Xidian University & Heriot-Watt University

# Artificial Transmission Lines

(B39HF | B31HD)

High Frequency Circuit Design

**Team Members:**


| Name | XDU ID | HWU ID |
|---|---|---|
| Wenchuan Zhang | 21012100092 | H00392711 |
| Yirui Fu | 21012100086 | H00392656 |
| Fengliang Mi | 21012100040 | H00392671 |
| Yihong Yang | 21012100009 | H00392698 |
| Yiqian Zhang | 21012100052 | H00393159 |
| Zhuoyu Wu | 21012100038 | H00392691 |
| Xunchi Ma | 21012100015 | H00392669 |
| Yang Yu | 21012100011 | H00392718 |

# Artificial Transmission Lines

## 1. INTRODUCTION

The purpose of this experiment is to construct an artificial transmission line composed of 15 cells, and to analyze the corresponding time series waveform and other parameters by changing its specific parameter values, such as generator frequency, shape of generator source, capacitance and inductance values and to explain it by the corresponding transmission line formula.

In order to match the impedance, we need $R = Z_L$. (1)

If $\omega^2 C^2 Z_L^2 \ll 1$, the load impedance is $Z_L = Z_0 = \sqrt{\frac{L}{C}}$. (2) and (3)

The phase constant β is $\beta = \omega\sqrt{LC} = 2\pi f\sqrt{LC}\ [rads/cell]$. (4)

Convert rads/cell into degrees/cell $\beta = 360 f\sqrt{LC}\ [°/cell]$ (5)

Convert this equation in the form of T $\beta = Tf\sqrt{LC} = \sqrt{LC}\ [seconds/cell]$ (6)

A low-loss transmission line can be considered to satisfy $R \ll \omega L$ and $G \ll \omega C$ (7)

By simplification $\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right)[Np/cell], \beta = \omega\sqrt{LC}[rad/cell]$ (8)

## 2. LAB ACTIVITY AND RESULTS

**1)** Define a 10kHz sine-wave signal from an AC source.

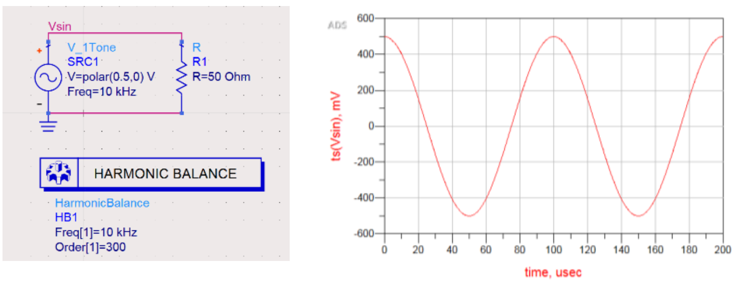



Figure 1. AC Source and Generator Waveform

From the figure we can see that the period of the generator is $T = 100\ \mu sec$.

Therefore, its period is $f = \frac{1}{T} = 10\ kHz$.

**2)** Define an artificial transmission line with 15 cells (15 470-μH inductors in series and 15 shunt 100 pF capacitors).

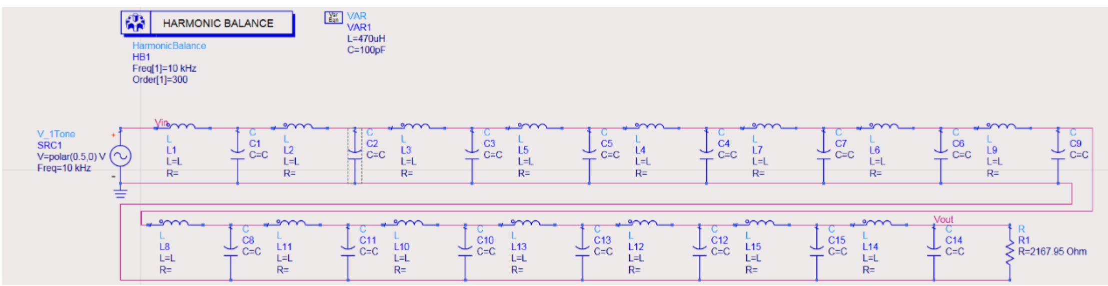



Figure 2. artificial transmission line with 15 cells

**a.** According to the Eq.(3), we can get the characteristic impedance:

$$Z_0 = Z_L = \sqrt{\frac{L}{C}} = \sqrt{\frac{470 \times 10^{-6}}{100 \times 10^{-12}}} = 2167.95\Omega$$

Therefore, the characteristic impedance is 2167.95 ohms, and to meet the impedance matching, the load impedance is 2167.95 ohms.

**b.** From Figure3, we can see that the signal source is the same as Lab Activity 1.

**c.**

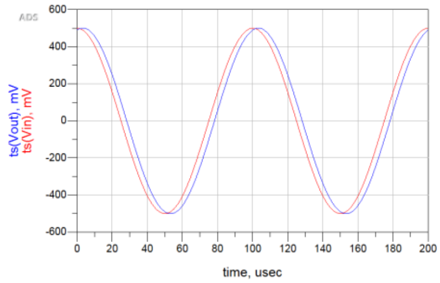



Figure 3. Input and Output Signal Waveforms

We can see from the figure that the input signal and the output signal are basically the same, but there is a certain phase difference between the two waveforms.

**d.** According to Eq.(2), we can get:

$$\omega^2 C^2 {Z_L}^2 = 0.000185 \ll 1$$

That means 10kHz satisfies Eq.(2).

**e.** According to Figure 4, we can find that the phase difference between the input and output is 11.723° . And the output is lagging with respect to the input terminal.

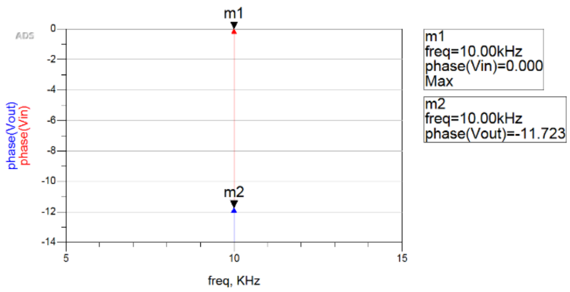



Figure 4. Phase Difference between the Input and Output

In theory, the phase difference between input and output is:

$$\varphi = 15\beta = 15 \times 360 f\sqrt{LC} = 15 \times 0.78046 \approx 11.707°$$

Compare the theoretical value (11.707° ) with the simulated value (11.723° ), we know that the simulation results and theoretical results are equal within the allowed error range.

**f.**

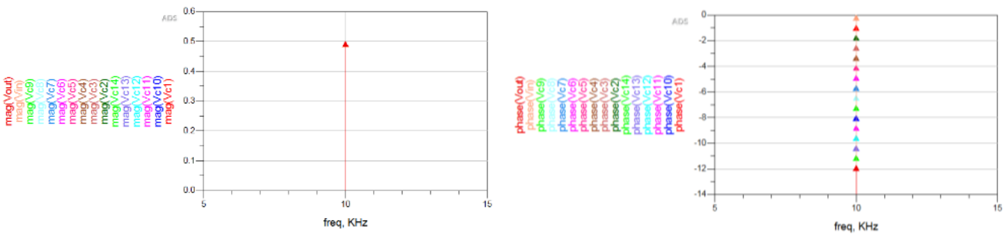



Figure 5. Amplitude and Phase Difference at every Junction

According to Figure 5, we can get the table below:

Table 1. Amplitude and Phase Difference at every Junction

| Junction | Simulated value | | Theoretical value | |
|---|---|---|---|---|
| | Amplitude | Phase (degree) | Amplitude | Phase (degree) |
| Input | 0 | 0 | 0 | 0 |
| Cell 1 | 0 | 0.783 | 0 | 0.781 |
| Cell 2 | 0 | 1.563 | 0 | 1.561 |
| Cell 3 | 0 | 2.347 | 0 | 2.340 |
| Cell 4 | 0 | 3.129 | 0 | 3.122 |
| Cell 5 | 0 | 3.911 | 0 | 3.902 |
| Cell 6 | 0 | 4.693 | 0 | 4.683 |
| Cell 7 | 0 | 5.475 | 0 | 5.463 |
| Cell 8 | 0 | 6.256 | 0 | 6.244 |
| Cell 9 | 0 | 7.038 | 0 | 7.024 |
| Cell 10 | 0 | 7.819 | 0 | 7.805 |
| Cell 11 | 0 | 8.600 | 0 | 8.585 |
| Cell 12 | 0 | 9.381 | 0 | 9.366 |
| Cell 13 | 0 | 10.162 | 0 | 10.146 |
| Cell 14 | 0 | 10.934 | 0 | 10.926 |
| Cell 15(Output) | 0 | 11.723 | 0 | 11.707 |

Using Table 1, we can easily compare the amplitude and phase differences between the theoretical and simulated values.

For the amplitude difference, we can see that the amplitude difference (relative to zero) at each node is 0, because there are no resistors and inductors in the transmission line. This satisfies our theoretical calculations very well.

For the phase difference, we can see that the theoretical value is basically consistent with the simulated value, and the relationship is linear with the increase of cells. This is also basically consistent with the theory.

**g.**

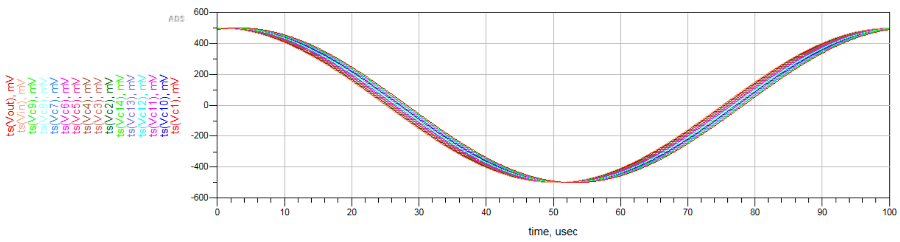



Figure 6. Waveform for Each Cell

According to Figure 6, The simulation value of the time delay for each cell is 0.25 usec. According to Eq.(6), the theoretical value of the time delay for each cell is:

$$\Delta t = Tf\sqrt{LC} = \sqrt{LC} = 2.2 \times 10^{-7} = 0.22\ [\mu sec/cell]$$

Comparing the theoretical value with the simulation value, we can see that the two are basically equal, and the total time delay will increase linearly with the increase of the cell.

**3)** Define an artificial transmission line with 15 cells (15 470-μH inductors in series and 15 shunt 1-nF capacitors).

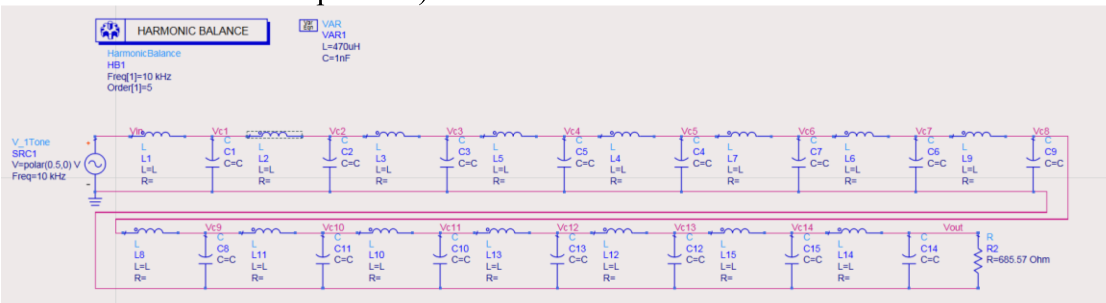



Figure 7. artificial transmission line with 15 cells

**a.** According to the Eq.(3), we can get the characteristic impedance:

$$Z_0 = Z_L = \sqrt{\frac{L}{C}} = \sqrt{\frac{470 \times 10^{-6}}{1 \times 10^{-9}}} = 685.57\Omega$$

Therefore, the characteristic impedance is 685.57 ohms, and to meet the impedance matching, the load impedance is 685.57 ohms.

**b.** From Figure8, we can see that the signal source is the same as Lab Activity 1.

**c.** We can see from the figure that the input signal and the output signal are basically the same, but there is a certain phase difference between the two waveforms.

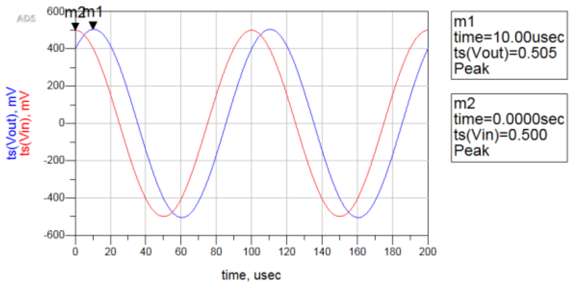



Figure 8. Input and Output Signal Waveforms

**d.** According to Eq.(2), we can get:

$$\omega^2 C^2 {Z_L}^2 = 0.00185 \ll 1.$$

That means 10kHz satisfies Eq.(2).

**e.** According to Figure 9, we can find that the phase difference between the input and output is 37.482° . And the output is lagging with respect to the input terminal.

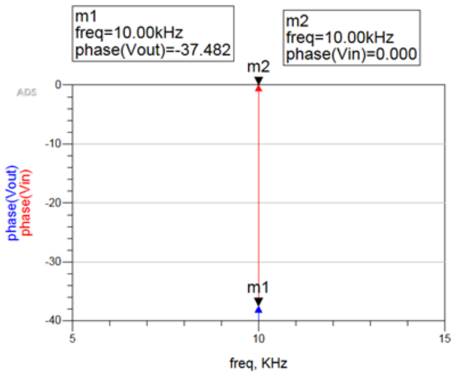



Figure 9. Phase Difference between the Input and Output

In theory, the phase difference between input and output is:

$$\Delta\varphi = 15\beta = 15 \times 360 f\sqrt{LC} \approx 37.02°$$

Compare the theoretical value (37.482° ) with the simulated value (37.02° ), we know that the simulation results and theoretical results are equal within the allowed error range.

**f.**

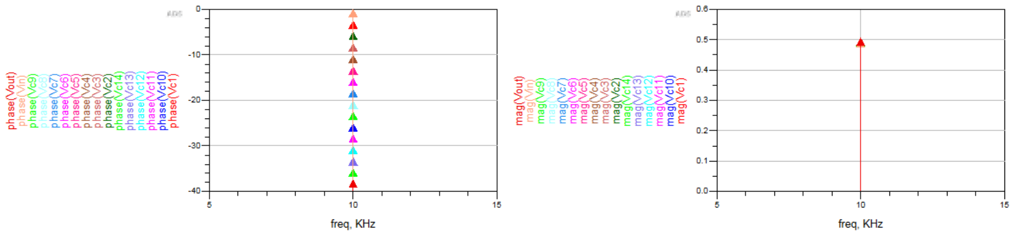



Figure 10. Amplitude and Phase Difference at every Junction

According to Figure 10, we can get the table below:

Table 2. Amplitude and Phase Difference at every Junction

| Junction | Simulated value | | Theoretical value | |
|---|---|---|---|---|
| | Amplitude | Phase (degree) | Amplitude | Phase (degree) |
| Input | 0 | 0 | 0 | 0 |
| Cell 1 | 0 | 2.495 | 0 | 2.468 |
| Cell 2 | 0 | 4.99 | 0 | 4.936 |
| Cell 3 | 0 | 7.485 | 0 | 7.404 |
| Cell 4 | 0 | 9.98 | 0 | 9.872 |
| Cell 5 | 0 | 12.475 | 0 | 12.34 |
| Cell 6 | 0 | 14.97 | 0 | 14.808 |
| Cell 7 | 0 | 17.465 | 0 | 17.276 |
| Cell 8 | 0 | 19.96 | 0 | 19.744 |
| Cell 9 | 0 | 22.455 | 0 | 22.212 |
| Cell 10 | 0 | 24.95 | 0 | 24.68 |
| Cell 11 | 0 | 27.445 | 0 | 27.148 |
| Cell 12 | 0 | 29.94 | 0 | 29.616 |
| Cell 13 | 0 | 32.435 | 0 | 32.084 |
| Cell 14 | 0 | 34.93 | 0 | 34.552 |
| Cell 15(Output) | 0 | 37.482 | 0 | 37.02 |

**g.**

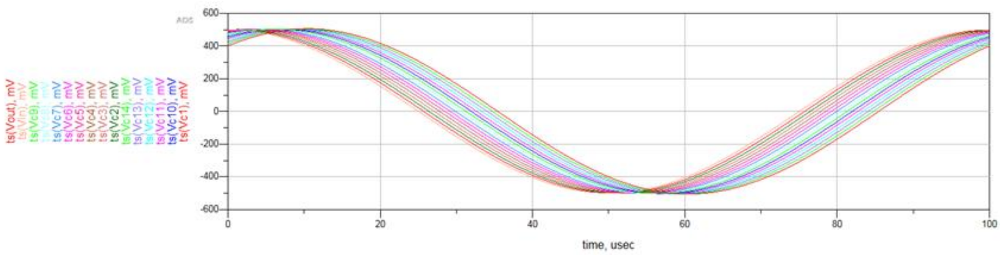



Figure 11. Waveform for Each Cell

According to Figure 6, The simulation value of the time delay for each cell is 0.66 usec. According to Eq.(6), the theoretical value of the time delay for each cell is:

$$\Delta t = Tf\sqrt{LC} = \sqrt{LC} = 6.86 \times 10^{-7} = 0.686\ [\mu sec/cell]$$

Comparing the theoretical value with the simulation value, we can see that the two are basically equal, and the total time delay will increase linearly with the increase of the cell.

**4)** Define an artificial transmission line with 15 cells (15 470-μH inductors in series and 15 shunt 4.7-nF capacitors).

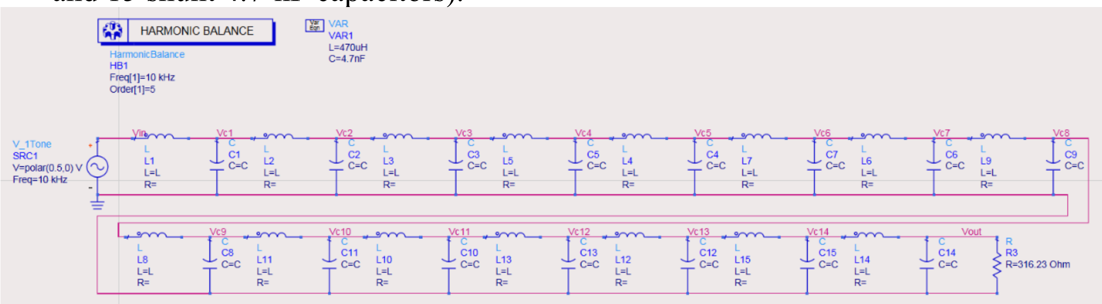



Figure 12. artificial transmission line with 15 cells

**a.** According to the Eq.(3), we can get the characteristic impedance:

$$Z_0 = Z_L = \sqrt{\frac{L}{C}} = \sqrt{\frac{470 \times 10^{-6}}{4.7 \times 10^{-9}}} = 316.23\ \Omega$$

Therefore, the characteristic impedance is 316.23 ohms, and to meet the impedance matching, the load impedance is 316.23 ohms.

**b.** From Figure3, we can see that the signal source is the same as Lab Activity 1.

**c.** We can see from the figure that the input signal and the output signal are basically the same, but there is a certain phase difference between the two waveforms.

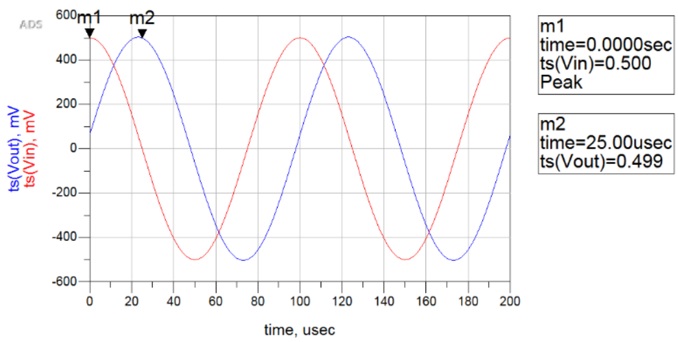



Figure 13. Input and Output Signal Waveforms

**d.** According to Eq.(2), we can get:

$$\omega^2 C^2 {Z_L}^2 = 0.00871 \ll 1$$

That means 10kHz satisfies Eq.(2).

**e.** According to Figure 14, we can find that the phase difference between the input and output is 82.916° . And the output is lagging with respect to the input terminal.

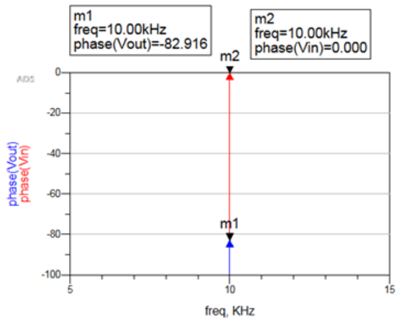



Figure 14. Phase Difference between the Input and Output

In theory, the phase difference between input and output is:

$$\Delta\varphi = 15\beta = 15 \times 360f\sqrt{LC} \approx 80.26°$$

Compare the theoretical value (82.916°) with the simulated value (80.26°), we know that the simulation results and theoretical results are equal within the allowed error range.

**f.**

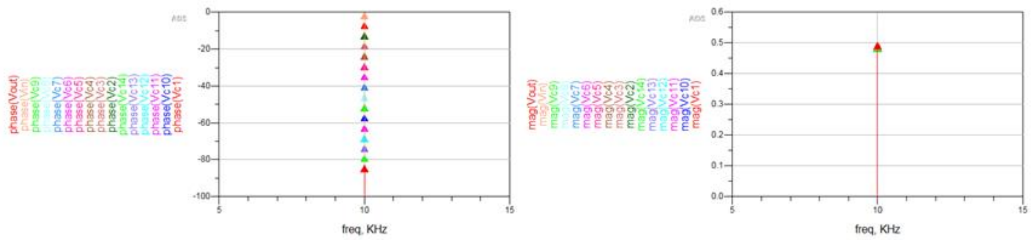



Figure 15. Amplitude and Phase Difference at every Junction

According to Figure 15, we can get the table below:

Table 3. Amplitude and Phase Difference at every Junction

| Junction | Simulated value | | Theoretical value | |
|---|---|---|---|---|
| | Amplitude | Phase (degree) | Amplitude | Phase (degree) |
| Input | 0 | 0 | 0 | 0 |
| Cell 1 | 0 | 5.528 | 0 | 5.351 |
| Cell 2 | 0 | 11.056 | 0 | 10.702 |
| Cell 3 | 0 | 16.584 | 0 | 16.053 |
| Cell 4 | 0 | 22.112 | 0 | 21.404 |
| Cell 5 | 0 | 27.64 | 0 | 26.755 |
| Cell 6 | 0 | 33.168 | 0 | 32.106 |
| Cell 7 | 0 | 38.696 | 0 | 37.457 |
| Cell 8 | 0 | 44.224 | 0 | 42.808 |
| Cell 9 | 0 | 49.752 | 0 | 48.159 |
| Cell 10 | 0 | 55.28 | 0 | 53.51 |
| Cell 11 | 0 | 60.808 | 0 | 58.861 |
| Cell 12 | 0 | 66.336 | 0 | 64.212 |
| Cell 13 | 0 | 71.864 | 0 | 69.563 |
| Cell 14 | 0 | 77.392 | 0 | 74.914 |
| Cell 15(Output) | 0 | 82.916 | 0 | 80.26 |

**g.**

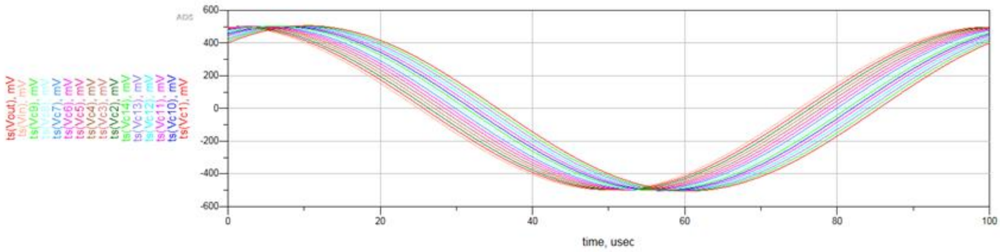



Figure 16. Waveform for Each Cell

According to Figure 6, The simulation value of the time delay for each cell is 1.5 usec. According to Eq.(6), the theoretical value of the time delay for each cell is:

$$\Delta t = Tf\sqrt{LC} = \sqrt{LC} = 1.486\ [\mu sec/cell]$$

Comparing the theoretical value with the simulation value, we can see that the two are basically equal, and the total time delay will increase linearly with the increase of the cell.

**5)** The transmission line will cause square wave distortion, resulting in the simulation output cannot display the corresponding input waveform. Square waves are composed of many sinusoidal waves of different frequencies, and the sinusoidal wave components of different frequencies propagate at different speeds along the transmission line. Therefore, after the matrix wave passes through the equivalent transmission line, the waveform will be deformed.

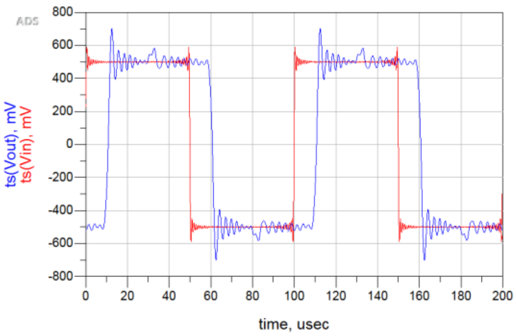



Figure17. the Input and Output Waveforms of 10kHz Square Wave

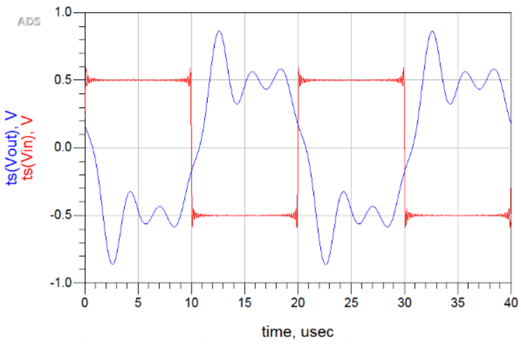



Figure18. the Input and Output Waveforms of 50kHz Square Wave

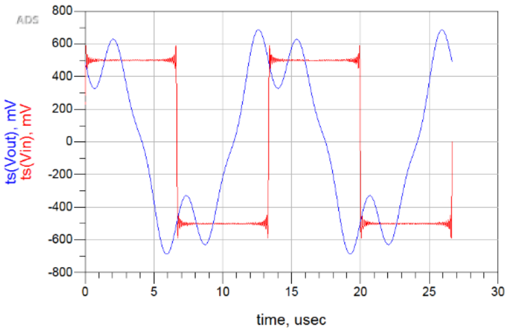



Figure19. the Input and Output Waveforms of 75kHz Square Wave

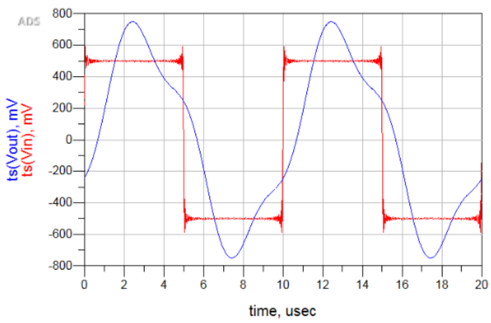



Figure20. the Input and Output Waveforms of 100kHz Square Wave

From the above four frequencies, when the signal source is a matrix square wave and the frequency increases, the output signal gradually approaches a sine wave. At 10kHz, we can see that the output waveform is basically the same as the square wave. However, as the frequency continues to increase, the output waveform gradually becomes distorted and approaches the sine wave. The reason is mainly because the square wave is composed of sine waves with different frequencies. As the frequency continues to increase, higher harmonics cannot pass through the inductor, and the final output signal will be presented as a sine wave.

**6)** Redefine a 100 kHz sine-wave signal as the generator.

**7)** Build the transmission line according to the experimental requirements.

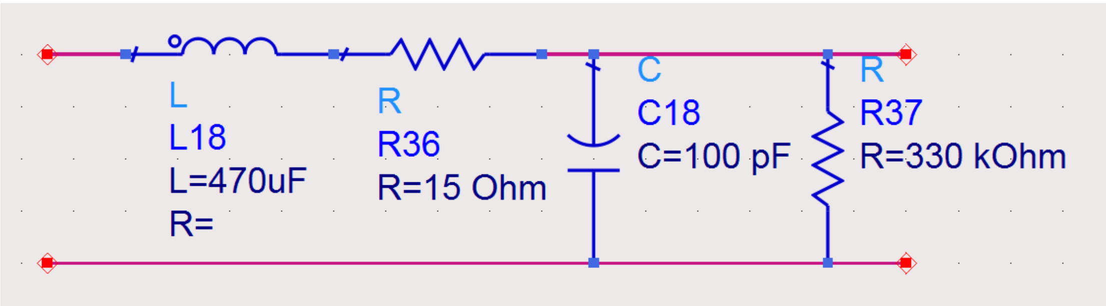



Figure 21. Transmission Line Cells in ADS Simulation

**a.** At frequency 100kHz, the characteristic impedance of this lossy transmission line is:

$$Z_0 = \sqrt{\frac{R + j\omega L}{S + j\omega C}} = 2168.08 - j2.77 \ \ \Omega$$

However, According to Eq.(7).

$$R = 15\Omega \ll \omega L = 295.16\Omega$$

$$G = 3.03 \times 10^{-6}\ S \ll \omega C = 6.28 \times 10^{-5}\ S$$

We can find that the low-loss conditions are satisfied at 100kHz. So, we can treat it as lossless circuit.

**b.** Because at 100kHz, the transmission line meets the low loss condition. The characteristic impedance is: $Z_0 = \sqrt{\frac{L}{C}} = 2167.95\Omega$

**c.** Impedance matching needs to meet the load and characteristic impedance equal. Therefore, the load is: $Z_L = Z_0 = 2167.95\ \Omega$.

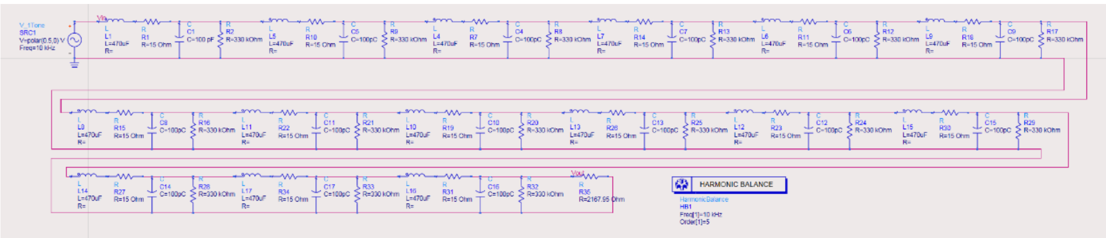


Figure 22. Artificial Transmission Line with 15 Cells

**d.**

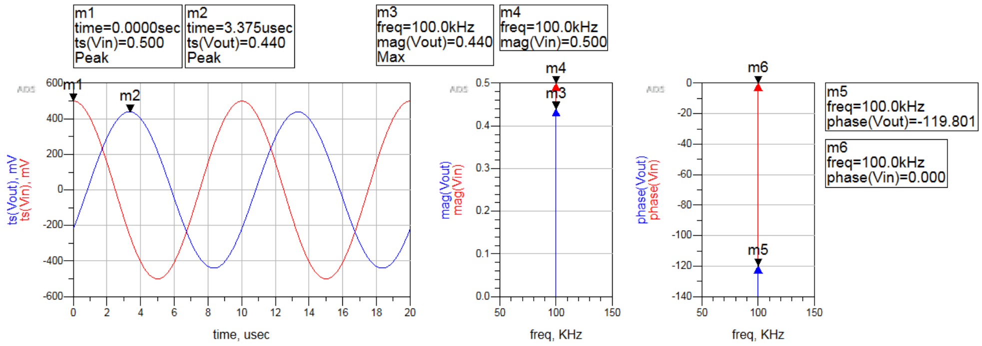



Figure 23. Amplitude and Phase Difference between the Input and Output Waveform for 100kHz

According to Figure 23, we can see that the simulation value of the amplitude difference is 0.06V and the simulation value of the phase difference is 119.802° .

According to Eq.(8),

$$\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) = 6.74 \times 10^{-3}\ [Np/cell]$$

$$\beta = \omega\sqrt{LC} = 0.136\ [rad/cell]$$

Therefore, the theoretical values of the amplitude and phase difference are:

$$\Delta V = V_0 - V_0 e^{-15\alpha} = 0.0481V$$

$$\Delta\varphi = 15\beta = 116.94°$$

By comparing the theoretical value with the actual value, we can see that the phase difference is basically the same, while the amplitude difference has a small error.
The error analysis is:

$$\frac{119.802 - 116.94}{116.94} \times 100\% = 2.39\%$$

**e.** The signal is distorted during transmission. We can see from Figure 24 that the amplitude of the output signal is different from the amplitude of the input signal, that is, the amplitude of the signal is attenuated.

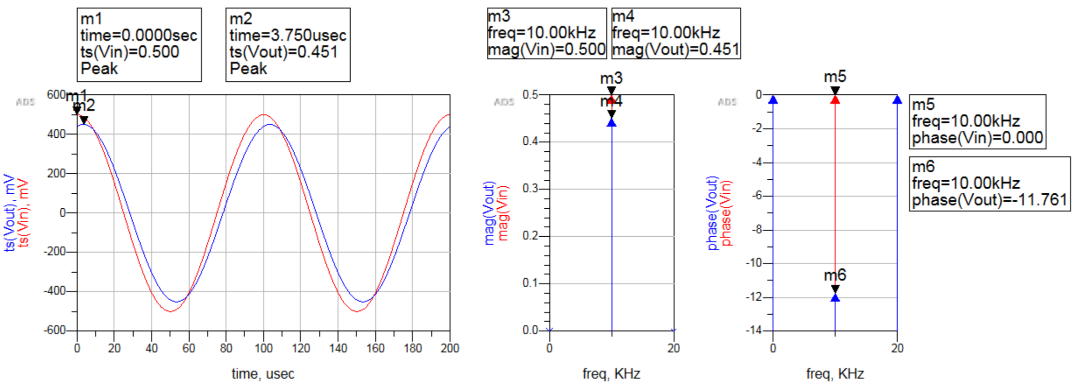



Figure 24. Amplitude and Phase Difference between the Input and Output Waveform for 10kHz

**For 10 kHz:**

According to the figure 24, we can see that the simulation value of the amplitude difference is 0.039V and the simulation value of the phase difference is 11.761°

Time delay :

The simulated time delay is: $t_s = \frac{11.761°}{f \times 360°} = 3.26 \times 10^{-6}\ sec$ .

The theoretical time delay is: $t_t = n\sqrt{LC} = 3.25 \times 10^{-6}\ sec$ .

Voltage amplitude :

The theoretical voltage amplitude difference is: $\frac{0.5-0.4519}{0.5} \times 100\% = 9.62\%$ .

The simulated voltage amplitude difference is: $\frac{0.5-0.451}{0.5} \times 100\% = 9.8\%$ .

**For 100 kHz:**

According to the figure 23, we can see that the simulation value of the amplitude difference is 0.06V and the simulation value of the phase difference is 119.802° .

Time delay :

The simulated time delay is: $t_s = \frac{119.802°}{f \times 360°} = 3.32 \times 10^{-6}\ sec$ .

The theoretical time delay is: $t_t = n\sqrt{LC} = 3.25 \times 10^{-6}\ sec$ .

Voltage amplitude :

The theoretical voltage amplitude difference is: $\frac{0.5-0.4519}{0.5} \times 100\% = 9.62\%$ .

The simulated voltage amplitude difference is: $\frac{0.5-0.44}{0.5} \times 100\% = 12\%$ .

By comparing the results of 10kHz and 100kHz, we can see that the change of frequency does not lead to a change in amplitude. However, the factor that really determines the amplitude is the loss constant α.

## 3. DISCUSSION AND CONCLUSION

In this lab, we have learned the relevant methods for calculating the characteristic impedance of a transmission line under lossless transmission line conditions. In lossless transmission line, if the input waveform is sine wave condition. The phase difference and amplitude of each junction are simulated, and the time delay is calculated theoretically. If the input waveform is square wave condition, we found that simulated square wave is composed of many sinusoidal waves of different frequencies and they will propagate at different speeds along the transmission line. Therefore, the transmission line will cause square wave distortion when propagating square wave. And as the frequency continues to increase, the output waveform gradually becomes distorted and approaches the sine wave.

In low-loss transmission line, need to satisfy $R \ll \omega L$ and $G \ll \omega C$. There is also distortion in low-loss transmission line conditions. And we found that it is the loss constant α determines the amplitude rather than frequency.

In this lab, we attempted to use ADS software to accomplish the circuit simulation of an artificial transmission line. By changing the transmission line parameters, we observed the corresponding effects and compare these observations with artificial transmission line theory.

This lab has deepened our understanding of transmission line theory. During the lab, the transmission line theory is verified by changing the wave frequency, impedance and capacitive reactance and the experimental results are explained and discussed scientifically by using the transmission line theory.

**Reference**


[1] https://blog.csdn.net/yinuoheqian123/article/details/130840375

[2] B39HF-B31HD Laboratory 1 - Artificial Transmission Lines (final)

**Appendix**

Pre-Lab Assignment

Q1.Solution:

$Z_0 = 50\Omega \quad C = 1nF$

We need to make sure that $\omega^2 C^2 Z_0^2 \ll 1$.

Therefore $\omega^2 \ll \frac{1}{C^2 Z_0^2}$

Which is $f \ll \frac{1}{2\pi C Z_0} = 3.183 \times 10^6 Hz$

The upper frequency limit is $3.183 \times 10^6 Hz$

Q2.Solution:

$f = 10\text{kHz} = 10^4 Hz \quad C = 1nF \quad L = 1nH \quad D_l = 2cm/cell$

Characteristic impedance $Z_0 = \sqrt{\frac{L}{C}} = 1\Omega$

Phase shift $2\pi f\sqrt{LC} = 6.283 \times 10^{-5}\ rads/cell$

Time delay $\sqrt{LC} = 10^{-9}\ seconds/cell$

$\beta = \frac{2\pi f\sqrt{LC}}{D_l} = 3.142 \times 10^{-3} rad/cm$

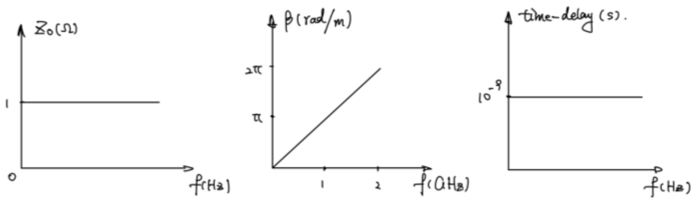
Q3.Solution:

$C = 1nF \quad L = 1nH$

Characteristic impedance $Z_0 = \sqrt{\frac{L}{C}} = 1\Omega$

Phase constant $\beta = \omega\sqrt{LC} = 2\pi f\sqrt{LC} = (2\pi \times 10^{-9})f$

Graphs below:




Discussion:

Characteristic impedance and time delay are independent from frequency.

Phase constant is linearly related to the frequency with a slope of $2\pi \times 10^{-9}\ rad \cdot s/m$.

Q4.Solution:

$t = 0, V = V_0 \cos(0) = V_0$

$V(z) = V_0 e^{-\gamma z},\ \gamma = j\beta = j2\pi f\sqrt{LC} = 0.0297\pi j$

Therefore $V(z) = V_0 e^{-0.0297\pi jz}$

$V(z) = Re\{V(z)\} = V_0 \cos(-0.0297\pi z)$

$\lambda = \frac{2\pi}{\beta} = 67.34\ /cell \quad n = \frac{15}{\lambda} = 0.223$