**Problem 2.13** In addition to not dissipating power, a lossless line has two important features: (1) it is dispertionless ($\mu_\mathrm{p}$ is independent of frequency) and (2) its characteristic impedance $Z_0$ is purely real. Sometimes, it is not possible to design a transmission line such that $R' \ll \omega L'$ and $G' \ll \omega C'$, but it is possible to choose the dimensions of the line and its material properties so as to satisfy the condition

$$R'C' = L'G' \quad \text{(distortionless line).}$$

Such a line is called a *distortionless* line because despite the fact that it is not lossless, it does nonetheless possess the previously mentioned features of the loss line. Show that for a distortionless line,

$$\alpha = R'\sqrt{\frac{C'}{L'}} = \sqrt{R'G'}\,, \qquad \beta = \omega\sqrt{L'C'}\,, \qquad Z_0 = \sqrt{\frac{L'}{C'}}\,.$$

**Solution:** Using the distortionless condition in Eq. (2.22) gives

$$\begin{aligned}
\gamma = \alpha + j\beta &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
&= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{G'}{C'} + j\omega\right)} \\
&= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{R'}{L'} + j\omega\right)} \\
&= \sqrt{L'C'}\left(\frac{R'}{L'} + j\omega\right) = R'\sqrt{\frac{C'}{L'}} + j\omega\sqrt{L'C'}.
\end{aligned}$$

Hence,

$$\alpha = \mathfrak{Re}(\gamma) = R'\sqrt{\frac{C'}{L'}}\,, \qquad \beta = \mathfrak{Im}(\gamma) = \omega\sqrt{L'C'}\,, \qquad u_\mathrm{p} = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}\,.$$

Similarly, using the distortionless condition in Eq. (2.29) gives

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{L'}{C'}}\sqrt{\frac{R'/L' + j\omega}{G'/C' + j\omega}} = \sqrt{\frac{L'}{C'}}\,.$$

**Problem 2.14** For a distortionless line [see Problem 2.13] with $Z_0 = 50\ \Omega$, $\alpha = 20$ (mNp/m), $u_p = 2.5 \times 10^8$ (m/s), find the line parameters and $\lambda$ at 100 MHz.

**Solution:** The product of the expressions for $\alpha$ and $Z_0$ given in Problem 2.6 gives

$$R' = \alpha Z_0 = 20 \times 10^{-3} \times 50 = 1 \quad (\Omega/\text{m}),$$

and taking the ratio of the expression for $Z_0$ to that for $u_p = \omega/\beta = 1/\sqrt{L'C'}$ gives

$$L' = \frac{Z_0}{u_p} = \frac{50}{2.5 \times 10^8} = 2 \times 10^{-7}\ (\text{H/m}) = 200 \quad (\text{nH/m}).$$

With $L'$ known, we use the expression for $Z_0$ to find $C'$:

$$C' = \frac{L'}{Z_0^2} = \frac{2 \times 10^{-7}}{(50)^2} = 8 \times 10^{-11}\ (\text{F/m}) = 80 \quad (\text{pF/m}).$$

The distortionless condition given in Problem 2.6 is then used to find $G'$.

$$G' = \frac{R'C'}{L'} = \frac{1 \times 80 \times 10^{-12}}{2 \times 10^{-7}} = 4 \times 10^{-4}\ (\text{S/m}) = 400 \quad (\mu\text{S/m}),$$

and the wavelength is obtained by applying the relation

$$\lambda = \frac{\mu_p}{f} = \frac{2.5 \times 10^8}{100 \times 10^6} = 2.5\ \text{m}.$$

**Problem 2.16** A transmission line operating at 125 MHz has $Z_0 = 40\ \Omega$, $\alpha = 0.02$ (Np/m), and $\beta = 0.75$ rad/m. Find the line parameters $R'$, $L'$, $G'$, and $C'$.

**Solution:** Given an arbitrary transmission line, $f = 125$ MHz, $Z_0 = 40\ \Omega$, $\alpha = 0.02$ Np/m, and $\beta = 0.75$ rad/m. Since $Z_0$ is real and $\alpha \neq 0$, the line is distortionless. From Problem 2.13, $\beta = \omega\sqrt{L'C'}$ and $Z_0 = \sqrt{L'/C'}$, therefore,

$$L' = \frac{\beta Z_0}{\omega} = \frac{0.75 \times 40}{2\pi \times 125 \times 10^6} = 38.2 \text{ nH/m}.$$

Then, from $Z_0 = \sqrt{L'/C'}$,

$$C' = \frac{L'}{Z_0^2} = \frac{38.2 \text{ nH/m}}{40^2} = 23.9 \text{ pF/m}.$$

From $\alpha = \sqrt{R'G'}$ and $R'C' = L'G'$,

$$R' = \sqrt{R'G'}\sqrt{\frac{R'}{G'}} = \sqrt{R'G'}\sqrt{\frac{L'}{C'}} = \alpha Z_0 = 0.02 \text{ Np/m} \times 40\ \Omega = 0.6\ \Omega\text{/m}$$

and

$$G' = \frac{\alpha^2}{R'} = \frac{(0.02 \text{ Np/m})^2}{0.8\ \Omega\text{/m}} = 0.5 \text{ mS/m}.$$