Time and Frequency Signal Analysis (B39SB - Part 1)

Elements of Communication Systems

Laplace Transform for Circuit Analysis

Lecturer:

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Contents

- Fourier series for periodic signals
- Fourier transform and continuous spectra
- Fundamentals of sampling
- LTI and filtering
- Modulation methods for signal transmission in analogue communications
- Double-sided Laplace transform
- Single side Laplace transform and circuit analysis
- s-domain representation of circuit elements
- Application of Laplace transform in circuit and system analysis (transfer function, block diagram, amplifiers, bode plot)

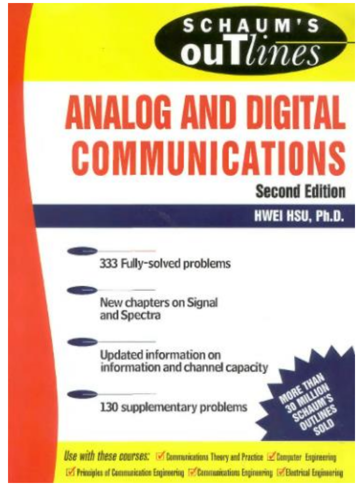
All course materials on Canvas – Module “B39SB Xidian - Part 1”

Learning Objectives

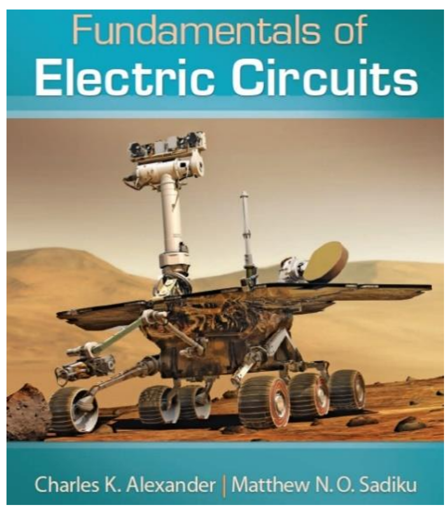
- To understand and apply trigonometric and complex Fourier series for periodic signals; Parseval’s theorem, power and energy signals
- To understand Fourier transform and application in signal analysis, frequency spectra, modulation theorem, instantaneous and natural sampling, Nyquist rate, frequency response, LTI systems, filters
- To understand amplitude and phase (phase and frequency modulation and demodulation methods in analogue communications
- To understand Laplace transform and theorems and application in circuit and system analysis
- To understand Laplace transform representation of circuit elements and application in circuit analysis
- To know how to obtain transfer functions for electrical circuits and systems
- To know how to obtain Bode plots from transfer functions
- To understand block diagrams and feedback systems
- To be able to use Matlab to solve the related problems

Suggested Textbooks (optional)

Elements of Communication Systems



Laplace Transform for Circuit Analysis

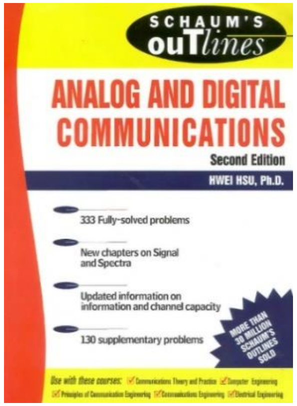


Almost every textbook on electric circuits contains chapters on Laplace circuit analysis. In particular, Chapters 15 and 16 from the Alexander-Sadiku textbook contain a good presentation of the topic.

Course Topics and Book Chapters

Signals, spectra, LTI systems, filtering.
Chapters 1 & 2

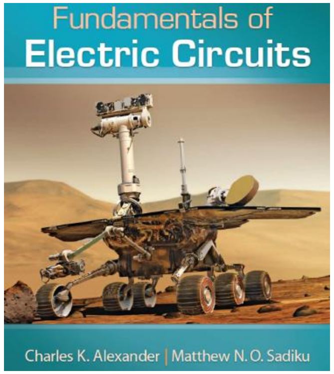
Amplitude and angle modulation (including phase and frequency modulation).
Chapters 3 & 4



Two-sided Laplace transform

One-sided Laplace transform and its applications to circuit analysis and transients.

Laplace transform methods for analysis of LTI systems and operational amplifiers. Bode plots.



Lecture slides

Assessment

- 70% from a 2-hour in-person exam
- 9% from one class test on 5 March (to be confirmed)
- 6% from 3 Matlab based labs
- 15% from the labs for Part 2 of the course.

Signals and Spectra

Fourier series and discrete spectra

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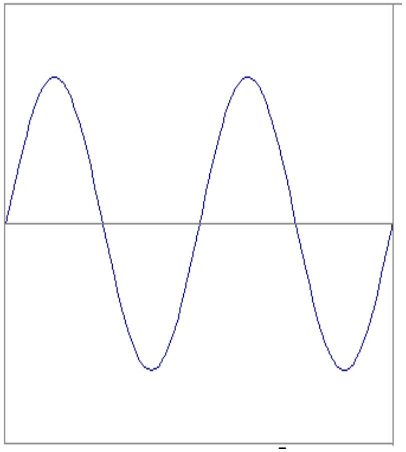
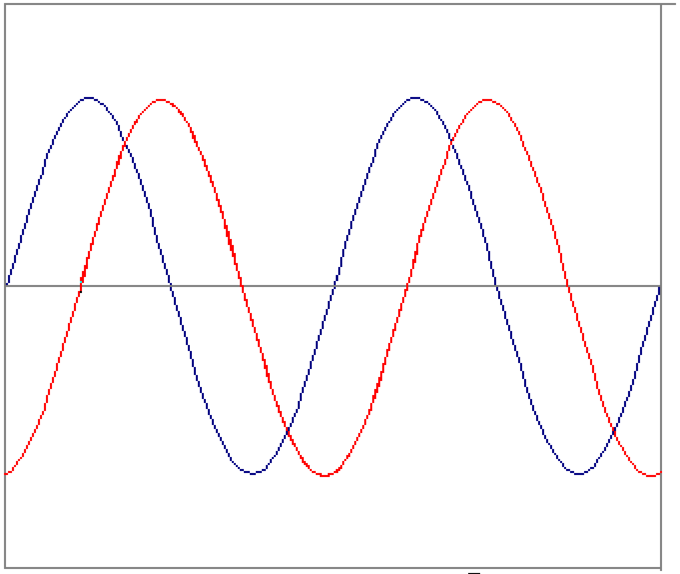
Periodic functions

- Definition: $f(t)$ is periodic if there exists T such that $f(t+T) = f(t)$
- Fundamental period of a function: smallest constant T_0 that satisfies $f(t+T_0) = f(t)$



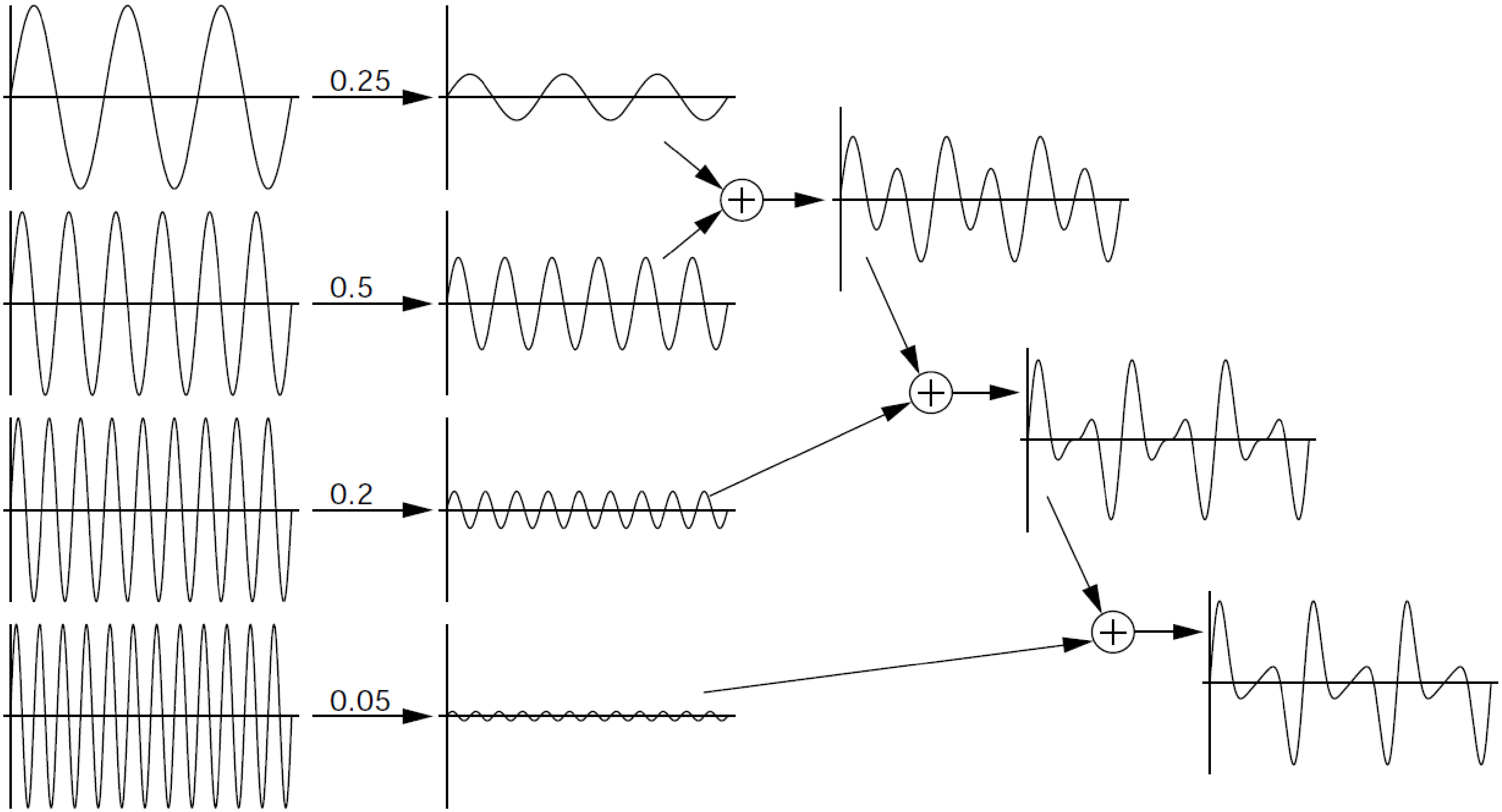
- **Amplitude**: max value of $f(t)$ in any period
- **Period**: T
- **Frequency**: $1/T$, cycles per second, Hz
- **Phase**: position of the function within a period



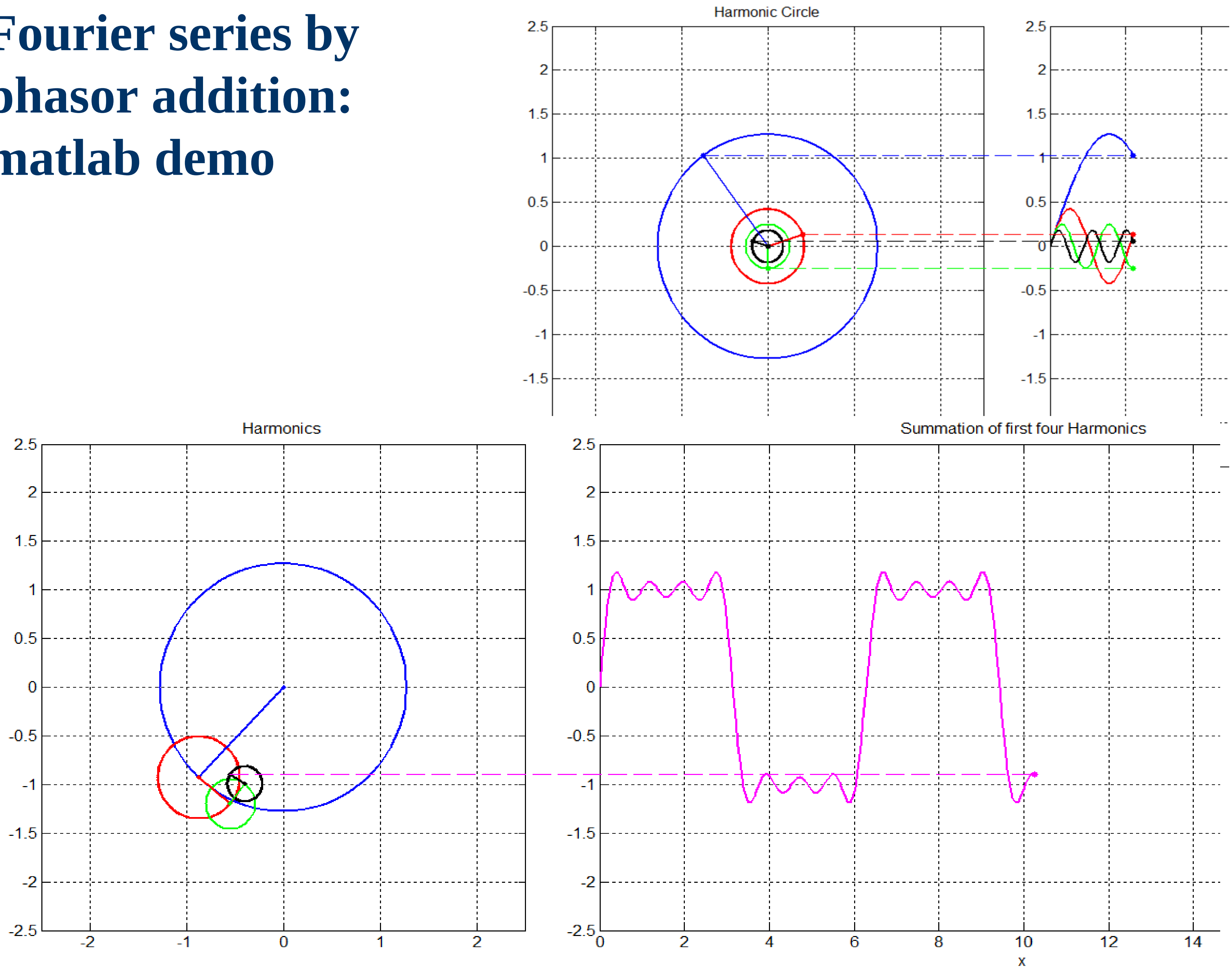
Jean Baptiste Joseph Fourier (1768-1830)

- had a crazy idea (1807):
- ***Any periodic function*** *can be rewritten as a* ***weighted sum of sines and cosines of different frequencies****.*
- Don't believe it?
  - Neither did Lagrange, Laplace, Poisson and other big wigs
  - Not translated into English until 1878!
- But it is true!
  - called **Fourier Series**

Fourier series



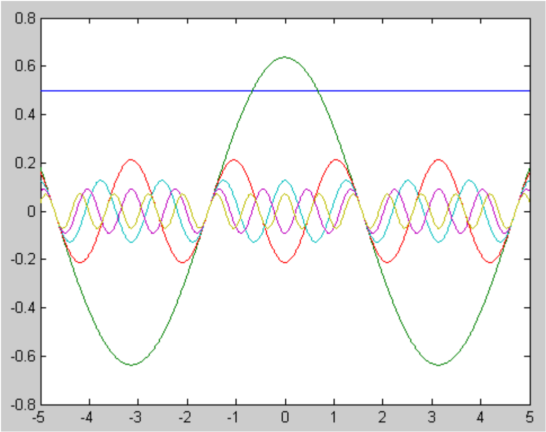
Fourier series by phasor addition: matlab demo

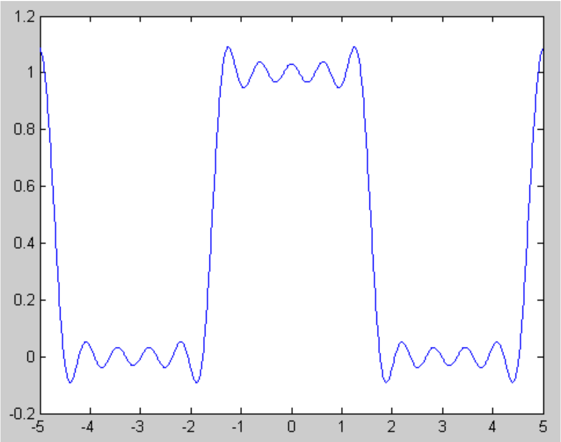


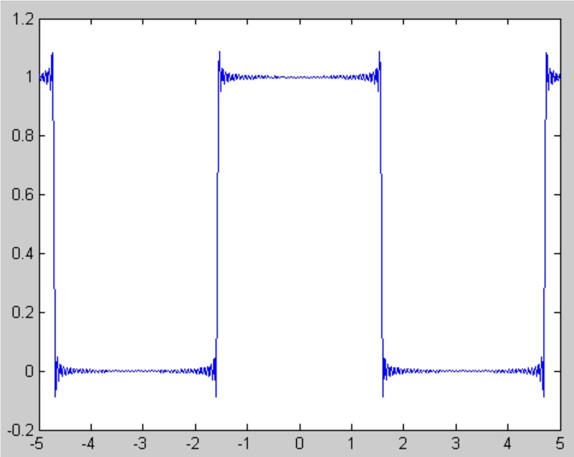
Fourier series

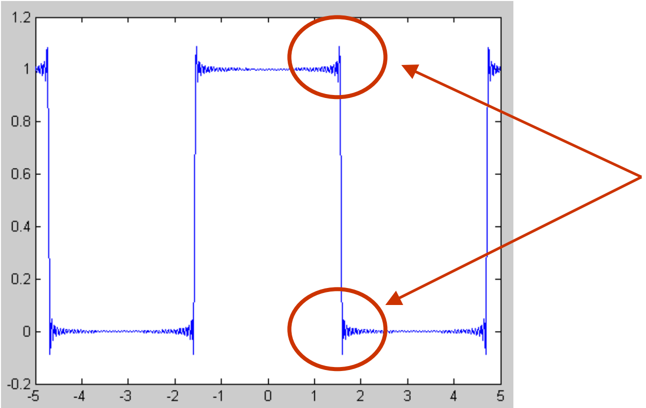
$$x(t) \equiv x(t+2\pi) \quad x(t) = a_0 + a_1 \cos t + b_1 \sin t + a_2 \cos 2t + \ldots$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jkt} \quad c_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} x(t) e^{-jkt} dt \quad j = \sqrt{-1}$$









9% overshot at a jump discontinuity: the so-called **Gibbs phenomenon**

Dealing with complex numbers

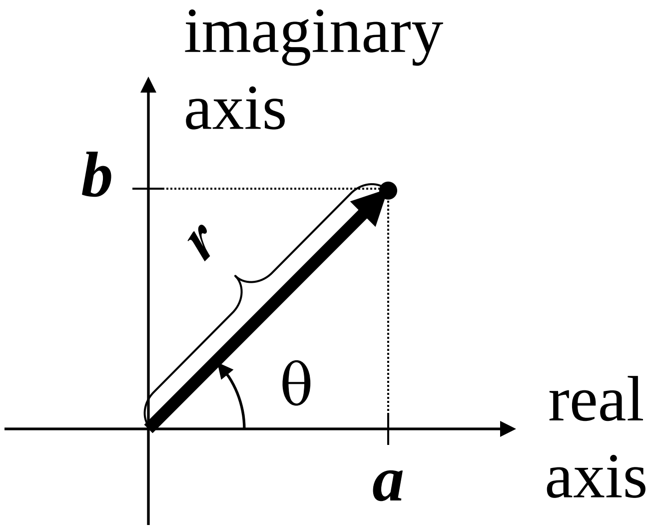
$$z = a + b\,j \qquad j = \sqrt{-1}$$

$$\bar{z} = a - b\,j \qquad |z|^2 = z\,\bar{z} = a^2 + b^2$$

Polar form:

$$z = a + jb \qquad r = \sqrt{a^2 + b^2}$$

$$z = r(\cos\theta + j\sin\theta) = re^{j\theta}$$



- a is the **real** part
- b is the **imaginary** part
- r is the **magnitude**
- θ is the **phase** (polar angle)

It is very convenient to use

$$\left\{ \exp\left(\frac{2\pi k\, j}{T} t \right) \right\}$$

instead of

$$\left\{ \cos\left(\frac{2\pi k}{T} t \right), \sin\left(\frac{2\pi k}{T} t \right) \right\}$$

Periodic signals

Let $x(t)$ be a periodic signal with period T

$$x(t+T) \equiv x(t) \qquad\qquad (*)$$

The fundamental period T_0 of $x(t)$ is the smallest positive value of T for which (*) is satisfied.

Two basic examples:

$$x(t) = \cos(\omega_0 t + \varphi) \qquad\qquad x(t) = e^{j\omega_0 t}$$

$\omega_0 = 2\pi/T = 2\pi\ f_0$ is called the fundamental angular frequency

Trigonometric Fourier series $x(t+T) \equiv x(t)$

$$x(t) = a_0 + a_1 \cos\omega_0 t + b_1 \sin\omega_0 t + a_2 \cos 2\omega_0 t + b_2 \sin 2\omega_0 t + \ldots$$

$$\omega_0 = 2\pi/T$$

$$x(t) = \underbrace{a_0}_{dc} + \underbrace{\sum_{n=1}^{\infty}\left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right)}_{ac \text{ components}}$$

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt$$

integrals over the period

$$a_n = \frac{2}{T}\int_0^T x(t)\cos(n\omega_0 t)\,dt \quad b_n = \frac{2}{T}\int_0^T x(t)\sin(n\omega_0 t)\,dt$$

Trigonometric Fourier series $x(t+T) \equiv x(t) \quad \omega_0 = 2\pi/T$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right) \Bigg| \int_0^T (\ldots) \cos m\omega_0 t dt$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right) \Bigg| \int_0^T (\ldots) \sin m\omega_0 t dt$$

$$\text{if} \quad m \neq 0 \quad \int_0^T a_0 \cos(m\omega_0 t) dt = 0$$

$$\int_0^T \cos(n\omega_0 t)\cos(m\omega_0 t) dt = 0 = \int_0^T \sin(n\omega_0 t)\sin(m\omega_0 t) dt, \text{ n≠m}$$

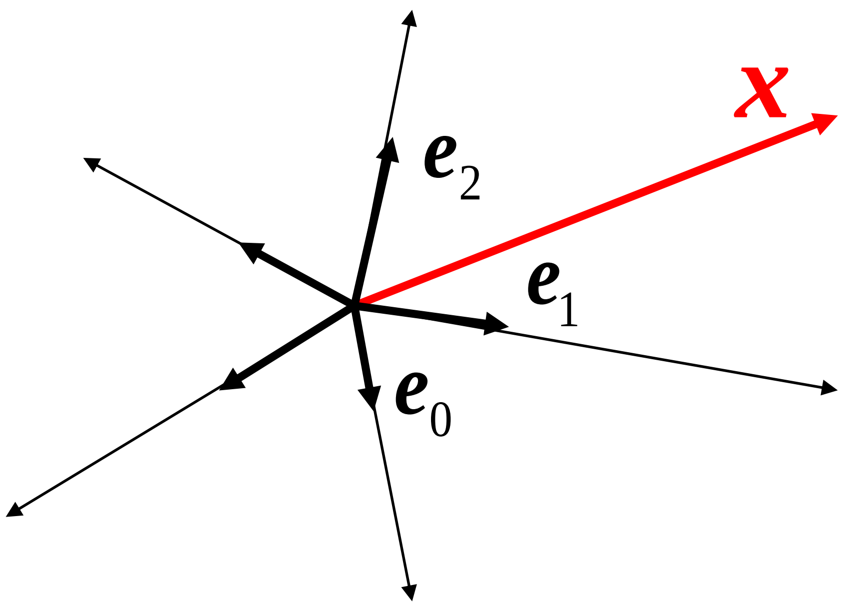
$$\text{for any} \quad m \neq n \quad \int_0^T \sin(n\omega_0 t)\cos(m\omega_0 t) dt = 0$$

Expansion w.r.t. an orthogonal basis (a general idea)

$$\boldsymbol{x} = c_0\boldsymbol{e}_0 + c_1\boldsymbol{e}_1 + c_2\boldsymbol{e}_2 + \ldots + c_n\boldsymbol{e}_n \mid \cdot\, \boldsymbol{e}_k$$

$\boldsymbol{e}_1$, $\boldsymbol{e}_2$, ….$\boldsymbol{e}_n$ are unit vectors

$$\boldsymbol{x}\cdot\boldsymbol{e}_k = c_k\boldsymbol{e}_k\cdot\boldsymbol{e}_k \qquad\qquad c_k = \boldsymbol{x}\cdot\boldsymbol{e}_k \big/ \left|\boldsymbol{e}_k\right|^2$$



Trigonometric Fourier series $x(t+T) \equiv x(t)$ $\omega_0 = 2\pi/T$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right)$$

$$= a_0 + \sum_{n=1}^{\infty} A_n \cos\left(n\omega_0 t + \varphi_n \right)$$

$$A_n = \sqrt{a_n^2 + b_n^2}, \quad \varphi_n = -\tan^{-1} \frac{b_n}{a_n}$$

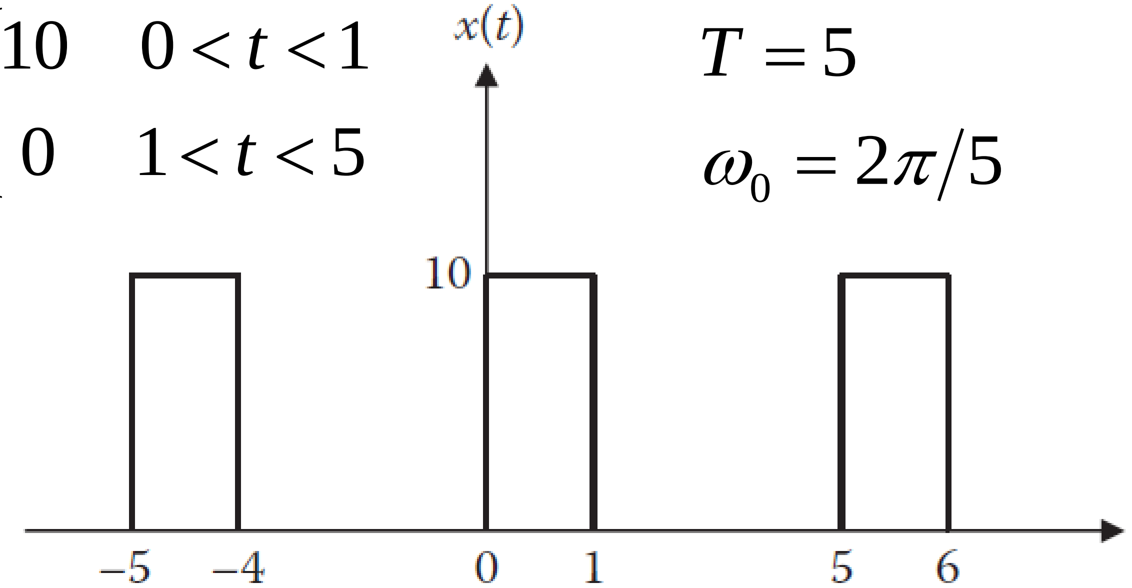
This is the so-called amplitude-phase (or polar) form

$$A_n \cos\left(n\omega_0 t + \varphi_n \right) = A_n \cos\varphi_n \cos n\omega_0 t - A_n \sin\varphi_n \sin n\omega_0 t$$

$$a_n = A_n \cos\varphi_n \qquad b_n = -A_n \sin\varphi_n$$

Example:
a periodic train of pulses

$$x(t)=\begin{cases}10 & 0<t<1\\ 0 & 1<t<5\end{cases}$$



$$T=5$$

$$\omega_0=2\pi/5$$

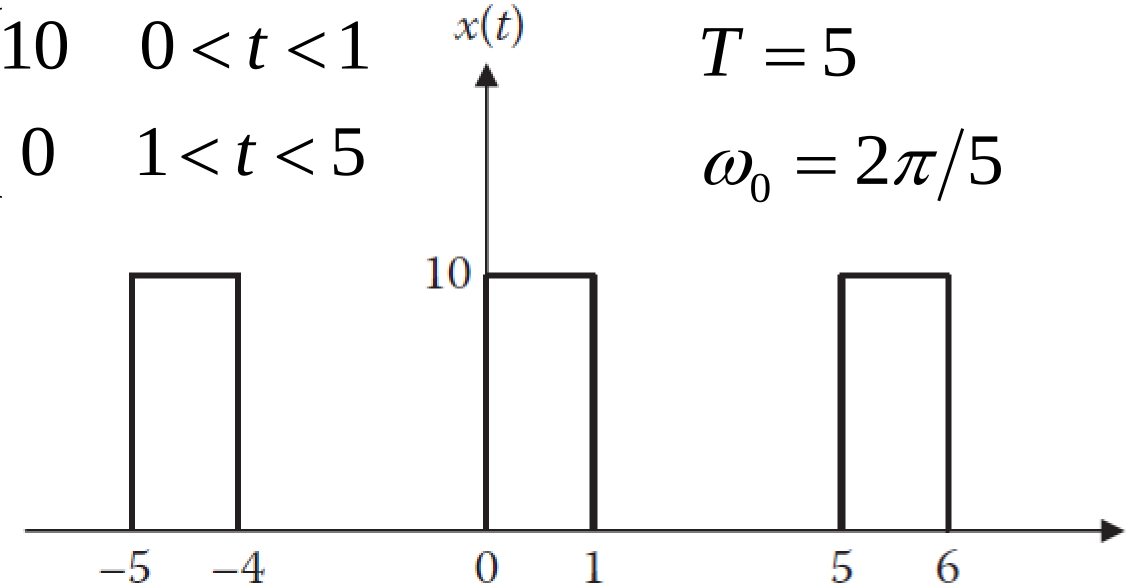
$$a_0=\frac{1}{T}\int_0^T x(t)\,dt=\frac{1}{5}10=2$$

$$a_n=\frac{2}{T}\int_0^T x(t)\cos(n\omega_0 t)\,dt=\frac{2}{5}\int_0^1 10\cos(n\omega_0 t)\,dt=4\frac{\sin n\omega_0}{n\omega_0}$$

$$b_n=\frac{2}{T}\int_0^T x(t)\sin(n\omega_0 t)\,dt=\frac{2}{5}\int_0^1 10\sin(n\omega_0 t)\,dt=4\frac{1-\cos n\omega_0}{n\omega_0}$$

Example:
a periodic train of pulses

$$x(t)=\begin{cases}10 & 0<t<1\\ 0 & 1<t<5\end{cases}$$



$$T=5$$

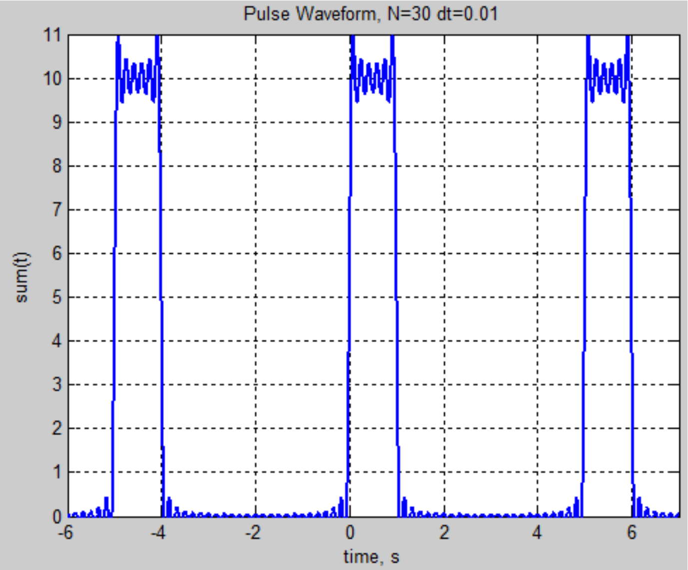
$$\omega_0=2\pi/5$$

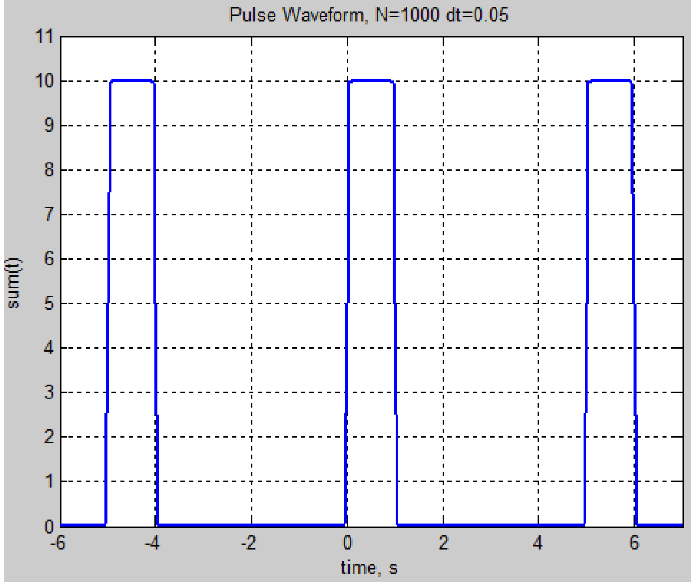
$$x(t)=a_0+\sum_{n=1}^{\infty}\left(a_n\cos n\omega_0 t+b_n\sin n\omega_0 t\right)=$$

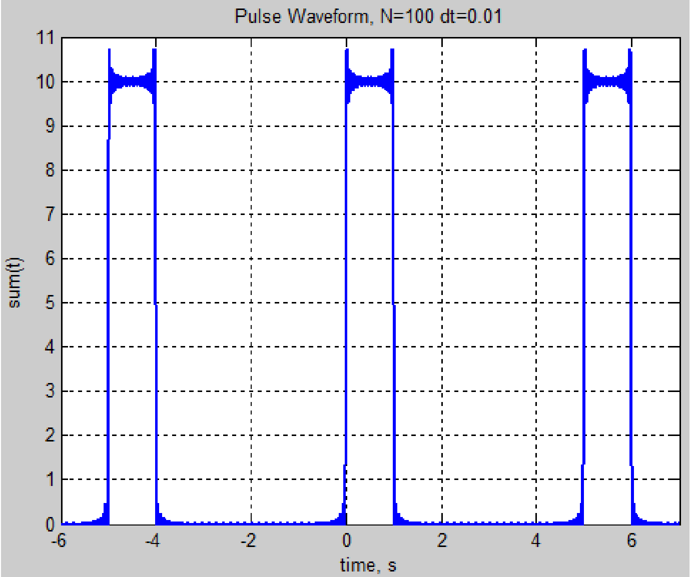
$$=2+\sum_{n=1}^{\infty}\left(4\frac{\sin n\omega_0}{n\omega_0}\cos n\omega_0 t+4\frac{1-\cos n\omega_0}{n\omega_0}\sin n\omega_0 t\right)$$

Example: a periodic train of pulses

$$x(t) = 2 + \sum_{n=1}^{\infty}\left(4\frac{\sin n\omega_0}{n\omega_0}\cos n\omega_0 t + 4\frac{1-\cos n\omega_0}{n\omega_0}\sin n\omega_0 t\right)$$

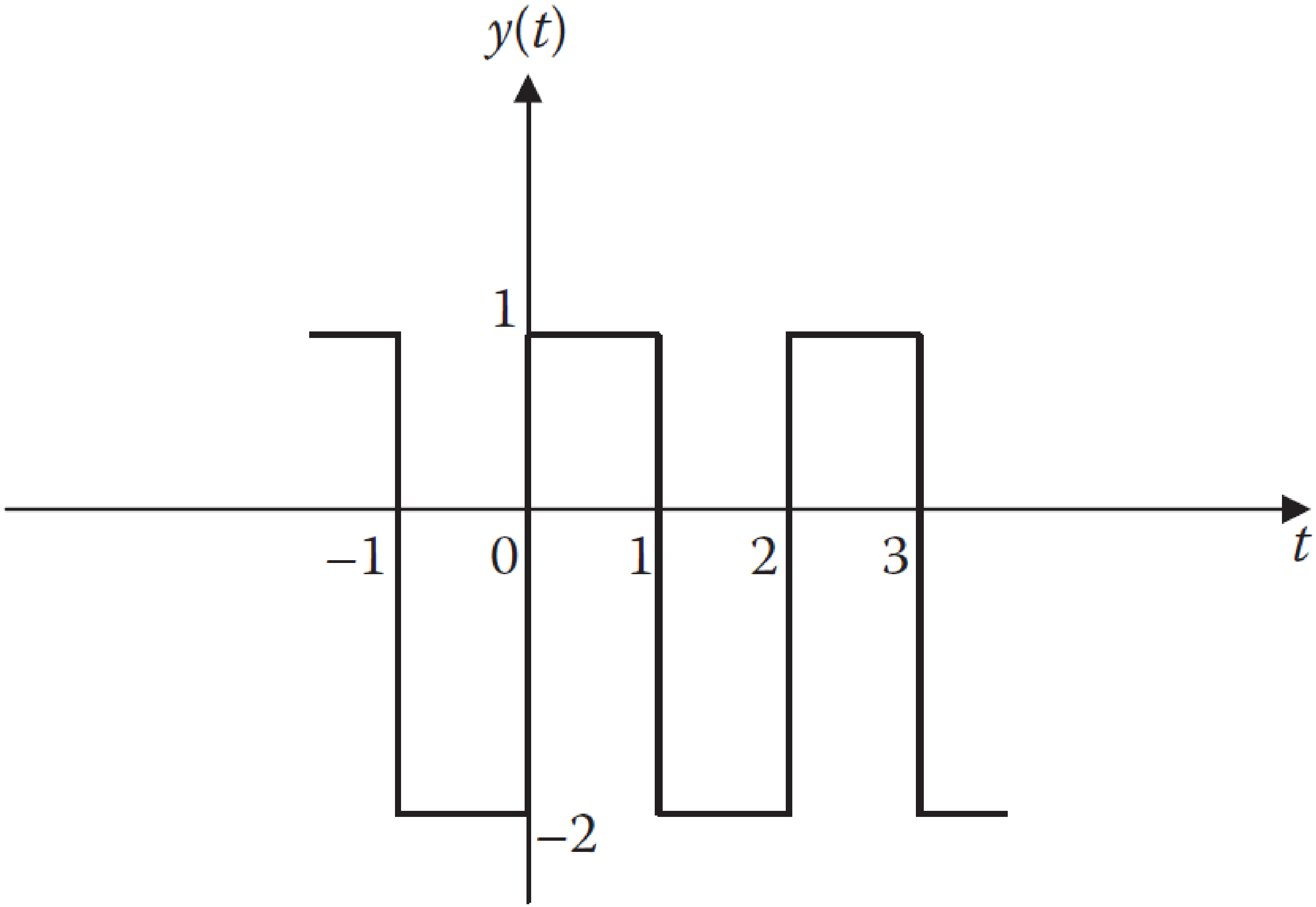






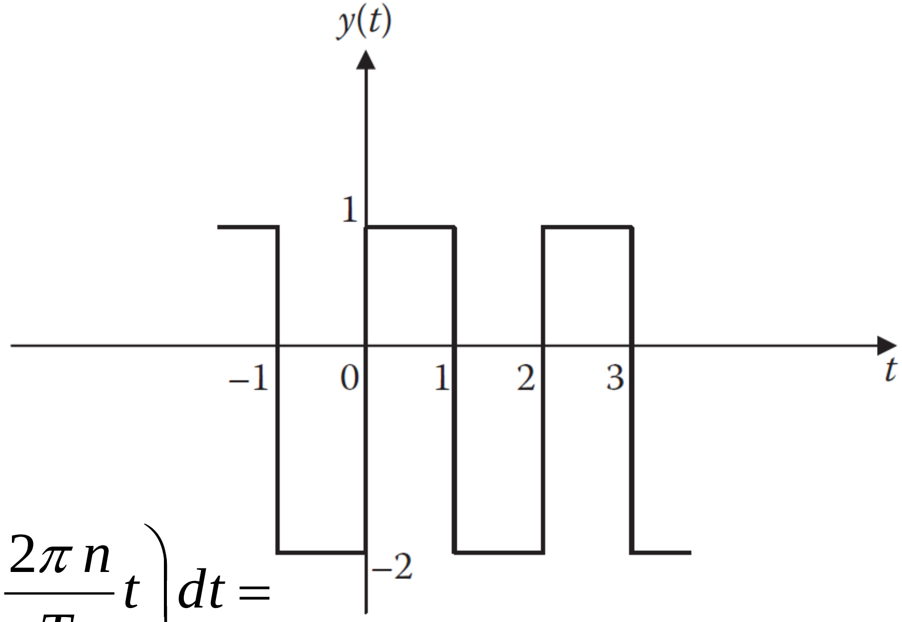
9% overshoot (or ringing) at a jump discontinuity: the so-called **Gibbs phenomenon**

Exercise 1. Obtain the Fourier series expansion for the waveform shown below



Exercise 1.

Obtain the Fourier series expansion for the waveform shown



$$a_0 = \frac{1}{T}\int_0^T y(t)\,dt = \frac{1}{2}\left(\int_0^1 dt - 2\int_1^2 dt\right) = -\frac{1}{2}$$

$$a_n = \frac{2}{T}\int_0^T y(t)\cos\left(\frac{2\pi n}{T}t\right)dt = \frac{1}{2}\int_0^2 y(t)\cos\left(\frac{2\pi n}{T}t\right)dt =$$

$$= \frac{2}{2}\left(\int_0^1 \cos(\pi nt)\,dt - 2\int_1^2 \cos(\pi nt)\,dt\right) = \frac{1}{2\pi n}\left(\sin(\pi nt)\Big|_0^1 - 2\sin(\pi nt)\Big|_1^2\right) = 0$$

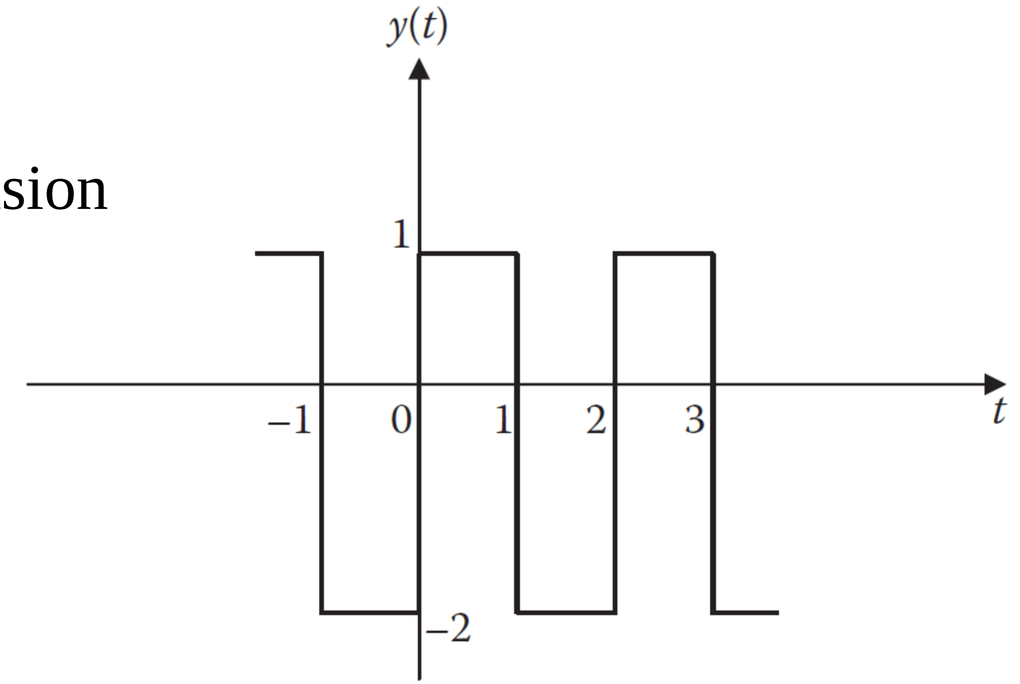
$$b_n = \frac{2}{T}\int_0^T y(t)\sin\left(\frac{2\pi n}{T}t\right)dt = \frac{1}{2}\int_0^2 y(t)\sin\left(\frac{2\pi n}{T}t\right)dt =$$

$$= \frac{2}{2}\left(-\int_0^1 \sin(\pi nt)\,dt + 2\int_1^2 \sin(\pi nt)\,dt\right) = \frac{2}{2\pi n}\left(-\cos(\pi nt)\Big|_0^1 + 2\cos(\pi nt)\Big|_1^2\right) =$$

$$= \frac{2}{2\pi n}\left(3 - 3(-1)^n\right)$$

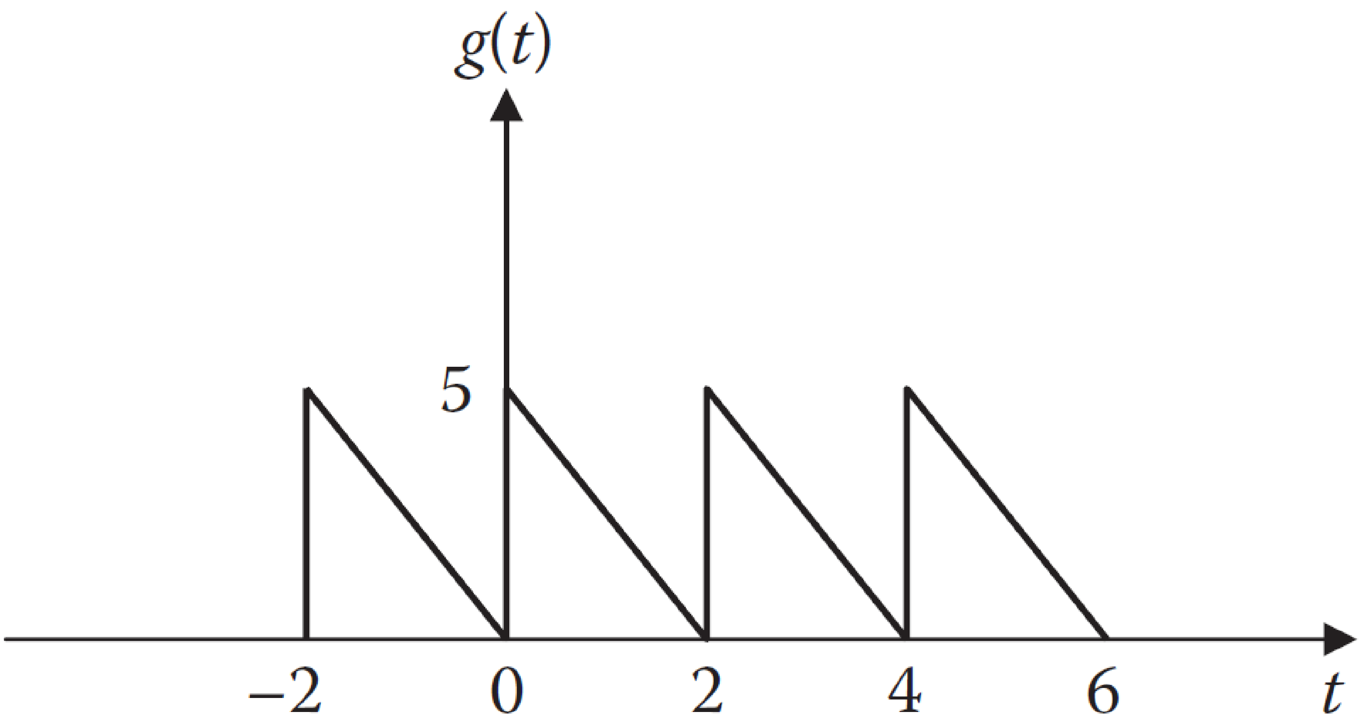
Exercise 1.

Obtain the Fourier series expansion for the waveform shown below



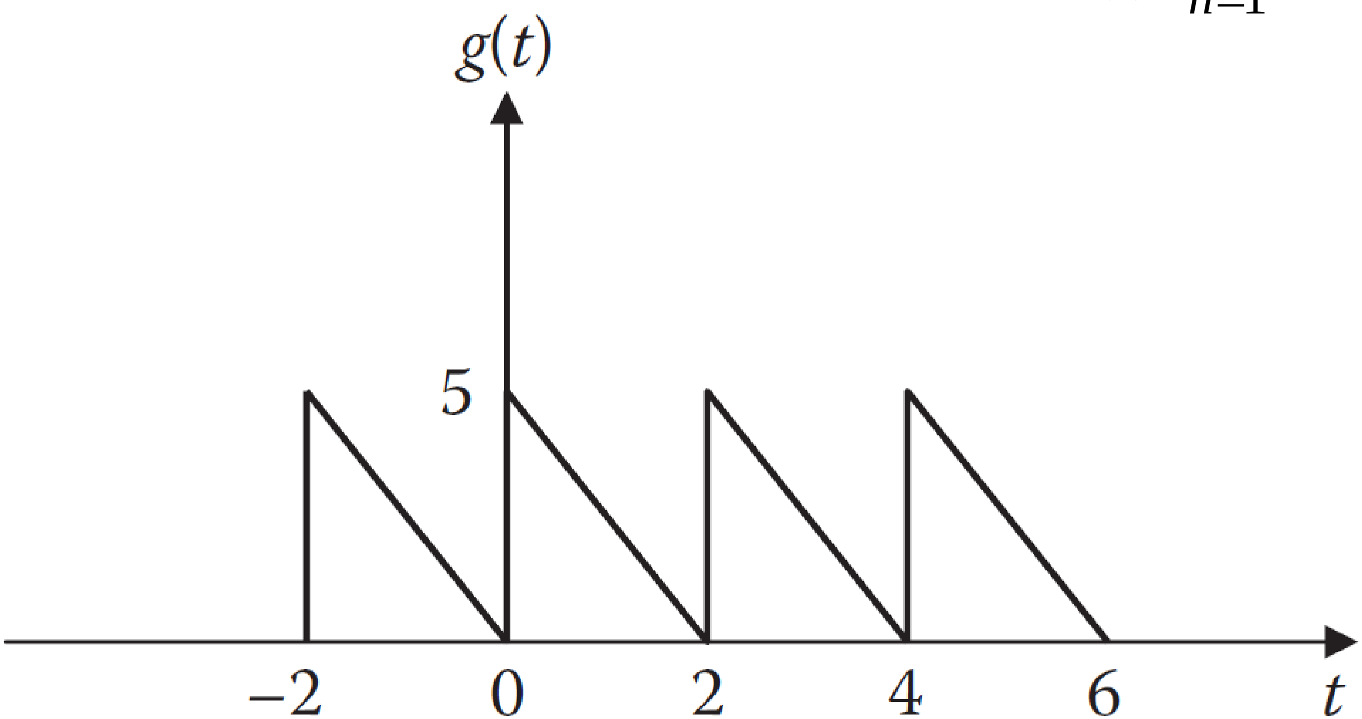
$$y(t) = -\frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{2\pi n}(3 - 3(-1)^n)\sin(n\pi t)$$

Exercise 2. Obtain the Fourier series expansion of the backward sawtooth waveform shown below



Exercise 2. Obtain the Fourier series expansion of the backward sawtooth waveform shown below

$$g(t) = 2.5 + \frac{5}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(n\pi t)$$



Complex exponential Fourier series

Let $x(t)$ be a periodic signal with fundamental period T

$$x(t+T) \equiv x(t)$$

Then we can expand $x(t)$ into the complex exponential Fourier series

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \qquad \omega_0 = 2\pi/T \qquad j = \sqrt{-1}}$$

$$c_n = \frac{1}{T}\int_{t_0}^{t_0+T} x(t) e^{-jn\omega_0 t} dt = \frac{1}{T}\int_0^T x(t) e^{-jn\omega_0 t} dt$$

Complex exponential Fourier series

$x(t)$ is a periodic signal with fundamental period T

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right)$$

$$e^{j\theta} = \cos\theta + j\sin\theta \qquad \cos\theta = \frac{1}{2}\left[e^{j\theta} + e^{-j\theta}\right] \qquad \sin\theta = \frac{1}{2j}\left[e^{j\theta} - e^{-j\theta}\right]$$

$$\cos n\omega_0 t = \frac{1}{2}\left[e^{jn\omega_0 t} + e^{-jn\omega_0 t}\right] \qquad \sin n\omega_0 t = \frac{1}{2j}\left[e^{jn\omega_0 t} - e^{-jn\omega_0 t}\right]$$

$$x(t) = a_0 + \frac{1}{2}\sum_{n=1}^{\infty} \left\{ a_n \left(e^{jn\omega_0 t} + e^{-jn\omega_0 t} \right) - jb_n \left(e^{jn\omega_0 t} - e^{-jn\omega_0 t} \right) \right\}$$

$$= a_0 + \frac{1}{2}\sum_{n=1}^{\infty} \left\{ (a_n - jb_n) e^{jn\omega_0 t} + (a_n + jb_n) e^{-jn\omega_0 t} \right\}$$

$$c_0 = a_0 \qquad c_n = \frac{1}{2}\left[a_n - jb_n\right] \qquad c_{-n} = c_n^* = \frac{1}{2}\left[a_n + jb_n\right]$$

Complex exponential Fourier series

$x(t)$ is a periodic signal with fundamental period T_0

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \qquad \omega_0 = 2\pi/T \qquad j = \sqrt{-1}$$

An example:

$$x(t) = 1 + \frac{1}{2}\cos 2\pi t + \cos 4\pi t + \frac{2}{3}\cos 6\pi t$$

$$= 1 + \frac{1}{4}\left(e^{j2\pi t} + e^{-j2\pi t}\right) + \frac{1}{2}\left(e^{j4\pi t} + e^{-j4\pi t}\right) + \frac{1}{3}\left(e^{j6\pi t} + e^{-j6\pi t}\right)$$

$$c_0 = 1, \quad c_1 = c_{-1} = 1/4, \quad c_2 = c_{-2} = 1/2, \quad c_3 = c_{-3} = 1/3$$

Complex exponential Fourier series: another example

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}}$$

Another example:

$$x(t) = 1 + \sin\omega_0 t + 2\cos\omega_0 t + \cos(2\omega_0 t + \pi/4)$$

$$x(t) = 1 + \frac{1}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right) + \left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$$

$$+ \frac{1}{2}\left(e^{j(2\omega_0 t + \pi/4)} + e^{-j(2\omega_0 t + \pi/4)}\right) = \ldots$$

Expansion w.r.t. orthogonal basis (a general idea)

$$\boldsymbol{x} = c_1\boldsymbol{e}_1 + c_2\boldsymbol{e}_2 + \ldots + c_n\boldsymbol{e}_n \mid \cdot \boldsymbol{e}_k$$

$$\boldsymbol{x} \cdot \boldsymbol{e}_k = c_k \boldsymbol{e}_k \cdot \boldsymbol{e}_k$$

$$c_k = \boldsymbol{x} \cdot \boldsymbol{e}_k \Big/ \left|\boldsymbol{e}_k\right|^2$$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}}$$

$$z = a + jb$$

$$z^* = a - jb$$

$$c_n = \int_0^T x(t) e^{-jn\omega_0 t}\, dt \Big/ \int_0^T e^{jn\omega_0 t} e^{-jn\omega_0 t} dt$$

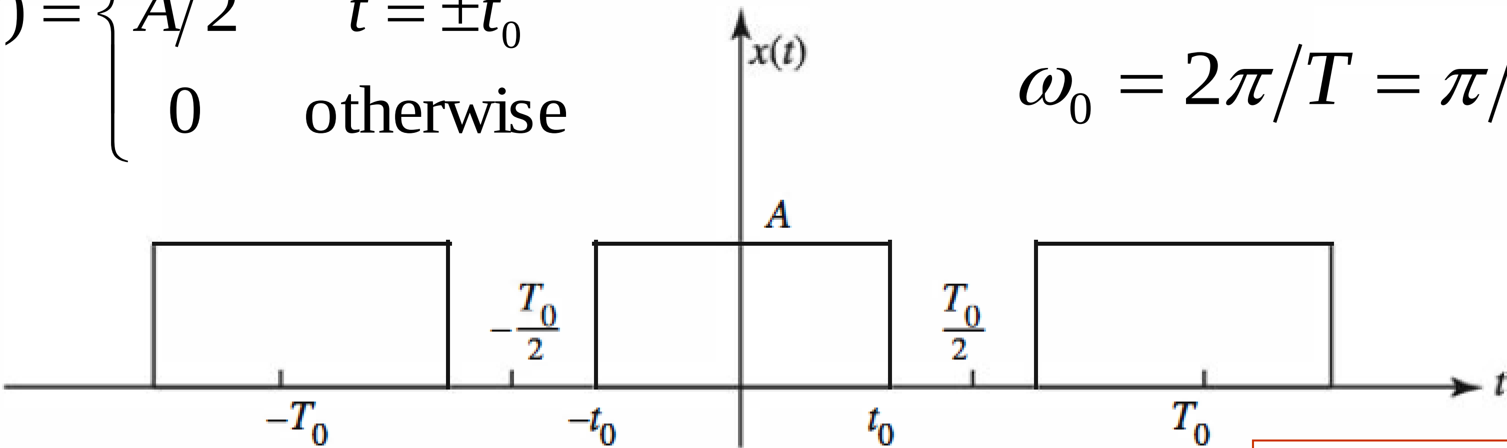
$$\boxed{= \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t}\, dt}$$

Fourier series of a rectangular signal train

$$x(t)=\begin{cases} A & |t|<t_0 \\ A/2 & t=\pm t_0 \\ 0 & \text{otherwise} \end{cases}$$

Let $A=1,\quad T_0=4,\quad t_0=1$

$$\omega_0 = 2\pi/T = \pi/2$$



$$c_0 = 1/2$$

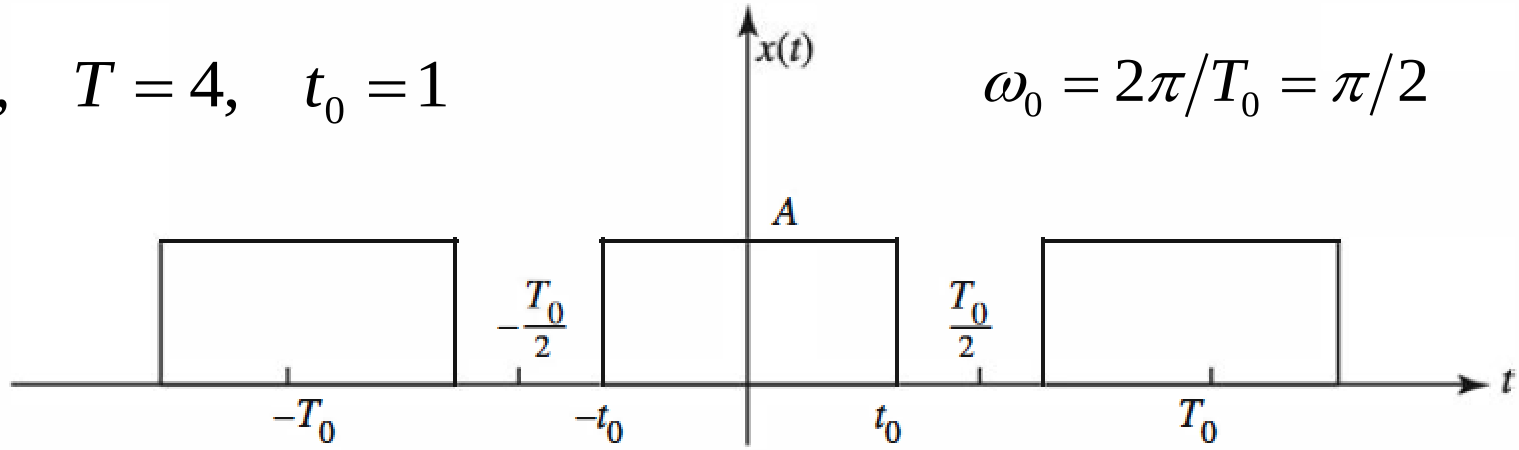
$$\boxed{\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}}$$

$$c_n = \frac{1}{T}\int_{-T_0/2}^{T_0/2} x(t)e^{-jn\omega_0 t}dt = \frac{1}{4}\int_{-t_0}^{t_0} e^{-j\pi nt/2}dt = \frac{1}{4}\int_{-1}^{1} e^{-j\pi nt/2}dt$$

$$= \frac{1}{-2j\pi n}\left[e^{-j\pi n/2} - e^{j\pi n/2}\right] = \frac{1}{\pi n}\sin(\pi n/2) = \frac{1}{2}\operatorname{sinc}\left(\frac{n}{2}\right)$$

Fourier series of a rectangular signal train

$A = 1, \quad T = 4, \quad t_0 = 1$ $\qquad\qquad$ $\omega_0 = 2\pi / T_0 = \pi / 2$



$$c_n = \frac{1}{2}\,\mathrm{sinc}\left(\frac{n}{2}\right)$$

$$\boxed{\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}}$$

$$x(t) = -\frac{1}{2} + \sum_{-\infty}^{\infty} \frac{1}{2}\, sinc\left(\frac{n}{2}\right) e^{j2\pi nt/4}$$

Frequency spectra

$$x(t)=\sum_{n=-\infty}^{\infty}c_n e^{jn\omega_0 t}$$

Fourier coefficients: $c_n=|c_n|e^{j\theta_n}$

$|c_n|$ is the amplitude and θ_n is the phase angle of c_n

Amplitude spectrum: a plot of $|c_n|$ versus the angular frequency $\omega=2\pi f$

Phase spectrum: a plot of θ_n versus ω

Example

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \qquad \omega_0 = 2\pi/T$$

$$\boxed{x(t) = t \quad -1 < t < 1, \quad x(t+2) = x(t)}$$

$$T = 2, \quad \omega_0 = 2\pi/T = \pi$$

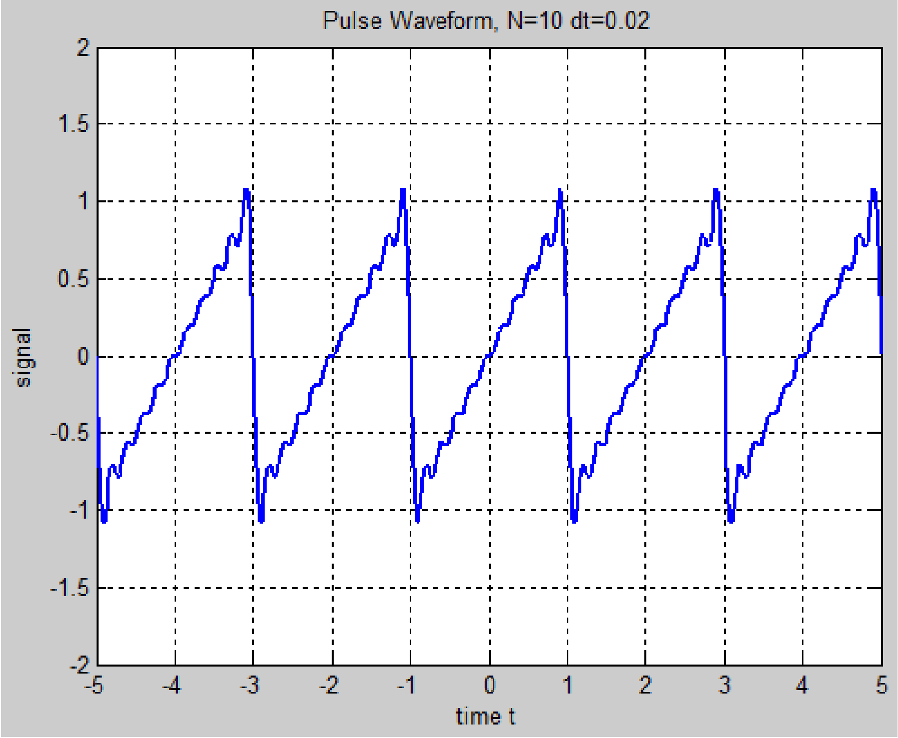
$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\omega_0 nt} dt = \frac{1}{2}\int_{-1}^{1} t\, e^{-j\pi nt} dt = \begin{cases} j(-1)^n/(n\pi) & n \neq 0 \\ 0 & n = 0 \end{cases}$$

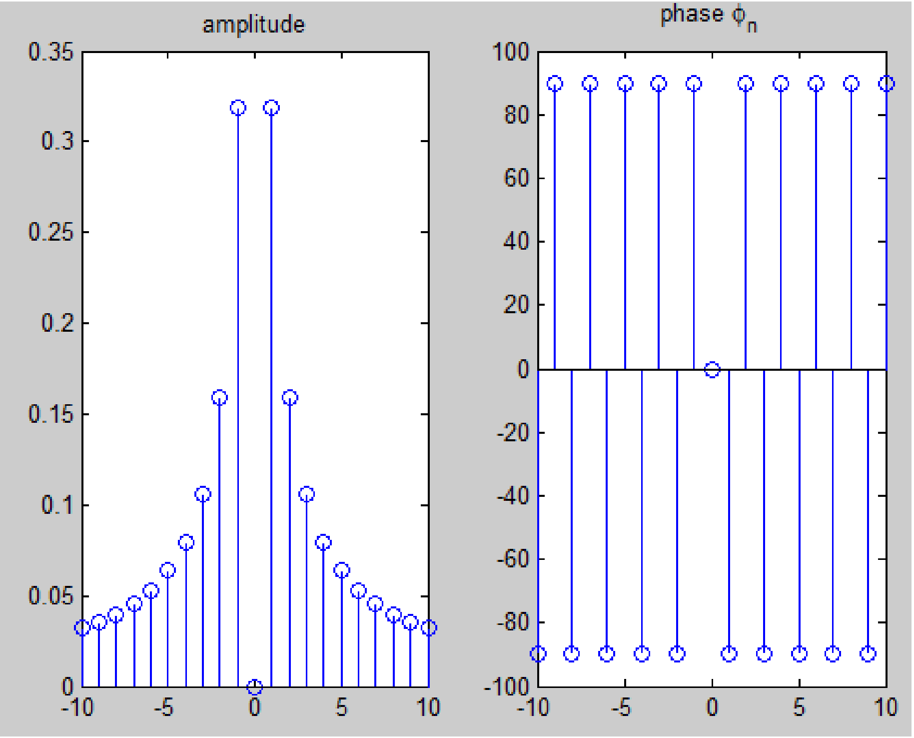
$$x(t) = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} (-1)^n \frac{j}{n\pi} e^{j\pi nt}$$

Example

$$x(t) = t \quad -1 < t < 1, \quad x(t+2n) = x(t) \qquad T = 2, \quad \omega_0 = 2\pi/T = \pi$$

$$x(t) = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} (-1)^n \frac{j}{n\pi} e^{j\pi n t}$$

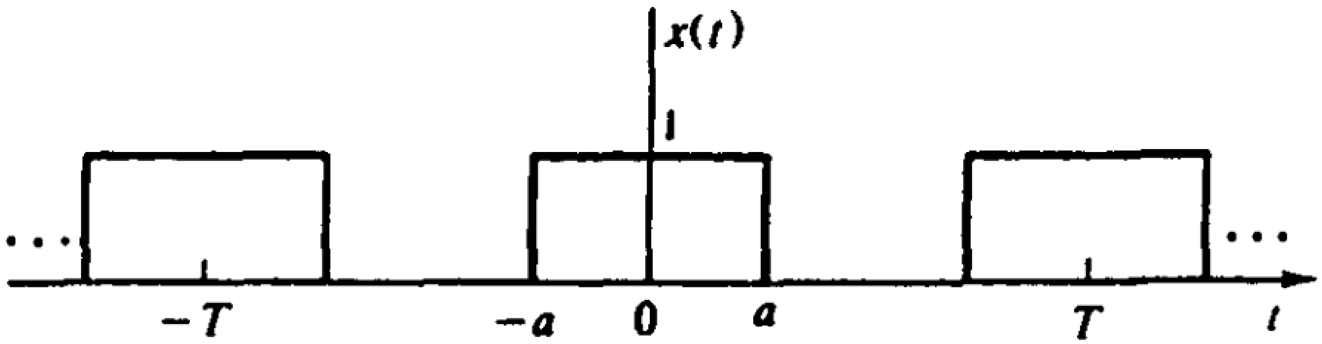




amplitude spectrum phase spectrum

Another example

Determine the complex Fourier series for



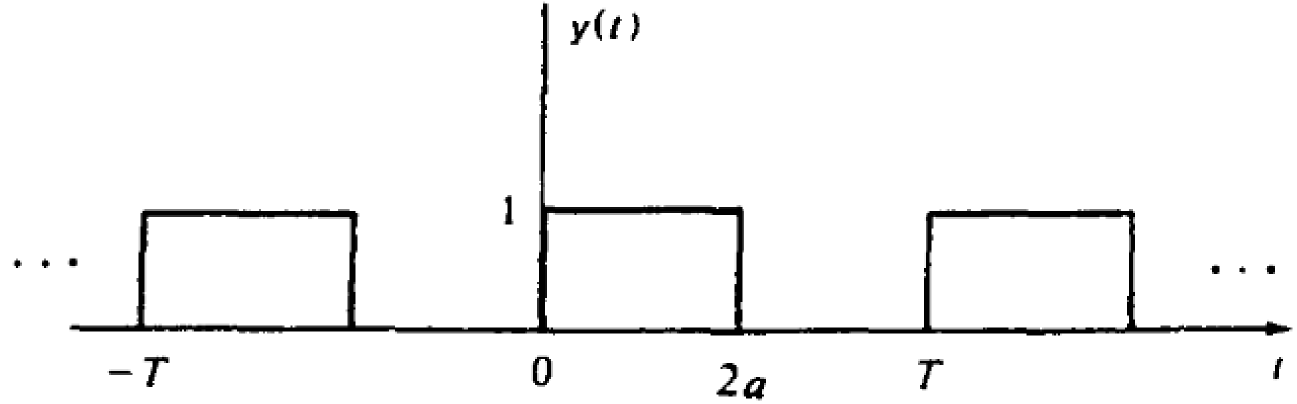
Solution. $$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \qquad \omega_0 = 2\pi/T_0$$

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-jn\omega_0 t} dt = \frac{1}{T}\int_{-a}^{a} e^{-jn\omega_0 t} dt \qquad c_0 = \frac{1}{T}\int_{-a}^{a} dt = \frac{2a}{T}$$

$$= \frac{1}{-jn\omega_0 T}\left[e^{-jn\omega_0 a} - e^{jn\omega_0 a}\right] = \frac{\sin n\omega_0 a}{n\pi}$$

One more example

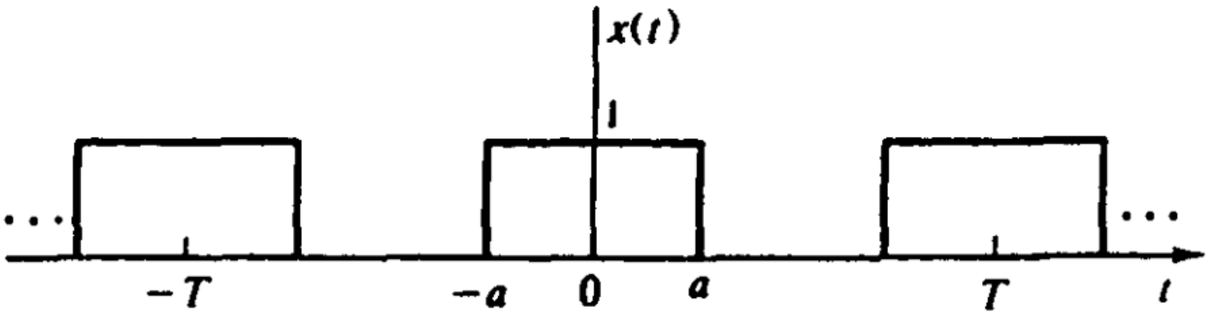
Determine the complex Fourier series for



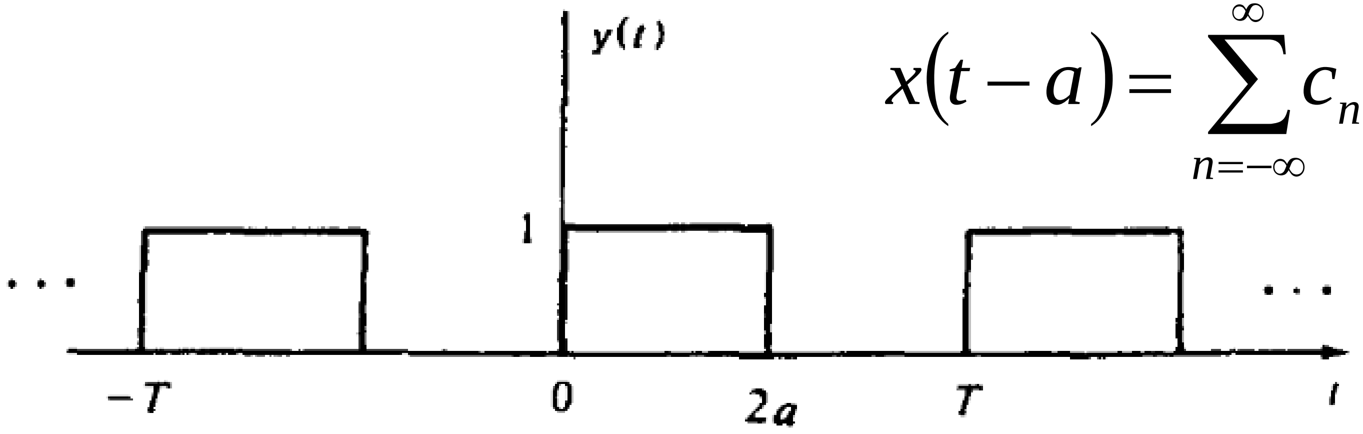
$$y(t) = \sum_{n=-\infty}^{\infty} d_n e^{jn\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T}$$

One more example

We know that $y(t) = x(t-a)$ $\qquad x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$



$$x(t-a) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 (t-a)}$$



$$d_n = c_n e^{-jn\omega_0 a} = \frac{\sin n\omega_0 a}{n\pi} e^{-jn\omega_0 a}$$

Exercise

Find the complex Fourier series for the signal:

$$x(t) = \cos \omega_0 t + \sin^2 \omega_0 t$$

Exercise

Find the complex Fourier series for the signal:

$$x(t) = \cos\omega_0 t + \sin^2 \omega_0 t$$

$$x(t) = \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) + \left[\frac{1}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)\right]^2$$

$$= \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t} - \frac{1}{4}\left(e^{j2\omega_0 t} - 2 + e^{-j2\omega_0 t}\right)$$

$$c_0 = \frac{1}{2}, \quad c_1 = \frac{1}{2}, \quad c_{-1} = \frac{1}{2}, \quad c_2 = c_{-2} = -\frac{1}{4}$$

Average power of a periodic signal. Parseval's theorem.

Average power (power content) of a periodic signal $x(t)$:

$$P = \frac{1}{T}\int_{0}^{T} \left|x(t)\right|^2 dt = \frac{1}{T}\int_{-T/2}^{T/2} \left|x(t)\right|^2 dt$$

Parseval's theorem

$$P = \frac{1}{T}\int_{-T/2}^{T/2} \left|x(t)\right|^2 dt = \sum_{k=-\infty}^{\infty} \left|c_k\right|^2$$

Parseval's theorem

$$\boxed{P = \frac{1}{T}\int_{-T/2}^{T/2} \left|x(t)\right|^2 dt = \sum_{k=-\infty}^{\infty} \left|c_k\right|^2}$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \qquad y(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t}$$

$$z = a + jb, \quad z^* = a - jb$$

$$\int_0^T x(t)\, y^*(t)\, dt = \int_0^T \left(\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}\right)\left(\sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t}\right)^* dt$$

Parseval's theorem

$$\boxed{P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt = \sum_{k=-\infty}^{\infty} |c_k|^2}$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \qquad y(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t}$$

$$z = a + jb, \quad z^* = a - jb$$

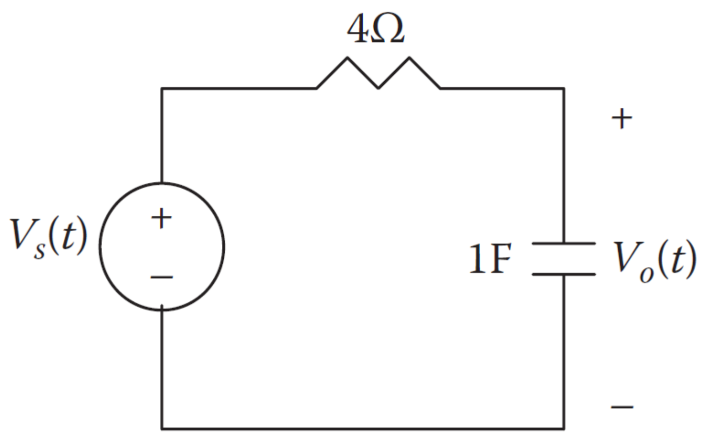
$$\int_0^T x(t)y^*(t)\,dt = \int_0^T \left(\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}\right)\left(\sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t}\right)^* dt$$

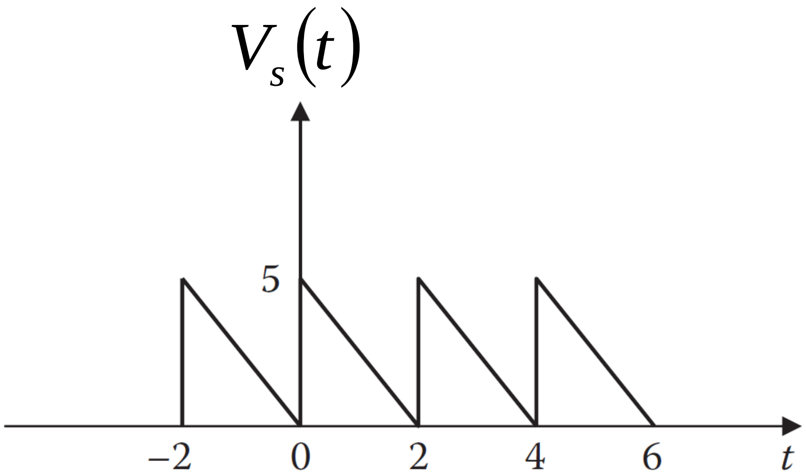
$$= \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} c_n d_k^* \int_0^T e^{j(n-k)\omega_0 t} dt \qquad \int_0^T e^{j(n-k)\omega_0 t} dt = \begin{cases} T & n = k \\ 0 & n \neq k \end{cases}$$

$$= T\sum_{n=-\infty}^{\infty} c_n d_n^*$$

$$\boxed{\int_0^T |x(t)|^2\, dt = T\sum_{n=-\infty}^{\infty} c_n c_n^* = T\sum_{n=-\infty}^{\infty} |c_n|^2}$$

Applications: Circuit Analysis





Phasors $\boldsymbol{V}_0$ and $\boldsymbol{V}_s$

$$\boldsymbol{V}_0 = \frac{1/(j\omega C)}{R+1/(j\omega C)}\boldsymbol{V}_s$$

$$= \frac{1}{1+j\omega RC}\boldsymbol{V}_s$$

$$V_s(t) = 5-2.5t \quad 0<t<2$$

$$T=2,\ \omega_0 = 2\pi/T = \pi$$

$$V_s(t) = 2.5 + \frac{5}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(n\pi t)$$