Operational Amplifies and Transfer Functions

Changhai Wang

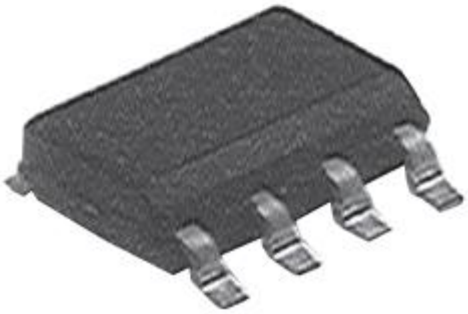
c.wang@hw.ac.uk

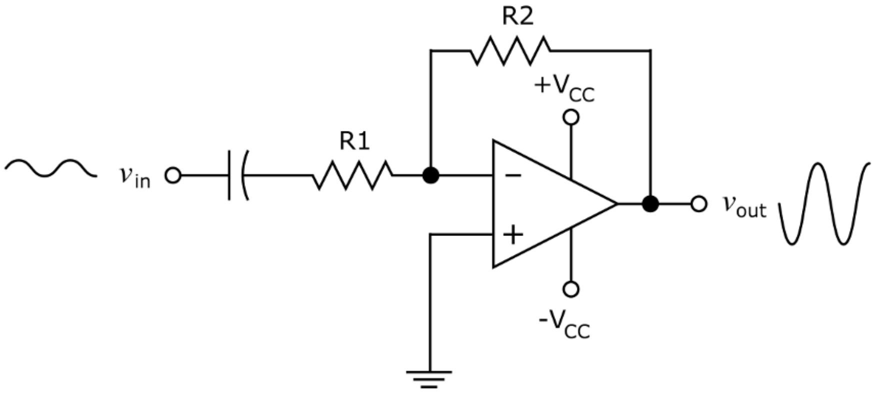
Integrating Circuits and Operational Amplifiers

- In 1952, Geoffrey Dummer, a British electronics engineer, presented his idea for combining multiple circuit elements onto a single piece of semiconductor material with no connecting wires.

- In the summer of 1958, Jack Kilby, a newly employed engineer at Texas Instruments working alone in a lab (his colleagues were on vacation but he wasn't, as he did not yet have the right to a summer vacation) was able to build multiple circuit components out of a single, monolithic piece of germanium (a semiconductor material), and lay metal connectors in patterns on top of it.

- The most popular type of IC is the ***operational amplifier***, nicknamed the ***op amp***, which is designed to amplify a weak signal. An op amp contains several transistors, resistors, and capacitors, and offers more robust performance than a single transistor. An op amp can provide uniform amplification over a much wider range of frequencies *(bandwidth)* than a single-transistor amplifier.

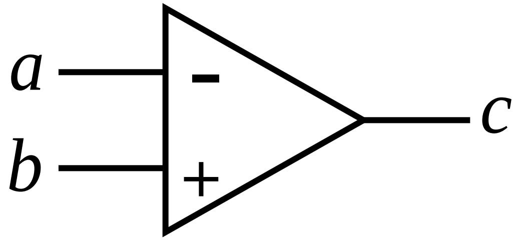




Operational Amplifiers

An *operational amplifier* or *op amp* is a circuit that takes an input voltage and amplifies it.

An **ideal op amp**: an ideal model of an operational amplifier



$$V_a = V_b$$
$$I_a = 0 = I_b$$

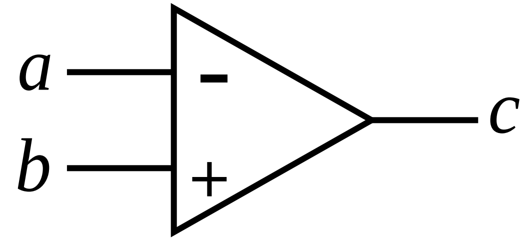
If you would like to know what is inside an *op amp*, watch the following video https://www.youtube.com/watch?v=Q3RMFpGGcZM, for example. Many other excellent explanations can be found on youtube.

However, in this course, we consider only with *ideal op amps*.

Operational Amplifiers

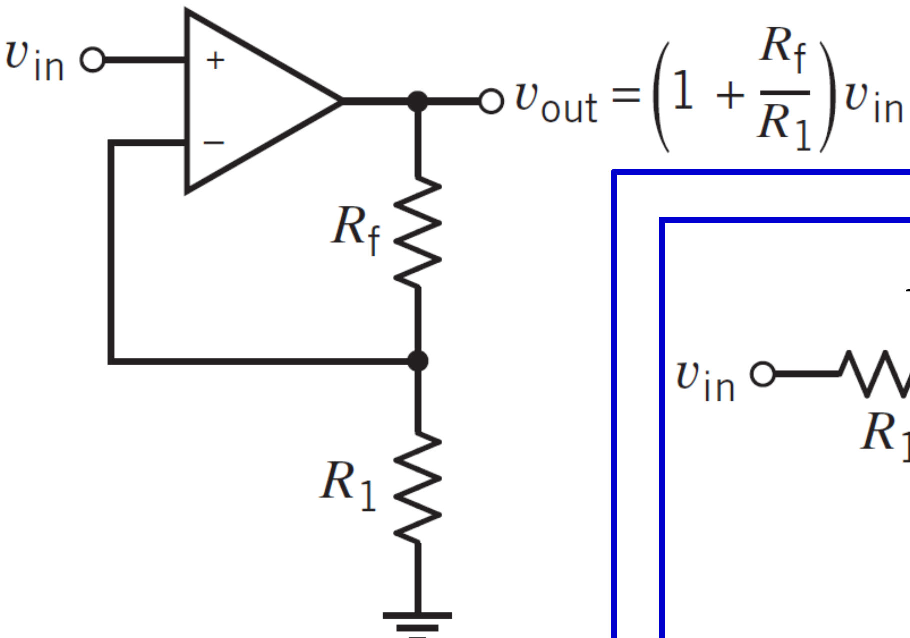
An *operational amplifier* or *op amp* is a circuit that takes an input voltage and amplifies it.

An **ideal op amp**: an ideal model of an operational amplifier



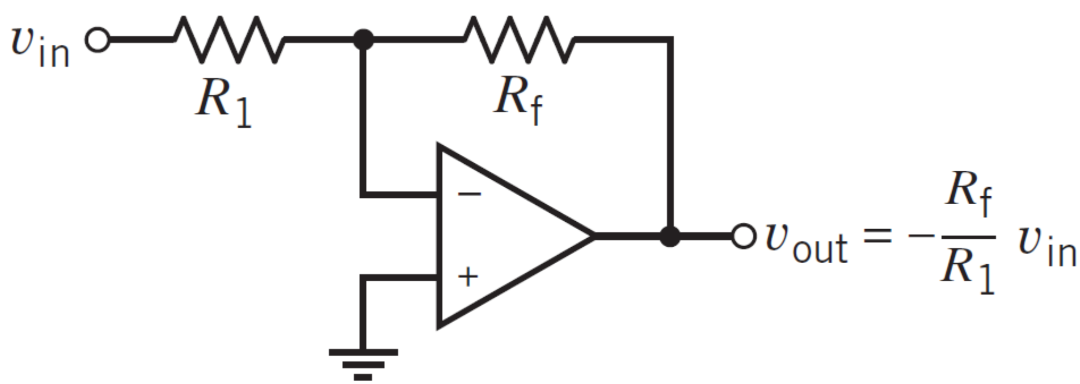
$$V_a = V_b$$
$$I_a = 0 = I_b$$

A noninverting amplifier

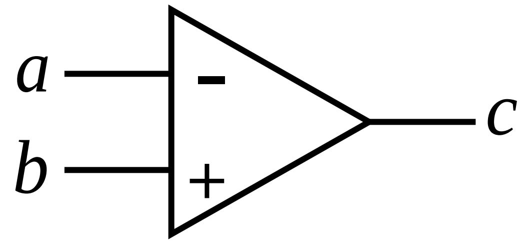


Use nodal analysis to study properties of these circuits

An inverting amplifier



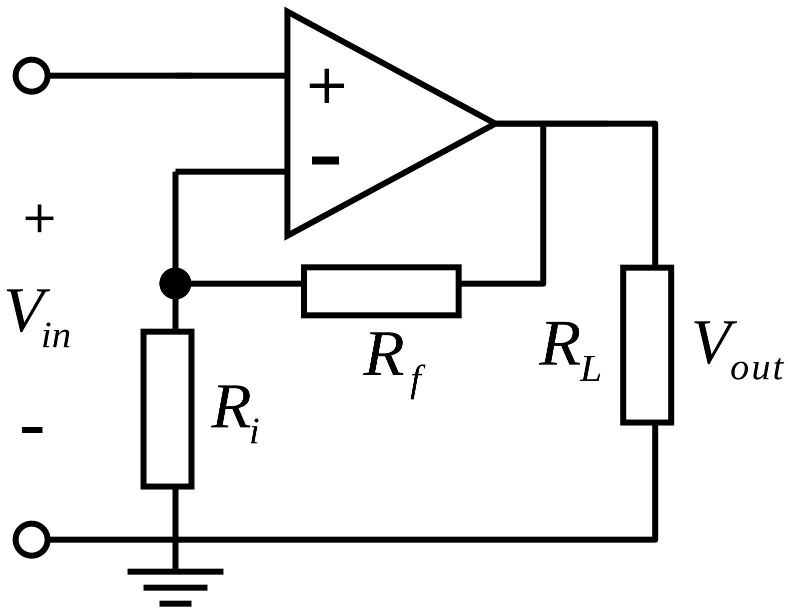
Operational Amplifiers



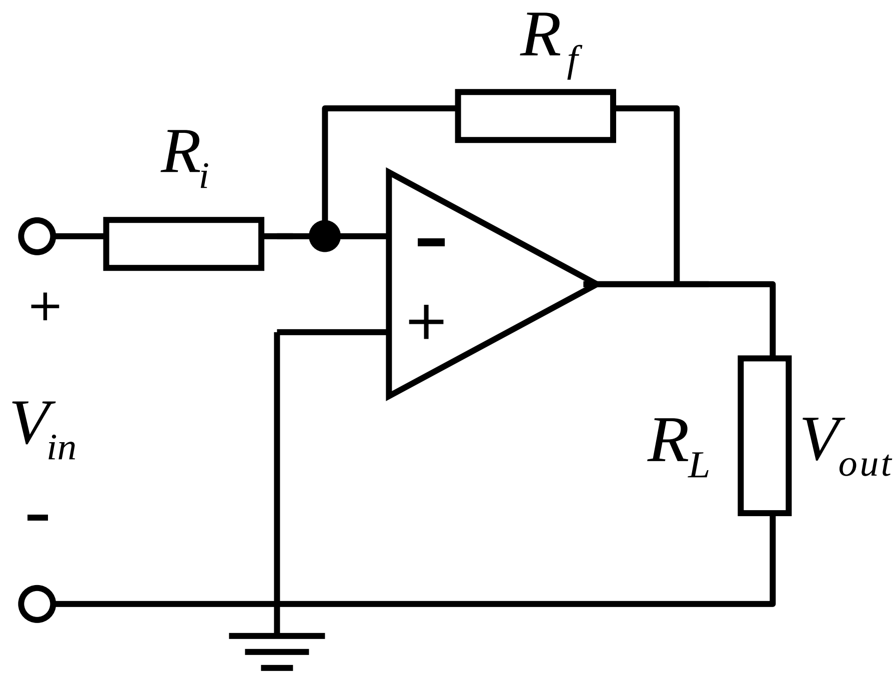
$$V_a = V_b$$

$$I_a = 0 = I_b$$

Use nodal analysis to study properties of these circuits



A noninverting amplifier

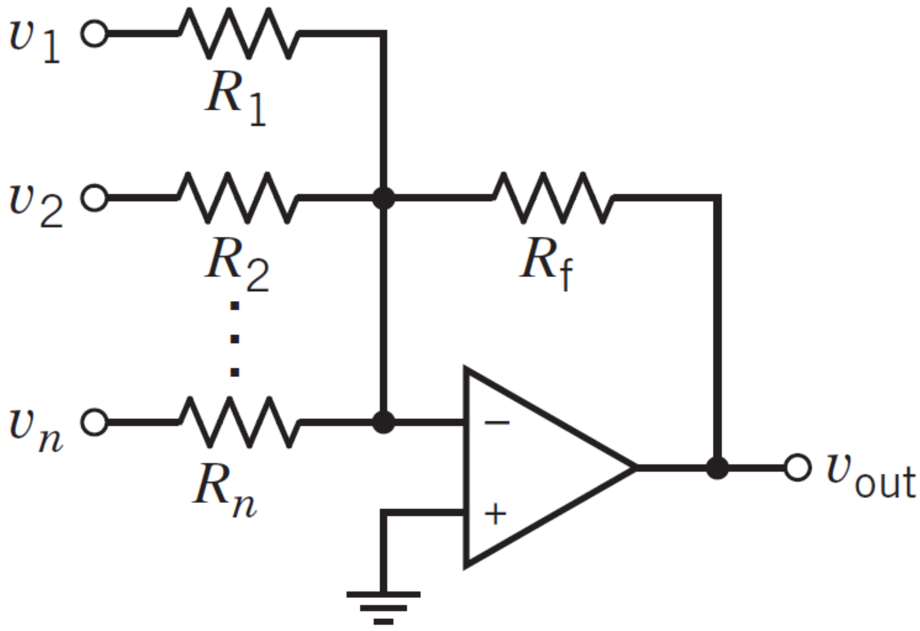


An inverting amplifier

$$\frac{V_{in}}{R_i} = \frac{V_{out} - V_{in}}{R_f} \qquad V_{out} = V_{in}\left(1 + \frac{R_f}{R_i}\right)$$

$$\frac{V_{in}}{R_i} = -\frac{V_{out}}{R_f} \qquad V_{out} = -\frac{R_f}{R_i} V_{in}$$

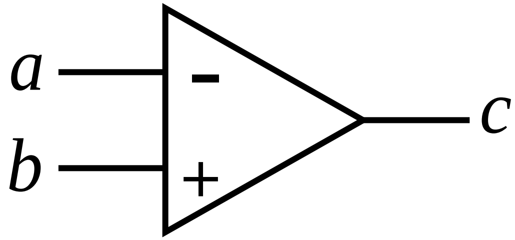
Circuit Design with Operational Amplifiers



$$v_{out} = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \ldots + \frac{R_f}{R_n} v_n\right)$$

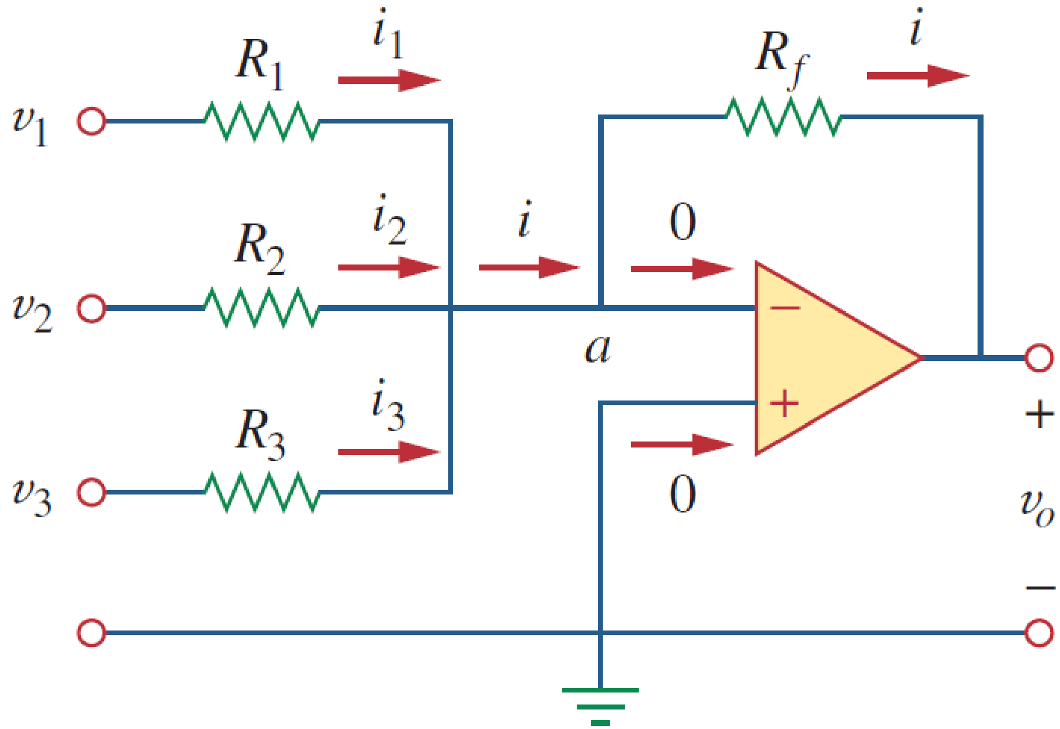
Summing amplifier

Summing Amplifier



$$V_a = V_b$$

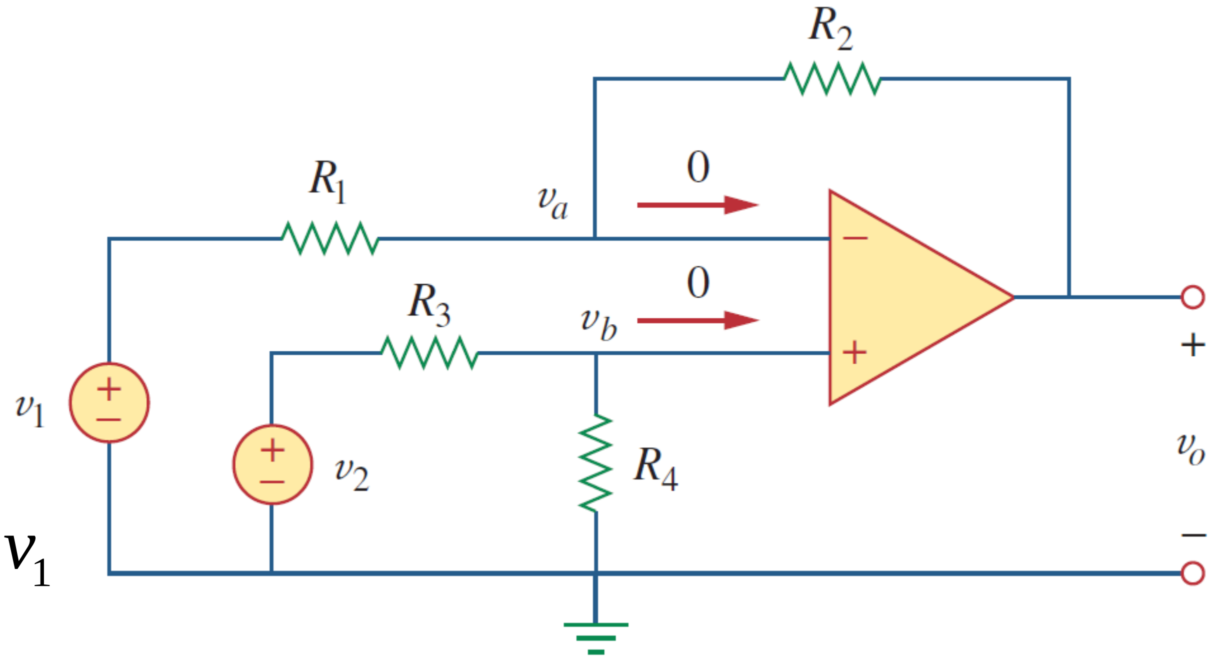
$$I_a = 0 = I_b$$



$$-\frac{v_o}{R_f} = i = i_1 + i_2 + i_3 = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}$$

$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 \right)$$

Difference Amplifier



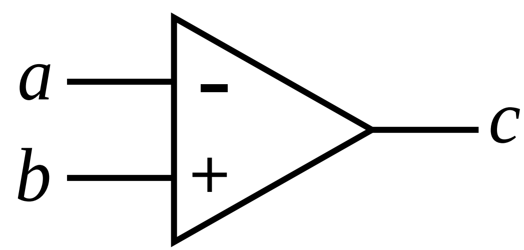
$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2}$$

$$v_o = \left(\frac{R_2}{R_1} + 1\right) v_a - \frac{R_2}{R_1} v_1$$

$$\frac{v_2 - v_b}{R_1} = \frac{v_b - 0}{R_2} \qquad v_b = \frac{R_4}{R_3 + R_4} v_2$$

$$v_a = v_b \quad \Rightarrow \quad v_o = \frac{R_2\left(1 + R_1/R_2\right)}{R_1\left(1 + R_3/R_4\right)} v_a - \frac{R_2}{R_1} v_1$$

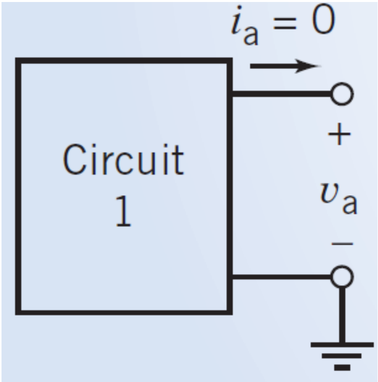
$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \Rightarrow \quad v_0 = \frac{R_2}{R_1}\left(v_2 - v_1\right)$$



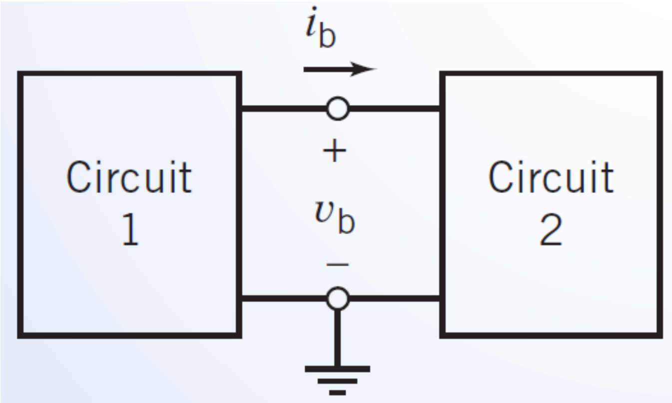
$$V_a = V_b$$

$$I_a = 0 = I_b$$

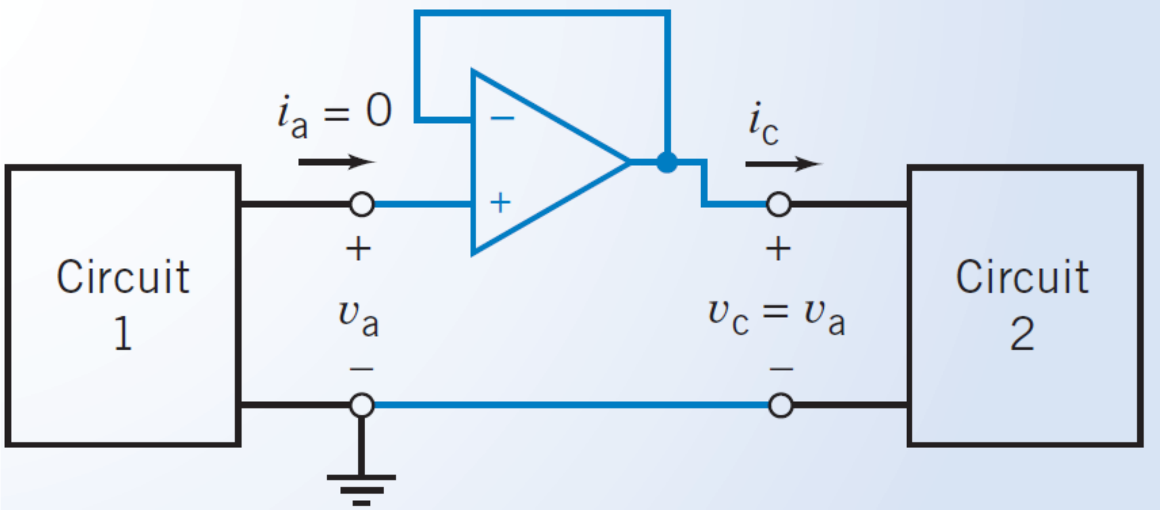
An example: Preventing Loading Using a Voltage Follower



Circuit 2 is connected to Circuit 1.

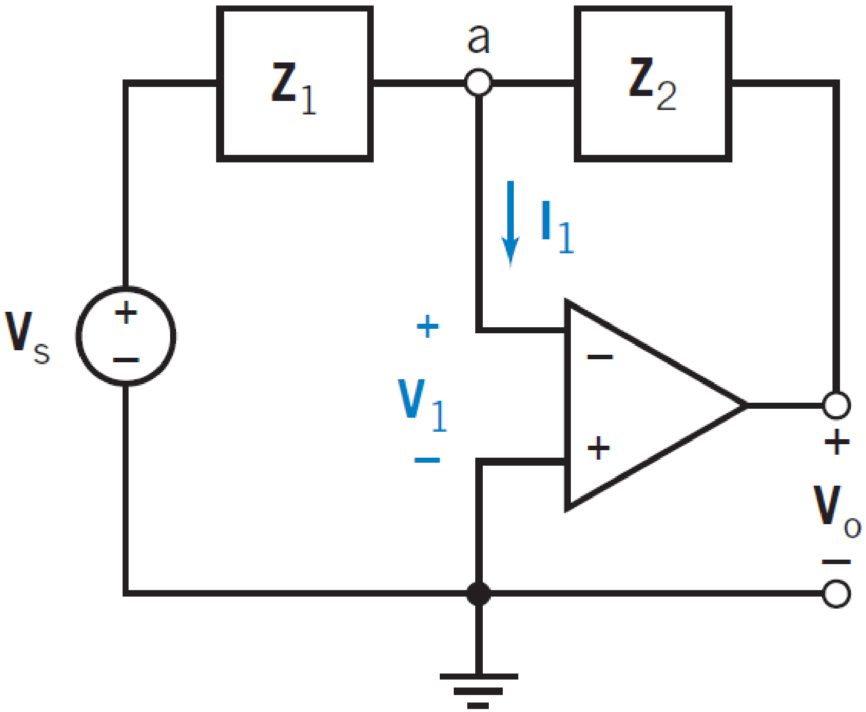


The output of 1 is used as the input to 2. Unfortunately, connecting c2 to 1 can change the output of circuit 1. This is called loading.



The voltage follower copies voltage v_a from the output of circuit 1 to the input of circuit 2 without disturbing circuit 1.

Op Amps in AC Circuits



Let us use nodal analysis:

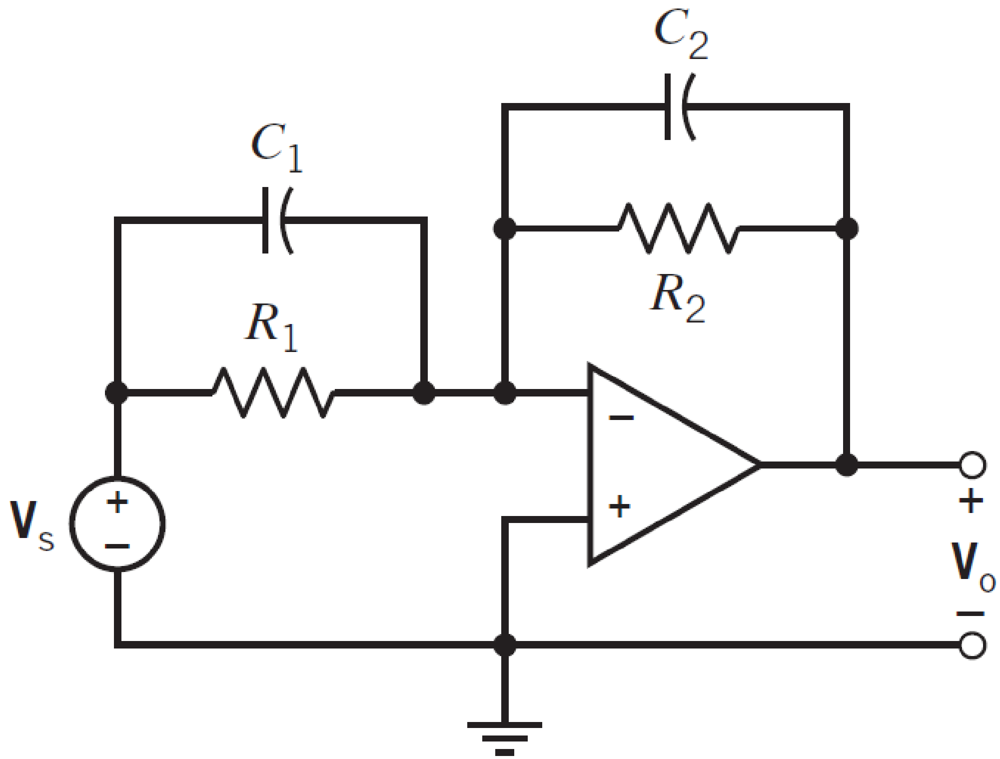
$$\frac{V_s - V_1}{Z_1} + \frac{V_o - V_1}{Z_2} = I_1$$

$$V_1 = 0 \quad I_1 = 0$$

$$\frac{V_s}{Z_1} + \frac{V_o}{Z_2} = 0$$

$$\frac{V_o}{V_s} = -\frac{Z_2}{Z_1}$$

Op Amps in AC Circuits



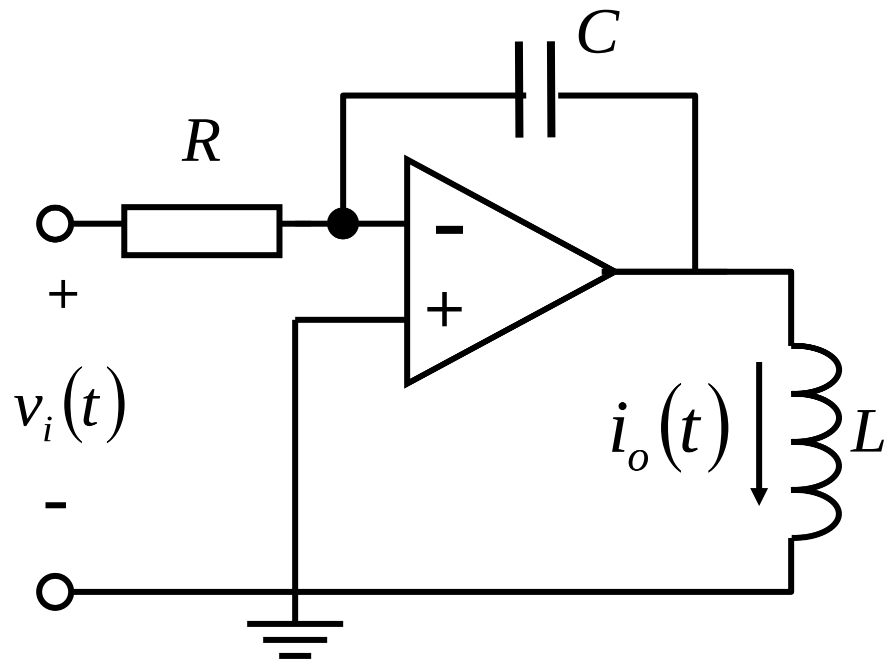
$$\frac{V_0}{V_s} = -\frac{Z_2}{Z_1}$$

$$Z = \frac{R/(sC)}{R + 1/sC} = \frac{R}{1 + sCR}$$

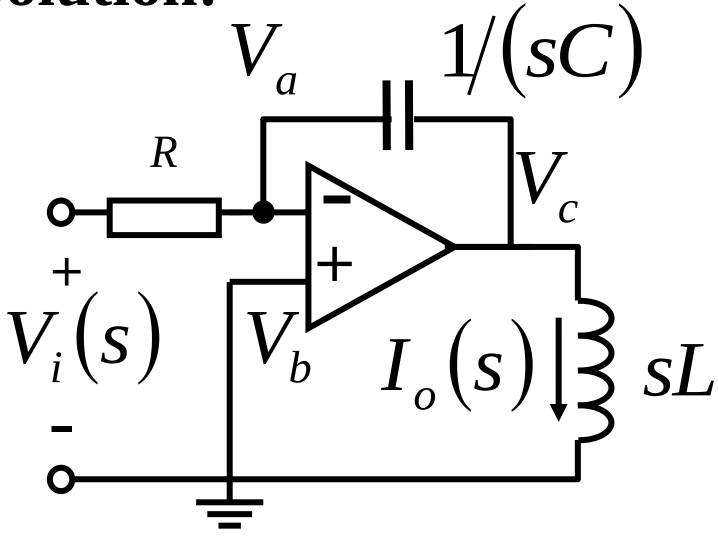
$$\frac{V_0}{V_s} = -\frac{Z_2}{Z_1} = -\frac{\dfrac{R_2}{1 + s\,C_2 R_2}}{\dfrac{R_2}{1 + s\,C_1 R_1}} = -\frac{R_2\left(1 + s\,C_1 R_1\right)}{R_1\left(1 + s\,C_2 R_2\right)}$$

Transfer function 1

Determine the transfer function $H(s) = I_o(s) / V_i(s)$. Assume all initial conditions are zero.



Solution:



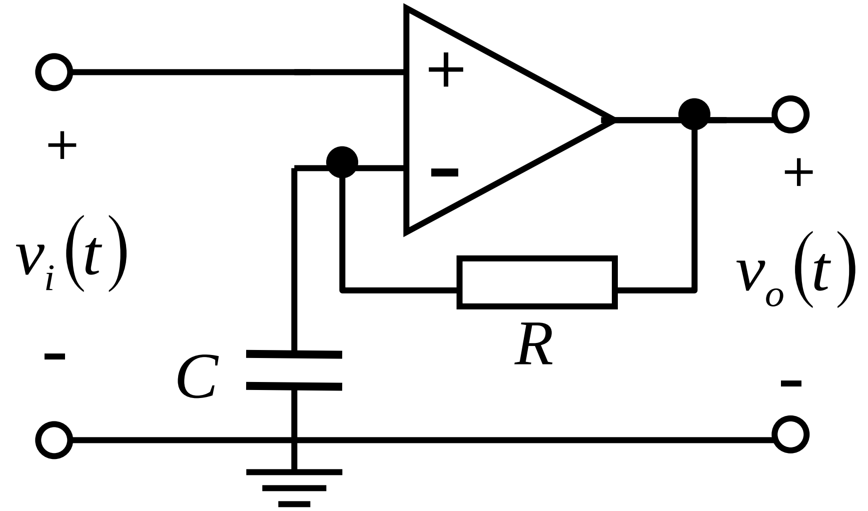
$$V_a = V_b = 0 \qquad I_o = V_c / (sL)$$

$$\frac{V_a - V_i}{R} + \frac{V_a - V_c}{1/(sC)} = 0$$

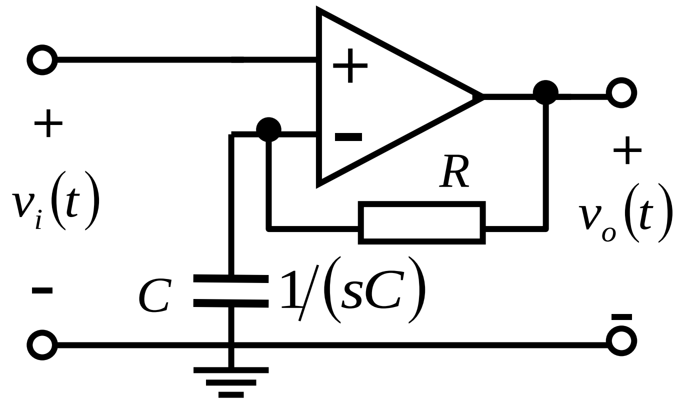
$$H(s) = \frac{I_o(s)}{V_i(s)} = -\frac{1}{s^2 RCL}$$

Transfer function 2

Determine the transfer function $H(s) = V_o(s) / V_i(s)$. Assume all initial conditions are zero.



Solution:



$$\frac{V_i - V_o}{R} + V_i(sC) = 0$$

$$V_o = V_i(1 + sRC)$$

$$H(s) = \frac{V_o}{V_i} = 1 + sRC$$