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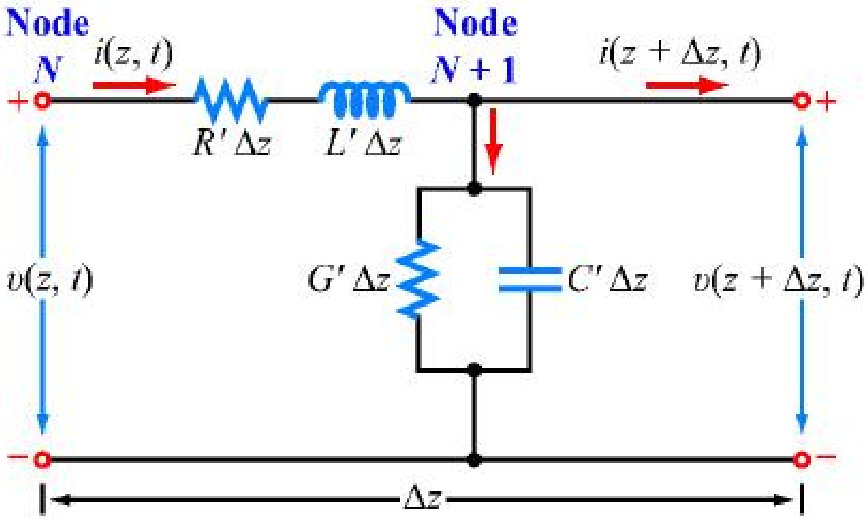
B39HF High Frequency Circuits

Lecture 3
Transmission-Line Equations

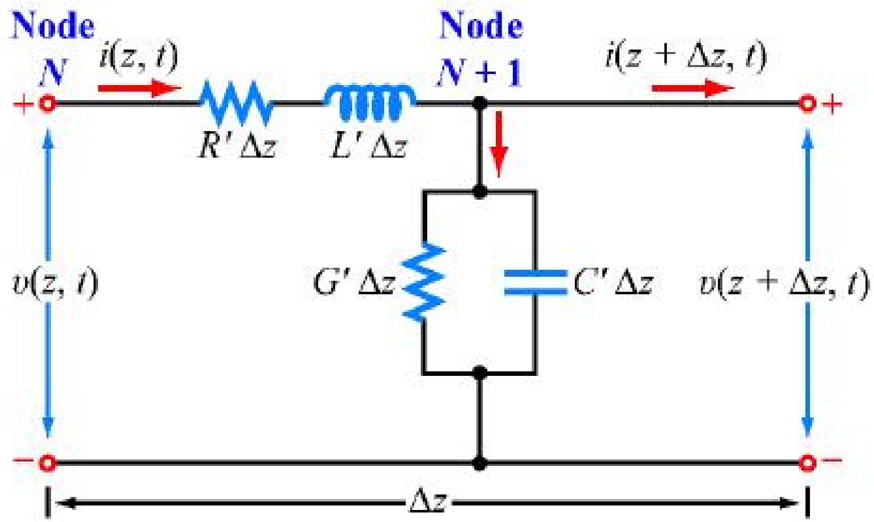
Transmission-Line Equations

A transmission line usually connects a source on one end to a load on the other. Before considering the complete circuit, however, we will develop general equations that describe the voltage across and current carried by the transmission line as a function of time *t* and spatial position *z.*



The quantities $v(z, t)$ and $i(z, t)$ denote the instantaneous voltage and current at the left end of the differential section (node *N*), and similarly $v(z+\triangle z, t)$ and $i(z+\triangle z, t)$ denote the same quantities at node (N + 1), located at the right end of the section.

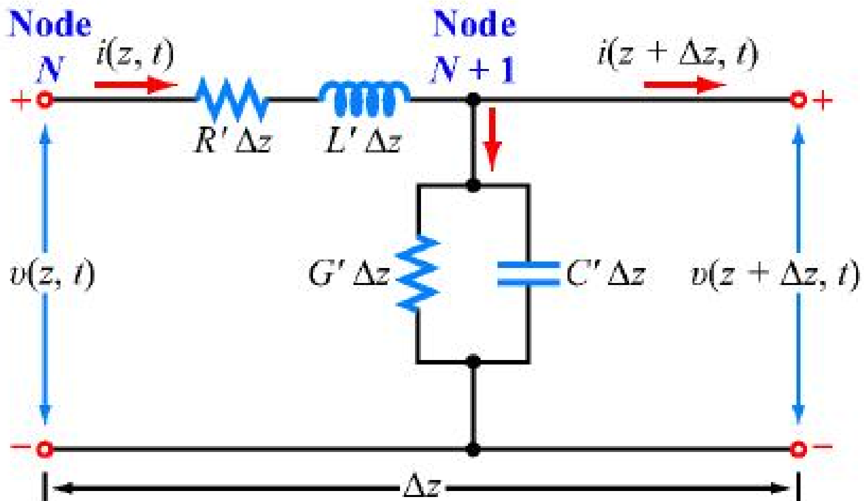
Transmission-Line Equations



Application of Kirchhoff's voltage law accounts for the voltage drop across the series resistance $R'\Delta z$ and inductance $L'\Delta z$

$$v(z,t) - R'\Delta z i(z,t) - L'\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

Transmission-Line Equations



$$v(z,t) - R'\Delta z i(z,t) - L'\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

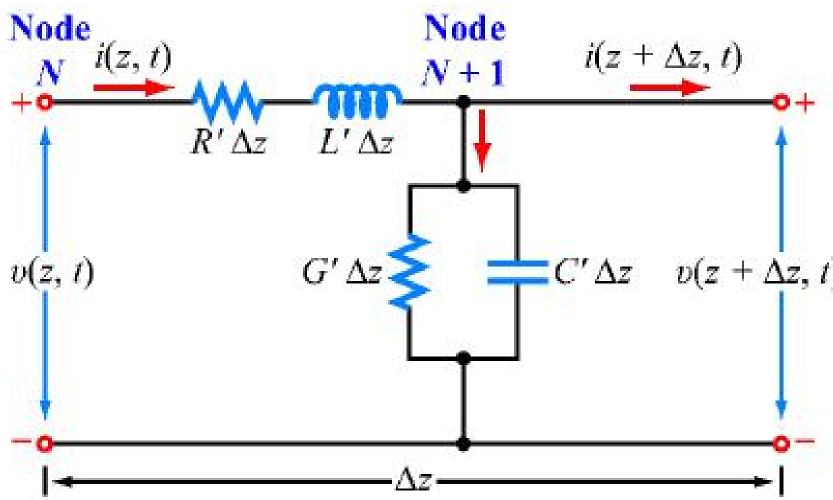
Upon dividing all terms by Δz and rearranging them, we obtain

$$-\left[\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z}\right] = R'i(z,t) + L'\frac{\partial i(z,t)}{\partial t}$$

In the limit as $\Delta z \to 0$, we obtain a differential equation:

$$-\frac{\partial v(z,t)}{\partial z} = R'i(z,t) + L'\frac{\partial i(z,t)}{\partial t}$$

Transmission-Line Equations



Similarly, Kirchhoff's current law accounts for current drawn from the upper line at node (N +1) by the parallel conductance $G' \Delta z$ and capacitance $C' \Delta z$:

$$i(z,t) - G'\Delta z v(z+\Delta z,t) - C'\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

Upon dividing all terms by Δz and taking the limit $\Delta z \to 0$, becomes a second-order differential equation:

$$-\frac{\partial i(z,t)}{\partial z} = G' v(z,t) + C' \frac{\partial v(z,t)}{\partial t}$$

Transmission-Line Equations

The first-order differential equations are the time-domain forms of the transmission-line equations, known as the telegrapher's equations.

$$-\frac{\partial v(z,t)}{\partial z} = R' i(z,t) + L' \frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\partial i(z,t)}{\partial z} = G' v(z,t) + C' \frac{\partial v(z,t)}{\partial t}$$

Transmission-Line Equations

Our primary interest is in sinusoidal steady-state conditions.We make use of the phasor representation with a cosine reference.Thus, we define

$$v(z,t) = \mathrm{Re}\left[\tilde{V}(z)e^{j\omega t}\right]$$

$$i(z,t) = \mathrm{Re}\left[\tilde{I}(z)e^{j\omega t}\right]$$

where $\tilde{V}(z)$ and $\tilde{I}(z)$ are the phasor counterparts of $v(z,t)$ and $I(z,t)$ respectively

Transmission-Line Equations

Upon substituting

$$v(z,t) = \Re e[\tilde{V}(z)e^{j\omega t}] \qquad i(z,t) = \Re e[\tilde{I}(z)e^{j\omega t}]$$

into

$$-\frac{\partial v(z,t)}{\partial z} = R' i(z,t) + L' \frac{\partial i(z,t)}{\partial t} \qquad -\frac{\partial i(z,t)}{\partial z} = G' v(z,t) + C' \frac{\partial v(z,t)}{\partial t}$$

utilizing the property given by $\frac{di}{dt} = \Re e[j\omega\tilde{I}e^{jwt}]$ that ∂/∂t in time domain is equivalent to multiplication by $j\omega$ in the phasor domain, we obtain :

(telegrapher's equations in phasor form)

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\tilde{I}(z)$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\tilde{V}(z)$$

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Lecture 4

Wave Propagation on a Transmission Line

Wave Propagation on a Transmission Line

The two first-order coupled equation

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\tilde{I}(z)$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\tilde{V}(z)$$

can be combined to give two second-order uncoupled wave equations,one for $\tilde{V}(z)$ and another for $\tilde{I}(z)$

The wave equation for $\tilde{V}(z)$ is derived by first differentiating both sides of $-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\tilde{I}(z)$ with respect to z, resulting in

$$-\frac{d^2\tilde{V}(z)}{dz^2} = (R' + j\omega L')\frac{d\tilde{I}(z)}{dz}$$

Wave Propagation on a Transmission Line

The wave equation for $\tilde{V}(z)$ is derived by first differentiating both sides of $-\frac{d\tilde{V}(z)}{dz} = (R^{'} + j\omega L^{'})\tilde{I}(z)$ with respect to z, resulting in

$$-\frac{d^2\tilde{V}(z)}{dz^2} = (R' + j\omega L')\frac{d\tilde{I}(z)}{dz}$$

Then, upon substituting $-\frac{d\tilde{I}(z)}{dz} = (G^{'} + j\omega C^{'})\tilde{V}(z)$, becomes

$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0$$

where,

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

Wave Propagation on a Transmission Line

The same steps to

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\tilde{I}(z)$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\tilde{V}(z)$$

in reverse order leads to

$$\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2\tilde{I}(z) = 0$$

where,

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

Wave Propagation on a Transmission Line

The second-order differential equations are called wave equations for $\tilde{V}(z)$ and $\tilde{I}(z)$ respectively,

$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0$$

$$\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2\tilde{I}(z) = 0$$

γ is called the complex propagation constant of the transmission line:

Wave Propagation on a Transmission Line

γ consists of a real part α ,called the attenuation constant of the line with units of Np/m, and an imaginary part β is called the phase constant of the line with units of rad/m.

$$\gamma = \alpha + j\beta$$

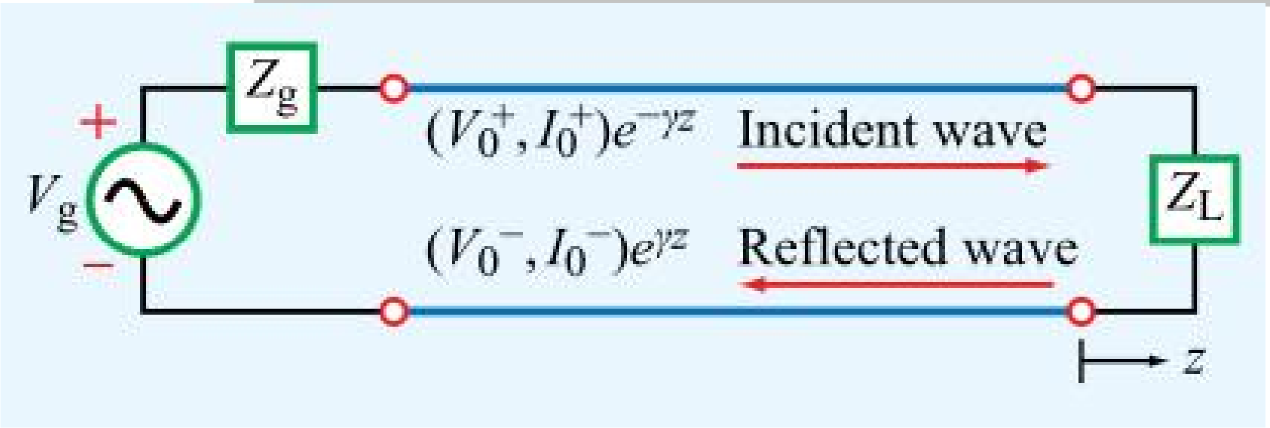
$$\alpha = \mathrm{Re}(\gamma) = \mathrm{Re}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right)(Np/m)$$

$$\beta = \mathrm{Im}(\gamma) = \mathrm{Im}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right)(rad/m)$$

For passive transmission lines, α is either zero or positive.

Most transmission lines, and all those considered in this chapter, are of the passive type.

Wave Propagation on a Transmission Line



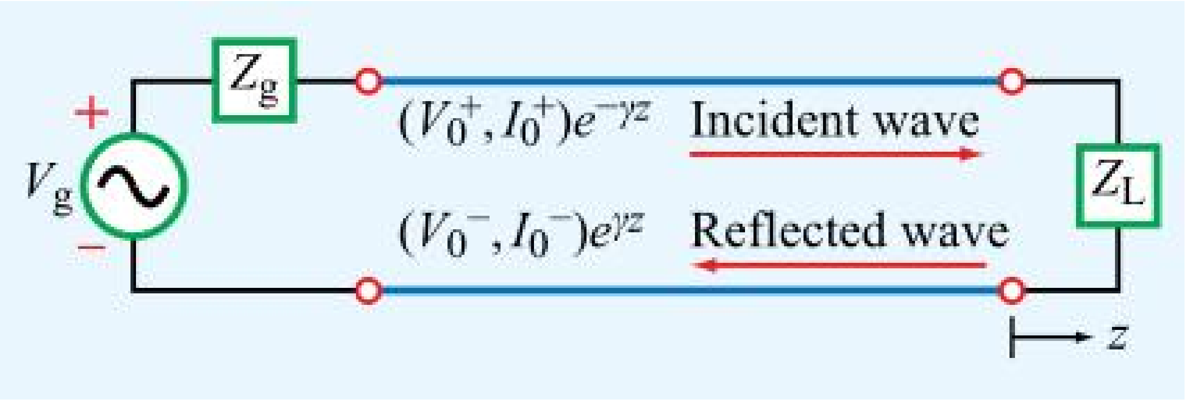
The wave equations $\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0$ and $\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2\tilde{I}(z) = 0$ have traveling wave solutions of the following form

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \left(V\right)$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \left(A\right)$$

The $e^{-\gamma z}$ term represents a wave propagating in the +z direction while the $e^{\gamma z}$ term represents a wave propagating in the $-z$ direction.

Wave Propagation on a Transmission Line



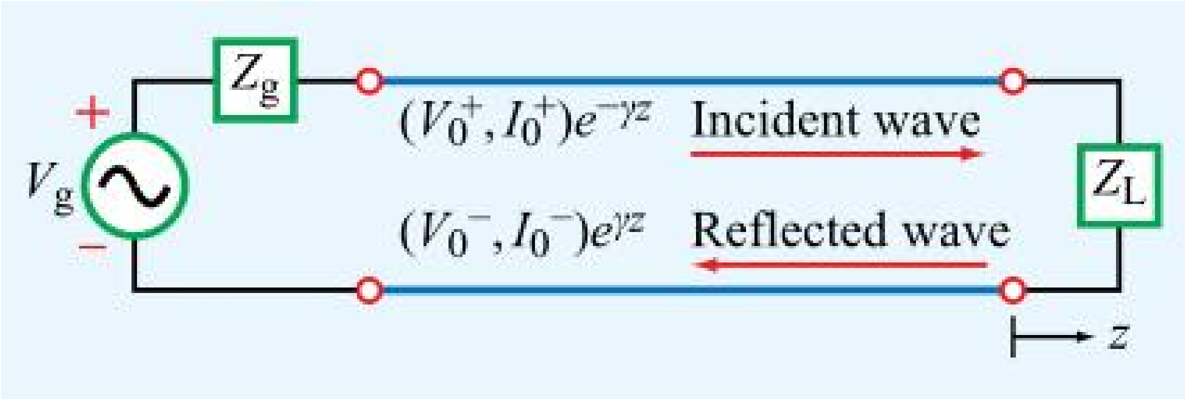
the wave amplitudes (V_0^+, I_0^+) the +z propagating wave

the wave amplitudes (V_0^-, I_0^-) the −z propagating wave.

We can easily relate the current wave amplitudes, I_0^+ and I_0^-, to the voltage wave amplitudes, V_0^+ and V_0^-, by using $\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} (V)$ in $-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\tilde{I}(z)$ and then solving for the current $\tilde{I}(z)$. The process leads to

$$\tilde{I}(z) = \frac{\gamma}{R' + j\omega L'}\left[V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right]$$

Wave Propagation on a Transmission Line



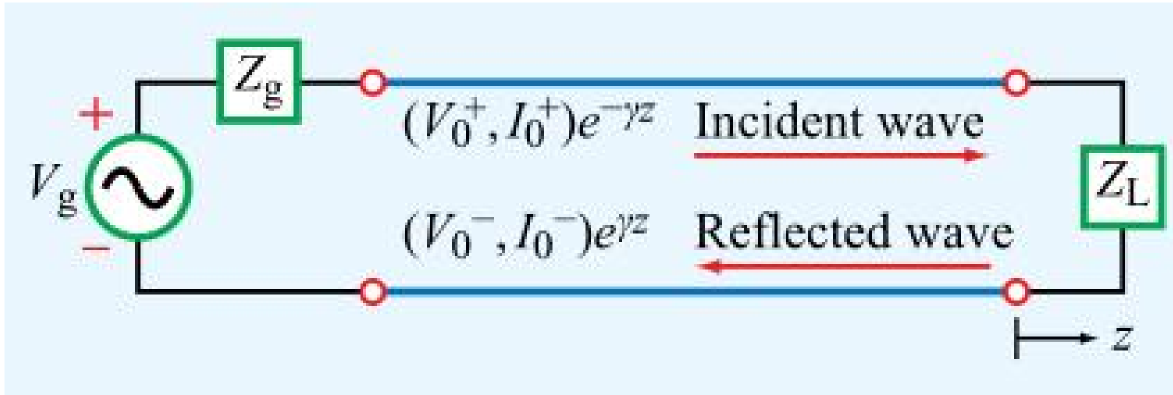
Comparison of each term with the corresponding term in $\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$ leads us to conclude that

$$\frac{V_0^+}{I_0^+} = Z_0 = -\frac{V_0^-}{I_0^-}$$

where,

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

Wave Propagation on a Transmission Line



$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

Z_0 is called the characteristic impedance of the line.

Note that Z_0 is equal to the ratio of the voltage amplitude to the current amplitude for each of the traveling waves individually (with an additional minus sign in the case of the −z propagating wave), it is not equal to the ratio of the total voltage $\tilde{V}(z)$ to the total current $\tilde{I}(z)$, unless one of the two waves is absent.

Wave Propagation on a Transmission Line

$$\frac{V_0^+}{I_0^+} = Z_0 = -\frac{V_0^-}{I_0^-}$$

It seems reasonable that the voltage-to-current ratios of the two waves V_0^+/I_0^+ and V_0^-/I_0^-, are both related to the same quantity, namely Z_0

Why is one of the ratios is the negative of the other?

The explanation, is based on a directional rule that specifies the relationships between the directions of the electric and magnetic fields of a TEM wave and its direction of propagation.

On a transmission line, the voltage is related to the electric field **E** and the current is related to the magnetic field **H.** To satisfy the directional rule, reversing the direction of propagation requires reversal of the direction (or polarity) of I relative to V. Hence, $V_0^-/I_0^- = -V_0^+/I_0^+$.

Wave Propagation on a Transmission Line

In terms of Z_0, $\tilde{I}(z) = \frac{\gamma}{R^{'}+j\omega L^{'}}[V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}]$ can be cast in the form

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

The combination of $\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} (V)$ and $\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$ now contains only two unknowns, namely V_0^+ and V_0^-.

Wave Propagation on a Transmission Line

In general, each is a complex quantity characterized by a magnitude and a phase angle:

$$V_0^+ = \left|V_0^+\right| e^{j\phi+}$$

$$V_0^- = \left|V_0^-\right| e^{j\phi-}$$

After substituting these definitions in $\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$ and using $\gamma = \alpha + j\beta$ to decompose γ into its real and imaginary parts,we can convert back to the time domain to obtain an expression for $v(z,t)$, the instantaneous voltage on the line:

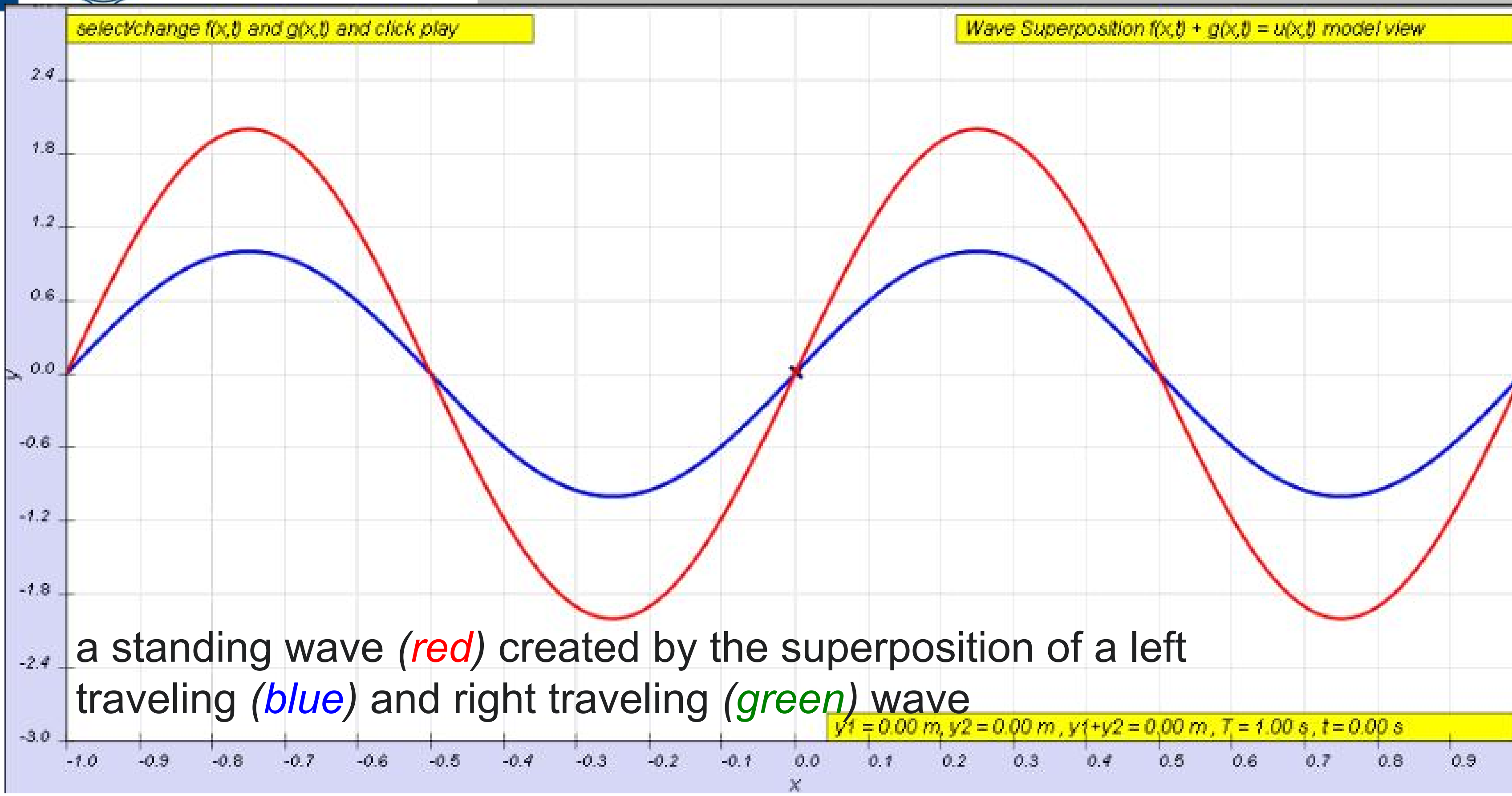
$$\begin{aligned} v(z,t) &= \mathrm{Re}\left[\tilde{V}(\mathrm{z}) e^{j\omega t}\right] \\ &= \mathrm{Re}\left[\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right) e^{j\omega t}\right] \\ &= \mathrm{Re}\left[\left|V_0^+\right| e^{j\phi^+} e^{j\omega t} e^{-(\alpha + j\beta)z} + \left|V_0^-\right| e^{j\phi^-} e^{j\omega t} e^{(\alpha + j\beta)z}\right] \\ &= \left|V_0^+\right| e^{-\alpha z} \cos\left(\omega t - \beta z + \phi^+\right) + \left|V_0^-\right| e^{\alpha z} \cos\left(\omega t + \beta z + \phi^-\right) \end{aligned}$$

Wave Propagation on a Transmission Line

$$
\begin{aligned}
v(z,t) &= \mathrm{Re}\left[\tilde{V}(z)\, e^{j\omega t}\right] \\
&= \mathrm{Re}\left[\left(V_0^{+} e^{-\gamma z} + V_0^{-} e^{\gamma z}\right) e^{j\omega t}\right] \\
&= \mathrm{Re}\left[\left|V_0^{+}\right| e^{j\phi^{+}} e^{j\omega t} e^{-(\alpha+j\beta)z} + \left|V_0^{-}\right| e^{j\phi^{-}} e^{j\omega t} e^{(\alpha+j\beta)z}\right] \\
&= \left|V_0^{+}\right| e^{-\alpha z} \cos\left(\omega t - \beta z + \phi^{+}\right) + \left|V_0^{-}\right| e^{\alpha z} \cos\left(\omega t + \beta z + \phi^{-}\right)
\end{aligned}
$$

The first term in the equation as a wave traveling in the +*z* direction(the coefficients of *t* and *z* have opposite signs) and the second term as a wave traveling in the −z direction.The factor $e^{-\alpha z}$ accounts for the attenuation of the +z propagating wave, and the factor $e^{\alpha z}$ accounts for the attenuation of the −z propagating wave.

The presence of two waves on the line propagating in opposite directions produces a standing wave.



a standing wave (*red*) created by the superposition of a left traveling (*blue*) and right traveling (*green*) wave

a standing wave, also known as a stationary wave, is a wave which oscillates in time but whose peak amplitude profile does not move in space.

The peak amplitude of the wave oscillations at any point in space is constant with time, and the oscillations at different points throughout the wave are in phase.

Wave Propagation on a Transmission Line

Both waves propagate with a phase velocity u_p given by

$$u_p = f\lambda = \frac{\omega}{\beta}$$

Because the wave is guided by the transmission line, λ often is called the guide wavelength.

Wave Propagation on a Transmission Line

To gain a physical understanding of what that means, we shall first examine the relatively simple but important case of a lossless line ($\alpha = 0$) and then extend the results to the more general case of a lossy transmission line ($\alpha \neq 0$).

In fact, we shall devote the next several sections to the study of lossless transmission lines because in practice many lines can be designed to exhibit very low-loss characteristics.

Example 2-1: Air Line

An air line is a transmission line in which air separates the two conductors, which renders $G' = 0$ because $\sigma = 0$. In addition, assume that conductors are made of a material with high conductivity so that $R' = 0$. For an air line with a characteristic impedance of 50Ω and a phase constant of $20\ rad/m$ at $700MHz$, ,find the line inductance L' and the line capacitance C'.

Solution: The following quantities are given:

$$Z_0 = 50\Omega$$

$$\beta = 20\text{rad/m}$$

$$f = 700MHz = 7\times10^8\ Hz$$

Wave Propagation on a Transmission line

With $R' = G' = 0$,

$$\beta = \Im m\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right)$$

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

reduce to

$$\beta = \mathrm{Im}\left[\sqrt{(j\omega L')(j\omega C')}\right] = \mathrm{Im}\left[j\omega\sqrt{L'C'}\right] = \omega\sqrt{L'C'}$$

$$Z_0 = \sqrt{\frac{j\omega L'}{j\omega C'}} = \sqrt{\frac{L'}{C'}}$$

The ratio of β to Z_0 is

$$\frac{\beta}{Z_0} = \omega C'$$

or

$$C' = \frac{\beta}{\omega Z_0} = \frac{20}{2\pi \times 7 \times 10^8 \times 50} = 9.09 \times 10^{-11} \left(F / \mathrm{m} \right)$$
$$= 90.\ 9 \left(pF/m \right)$$

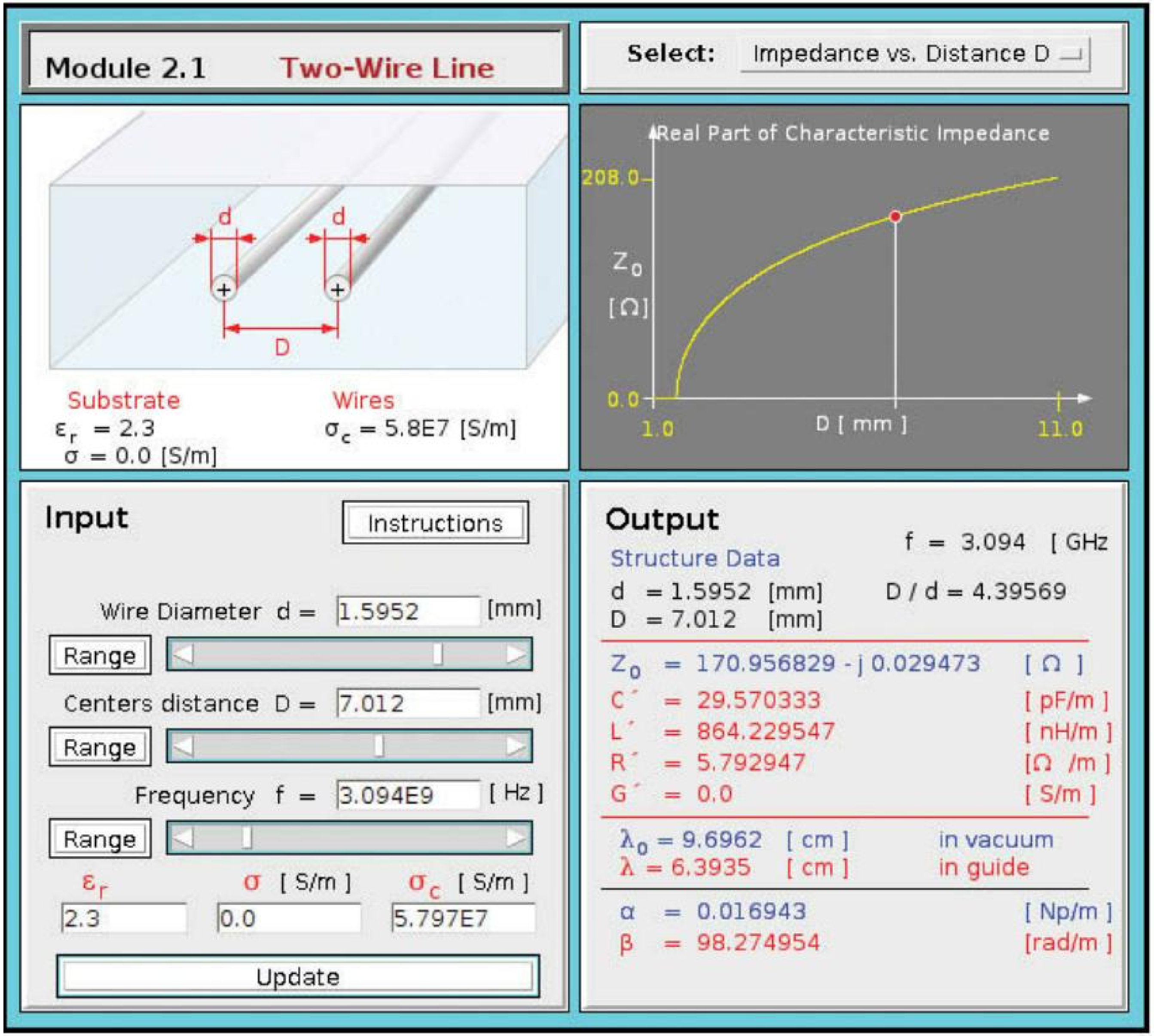
From $Z_0 = \sqrt{L'/C'}$, it follows that

$$L' = Z_0^2 C' = \left(50\right)^2 \times 90.9 \times 10^{-12} = 2.27 \times 10^{-7} \ \left(H/m\right)$$
$$= 227 \left(\mathrm{n}H/m\right)$$

Wave Propagation on a Transmission Line

Module2.1

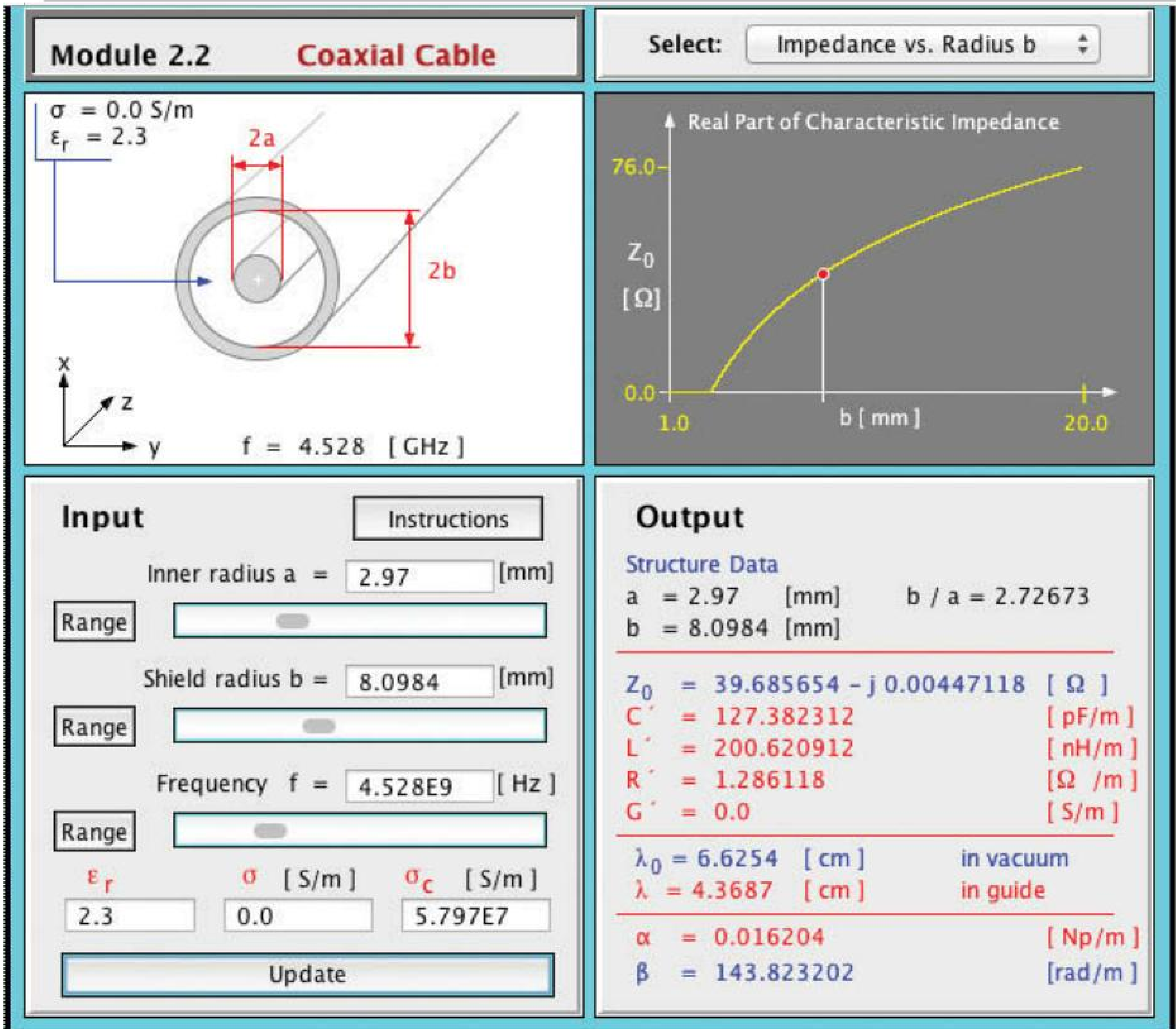
The input data specifies the geometric and electric parameters of a two-wire transmission line. The output includes the calculated values for the line parameters, characteristic impedance Z_0, and attenuation and phase constants, as well as plots of Z_0 as a function of d and D.



Wave Propagation on a Transmission Line

Module2.2

Coaxial Cable Except for changing the geometric parameters to those of a coaxial transmission line, this module offers the same output information as Module 2.1.



Exercise 2-3:

Verify that $\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}(V)$ indeed provides a solution to the wave equation $\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0$.

Answer:

$$\frac{d\tilde{V}(z)}{dz} = V_0^+ e^{-\gamma z}(-\gamma) + V_0^- e^{\gamma z}\cdot(\gamma)$$

$$\frac{d^2\tilde{V}(z)}{dz^2} = V_0^+ e^{-\gamma z}\gamma^2 + V_0^- e^{\gamma z}\gamma^2$$

$$= \gamma^2\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right)$$

$$= \gamma^2\tilde{V}(z)$$

Hence, $\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}(V)$ indeed provides a solution to the wave equation $\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0$.

Exercise 2-4:

A two-wire air line has the following line parameters: $R' = 0.404(m\Omega/m)$, $L' = 2.0(\mu H/m)$, $G' = 0$, and $C' = 5.56(pF/m)$. For operation at 5 kHz, determine (a) the attenuation constant α, (b) the phase constant β, (c) the phase velocity u_p, and (d) the characteristic impedance Z_0.

Answer:

The following quantities are given:

$$R' = 0.404\times10^{-3}\,(\Omega/m)$$

$$L' = 2.0\times10^{-6}\,(H/m)$$

$$G' = 0$$

$$C' = 5.56\times10^{-12}\,(F/m)$$

Exercise

(a) $\alpha = \mathrm{Re}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) = 3.37\times10^{-7}\ (Np/m)$

(b) $\beta = \mathrm{Im}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) = 1.05\times10^{-4}\ (rad/m)$

(c) $u_p = \dfrac{\omega}{\beta} = \dfrac{2\pi f}{\beta} = 3.0\times10^{8}\ (m/s)$

(d) $Z_0 = \sqrt{\dfrac{R' + j\omega L'}{G' + j\omega C'}} = (600 - j1.9)\ (\Omega)$

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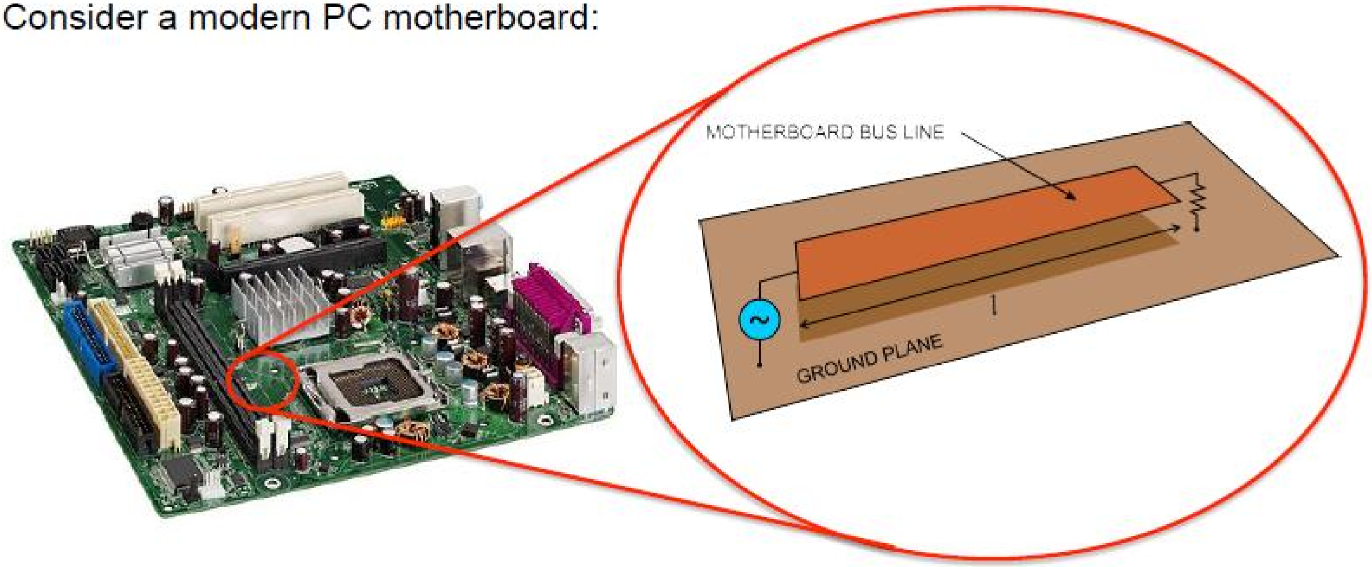
Lecture 5

The Lossless Microstrip Line

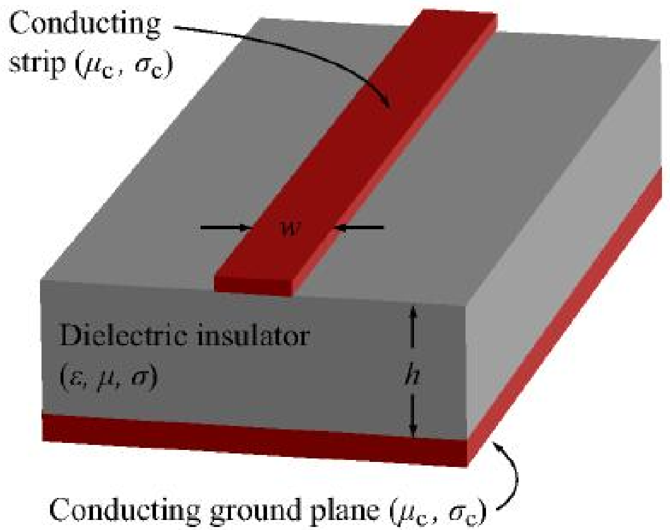
The Lossless Microstrip Line

Consider a modern PC motherboard:



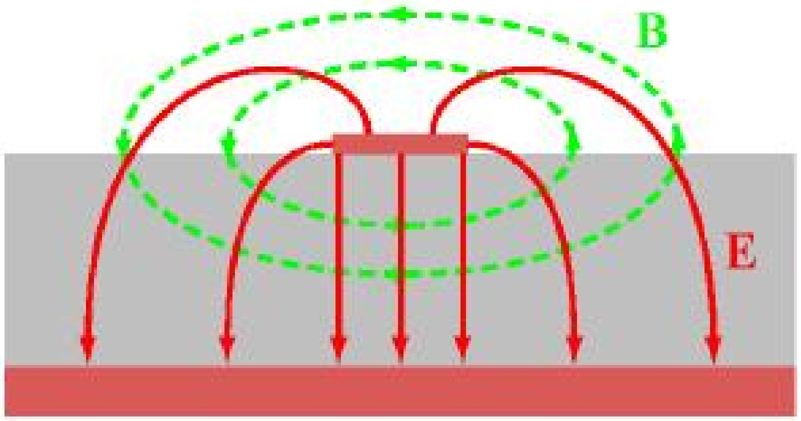
- Microstrip technology is the most commonly used TL in practice, due to ease of implementation and low-cost PCB fabrication.
- Transmission line is defined by a top metallic strip elevated above some type of material.
- This material is typically a dielectric material of low loss and it is attached to a metallic ground plane.

The Lossless Microstrip Line



- Presence of charges of opposite polarity on its two conducting sides gives rise to electric field lines.
- Considering a HF source, fields are time-varying.
- Due to Maxwell’s Equations a magnetic field is also field generated.
- The microstrip line has two geometric parameters: the width of the elevated strip, *w*, and the thickness (height) of the dielectric layer, *h*.

The Lossless Microstrip Line



- Patterns of E and H (or B) are not always perpendicular.
- This does not define a pure transverse electromagnetic wave (TEM).
- Around the regions of the conductors, field lines have highest intensity and are generally orthogonal.
- Microstrip is considered a quasi-TEM transmission line (TL).
- Can apply fundamental TL theory to practical high frequency circuit design when using microstrip.

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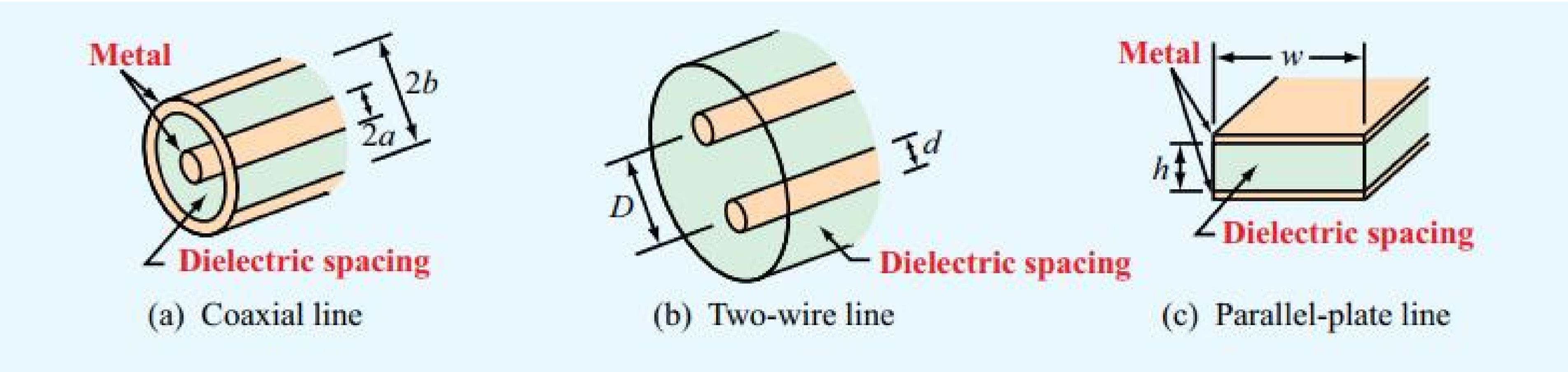
The Lossless Microstrip Line



For the coaxial, two-wire, and parallel-plate lines, the field lines are confined to the region between the conductors. A characteristic attribute of such transmission lines is that the phase velocity of a wave traveling along any one of them is given by

$$u_p = \frac{c}{\sqrt{\varepsilon_r}}$$

where *c* is the velocity of light in free space and ε_r is the relative permittivity of the dielectric medium between the conductors.

The Lossless Microstrip Line

In the microstrip line, nonuniform mixture can be accounted for by defining an effective relative permittivity ε_{eff} such that the phase velocity is given by an expression that resembles, namely

$$u_p = \frac{c}{\sqrt{\varepsilon_{eff}}}$$

It is possible to use curve-fit approximations to rigorous solutions to arrive at the following set of expressions:

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \left(\frac{\varepsilon_r - 1}{2}\right)\left(1 + \frac{10}{s}\right)^{-xy}$$

where s is the width-to-thickness ratio

$$s = \frac{w}{h}$$

The Lossless Microstrip Line

and x and y are intermediate variables given by

$$x = 0.56\left[\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3}\right]^{0.05}$$

$$y = 1 + 0.02\ln\left(\frac{s^4 + 3.7\times10^{-4}s^2}{s^4 + 0.43}\right) + 0.05\ln\left(1 + 1.7\times10^{-4}s^3\right)$$

The characteristic impedance of the microstrip line is given by

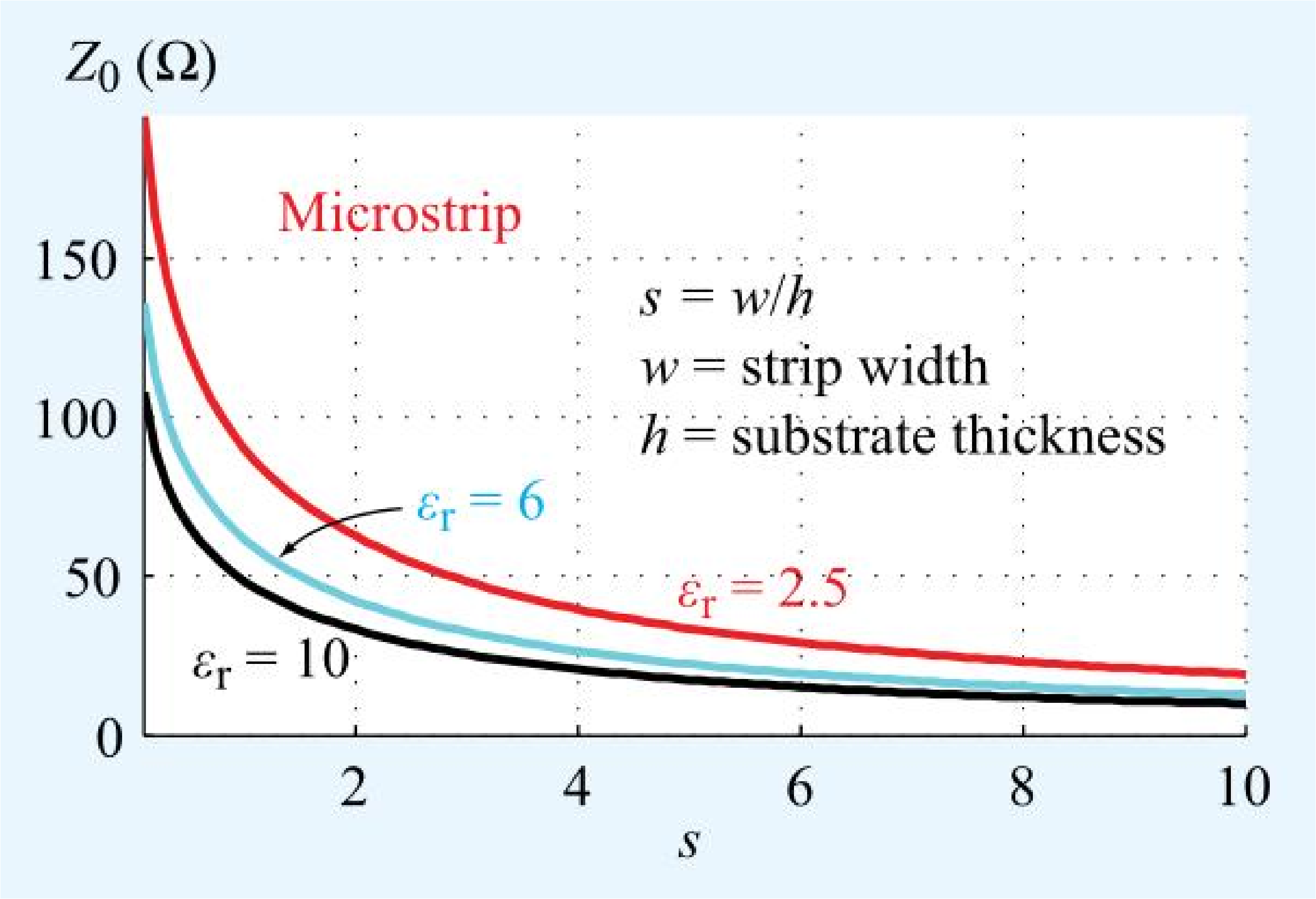
$$Z_0 = \frac{60}{\sqrt{\varepsilon_{eff}}}\ln\left\{\frac{6 + (2\pi - 6)e^{-t}}{s} + \sqrt{1 + \frac{4}{s^2}}\right\}$$

with

$$t = \left(\frac{30.67}{s}\right)^{0.75}$$

The Lossless Microstrip Line

Figure 2-11 displays plots of Z_0 as a function of s for various types of dielectric materials：



The Lossless Microstrip Line

The corresponding line and propagation parameters are given by

$$R' = 0 \qquad \text{(Because } \sigma_c = \infty\text{)}$$

$$G' = 0 \qquad \text{(Because } \sigma = \infty\text{)}$$

$$\alpha = 0 \qquad \text{(Because } R' = G' = 0\text{)}$$

$$L' = Z_0^2 C'$$

$$C' = \frac{\sqrt{\varepsilon_{eff}}}{Z_0^2 c}$$

$$\beta = \frac{\omega}{c}\sqrt{\varepsilon_{eff}}$$

The Lossless Microstrip Line

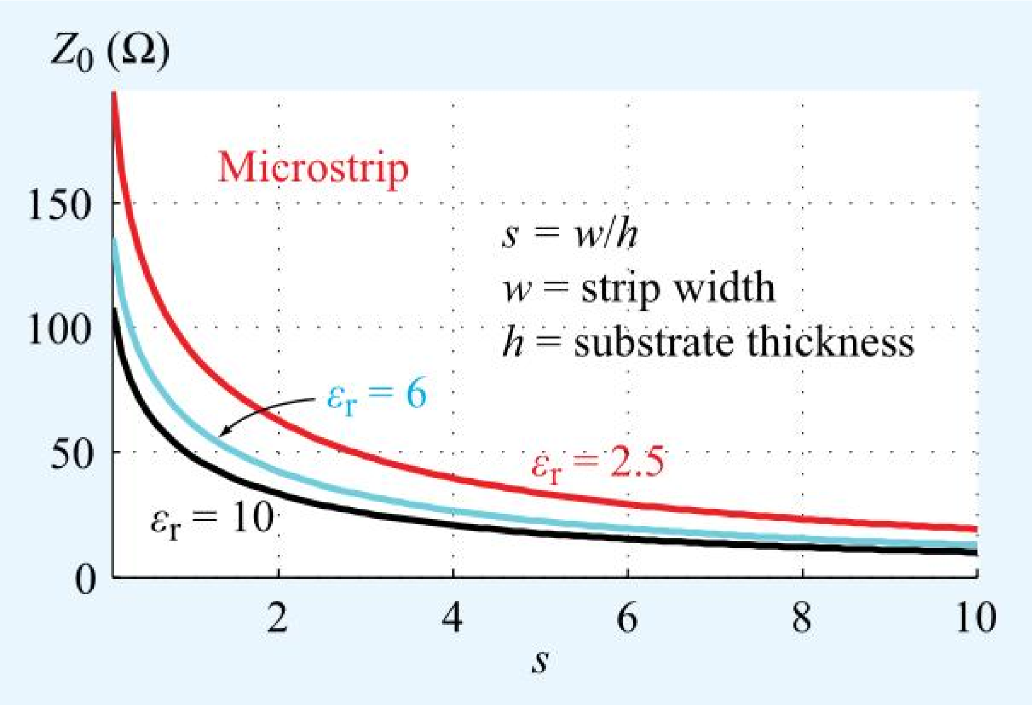
The preceding expressions allow us to compute the values of Z_0 and the other propagation parameters when given values for ε_r, h, and ω. This is exactly what is needed in order to analyze a circuit containing a microstrip transmission line.

To perform the reverse process, namely to design a microstrip line by selecting values for its ω and h such that their ratio yields the required value of Z_0 . we need to express s in terms of Z_0 given by

$$Z_0 = \frac{60}{\sqrt{\varepsilon_{eff}}} \ln\left\{\frac{6+(2\pi-6)e^{-t}}{s} + \sqrt{1+\frac{4}{s^2}}\right\}$$

is rather complicated, so inverting it to obtain an expression for s in terms Z_0 is rather difficult.

The Lossless Microstrip Line



An alternative option is to generate a family of curves similar to those displayed in figure and to use them to estimate s for a specified value of Z_0.

The Lossless Microstrip Line

A logical extension of the graphical approach is to generate curve-fit expressions that provide high-accuracy estimates of *s*

(a) For $Z_0 \le (44 - 2\varepsilon_r)\ \Omega$

$$s = \frac{\omega}{h} = \frac{2}{\pi}\left\{(q-1) - \ln(2q-1) + \frac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(q-1) + 0.29 - \frac{0.52}{\varepsilon_r}\right]\right\}$$

where, $q = \dfrac{60\pi^2}{Z_0\sqrt{\varepsilon_r}}$

The error associated with the following formulas is less than 2%:

The Lossless Microstrip Line

A logical extension of the graphical approach is to generate curve-fit expressions that provide high-accuracy estimates of *s* :

(b) For $Z_0 \geq \left(44 - 2\varepsilon_r\right)\Omega$

$$s = \frac{\omega}{h} = \frac{8\mathrm{e}^{\mathrm{p}}}{\mathrm{e}^{2\mathrm{p}} - 2}$$

where,

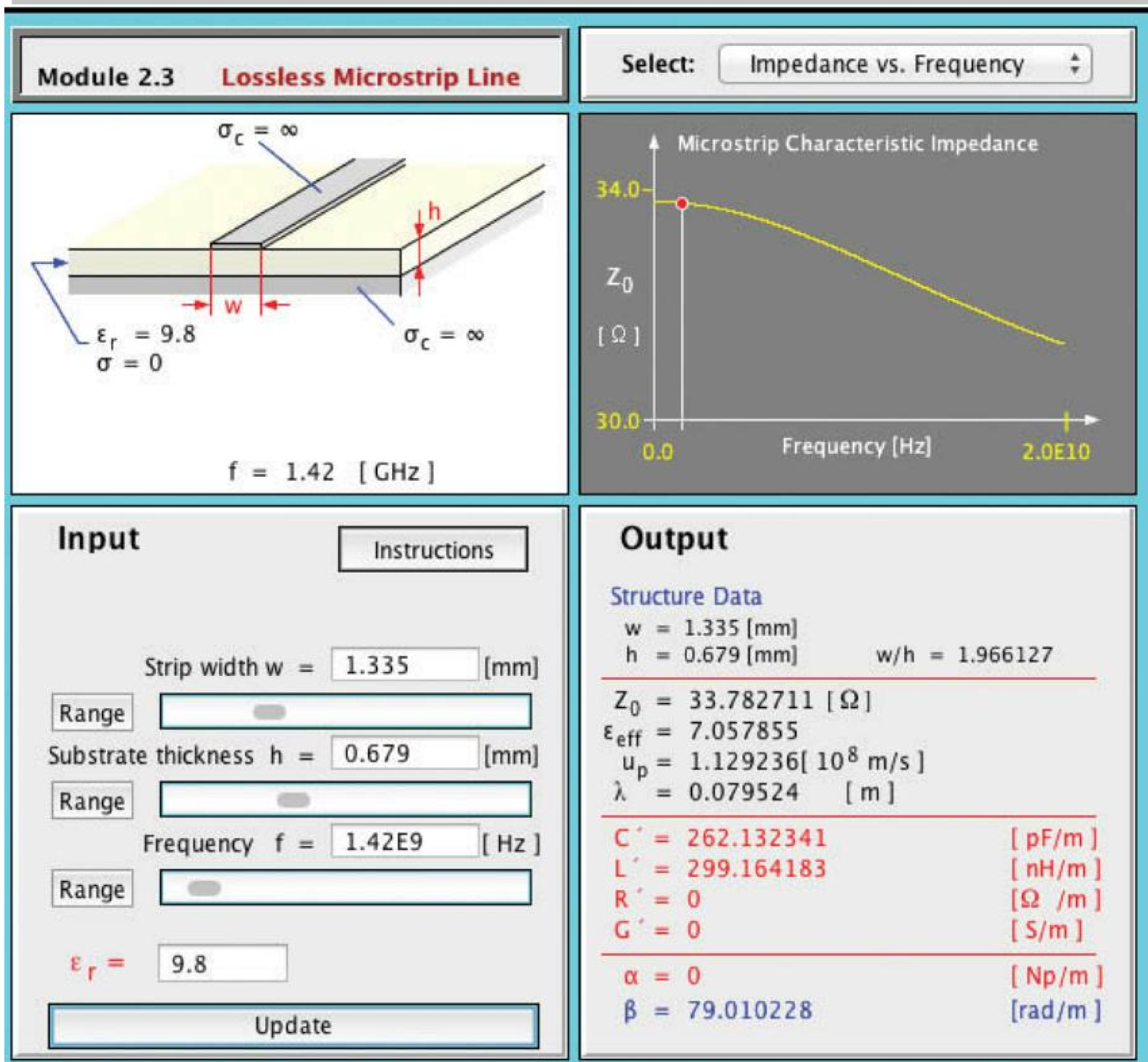
$$p = \sqrt{\frac{\varepsilon_r + 1}{2}} \frac{Z_0}{60} + \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left(0.23 + \frac{0.12}{\varepsilon_r}\right)$$

The Lossless Microstrip Line

Module 2.3

Lossless Microstrip Line

The output panel lists the values of the transmission-line parameters and displays the variation of Z_0 and ε_{eff} with h and ω.



Example 2-2: Microstrip Line

A 50Ω microstrip line uses a 0.5 mm thick sapphire substrate with $\varepsilon_r = 9$. What is the width of its copper strip?

Solution: Since $Z_0 = 50 > 44 - 18 = 32$, we should use

$$p = \sqrt{\frac{\varepsilon_r + 1}{2}} * \frac{Z_0}{60} + \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left(0.23 + \frac{0.12}{\varepsilon_r}\right)$$

$$= \sqrt{\frac{9+1}{2}} * \frac{50}{60} + \left(\frac{9-1}{9+1}\right)\left(0.23 + \frac{0.12}{9}\right) = 0.26$$

$$s = \frac{\omega}{h} = \frac{8e^{p}}{e^{2p} - 2} = \frac{8e^{2.06}}{e^{4.12} - 2} = 1.056$$

The Lossless Microstrip Line

Hence, $\omega = sh = 1.056 \times 0.5\ mm = 0.53\ mm$

To check our calculations, we use s = 1.056 to calculate Z_0 to verify that the value we obtained is indeed equal or close to 50Ω. With ε_r = 9,

$$x = 0.56\left[\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3}\right]^{0.05}$$

$$y = 1 + 0.02\ln\left(\frac{s^4 + 3.7*10^{-4}s^2}{s^4 + 0.43}\right) + 0.05\ln\left(1 + 1.7*10^{-4}s^3\right)$$

$$s = \frac{w}{h}$$

$$t = \left(\frac{30.67}{s}\right)^{0.75}$$

The Lossless Microstrip Line

$$\varepsilon_{eff}=\frac{\varepsilon_r+1}{2}+\left(\frac{\varepsilon_r-1}{2}\right)\left(1+\frac{10}{s}\right)^{-xy}$$

$$Z_0=\frac{60}{\sqrt{\varepsilon_{eff}}}\ln\left\{\frac{6+(2\pi-6)e^{-t}}{s}+\sqrt{1+\frac{4}{s^2}}\right\}$$

yield,

$$x=0.55 \qquad y=0.99 \qquad \varepsilon_{\text{eff}}=6.11$$

$$t=12.51 \qquad Z_0=49.93\,\Omega$$

The calculated value of Z_0 is, for all practical purposes, equal to the value specified in the problem statement.