1.1. Sketch and label each of the signals defined below:

a. $$x_a(t) = \begin{cases} 0\,, & t < 0 \ \text{ or } \ t > 4 \\ 2\,, & 0 < t < 1 \\ 1\,, & 1 < t < 2 \\ t-1\,, & 2 < t < 3 \\ 2\,, & 3 < t < 4 \end{cases}$$

1.2. Consider the signals shown in Fig. P.1.2. For each signal write the analytical description in segmented form similar to the descriptions of the signals in Problem 1.1.

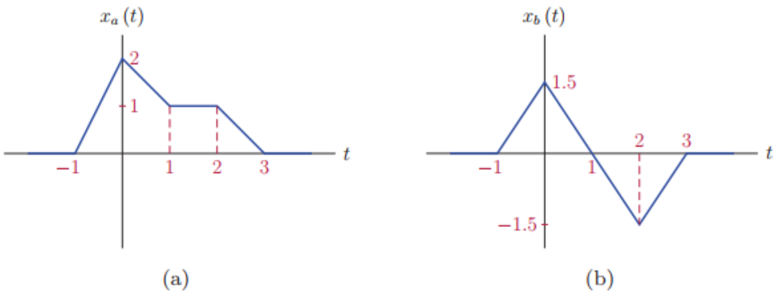

Figure P. 1.2

1.3. Using the two signals $x_a(t)$ and $x_b(t)$ given in Fig. P.1.2, compute and sketch the signals specified below:

a. $g_1(t) = x_a(t) + x_b(t)$

b. $g_2(t) = x_a(t)\; x_b(t)$

1.4. For the signal $x(t)$ shown in Fig. P.1.4, compute the following:

b. $g_2(t) = x(2t)$

e. $g_5(t) = x\left(\frac{t-1}{3}\right)$

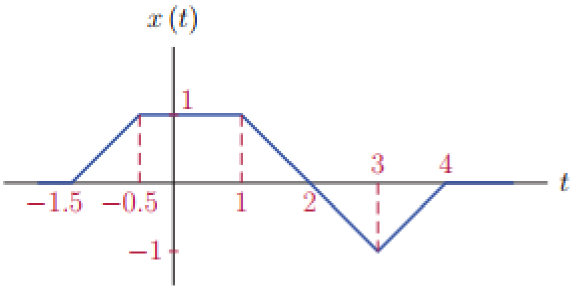

Figure P. 1.4

1.5. Consider the signal

$$x(t) = \left(e^{-t} - e^{-2t}\right) u(t)$$

Determine and sketch the following signals derived from $x(t)$ through signal operations:

a. $g_1(t) = x(2t-1)$

1.6. Let b be a positive constant. Show that

$$\delta(bt) = \frac{1}{b}\,\delta(t)$$

Hint: Start with Eqn. (1.23) that expresses the impulse function as the limit of a pulse $q(t)$ with height $1/a$ and unit area. Apply time scaling to $q(t)$ and then take the limit as $a \to 0$.

1.7. Consider again Eqn. (1.23) that expresses the impulse function as the limit of a pulse $q(t)$ with height $1/a$ and area equal to unity. Show that, for small values of a, we have

$$\int_{-\infty}^{\infty} f(t)\, q(t - t_1)\, dt \approx f(t_1)$$

where $f(t)$ is any function that is continuous in the interval $t_1 - a/2 < t < t_1 + a/2$. Afterwards, by taking the limit of this result, show that the sifting property of the impulse function holds.

1.8. Sketch each of the following functions.

a. $\delta(t) + \delta(t-1) + \delta(t-2)$

1.9. Sketch each of the following functions in the time interval $-1 \le t \le 5$. Afterwards use the waveform explorer program "`wav_demo1.m`" to check your results.

c. $u(t) + r(t-2) - u(t-3) - r(t-4)$

e. $\Lambda(t) + 2\Lambda(t-1) + 1.5\Lambda(t-3) - \Lambda(t-4)$

1.14. Time derivative of the unit-impulse function $\delta(t)$ is called a *doublet*. Given a function $f(t)$ that is continuous at $t = 0$ and also has continuous first derivative at $t = 0$, show that

$$\int_{-\infty}^{\infty} f(t)\, \delta'(t)\, dt = -f'(0) = -\left.\frac{df(t)}{dt}\right|_{t=0}$$

Hint: Use integration by parts, and then apply the sifting property of the impulse function.

1.16. Using Euler's formula, prove the following identities:

a. $\cos(a) = \frac{1}{2} e^{ja} + \frac{1}{2} e^{-ja}$

b. $\sin(a) = \frac{1}{2j} e^{ja} - \frac{1}{2j} e^{-ja}$

c. $\frac{d}{da} [\cos(a)] = -\sin(a)$

d. $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

e. $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

f. $\cos^2(a) = \frac{1}{2} + \frac{1}{2}\cos(2a)$