B39SB Time and Frequency Signal Analysis - Week 10 Tutorial

P1. Given that $x_1(t)=e^{-t}$ and $x_2(t)=\cos(5t)$, determine the Laplace transforms of the following functions

a) $y_1(t)=x_1(t-a)u(t-a)$

b) $y_2(t)=x_1(t)+x_2(t)$

c) $y_3(t)=x_1(t)*x_2(t)$

Solution. $X_1(s)=L[x_1(t)]=\int_0^\infty e^{-t}e^{-st}dt=-\frac{1}{s+1}e^{-(s+1)t}\Big|_0^\infty=\frac{1}{s+1}$.

Therefore, by the time shift property, we have $L[y_1(t)]=L[x_1(t-a)u(t-a)]=\frac{1}{s+1}e^{-as}$. This can be also verified directly:

$$\int_a^\infty e^{-t+a}e^{-st}dt=-\frac{e^{a}}{s+1}e^{-(s+1)t}\Bigg|_a^\infty=\frac{e^{-as}}{s+1}$$

We know that $X_2(s)=L[x_2(t)]=\frac{s}{s^2+25}$. Therefore $L[y_2(t)]=L[x_1(t)+x_2(t)]=\frac{1}{s+1}+\frac{s}{s^2+25}$.

Finally let us use the convolution property of the Laplace transform:

$$L[y_3(t)]=L[x_1(t)*x_2(t)]=X_1(s)X_2(s)=\frac{1}{s+1}\times\frac{s}{s^2+25}=\frac{s}{(s+1)(s^2+25)}.$$

P2. Use the Laplace transform to solve the follwoing differential equation

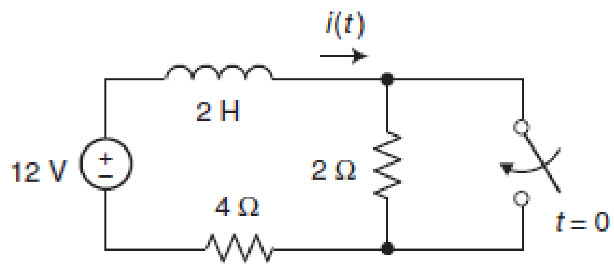
$$5\frac{dx}{dt}+2x-1=0,\quad t>0,\quad x(0)=3\,.$$

Solution. Applying Laplace transform and using $L[dx/dt]=sX(s)-x(0)$ yields

$$5(sX(s)-3)+2X(s)-\frac{1}{s}=0,\quad X(s)=\frac{15+1/s}{5s+2}=\frac{15s+1}{s(5s+2)}=\frac{3s+1/5}{s(s+2/5)}=\frac{A}{s}+\frac{B}{s+2/5}=\frac{1/2}{s}+\frac{5/2}{s+2/5}$$

$$x(t)=\frac{1}{2}+\frac{5}{2}e^{-\frac{2}{5}t},\quad t>0$$

P3. The circuit shown below is at steady state before the switch closes at time $t=0$. Determine the inductor current after the switch is closed.



Solution. $i(0)=2\text{A}, \qquad V(s)=L\left(sI(s)-i(0)\right).$

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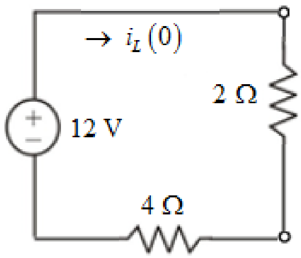

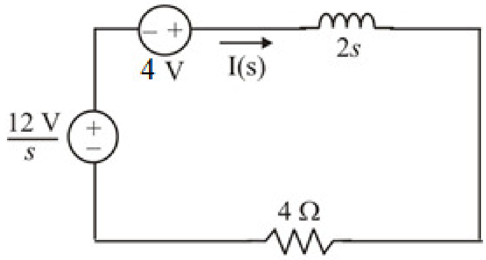

$$2sI(s)+4I(s)=\frac{12}{s}+4 \qquad I(s)=\frac{12+4s}{s(2s+4)}=\frac{A}{s}+\frac{B}{2s+4}=\frac{3}{s}-\frac{2}{2s+4}=\frac{3}{s}-\frac{1}{s+2}$$

$$i(t)=3u(t)-e^{-3t}u(t)=\left(3-e^{-3t}\right)u(t)\text{A}$$

P4. Let $F(s)=\dfrac{6s+5}{s^2+2s+1}$ be the one-sided Laplace transfrom of signal $f(t)$. Find $f(0)$ and $f(\infty)$.

Solution.

$$f(0)=\lim_{s\to\infty} sF(s)=\lim_{s\to\infty}\left[\frac{s(6s+5)}{s^2+2s+1}\right]=6 \qquad f(\infty)=\lim_{s\to 0} sF(s)=\lim_{s\to 0}\left[\frac{s(6s+5)}{s^2+2s+1}\right]=0$$