B39SB Time and Frequency Signal Analysis - Week 11 Tutorial

P1. Two LTI systems are arranged in series with $h_1(t)=4\delta(t)$ and $h_2(t)=e^{-4t}u(t)$. Find the impulse response of the entire system.

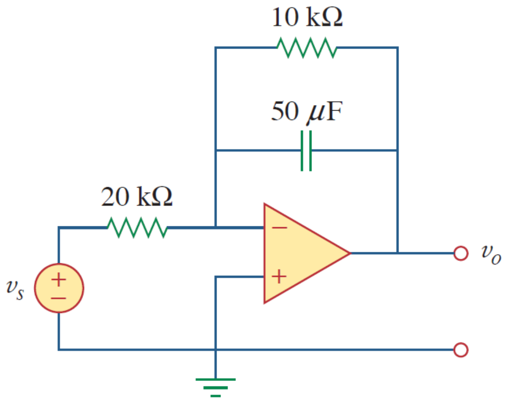
Solution. $H_1(s)=L[h_1(t)]=4,\quad H_2(s)=L[h_2(t)]=\dfrac{1}{s+4},\quad H_1(s)H_2(s)=\dfrac{4}{s+4}.$

Therefore $h(t)=L^{-1}\left[\dfrac{4}{s+4}\right]=4e^{-4t}u(t)$.

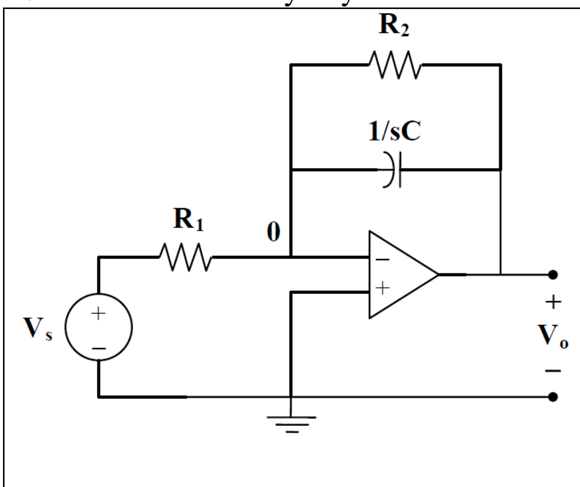
P2. An active filter has the transfer function $H(s)=\dfrac{k}{s^2+s(4-k)+1}$. For what values of k is the filter stable?

Solution. For the circuit to be stable, the poles must be located in the left half of the s plane. This implies that $4-k>0$ or $k<4$.

P3. (from Alexander-Sadiku textbook) $v_s(t)=3e^{-5t}u(t)\,\mathrm{V}$. Find $v_o(t)$ for $t>0$.



Solution. Nodal analysis yields



$$\frac{V_s-0}{R_1}=\frac{0-V_0}{R_2}+(0-V_0)sC$$

$$V_s=-R_1\left(\frac{1}{R_2}+sC\right)V_0 \qquad H(s)=\frac{V_0}{V_s}=\frac{-1}{sR_1C+R_1/R_2}=\frac{-1}{s+2}$$

$$V_s=\frac{3}{s+5} \qquad V_0=\frac{-3}{(s+5)(s+2)}=\frac{A}{s+2}+\frac{B}{s+5}$$

$$A=-1 \quad B=1 \qquad V_0=\frac{1}{s+5}-\frac{1}{s+2}$$

$$v_0(t)=\left(e^{-5t}-e^{-2t}\right)u(t)\,\mathrm{V}$$

P4. The transfer function of a circuit is given by $H(s)=\dfrac{s+3}{s^2+4s+5}$. Find the output when (a) the input is $u(t)$; (b) the input is $6te^{-2t}u(t)$.

Solution. (a) We have $Y(s)=H(s)X(s)=\dfrac{s+3}{s\left(s^2+4s+5\right)}=\dfrac{s+3}{s\left((s+2)^2+1\right)}=\dfrac{A}{s}+\dfrac{Bs+C}{s^2+4s+5}$

$$s+3=A\left(s^2+4s+5\right)+s(Bs+C)$$

Equating coefficients:

$$s^0:\quad 3=5A \quad\Rightarrow\quad A=3/5$$

$$s^1:\quad 1=4A+C \quad\Rightarrow\quad C=1-4A=-7/5$$

$$s^2:\quad 0=A+B \quad\Rightarrow\quad B=-A=-3/5$$

$$Y(s)=\frac{3/5}{s}-\frac{1}{5}\frac{3s+7}{s^2+4s+5}=\frac{0.6}{s}-\frac{1}{5}\frac{3(s+2)+1}{(s+2)^2+1}$$

$$y(t)=\left[0.6-0.6e^{-2t}\cos t-0.2e^{-2t}\sin t\right]u(t)$$

(b)

$$x(t)=6te^{-2t}\quad\Rightarrow\quad X(s)=\frac{6}{(s+2)^2}$$

$$Y(s)=H(s)X(s)=\frac{6(s+3)}{(s+2)^2\left(s^2+4s+5\right)}=\frac{A}{s+2}+\frac{B}{(s+2)^2}+\frac{Cs+D}{s^2+4s+5}$$

Equating coefficients:

$$s^3:\quad 0=A+C\quad\Rightarrow\quad C=-A$$

$$s^2:\quad 0=6A+B+4C+D=2A+B+D$$

$$s^1:\quad 6=13A+4B+4C+4D=9A+9B+4D$$

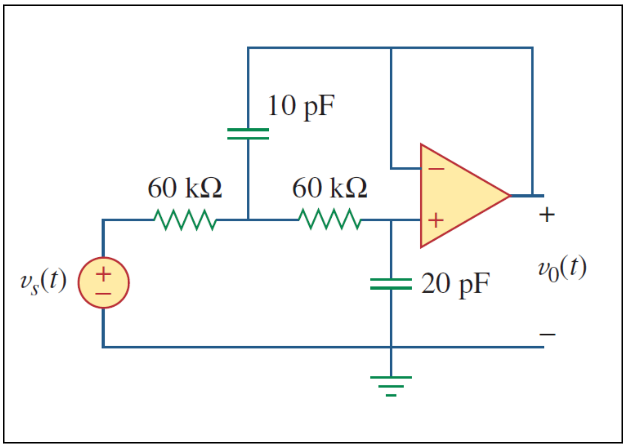
$$s^0:\quad 18=10A+5B+4D=2A+B$$

$$A=6\quad B=6\quad C=-6\quad D=-18$$

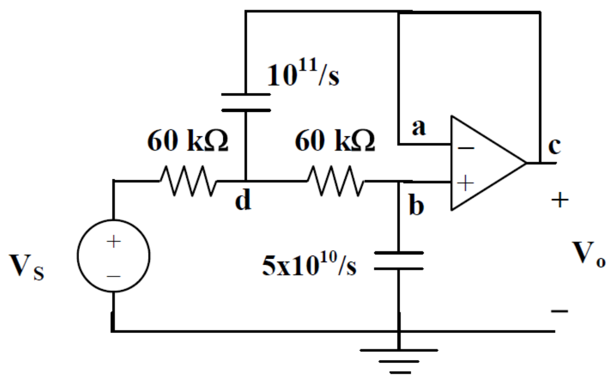
$$Y(s)=\frac{6}{s+2}+\frac{6}{(s+2)^2}-\frac{6(s+2)}{(s+2)^2+1}-\frac{6}{(s+2)^2+1}$$

$$y(t)=\left[6e^{-2t}+6te^{-2t}-6e^{-2t}\cos t-6e^{-2t}\sin t\right]u(t)$$

P5. (from Alexander-Sadiku textbook) Find $H(s)=V_0(s)/V_s(s)$.



Solution. We use nodal analysis.



$$V_0 = V_c = V_a = V_b$$

Let us write KCL equations at nodes **b** and **d**.

$$\frac{V_b - V_d}{60k} + \frac{V_b - 0}{5\times10^{10}/s} = 0$$

$$\frac{V_d - V_s}{60k} + \frac{V_d - V_c}{10^{11}/s} + \frac{V_d - V_b}{60k} = 0$$

...

$$H(s) = V_0(s)/V_s(s)$$

$$= \frac{1}{7.2\times10^{-13}s^2 + 2.4\times10^{-6}s + 1}$$