西安電子科技大學

XIDIAN UNIVERSITY

B39HF High Frequency Circuits

Lecture 12
Transients on Transmission Lines

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Introduction

Transients on Transmission Line

- In previous chapters, the analysis is focused on single-frequency, time-harmonic signals under steady-state conditions, where the *impedance-matching* and *Smith chart* are useful for a wide range of applications.
- However, *impedance-matching* and *Smith chart* are inappropriate for analysis of *digital or wideband signals* which exist in *digital chips, circuits and computer network*.
- Then, how to analysis these signals? *The transient response of the transmission-line.*

Transients on Transmission Line

What is the *Transient response* mean?

The *transient response* of a voltage pules on a transmission line is a time record of its back and forth travel between the sending receiving end of the line, taking into account all the multiple reflection (echoes) at both ends.

Transients on Transmission Line

- Consider a single rectangular pulse of amplitude V_0 and duration τ [Fig. (a)]:

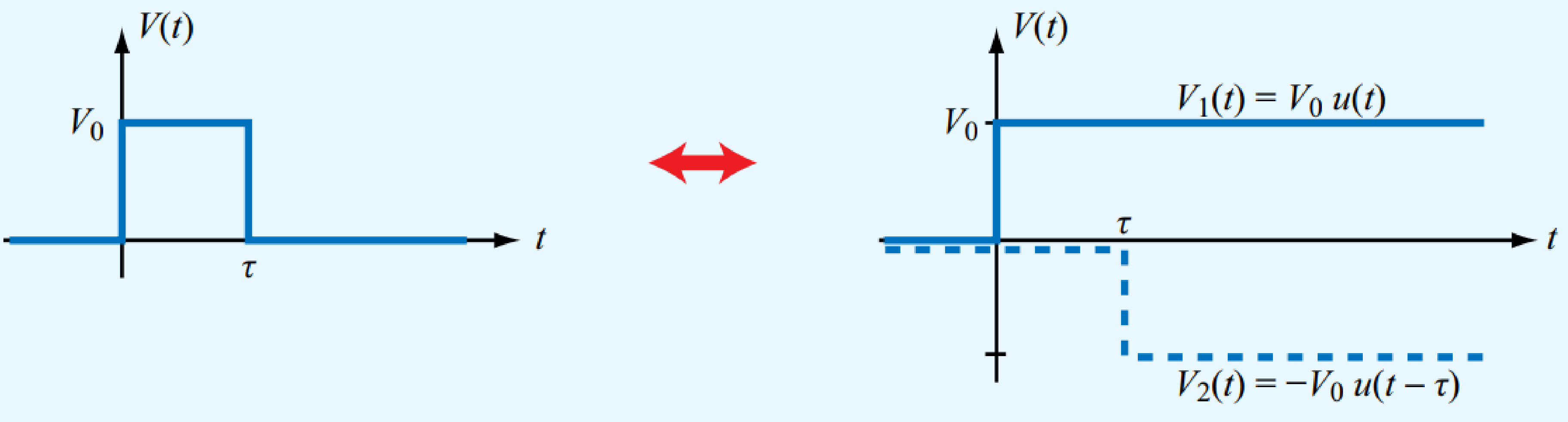
$$V(t)=\begin{cases}V_0 & 0\le t\le\tau\\ 0 & Otherwise\end{cases}$$

it can be decomposed into the sum of two unit step functions [Fig. (b)]:

$$V(t)=V_1(t)+V_2(t)=V_0u(t)-V_0u(t-\tau)$$

where the step function is:

$$u(x)=\begin{cases}1 & for \quad x>0\\ 0 & for \quad x<0\end{cases}$$

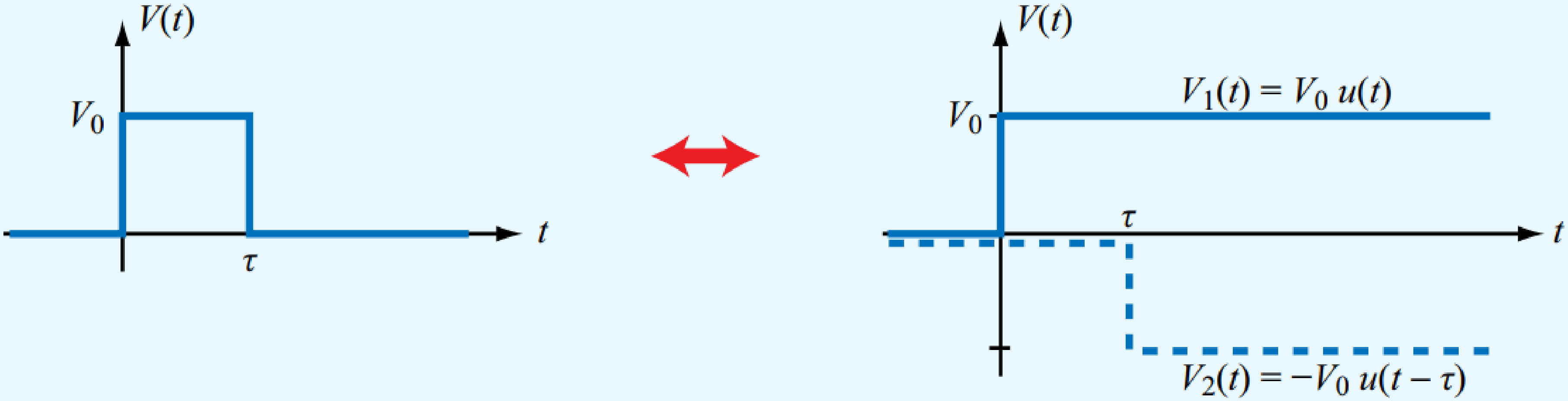


(a) Pulse of duration τ (b) $V(t) = V_1(t) + V_2(t)$

Transients on Transmission Line

$$V(t)=V_1(t)+V_2(t)=V_0u(t)-V_0u(t-\tau)$$

- *The first component*: $V_1(t)=V_0u(t)$, a dc voltage whose amplitude is V_0 is switched on at $t=0$ and retains its value indefinitely;
- *The second component*: $V_2(t)=-V_0u(t-\tau)$, a dc voltage whose amplitude is $-V_0$ is switched on at $t=\tau$ and retains that way indefinitely;
- The sum of $V_1(t)$ and $V_2(t)$ is equal to V_0.
- The decomposition of $V(t)$ allows the analysis of the transient behavior be regarded as a superposition of two dc signals..



(a) Pulse of duration τ (b) $V(t)=V_1(t)+V_2(t)$

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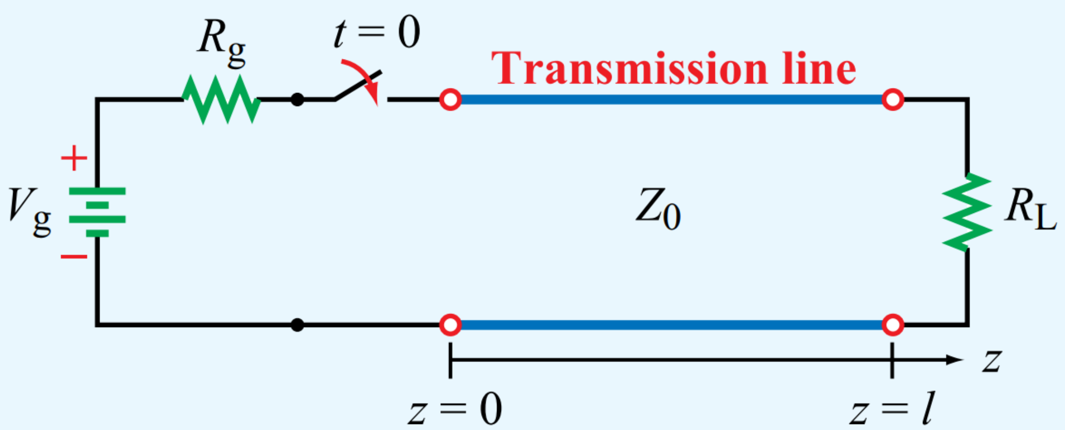
Transient Response to a Step Function

Transient Response to a Step Function



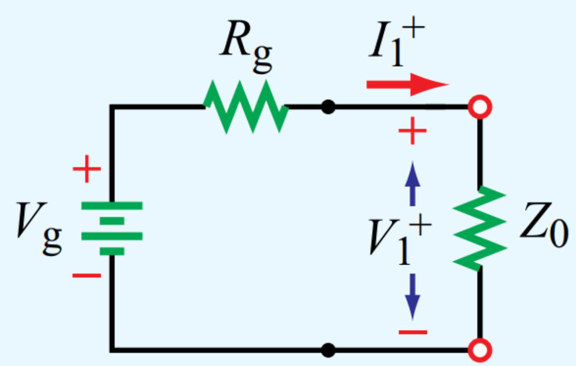
(a) Transmission-line circuit

- The *generator* contains:
  - V_g: dc voltage source
  - R_g: internal resistance of generator
- The *transmission line*:
  - l: length
  - Z_0: characteristic impedance
- The *load*:
  - R_L: a purely resistive

➢ Note that the definition of $z = 0$ is different from that previous, it is more convenient to define it as the location of the source here.

➢ After the switch closing, the variation of the corresponding parameters will be discussed in detail.

Transient Response to a Step Function



(b) Equivalent circuit at $t = 0^+$

- The superscript $[\cdot]^+$ denotes the fact that the wave is traveling in the $+z$ direction.
- *T* is the time it takes the wave to travel the full length of the line.

$$T = \frac{l}{u_p}$$

- Three instances in time for a circuit in which $R_g = 4Z_0$ and $R_L = 2Z_0$ will be discussed next.

Assuming that the switch is closed at $t = 0$, the *initial condition* is shown in Fig (b).

In absence of a signal on the line at $t = 0$, the input impedance of the line is unaffected by the load impedance R_L, hence the load of the generator circuit is Z_0, then at the sending end, there are:

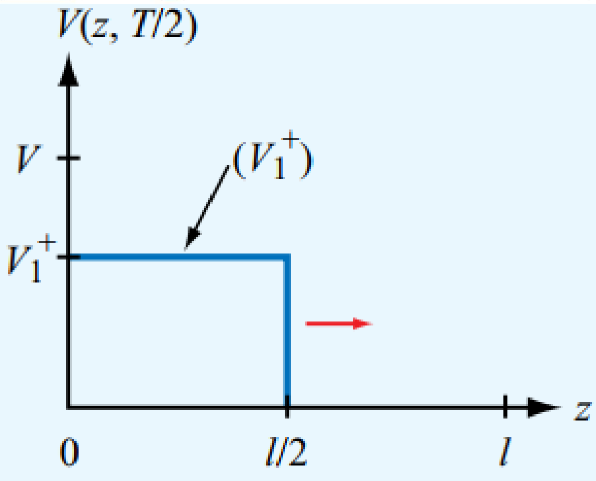
$$I_1^+ = \frac{V_g}{R_g + Z_0}$$

$$V_1^+ = I_1^+ Z_0 = \frac{V_g Z_0}{R_g + Z_0}$$

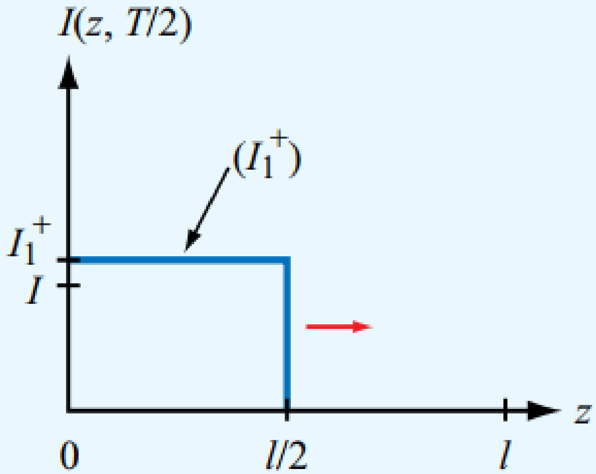
V_1^+ and I_1^+ constitutes a wave that travels along the line with a velocity of:

$$u_p = \frac{1}{\sqrt{\mu\varepsilon}}$$

Transient Response to a Step Function



(a) $V(z)$ at $t = T/2$



(d) $I(z)$ at $t = T/2$

1. At time $t_1 = T/2$:

By time t_1, the wave has traveled halfway down the line, hence the voltage on $\left[0, \frac{l}{2}\right)$ is equal to V_1^+ while that on $\left[\frac{l}{2}, l\right)$ is zero.

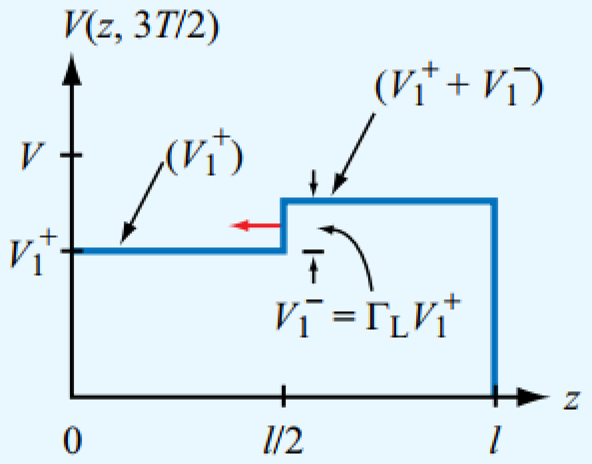
$$V(z, T/2) = \begin{cases} V_1^+ & 0 \le z < l/2 \\ 0 & l/2 \le z \le l \end{cases}$$

$$I(z, T/2) = \begin{cases} I_1^+ & 0 \le z < l/2 \\ 0 & l/2 \le z \le l \end{cases}$$

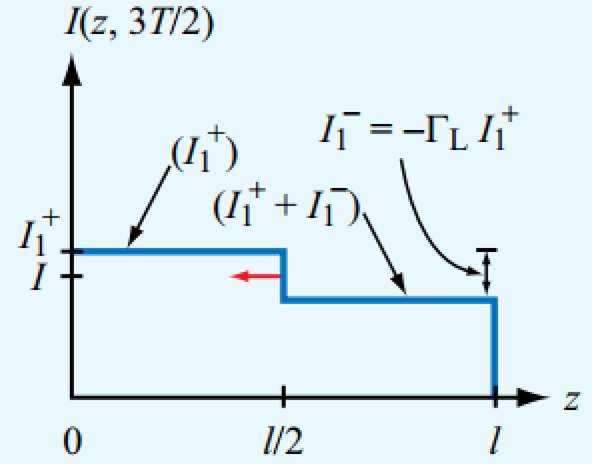
The voltage [Fig. (a)] and current [Fig. (d)] corresponding to the location z on transmission line is shown on the left [See in P. 116, Figure 2-41].

No reflection effect could be considered and no reflected voltage exists at this time.

Transient Response to a Step Function



(b) $V(z)$ at $t = 3T/2$

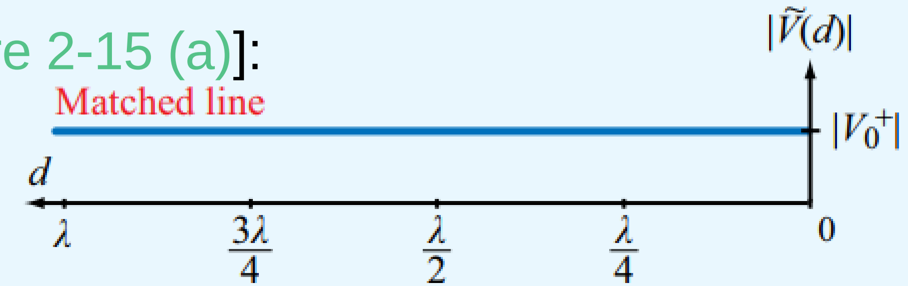


(e) $I(z)$ at $t = 3T/2$

2. At time $t_2 = 3T/2$:

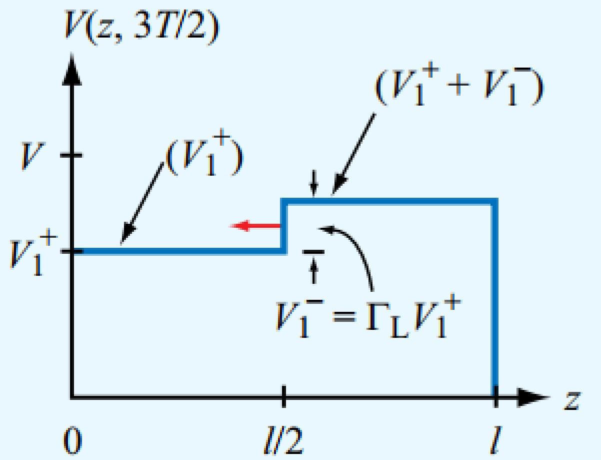
At $t = T$, as defined previously $T = \frac{l}{u_p}$, the wave reaches the load at $z = l$, hence the reflection effect should be considered here.

- When $R_L = Z_0$, no reflection exists at the load side ($|\Gamma_L| = 0$), no reflected wave present and no standing waves, and $|\tilde{V}(d)| = |V_0^+|$ for all values of l. [See in P.71] and the voltage on the line is [P.72 Figure 2-15 (a)]:

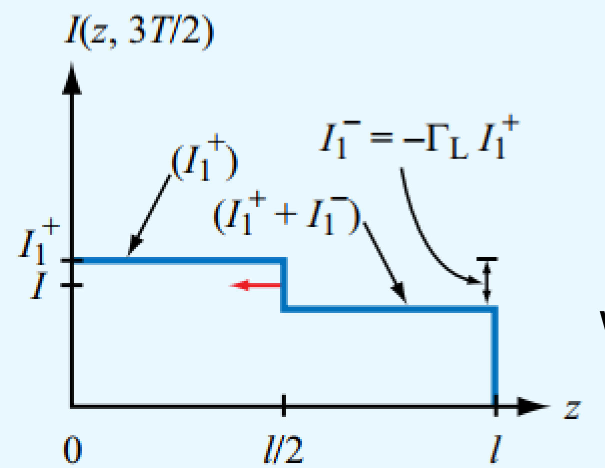


(a) $Z_L = Z_0$

Transient Response to a Step Function



(b) $V(z)$ at $t = 3T/2$



(e) $I(z)$ at $t = 3T/2$

2. At time $t_2 = 3T/2$:

- Here we focus on the common situation where $R_L \neq Z_0$, and make a discussion on the variety of voltage on the transmission line in the presence of the reflection effect ($|\Gamma_L| \neq 0$). Substituting the assuming value mentioned above: $R_g = 4Z_0$; $R_L = 2Z_0$, then the reflection coefficient of the load is:

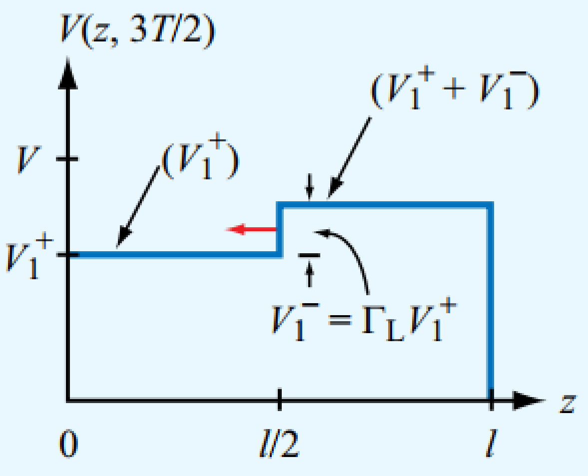
$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = \frac{2Z_0 - Z_0}{2Z_0 + Z_0} = \frac{1}{3}$$

Then the mismatch generates a reflected wave with:

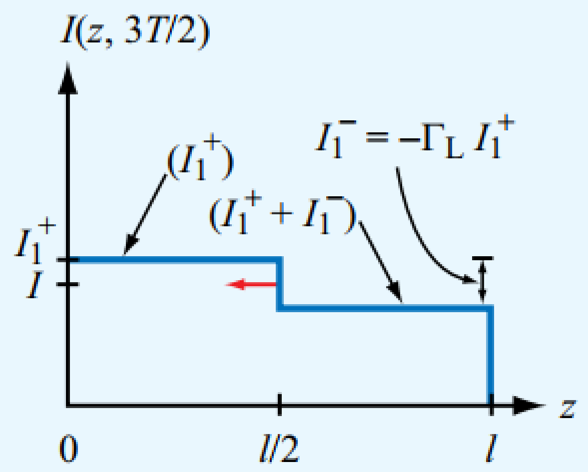
$$V_1^- = \Gamma_L V_1^+ \quad ; \quad I_1^- = -\Gamma_L I_1^+$$

- The superscript $[\cdot]^-$ denotes the fact that the wave is traveling in the $-z$ direction.

Transient Response to a Step Function



(b) $V(z)$ at $t = 3T/2$

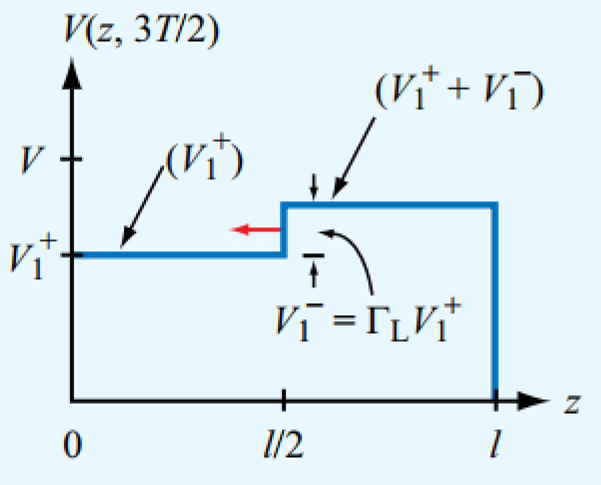


(e) $I(z)$ at $t = 3T/2$

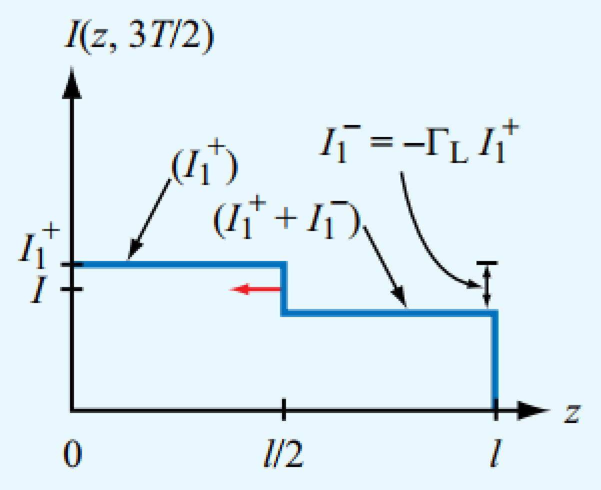
2. At time $t_2 = 3T/2$:

After this first reflection, the voltage and current on the line consists of the sum of two waves: the initial wave V_1^+, I_1^+ and the reflected wave V_1^- and I_1^-, and we analyze the voltage and current on the transmission line by the analysis the two waves respectively and then superimposing the voltages and currents of them to obtain the location-related voltage and currents on the transmission line.

Transient Response to a Step Function



(b) $V(z)$ at $t = 3T/2$



(e) $I(z)$ at $t = 3T/2$

2. At time $t_2 = 3T/2$:

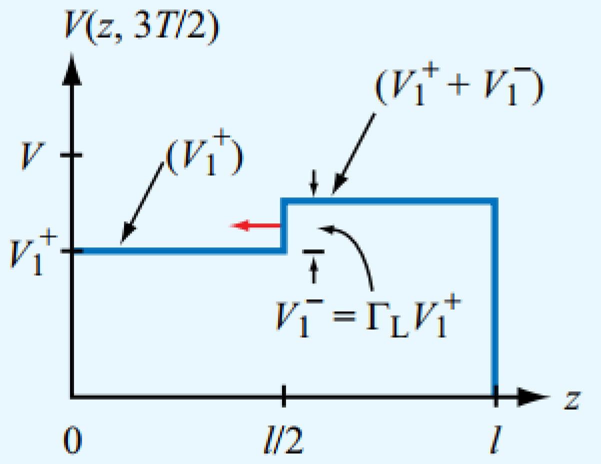
- Reflected wave V_1^- , I_1^- :

The first reflection occurs at $t = T$. Similar to the analyses at $t_1 = T/2$, from $t = T$ when the reflected wave occurs to the reference time $t_2 = 3T/2$, the distance the reflected wave traveling is half of the transmission line length, which can be expressed as:

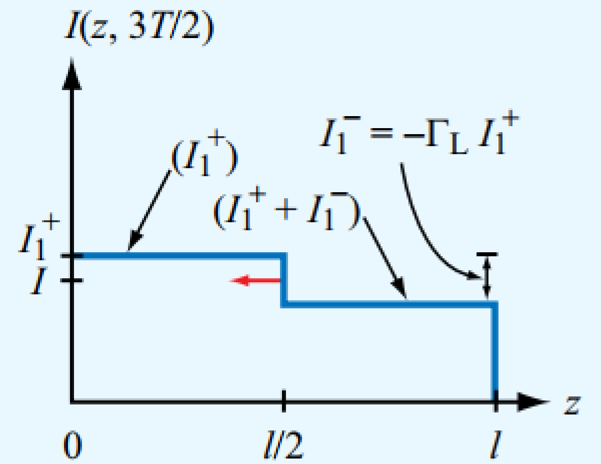
$$V_1^-(z) = \begin{cases} 0 & 0 \le z < l/2 \\ V_1^- & l/2 \le z \le l \end{cases}$$

$$I_1^-(z) = \begin{cases} 0 & 0 \le z < l/2 \\ -I_1^- & l/2 \le z \le l \end{cases}$$

Transient Response to a Step Function



(b) $V(z)$ at $t = 3T/2$



(e) $I(z)$ at $t = 3T/2$

2. At time $t_2 = 3T/2$:

- Reflected wave V_1^- , I_1^- :

  Upon substituting the assumed values, that is:

$$R_g = 4Z_0$$

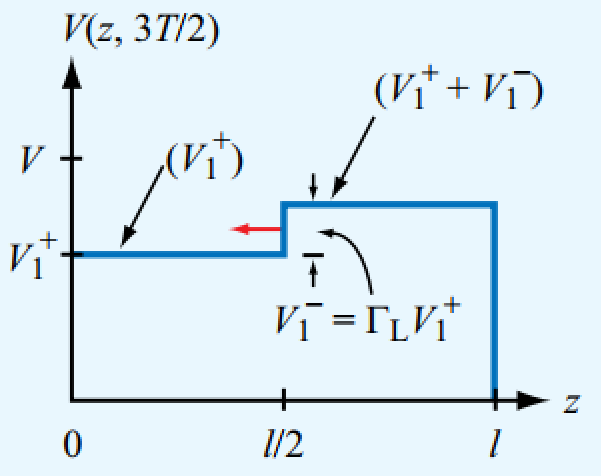
$$R_L = 2Z_0$$

  it can be obtained that:

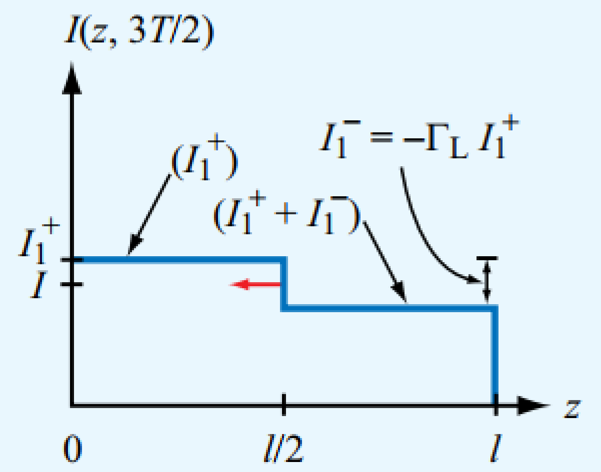
$$V_1^-(z) = \begin{cases} 0 & 0 \le z < l/2 \\ \dfrac{1}{3}V_1^+ & l/2 \le z \le l \end{cases}$$

$$I_1^-(z) = \begin{cases} 0 & 0 \le z < l/2 \\ -\dfrac{1}{3}I_1^+ & l/2 \le z \le l \end{cases}$$

Transient Response to a Step Function



(b) $V(z)$ at $t = 3T/2$



(e) $I(z)$ at $t = 3T/2$

2. At time $t_2 = 3T/2$:

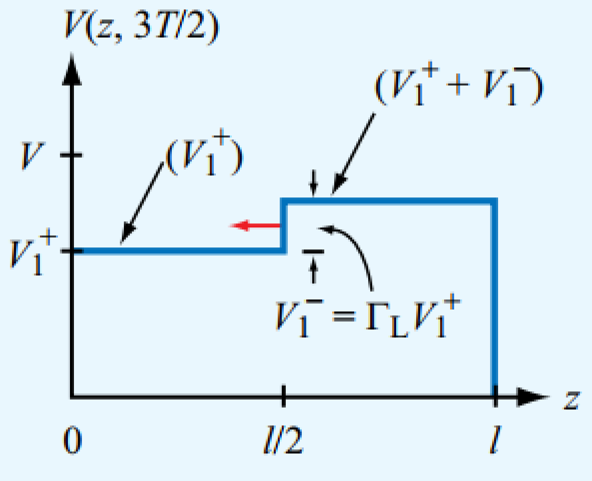
- Initial wave V_1^+ , I_1^+ :

After the initial wave achieve the load at $t = T$, the voltage and current distribution on the transmission line owing to the initial wave can be expressed as:

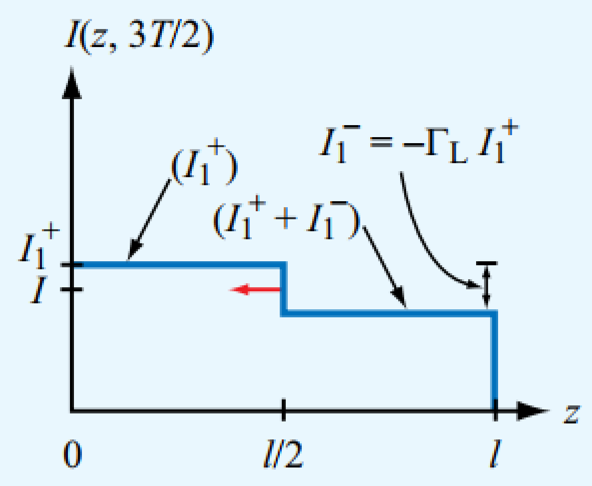
$$V_1^+(z) = V_1^+;(0 \le z \le l)$$

$$I_1^+(z) = I_1^+;(0 \le z \le l)$$

Transient Response to a Step Function



(b) $V(z)$ at $t = 3T/2$



(e) $I(z)$ at $t = 3T/2$

2. At time $t_2 = 3T/2$:

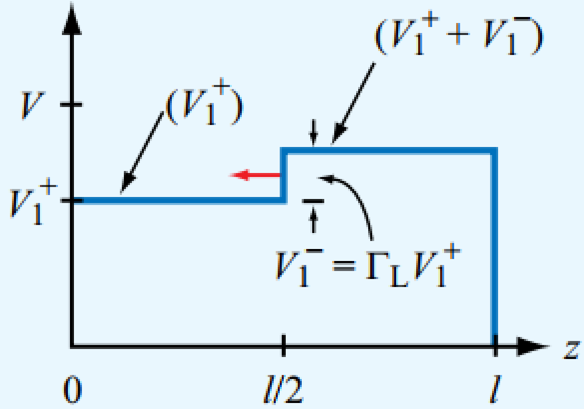
- Voltage and current on the transmission line:

Voltage and current distribution on the transmission line at $t_2 = 3T/2$ can be expressed as the superimpositions of V_1^+ and V_1^-, I_1^+ and I_1^-, respectively, hence:

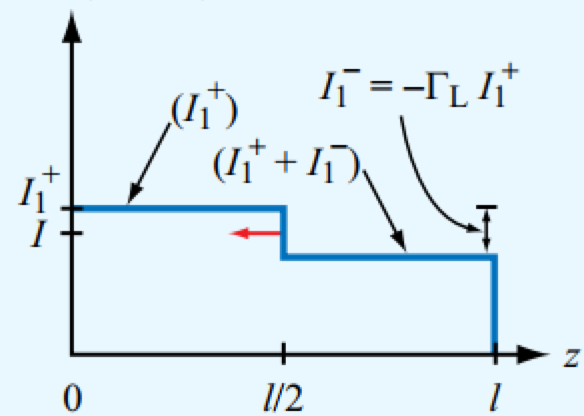
$$V(z,3T/2)=\begin{cases} V_1^+ & 0 \le z < l/2 \\ V_1^+ + V_1^- = \dfrac{4}{3}V_1^+ & l/2 \le z \le l \end{cases}$$

$$I(z,3T/2)=\begin{cases} I_1^+ & 0 \le z < l/2 \\ I_1^+ + I_1^- = \dfrac{2}{3}V_1^+ & l/2 \le z \le l \end{cases}$$

Transient Response to a Step Function



(b) $V(z)$ at $t = 3T/2$

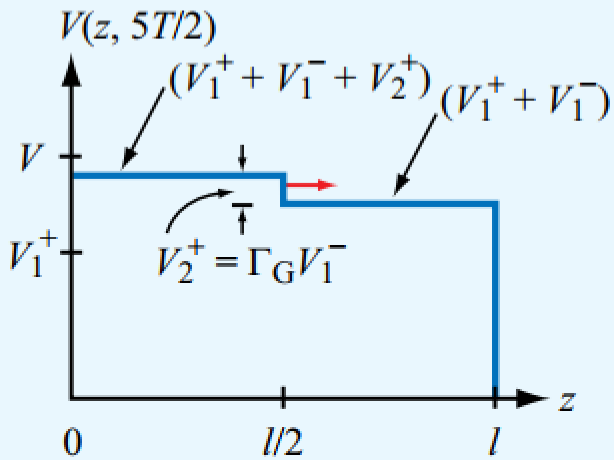


(e) $I(z)$ at $t = 3T/2$

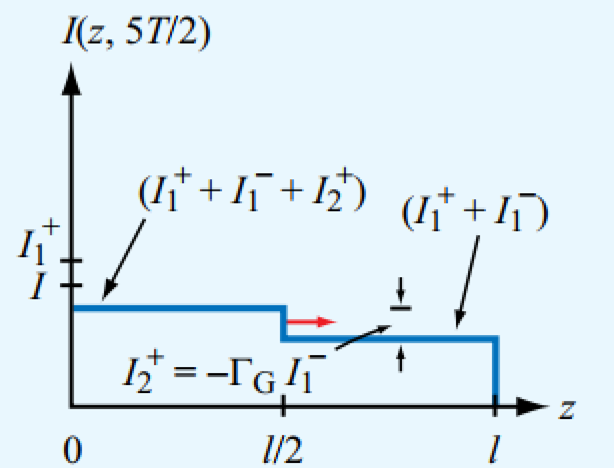
2. At time $t_2 = 3T/2$:

According to the analysis of the voltage and current variety associated with the location along the lossless transmission line, and to the assumption of the value of the load and the characteristic impedance of line, the voltage [Fig. b] on the line and its corresponding current [Fig. e] are shown in left [See in P. 116, Figure 2-41].

Transient Response to a Step Function



(c) $V(z)$ at $t = 5T/2$



(f) $I(z)$ at $t = 5T/2$

3. At time $t_3 = 5T/2$:

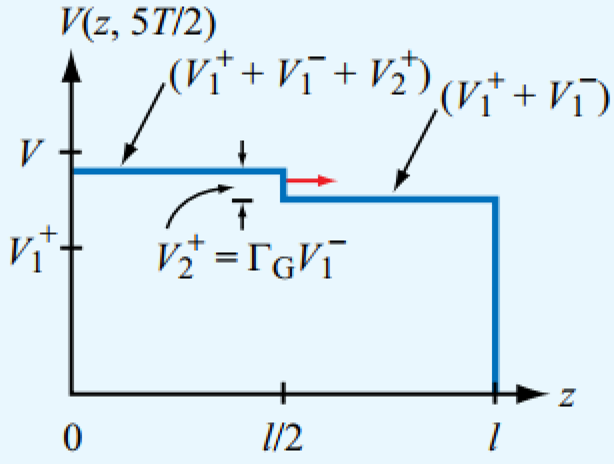
At $t = T$, the initial wave reaches the load side for the first time, and the reflected wave is generated, and at $t = 2T$ the first-reflected wave travels back along $-z$ and reaches the sending end. The generator has internal resistance which can reflect the reflected wave when mismatching.

- When $R_g = Z_0$, no reflection will occur at the sending end ($|\Gamma_g| = 0$), in this case,

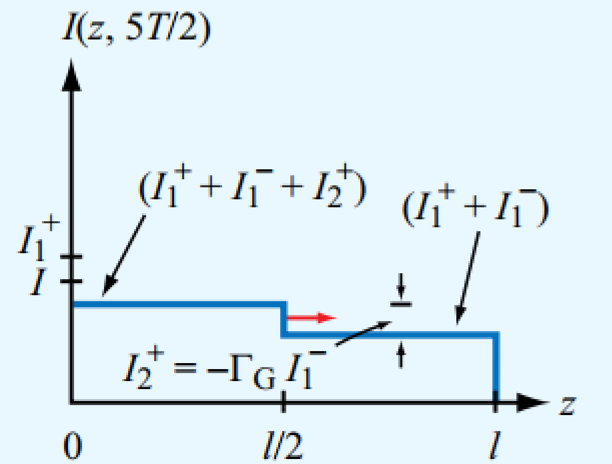
$$V(z) = V_1^+ + V_1^- = \frac{4}{3} V_1^+; (0 \le z \le l)$$

$$I(z) = I_1^+ + I_1^- = \frac{2}{3} I_1^+; (0 \le z \le l)$$

Transient Response to a Step Function



(c) $V(z)$ at $t = 5T/2$



(f) $I(z)$ at $t = 5T/2$

3. At time $t_3 = 5T/2$:

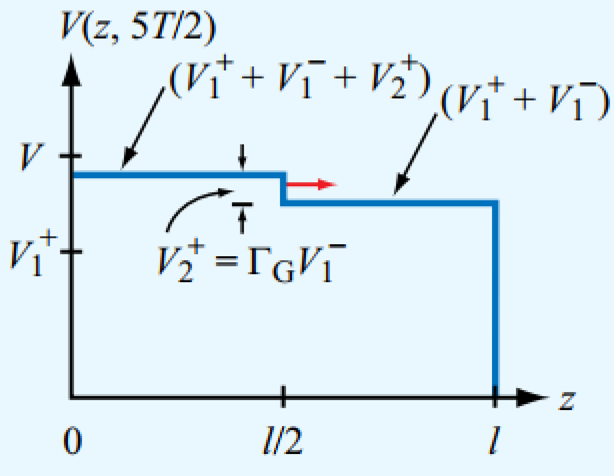
- Here we also focus on the common situation where $R_g \neq Z_0$, the mismatching of the internal resistance will also induce reflect wave toward $+z$ direction. Denote the reflection coefficient generated by the internal resistance of the generator as Γ_g and that is:

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

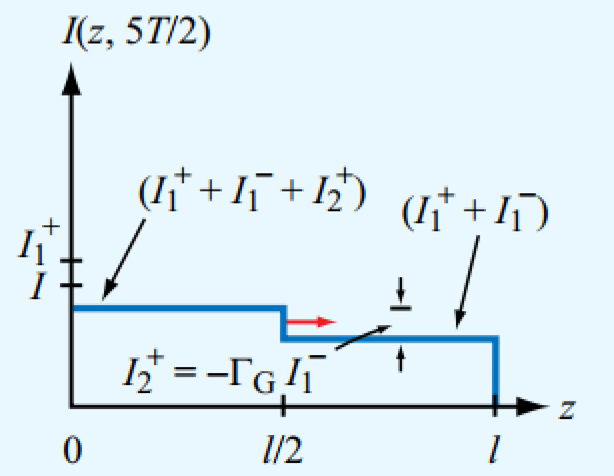
In substituting the assuming value mentioned above: $R_g = 4Z_0$, then the reflection coefficient of the generator resistance is:

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{4Z_0 - Z_0}{4Z_0 + Z_0} = \frac{3}{5}$$

Transient Response to a Step Function



(c) $V(z)$ at $t = 5T/2$

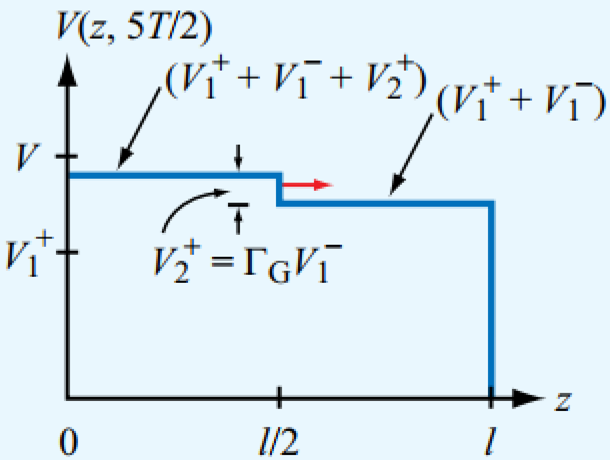


(f) $I(z)$ at $t = 5T/2$

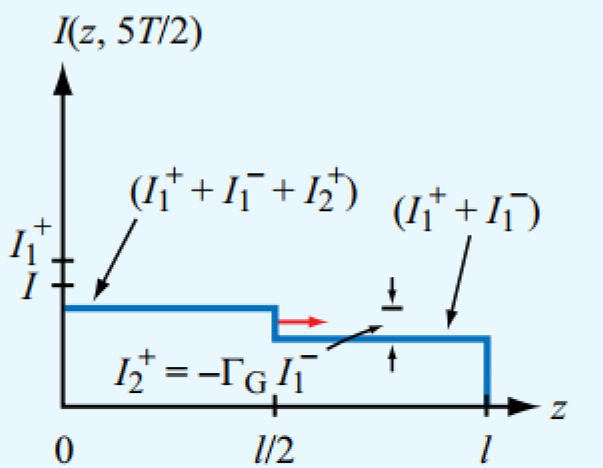
3. At time $t_3 = 5T/2$:

After this second reflection, the voltage and current on the line consists of the sum of three waves: the initial wave V_1^+ and I_1^+ , the reflected wave V_1^- and I_1^-, and the second reflected wave V_2^+ and I_2^+, hence the analysis could be simplified by the analyzing these three waves respectively and then superimposing them.

Transient Response to a Step Function



(c) $V(z)$ at $t = 5T/2$



(f) $I(z)$ at $t = 5T/2$

3. At time $t_3 = 5T/2$:

What waves existing in transmission line now?

- Initial wave: V_1^+, I_1^+;
- First reflected wave: V_1^-, I_1^-;
- Second reflected wave: V_2^+, I_2^+;

And the amplitude relationships between these three waves are:

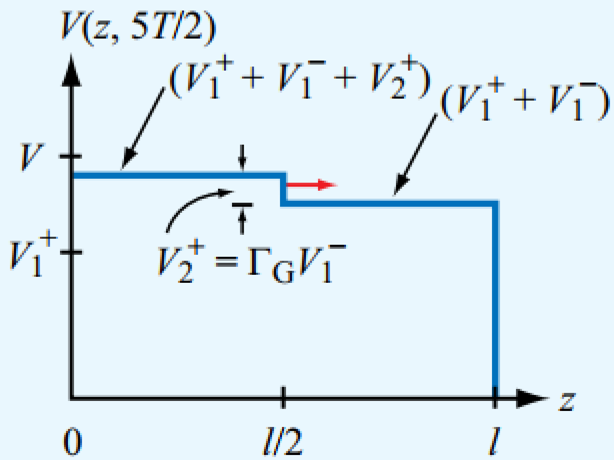
Voltage:

$$V_1^- = \Gamma_L V_1^+$$
$$V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+$$

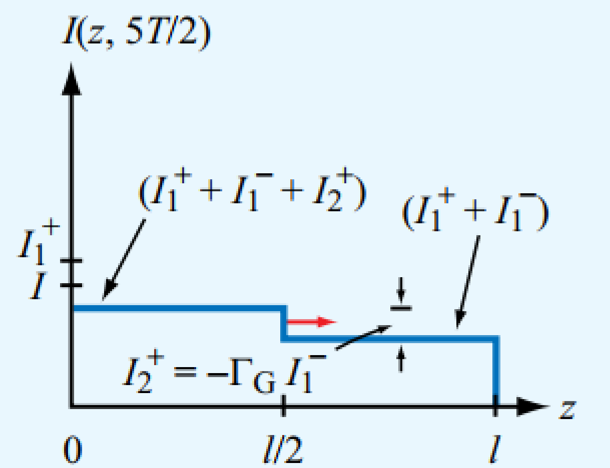
Current:

$$I_1^- = -\Gamma_L I_1^+$$
$$I_2^+ = -\Gamma_g I_1^- = \Gamma_g \Gamma_L I_1^+$$

Transient Response to a Step Function



(c) $V(z)$ at $t = 5T/2$



(f) $I(z)$ at $t = 5T/2$

3. At time $t_3 = 5T/2$:

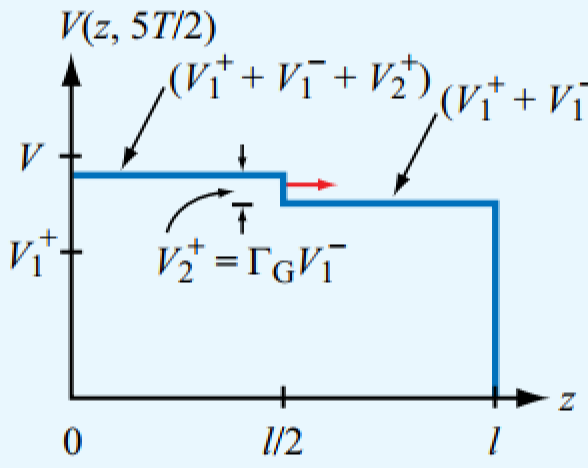
- Reflected wave V_2^+ , I_2^+ :

The second reflection occurs at $t = 2T$. Similar to the analysis at $t_2 = 3T/2$, from $t = 2T$ when the second reflected wave occurs at the reference time $t_2 = 5T/2$, the distance the second reflected wave traveling is half of the transmission line length, which can be expressed as:

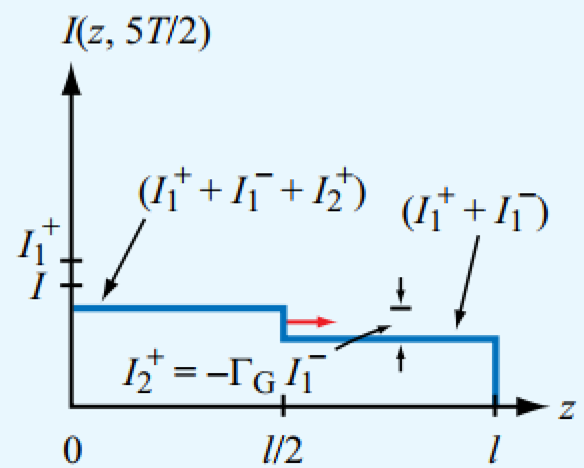
$$V_2^+(z) = \begin{cases} V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+ = \dfrac{1}{5} V_1^+ & 0 \le z < l/2 \\ 0 & l/2 \le z \le l \end{cases}$$

$$I_2^+(z) = \begin{cases} I_2^+ = -\Gamma_g I_1^- = \Gamma_g \Gamma_L I_1^+ = \dfrac{1}{5} I_1^+ & 0 \le z < l/2 \\ 0 & l/2 \le z \le l \end{cases}$$

Transient Response to a Step Function



(c) $V(z)$ at $t = 5T/2$



(f) $I(z)$ at $t = 5T/2$

3. At time $t_3 = 5T/2$:

➢ Reflected wave V_1^- , I_1^- :

After the first reflected wave achieve the sending end at $t = 2T$, the voltage and current distribution on the transmission line owing to the first reflected wave can be expressed as:

$$V_1^-(z) = V_1^- = \Gamma_L V_1^+ = \frac{1}{3} V_1^+ ; (0 \le z \le l)$$

$$I_1^-(z) = I_1^- = -\Gamma_L I_1^+ = -\frac{1}{3} I_1^+ ; (0 \le z \le l)$$

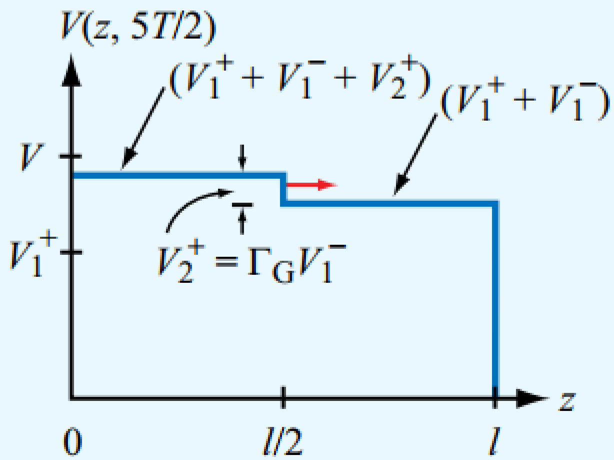
➢ Initial wave V_1^+ , V_1^+ :

The voltage and current on the transmission line owing to the initial wave can be expressed as:

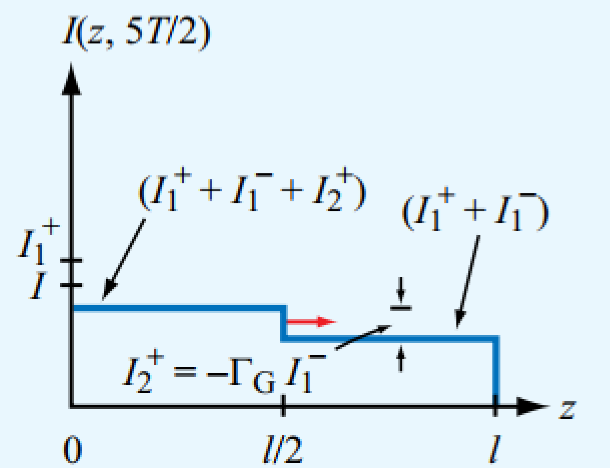
$$V_1^+(z) = V_1^+ ; (0 \le z \le l)$$

$$I_1^+(z) = I_1^+ ; (0 \le z \le l)$$

Transient Response to a Step Function



(c) $V(z)$ at $t = 5T/2$



(f) $I(z)$ at $t = 5T/2$

3. At time $t_3 = 5T/2$:

- Voltage on the transmission line:

Voltage distribution on the transmission line at $t_3 = 5T/2$ can be expressed as a superimposition of V_1^+, V_1^- and V_2^+, hence:

$$V\left(z,5T/2\right)=\begin{cases} V_1^+ + V_1^- + V_2^+ = \dfrac{23}{15}V_1^+ & 0 \le z < l/2 \\ V_1^+ + V_1^- = \dfrac{4}{3}V_1^+ & l/2 \le z \le l \end{cases}$$

$$I\left(z,5T/2\right)=\begin{cases} I_1^+ + I_1^- + I_2^+ = \dfrac{13}{15}I_1^+ & 0 \le z < l/2 \\ I_1^+ + I_1^- = \dfrac{2}{3}I_1^+ & l/2 \le z \le l \end{cases}$$

The voltage [Fig. c] on the line and its corresponding current [Fig. f] at time $t_3 = 5T/2$ are shown in left [See in P. 116, Figure 2-41].

Transient Response to a Step Function

- So far, we have examined the transient response of the voltage wave $V(z,t)$ and the current $I(z,t)$ at three time for examples. It should be noticed that at either end of the line the reflected voltage is related to the incident voltage by the reflection coefficient at that end, the reflected current is related to the incident current by the negative reflection coefficient [See in P. 168, Equation (2.61)].
- The multiple-reflection process continues indefinitely, and the ultimate value that $V(z,t)$ when t approaches $+\infty$ is the same at all locations on the transmission line.
- how to express the voltage $V(z,t)$ and current $I(z,t)$ on the transmission line when $t \to +\infty$?

Transient Response to a Step Function

- Firstly, we will discuss the voltage (V_∞) when $t \to +\infty$:

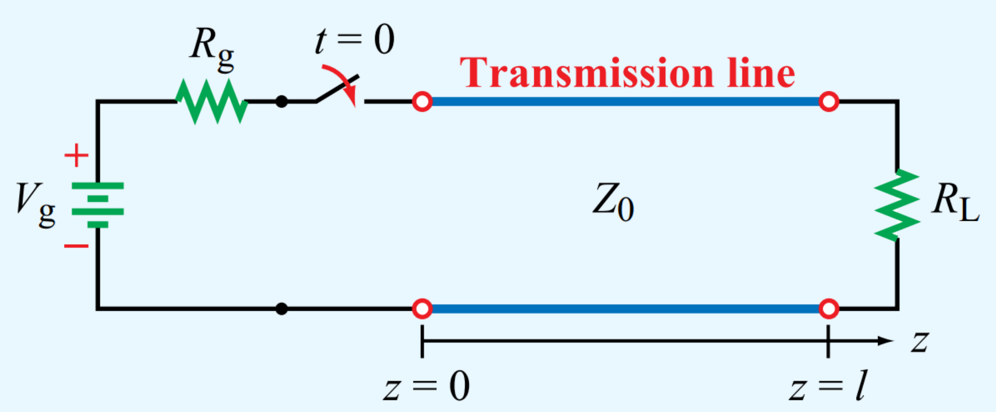
$$
\begin{aligned}
V_\infty &= V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + \cdots \\
&= V_1^+\left[1+\Gamma_L+\Gamma_L\Gamma_g+\Gamma_L^2\Gamma_g+\Gamma_L^2\Gamma_g^2+\Gamma_L^3\Gamma_g^2+\cdots\right] \\
&= V_1^+\left[\left(1+\Gamma_L\right)\left(1+\Gamma_L\Gamma_g+\Gamma_L^2\Gamma_g^2+\cdots\right)\right] \\
&= V_1^+\left[\left(1+\Gamma_L\right)\left(1+x+x^2+\cdots\right)\right]
\end{aligned}
$$

where $x = \Gamma_L\Gamma_g$. The series inside the square bracket is the geometric series of the function:

$$\frac{1}{1-x} = 1+x+x^2+\cdots \qquad for\ |x|<1$$

as we already know that the

$$|\Gamma| \in [0,1]$$



(a) Transmission-line circuit

Transient Response to a Step Function

- Firstly, we will discuss the voltage (V_∞) when $t \rightarrow +\infty$:

For $|\Gamma| = 1$ at both ends can not always hold, hence Γ_L and Γ_g will not equal to 1 at same time. Hence $\Gamma_L\Gamma_g$ will always satisfy the condition that $|x| = |\Gamma_L \Gamma_g| < 1$. By implying the geometric series to the voltage expression V_∞, which can be rewritten in compact form as:

$$V_\infty = V_1^+ \frac{1+\Gamma_L}{1-\Gamma_L\Gamma_g}$$

V_1^+, Γ_L and Γ_g in the equation of V_∞ have been defined, previously.

$$V_1^+ = I_1^+ Z_0 = \frac{V_g Z_0}{R_g + Z_0} \qquad \Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} \qquad \Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

Transient Response to a Step Function

$$V_\infty = V_1^+ \frac{1+\Gamma_L}{1-\Gamma_L\Gamma_g} \qquad V_1^+ = I_1^+ Z_0 = \frac{V_g Z_0}{R_g + Z_0} \qquad \Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} \qquad \Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

- Firstly, we will discuss the voltage (V_∞) when $t \to +\infty$:

  Upon replacing the V_1^+, Γ_L and Γ_g, we obtain:

$$V_\infty = \frac{V_g R_L}{R_g + R_L}$$

- The voltage V_∞ is called the *steady-state voltage* on the line;
- Notice that the expression of voltage when $t \to +\infty$ is exactly what we expect on the basis of dc analysis of the circuit, wherein we treat the transmission line as simply a connecting wire between the generator circuit and the load.
- The corresponding *steady-state current* is:

$$I_\infty = \frac{V_\infty}{R_L} = \frac{V_g}{R_g + R_L}$$

Transient Response to a Step Function

Time Domain Analysis of Transmission Lines

(a) Initiation Condition:

$$I_1^+ = \frac{V_g}{Z_g + Z_0}$$

$$V_1^+ = I_1^+ Z_0 = \frac{V_g Z_0}{Z_g + Z_0}$$

(b) At the Ends of the Transmission Line:

$$V_1^- = \Gamma_L V_1^+$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$I_1^- = -\Gamma_L I_1^+$$

$$V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+$$

$$\Gamma_g = \frac{Z_g - Z_0}{Z_L + Z_0}$$

$$I_2^+ = -\Gamma_g I_1^- = \Gamma_g \Gamma_L I_1^+$$

Transient Response to a Step Function

(c) Steady-State Voltage and Current Equations on the Transmission Line:

$$V_\infty = \frac{V_g Z_L}{Z_g + Z_L}$$

$$I_\infty = \frac{V_\infty}{Z_L} = \frac{V_g}{Z_g + Z_L}$$

Transient Response to a Step Function

- It is a complicated and tedious process to keep tracking of the voltage/current waves as they bounce back and forth on the line at anytime.

? Is there any convenient method to reflect the varieties of these parameters?

- The answer to this question is Yes. That is the *bounce diagrams*, which will be introduced in the following part.

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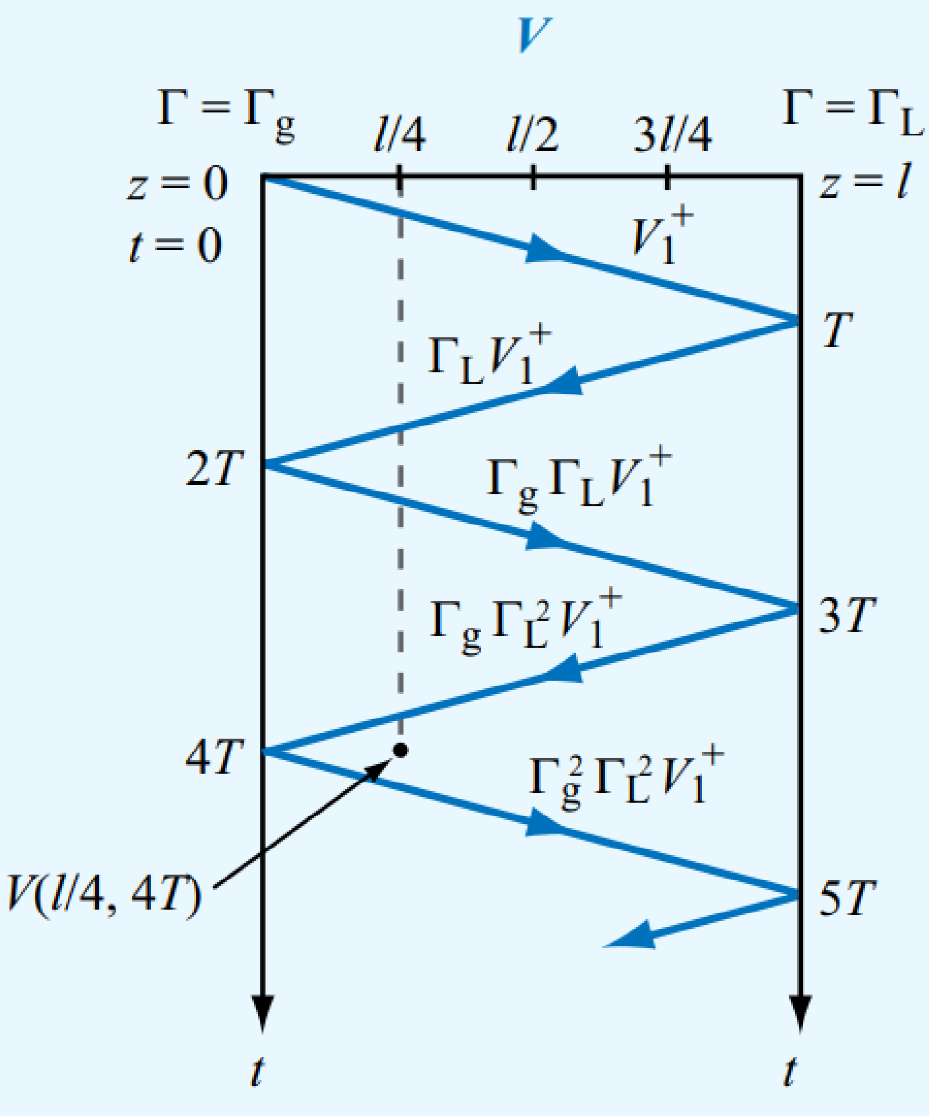
Bounce Diagrams

Bounce Diagrams



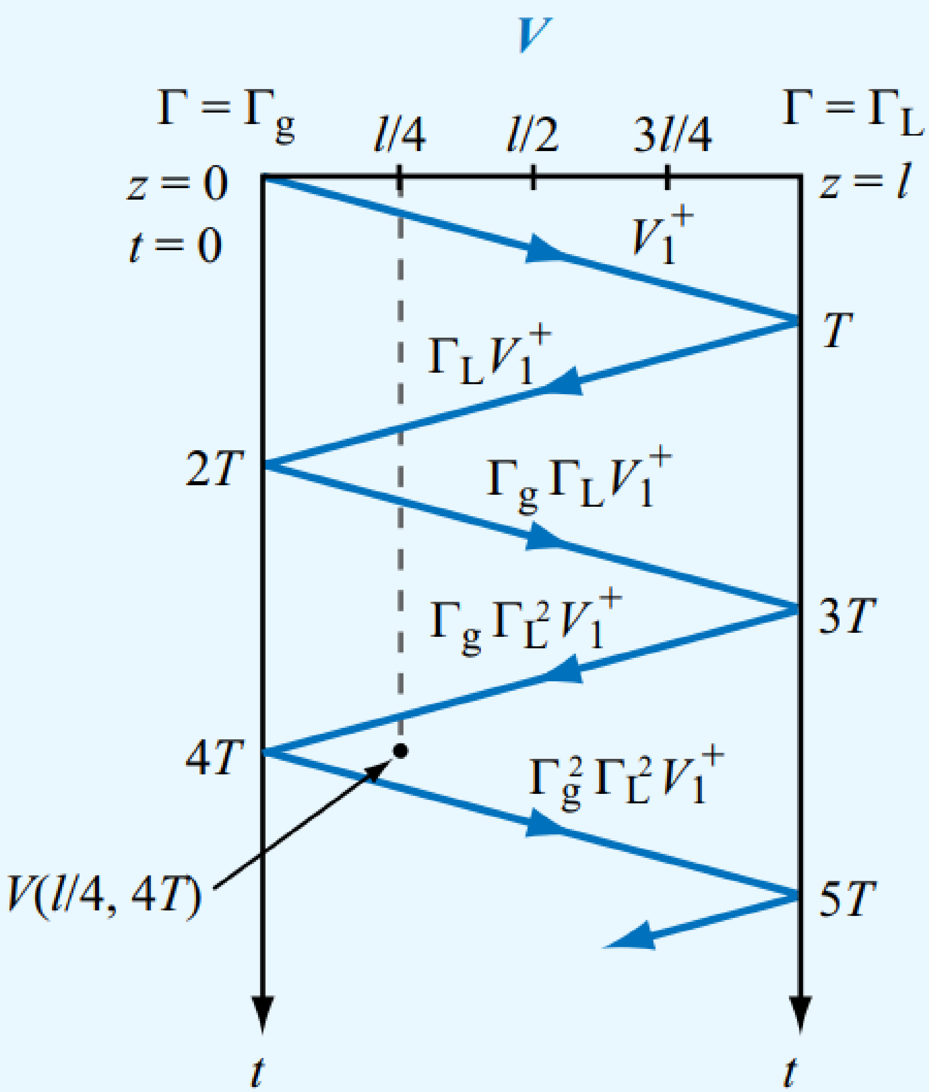
(a) Voltage bounce diagram

Bounce Diagrams: [Voltage]

Fig (a) [P. 118, Figure 2-42 (a)] is an example of the voltage, the figure placed here for the illustration of the bounce diagram.

- *Horizontal axes*: position along the transmission line;
- *Vertical axes*: time;
- *Zigzag line (blue)*: the progress of the voltage wave on the line;
- *Upper left*: $\Gamma = \Gamma_g$; origin of location z and time t;
- *Upper right*: $\Gamma = \Gamma_L$; end of location z.

Bounce Diagrams



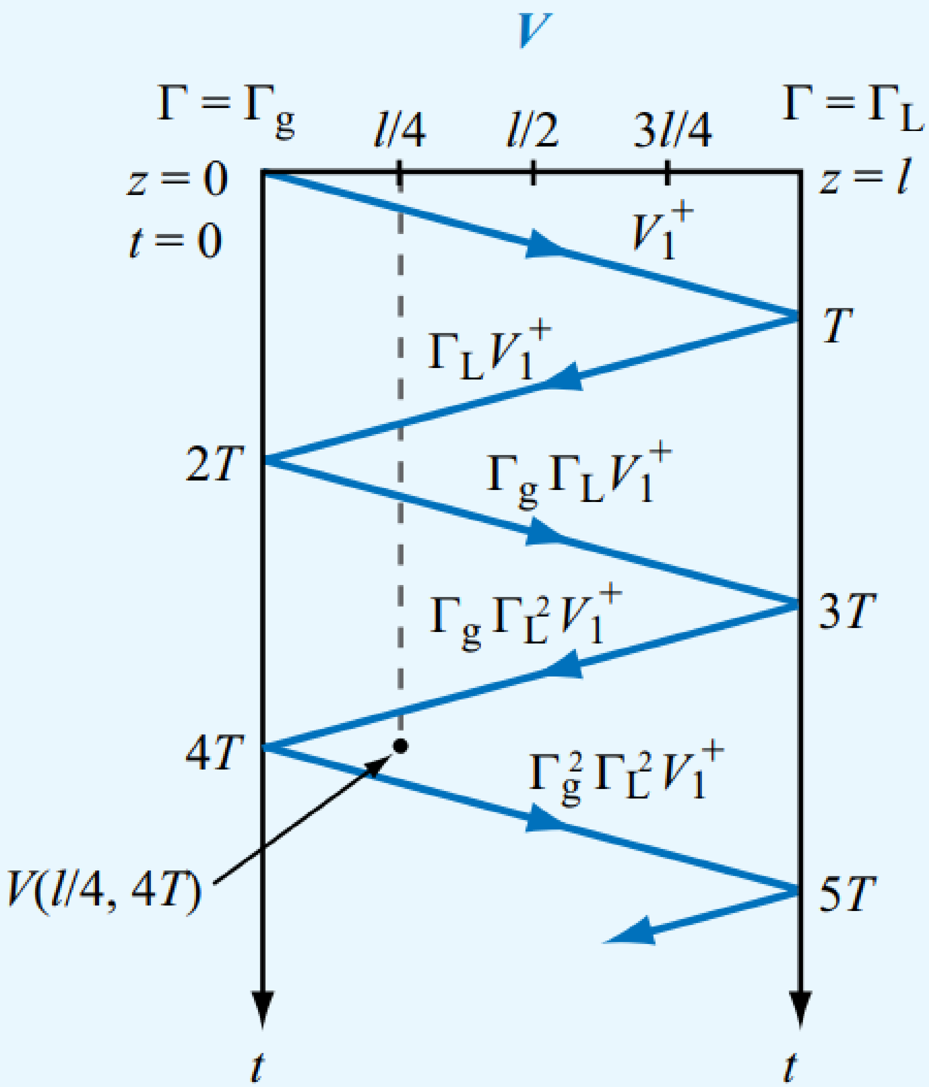
(a) Voltage bounce diagram

Bounce Diagrams: [Voltage]

Track process:

- The incident wave V_1^+ starts at $z = t = 0$ and travels in the $+z$ direction until it reaches the load at $z = l$ at time $t = T$. The notation at upper right ($\Gamma = \Gamma_L$) of bounce diagram indicating the voltage-related reflection coefficients.
- At the end of the first straight-line segment of the zigzag line, a second line is drawn to represent the reflected voltage wave $V_1^- = \Gamma_L V_1^+$.

Bounce Diagrams



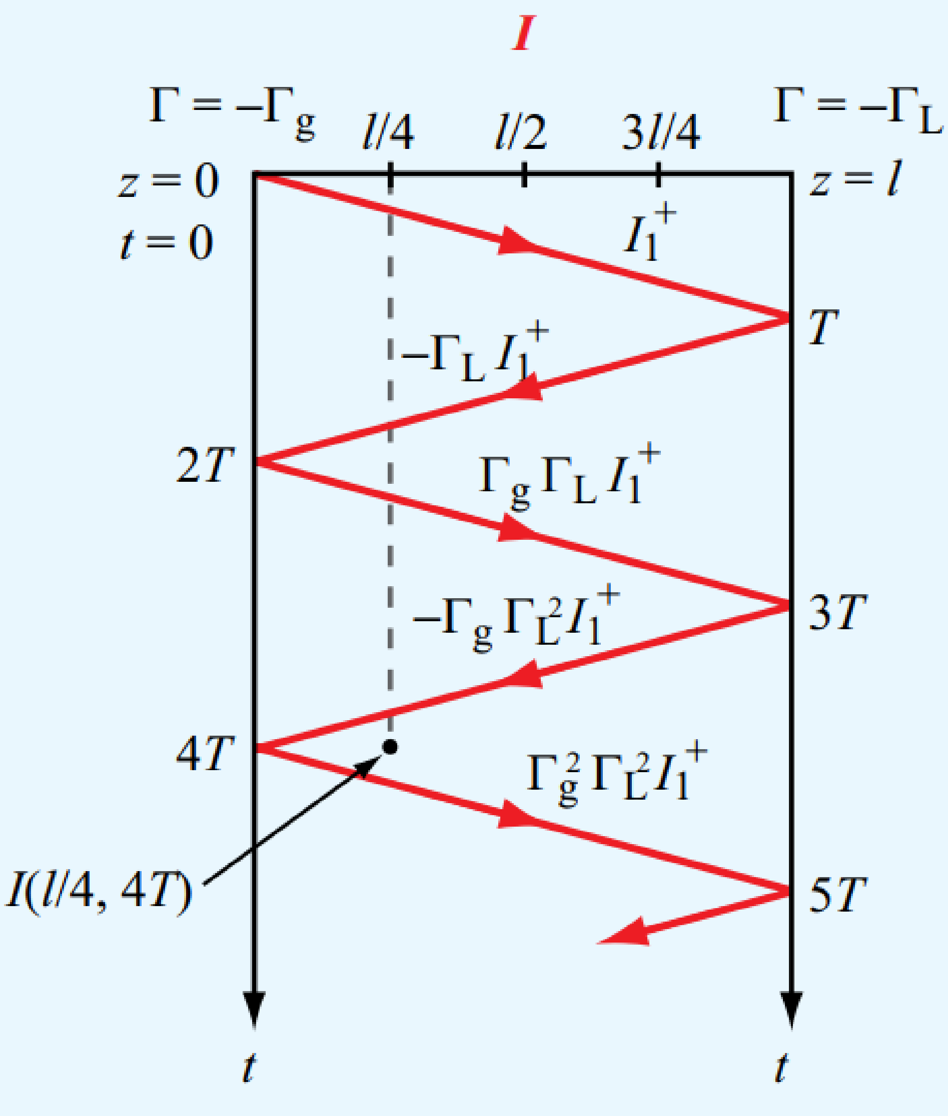
(a) Voltage bounce diagram

Bounce Diagrams: [Voltage]

Track process:

- The amplitude of each *new straight-line* segment equals the product of the amplitude of the *preceding straight-line* segment and the *reflection coefficient* at that end of the line.

Bounce Diagrams



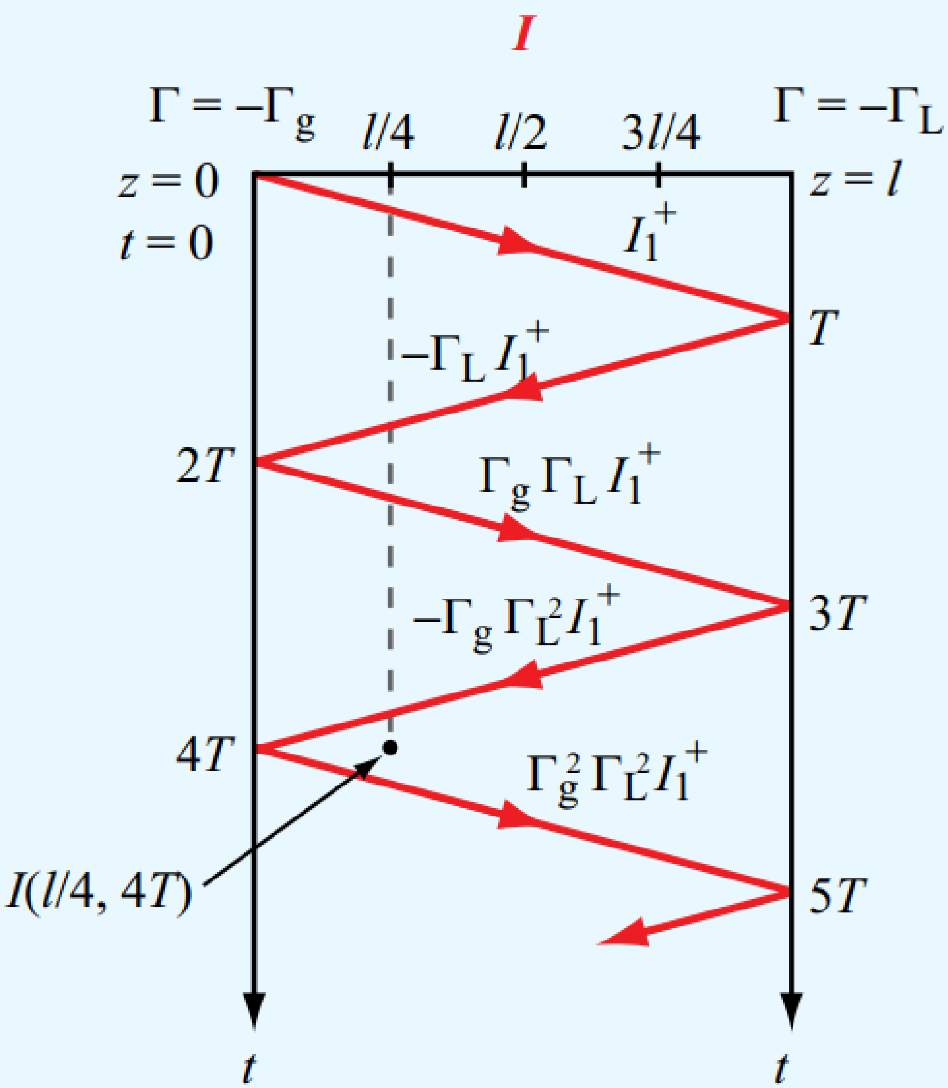
(b) Current bounce diagram

Bounce Diagrams: [Current]

Fig (b) [P. 118, Figure 2-42 (b)] is an example of the current. Similar to those of voltage, the parameters in the left current diagram are:

- *Horizontal axes*: position along the transmission line;
- *Vertical axes*: time;
- *Zigzag line (red)*: the progress of the current wave on the line;
- *Upper left*: $\Gamma = -\Gamma_g$; origin of location z and time t;
- *Upper right*: $\Gamma = -\Gamma_L$; end of location z.

Bounce Diagrams



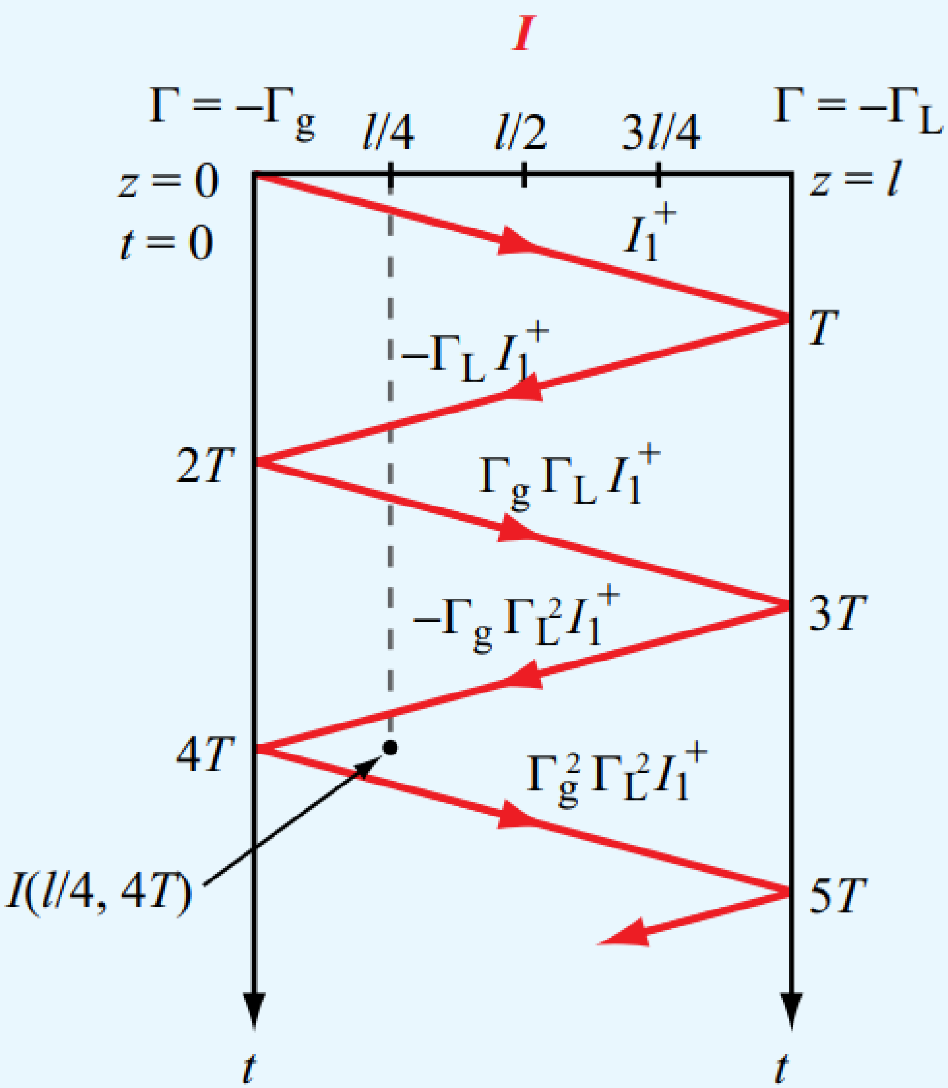
(b) Current bounce diagram

Bounce Diagrams: [Current]

Track process:

- The incident wave I_1^+ starts at $z = t = 0$ and travels in the $+z$ direction until it reaches the load at $z = l$ at time $t = T$. The notation at upper right ($\Gamma = -\Gamma_L$) of bounce diagram indicating the current-related reflection coefficients.
- At the end of the first straight-line segment of the zigzag line, a second line is drawn to represent the reflected current wave $I_1^- = -\Gamma_L I_1^+$.

Bounce Diagrams



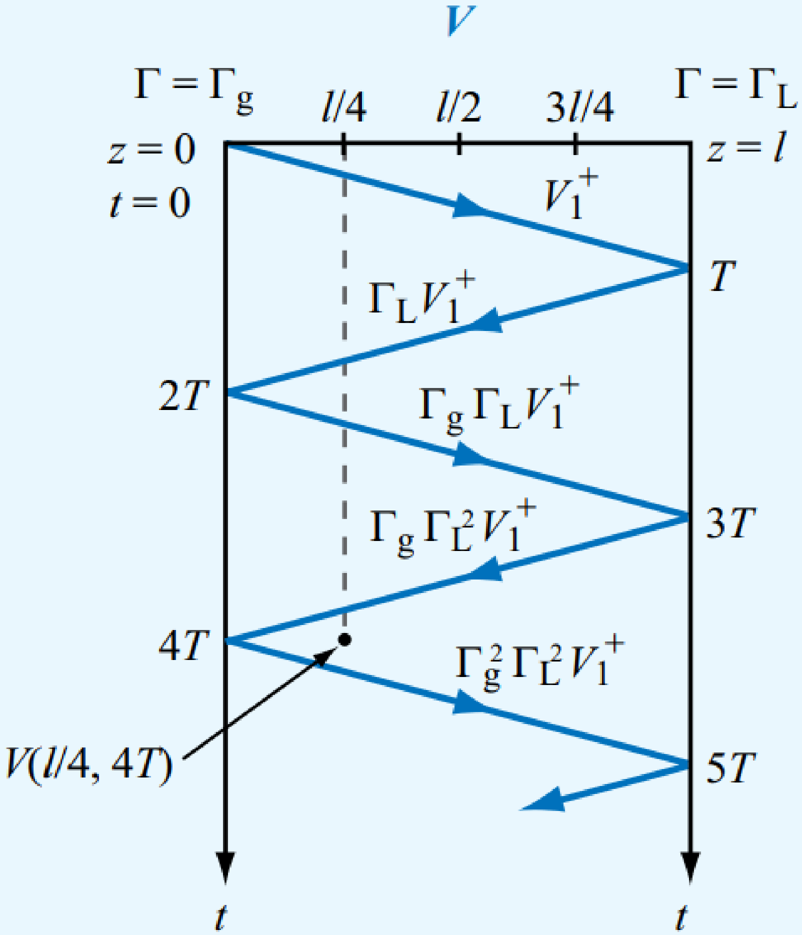
(b) Current bounce diagram

Bounce Diagrams: [Current]

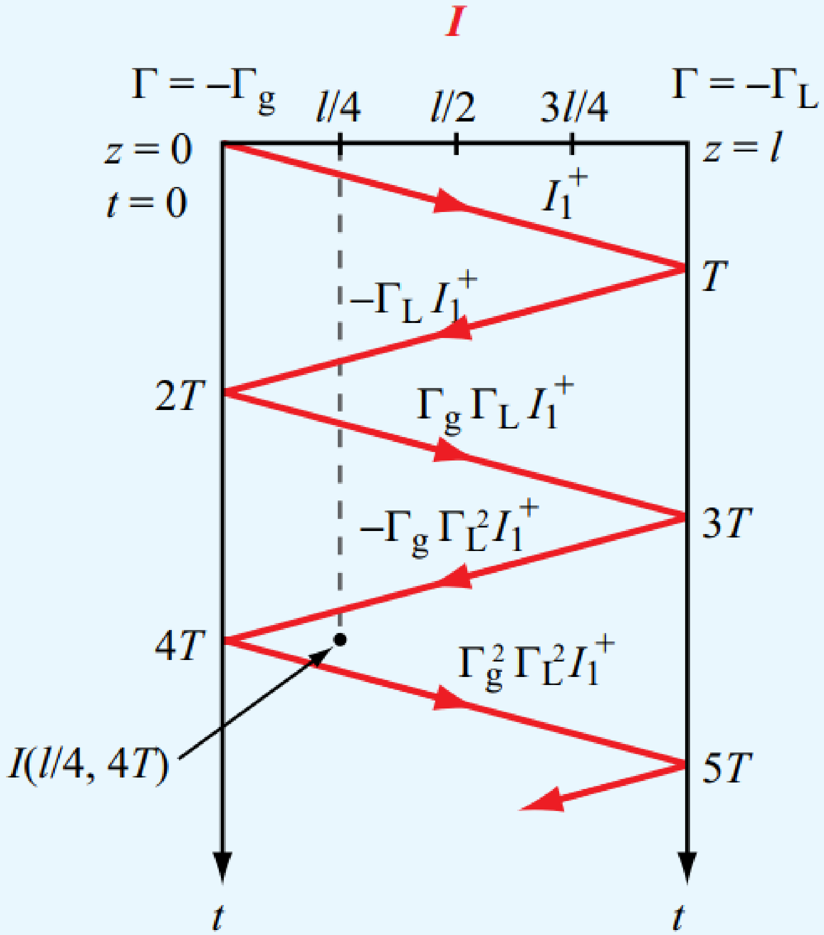
Track process:

- As for the current, the amplitude of each *new straight-line* segment adheres to the same principle except for the reversal of the signs of Γ_L and Γ_g at the top of the bounce diagram.

Bounce Diagrams



(a) Voltage bounce diagram



(b) Current bounce diagram

Generator: $\Gamma = \Gamma_g$

Load: $\Gamma = \Gamma_L$

Generator: $\Gamma = -\Gamma_g$

Load: $\Gamma = -\Gamma_L$

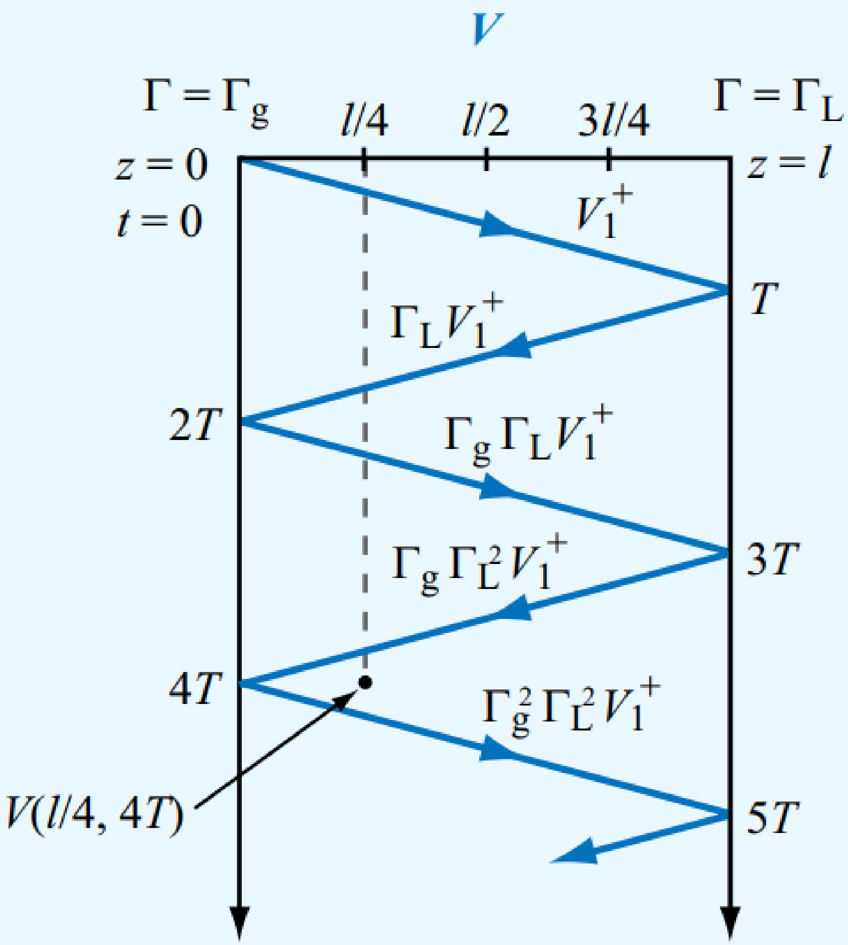
Bounce Diagrams

After the introduction of the bounce diagram, then the question is that how to determine the voltage/current by the diagram at any point and any time?

► Using the bounce diagram, the total voltage (or current) at any point z_1 and time t_1 can be determined by drawing a vertical line through point z_1, then adding the voltage (or currents) of all the zigzag segment intersected by that line between $t = 0$ and $t = t_1$.

Bounce Diagrams

Example: To find the voltage at $z = l/4$ and $t = 4T$ for example



(a) Voltage bounce diagram

Solution:

Draw a dashed vertical line in the voltage-bounce diagram through $z = l/4$ and extend the dashed vertical line from $t = 0$ to $t = 4T$.

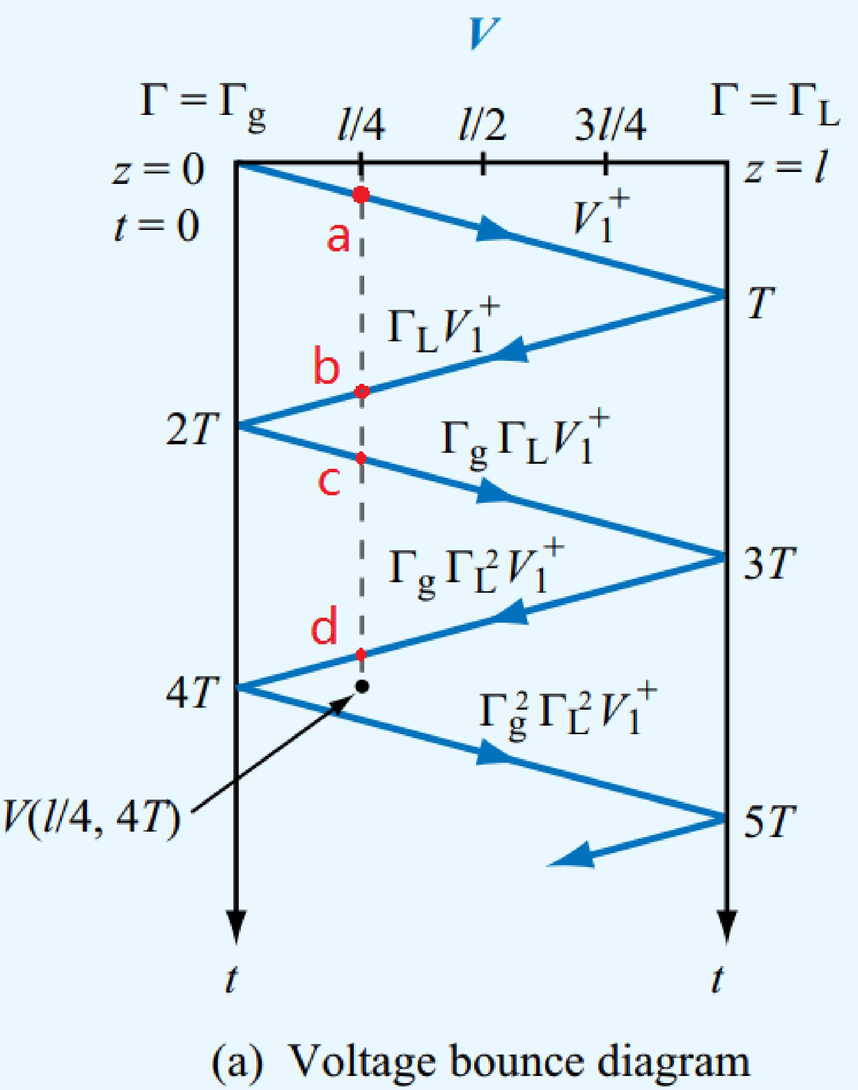
Notice that the dashed line intersects four line segments.

The total voltage at $z = l/4$ and $t = 4T$ therefore is:

$$V\left(l/4, 4T\right) = V_1^+ + \Gamma_L V_1^+ + \Gamma_g \Gamma_L V_1^+ + \Gamma_g \Gamma_L^2 V_1^+$$
$$= V_1^+ \left(1 + \Gamma_L + \Gamma_g \Gamma_L + \Gamma_g \Gamma_L^2\right)$$

Bounce Diagrams

Example: To find the voltage at $z = l/4$ and $t = 4T$ for example



(a) Voltage bounce diagram

Solution:

The time variation of $V(z,t)$ at a specific location z can be obtained by plotting the values of $V(z,t)$ along the (dashed) vertical line passing through z.

The time denoted in red point are:

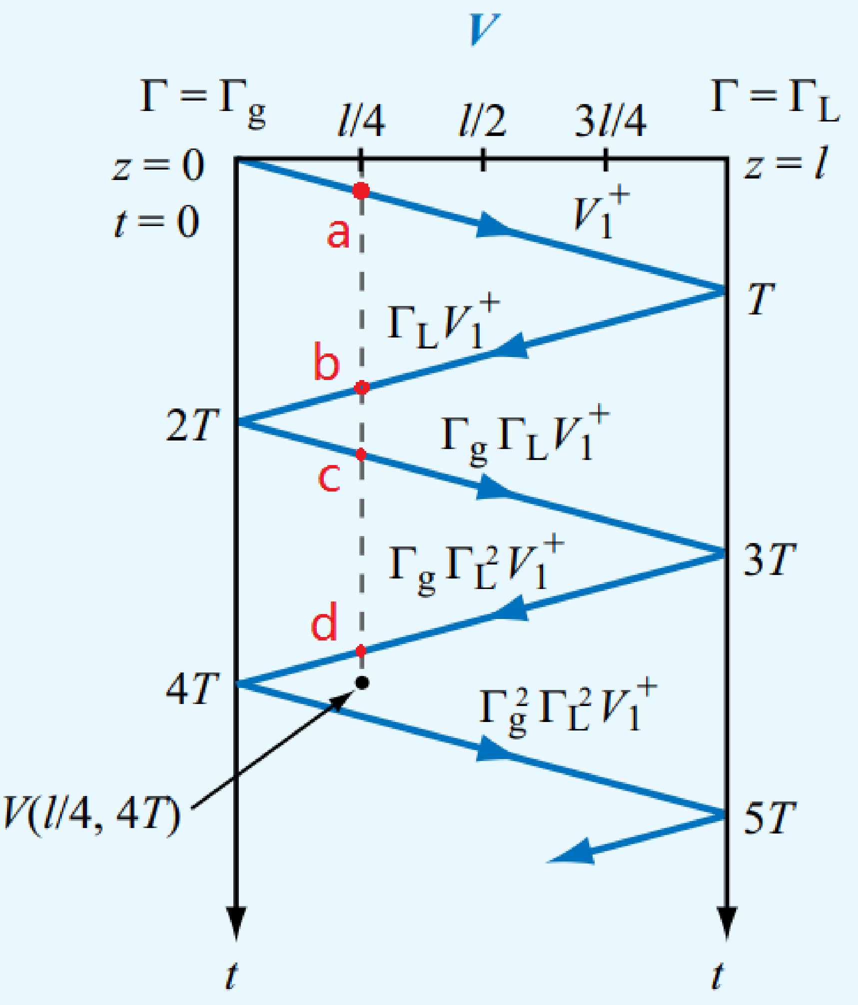
$$t_a = \frac{1}{4}T; t_b = \frac{7}{4}T; t_c = \frac{9}{4}T; t_d = \frac{15}{4}T$$

- For $0 \le t < t_a$; the dashed line intersects no segment, hence the voltage is:

$$V(l/4, t) = 0 \quad for \quad 0 \le t < t_a$$

Bounce Diagrams

Example: To find the voltage at $z = l/4$ and $t = 4T$ for example



(a) Voltage bounce diagram

Solution:

- For $t_a \le t < t_b$; the dashed line intersects one segment, hence the voltage is:

$$V(l/4,t) = V_1^+ \quad for \quad t_a \le t < t_b$$

- For $t_b \le t < t_c$; the dashed line intersects two segments, hence the voltage is:

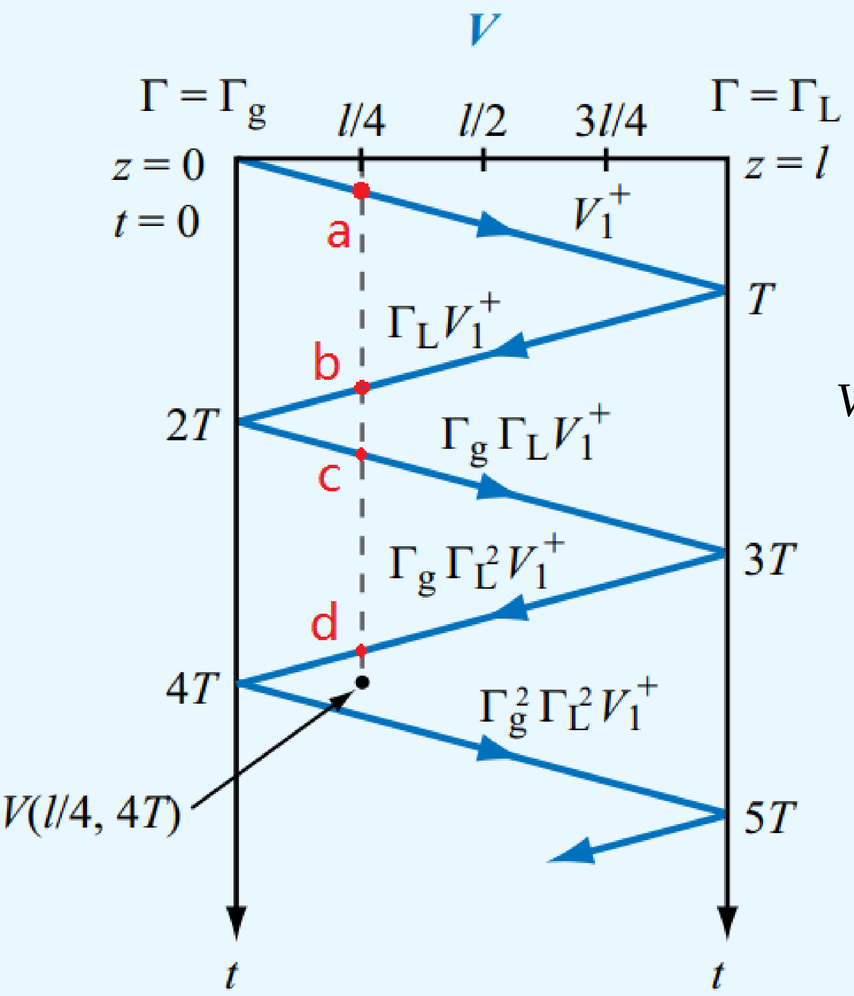
$$V(l/4,t) = (1+\Gamma_L)V_1^+ \quad for \quad t_b \le t < t_c$$

- For $t_c \le t < t_d$; the dashed line intersects three segments, hence the voltage is:

$$V(l/4,t) = (1+\Gamma_L+\Gamma_L\Gamma_g)V_1^+ \quad for \quad t_c \le t < t_d$$

Bounce Diagrams

Example: To find the voltage at $z = l/4$ and $t = 4T$ for example



(a) Voltage bounce diagram

Solution:

- For $t_d \le t < 4T$; the dashed line intersects four segments, hence the voltage is:

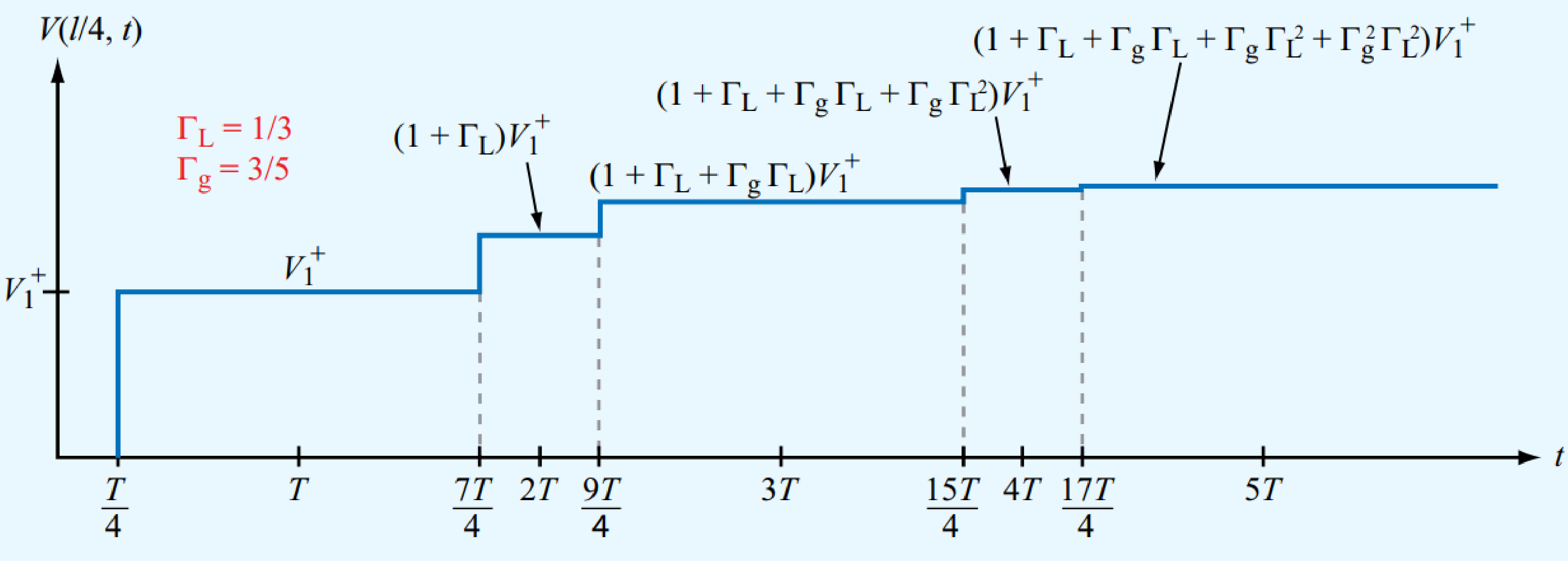
$$V(l/4,t) = \left(1+\Gamma_L+\Gamma_L\Gamma_g+\Gamma_L^2\Gamma_g\right)V_1^+ \quad for \quad t_d \le t < 4T$$

After finish analyzing separately, give an arrangement of the voltage and draw the variation of V as a function of time at $z = l/4$ for a circuit with $\Gamma_g = 3/5$ and $\Gamma_L = 1/3$.

Bounce Diagrams

Example: To find the voltage at $z = l/4$ and $t = 4T$ for example

$$V(l/4,t)=\begin{cases} 0 & for \quad 0 \le t < t_a \\ V_1^+ & for \quad t_a \le t < t_b \\ (1+\Gamma_L)V_1^+ & for \quad t_b \le t < t_c \\ (1+\Gamma_L+\Gamma_L\Gamma_g)V_1^+ & for \quad t_c \le t < t_d \\ (1+\Gamma_L+\Gamma_L\Gamma_g+\Gamma_L^2\Gamma_g)V_1^+ & for \quad t_d \le t < 4\mathrm{T} \end{cases}$$



(c) Voltage versus time at $z = l/4$

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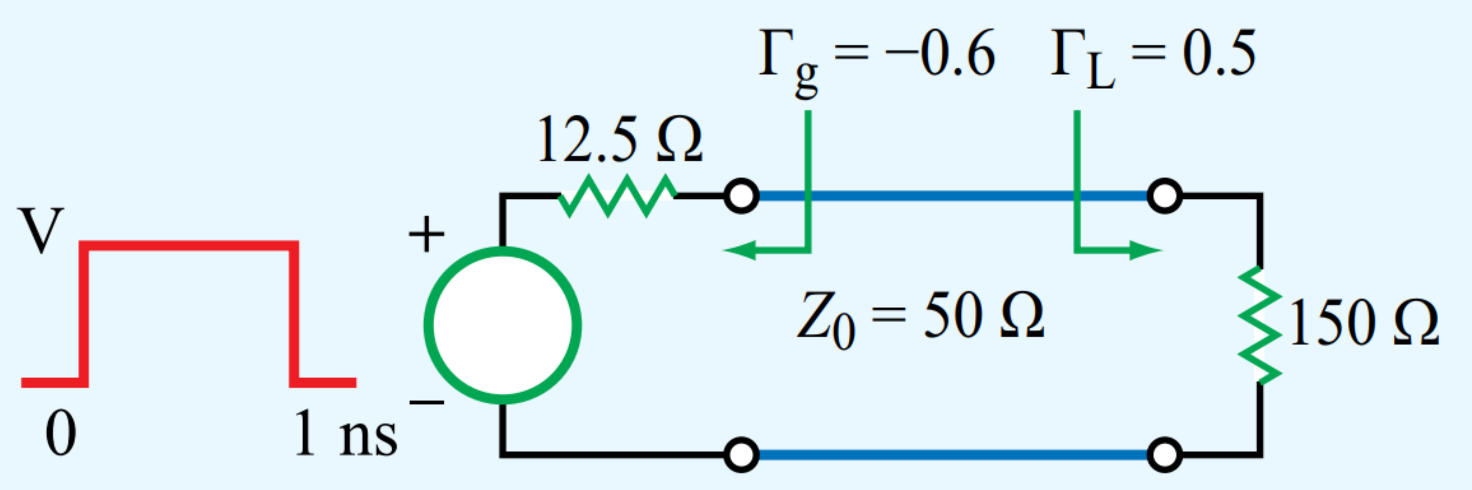
Exercises

Exercises

Example 2-15: Pulse Propagation

Q: The transmission-line circuit of Fig. 2-43(a) is excited by a rectangular pulse of duration $\tau = 1\text{ns}$ that starts at $t = 0$. Establish the waveform of voltage response at the load, given that the pulse amplitude is $5V$, the phase velocity is c, and the length of the line is 0.6m.



(a) Pulse circuit

Example 2-15: Pulse Propagation

Solution:

The one-way propagation time is:

$$T = \frac{l}{c} = \frac{0.6}{3 \times 10^8} = 2\,\text{ns}$$

According to the circuit, values of the resistances of the generator ($R_g = 12.5\Omega$) and the load ($R_L = 150\Omega$) can be obtained, then the corresponding reflection coefficients are:

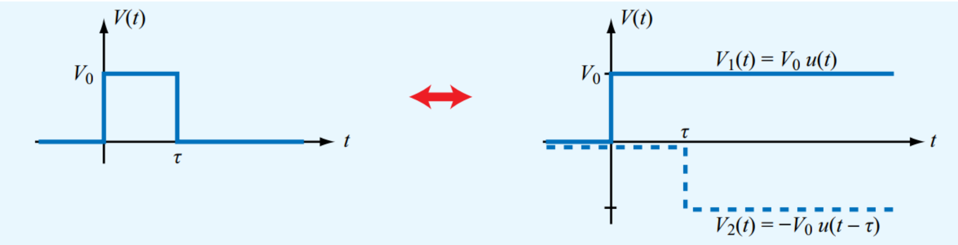
$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = \frac{150 - 50}{150 + 50} = 0.5$$

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{12.5 - 50}{12.5 + 50} = -0.6$$

Example 2-15: Pulse Propagation

Solution:

As have been mentioned previously [P.115 Figure 2-39 (a) and Equation (2.147)],



(a) Pulse of duration τ (b) $V(t) = V_1(t) + V_2(t)$

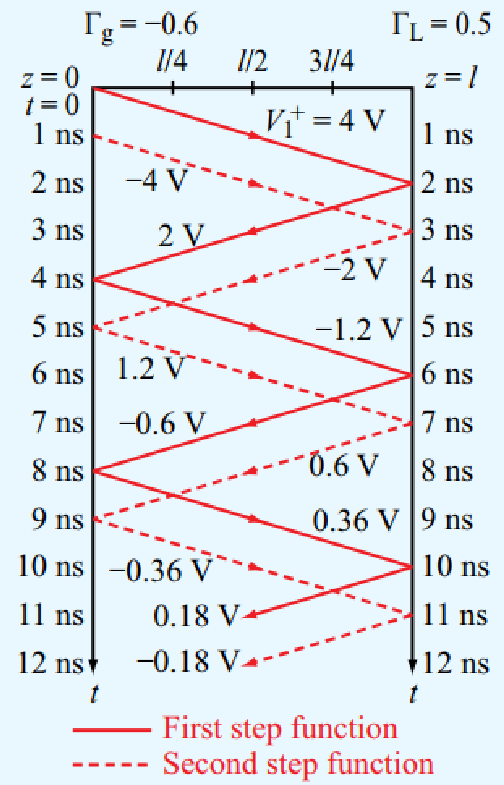
$$V(t) = V_1(t) + V_2(t) = V_0 u(t) - V_0 u(t - \tau)$$

Hence the pulse in this question can be expressed as:

$$V(t) = V_1(t) + V_2(t) = 5u(t) - 5u(t-1)$$

According to this decomposition, we will discuss the variation voltage (at the load) on the transmission line respectively.

Exercises



(b) Bounce diagram

Example 2-15: Pulse Propagation

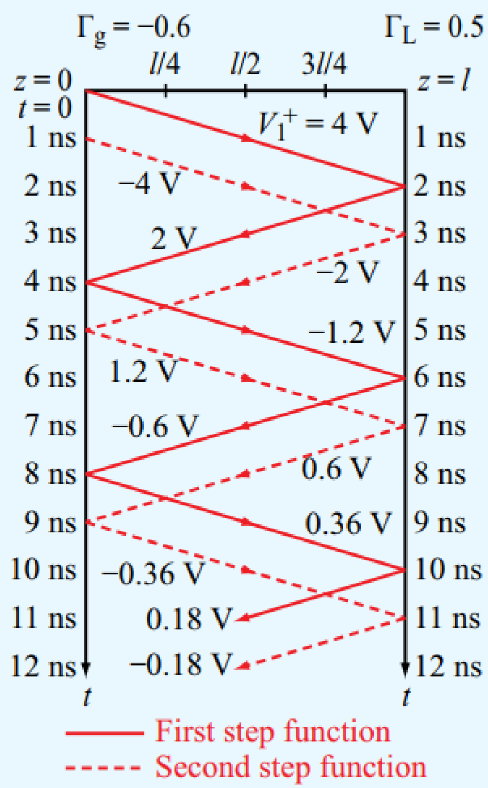
Solution: [Bounce diagrams: Fig. 2-43(b)]

For $V_1 = 5u(t)$: [Solid line]

According to the analysis in the previous example, the intersections of the zigzag segments with the vertical line $z = l$ indicating the components of the voltage, but what should be noticed is that at the load, *the voltages of incident wave and its corresponding reflected wave is occurring simultaneously on the line*, hence:

$$V_1(t) = \begin{cases} 0 & for \quad 0 \le t < T \\ V_1^+ + V_1^- = (1+\Gamma_L)V_1^+ & for \quad T \le t < 3T \\ V_1^+ + V_1^- + V_2^+ + V_2^- = (1+\Gamma_L+\Gamma_L\Gamma_g+\Gamma_L^2\Gamma_g)V_1^+ & for \quad 3T \le t < 5T \\ V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- = (1+\Gamma_L+\Gamma_L\Gamma_g+\Gamma_L^2\Gamma_g+\Gamma_L^2\Gamma_g^2+\Gamma_L^3\Gamma_g^2)V_1^+ & for \quad 5T \le t < 7T \end{cases}$$

Exercises



(b) Bounce diagram

Example 2-15: Pulse Propagation

Solution: [Bounce diagrams: Fig. 2-43(b)]

For $V_2 = -5u(t-1)$:[Dashed line]

The analysis of V_2 is similar to that of V_1, what should be noticed is that the amplitude of V_2 is opposite to that of V_1 and the variation of V_2-related voltage on transmission line has a time delay of 1ns comparing to that of V_1.

Hence we have:

$$V_2(t) = \begin{cases} 0 & for \quad 0 \le t < 1+T \\ \left(-V_1^+\right) + \left(-V_1^-\right) = -\left(1+\Gamma_L\right)V_1^+ & for \quad 1+T \le t < 1+3T \\ \left(-V_1^+\right) + \left(-V_1^-\right) + \left(-V_2^+\right) + \left(-V_2^-\right) = -\left(1+\Gamma_L+\Gamma_L\Gamma_g+\Gamma_L^2\Gamma_g\right)V_1^+ & for \quad 1+3T \le t < 1+5T \\ \left(-V_1^+\right) + \left(-V_1^-\right) + \left(-V_2^+\right) + \left(-V_2^-\right) + \left(-V_3^+\right) + \left(-V_3^-\right) = & \\ \quad -\left(1+\Gamma_L+\Gamma_L\Gamma_g+\Gamma_L^2\Gamma_g+\Gamma_L^2\Gamma_g^2+\Gamma_L^3\Gamma_g^2\right)V_1^+ & for \quad 1+5T \le t < 1+7T \end{cases}$$

Example 2-15: Pulse Propagation

Solution:

For the first step function, the initial voltage is given by:

$$V_1^+ = \frac{V_{01}Z_0}{R_g + Z_0} = \frac{5\times 50}{12.5+50} = 4V$$

The initial voltage of the delayed step function is:

$$V_1^+ = -\frac{V_{01}Z_0}{R_g + Z_0} = -\frac{5\times 50}{12.5+50} = -4V$$

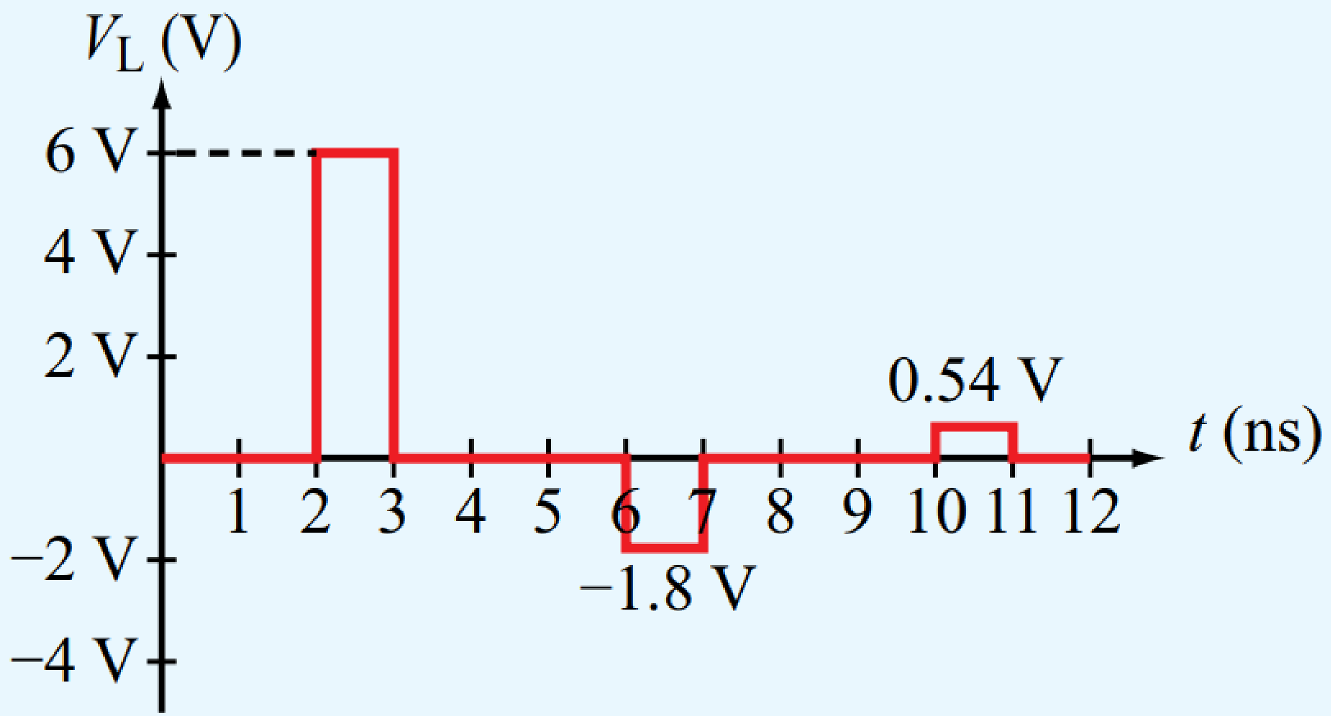
Sort the voltage and substitute it into the circuit parameters:

$$V(t) = \begin{cases} 0 & 0 \le t < 2 \\ V_1^+ + \Gamma_L V_1^+ = 6V & 2 \le t < 3 \\ \left(V_1^+ + \Gamma_L V_1^+\right) + \left(-V_1^+ - \Gamma_L V_1^+\right) = 0 & 3 \le t < 6 \\ \left(V_1^+ + \Gamma_L V_1^+ + \Gamma_L \Gamma_g V_1^+ + \Gamma_L^2 \Gamma_g V_1^+\right) + \left(-V_1^+ - \Gamma_L V_1^+\right) = -1.8V & 6 \le t < 7 \\ \left(V_1^+ + \Gamma_L V_1^+ + \Gamma_L \Gamma_g V_1^+ + \Gamma_L^2 \Gamma_g V_1^+\right) + \left(-V_1^+ - \Gamma_L V_1^+ - \Gamma_L \Gamma_g V_1^+ - \Gamma_L^2 \Gamma_g V_1^+\right) = 0 & 7 \le t < 10 \\ \left(V_1^+ + \Gamma_L V_1^+ + \Gamma_L \Gamma_g V_1^+ + \Gamma_L^2 \Gamma_g V_1^+ + \Gamma_L^2 \Gamma_g^2 V_1^+ + \Gamma_L^3 \Gamma_g^2 V_1^+\right) + \left(-V_1^+ - \Gamma_L V_1^+ - \Gamma_L \Gamma_g V_1^+ - \Gamma_L^2 \Gamma_g V_1^+\right) = 0.54V & 10 \le t < 11 \\ 0 & 11 \le t < 14 \end{cases}$$

Example 2-15: Pulse Propagation

Solution:

Using the information displayed in the bounce diagram, it is straightforward to generate the voltage response shown in Fig. 2-43(c).



(c) Voltage waveform at the load

Exercises

Example 2-16: Time-Domain Reflectometer

Q: A time-domain reflectometer (TDR) is an instrument used to locate faults on a transmission line. Consider, for example, a long underground or undersea cable that gets damaged at some distance d from the sending end of the line. The damage may alter the electrical properties or the shape of the cable, causing it to exhibit at the fault location an impedance R_{Lf}. A TDR sends a step voltage down the line, and by observing the voltage at the sending end as a function of time, it is possible to determine the location of the fault and its severity.

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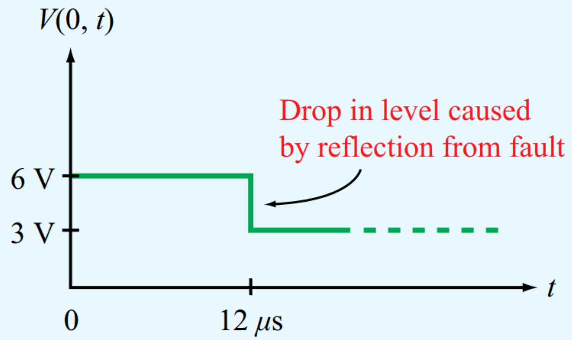
Q: If the voltage waveform shown in Fig. 2-44(a) is seen on an oscilloscope connected to the input of a 75Ω matched transmission line, determine:

(a) the generator voltage;

(b) the location of the fault;

(c) the fault shunt resistance.

The line's insulating material is Teflon with $\varepsilon_r = 2.1$.



(a) Observed voltage at the sending end

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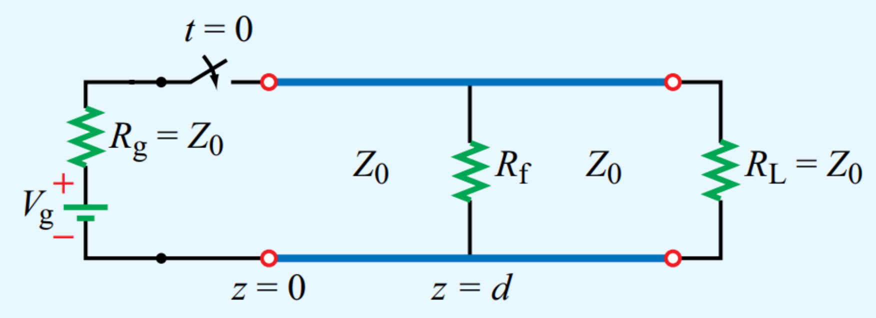
Solution:

(a):Since the line is properly matched, $R_g = R_L = Z_0$. In Fig. 2-44(b), the fault located a distance d from the sending end is represented by a shunt resistance R_f. For a matched line, according to Eq. (2.149b) that:

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{V_g Z_0}{2Z_0} = \frac{V_g}{2}$$



(b) The fault at $z = d$ is represented by a fault resistance R_f

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Solution:

(a): According to Fig. 2-44(a), $V_1^+ = 6V$. Hence,

$$V_g = 2V_1^+ = 12V$$

(b): The propagation velocity on the line is:

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times10^8}{\sqrt{2.1}} = 2.07\times10^8 \text{ m/ s}$$

For a fault at a distance d, the round-trip time delay of the echo is

$$\Delta t = \frac{2d}{u_p}$$

From Fig. 2-44(a), $\Delta t = 12\mu s$. Hence,

$$d = \frac{\Delta t}{2}u_p = \frac{12\times10^{-6}}{2}\times2.07\times10^8 = 1242\,\text{m}$$

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Solution:

(c): The change in level of $V(0,t)$ shown in Fig. 2-44(a) represents V_1^-.

Thus,

$$V_1^- = \Gamma_f V_1^+ = -3V$$

or

$$\Gamma_f = \frac{-3}{6} = -0.5$$

where Γ_f is the reflection coefficient due to the fault load R_{Lf}, that appears at $z = d$. According to Eq. (2.59)

$$\Gamma_f = \frac{R_{Lf} - Z_0}{R_{Lf} + Z_0}$$

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Solution:

(c): we could obtain the R_{Lf}:

$$R_{Lf}=\frac{1+\Gamma_f}{1-\Gamma_f}Z_0=25\Omega$$

This fault load is composed of the fault shunt resistance R_f and the characteristic impedance Z_0 of the line to the right of the fault:

$$\frac{1}{R_{Lf}}=\frac{1}{R_f}+\frac{1}{Z_0}$$

So the shunt resistance must be 37.5Ω.

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Thanks