COTG-134. Grupo de Redes de Computadores (GRC).

J. de Curtò y DíAz.

I. de Zarzà y Cubero.

26-05-2022.

c@decurto.be
z@dezarza.be

Signature and Log-signature for the Study of Empirical Distributions Generated with GANs.

De Curtò y DíAz.
De Zarzà y Cubero.

Signature and Log-signature for the Study of Empirical Distributions Generated with GANs.

https://arxiv.org/abs/2203.03226

https://github.com/decurtoydiaz/signatures

Learning with Signatures. On this manuscript we extend the metrics to be used for classification tasks.

https://arxiv.org/abs/2204.07953

https://github.com/decurtoydiaz/learning_with_signatures/

c@decurto.be
z@dezarza.be

On the Applicability of the Hadamard as an Input Modulator for Problems of Classification.

De Curtò y DíAz.
De Zarzà y Cubero.

C++ library that includes a complete learning framework.

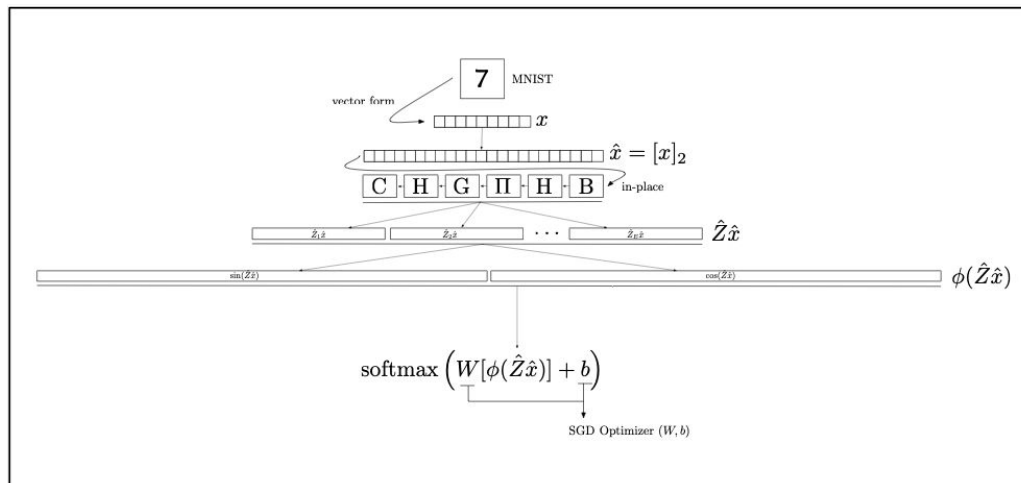https://doi.org/10.24433/CO.3851581.v1

https://github.com/curto2/mckernel
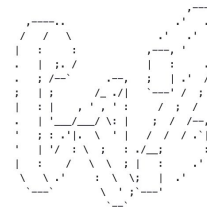
Figure 1: **Diagram of McKernel**. We visually describe $\text{softmax}(W\tilde{x}+b)$ where $\tilde{x} = \text{mckernel}(x)$. The original image is padded in form of long vector to the nearest power of 2, mapping $\hat{Z}$ is applied in-place. Calibration C defines the choice of Kernel. The tensor is expanded by the number of Kernel Expansions E building a network with high compositionality. Finally, use real feature map ϕ, Equation 2. SGD Optimizer finds appropriate weights W and bias b. Compute $\hat{Z}$ on-the-fly keeping same seed both for training and testing.

c@decurto.be
z@dezarza.be

On the Applicability of the Hadamard as an Input Modulator for Problems of Classification.
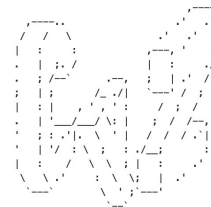
De Curtò y DíAz.
De Zarzà y Cubero.

Highlights

● C++ fast open-source Hadamard that works for any input size. Widely deployed in Signal Processing, Communications and Compressed Sensing.

● Approximate kernel expansions in log-linear time.

● Useful to foster new DL architectures with better human-induced/mathematical priors.

● DL research framework that offers multiple open questions. Domains that could be further explored are: end-to-end training, self-supervised learning, meta-learning, integration with evolution strategies, NAS reducing substantially the search space and many others.

c@decurto.be
z@dezarza.be

Signature and Log-signature for the Study of Empirical Distributions Generated with GANs.
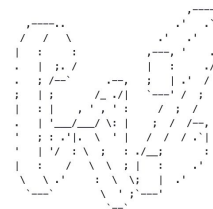
De Curtò y DíAz.
De Zarzà y Cubero.

Key contributions:

- We propose two frameworks to assess GAN convergence based on analytical measures, one based on statistical tests and the other leveraging tools from harmonic analysis (a recently proposed generalization of Fourier).

- The metrics are efficient and effective and can be computed in seconds.

Since its first appearance in 2014, measures to assess GAN convergence have been mainly based on empirical evaluations (MS-SSIM and FID).
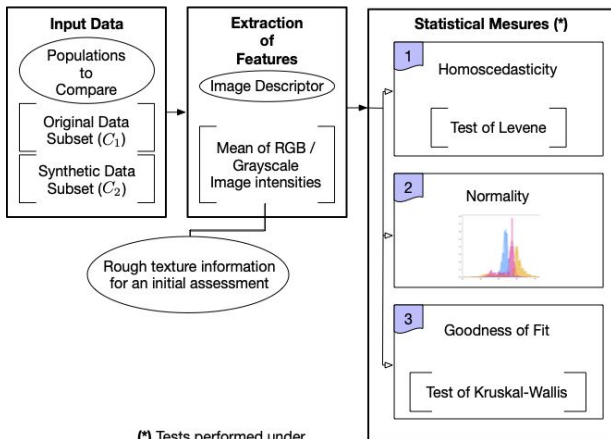
Goodfellow, I., Pouget-Abadie, J., Mirza, M., Xu, B., Warde-Farley, D., Ozair, S., Courville, A., Bengio, Y.: Generative adversarial networks. NIPS **27** (2014)

c@decurto.be
z@dezarza.be

Signature and Log-signature for the Study of Empirical Distributions Generated with GANs.

De Curtò y DíAz.
De Zarzà y Cubero.

Input Data
- Populations to Compare
- Original Data Subset (C_1)
- Synthetic Data Subset (C_2)

Extraction of Features
- Image Descriptor
- Mean of RGB / Grayscale Image intensities
- Rough texture information for an initial assessment

Statistical Mesures (*)
1. Homoscedasticity — Test of Levene
2. Normality
3. Goodness of Fit — Test of Kruskal-Wallis

(*) Tests performed under the following assumptions:

p-value 0.05
H_0: equality of variances / Normality

| Test	Population	Result	Interpretation
1	C_1 and C_2	✓	(a)
		✗	(b)
2	C_2	✓	(c)
		✗	(d)
3	C_1 and C_2	✓	(e)
		✗	(f)
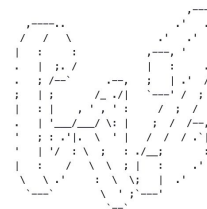
Description and interpretation of statistical measures are provided in Table I:

(a) Necessary condition but not sufficient to assert that both populations originate from the same distribution.

(b) There is not enough statistical evidence to attest both populations samples originate from the same distribution.

(c) With high probability the synthetic distribution generated is still close enough to the initial distribution of noise from the GAN architecture. The samples may not show enough fidelity, and there is probably bad generalization behavior.

(d) The synthetic distribution is far from the initial distribution of noise and has deviated from the original Normal, and may be close to the target distribution.

(e) If (a) then there is enough statistical evidence to confirm that both populations originate from the same distribution given this image descriptor. If (a) is not fulfilled, then we can only ascertain that the synthetic population is a good approximation.

(f) There is not enough statistical evidence to attest both populations are from the same distribution.

c@decurto.be
z@dezarza.be

Signature and Log-signature for the Study of Empirical Distributions Generated with GANs.
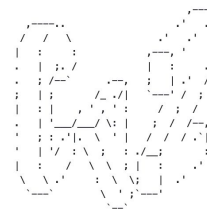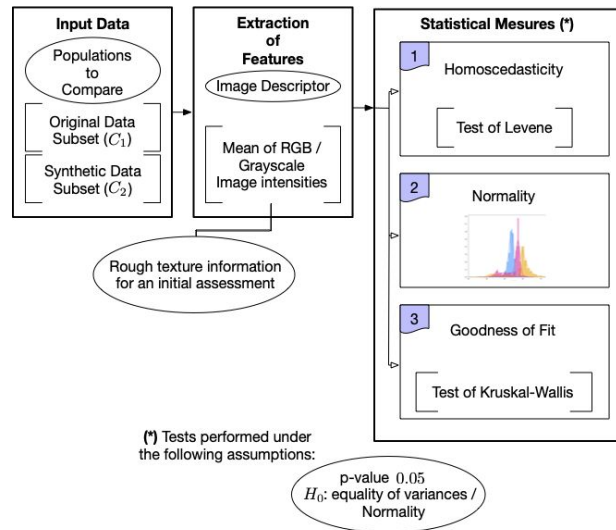
De Curtò y DíAz.
De Zarzà y Cubero.

Table 4: Evaluation of the statistical test measures of homoscedasticity (T1), normality (T2) and goodness of fit (T3) on AFHQ and MetFaces using state-of-the-art pretrained models of Stylegan2-ada [21] and Stylegan3-ada [22] and NASA Perseverance.

Model	Dataset		T1	T2	T3
	NASA Perseverance		✗	✓	✗
Stylegan2-ada	AFHQ	Cat	✗	✗	✓
		Dog	✗	✓	✗
		Wild	✗	✗	✓
r-Stylegan3-ada	MetFaces		✗	✗	✗
t-Stylegan3-ada			✗	✗	✗

c@decurto.be
z@dezarza.be

7

Signature and Log-signature for the Study of Empirical Distributions Generated with GANs.

De Curtò y DíAz.
De Zarzà y Cubero.

Following [2], the truncated signature of order N of the path $\mathbf{x}$ is defined as a collection of coordinate iterated integrals

$$S^N(\mathbf{x}) = \left(\left(\int\limits_{0<t_1<\cdots<t_a<1} \cdots \int \prod_{c=1}^{a} \frac{df_{z_c}}{dt}(t_c)dt_1 \cdots dt_a \right)_{1\leq z_1,\ldots,z_a \leq d} \right)_{1\leq a \leq N}. \quad (1)$$

Bonnier, P., Kidger, P., Arribas, I.P., Salvi, C., Lyons, T.: Deep signature transforms. NIPS (2019)

Lyons, T.: Rough paths, signatures and the modelling of functions on streams. Proceedings of the International Congress of Mathematicians (2014)
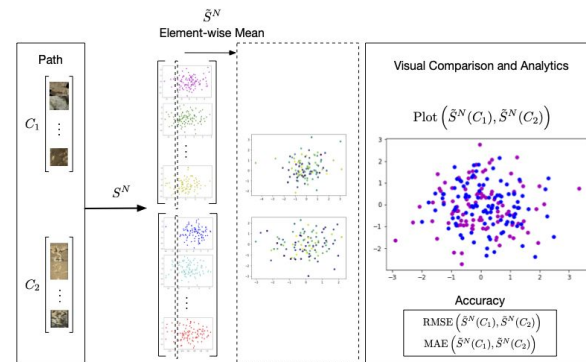
Model	Dataset		RMSE $\tilde{S}^3$	MAE $\tilde{S}^3$	RMSE log $\tilde{S}^3$	MAE log $\tilde{S}^3$
Stylegan2-ada	AFHQ	Cat	61450	45968	29201	22297
		Dog	38861	30441	31686	24612
		Wild	33306	25578	26622	20359
r-Stylegan3-ada	MetFaces		33247	23428	25685	18071
			34977	22799	24707	16539
t-Stylegan3-ada			30894	19872	21560	13761
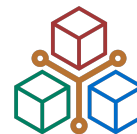
c@decurto.be
z@dezarza.be

Signature and Log-signature for the Study of Empirical Distributions Generated with GANs.

De Curtò y DíAz.
De Zarzà y Cubero.

Definition 4. *Given n components of the element-wise mean of the signatures $\{y^{(c)}\}_{c=1}^n \subseteq T(\mathbb{R}^d)$ from the model chosen as a source of synthetic samples and the same number of components of the element-wise mean of the signatures $\{x^{(c)}\}_{c=1}^n \subseteq T(\mathbb{R}^d)$ from the original distribution, then we define the Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE) by*

$$\text{RMSE}\left(\left\{x^{(c)}\right\}_{c=1}^n, \left\{y^{(c)}\right\}_{c=1}^n\right) = \sqrt{\frac{1}{n}\sum_{c=1}^n \left(y^{(c)} - x^{(c)}\right)^2},$$

and

$$\text{MAE}\left(\left\{x^{(c)}\right\}_{c=1}^n, \left\{y^{(c)}\right\}_{c=1}^n\right) = \frac{1}{n}\sum_{c=1}^n \left|y^{(c)} - x^{(c)}\right|.$$

The case for log-signature is analogous.

Fig. 8: Spectrum of the element-wise mean of the signatures (left) and log-signatures (right) of order 3 and size 64×64 of original ('o') against synthetic ('x') samples.

Model	FID	RMSE $\tilde{S}^3$
Stylegan2-ada	15.22	33247
r-Stylegan3-ada	15.33	34977
t-Stylegan3-ada	**15.11**	**30894**

c@decurto.be
z@dezarza.be

9

Signature and Log-signature for the Study of Empirical Distributions Generated with GANs.

De Curtò y DíAz.
De Zarzà y Cubero.

Fig. 14: Visualization of PCA Adaptive t-SNE on original (top) versus synthetic (bottom) samples of MetFaces using Stylegan2-ada (b), r-Stylegan3-ada (c) and t-Stylegan3-ada (d).
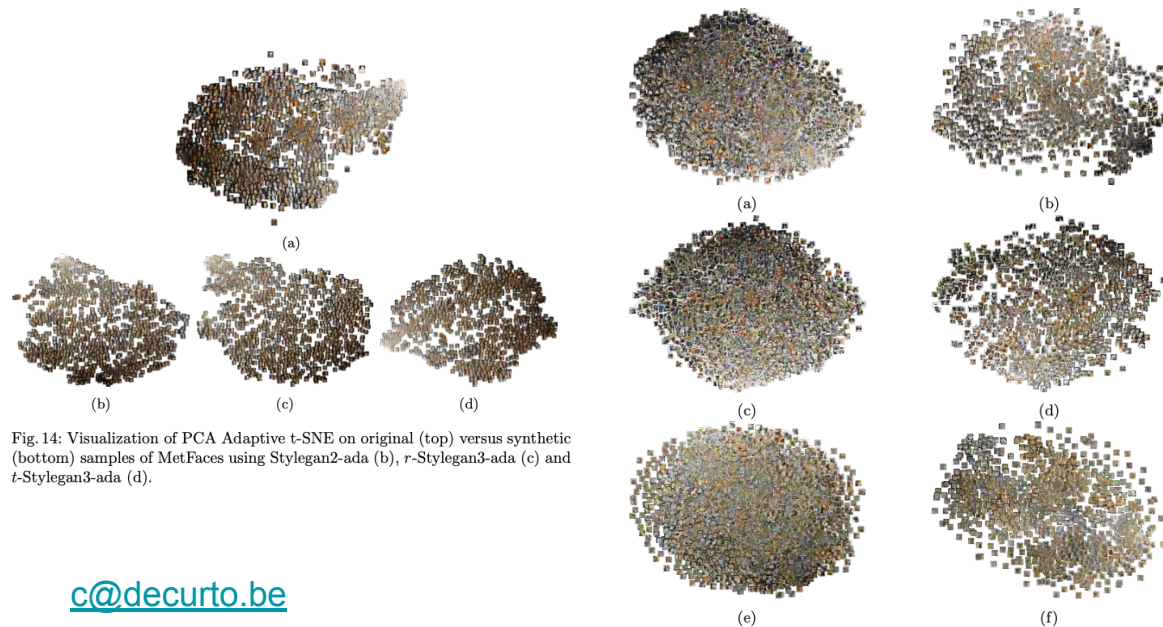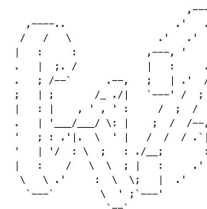

Fig. 13: Visualization of PCA Adaptive t-SNE on original (left) versus synthetic (right) samples of AFHQ Cat (a,b), Dog (c,d) and Wild (e,f) using Stylegan2-ada.
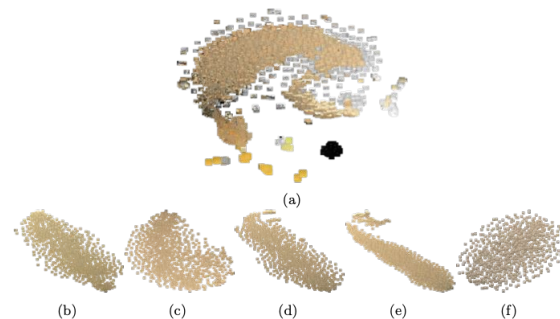

Fig. 15: Visualization of PCA Adaptive t-SNE on original (top) versus synthetic (bottom) samples of NASA Perseverance using Stylegan2-ada across several epoch iterations: 193 (b), 371 (c), 596 (d), 798 (e) and 983 (d).

c@decurto.be
z@dezarza.be

10

De Curtò y DíAz.
De Zarzà y Cubero.

Monthly Update

- Writing another manuscript: Overview on a topic of Unmanned Aerial Vehicles.
- Submission to Transactions on Neural Networks and Learning Systems.
- Submission to Software Impacts (Source Code).
- Transversal course on Tools for Academic Research: LaTeX. -> completed
- Specific course on Deep Learning.

c@decurto.be
z@dezarza.be