西安電子科技大學
XIDIAN UNIVERSITY

B39HF High Frequency Circuits

Lecture 1 Introduction

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Outline and Review

- Development of Electromagnetics
- Dimensions, Units, and Notation
- The Nature of Electromagnetism
- Traveling Waves
- The Electromagnetic Spectrum
- Review of Complex Numbers
- Review of Phasors

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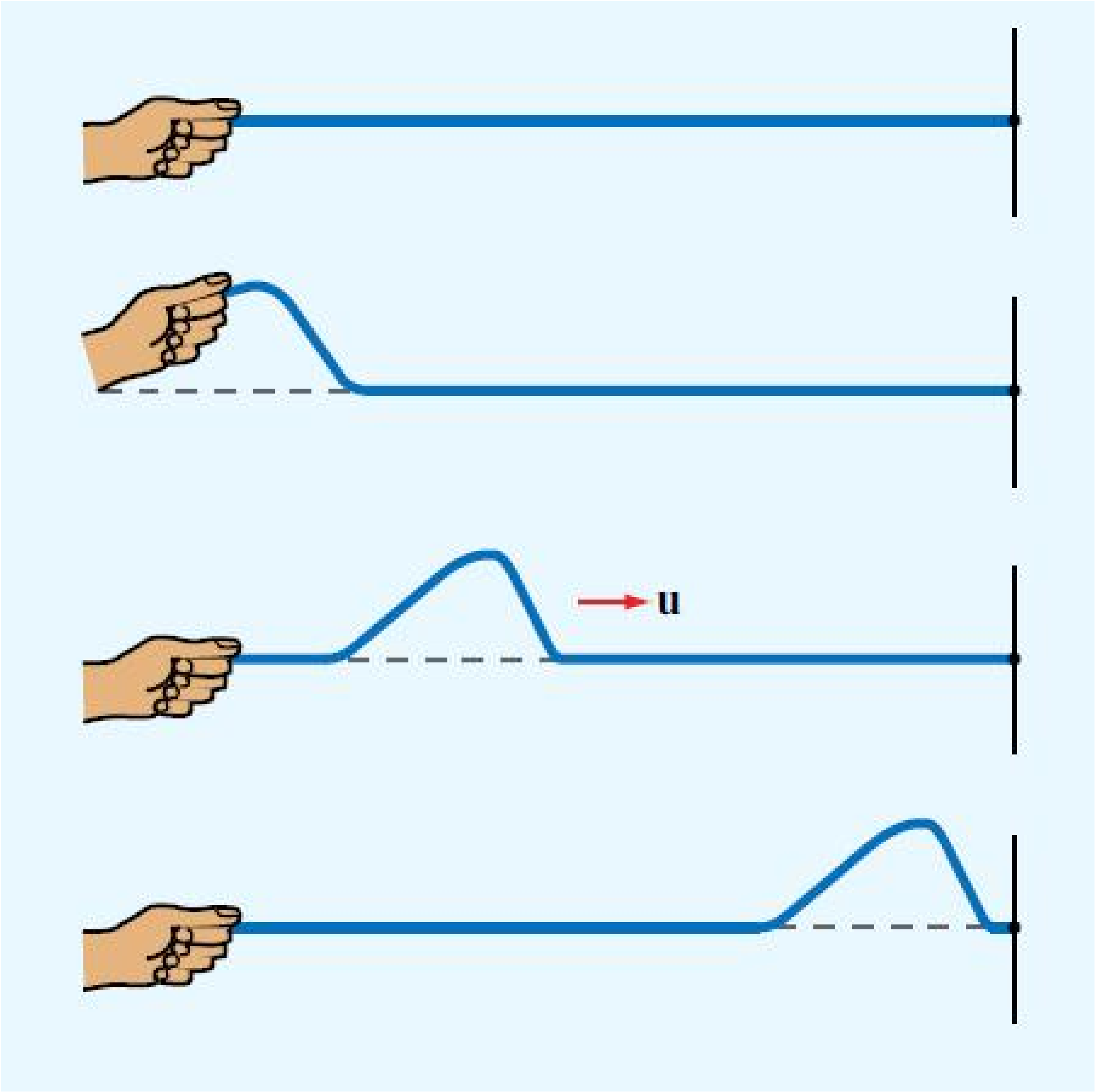
Traveling Waves

A one-dimensional wave traveling on a string



Traveling Waves

- **Two-dimensional wave:** propagates out across a surface, like the ripples on a pond , and its disturbance can be described by two space variables.
- **Three-dimensional wave:** propagates through a volume and its disturbance may be a function of all three space variables. may take on many different shapes; include ***plane waves***, ***cylindrical waves***, and ***spherical waves***.

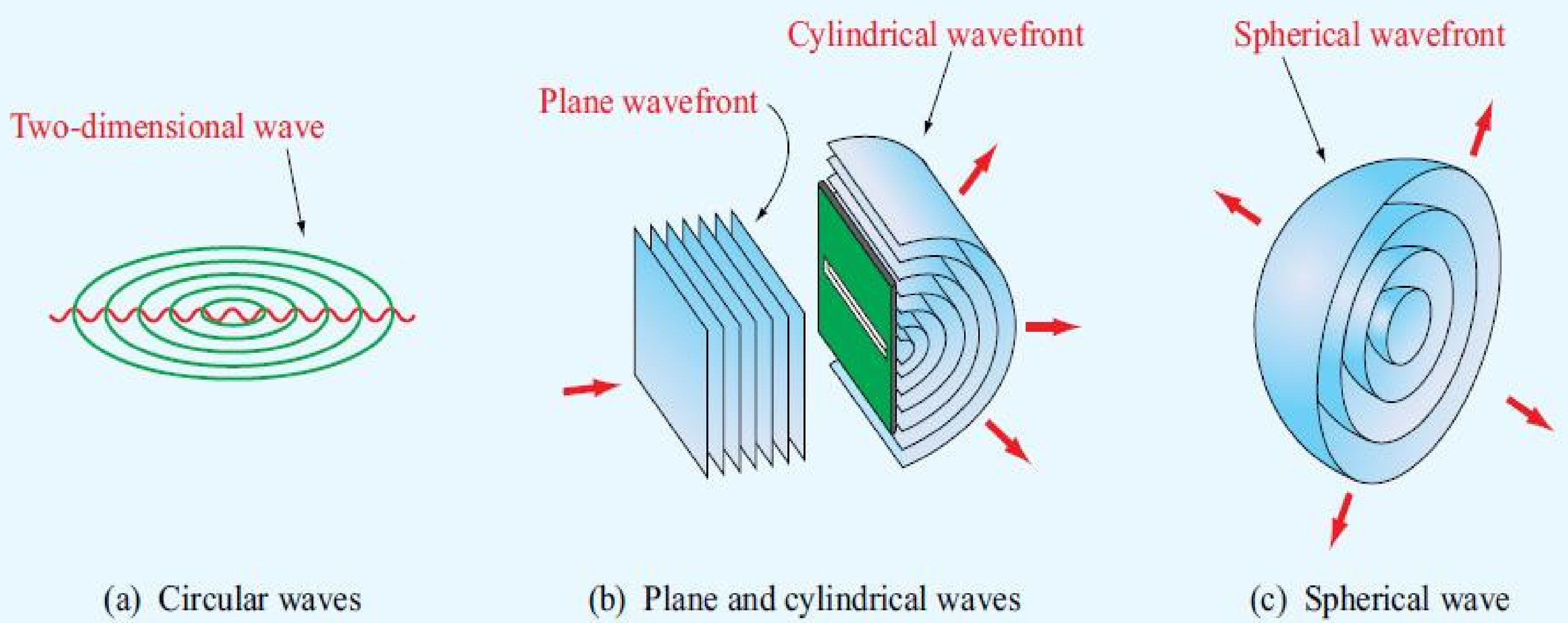
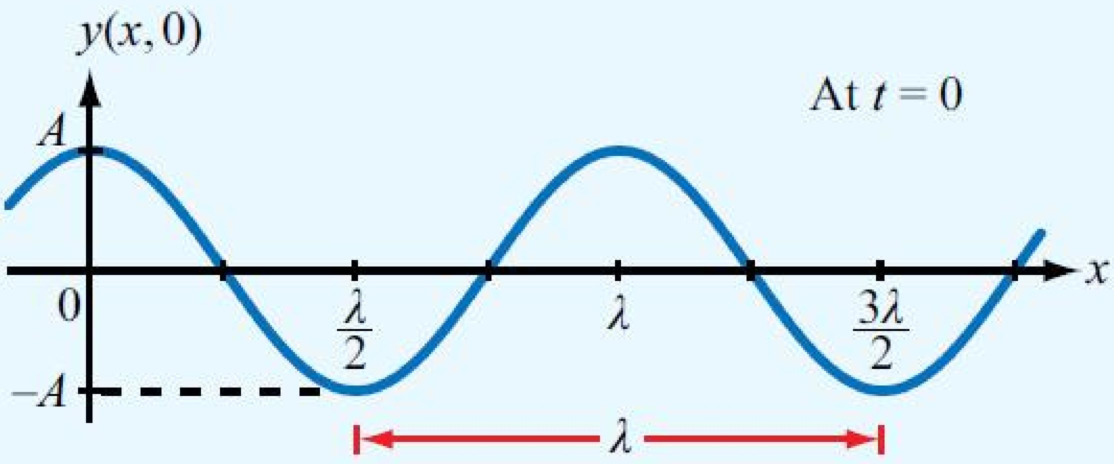
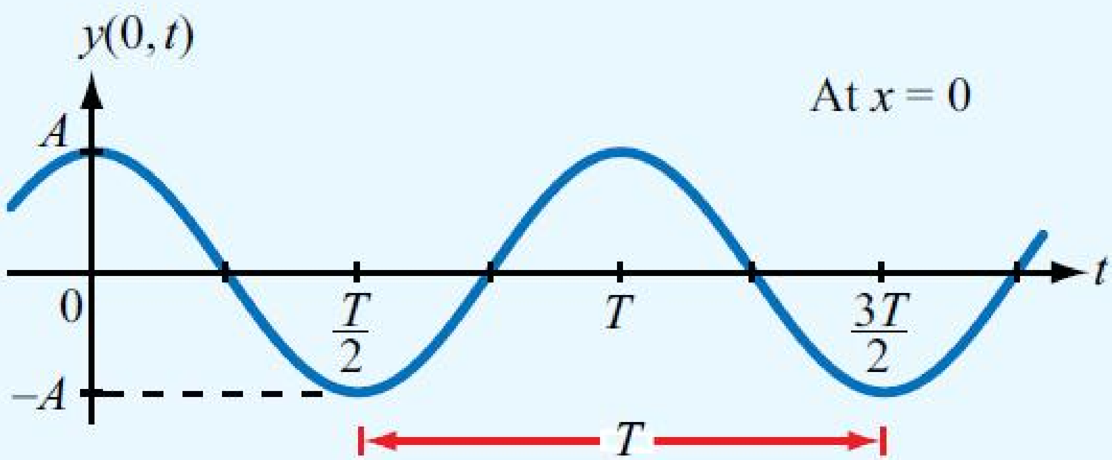


Figure 1-11 Examples of two-dimensional and three-dimensional waves: (a) circular waves on a pond, (b) a plane light wave exciting a cylindrical light wave through the use of a long narrow slit in an opaque screen, and (c) a sliced section of a spherical wave.

Traveling Waves

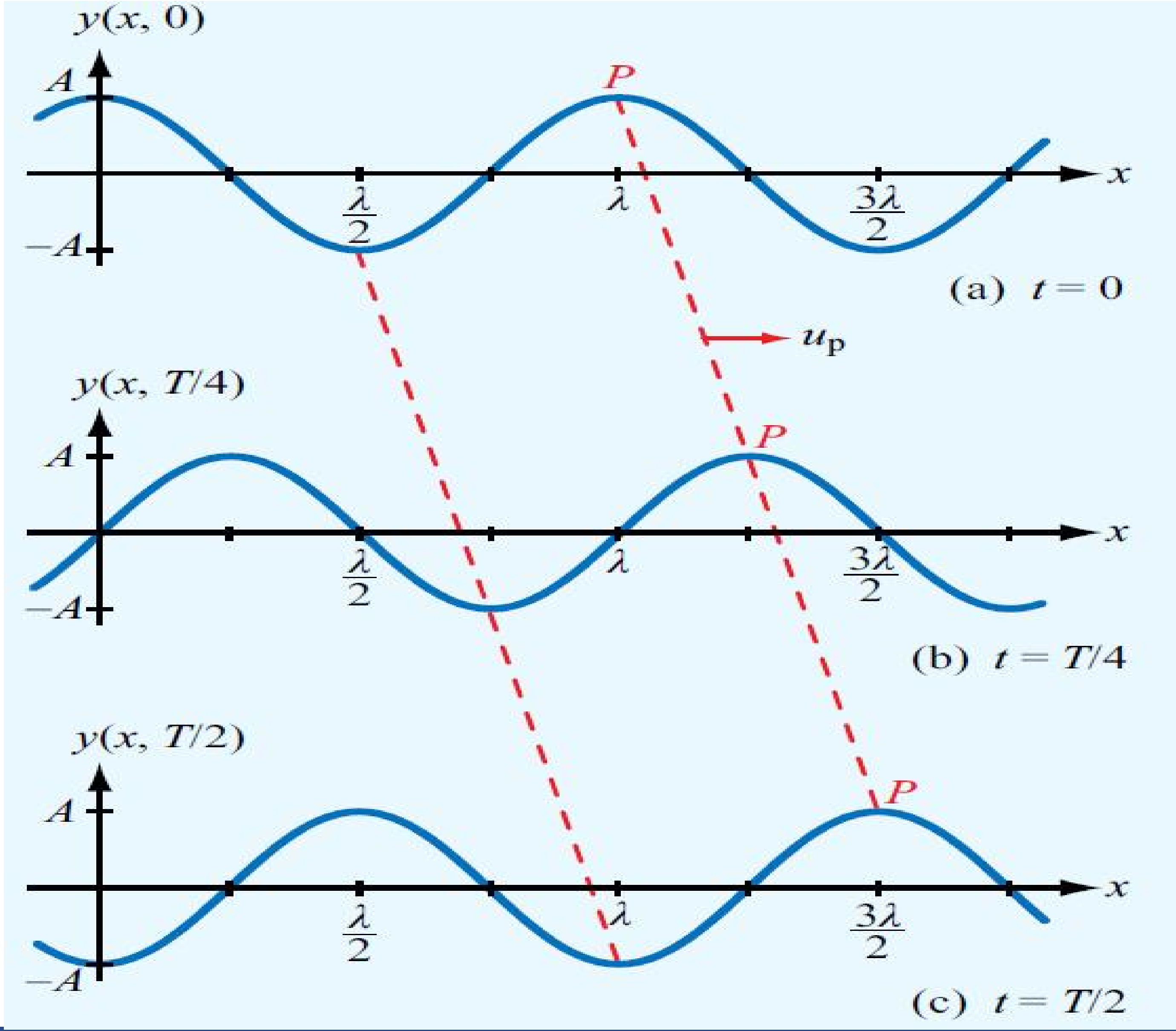


(a) $y(x, t)$ versus x at $t = 0$



(b) $y(x, t)$ versus t at $x = 0$

Traveling Waves



Sinusoidal Waves in a Lossless Medium

Phase of the Wave $\rightarrow \phi(x,t)$

$$y(x,t) = A\cos(\overbrace{\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_o}^{\phi(x,t)})$$

Amplitude

Time Period

Spatial Wavelength

Reference Phase

Let us first analyze the simple case when $\varphi_0 = 0$:

$$y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right) \qquad \text{(m)}$$

Traveling Waves

At the peaks of the wave pattern, the phase *φ(x, t)* is equal to zero or multiples of 2*π* radians. Thus,

$$\phi(x,t)=\frac{2\pi t}{T}-\frac{2\pi x}{\lambda}=2n\pi, \quad n=0,1,2,\ldots$$

Had we chosen any other fixed height of the wave, say y_0,
and monitored its movement as a function of *t* and *x*, this again would have been equivalent to setting the phase *φ(x, t)* constant such that

$$y(x,t)=y_0=A\cos\left(\frac{2\pi t}{T}-\frac{2\pi x}{\lambda}\right)$$

$$\frac{2\pi t}{T}-\frac{2\pi x}{\lambda}=\cos^{-1}\left(\frac{y_0}{A}\right)= \text{ constant}$$

$$\frac{2\pi}{T}-\frac{2\pi}{\lambda}\frac{dx}{dt}=0$$

which gives the ***phase velocity*** *u*p as $u_{\mathrm{p}}=\frac{dx}{dt}=\frac{\lambda}{T} \qquad \text{(m/s)}$

Traveling Waves

Phase Velocity

$$u_p = \frac{\lambda}{T}$$

A traveling wave is characterized by a spatial wavelength λ, a time period T and a phase velocity $u_p=\lambda/T$.

Traveling Waves

phase velocity

$$u_p = \frac{dx}{dt} = \frac{\lambda}{T} \qquad \text{(m/s)}$$

The phase velocity, also called the ***propagation velocity***, is ***the velocity of the wave pattern*** as it moves across the water surface.

$$f = \frac{1}{T} \qquad \text{(Hz)}$$

$$u_p = f\lambda \qquad \text{(m/s)}$$

with $y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right) \qquad \text{(m)}$

$$y(x,t) = A\cos\left(2\pi f t - \frac{2\pi}{\lambda}x\right) = A\cos(\omega t - \beta x),$$

(wave moving along $+x$ direction)

$$y(x,t) = A\cos(\omega t + \beta x).$$

(wave moving along $-x$ direction)

$$\omega = 2\pi f \qquad \text{(rad/s)}$$

$$\beta = \frac{2\pi}{\lambda} \qquad \text{(rad/m)}$$

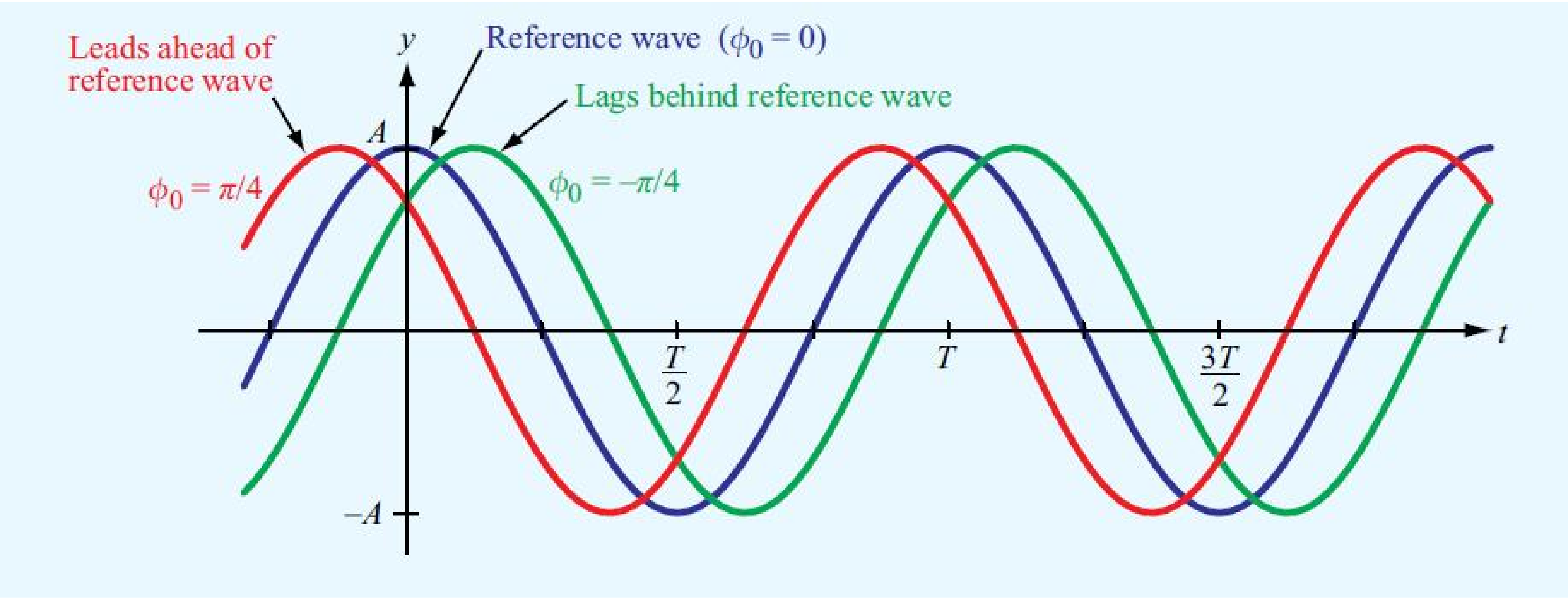
Direction of Travel

Determined by inspection of both t and x terms in phase of the wave

- If one sign is positive and the other negative, then the wave is traveling in the positive x-direction

- If both signs are positive or both are negative, then the wave is traveling in the negative x-direction

Traveling Waves



The constant reference ϕ_0 has no influence on either <u>velocity</u> or the <u>direction</u> of wave propagation

Traveling Waves

Sinusoidal Wave in a Lossless Medium

- Alternate Form

$$y(x,t) = A\cos(\omega t - \beta x + \phi_o)$$

Angular Velocity

Phase Constant or Wavenumber

Sinusoidal Waves in a Lossy Medium

If a wave is traveling in the x direction in a ***lossy medium***, its amplitude decreases as $e^{-\alpha x}$. This factor is called the ***attenuation factor***, and α is called the ***attenuation constant*** of the medium and its unit is neper per meter (Np/m).

Attenuation constant (Neper/meter)

$$y(x,t) = \underbrace{Ae^{-\alpha x}}_{\text{Attenuation Factor}} \cos(\omega t - \beta x + \phi_o)$$

Sinusoidal Waves in a Lossy Medium

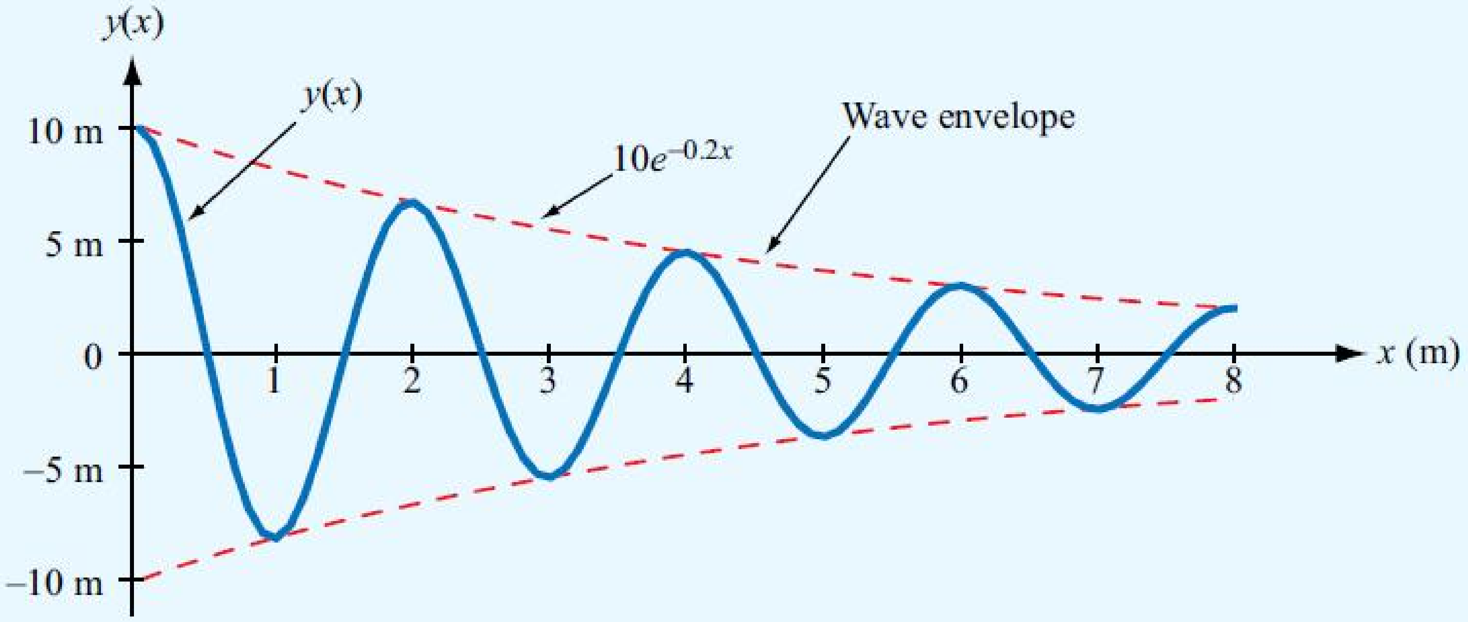


Figure 1-15 Plot of $y(x) = (10e^{-0.2x} \cos \pi x)$ meters. Note that the envelope is bounded between the curve given by $10e^{-0.2x}$ and its mirror image.

Example

The electric field of a traveling electromagnetic wave is given by

$E(z, t) = 10\cos(\pi \times 10^7 t + \pi z/15 + \pi/6)$ (V/m).

Determine

(a) the direction of wave propagation

(b) The wave frequency f

(c) its wavelength λ

(d) its phase velocity u_p.

Answer: (a) $-z$ direction, (b) $f = 5$ MHz, (c) $\lambda = 30$ m,

(d) $u_p = 1.5 \times 10^8$ m/s.

Sinusoidal Waves in a Lossy Medium

Example

A laser beam of light propagating through the atmosphere is characterized by an electric field given by

$$E(x,t) = 150e^{-0.03x}\cos(3\times 10^{15}t - 10^{7}x) \qquad \text{(V/m)},$$

where x is the distance from the source in meters. The attenuation is due to absorption by atmospheric gases. Determine

(a) the direction of wave travel,

(b) the wave velocity, and

(c) the wave amplitude at a distance of 200 m.

Sinusoidal Waves in a Lossy Medium

Solution: (a) Since the coefficients of t and x in the argument of the cosine function have opposite signs, the wave must be traveling in the $+x$ direction.

(b)

$$u_p = \frac{\omega}{\beta} = \frac{3 \times 10^{15}}{10^7} = 3 \times 10^8 \text{ m/s},$$

which is equal to c, the velocity of light in free space.

(c) At $x = 200$ m, the amplitude of $E(x, t)$ is

$$150e^{-0.03\times200} = 0.37 \qquad \text{(V/m)}.$$

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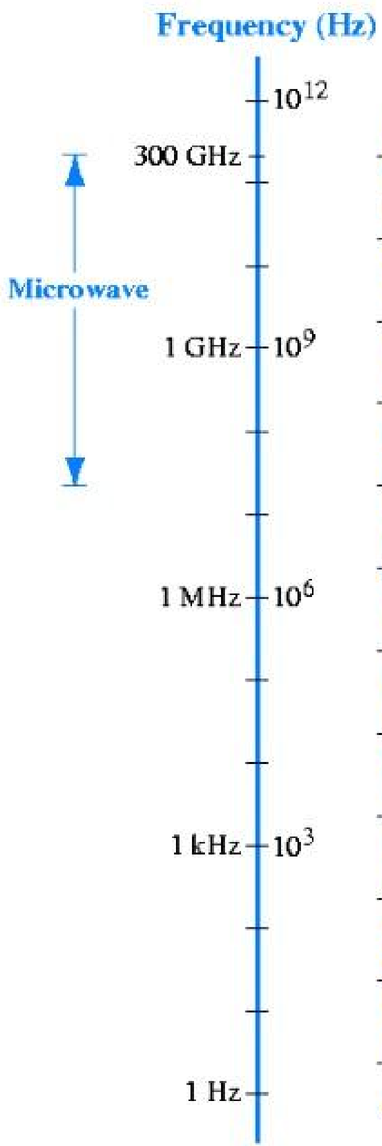
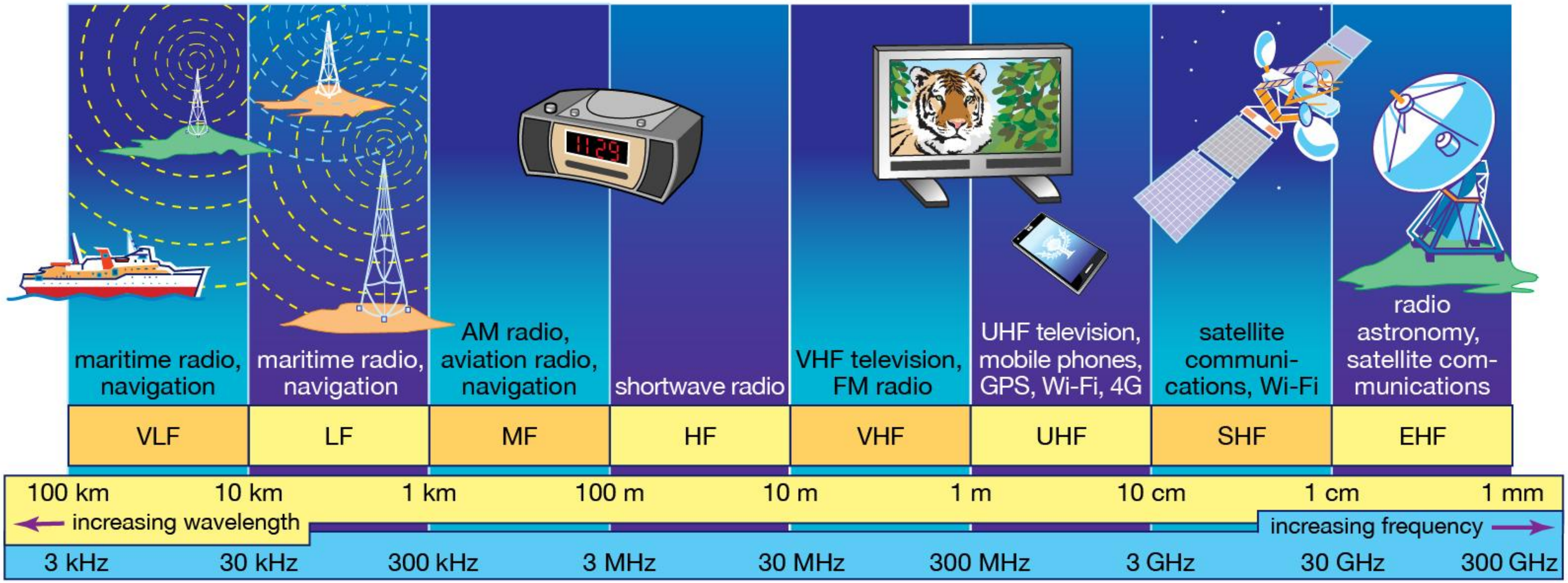
The Electromagnetic Spectrum



Band	Applications
Extremely High Frequency EHF (30 - 300 GHz)	Radar, advanced communication systems, remote sensing, radio astronomy
Super High Frequency SHF (3 - 30 GHz)	Radar, satellite communication systems, aircraft navigation, radio astronomy, remote sensing
Ultra High Frequency UHF (300 MHz - 3 GHz)	TV broadcasting, radar, radio astronomy, microwave ovens, cellular telephone
Very High Frequency VHF (30 - 300 MHz)	TV and FM broadcasting, mobile radio communication, air traffic control
High Frequency HF (3 - 30 MHz)	Short wave broadcasting
Medium Frequency MF (300 kHz - 3 MHz)	AM broadcasting
Low Frequency LF (30 - 300 kHz)	Radio beacons, weather broadcast stations for air navigation
Very Low Frequency VLF (3 - 30 kHz)	Navigation and position location
Ultra Low Frequency ULF (300 Hz - 3 kHz)	Audio signals on telephone
Super Low Frequency SLF (30 - 300 Hz)	Ionospheric sensing, electric power distribution, submarine communication
Extremely Low Frequency ELF (3 - 30 Hz)	Detection of buried metal objects
$f < 3$ Hz	Magnetotelluric sensing of the earth's structure

The Electromagnetic Spectrum



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Review of Complex Numbers

Any ***complex number*** z can be expressed in ***rectangular form*** as

$$z = x + jy$$

where x and y are the ***real*** (Re) and ***imaginary*** (Im) parts of z, respectively, and $j=\sqrt{-1}$. That is,

$$x = \mathrm{Re}(z),\ y = \mathrm{Im}(z).$$

Alternatively, z may be cast in ***polar form*** as

$$z = |z|e^{j\theta} = |z|\angle\theta$$

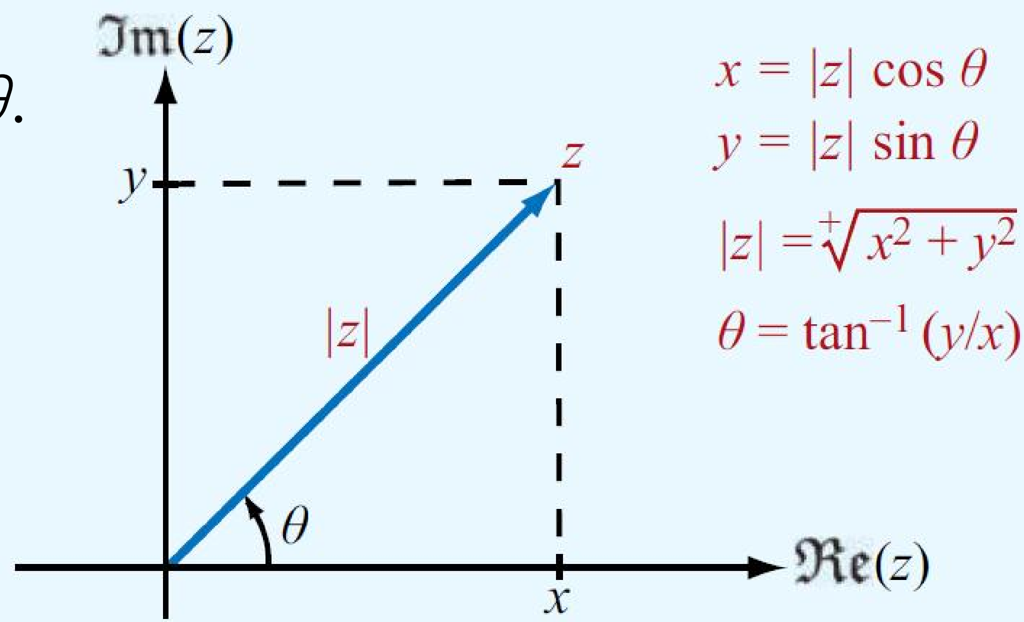
where $|z|$ is the magnitude of z, θ is its phase angle, and $\angle\theta$ is a useful shorthand representation for $e^{j\theta}$. Applying ***Euler's identity***,

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Review of Complex Numbers

$$z = |z| e^{j\theta} = |z| \cos\theta + j|z| \sin\theta.$$

The magnitude $|z|$ is equal to the positive square root of the product of z and its complex conjugate: $|z| = \sqrt[+]{zz^*}$



The ***complex conjugate*** of z, denoted with a star superscript (or asterisk), is obtained by replacing j (wherever it appears) with $-j$, so that

$$z^* = (x + jy)^* = x - jy = |z| e^{-j\theta} = |z| \angle -\theta$$

Example

Given two complex numbers $V = 3 - j4$, $I = -(2 + j3)$,

(a) express V and I in polar form, and find (b) VI, (c) VI^*, (d) V/I, and (e) $\sqrt{I}$.

Solution

(a)

$$|V| = \sqrt[+]{VV^*} = \sqrt[+]{(3-j4)(3+j4)}$$

$$= \sqrt[+]{9+16} = 5,$$

$$\theta_v = \tan^{-1}(-4/3) = -53.1°$$

$$V = |V| e^{j\theta v} = 5e^{-j53.1°} = |5| \angle -53.1°$$

$$|I| = \sqrt[+]{2^2+3^2} = \sqrt[+]{13} = 3.61$$

Since $I = (-2 - j3)$ is in the third quadrant in the complex plane

$$\theta_I = +180° + \tan^{-1}(3/2) = 236.3°$$

$$I = 3.61\angle 236.3°$$

(b)

$$VI = 5e^{-j53.1°} \times 3.61e^{j236.3°}$$

$$= 18.03e^{j(236.3°-53.1°)}$$

$$= 18.03e^{j183.2°}$$

(c)

$$VI^* = 5e^{-j53.1°} \times 3.61e^{-j236.3°} = 18.03e^{-j289.4°} = 18.03e^{j70.6°}$$

(d)

$$\frac{V}{I} = \frac{5e^{-j53.1°}}{3.61e^{j236.3°}} = 1.39e^{-j289.4°} = 1.39e^{j70.6°}$$

(e)

$$\sqrt{I} = \sqrt{3.61e^{j236.3°}} = \pm\sqrt{3.61}e^{j236.3°/2} = \pm 1.90e^{j118.15°}$$

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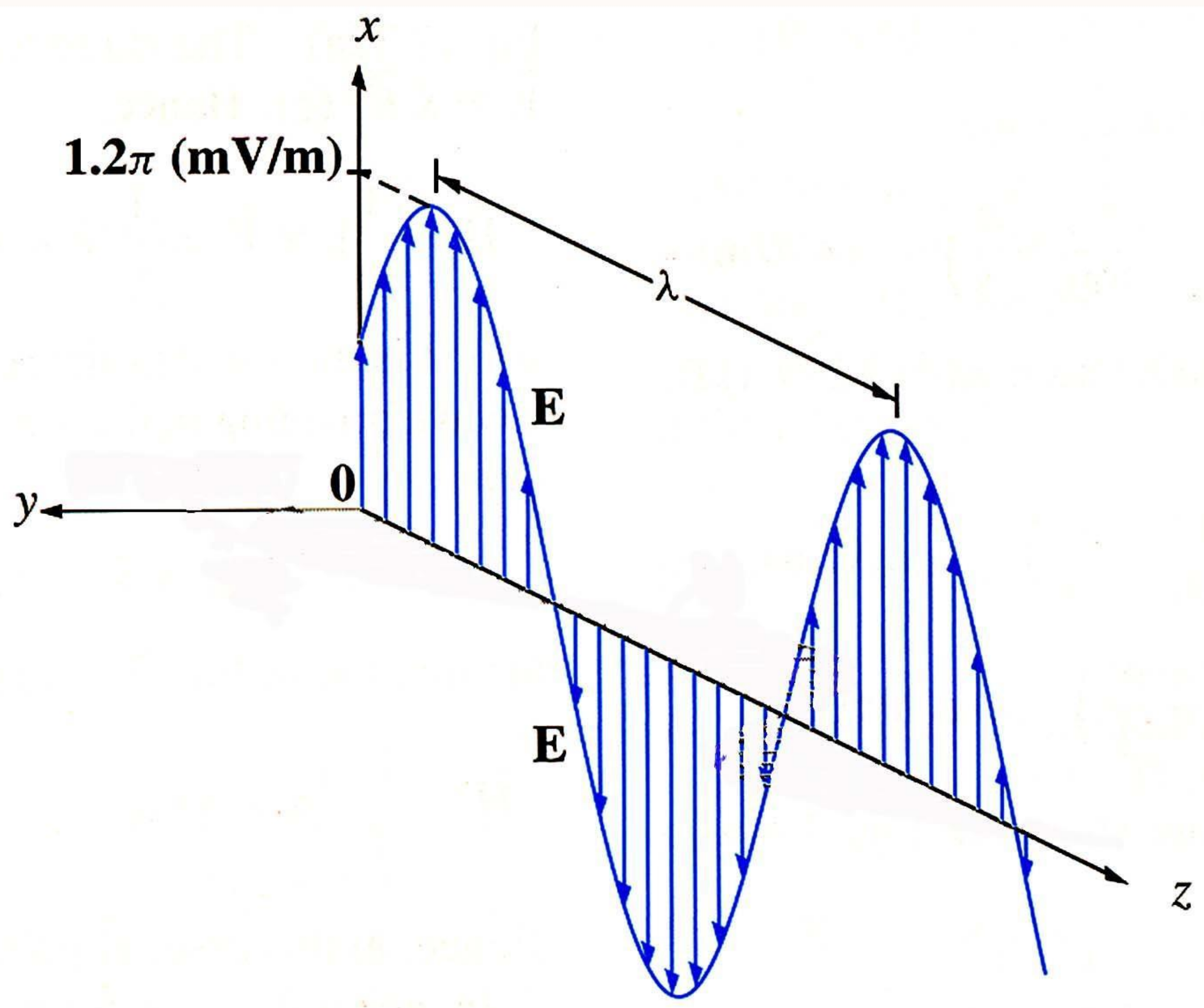
Traveling Wave for Electric Field or Voltages



Traveling Wave for Electric Field or Voltages

$$\mathbf{E}(x,y,z;t) = \mathrm{Re}[\underbrace{\widetilde{\mathbf{E}}(x,y,z)}_{\text{Vector Phasor}} e^{j\omega t}]$$

Instantaneous

Vector Phasor

Phasors

- Phasors are useful when analyzing periodic functions in linear systems
- A simple cosine can be represented as a constant in the phasor domain
- Complicated time domain functions can be represented as an addition of sin and cosine functions (Fourier series)
- Analyzing the frequency content of the wave
- Using a cosine as a reference:

Review of Phasors

The simple RC circuit shown in **Fig. 1-20** contains a sinusoidally time-varying voltage source given by

$$v_s(t) = V_0 \sin(wt + \phi_0)$$

Application of Kirchhoff's voltage law gives the following loop equation:

$$Ri(t) + \frac{1}{C}\int i(t)dt = v_s(t)$$

Our objective is to obtain an expression for the current $i(t)$.

We can do this by solving equation in the time domain, which is somewhat cumbersome because the forcing function $v_s(t)$ is a sinusoid.

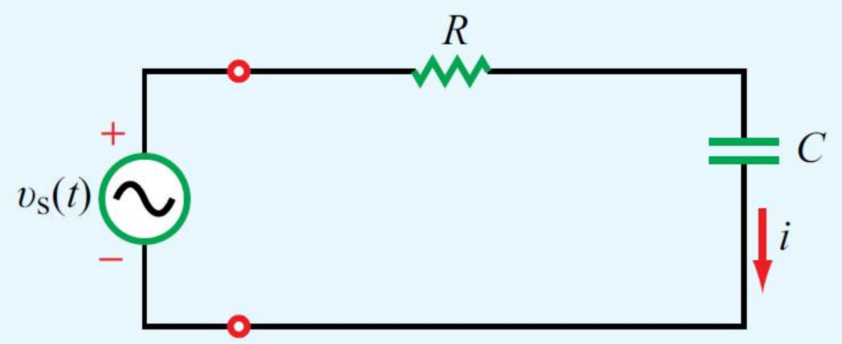


Figure 1-20 RC circuit connected to a voltage source $v_S(t)$.

Solution

Step 1: Adopt a cosine reference

$$v_s(t) = V_0 \sin(wt + \phi_0)$$

$$= V_0 \cos(\frac{\pi}{2} - wt - \phi_0)$$

$$= V_0 \cos(wt + \phi_0 - \frac{\pi}{2})$$

Step 2: Express time-dependent variables as phasors

$$z(t) = \mathrm{Re}\left[\tilde{Z} e^{j\omega t}\right]$$

$$v_s(t) = \mathrm{Re}\left[V_0 e^{j(\omega t + \phi_0 - \frac{\pi}{2})}\right]$$

$$= \mathrm{Re}\left[V_0 e^{j(\phi_0 - \frac{\pi}{2})} e^{j\omega t}\right] = \mathrm{Re}\left[\tilde{V}_s e^{j\omega t}\right]$$

$$\tilde{V}_s = V_0 e^{j(\phi_0 - \frac{\pi}{2})}$$

$$i(t) = \mathrm{Re}(\tilde{I} e^{j\omega t})$$

$$\frac{di}{dt} = \frac{d}{dt}\left[\mathrm{Re}(\tilde{I} e^{j\omega t})\right]$$

$$= \mathrm{Re}\left[\frac{d}{dt}(\tilde{I} e^{j\omega t})\right] = \mathrm{Re}\left[j\omega \tilde{I} e^{j\omega t}\right]$$

$$\int i\,dt = \int \mathrm{Re}(\tilde{I} e^{j\omega t})\,dt$$

$$= \mathrm{Re}(\int \tilde{I} e^{j\omega t}\,dt) = \mathrm{Re}(\frac{\tilde{I}}{j\omega} e^{j\omega t})$$

Solution

Step 3: Recast the differential / integral equation in phasor form

$$R\,\mathrm{Re}(\tilde{I}e^{j\omega t}) + \frac{1}{C}\mathrm{Re}\left(\frac{\tilde{I}}{j\omega}e^{j\omega t}\right) = \mathrm{Re}\left[\widetilde{V}_s e^{j\omega t}\right]$$

$$\mathrm{Re}\left\{\left[\left(R + \frac{1}{j\omega C}\right)\tilde{I} - \widetilde{V}_s\right]e^{j\omega t}\right\} = 0$$

$$\mathrm{Im}\left\{\left[\left(R + \frac{1}{j\omega C}\right)\tilde{I} - \widetilde{V}_s\right]e^{j\omega t}\right\} = 0$$

$$\tilde{I}\left(R + \frac{1}{j\omega C}\right) = \widetilde{V}_s$$ **(phasor domain).**

Step 4: Solve the phasor-domain equation

$$\tilde{I} = \frac{\widetilde{V}_s}{R + 1/(j\omega C)}$$

$$\tilde{I} = V_0 e^{j(\phi_0 - \frac{\pi}{2})}\left[\frac{j\omega C}{1 + j\omega RC}\right]$$

$$= V_0 e^{j(\phi_0 - \frac{\pi}{2})}\left[\frac{\omega C e^{j\pi/2}}{\sqrt{1+\omega^2 R^2 C^2}\, e^{j\phi_1}}\right]$$

$$= \frac{V_0 \omega C}{\sqrt{1+\omega^2 R^2 C^2}} e^{j(\phi_0 - \phi_1)}$$

Solution

Step 5: Find the instantaneous value

$$i(t) = \mathrm{Re}\left[\tilde{I} e^{j\omega t}\right]$$

$$= \mathrm{Re}\left[\frac{V_0 \omega C}{\sqrt{1+\omega^2 R^2 C^2}} e^{j(\phi_0 - \phi_1)} e^{j\omega t}\right]$$

$$= \frac{V_0 \omega C}{\sqrt{1+\omega^2 R^2 C^2}} \cos(\omega t + \phi_0 - \phi_1)$$

In summary, we converted all time-varying quantities into the phasor domain, solved for the phasor $\tilde{I}$ of the desired instantaneous current $i(t)$, and then converted back to the time domain to obtain an expression for $i(t)$.

Review of Phasors

Table 1-5 Time-domain sinusoidal functions $z(t)$ and their cosine-reference phasor-domain counterparts $\tilde{Z}$, where $z(t) = \mathfrak{Re}\ [\tilde{Z}e^{j\omega t}]$.

$z(t)$		$\tilde{Z}$
$A\cos\omega t$	$\leftrightarrow$	A
$A\cos(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j\phi_0}$
$A\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{j(\beta x+\phi_0)}$
$Ae^{-\alpha x}\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{-\alpha x}e^{j(\beta x+\phi_0)}$
$A\sin\omega t$	$\leftrightarrow$	$Ae^{-j\pi/2}$
$A\sin(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j(\phi_0-\pi/2)}$
$\frac{d}{dt}(z(t))$	$\leftrightarrow$	$j\omega\tilde{Z}$
$\frac{d}{dt}[A\cos(\omega t + \phi_0)]$	$\leftrightarrow$	$j\omega Ae^{j\phi_0}$
$\int z(t)\ dt$	$\leftrightarrow$	$\frac{1}{j\omega}\tilde{Z}$
$\int A\sin(\omega t + \phi_0)\ dt$	$\leftrightarrow$	$\frac{1}{j\omega}Ae^{j(\phi_0-\pi/2)}$

Summary

Concepts

- Electromagnetics is the study of electric and magnetic phenomena and their engineering applications.
- The International System of Units consists of the six fundamental dimensions listed in Table 1-1. The units of all other physical quantities can be expressed in terms of the six fundamental units.
- The four fundamental forces of nature are the nuclear, weak-interaction, electromagnetic, and gravitational forces.
- The source of the electric field quantities $\mathbf{E}$ and $\mathbf{D}$ is the electric charge q. In a material, $\mathbf{E}$ and $\mathbf{D}$ are related by $\mathbf{D} = \epsilon\mathbf{E}$, where ϵ is the electrical permittivity of the material. In free space, $\epsilon = \epsilon_0 \approx (1/36\pi) \times 10^{-9}$ (F/m).
- The source of the magnetic field quantities $\mathbf{B}$ and $\mathbf{H}$ is the electric current I. In a material, $\mathbf{B}$ and $\mathbf{H}$ are related by $\mathbf{B} = \mu\mathbf{H}$, where μ is the magnetic permeability of the medium. In free space, $\mu = \mu_0 = 4\pi \times 10^{-7}$ (H/m).
- Electromagnetics consists of three branches: (1) electrostatics, which pertains to stationary charges, (2) magnetostatics, which pertains to dc currents, and (3) electrodynamics, which pertains to time-varying currents.
- A traveling wave is characterized by a spatial wavelength λ, a time period T, and a phase velocity $u_{\mathrm{p}} = \lambda/T$.
- An electromagnetic (EM) wave consists of oscillating electric and magnetic field intensities and travels in free space at the velocity of light $c = 1/\sqrt{\epsilon_0\mu_0}$. The EM spectrum encompasses gamma rays, X-rays, visible light, infrared waves, and radio waves.
- Phasor analysis is a useful mathematical tool for solving problems involving time-periodic sources.