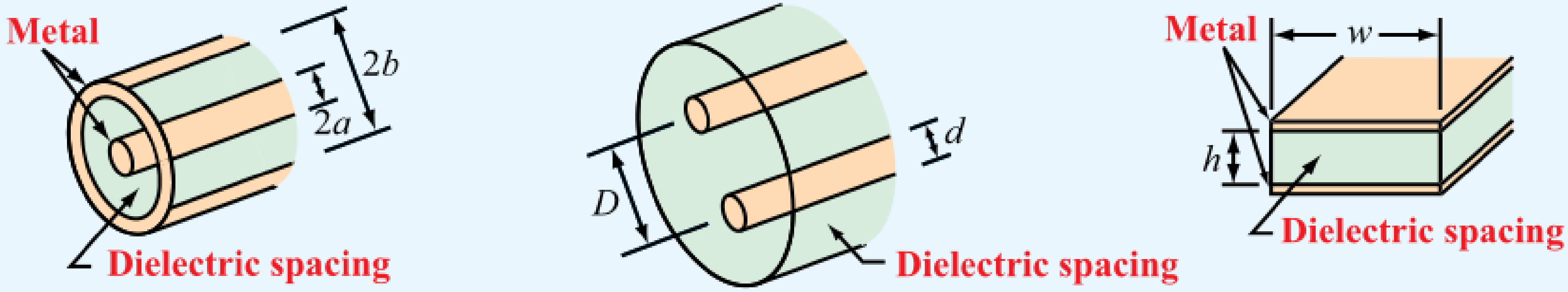
(c) Parallel-plate line



(c) Parallel-plate line

ω = width of each plate, *m*

h = thickness of insulation between plates, *m*



(a) Coaxial line (b) Two-wire line (c) Parallel-plate line

R'	$\dfrac{R_s}{2\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$	$\dfrac{2R_s}{\pi d}$	$\dfrac{2R_s}{w}$	Ω/m
L'	$\dfrac{\mu}{2\pi}\ln(b/a)$	$\dfrac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\dfrac{\mu h}{w}$	H/m
G'	$\dfrac{2\pi\sigma}{\ln(b/a)}$	$\dfrac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\dfrac{\sigma w}{h}$	S/m
C'	$\dfrac{2\pi\epsilon}{\ln(b/a)}$	$\dfrac{\pi\epsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\dfrac{\epsilon w}{h}$	F/m

Notes: (1) Refer to **Fig. 2-4** for definitions of dimensions. (2) μ, ϵ, and σ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\pi f \mu_c/\sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(D/d)^2 \gg 1$, then $\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right] \approx \ln(2D/d)$.

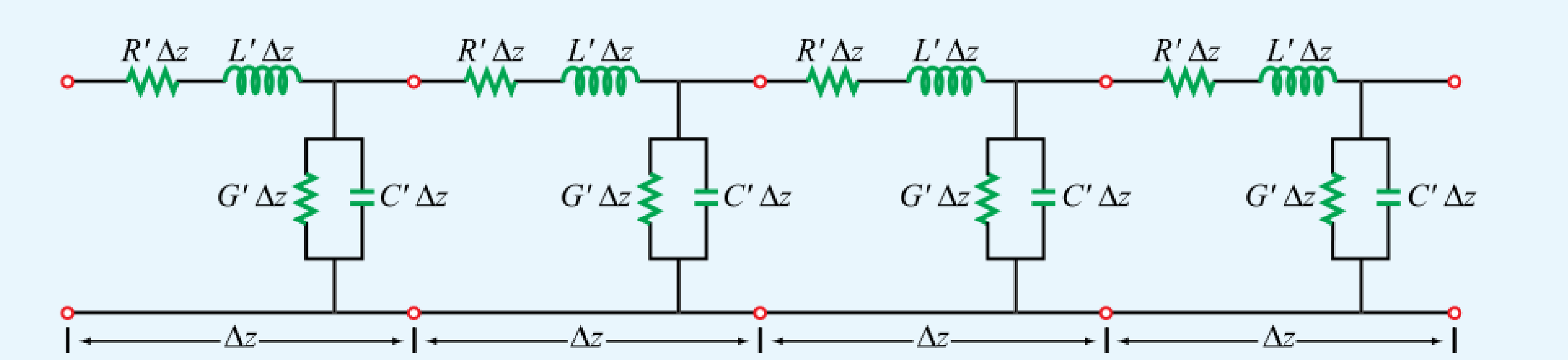
Lumped-Element Model

The pertinent constitutive parameters apply to all three lines and consist of two groups:

(1) μ_c and σ_c are the magnetic permeability and electrical conductivity of the conductors, and

(2) ε, μ, and σ are the electrical permittivity, magnetic permeability, and electrical conductivity of the insulation material separating them.

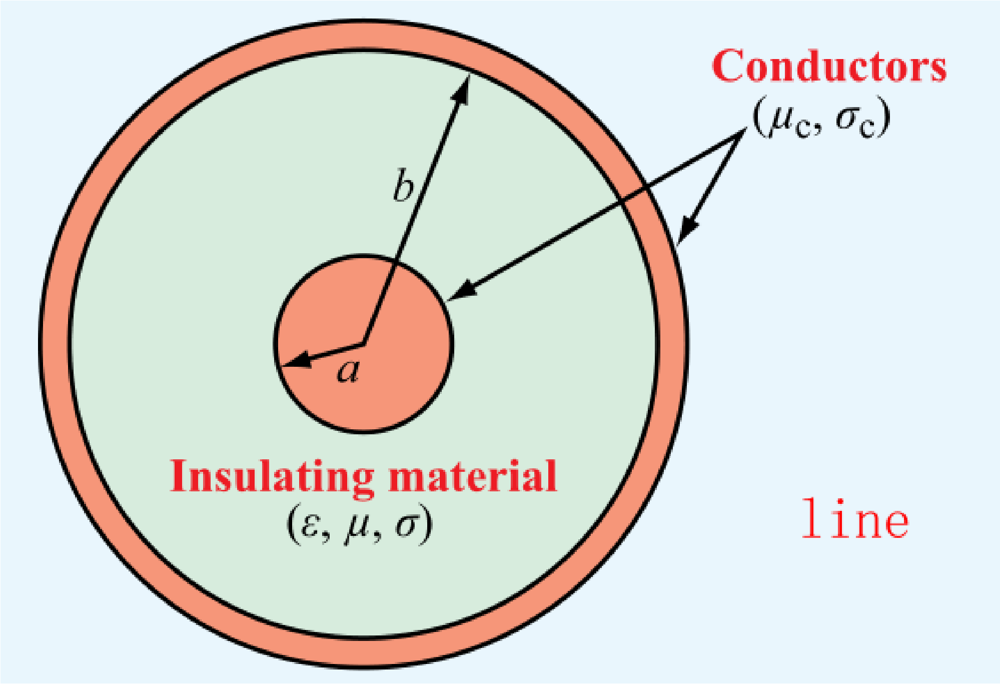
The lumped-element model shown in Fig. 2-6(c) reflects the physical phenomena associated with the currents and voltages on any TEM transmission line. It consists of two in-series elements, R' and L', and two shunt elements, G' and C'.



(c) Each section is represented by an equivalent circuit.

Lumped-Element Model

To explain the lumped-element model, consider a small section of a coaxial line:



Cross section of a coaxial line with inner conductor of radius a and outer conductor of radius b. The conductors have magnetic permeability μ_c and conductivity σ_c, and the spacing material between the conductors has permittivity ε,permeability μ, and conductivity σ.

Lumped-Element Model

When a voltage source is connected across the terminals connected to the two conductors at the sending end of the line, currents flow through the conductors, primarily along the outer surface of the inner conductor and the inner surface of the outer conductor.

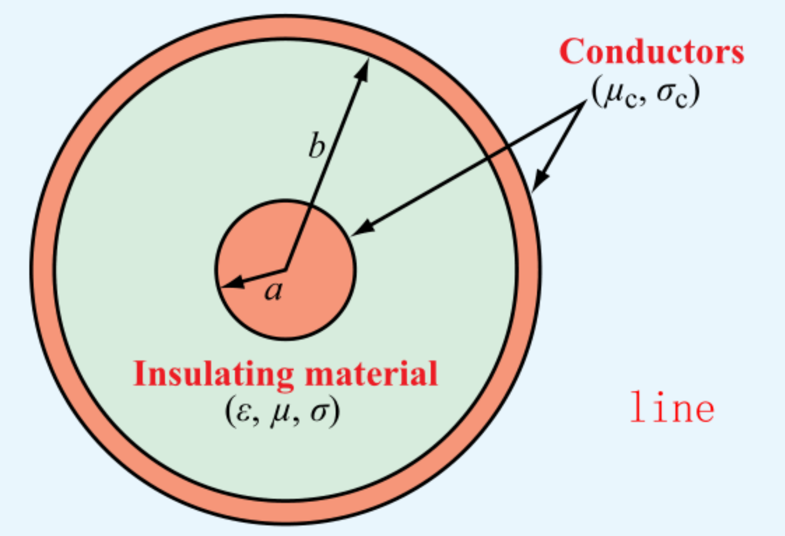
The coaxial line resistance R' accounts for the combined resistance per unit length of the inner and outer conductors:

(coax line)

$$R' = \frac{R_S}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right) \quad (\Omega/m)$$

where,

$$Rs = \sqrt{\frac{\pi f \mu_c}{\sigma_c}}$$



For a perfect conductor with $\sigma_c = \infty$ or a high-conductivity material such that $(f\mu_c/\sigma_c) \ll 1$, R_s approaches zero, and so does R'.

Lumped-Element Model

Line inductance L', which accounts for the joint inductance of both conductors per unit length of a coaxial line is :

$$L' = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \quad (H/m)$$

Line conductance G', accounts for current flow between the outer and inner conductors, per unit length is :

$$G' = \frac{2\pi\sigma}{\ln(\mathrm{b}/a)} \quad (S/m)$$

If the material separating the inner and outer conductors is a perfect dielectric with σ=0, then G' = 0.

Capacitance is defined as the ratio of the charge to the voltage difference. For the coaxial line, the capacitance per unit length is:

$$C' = \frac{2\pi\varepsilon}{\ln(\mathrm{b}/\mathrm{a})} \quad (F/m)$$

Lumped-Element Model

All TEM transmission lines satisfy:

$$L'C' = \mu\varepsilon \quad \text{(all TEM lines)}$$

$$\frac{G'}{C'} = \frac{\sigma}{\varepsilon} \quad \text{(all TEM lines)}$$

Exercise

Concept Question 2-1:

What is a transmission line? When should transmission-line effects be considered, and when may they be ignored?

Concept Question 2-2:

What is the difference between dispersive and nondispersive transmission lines? What is the practical significance of dispersion?

Concept Question 2-3:

What constitutes a TEM transmission line?

Concept Question 2-4:

What purpose does the lumped-element circuit model serve? How are the line parameters R', L', G' and C' related to the physical and electromagnetic constitutive properties of the transmission line?

Exercise

Concept Question 2-1:

What is a transmission line? When should transmission-line effects be considered, and when may they be ignored?

Answer:

(2) The factors that determine whether or not we should treat the wires as a transmission line are governed by the length of the line l and the frequency f of the signal provided by the generator.

When l/λ is very small, transmission-line effects may be ignored, but when $l/\lambda \geq 0.01$, it may be necessary to account not only for the phase shift due to the time delay.

Exercise

Concept Question 2-2:

What is the difference between dispersive and nondispersive transmission lines? What is the practical significance of dispersion?

Answer:

(1) A dispersionless line does not distort signals passing through it regardless of its length, whereas a dispersive line distorts the shape of the input pulses because the different frequency components propagate at different velocities. The degree of distortion is proportional to the length of the dispersive line.

(2) A dispersive transmission line is one on which the wave velocity is not constant as a function of the frequency f. This means that the shape of a rectangular pulse, will be distorted as it travels down the line because different frequency components will not propagate at the same frequency.

Exercise

Concept Question 2-3:

What constitutes a TEM transmission line?

Answer:

Waves propagating along these lines are characterized by electric and magnetic fields that are entirely transverse to the direction of propagation. Such an orthogonal configuration is called a TEM mode. A common feature among TEM lines is that they consist of two parallel conducting surfaces.

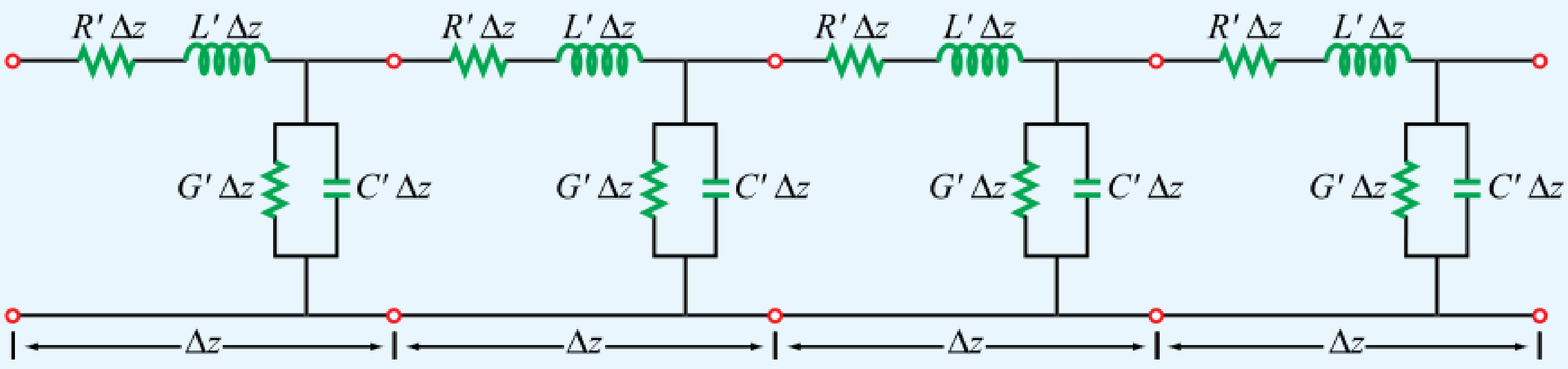
Exercise

Concept Question 2-4:

What purpose does the lumped-element circuit model serve? How are the line parameters R', L', G' and C' related to the physical and electromagnetic constitutive properties of the transmission line?

Answer:

The lumped-element model shown in Fig. 2-6(c) reflects the physical phenomena associated with the currents and voltages on any TEM transmission line.



(c) Each section is represented by an equivalent circuit.

Exercise

Concept Question 2-4:

What purpose does the lumped-element circuit model serve? How are the line parameters R', L', G' and C' related to the physical and electromagnetic constitutive properties of the transmission line?

Answer:

(2) Expressions for the line parameters R', L', G', and C' are given in Table.

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\epsilon}{\ln(b/a)}$	$\frac{\pi\epsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\epsilon w}{h}$	F/m

For each of these lines, the expressions are functions of two sets of parameters: (1) geometric parameters defining the cross-sectional dimensions of the given line and (2) the electromagnetic constitutive parameters of the conducting and insulating materials.

Exercise 2-1

Use Table to evaluate the line parameters of a two-wire air line with wires of radius 1mm, separated by a distance of 2 cm. The wires may be treated as perfect conductors with $\sigma_c = \infty$.

Exercise 2-2

Calculate the transmission line parameters at 1 MHz for a coaxial air line with inner and outer conductor diameters of 0.6 cm and 1.2 cm, respectively.The conductors are made of copper (see Appendix B for μ_c and σ_c of copper).

Table B-1 RELATIVE PERMITTIVITY ϵ_r OF COMMON MATERIALS[a]

$$\epsilon = \epsilon_r \epsilon_0 \text{ and } \epsilon_0 = 8.854 \times 10^{-12} \text{ F/m.}$$

Material	Relative Permittivity, ϵ_r	Material	Relative Permittivity, ϵ_r
Vacuum	1	Dry soil	2.5–3.5
Air (at sea level)	1.0006	Plexiglass	3.4
Styrofoam	1.03	Glass	4.5–10
Teflon	2.1	Quartz	3.8–5
Petroleum oil	2.1	Bakelite	5
Wood (dry)	1.5–4	Porcelain	5.7
Paraffin	2.2	Formica	6
Polyethylene	2.25	Mica	5.4–6
Polystyrene	2.6	Ammonia	22
Paper	2–4	Seawater	72–80
Rubber	2.2–4.1	Distilled water	81

[a]These are low-frequency values at room temperature (20° C).

Note: For most metals, $\epsilon_r \simeq 1$.

Table B-2 CONDUCTIVITY σ OF SOME COMMON MATERIALS[a]

Material	Conductivity, σ (S/m)	Material	Conductivity, σ (S/m)
Conductors		**Semiconductors**	
Silver	6.2×10^7	Pure germanium	2.2
Copper	5.8×10^7	Pure silicon	4.4×10^{-4}
Gold	4.1×10^7	**Insulators**	
Aluminum	3.5×10^7	Wet soil	$\sim 10^{-2}$
Tungsten	1.8×10^7	Fresh water	$\sim 10^{-3}$
Zinc	1.7×10^7	Distilled water	$\sim 10^{-4}$
Brass	1.5×10^7	Dry soil	$\sim 10^{-4}$
Iron	10^7	Glass	10^{-12}
Bronze	10^7	Hard rubber	10^{-15}
Tin	9×10^6	Paraffin	10^{-15}
Lead	5×10^6	Mica	10^{-15}
Mercury	10^6	Fused quartz	10^{-17}
Carbon	3×10^4	Wax	10^{-17}
Seawater	4		
Animal body (average)	0.3 (poor cond.)		

[a]These are low-frequency values at room temperature (20° C).

Table B-3 RELATIVE PERMEABILITY μ_r OF SOME COMMON MATERIALS[a]

$\mu = \mu_r \mu_0$ and $\mu_0 = 4\pi \times 10^{-7}$ H/m.

Material	Relative Permeability, μ_r
Diamagnetic	
Bismuth	$0.99983 \simeq 1$
Gold	$0.99996 \simeq 1$
Mercury	$0.99997 \simeq 1$
Silver	$0.99998 \simeq 1$
Copper	$0.99999 \simeq 1$
Water	$0.99999 \simeq 1$
Paramagnetic	
Air	$1.000004 \simeq 1$
Aluminum	$1.00002 \simeq 1$
Tungsten	$1.00008 \simeq 1$
Titanium	$1.0002 \simeq 1$
Platinum	$1.0003 \simeq 1$
Ferromagnetic (nonlinear)	
Cobalt	250
Nickel	600
Mild steel	2,000
Iron (pure)	4,000–5,000
Silicon iron	7,000
Mumetal	$\sim 100,000$
Purified iron	$\sim 200,000$

[a]These are typical values; actual values depend on material variety.

Exercise

Exercise 2-1

Use Table to evaluate the line parameters of a two-wire air line with wires of radius 1mm, separated by a distance of 2 cm. The wires may be treated as perfect conductors with $\sigma_c = \infty$.

Answer:

From the table:

$$\mu = 4\pi \times 10^{-7}\left(H/m\right)$$

Table B-3 RELATIVE PERMEABILITY μ_r OF SOME COMMON MATERIALS[a]

$\mu = \mu_r\mu_0$ and $\mu_0 = 4\pi \times 10^{-7}$ H/m.

Material	Relative Permeability, μ_r
Diamagnetic	
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Ferromagnetic (nonlinear)	
Cobalt	250
Nickel	600
Mild steel	2,000
Iron (pure)	4,000–5,000
Silicon iron	7,000
Mumetal	$\sim 100,000$
Purified iron	$\sim 200,000$

[a]These are typical values; actual values depend on material variety.

Exercise

Table B-1 RELATIVE PERMITTIVITY ϵ_r OF COMMON MATERIALS[a]

$\epsilon = \epsilon_r\epsilon_0$ and $\epsilon_0 = 8.854 \times 10^{-12}$ F/m.

Material	Relative Permittivity, ϵ_r	Material	Relative Permittivity, ϵ_r
Vacuum	1	Dry soil	2.5–3.5
Air (at sea level)	1.0006	Plexiglass	3.4
Styrofoam	1.03	Glass	4.5–10
Teflon	2.1	Quartz	3.8–5
Petroleum oil	2.1	Bakelite	5
Wood (dry)	1.5–4	Porcelain	5.7
Paraffin	2.2	Formica	6
Polyethylene	2.25	Mica	5.4–6
Polystyrene	2.6	Ammonia	22
Paper	2–4	Seawater	72–80
Rubber	2.2–4.1	Distilled water	81

[a]These are low-frequency values at room temperature (20° C).

Note: For most metals, $\epsilon_r \simeq 1$.

From the above table:

$$\varepsilon = 1.006 \times \left(8.854 \times 10^{-12}\right)\left(F/m\right)$$

Exercise

The following quantities are given:

$$\sigma_C = \infty \qquad \sigma = 0 \qquad d = 2\times10^{-3}\,(m)$$

$$\mu = 4\pi\times10^{-7}\,(H/m) \qquad D = 2\times10^{-2}\,(m) \qquad Rs = \sqrt{\frac{\pi f \mu_c}{\sigma_c}} = 0$$

$$\varepsilon = 1.006\times\left(8.854\times10^{-12}\right)(F/m)$$

Exercise

Hence,

$$R' = \frac{2R_S}{\pi d} = 0$$

$$L' = \frac{\mu}{\pi} \ln\left[(D/d) + \sqrt{(D/d)^2 - 1} \right] = 1.20(\mu H/m)$$

$$G' = \frac{\pi\sigma}{\ln\left[(D/d) + \sqrt{(D/d)^2 - 1} \right]} = 0$$

$$C' = \frac{\pi\varepsilon}{\ln\left[(D/d) + \sqrt{(D/d)^2 - 1} \right]} = 9.29(pH/m)$$

Exercise 2-2

Calculate the transmission line parameters at 1 MHz for a coaxial air line with inner and outer conductor diameters of 0.6 cm and 1.2 cm, respectively.The conductors are made of copper (see Appendix B for μ_c and σ_c of copper).

Table B-3 RELATIVE PERMEABILITY μ_r OF SOME COMMON MATERIALS[a]

$\mu = \mu_r\mu_0$ and $\mu_0 = 4\pi \times 10^{-7}$ H/m.

Material	Relative Permeability, μ_r
Diamagnetic	
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Copper	$0.99999 \simeq 1$
Water	$0.99999 \simeq 1$
Paramagnetic	
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Tungsten	$1.00008 \simeq 1$
Titanium	$1.0002 \simeq 1$
Platinum	$1.0003 \simeq 1$
Ferromagnetic (nonlinear)	
Cobalt	250
Nickel	600
Mild steel	2,000
Iron (pure)	4,000–5,000
Silicon iron	7,000
Mumetal	$\sim 100,000$
Purified iron	$\sim 200,000$

[a]These are typical values; actual values depend on material variety.

Answer:

From the table:

$$\mu_c = 4\pi \times 10^{-7} \left(H/m \right)$$

$$\mu = 4\pi \times 10^{-7} \left(H/m \right)$$

Exercise

Table B-2 CONDUCTIVITY σ OF SOME COMMON MATERIALS[a]

Material	Conductivity, σ (S/m)	Material	Conductivity, σ (S/m)
Conductors		**Semiconductors**	
Silver	6.2×10^7	Pure germanium	2.2
Copper	5.8×10^7	Pure silicon	4.4×10^{-4}
Gold	4.1×10^7	**Insulators**	
Aluminum	3.5×10^7	Wet soil	$\sim 10^{-2}$
Tungsten	1.8×10^7	Fresh water	$\sim 10^{-3}$
Zinc	1.7×10^7	Distilled water	$\sim 10^{-4}$
Brass	1.5×10^7	Dry soil	$\sim 10^{-4}$
Iron	10^7	Glass	10^{-12}
Bronze	10^7	Hard rubber	10^{-15}
Tin	9×10^6	Paraffin	10^{-15}
Lead	5×10^6	Mica	10^{-15}
Mercury	10^6	Fused quartz	10^{-17}
Carbon	3×10^4	Wax	10^{-17}
Seawater	4		
Animal body (average)	0.3 (poor cond.)		

[a]These are low-frequency values at room temperature (20° C).

From the above table:

$$\sigma_C = 5.8 \times 10^7 \left(S/m \right)$$

Exercise

Table B-1 RELATIVE PERMITTIVITY ϵ_r OF COMMON MATERIALS[a]

$\epsilon = \epsilon_r \epsilon_0$ and $\epsilon_0 = 8.854 \times 10^{-12}$ F/m.

Material	Relative Permittivity, ϵ_r	Material	Relative Permittivity, ϵ_r
Vacuum	1	Dry soil	2.5–3.5
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Teflon	2.1	Quartz	3.8–5
Petroleum oil	2.1	Bakelite	5
Wood (dry)	1.5–4	Porcelain	5.7
Paraffin	2.2	Formica	6
Polyethylene	2.25	Mica	5.4–6
Polystyrene	2.6	Ammonia	22
Paper	2–4	Seawater	72–80
Rubber	2.2–4.1	Distilled water	81

[a]These are low-frequency values at room temperature (20° C).

Note: For most metals, $\epsilon_r \simeq 1$.

From the above table:

$$\varepsilon = 1.006 \times \left(8.854 \times 10^{-12}\right)\left(F/m\right)$$

Exercise

The following quantities are given:

$$f = 1\times10^{6}\,(Hz) \qquad \sigma = 0 \qquad a = 3\times10^{-3}\,(m)$$

$$b = 6\times10^{-3}\,(m) \qquad \sigma_C = 5.8\times10^{7}\,(S/m) \qquad \mu_c = 4\pi\times10^{-7}\,(H/m)$$

$$\varepsilon = 1.006\times\left(8.854\times10^{-12}\right)(F/m) \qquad \mu = 4\pi\times10^{-7}\,(H/m)$$

Exercise

Hence,

$$Rs = \sqrt{\frac{\pi f \mu_c}{\sigma_c}} = 2.61\times10^{-4}\,(\Omega)$$

$$R' = \frac{R_S}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right) = 2.07(\Omega/m)$$

$$L' = \frac{\mu}{2\pi}\ln(b/a) = 0.14(\mu H/m)$$

$$G' = \frac{2\pi\sigma}{\ln(b/a)} = 0$$

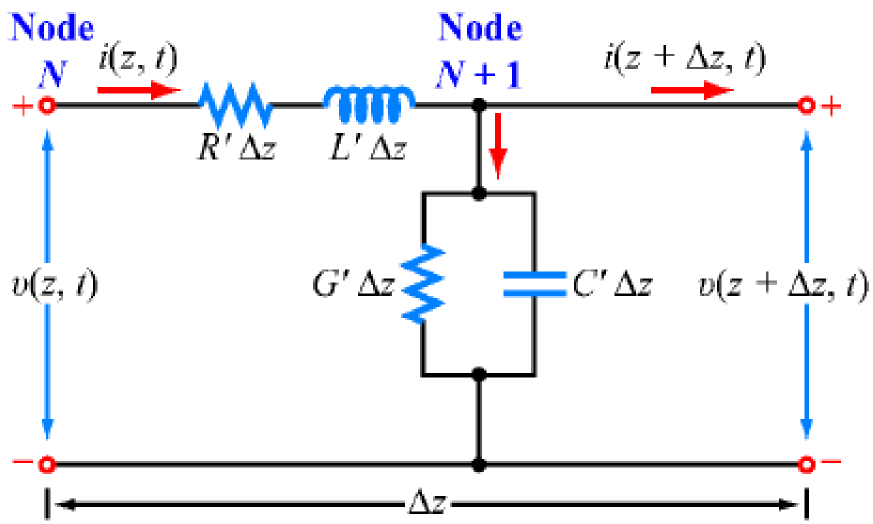
$$C' = \frac{2\pi\varepsilon}{\ln(b/a)} = 80.3(pH/m)$$

西安电子科技大学
XIDIAN UNIVERSITY

Lecture 3
Transmission-Line Equations

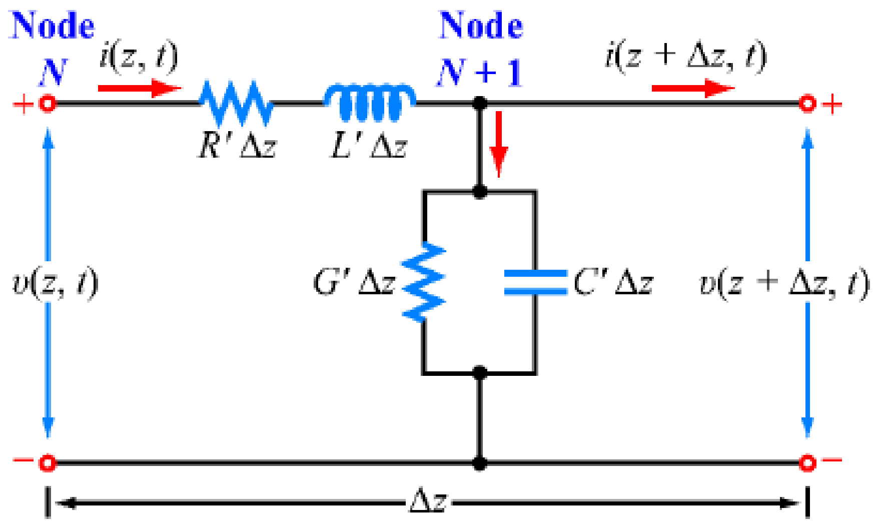
Transmission-Line Equations

A transmission line usually connects a source on one end to a load on the other. Before considering the complete circuit, however, we will develop general equations that describe the voltage across and current carried by the transmission line as a function of time *t* and spatial position *z.*



The quantities $v(z,t)$ and $i(z,t)$ denote the instantaneous voltage and current at the left end of the differential section (node *N*), and similarly $v(z+\Delta z,t)$ and $i(z+\Delta z,t)$ denote the same quantities at node (*N* + 1), located at the right end of the section.

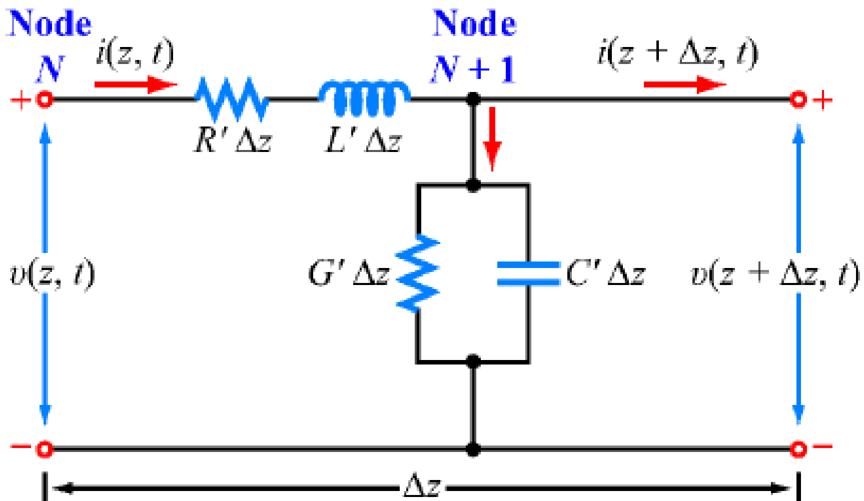
Transmission-Line Equations



Application of Kirchhoff's voltage law accounts for the voltage drop across the series resistance $R'\Delta z$ and inductance $L'\Delta z$

$$v(z,t) - R'\Delta z i(z,t) - L'\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

Transmission-Line Equations



$$v(z,t)-R'\Delta z\, i(z,t)-L'\Delta z\frac{\partial i(z,t)}{\partial t}-v(z+\Delta z,t)=0$$

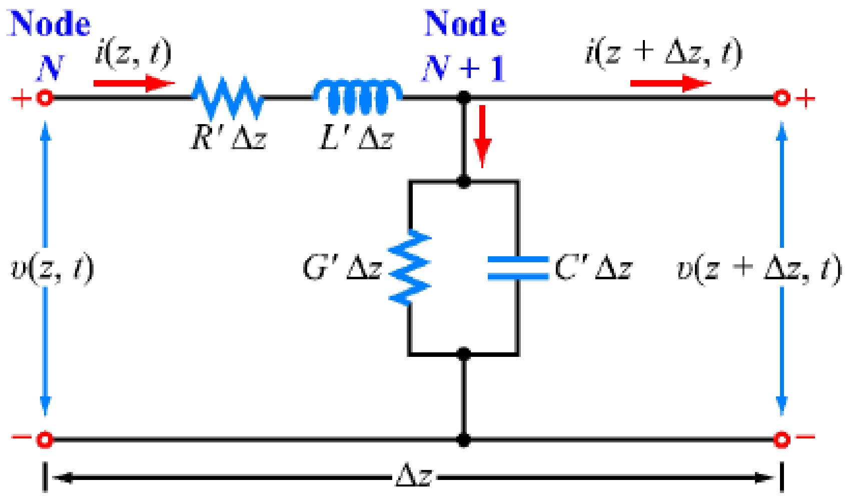
Upon dividing all terms by Δz and rearranging them, we obtain

$$-\left[\frac{v(z+\Delta z,t)-v(z,t)}{\Delta z}\right]=R' i(z,t)+L'\frac{\partial i(z,t)}{\partial t}$$

In the limit as $\Delta z \rightarrow 0$, we obtain a differential equation:

$$-\frac{\partial v(z,t)}{\partial z}=R' i(z,t)+L'\frac{\partial i(z,t)}{\partial t}$$

Transmission-Line Equations



Similarly, Kirchhoff's current law accounts for current drawn from the upper line at node (N +1) by the parallel conductance $G'\Delta z$ and capacitance $C'\Delta z$:

$$i(z,t) - G'\Delta z v(z+\Delta z,t) - C'\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

Upon dividing all terms by Δz and taking the limit $\Delta z \to 0$, becomes a second-order differential equation:

$$-\frac{\partial i(z,t)}{\partial z} = G'v(z,t) + C'\frac{\partial v(z,t)}{\partial t}$$

Transmission-Line Equations

The first-order differential equations are the time-domain forms of the transmission-line equations, known as the telegrapher's equations.

$$-\frac{\partial v(z,t)}{\partial z} = R'i(z,t) + L'\frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\partial i(z,t)}{\partial z} = G'v(z,t) + C'\frac{\partial v(z,t)}{\partial t}$$

Transmission-Line Equations

Our primary interest is in sinusoidal steady-state conditions.We make use of the phasor representation with a cosine reference.Thus, we define

$$v(z,t) = \Re e\left[\tilde{V}(z)e^{j\omega t}\right]$$

$$i(z,t) = \Re e\left[\tilde{I}(z)e^{j\omega t}\right]$$

where $\tilde{V}(z)$ and $\tilde{I}(z)$ are the phasor counterparts of $v(z,t)$ and $I(z,t)$ respectively

Transmission-Line Equations

Upon substituting

$$v(z,t) = \Re e\left[\tilde{V}(z)e^{j\omega t}\right] \qquad i(z,t) = \Re e\left[\tilde{I}(z)e^{j\omega t}\right]$$

into

$$-\frac{\partial v(z,t)}{\partial z} = R' i(z,t) + L' \frac{\partial i(z,t)}{\partial t} \qquad -\frac{\partial i(z,t)}{\partial z} = G' v(z,t) + C' \frac{\partial v(z,t)}{\partial t}$$

utilizing the property given by $\frac{di}{dt} = \Re e\left[j\omega \tilde{I} e^{jwt}\right]$ that ∂/∂t in time domain is equivalent to multiplication by $j\omega$ in the phasor domain, we obtain :

(telegrapher's equations in phasor form)

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\tilde{I}(z)$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\tilde{V}(z)$$