Signature and Log-signature for the Study of Empirical Distributions Generated with GANs.

J. de Curtò y DíAz.
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https://arxiv.org/abs/2203.03226

https://github.com/decurtoydiaz/signatures

Acknowledgements

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Key contributions:

- We propose two frameworks to assess GAN convergence based on analytical measures, one based on statistical tests and the other leveraging tools from harmonic analysis (a recently proposed generalization of Fourier).

Since its first appearance in 2014, measures to assess GAN convergence have been mainly based on empirical evaluations (MS-SSIM and FID).

- We also propose a PCA-Adaptive t-SNE for image visualization on several domains.

Goodfellow, I., Pouget-Abadie, J., Mirza, M., Xu, B., Warde-Farley, D., Ozair, S., Courville, A., Bengio, Y.: Generative adversarial networks. NIPS **27** (2014)

van der Maaten, L., Hinton, G.: Visualizing data using t-SNE. JMLR **9**(86), 2579–2605 (2008)

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Visualization pipeline.

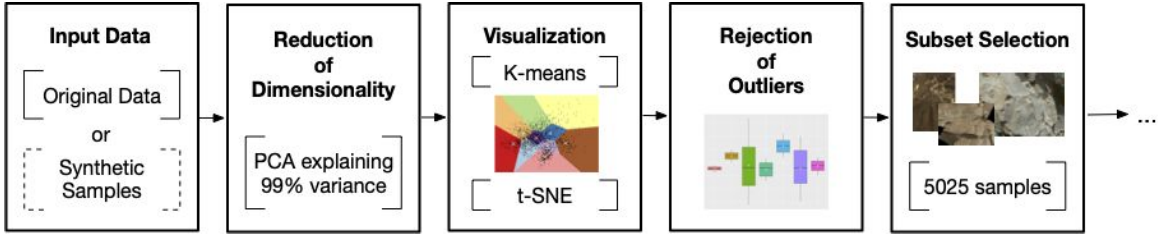

Fig. 2: K-means Clustering (left) and t-SNE (right) on images from NASA Curiosity (upper figures) and Perseverance (lower figures).

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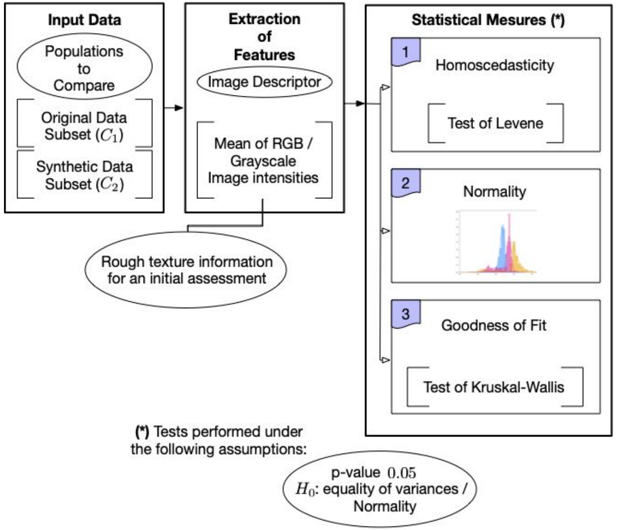

Table 4: Evaluation of the statistical test measures of homoscedasticity (T1), normality (T2) and goodness of fit (T3) on AFHQ and MetFaces using state-of-the-art pretrained models of Stylegan2-ada [21] and Stylegan3-ada [22] and NASA Perseverance.

Model	Dataset		T1	T2	T3
Stylegan2-ada	NASA Perseverance		✗	✓	✗
	AFHQ	Cat	✗	✗	✓
		Dog	✗	✓	✗
		Wild	✗	✗	✓
	MetFaces		✗	✗	✗
r-Stylegan3-ada			✗	✗	✗
t-Stylegan3-ada			✗	✗	✗

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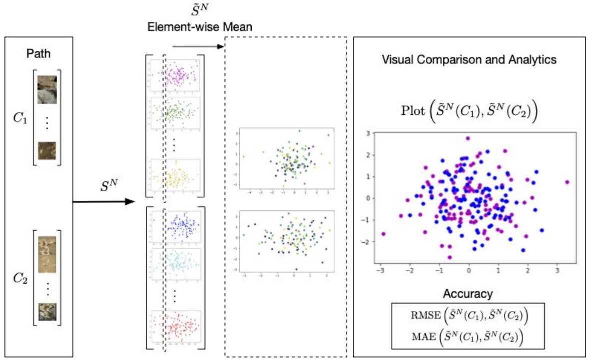
Following [2], the truncated signature of order N of the path $\mathbf{x}$ is defined as a collection of coordinate iterated integrals

$$\mathrm{S}^N(\mathbf{x}) = \left(\left(\int \cdots \int_{0<t_1<\cdots<t_a<1} \prod_{c=1}^{a} \frac{\mathrm{d} f_{z_c}}{\mathrm{d}t}(t_c)\mathrm{d}t_1 \cdots \mathrm{d}t_a \right)_{1\le z_1,\ldots,z_a \le d} \right)_{1\le a\le N} . \quad (1)$$



Bonnier, P., Kidger, P., Arribas, I.P., Salvi, C., Lyons, T.: Deep signature transforms. NIPS (2019)

Lyons, T.: Rough paths, signatures and the modelling of functions on streams. Proceedings of the International Congress of Mathematicians (2014)

Model	Dataset		RMSE $\tilde{S}^3$	MAE $\tilde{S}^3$	RMSE log $\tilde{S}^3$	MAE log $\tilde{S}^3$
Stylegan2-ada	AFHQ	Cat	61450	45968	29201	22297
		Dog	38861	30441	31686	24612
		Wild	33306	25578	26622	20359
	MetFaces		33247	23428	25685	18071
r-Stylegan3-ada			34977	22799	24707	16539
t-Stylegan3-ada			**30894**	**19872**	**21560**	**13761**

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Definition 4. *Given n components of the element-wise mean of the signatures $\{y^{(c)}\}_{c=1}^{n} \subseteq T(\mathbb{R}^d)$ from the model chosen as a source of synthetic samples and the same number of components of the element-wise mean of the signatures $\{x^{(c)}\}_{c=1}^{n} \subseteq T(\mathbb{R}^d)$ from the original distribution, then we define the Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE) by*

$$\mathrm{RMSE}\left(\left\{x^{(c)}\right\}_{c=1}^{n}, \left\{y^{(c)}\right\}_{c=1}^{n}\right) = \sqrt{\frac{1}{n}\sum_{c=1}^{n}\left(y^{(c)} - x^{(c)}\right)^2},$$

and

$$\mathrm{MAE}\left(\left\{x^{(c)}\right\}_{c=1}^{n}, \left\{y^{(c)}\right\}_{c=1}^{n}\right) = \frac{1}{n}\sum_{c=1}^{n}|y^{(c)} - x^{(c)}|.$$

The case for log-signature is analogous.

J. de Curtò, I. de Zarzà, and H. Yan, "Signature and Log-signature for the Study of Empirical Distributions Generated with GANs," *arXiv:2203.03226*, 2022.

Fig. 8: Spectrum of the element-wise mean of the signatures (left) and log-signatures (right) of order 3 and size 64×64 of original ('o') against synthetic ('x') samples.

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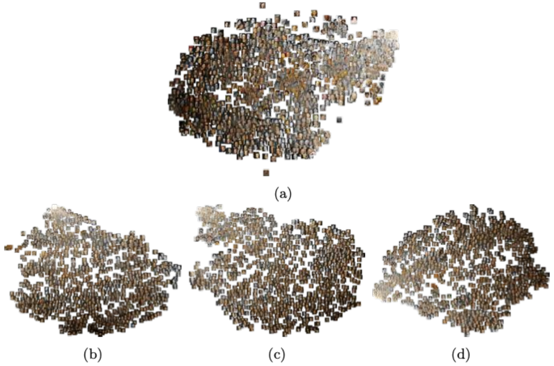

Fig. 14: Visualization of PCA Adaptive t-SNE on original (top) versus synthetic (bottom) samples of MetFaces using Stylegan2-ada (b), *r*-Stylegan3-ada (c) and *t*-Stylegan3-ada (d).

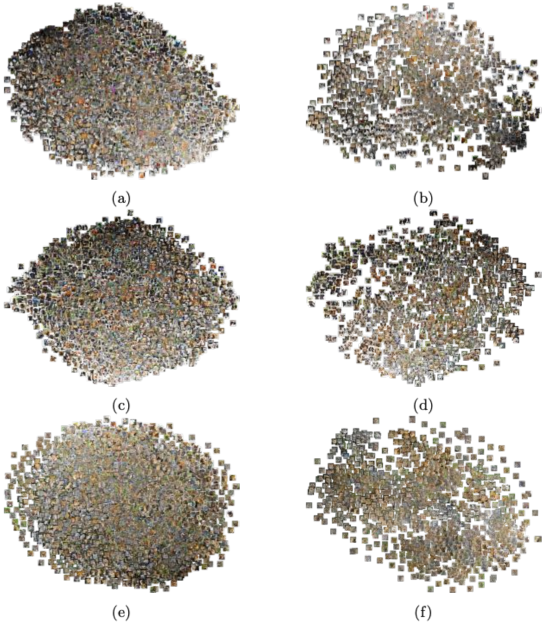

Fig. 13: Visualization of PCA Adaptive t-SNE on original (left) versus synthetic (right) samples of AFHQ Cat (a,b), Dog (c,d) and Wild (e,f) using Stylegan2-ada.

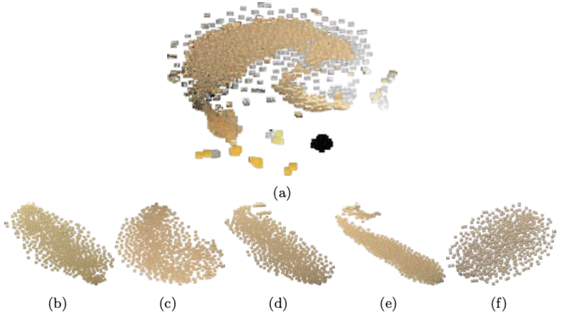

Fig. 15: Visualization of PCA Adaptive t-SNE on original (top) versus synthetic (bottom) samples of NASA Perseverance using Stylegan2-ada across several epoch iterations: 193 (b), 371 (c), 596 (d), 798 (e) and 983 (d).

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