Amplitude and Angle Modulation

Double-Sideband and Single-Sideband Modulation Schemes, Phase and Frequency Modulation

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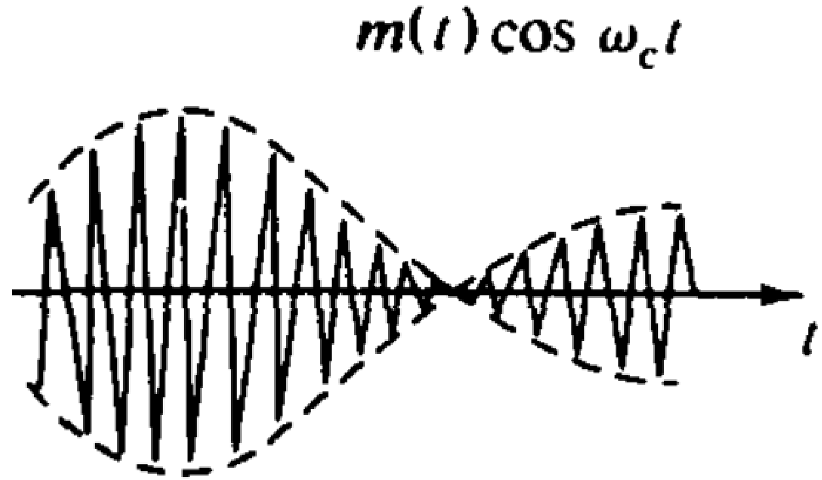
Modulation

Modulation is the process by which some characteristic of a carrier signal is varied in accordance with a modulating signal.

$$x_c(t) = A(t)\cos[\omega_c t + \varphi(t)] \qquad \omega_c = 2\pi\, f_c$$

Amplitude Modulation

$$x_c(t) = m(t)\cos\omega_c t$$



Modulation

<u>Modulation is used to shift signal spectra.</u>

If several signals, all occupying the same frequency band, are transmitted simultaneously over the same transmission medium, they will all interfere; it will be impossible to separate or retrieve them at a receiver.

<u>Demodulation</u> consists of another spectral shift required to restore the signal to its original band. Both modulation and demodulation implement spectral shifting.

Double-Sideband Modulation

$$x_{\mathrm{DSB}}(t) = \underbrace{m(t)}_{\text{message signal}}\underbrace{\cos\omega_c t}_{\text{carrier}}$$

$$m(t) \xrightarrow{\mathcal{F}} M(\omega)$$

Modulation Theorem

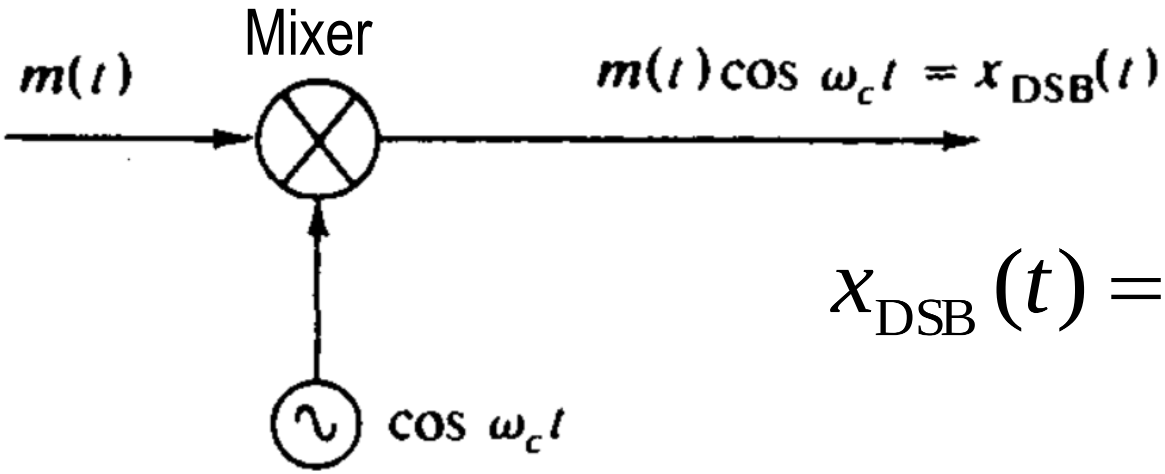
$$x_{\mathrm{DSB}}(t) \xrightarrow{\mathcal{F}} X_{\mathrm{DSB}}(\omega) = \frac{1}{2}M(\omega-\omega_c) + \frac{1}{2}M(\omega+\omega_c)$$

How to prove:

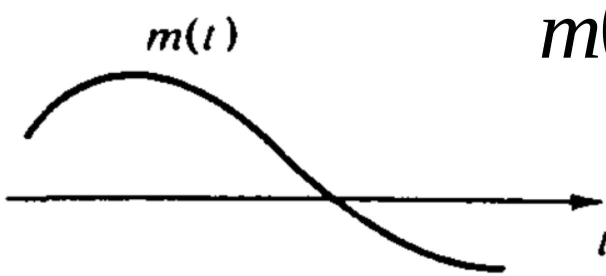
$$\cos\omega_c t = \tfrac{1}{2}\left(e^{j\omega_c t} + e^{j\omega_c t}\right) \qquad M(\omega) = \mathcal{F}\left[m(t)\right] = \int_{-\infty}^{\infty} m(t)e^{-j\omega t}dt$$

$$\mathcal{F}\left[m(t)e^{j\omega_c t}\right] = \int_{-\infty}^{\infty} m(t)\,e^{j\omega_c t}e^{-j\omega t}dt = \int_{-\infty}^{\infty} m(t)\,e^{-j(\omega-\omega_c)t}dt = M(\omega-\omega_c)$$

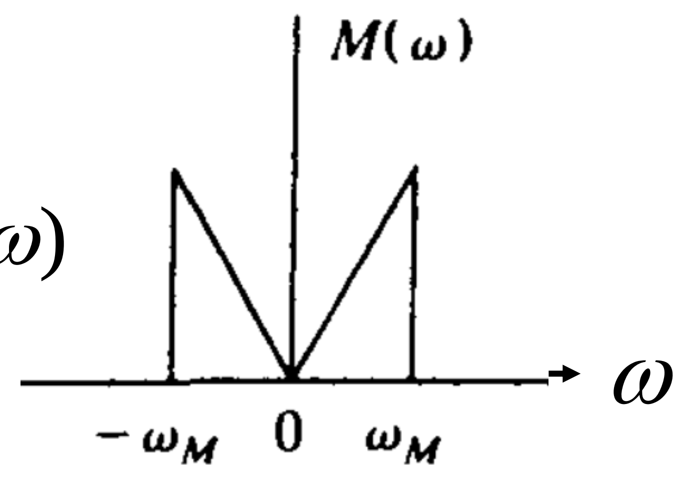
Double-Sideband Modulation



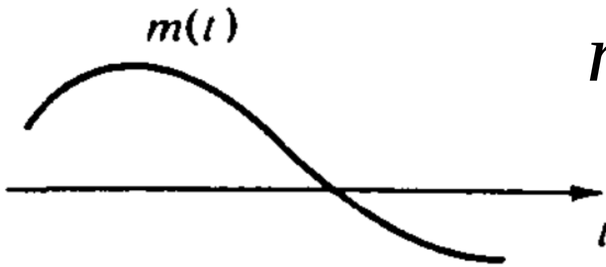
$$x_{\mathrm{DSB}}(t) = \underbrace{m(t)}_{\text{message signal}}\underbrace{\cos\omega_c t}_{\text{carrier}}$$



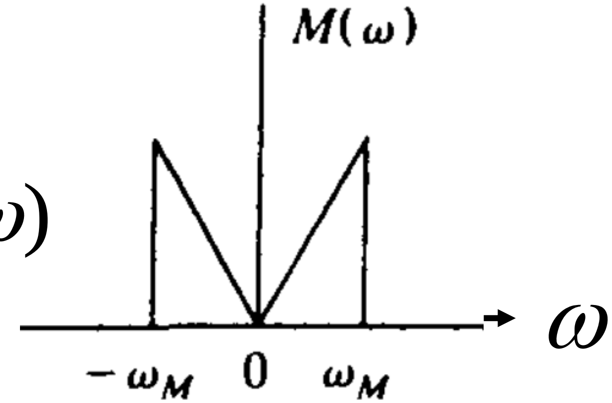
$$m(t) \xleftrightarrow{\mathrm{F}} M(\omega)$$



Double-Sideband Modulation

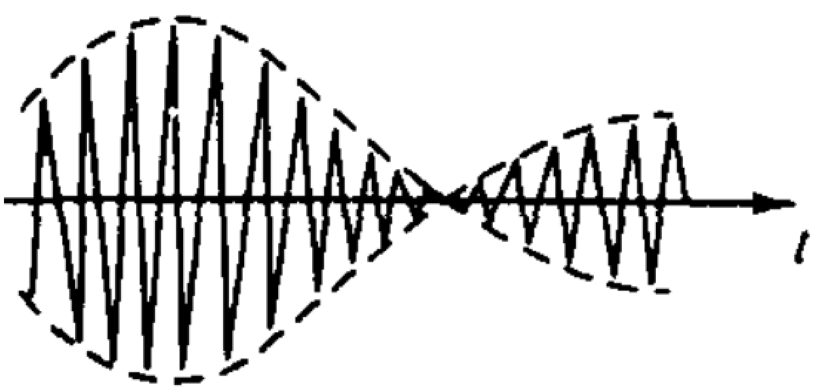


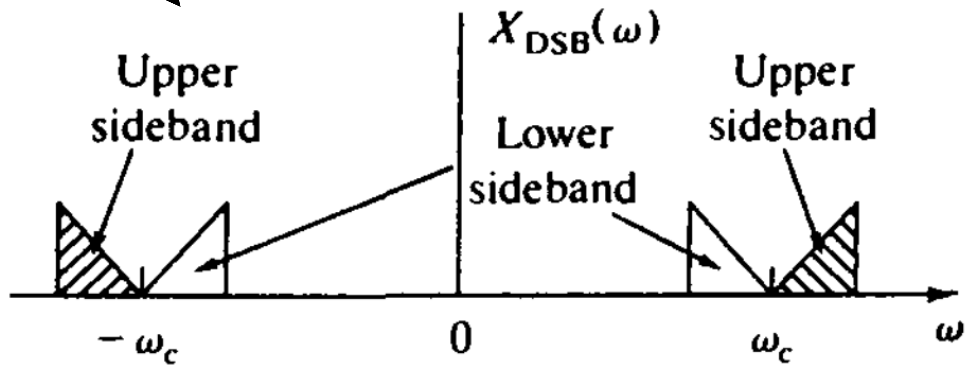
$$m(t) \xleftrightarrow{F} M(\omega)$$



$$x(t) \xleftrightarrow{F} X(\omega)$$

$m(t)\cos\omega_c t$

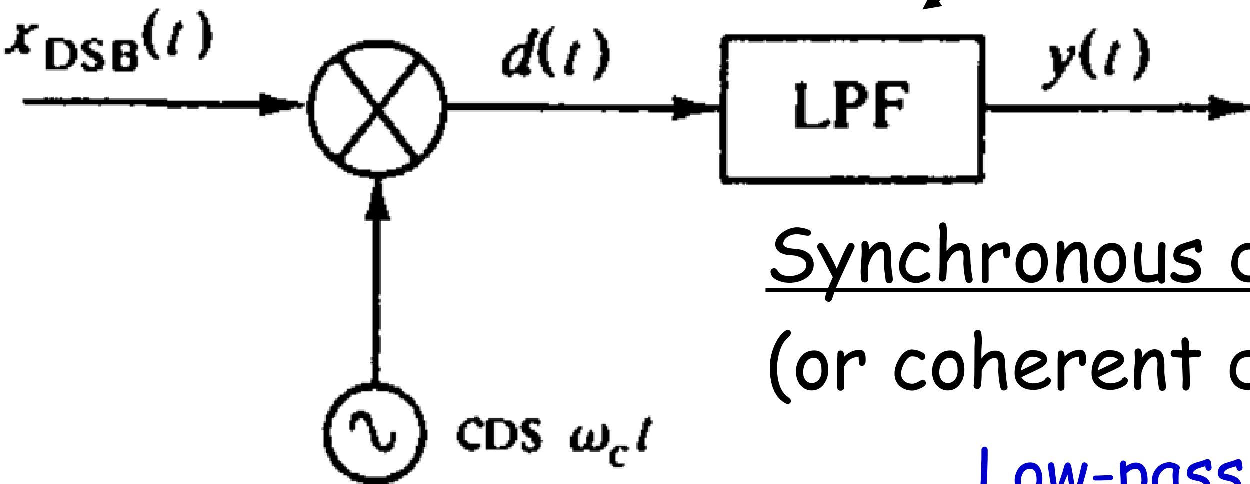




$$X_{\mathrm{DSB}}(\omega) = \frac{1}{2}M(\omega-\omega_c) + \frac{1}{2}M(\omega+\omega_c)$$

Demodulation of DSB Signals

Low-pass filtering



<u>Synchronous demodulation</u>

(or coherent demodulation)

$$x_{\text{DSB}}(t) = m(t)\cos\omega_c t$$

Low-pass filtering (LPF) removes this term

$$d(t) = x_{\text{DSB}}\cos\omega_c t = m(t)\cos^2\omega_c t = m(t)\left(\frac{1}{2} + \frac{1}{2}\cos 2\omega_c t\right)$$

For demodulation, we need to generate a local carrier at the receiver in frequency and phase coherence (synchronism) with the carrier used at the modulator. **Both phase and frequency synchronism are extremely critical.**

Demodulation of DSB Signals

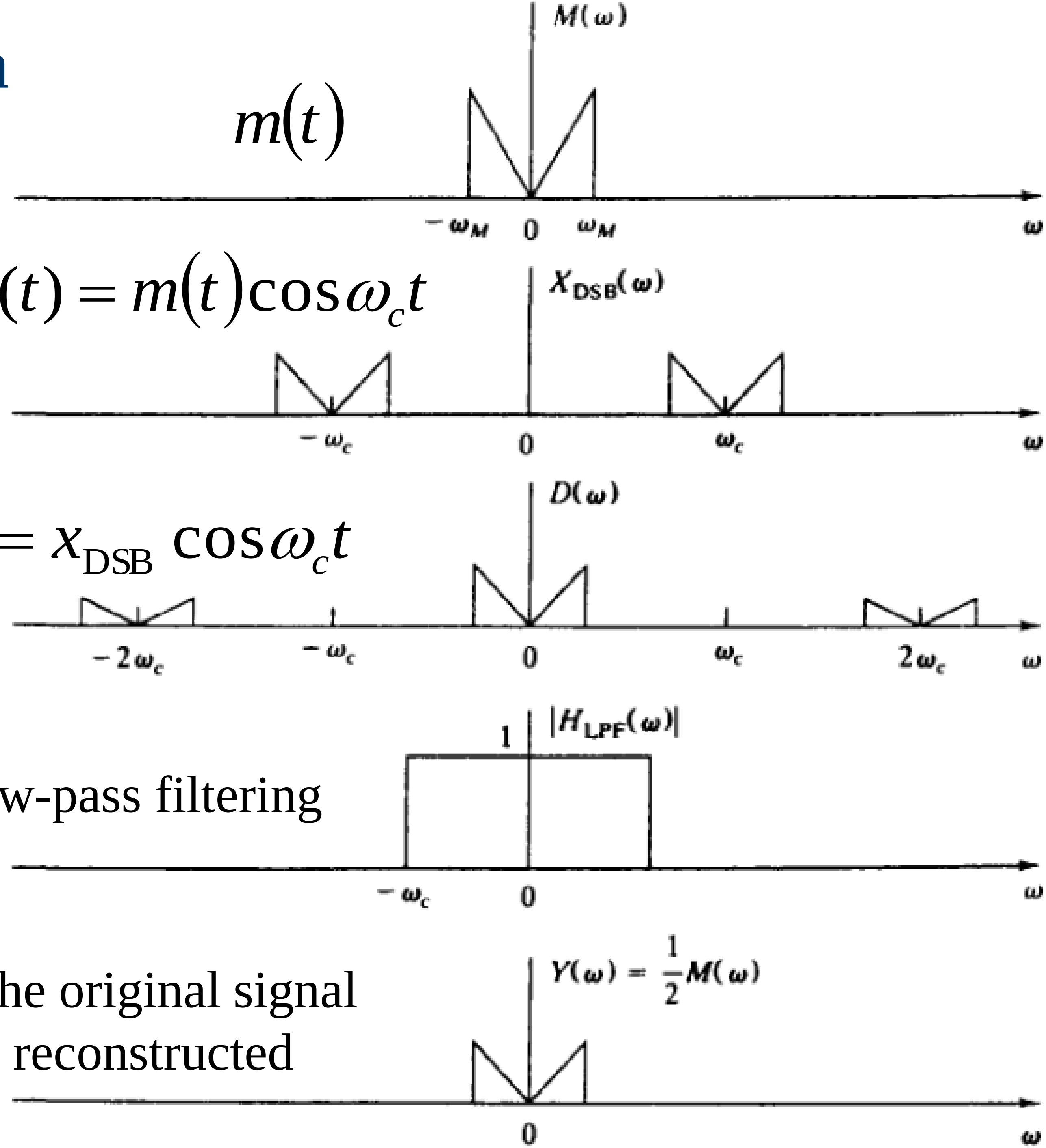
$$m(t)$$

$$x_{\text{DSB}}(t) = m(t)\cos\omega_c t$$

$$d(t) = x_{\text{DSB}}\cos\omega_c t$$

Low-pass filtering

The original signal is reconstructed



Demodulation of DSB Signals: Phase Error

$$x_{\text{DSB}}(t) = m(t)\cos\omega_c t$$

$$d(t) = [m(t)\cos\omega_c t]\cos(\omega_c t + \varphi)$$

$$= \frac{1}{2}m(t)[\cos\varphi + \cos(2\omega_c t + \varphi)]$$

$$= \boxed{\frac{1}{2}m(t)\cos\varphi} + \boxed{\frac{1}{2}m(t)\cos(2\omega_c t + \varphi)}$$

To be filtered out
by a low-pass filter

The output $\boxed{\frac{1}{2}m(t)\cos\varphi}$ is completely lost when $\varphi = \pm\frac{\pi}{2}$

Demodulation of DSB Signals: Frequency Error

$$x_{\mathrm{DSB}}(t) = m(t)\cos\omega_c t$$

$$e(t) = [m(t)\cos\omega_c t]\cos(\omega_c + \Delta\omega)t = (1/2)m(t)[\cos(\Delta\omega)t + \cos(2\omega_c + \Delta\omega)t]$$

The spectrum of $(1/2)m(t)\cos(2\omega_c + \Delta\omega)t$ is centred at $\pm(2\omega_c + \Delta\omega)$ And can be filtered out by LPF.

The spectrum of $(1/2)m(t)\cos(\Delta\omega)t$ is centred at $\pm\Delta\omega$

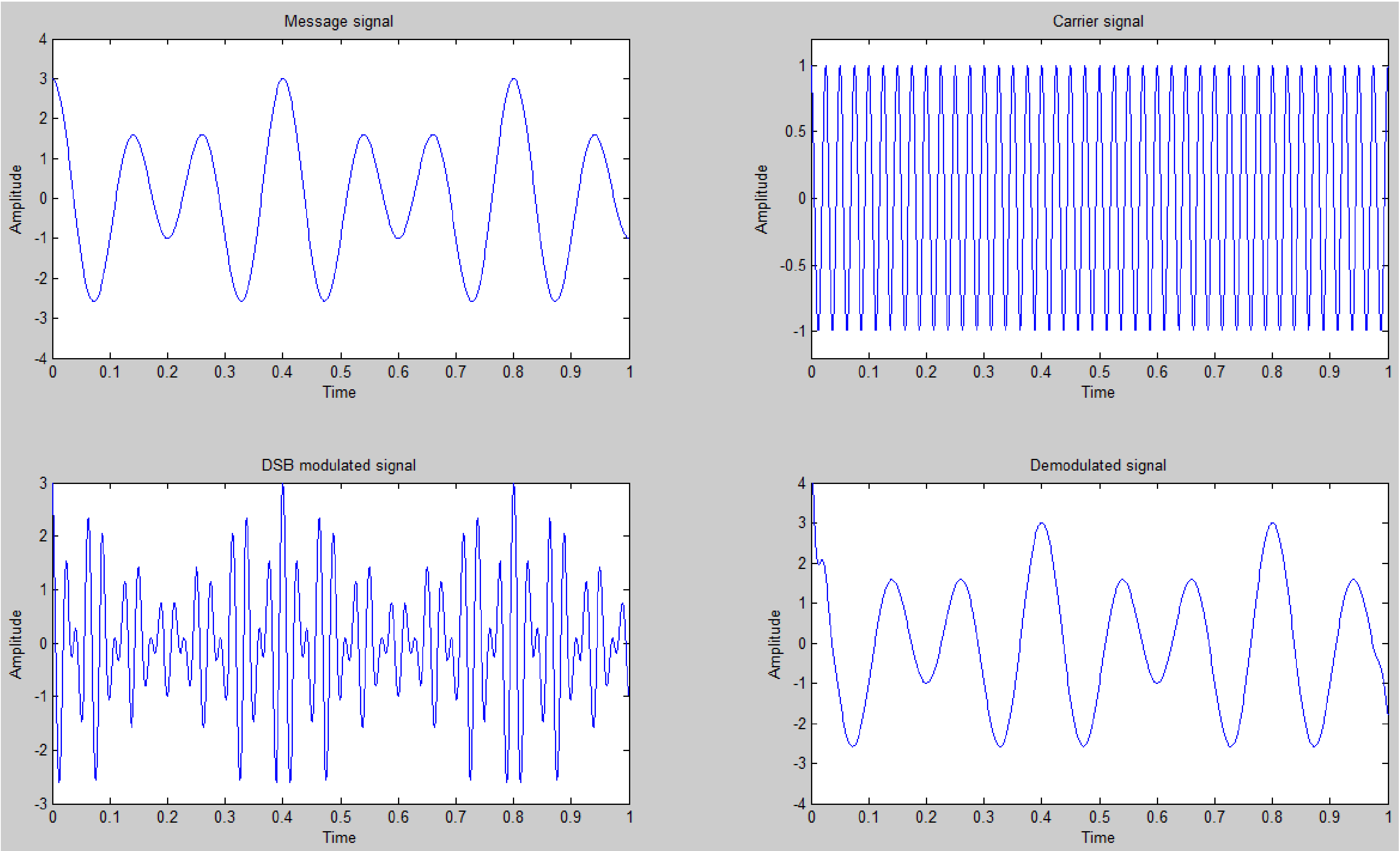
It represents $m(t)$ multiplied by time-varying gain $\cos(\Delta\omega)t$

If, for example, the transmitter and the receiver carrier frequencies differ just by 1 Hz, the output will be the desired signal $m(t)$ multiples by a time-varying signal whose gain goes from the maximum to 0 every half-second. This kind of distortion (called the *beat effect*) is beyond repair.

matlab demo: dsb.m

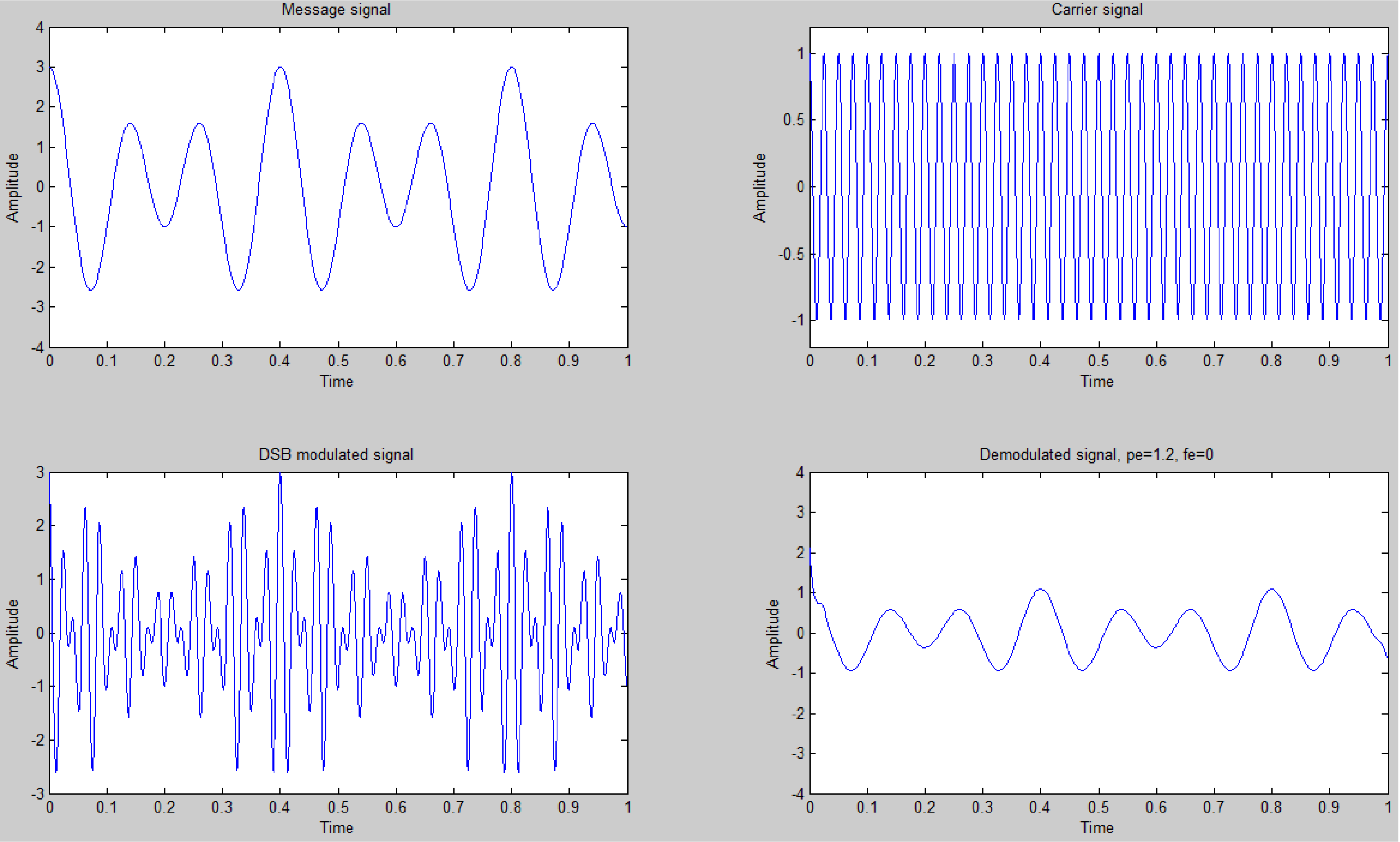
Demodulation of DSB Signals

```
pe = 0.0; % phase error in radians
fe = 0.0; % frequency error in Hz
```



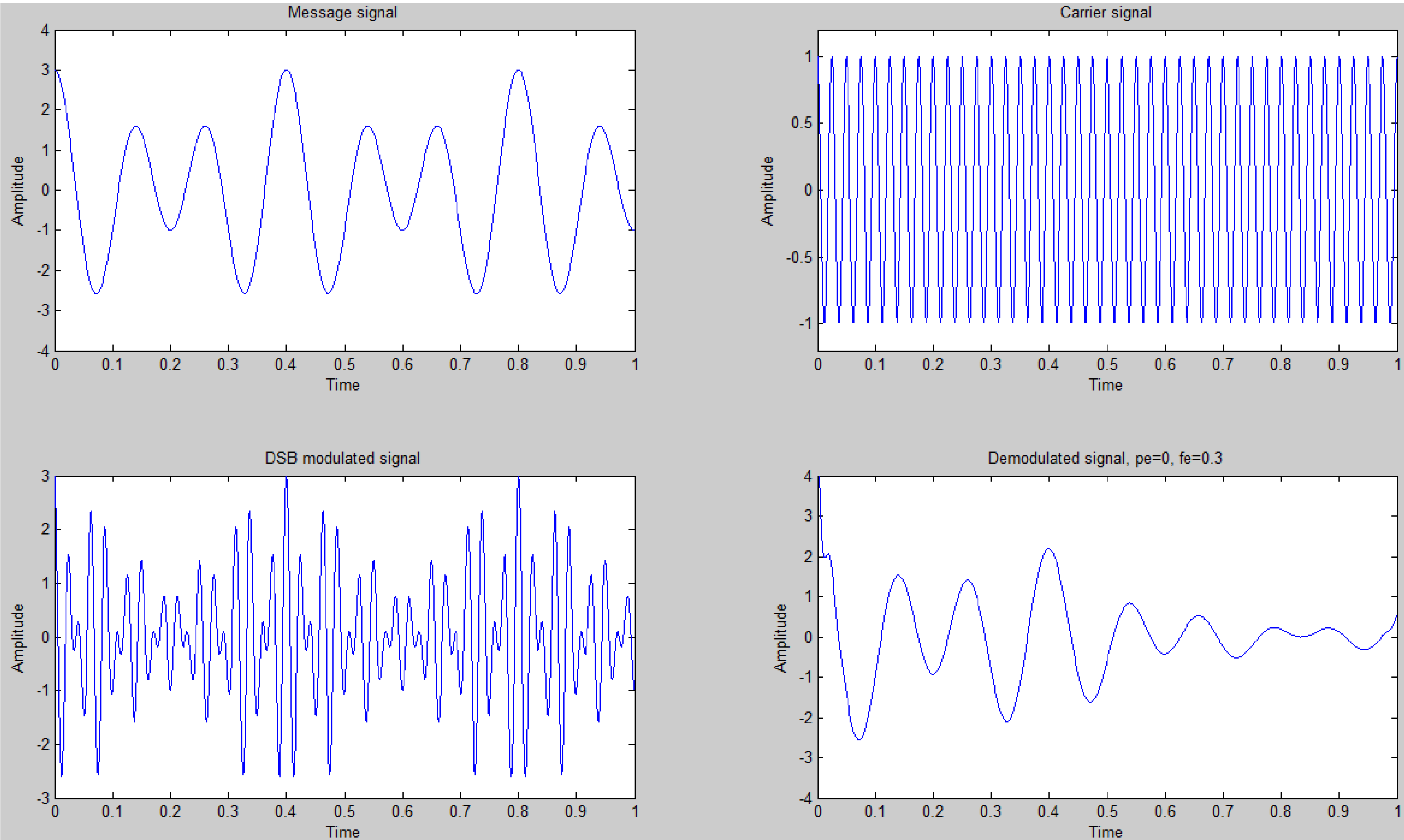
Demodulation of DSB Signals

```
pe = 1.2; % phase error in radians
fe = 0.0; % frequency error in Hz
```



Demodulation of DSB Signals

```
pe = 0.0; % phase error in radians
fe = 0.3; % frequency error in Hz
```



Ordinary Amplitude Modulation

For the scheme just discussed, a receiver must generate a carrier in frequency and phase synchronism with the carrier at a transmitter that may be located hundreds or thousand of miles away. This calls for a sophisticated receiver, which could be quite costly.

The other alternative is for the transmitter to transmit a carrier $A\cos\omega_c t$ along with the modulated signal $m(t)\cos\omega_c t$

$$x_{\mathrm{AM}}(t) = m(t)\cos\omega_c t + A\cos\omega_c t = [m(t) + A]\cos\omega_c t$$

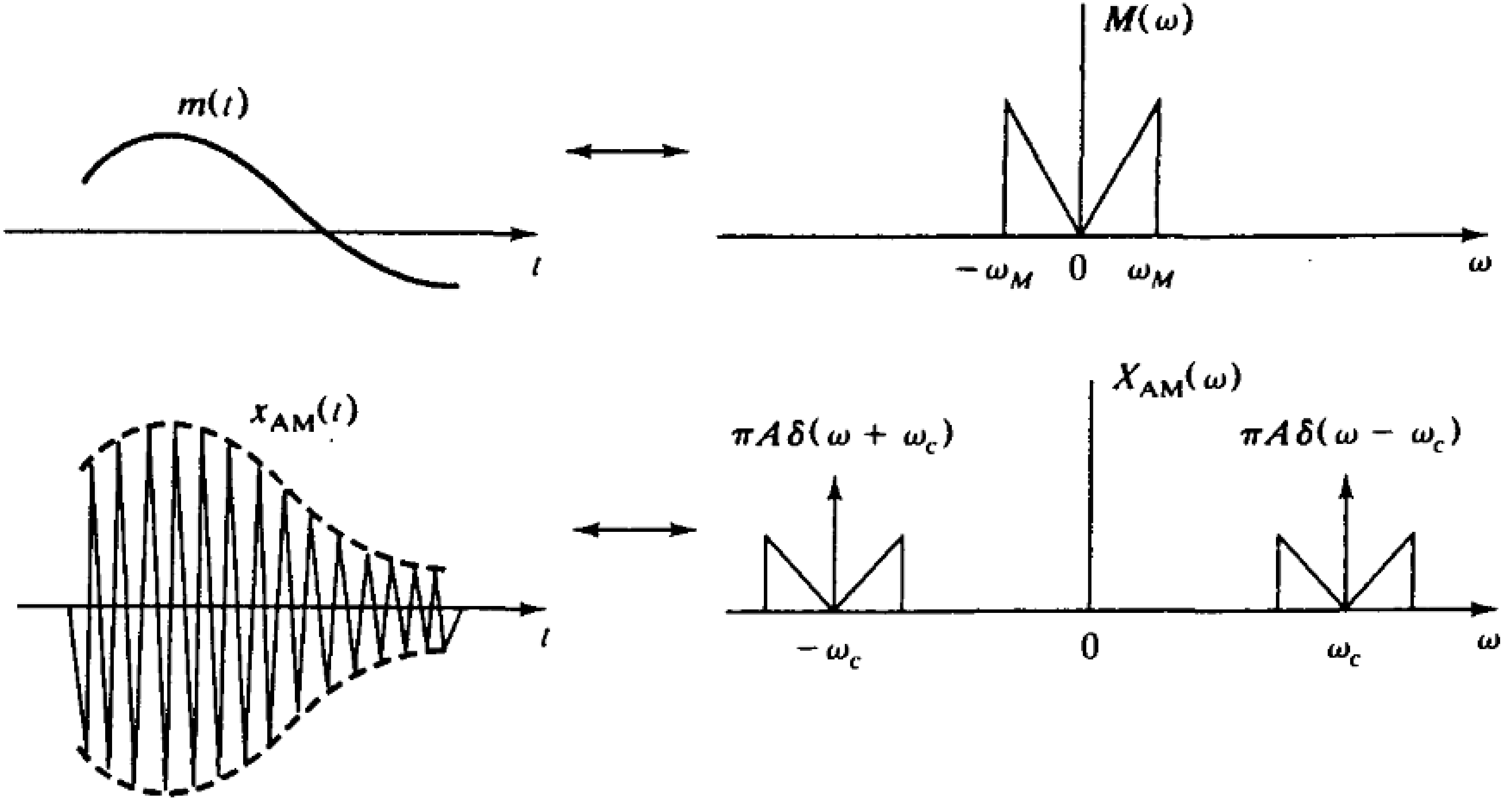
$$x_{\mathrm{AM}}(t) \xleftrightarrow{\mathcal{F}} X_{\mathrm{AM}}(\omega)$$

$$= \frac{1}{2}M(\omega-\omega_c) + \frac{1}{2}M(\omega+\omega_c) + \pi A[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)]$$

Ordinary Amplitude Modulation

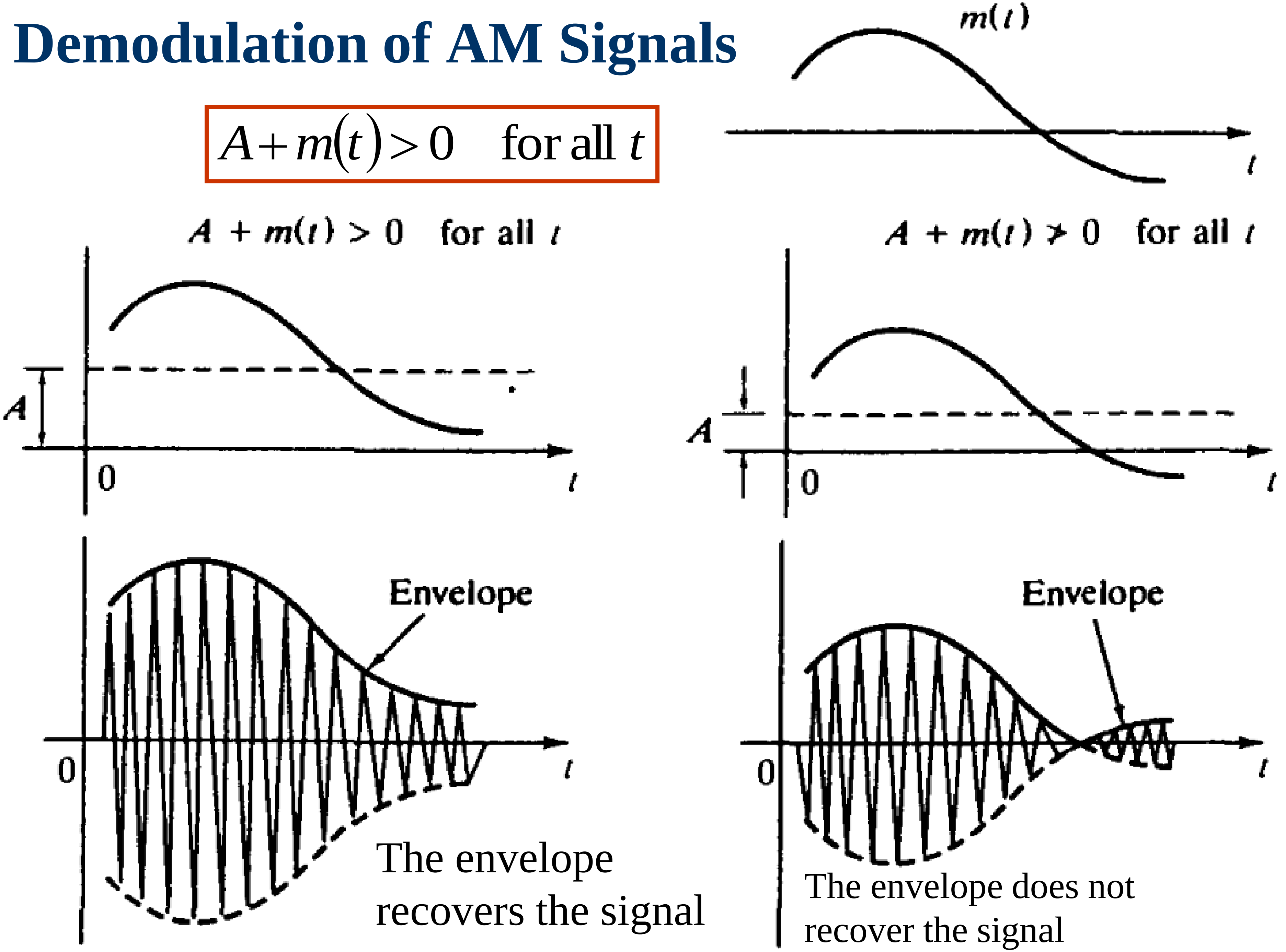
$$x_{\mathrm{AM}}(t) = [m(t) + A]\cos\omega_c t \qquad x_{\mathrm{AM}}(t) \xleftrightarrow{\mathcal{F}} X_{\mathrm{AM}}(\omega) =$$

$$\frac{1}{2}M(\omega-\omega_c) + \frac{1}{2}M(\omega+\omega_c) + \pi A[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)]$$



Demodulation of AM Signals

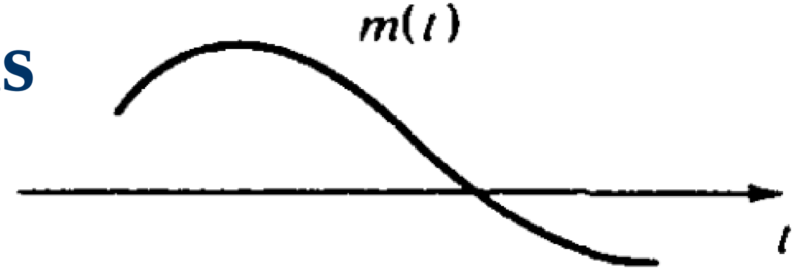
$$\boxed{A + m(t) > 0 \quad \text{for all } t}$$



The envelope recovers the signal

The envelope does not recover the signal

Demodulation of AM Signals



$$\boxed{A + m(t) > 0 \quad \text{for all } t}$$

Let m_p be the peak amplitude (positive or negative) of $m(t)$. The modulation index μ is defined as

$$\mu = m_p / A$$

$0 \le \mu \le 1$ is the required condition for demodulation of AM by an envelope detector.

When A< m_p we have μ >1 (overmodulation). In this case, we need to use synchronous demodulation.

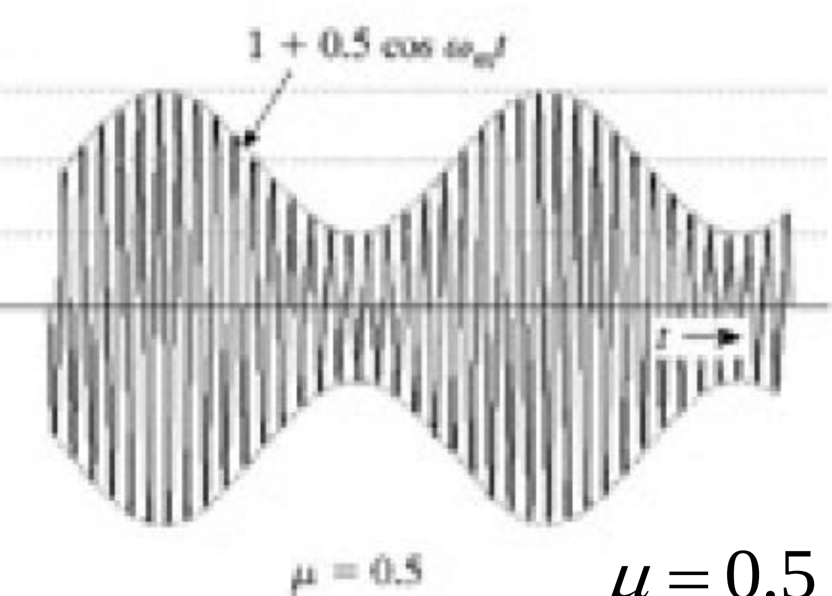
Example

$$m(t) = B\cos\omega_m t$$

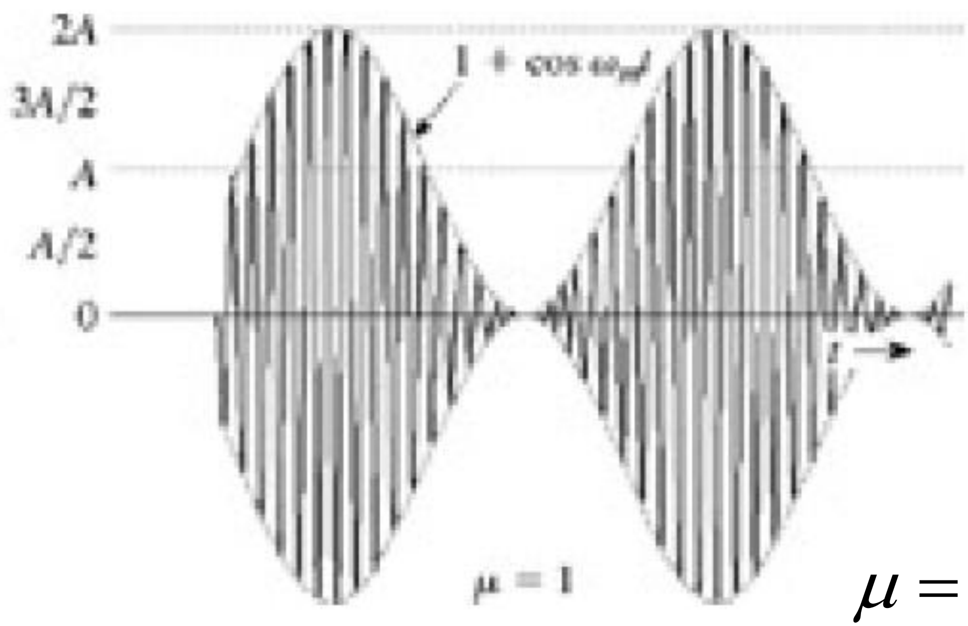
This case is referred as <u>tone modulation</u> because the modulation signal is a pure sinusoid (or tone)

$$m_p = B \quad \mu = B/A \quad B = \mu A$$

$$x_{\text{AM}}(t) = [m(t) + A]\cos\omega_c t = A[1 + \mu\cos\omega_m t]\cos\omega_c t$$

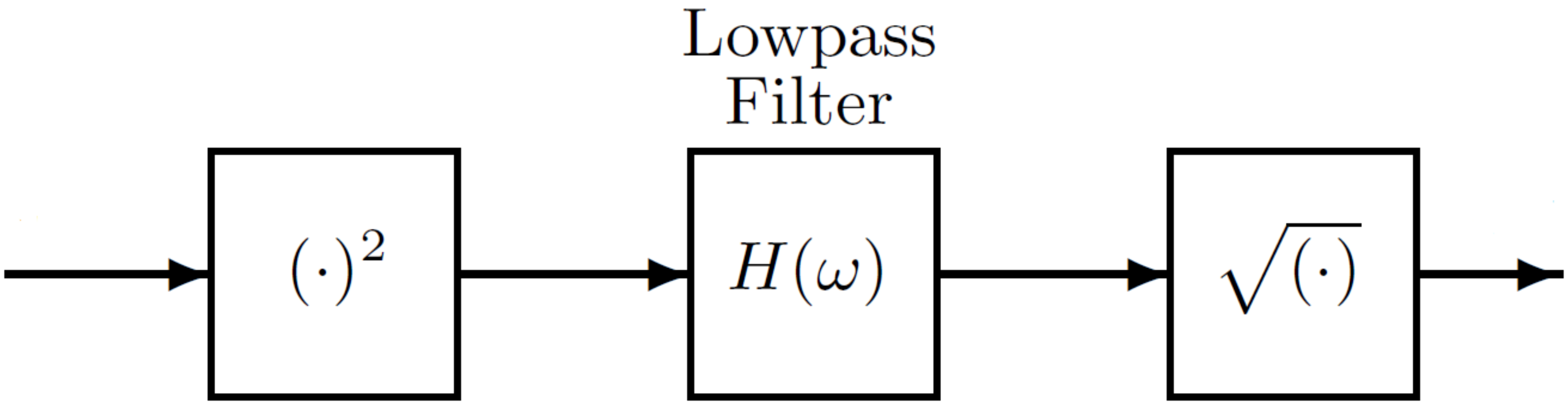


$\mu = 0.5$



$\mu = 1$

Square-Law Demodulation



$$x_{\mathrm{AM}}(t) = [m(t) + A]\cos\omega_c t = A[1 + \mu\cos\omega_m t]\cos\omega_c t$$

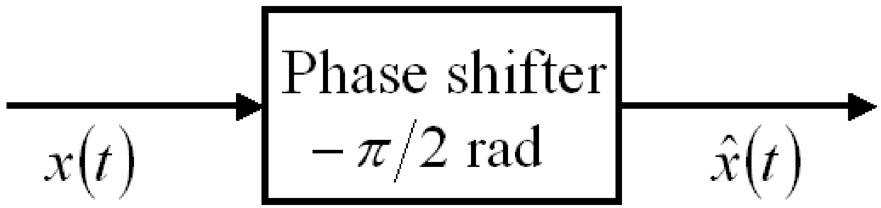
$$[x_{\mathrm{AM}}(t)]^2 = A^2[1 + \mu\cos\omega_m t]^2 \cos^2\omega_c t$$

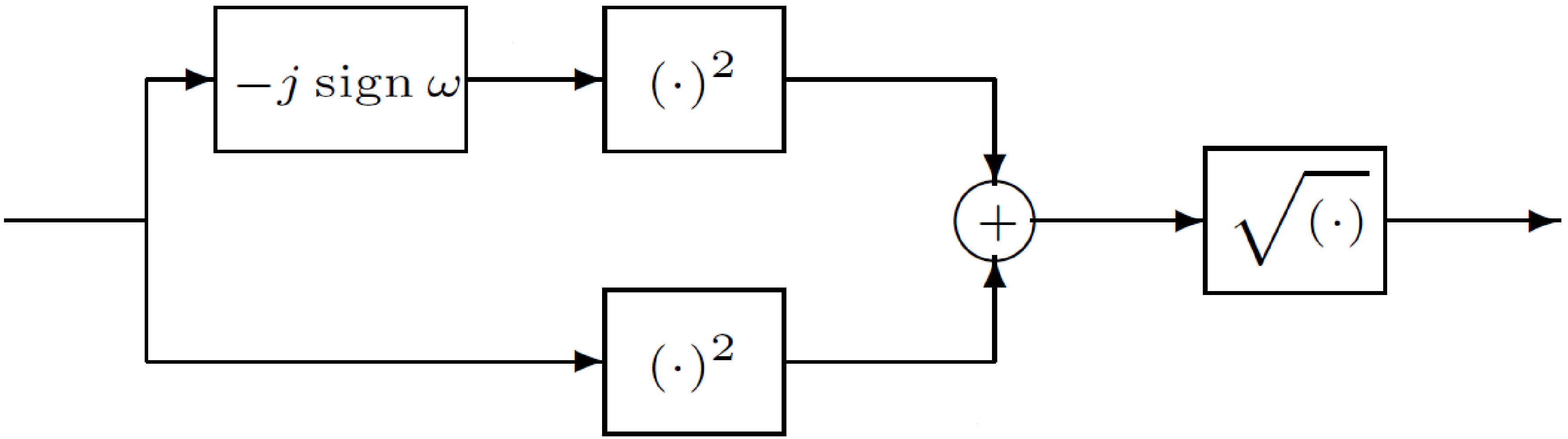
$$= \frac{1}{2}A^2[1 + \mu\cos\omega_m t]^2 + \boxed{\frac{1}{2}A^2[1 + \mu\cos\omega_m t]^2 \cos 2\omega_c t}$$

Can be removed by LPF

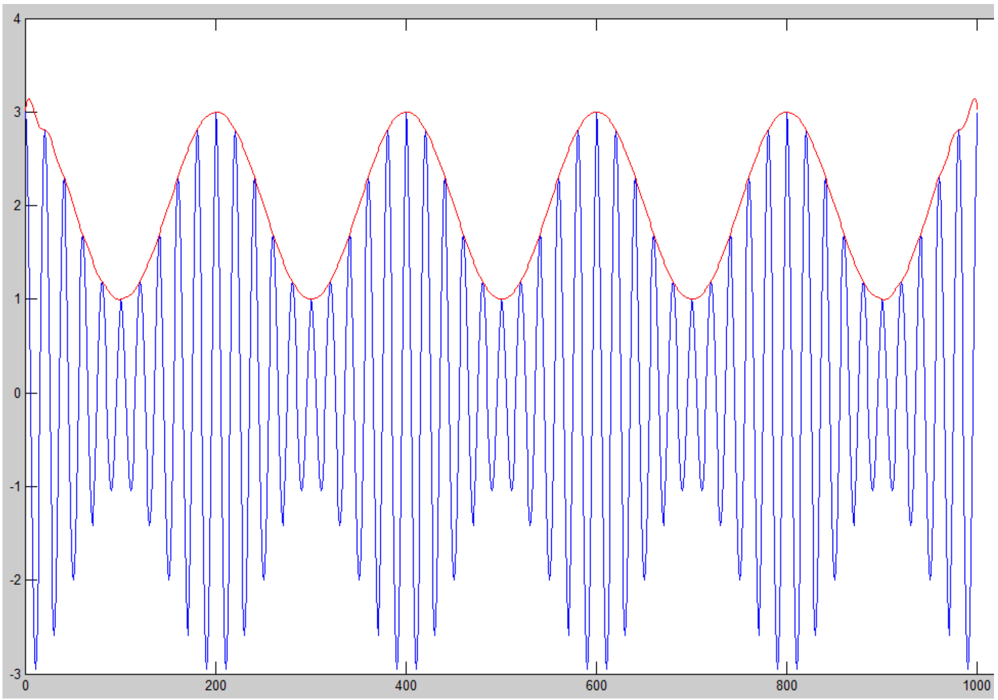
Envelope detector using the Hilbert transform

$$H(\omega) = -j\,\mathrm{sgn}(\omega) = \begin{cases} e^{-j\pi/2} & \omega > 0 \\ e^{j\pi/2} & \omega < 0 \end{cases}$$



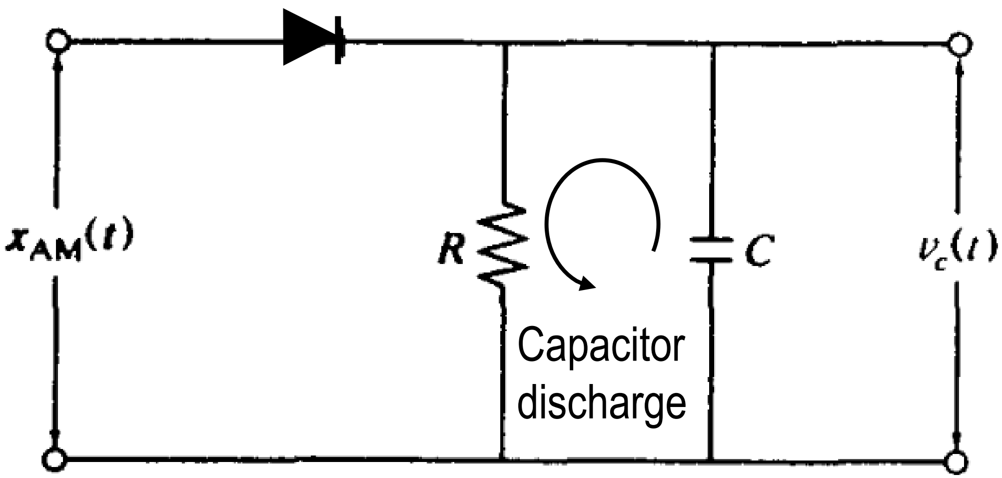


Envelope detector using the Hilbert transform

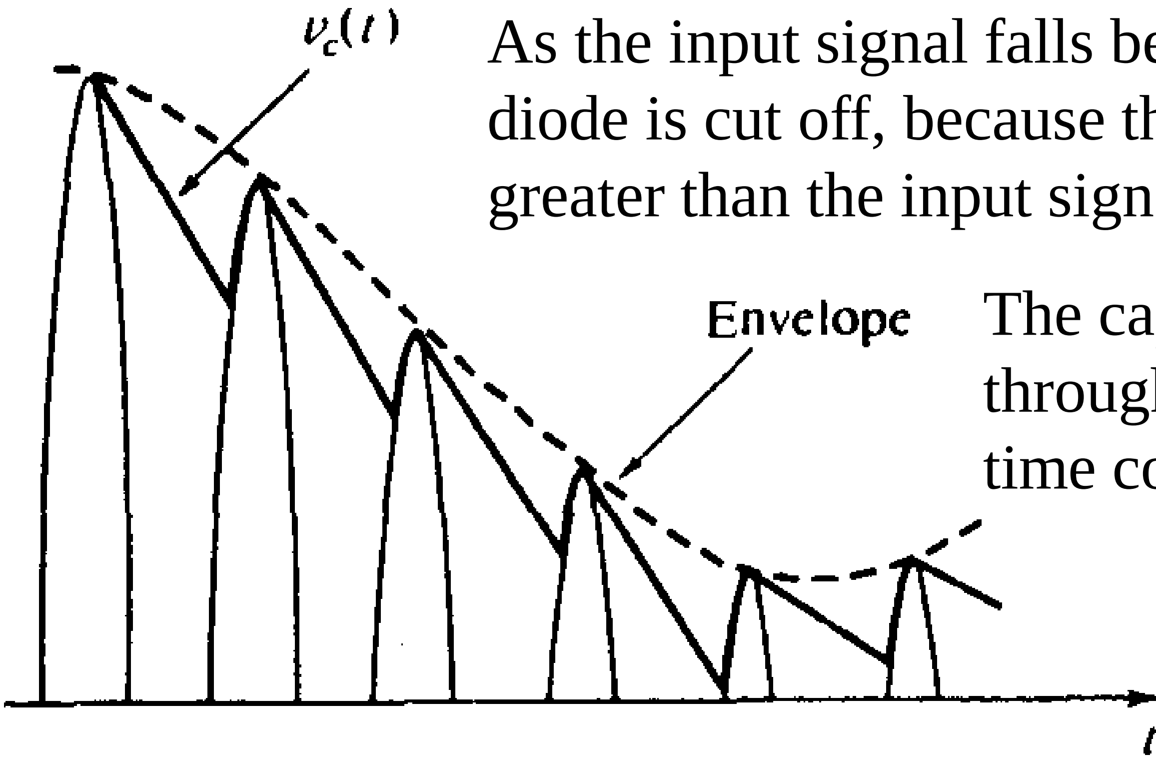



```
plot(AM, 'b');
hold on;
envelope = abs(hilbert(AM));
plot(envelope, 'r');
hold off
```

Demodulation by means of envelope detector



During the positive cycle of the input signal, the diode conducts and the capacitor *C* charges up to the peak voltage of the input signal.



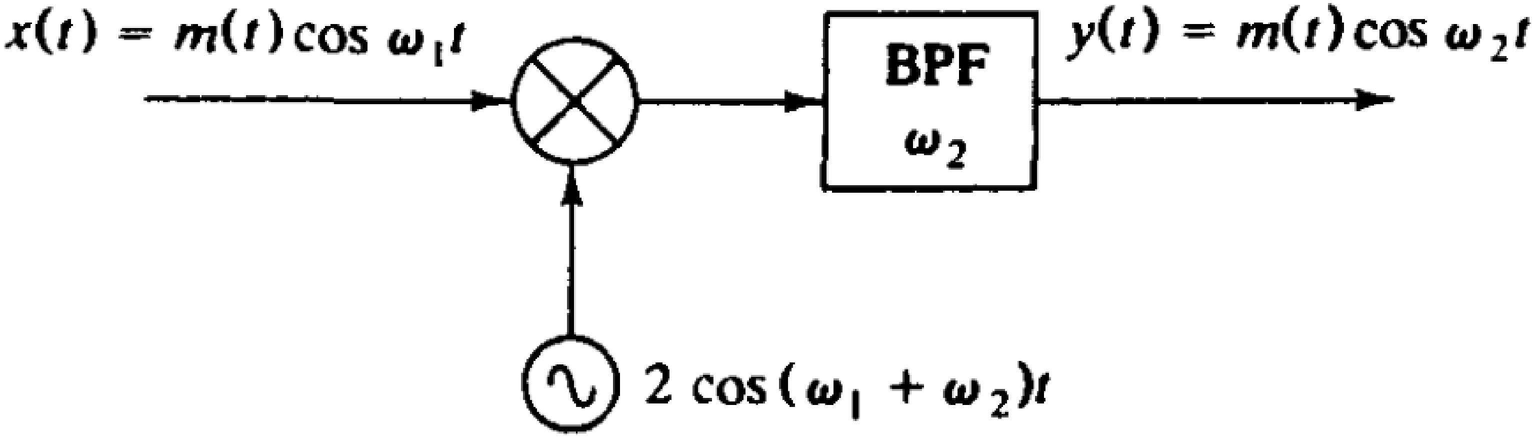
As the input signal falls below this peak value, the diode is cut off, because the capacitor voltage is greater than the input signal voltage.

The capacitor is now discharging through *R* at a slow rate (with a time constant *RC*).

Then the capacitor is charging again, etc.

Frequency Translation and Mixing

It is often desirable to translate or shift the modulated signal to a new frequency band.



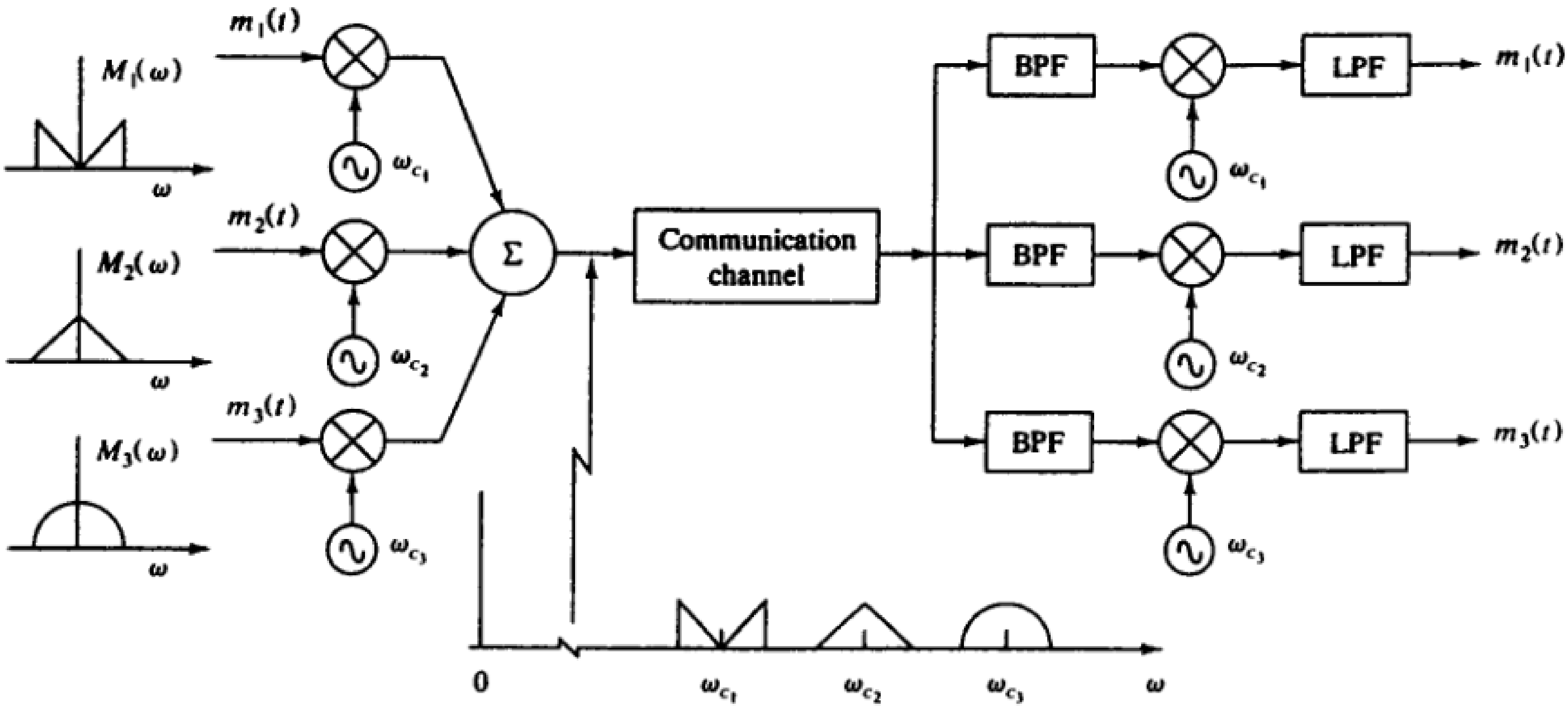
$$m(t)\cos\omega_1 t \times 2\cos(\omega_1+\omega_2)t$$
$$= m(t)\cos\omega_2 t + \underbrace{m(t)\cos(2\omega_1+\omega_2)t}_{\text{removed by BPF}}$$

$$2\cos\alpha\cos\beta = \cos(\alpha-\beta)+\cos(\alpha+\beta)$$

Mutltiplexing is combining several message signals into a composite signal for transmission over a common channel.

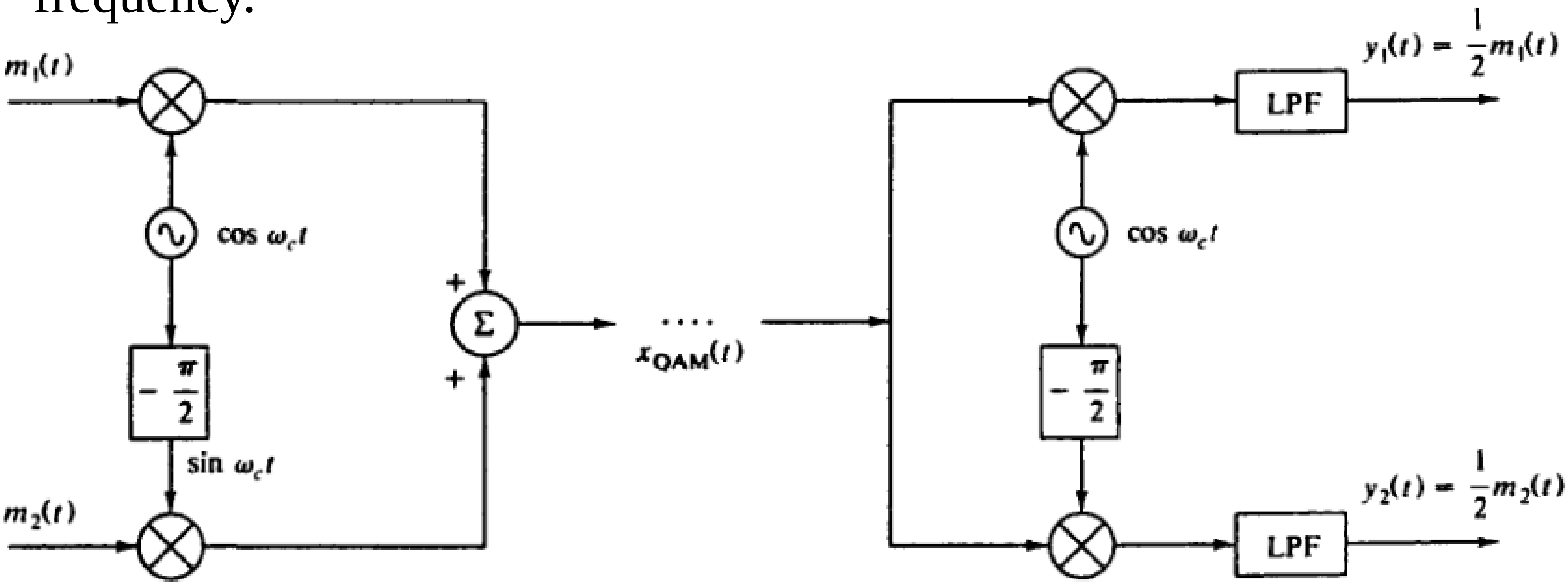
Time-division multiplexing (TDM): the signals are separated in time.

Frequency-division multiplexing (FDM):
the signals are separated in frequency.



Quadrature Multiplexing
(quadrature amplitude modulation or QAM)

The <u>orthogonality</u> of sine and cosine makes it possible to transmit and receive two different signals simultaneously at the same carrier frequency.

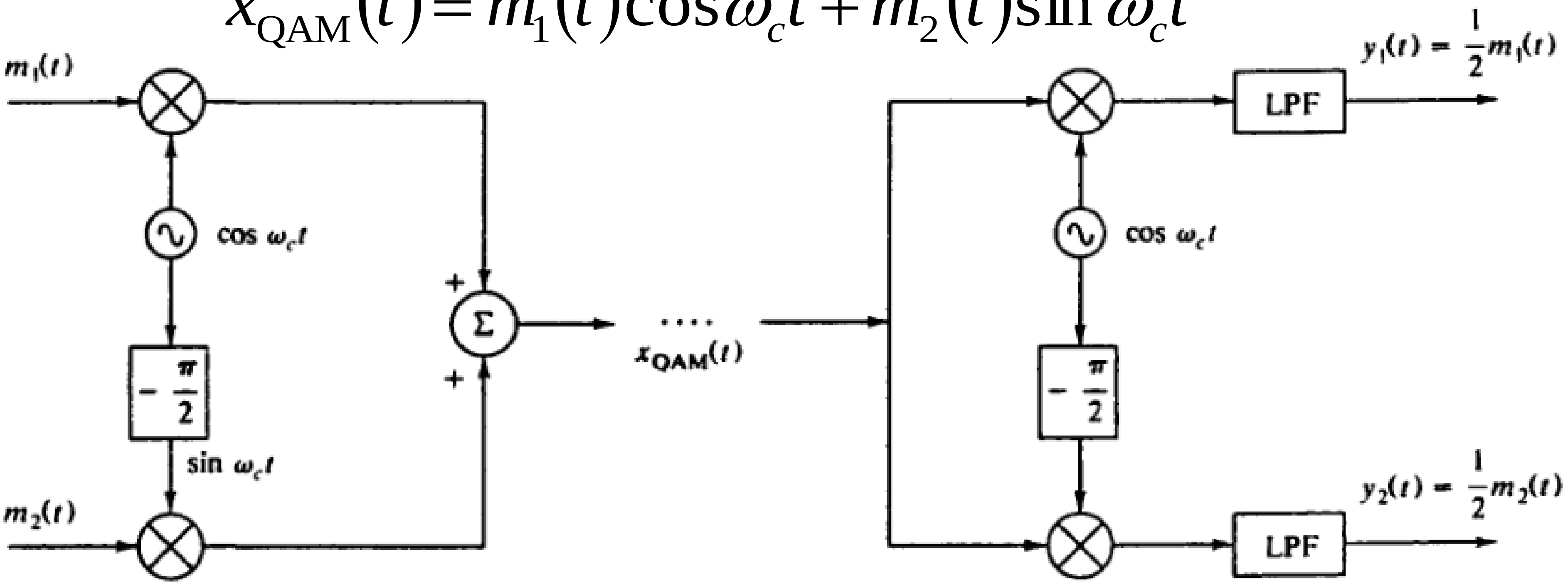


$$x_{\mathrm{QAM}}(t) = m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t$$

matlab demo: qam_sol.m

Quadrature amplitude modulation (QAM)

$$x_{\text{QAM}}(t) = m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t$$



$$x_{\text{QAM}}(t)\cos\omega_c t = m_1(t)\cos^2\omega_c t + m_2(t)\sin\omega_c t\cos\omega_c t$$

$$= \tfrac{1}{2}m_1(t) + \tfrac{1}{2}m_1(t)\cos 2\omega_c t + \tfrac{1}{2}m_2(t)\sin 2\omega_c t$$

$$x_{\text{QAM}}(t)\sin\omega_c t = m_1(t)\cos\omega_c t\sin\omega_c t + m_2(t)\sin^2\omega_c t$$

$$= \tfrac{1}{2}m_1(t)\sin 2\omega_c t + \tfrac{1}{2}m_2(t) - \tfrac{1}{2}m_2(t)\cos 2\omega_c t$$

Quadrature amplitude modulation (QAM)

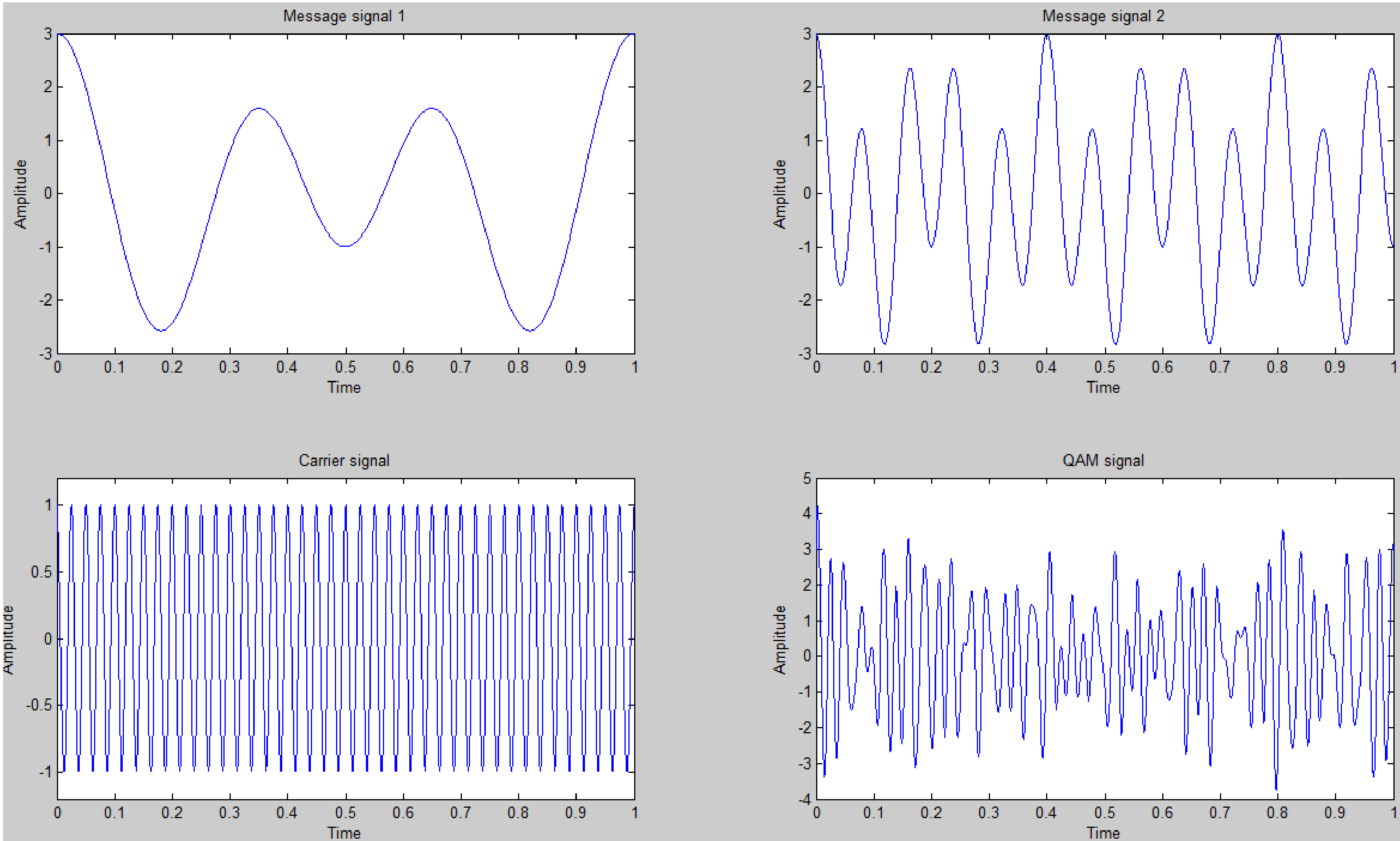
$$x_{\mathrm{QAM}}(t)\cos\omega_c t = m_1(t)\cos^2\omega_c t + m_2(t)\sin\omega_c t\cos\omega_c t$$
$$= \tfrac{1}{2}m_1(t) + \tfrac{1}{2}m_1(t)\cos 2\omega_c t + \tfrac{1}{2}m_2(t)\sin 2\omega_c t$$
$$x_{\mathrm{QAM}}(t)\sin\omega_c t = m_1(t)\cos\omega_c t\sin\omega_c t + m_2(t)\sin^2\omega_c t$$
$$= \tfrac{1}{2}m_1(t)\sin 2\omega_c t + \tfrac{1}{2}m_2(t) - \tfrac{1}{2}m_2(t)\cos 2\omega_c t$$

All terms at $2\omega_c$ are filtered out by the low-pass filter, yielding

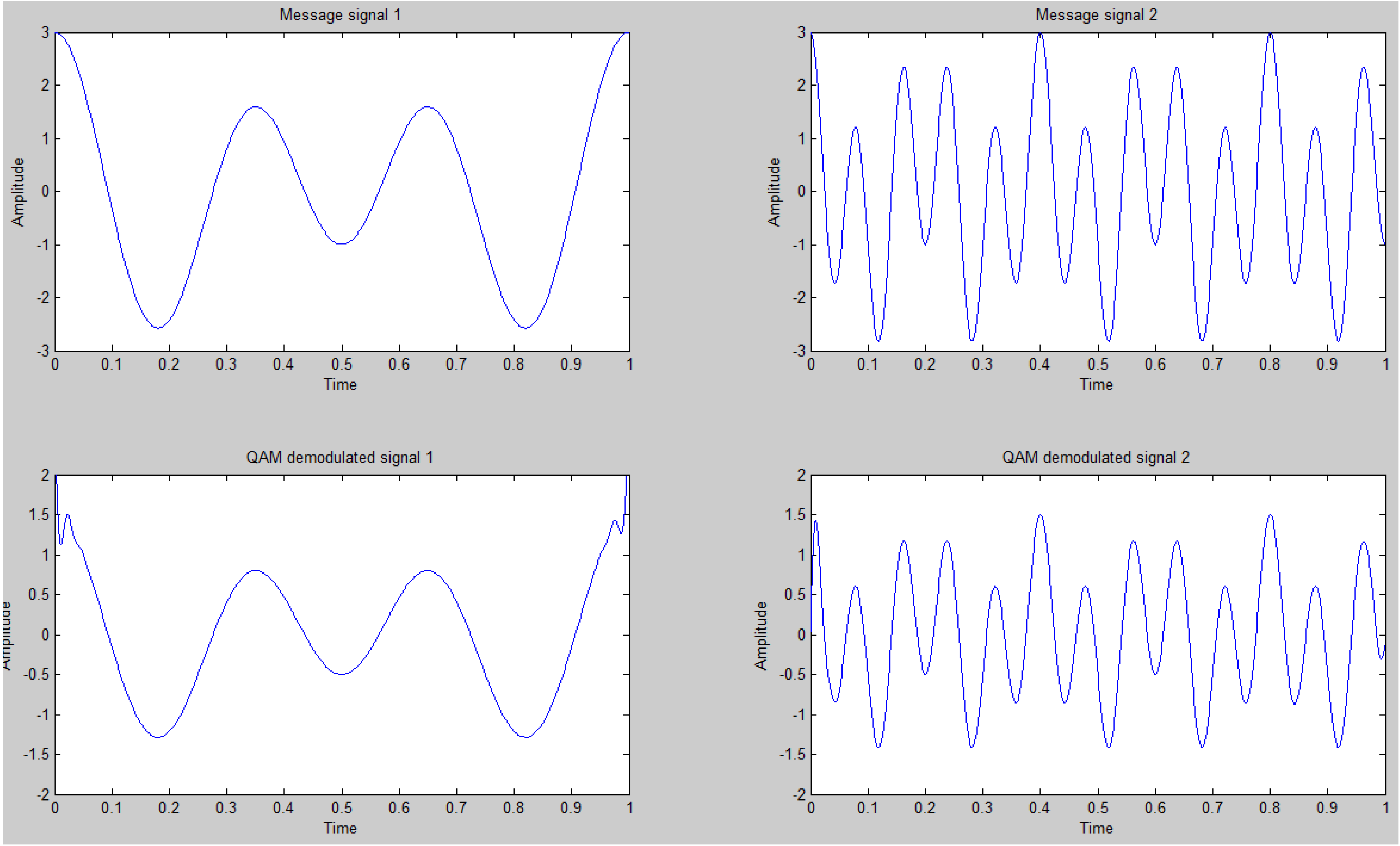
$$y_1(t) = \tfrac{1}{2}m_1(t) \quad \text{and} \quad y_2(t) = \tfrac{1}{2}m_2(t)$$

Quadrature multiplexing is an efficient method of transmitting two message signals within the same bandwidth. It is used in the transmission of colour information signals in commercial TV broadcasts.

Quadrature amplitude modulation (QAM)



Quadrature amplitude modulation (QAM)



Angle Modulation

Phase and Frequency Modulation

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Modulation

Modulation is the process by which some characteristic of a carrier signal is varied in accordance with a modulating signal.

Amplitude Modulation: $x_c(t) = A(t)\cos\omega_c t$

Angle modulation: the modulated carrier is represented by

$$\boxed{x_c(t) = A\cos[\omega_c t + \varphi(t)]}$$

Phase modulation: $\varphi(t) = k_p m(t)$ k_p- phase deviation constant

Frequency modulation: $\dfrac{d\varphi(t)}{dt} = k_f\, m(t)$ k_f - frequency deviation constant

Angle Modulation: $x_c(t) = A\cos[\omega_c t + \varphi(t)]$

$$x_c(t) = A\cos\theta(t) \qquad \theta(t) = \omega_c t + \varphi(t)$$

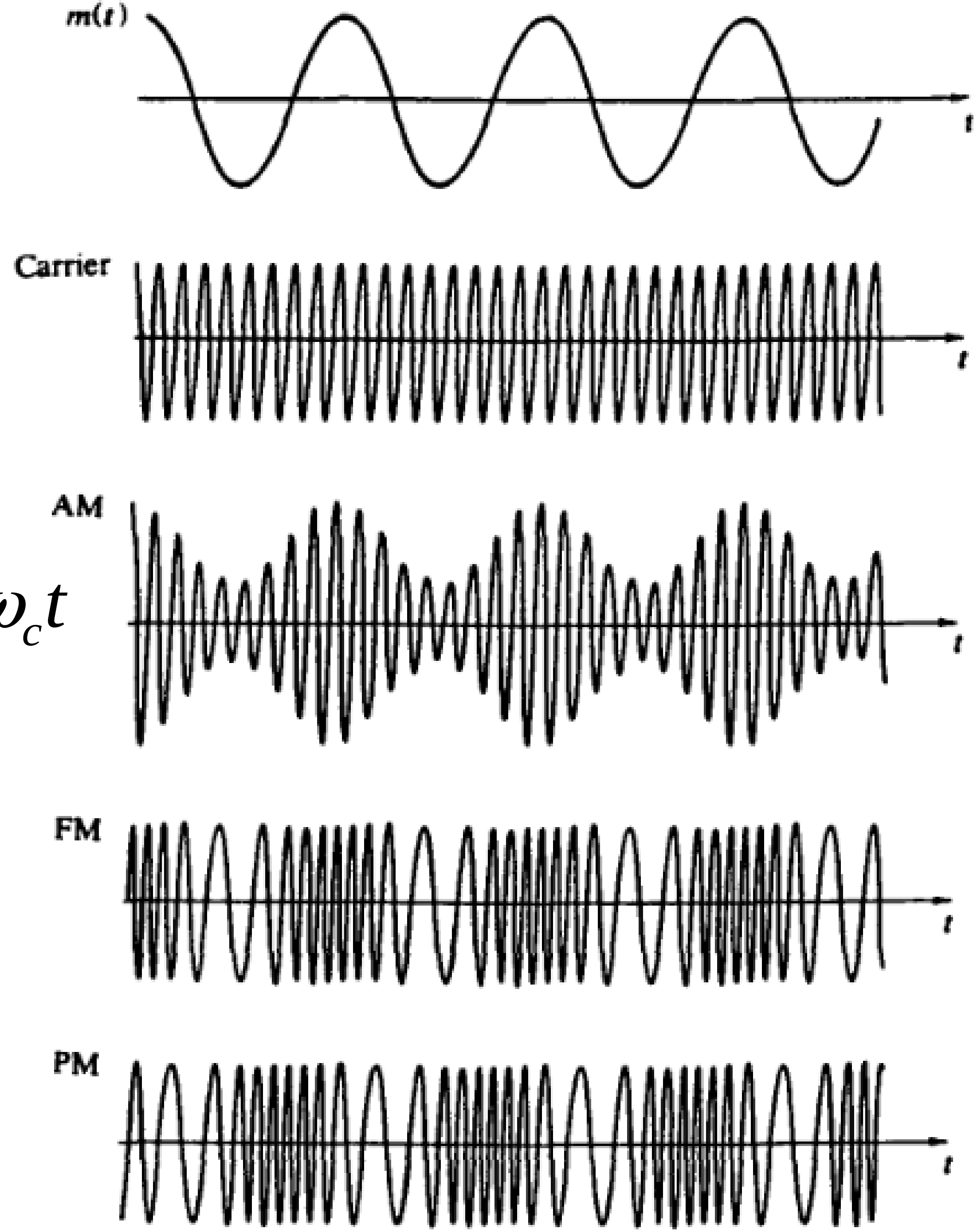
$$\omega_i = \frac{d\theta(t)}{dt} = \omega_c + \frac{d\varphi(t)}{dt}$$ instantaneous frequency

PM: $\varphi(t) = k_p m(t) \qquad \omega_i = \omega_c + k_p \dfrac{dm(t)}{dt}$

FM: $\dfrac{d\varphi(t)}{dt} = k_f\, m(t) \qquad \omega_i = \omega_c + k_f\, m(t)$

$$x_{FM}(t) = A\cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(t)dt\right]$$

Modulation



AM

$$x_c(t) = [m(t) + A]\cos\omega_c t$$

FM $$\frac{d\varphi(t)}{dt} = k_f\, m(t)$$

PM $$\varphi(t) = k_p m(t)$$