西安电子科技大学
XIDIAN UNIVERSITY

B39HF High Frequency Circuits

Lecture 5
The Lossless Microstrip Line

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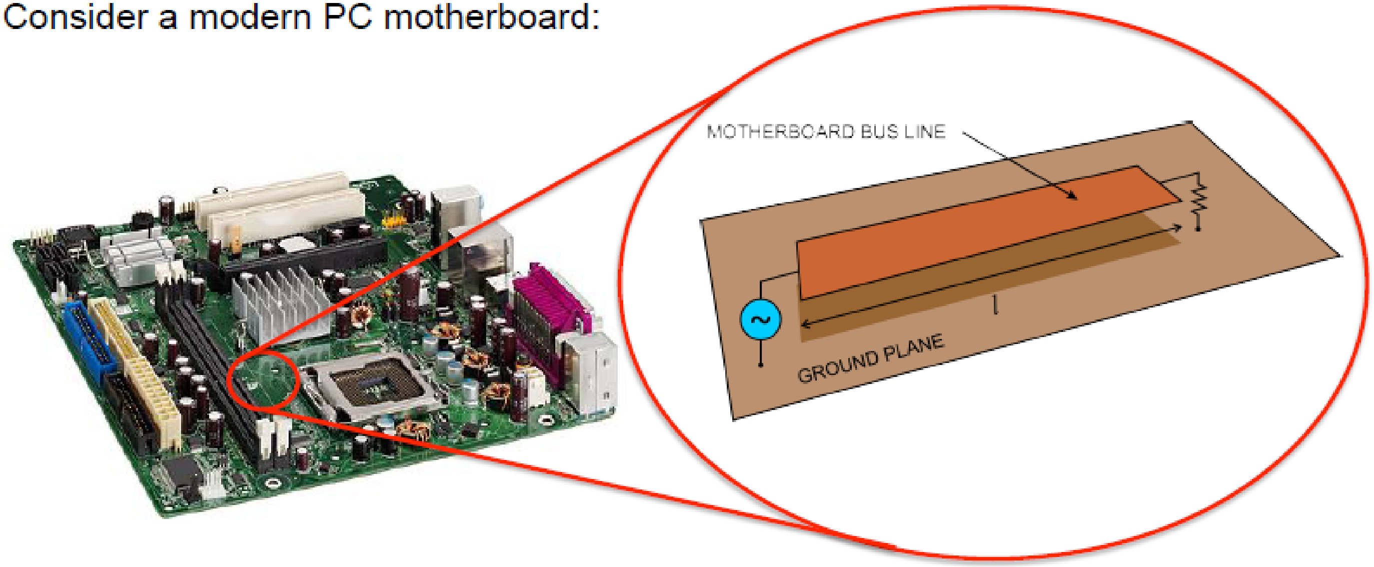
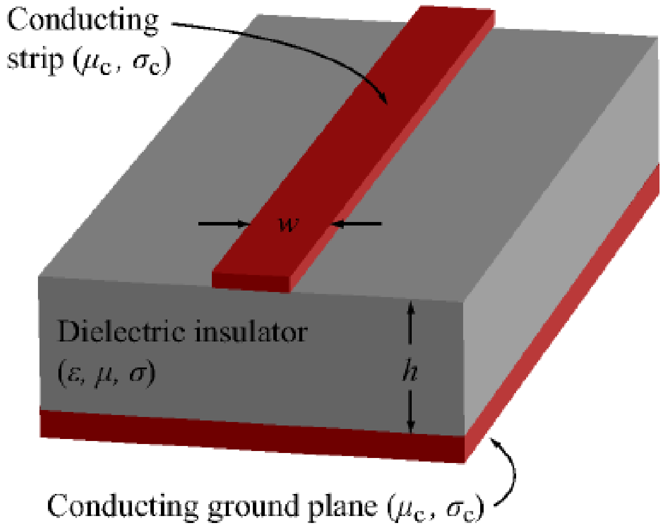
The Lossless Microstrip Line

Consider a modern PC motherboard:



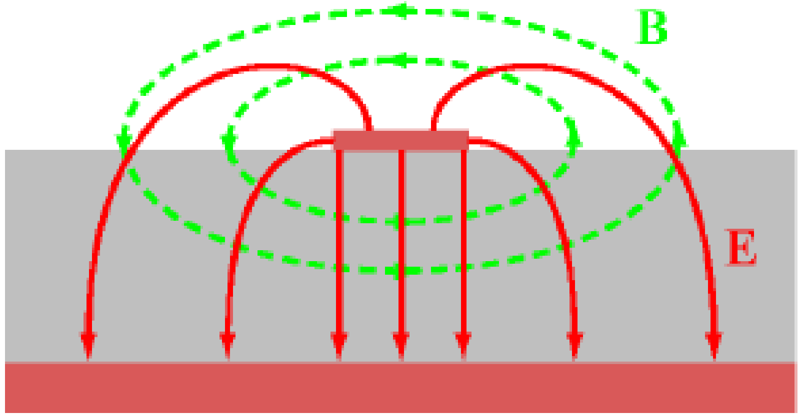
- Microstrip technology is the most commonly used TL in practice, due to ease of implementation and low-cost PCB fabrication.
- Transmission line is defined by a top metallic strip elevated above some type of material.
- This material is typically a dielectric material of low loss and it is attached to a metallic ground plane.

The Lossless Microstrip Line



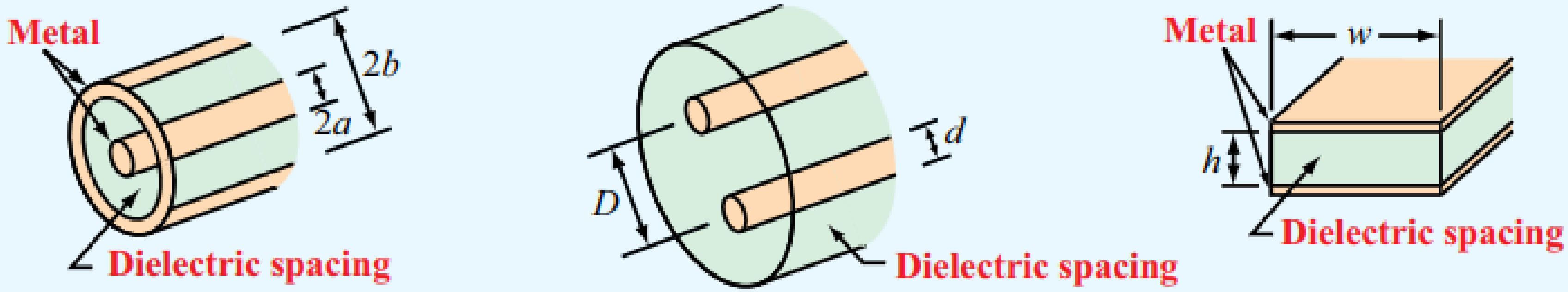
- Presence of charges of opposite polarity on its two conducting sides gives rise to electric field lines.
- Considering a HF source, fields are time-varying.
- Due to Maxwell's Equations a magnetic field is also field generated.
- The microstrip line has two geometric parameters: the width of the elevated strip, w, and the thickness (height) of the dielectric layer, h.

The Lossless Microstrip Line



- Patterns of E and H (or B) are not always perpendicular.
- This does not define a pure transverse electromagnetic wave (TEM).
- Around the regions of the conductors, field lines have highest intensity and are generally orthogonal.
- Microstrip is considered a quasi-TEM transmission line (TL).
- Can apply fundamental TL theory to practical high frequency circuit design when using microstrip.

The Lossless Microstrip Line



(a) Coaxial line (b) Two-wire line (c) Parallel-plate line

For the coaxial, two-wire, and parallel-plate lines, the field lines are confined to the region between the conductors. A characteristic attribute of such transmission lines is that the phase velocity of a wave traveling along any one of them is given by

$$u_p = \frac{c}{\sqrt{\varepsilon_r}}$$

where c is the velocity of light in free space and ε_r is the relative permittivity of the dielectric medium between the conductors.

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In the microstrip line, nonuniform mixture can be accounted for by defining an effective relative permittivity ε_{eff} such that the phase velocity is given by an expression that resembles, namely

$$u_p = \frac{c}{\sqrt{\varepsilon_{eff}}}$$

It is possible to use curve-fit approximations to rigorous solutions to arrive at the Following set of expressions:

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \left(\frac{\varepsilon_r - 1}{2}\right)\left(1 + \frac{10}{s}\right)^{-xy}$$

where s is the width-to-thickness ratio

$$s = \frac{w}{h}$$

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and x and y are intermediate variables given by

$$x = 0.56\left[\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3}\right]^{0.05}$$

$$y = 1 + 0.02\ln\left(\frac{s^4 + 3.7\times 10^{-4}s^2}{s^4 + 0.43}\right) + 0.05\ln\left(1 + 1.7\times 10^{-4}s^3\right)$$

The characteristic impedance of the microstrip line is given by

$$Z_0 = \frac{60}{\sqrt{\varepsilon_{eff}}}\ln\left\{\frac{6 + (2\pi - 6)e^{-t}}{s} + \sqrt{1 + \frac{4}{s^2}}\right\}$$

with

$$t = \left(\frac{30.67}{s}\right)^{0.75}$$

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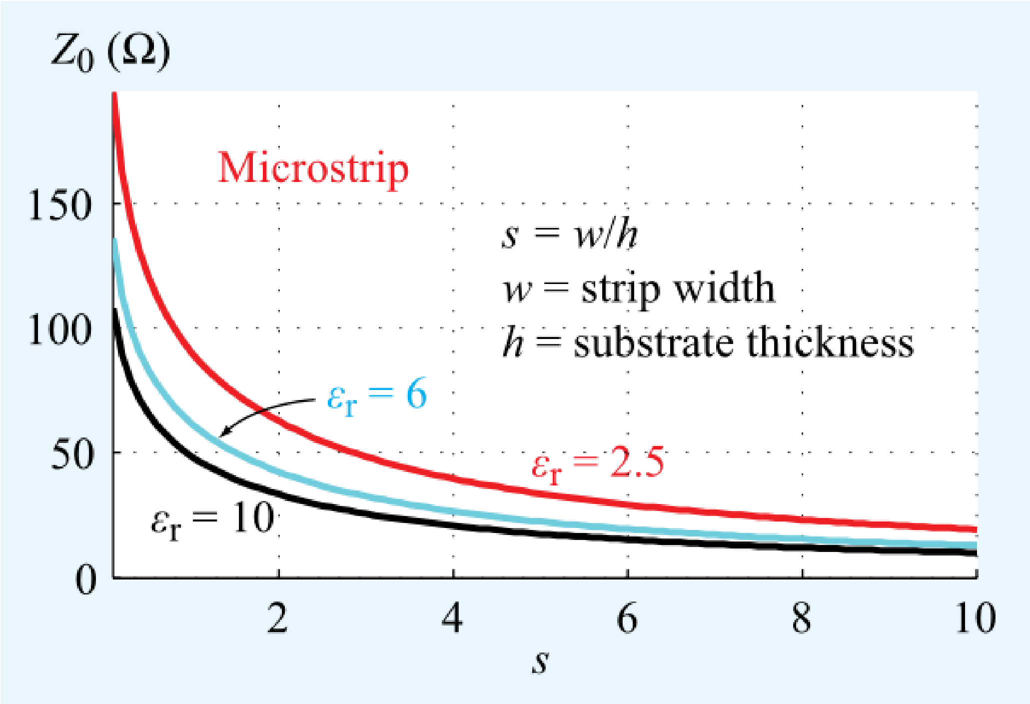
Figure 2-11 displays plots of Z_0 as a function of s for various types of dielectric materials：



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The corresponding line and propagation parameters are given by

$$R' = 0 \quad \text{(Because } \sigma_c = \infty\text{)}$$

$$G' = 0 \quad \text{(Because } \sigma = \infty\text{)}$$

$$\alpha = 0 \quad \text{(Because } R' = G' = 0\text{)}$$

$$L' = Z_0^2 C'$$

$$C' = \frac{\sqrt{\varepsilon_{eff}}}{Z_0^2 c}$$

$$\beta = \frac{\omega}{c}\sqrt{\varepsilon_{eff}}$$

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The preceding expressions allow us to compute the values of Z_0 and the other propagation parameters when given values for ε_r, h, and ω. This is exactly what is needed in order to analyze a circuit containing a microstrip transmission line.

To perform the reverse process, namely to design a microstrip line by selecting values for its ω and h such that their ratio yields the required value of Z_0 (to satisfy design specifications), we need to express s in terms of Z_0 . The expression for Z_0 given by $Z_0 = \frac{60}{\sqrt{\varepsilon_{eff}}} \ln\left\{\frac{6+(2\pi-6)e^{-t}}{s} + \sqrt{1+\frac{4}{s^2}}\right\}$ is rather complicated, so inverting it to obtain an expression for s in terms of Z_0 is rather difficult.

The Lossless Microstrip Line

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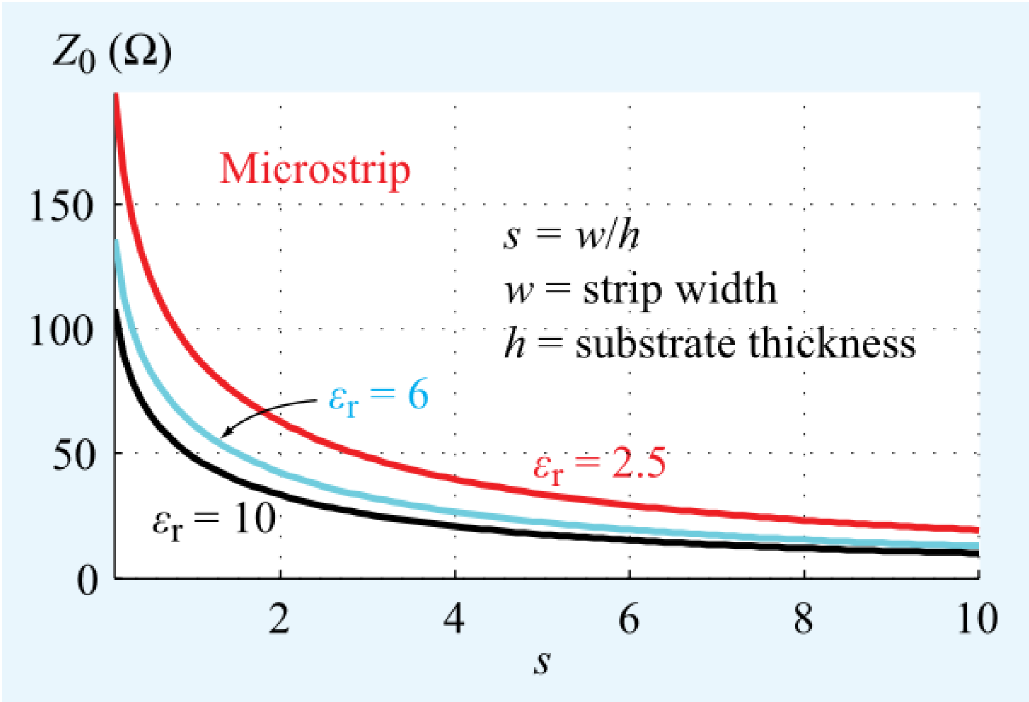


An alternative option is to generate a family of curves similar to those displayed in figure and to use them to estimate s for a specified value of Z_0.

The Lossless Microstrip Line

A logical extension of the graphical approach is to generate curve-fit expressions that provide high-accuracy estimates of s，The error associated with the following formulas is less than 2%:

(a) For $Z_0 \le (44 - 2\varepsilon_r)\Omega$

$$s = \frac{\omega}{h} = \frac{2}{\pi}\left\{(q-1) - \ln(2q-1) + \frac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(q-1) + 0.29 - \frac{0.52}{\varepsilon_r}\right]\right\}$$

where, $q = \dfrac{60\pi^2}{Z_0\sqrt{\varepsilon_r}}$

The error associated with the following formulas is less than 2%:

The Lossless Microstrip Line

A logical extension of the graphical approach is to generate curve-fit expressions that provide high-accuracy estimates of s :

(b) For $Z_0 \geq (44 - 2\varepsilon_r)\Omega$

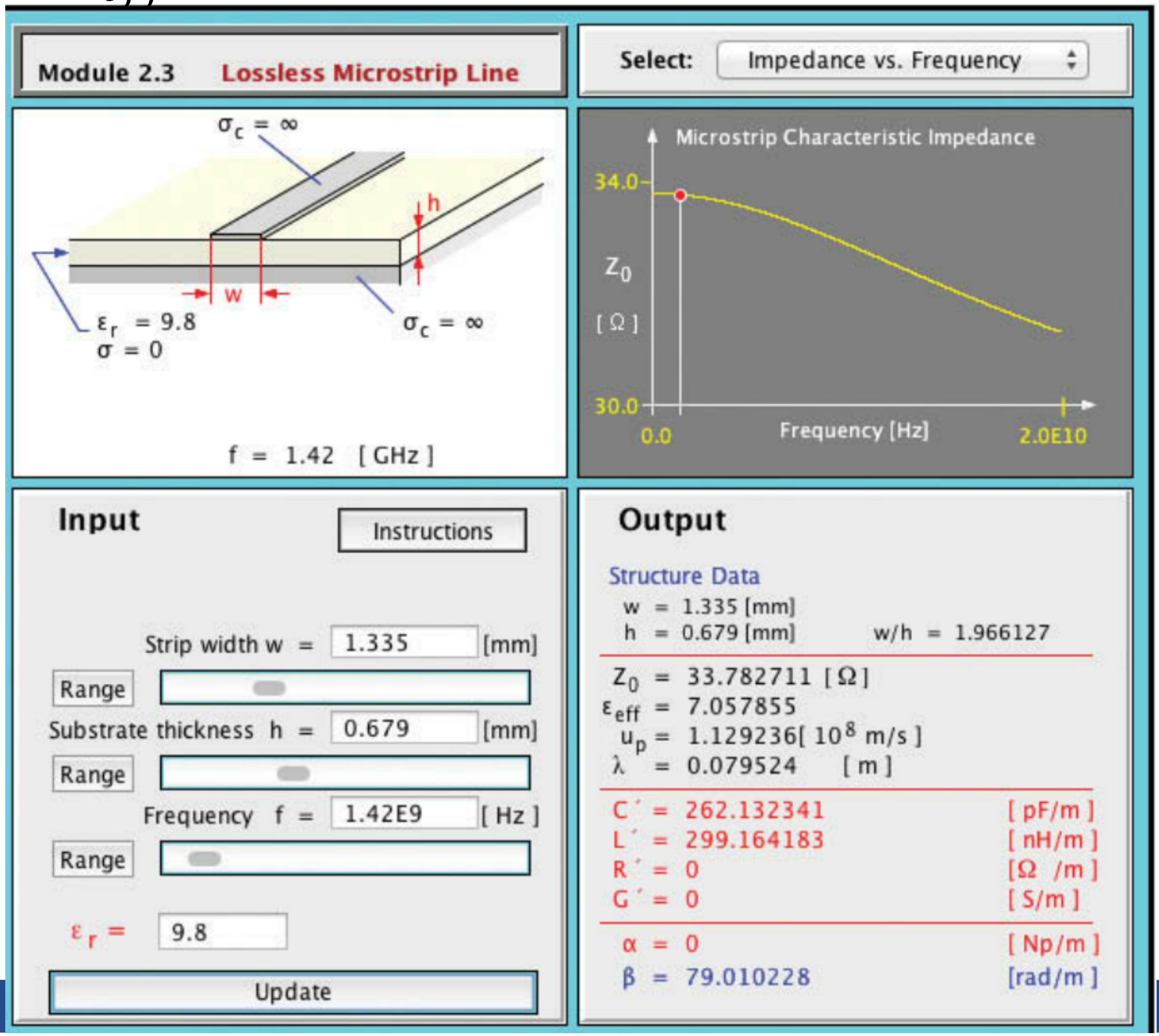
$$s = \frac{\omega}{h} = \frac{8e^{p}}{e^{2p} - 2}$$

where, $$p = \sqrt{\frac{\varepsilon_r + 1}{2}\frac{Z_0}{60}} + \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left(0.23 + \frac{0.12}{\varepsilon_r}\right)$$

The Lossless Microstrip Line

Module 2.3 Lossless Microstrip Line

The output panel lists the values of the transmission-line parameters and displays the variation of Z_0 and ε_{eff} with h and ω.



Example 2-2: Microstrip Line

A 50Ω microstrip line uses a 0.5 mm thick sapphire substrate with $\varepsilon_r = 9$. What is the width of its copper strip?

Solution: Since $Z_0 = 50 > 44 - 18 = 32$, we should use

$$p = \sqrt{\frac{\varepsilon_r + 1}{2}} * \frac{Z_0}{60} + \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left(0.23 + \frac{0.12}{\varepsilon_r}\right)$$

$$= \sqrt{\frac{9+1}{2}} * \frac{50}{60} + \left(\frac{9-1}{9+1}\right)\left(0.23 + \frac{0.12}{9}\right) = 2.06$$

$$s = \frac{\omega}{h} = \frac{8e^{p}}{e^{2p} - 2} = \frac{8e^{2.06}}{e^{4.12} - 2} = 1.056$$

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Hence, $\omega = sh = 1.056 \times 0.5\ mm = 0.53\ mm$

To check our calculations, we use $s = 1.056$ to calculate Z_0 to verify that the value we obtained is indeed equal or close to 50Ω. With ε_r = 9,

$$x = 0.56\left[\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3}\right]^{0.05}$$

$$y = 1 + 0.02\ln\left(\frac{s^4 + 3.7*10^{-4}s^2}{s^4 + 0.43}\right) + 0.05\ln\left(1 + 1.7*10^{-4}s^3\right)$$

$$s = \frac{w}{h}$$

$$t = \left(\frac{30.67}{s}\right)^{0.75}$$

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$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \left(\frac{\varepsilon_r - 1}{2}\right)\left(1 + \frac{10}{s}\right)^{-xy}$$

$$Z_0 = \frac{60}{\sqrt{\varepsilon_{eff}}} \ln\left\{\frac{6 + (2\pi - 6)e^{-t}}{s} + \sqrt{1 + \frac{4}{s^2}}\right\}$$

yield,

$x = 0.55$ $y = 0.99$ $\varepsilon_{\text{eff}} = 6.11$

$t = 12.51$ $Z_0 = 49.93\Omega$

The calculated value of Z_0 is, for all practical purposes, equal to the value specified in the problem statement.