Cache Exercises, Virtual Memory

Exercise 1

The heart of the recent hit game *SimAquarium* is a tight loop that calculates the average position of 256 algae. You are evaluating its cache performance on a machine with a 1024-byte direct-mapped data cache with 16-byte blocks ($B = 16$). You are given the following definitions:

```
1   struct algae_position {
2       int x;
3       int y;
4   };
5
6   struct algae_position grid[16][16];
7   int total_x = 0, total_y = 0;
8   int i, j;
```

You should also assume the following:

- `sizeof(int) == 4`.
- `grid` begins at memory address 0.
- The cache is initially empty.
- The only memory accesses are to the entries of the array `grid`. Variables `i`, `j`, `total_x`, and `total_y` are stored in registers.

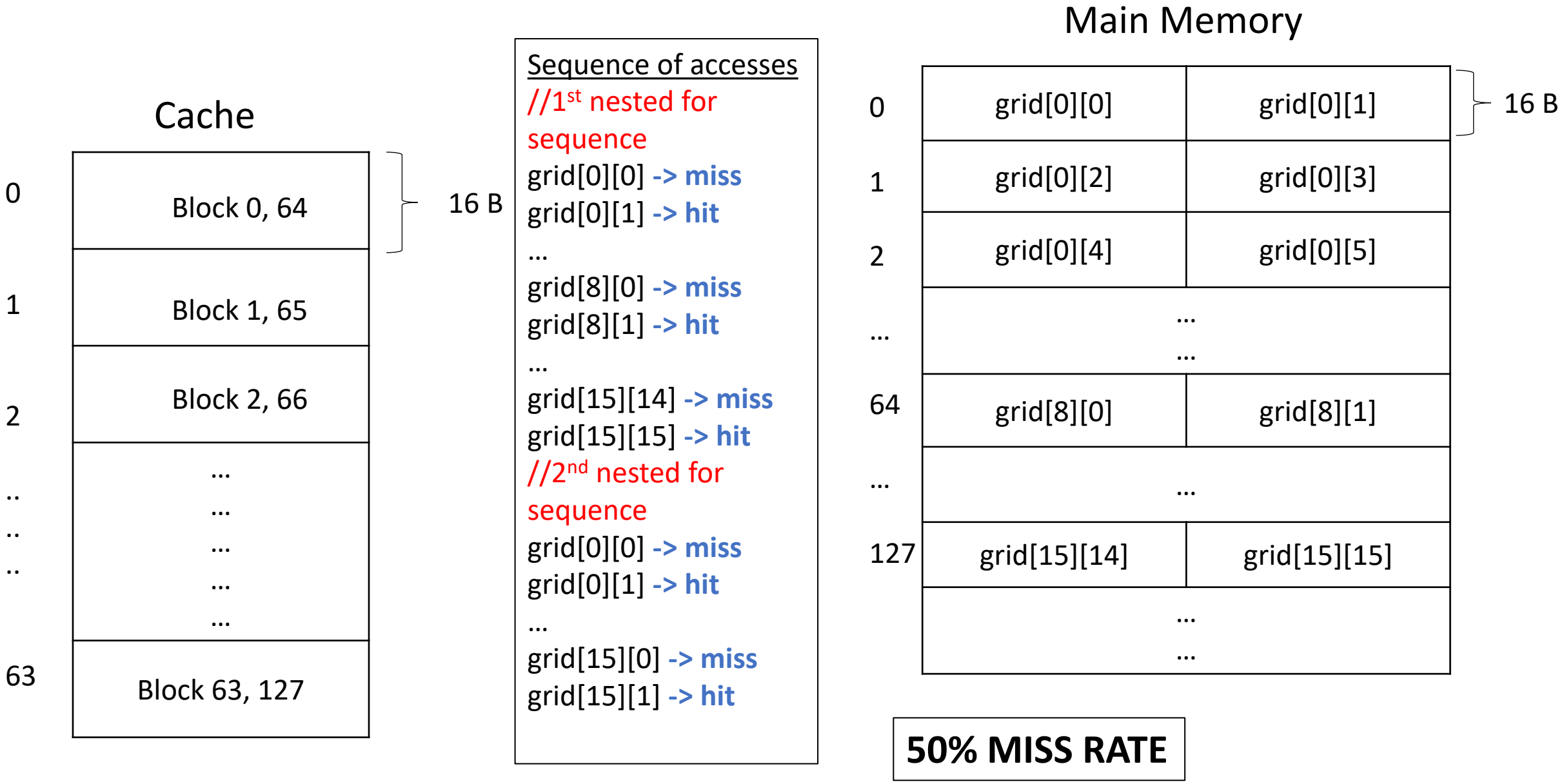
Determine the cache performance for the following code:

```
1       for (i = 0; i < 16; i++) {
2           for (j = 0; j < 16; j++) {
3               total_x += grid[i][j].x;
4           }
5       }
6
7       for (i = 0; i < 16; i++) {
8           for (j = 0; j < 16; j++) {
9               total_y += grid[i][j].y;
10          }
11      }
```

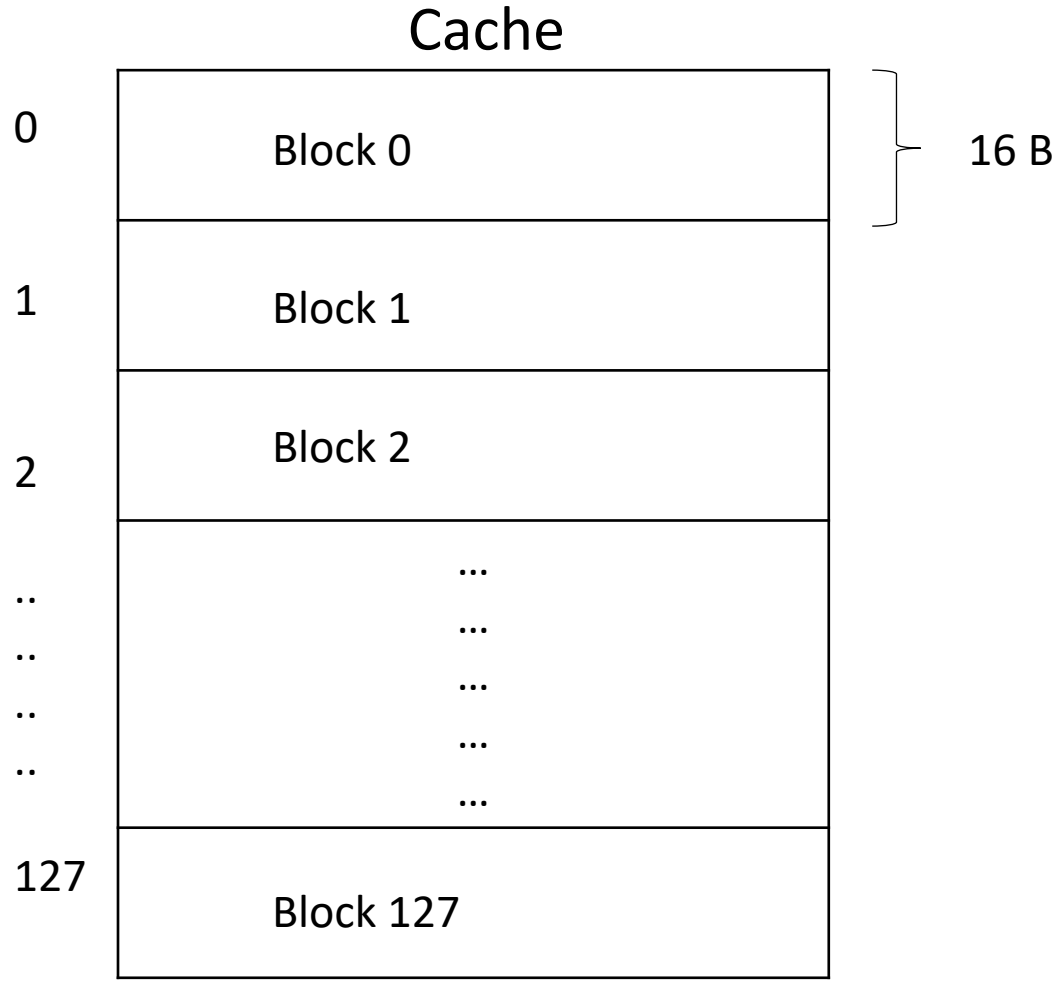
- What is the total number of reads?
- What is the total number of reads that miss in the cache?
- What is the miss rate?
- Will increasing the Cache size improve the performance?
- Will increasing the Block size improve performance?

Randal E. Bryant and David R. O'Hallaron. 2010. Computer Systems: A Programmer's Perspective (2nd. ed.). Addison-Wesley Publishing Company, USA.

Visualize the Cache and Main Memory



If we double the cache



Sequence of accesses
//1st nested for sequence
grid[0][0] -> miss
grid[0][1] -> hit
…
grid[8][0] -> miss
grid[8][1] -> hit
…
grid[15][14] -> miss
grid[15][15] -> hit
//2nd nested for sequence
grid[0][0] -> hit
grid[0][1] -> hit
…
grid[8][0] -> hit
grid[8][1] -> hit
…
grid[15][0] -> hit
grid[15][1] -> hit

25% MISS RATE

- What is the total number of reads?
  - 512
- What is the total number of reads that miss in the cache?
  - 256
- What is the miss rate?
  - 50%
- Will doubling the Cache size improve the performance?
  - Yes
  - What will the number of misses in this case?
    - 128 misses for the first nested for loop sequence. 0 misses for the next nested for loop sequence.
- Will increasing the Block size improve performance?
  - Yes

Exercise 2

```
1       for (i = 0; i < 16; i++){
2           for (j = 0; j < 16; j++) {
3               total_x += grid[j][i].x;
4               total_y += grid[j][i].y;
5           }
6       }
```

A. What is the total number of reads?

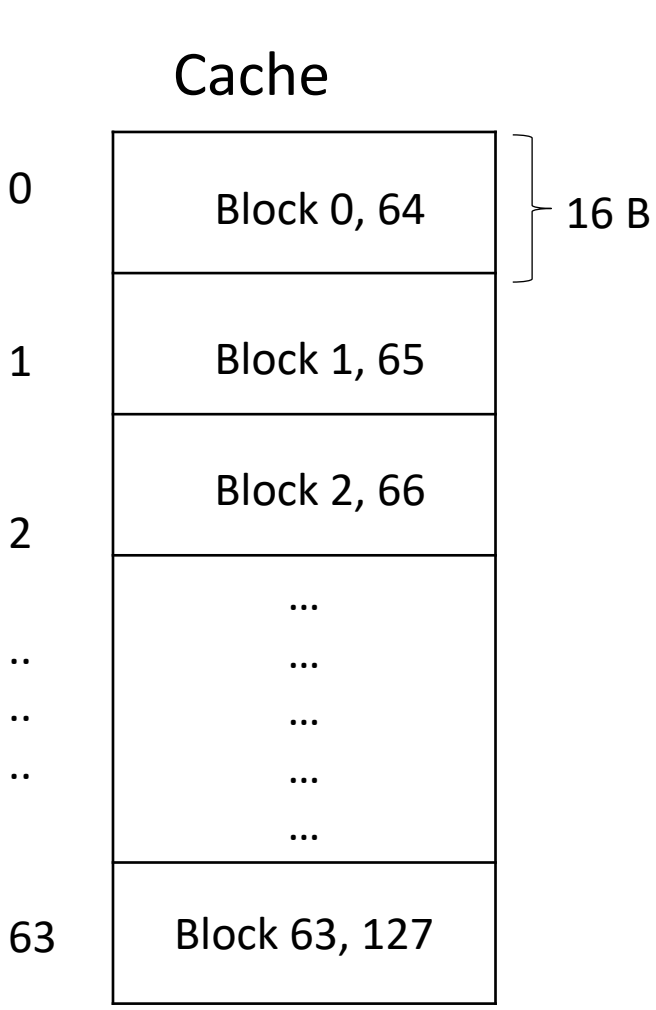
B. What is the total number of reads that miss in the cache?

C. What is the miss rate?

D. What would the miss rate be if the cache were twice as big?

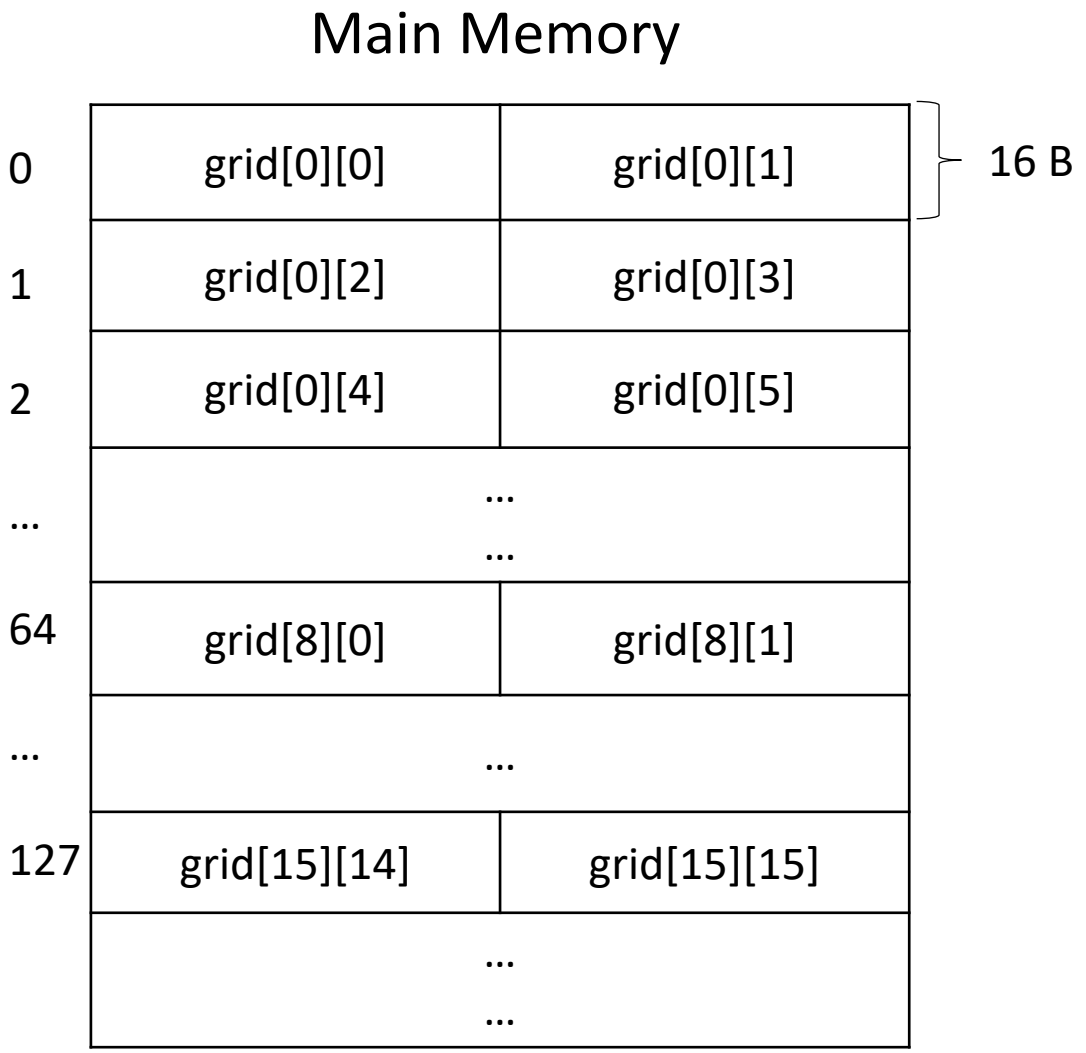
Randal E. Bryant and David R. O'Hallaron. 2010. Computer Systems: A Programmer's Perspective (2nd. ed.). Addison-Wesley Publishing Company, USA.

Visualize the Cache and Main Memory

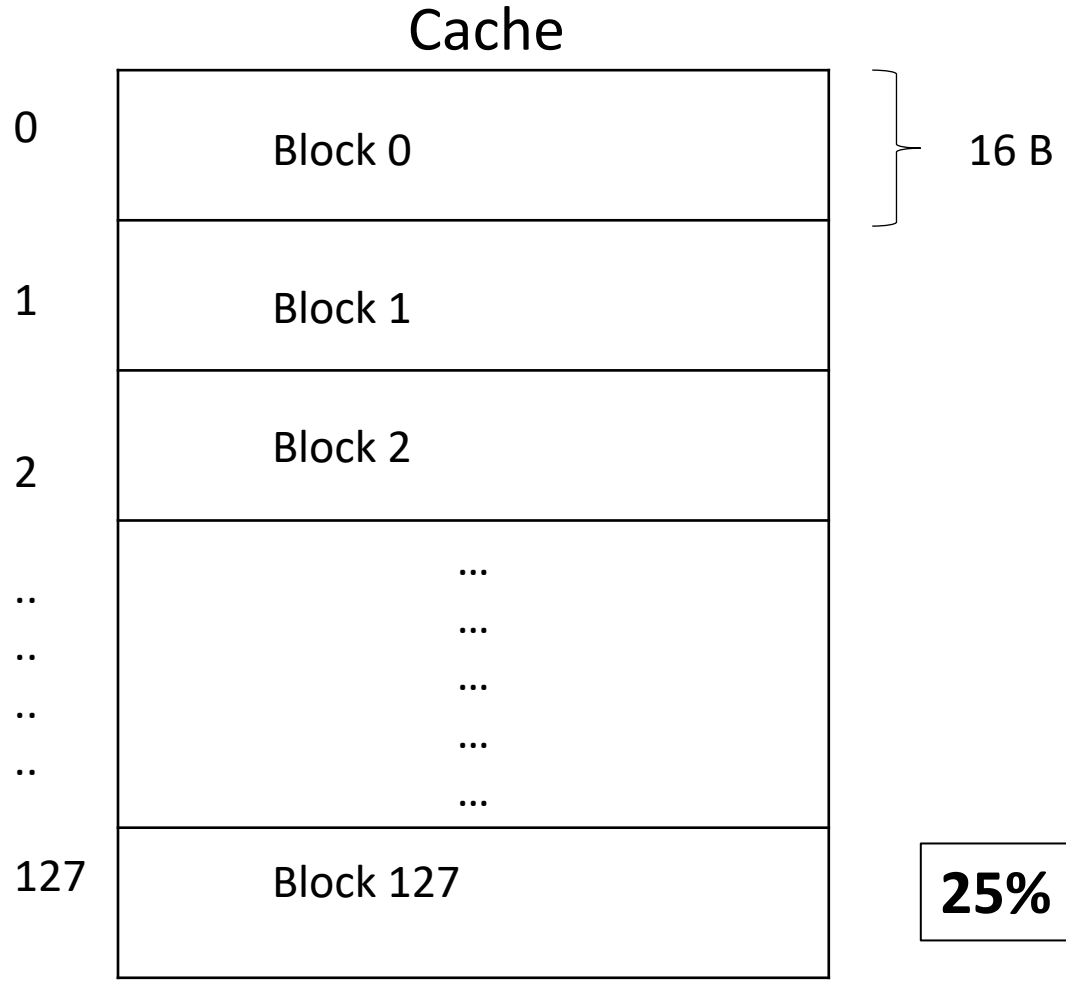


Sequence of accesses

grid[0][0] -> **miss**
grid[0][0] -> **hit**
grid[1][0] -> **miss**
grid[1][0] -> **hit**
…
grid[8][0] -> **miss**
grid[8][0] -> **hit**
…
grid[15][0] -> **miss**
grid[15][0] -> **hit**
grid[0][1] -> **miss**
grid[0][1] -> **hit**
…
grid[15][1] ->**miss**
grid[15][1] -> **hit**
…



If we double the cache



25% MISS RATE

Sequence of accesses

grid[0][0] -> miss
grid[0][0] -> hit
grid[1][0] -> miss
grid[1][0] -> hit
…
grid[8][0] -> miss
grid[8][0] -> hit
…
grid[15][0] -> miss
grid[15][0] -> hit
grid[0][1] -> hit
grid[0][1] -> hit
…
grid[15][1] -> hit
grid[15][1] -> hit
grid [0][2] -> miss
grid [0][2] -> hit
…
grid[0][3] -> hit
grid[0][3] -> hit
…

Exercise 2 - Answers

```
1       for (i = 0; i < 16; i++){
2           for (j = 0; j < 16; j++) {
3               total_x += grid[j][i].x;
4               total_y += grid[j][i].y;
5           }
6       }
```

A. What is the total number of reads? 512

B. What is the total number of reads that miss in the cache? 256

C. What is the miss rate? 50%

D. What would the miss rate be if the cache were twice as big? 25%

Randal E. Bryant and David R. O'Hallaron. 2010. Computer Systems: A Programmer's Perspective (2nd. ed.). Addison-Wesley Publishing Company, USA.

Exercise 3 – Try to solve it

```
1       for (i = 0; i < 16; i++){
2           for (j = 0; j < 16; j++) {
3               total_x += grid[i][j].x;
4               total_y += grid[i][j].y;
5           }
6       }
```

A. What is the total number of reads?

B. What is the total number of reads that miss in the cache?

C. What is the miss rate?

D. What would the miss rate be if the cache were twice as big?

Sequence of accesses
grid[0][0]
grid[0][0]
grid[0][1]
grid[0][1]
…
grid[15][14]
grid[15][14]
grid[15][15]
grid[15][15]

Randal E. Bryant and David R. O'Hallaron. 2010. Computer Systems: A Programmer's Perspective (2nd. ed.). Addison-Wesley Publishing Company, USA.

Exercise 3 – Answers

```
1       for (i = 0; i < 16; i++){
2           for (j = 0; j < 16; j++) {
3               total_x += grid[i][j].x;
4               total_y += grid[i][j].y;
5           }
6       }
```

A. What is the total number of reads? 512

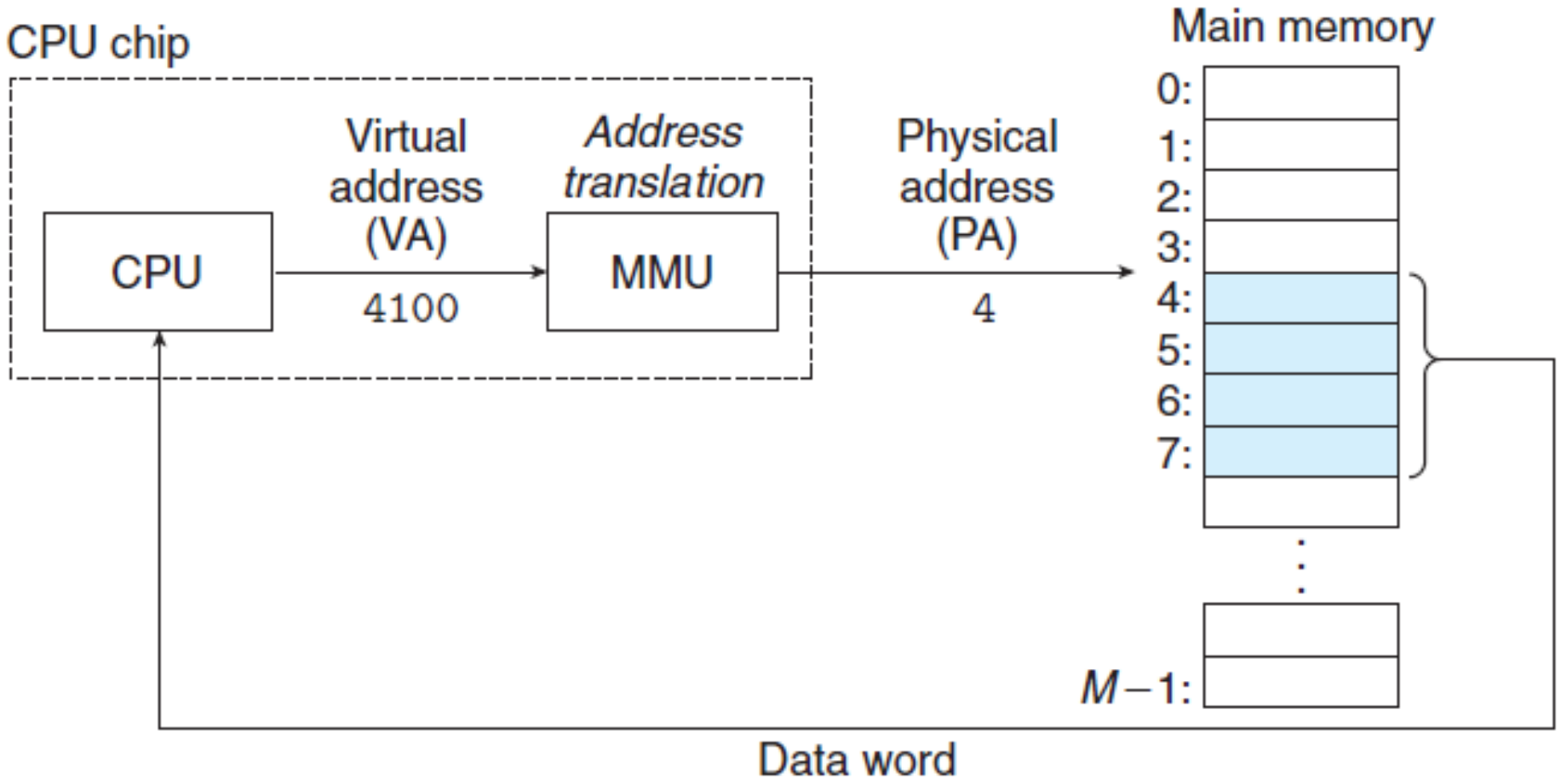
B. What is the total number of reads that miss in the cache? 128

C. What is the miss rate? 25%

D. What would the miss rate be if the cache were twice as big? No

Randal E. Bryant and David R. O'Hallaron. 2010. Computer Systems: A Programmer's Perspective (2nd. ed.). Addison-Wesley Publishing Company, USA.

Virtual Memory



Randal E. Bryant and David R. O'Hallaron. 2010. Computer Systems: A Programmer's Perspective (2nd. ed.). Addison-Wesley Publishing Company, USA.

Consider a Scenario

- Main Memory
  - Size of Main Memory = 64 Bytes
  - Frame Size = 4 Bytes
  - Number of Frames = 64/4 = 16

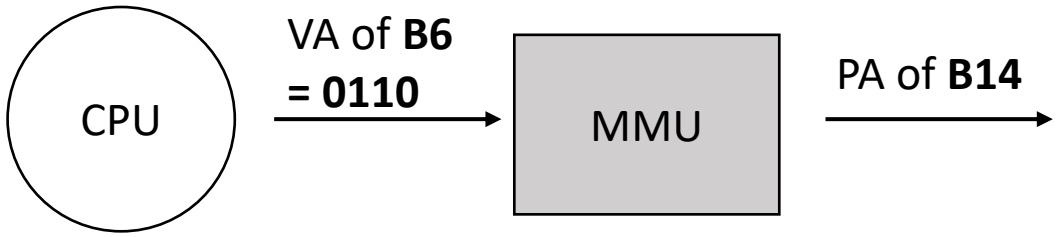
- Process
  - Size of Process = 16 Bytes
  - Page Size = 4 Bytes
  - Number of Pages = 16/4 = 4

Paging

How does the MMU get the Physical address of B14 in Main Memory?!!

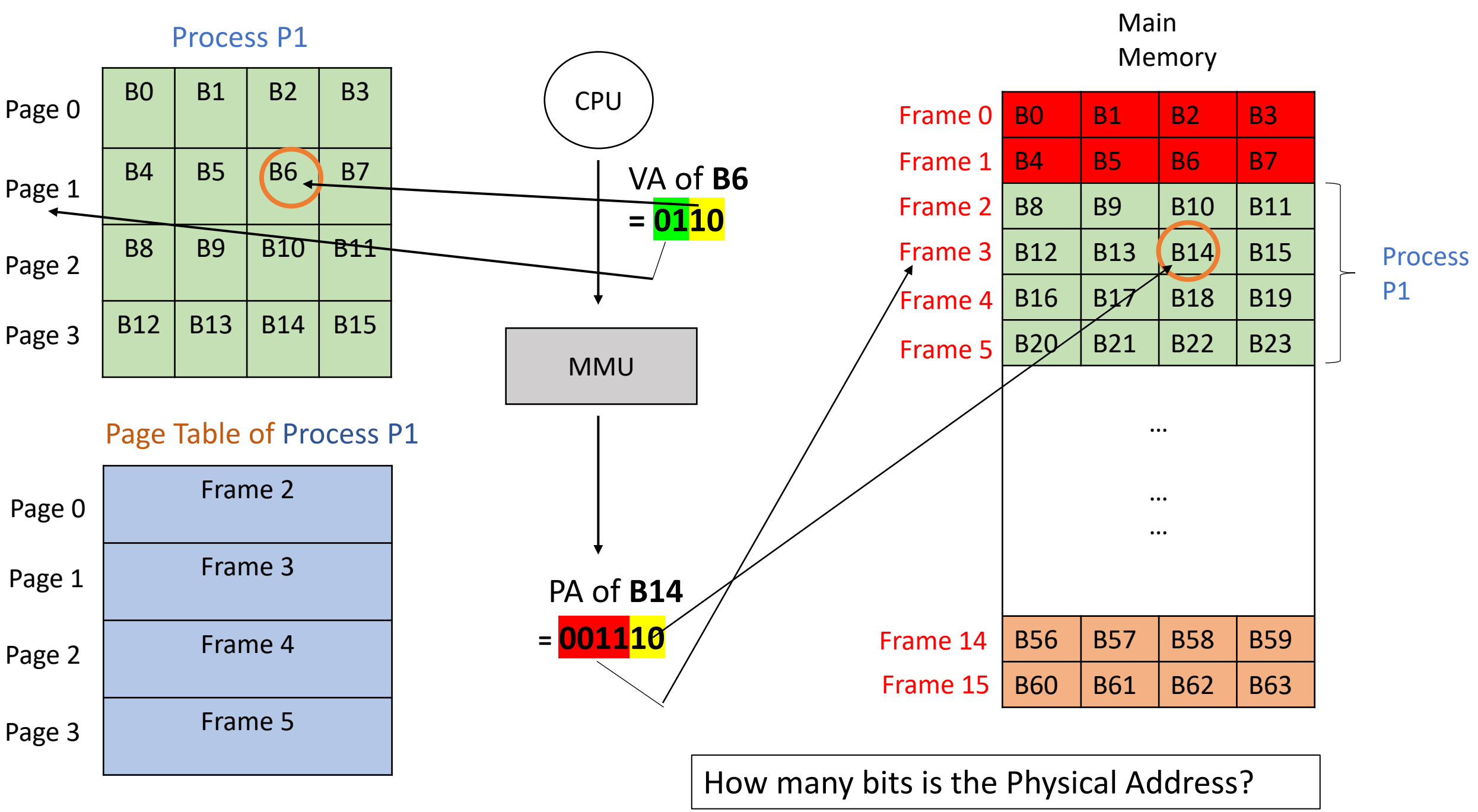
Process P1

Page 0	B0	B1	B2	B3
Page 1	B4	B5	B6	B7
Page 2	B8	B9	B10	B11
Page 3	B12	B13	B14	B15



Main Memory

Frame 0	B0	B1	B2	B3	
Frame 1	B4	B5	B6	B7	
Frame 2	B8	B9	B10	B11	Process P1
Frame 3	B12	B13	B14	B15	
Frame 4	B16	B17	B18	B19	
Frame 5	B20	B21	B22	B23	
	...				
Frame 14	B56	B57	B58	B59	
Frame 15	B60	B61	B62	B63	


Process P1
Page 0
Page 1
Page 2
Page 3
B0
B1
B2
B3
B4
B5
B6
B7
B8
B9
B10
B11
B12
B13
B14
B15
CPU
VA of B6
= 0110
MMU
Page Table of Process P1
Page 0
Page 1
Page 2
Page 3
Frame 2
Frame 3
Frame 4
Frame 5
PA of B14
= 001110
Main
Memory
Frame 0
Frame 1
Frame 2
Frame 3
Frame 4
Frame 5
Frame 14
Frame 15
B0
B1
B2
B3
B4
B5
B6
B7
B8
B9
B10
B11
B12
B13
B14
B15
B16
B17
B18
B19
B20
B21
B22
B23
...
...
...
B56
B57
B58
B59
B60
B61
B62
B63
Process
P1
How many bits is the Physical Address?

Related Links

- You can refer to the following links to know more about virtual memory, how frames are allocated to processes and about the page replacement policies.

  - Allocation of frames in Operating Systems
  - Page Replacement Algorithms in Operating Systems
  - Virtual Memory

Some Important Byte Conversions

- $1\,KB = 2^{10}B$
- $1\,MB = 2^{20}B$
- $1\,GB = 2^{30}B$
- $1\,TB = 2^{40}$

Example:

- $2^{34}\,B$
  - $2^{30} * 2^{4}\,B = 2^{4}\,GB = 16\,GB$

Exercise

- Physical Address Space (PAS)
  - Size of Main Memory
- Virtual Address Space (VAS)
  - Size of the process

- PAS = 128 KB
- VAS = 128 KB

- How many bits for PA?
  - 17
- How many bits for VA?
  - 17

Exercise

- Physical Address Space (PAS)
  - Size of Main Memory
- Virtual Address Space (VAS)
  - Size of the process

- PAS = 128 KB
- VAS = 128 KB
- Page Size = 4 KB

- How many bits for the page offset?
  - 12 bits

Exercise

- Physical Address Space (PAS)
  - Size of Main Memory
- Virtual Address Space (VAS)
  - Size of the process

- PAS = 128 KB
- VAS = 128 KB
- Page Size = 4 KB

- How many bits for the page number?
  - (17 – 12) bits = 5 bits
- How many Pages?
  - 2^5= 32

Exercise

- Physical Address Space (PAS)
  - Size of Main Memory
- Virtual Address Space (VAS)
  - Size of the process

- PAS = 128 KB
- VAS = 128 KB
- Page Size = 4 KB

- How many bits for the Frame number?
  - (17 – 12) bits = 5 bits
- How many Frames?
  - 2^5= 32

Exercise

- Physical Address Space (PAS)
  - Size of Main Memory
- Virtual Address Space (VAS)
  - Size of the process

- PAS = 128 KB
- VAS = 128 KB
- Page Size = 4 KB

- How many bits for a Page Table Entry (PTE)?
  - 5 bits

**For this example, we assume that a page table entry contains only the bits to represent the Frame Number.*

Exercise

- Physical Address Space (PAS)
  - Size of Main Memory
- Virtual Address Space (VAS)
  - Size of the process

- PAS = 128 KB
- VAS = 128 KB
- Page Size = 4 KB

- What is the Size of the Page Table?
  - (Number of Pages * Size of a Page Table Entry) = $2^5 * 5$ bits

**For this example, we assume that a page table entry contains only the bits to represent the Frame Number.*

Exercise 2

[$] *For this example, we assume that a page table entry contains only the bits to represent the Frame Number.*

VAS	PAS	VA	PA	Page Size	Page Offset	# of Virtual Pages	# of Physical Pages(or frames)	PTE[$]	Size of Page Table
1 MB					10		256		

Exercise 2

[$] *For this example, we assume that a page table entry contains only the bits to represent the Frame Number.*

VAS	PAS	VA	PA	Page Size	Page Offset	# of virtual Pages	# of Physical Pages (or Frames)	PTE[$]	Size of Page Table
1 MB				2^{10}	10		256		

Exercise 2

[$]For this example, we assume that a page table entry contains only the bits to represent the Frame Number.

VAS	PAS	VA	PA	Page Size	Page Offset	# of Virtual Pages	# of Frames	PTE[$]	Size of Page Table
1 MB	**# of Frames * Frame Size = 256 KB**			2^{10}	10		256		

Exercise 2

[$]For this example, we assume that a page table entry contains only the bits to represent the Frame Number.

VAS	PAS	VA	PA	Page Size	Page Offset	# of Virtual Pages	# of Frames	PTE[$]	Size of Page Table
1 MB = 2^20 Bytes	# of Frames * Frame Size = 256 KB =2^18 Bytes	**20 bits**	**18 bits**	2^{10}	10		256		

Exercise 2

[$] *For this example, we assume that a page table entry contains only the bits to represent the Frame Number.*

VAS	PAS	VA	PA	Page Size	Page Offset	# of Virtual Pages	# of Frames	PTE[$]	Size of Page Table
1 MB = 2^20 Bytes	# of Frames * Frame Size = 256 KB =2^18 Bytes	20 bits	18 bits	2^{10}	10	$\mathbf{2^{(20-10)} = 2^{10}}$	256		

Exercise 2

[$] *For this example, we assume that a page table entry contains only the bits to represent the Frame Number.*

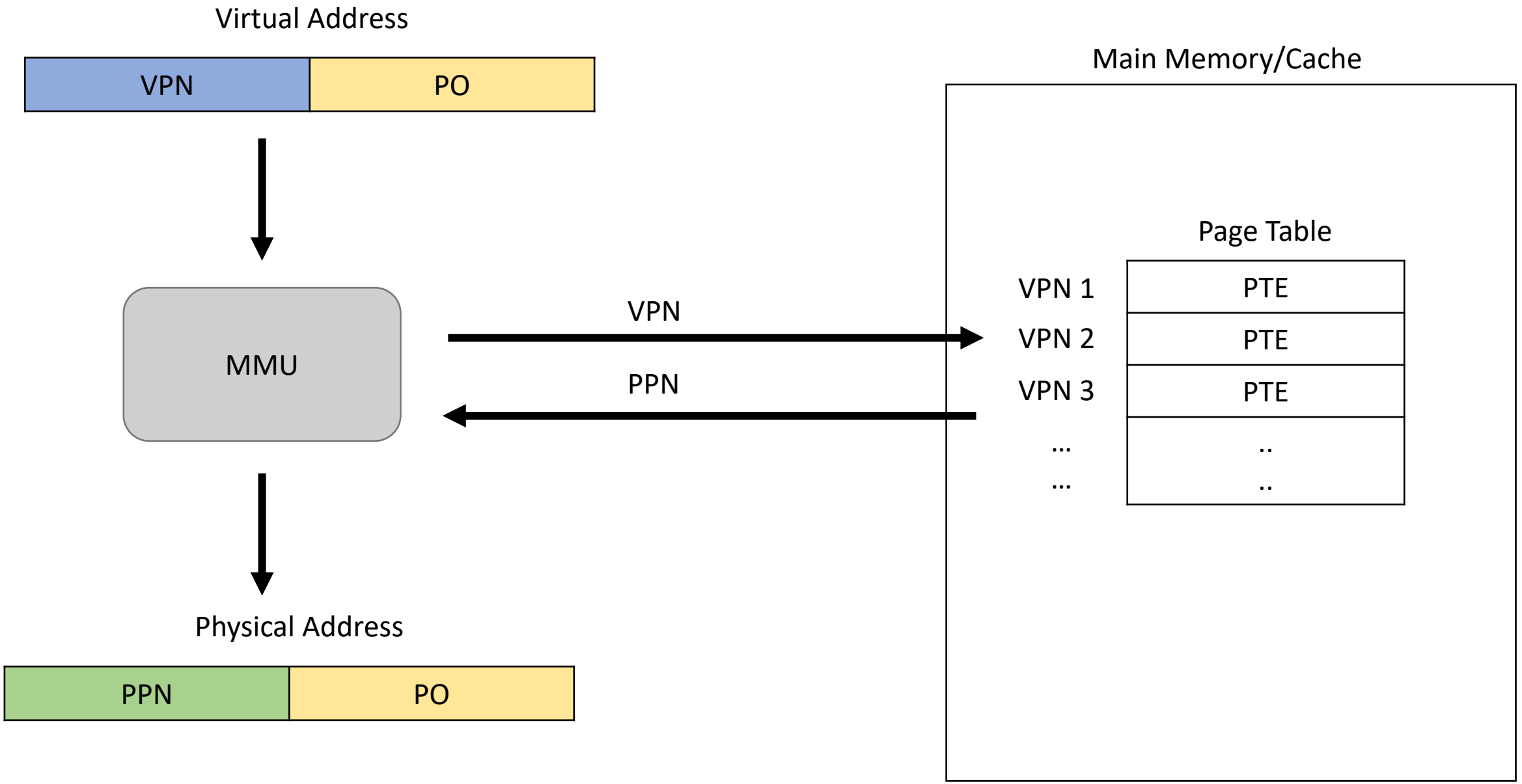
VAS	PAS	VA	PA	Page Size	Page Offset	# of Virtual Pages	# of Frames	PTE[$]	Size of Page Table
1 MB = 2^20 Bytes	# of Frames * Frame Size = 256 KB =2^18 Bytes	20 bits	18 bits	2^{10}	10	$2^{(20-10)} = 2^{10}$	256 = 2^8	**8 bits**	

Exercise 2

[$] For this example, we assume that a page table entry contains only the bits to represent the Frame Number.

VAS	PAS	VA	PA	Page Size	Page Offset	# of Virtual Pages	# of Frames	PTE[$]	Size of Page Table
1 MB = 2^20 Bytes	# of Frames * Frame Size = 256 KB =2^18 Bytes	20 bits	18 bits	2^{10}	10	$2^{(20-10)} = 2^{10}$	256 = 2^8	8 bits	$\mathbf{2^{10} * 8\ bits}$

TLB


Virtual Address
VPN
PO
MMU
VPN
PPN
Physical Address
PPN
PO
Main Memory/Cache
Page Table
VPN 1
VPN 2
VPN 3
...
...
PTE
PTE
PTE
..
..

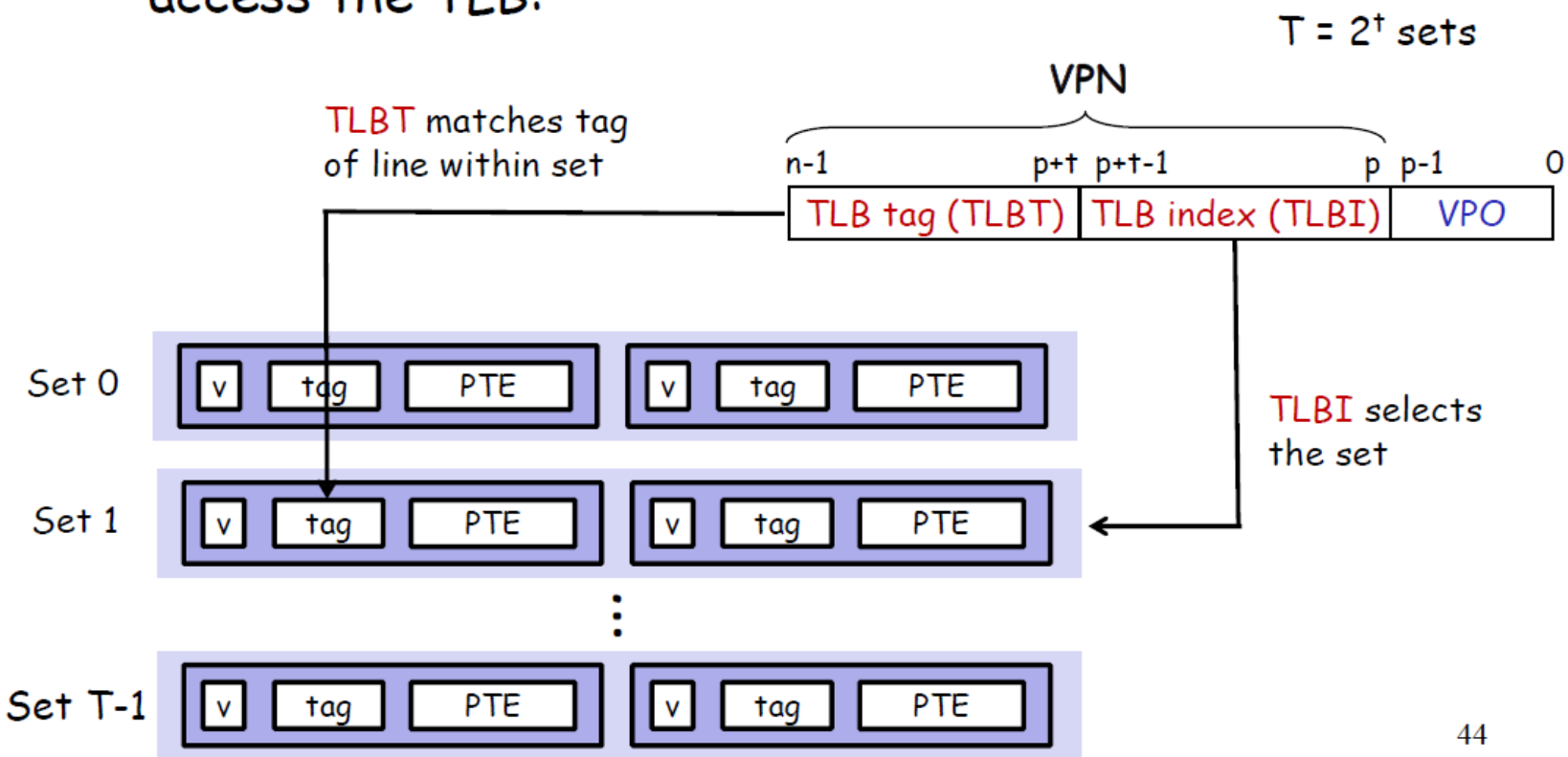
Speeding up Translation with a TLB

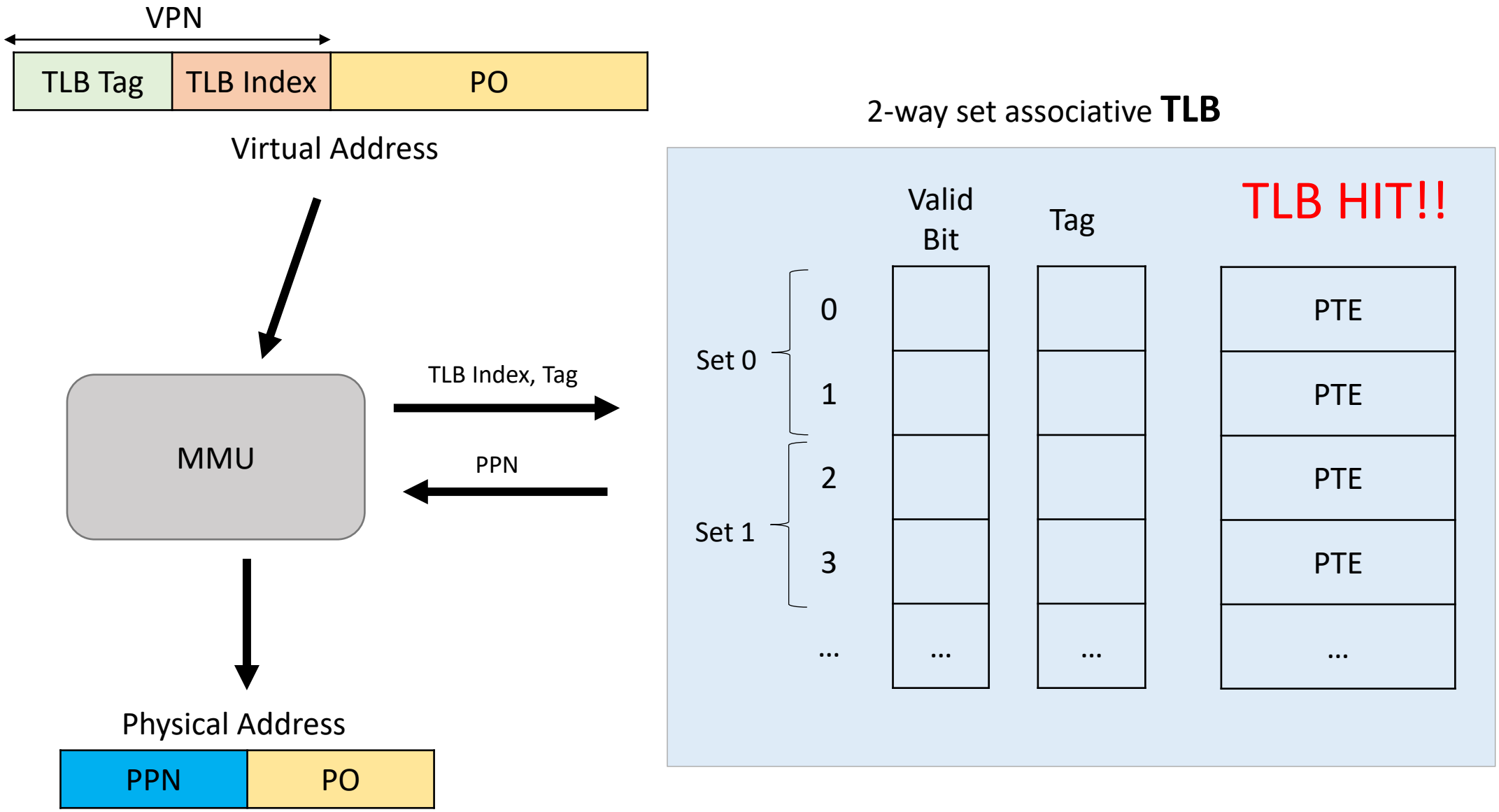
- Page table entries (PTEs) are cached in L1 like any other memory word
  - PTEs may be evicted by other data references
  - PTE hit still requires a small L1 delay

- Solution: *Translation Lookaside Buffer* (TLB)
  - Small set-associative hardware cache in MMU
  - Maps virtual page numbers to physical page numbers
  - Contains complete page table entries for small number of pages

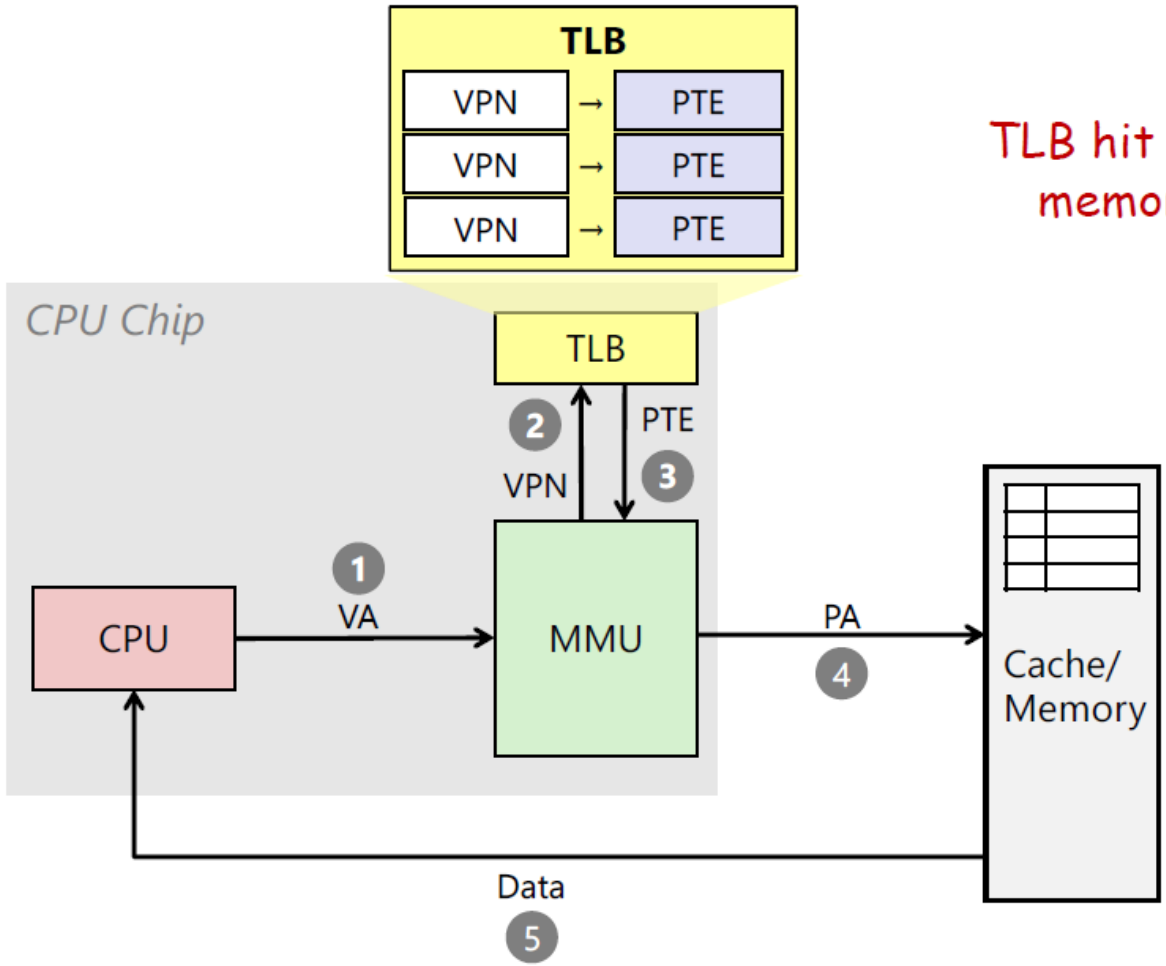
Accessing the TLB

- MMU uses the VPN portion of the virtual address to access the TLB:



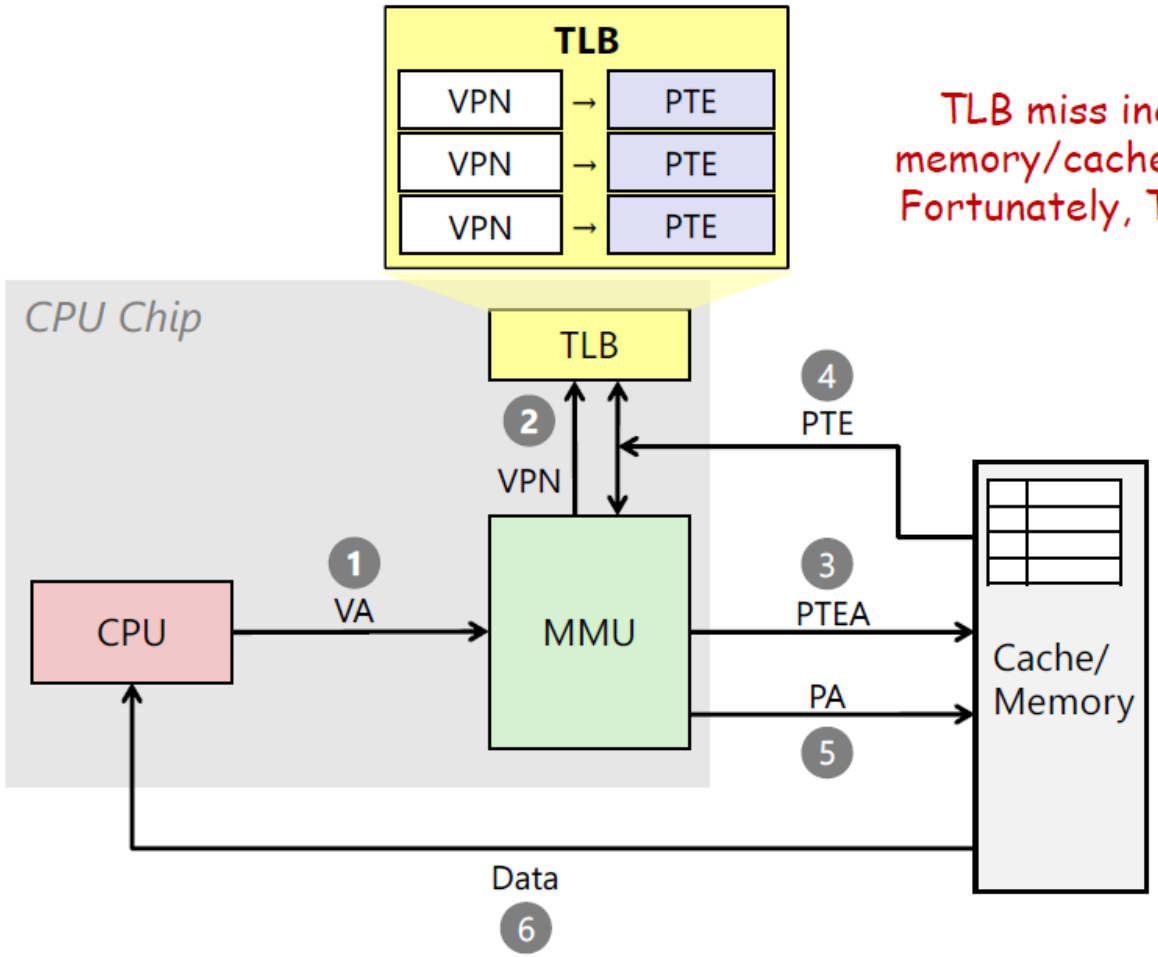

VPN
TLB Tag
TLB Index
PO
Virtual Address
MMU
TLB Index, Tag
PPN
Physical Address
PPN
PO
2-way set associative TLB
TLB HIT!!
Valid Bit
Tag
Set 0
Set 1
0
1
2
3
...
PTE
PTE
PTE
PTE
...
...
...

TLB Hit



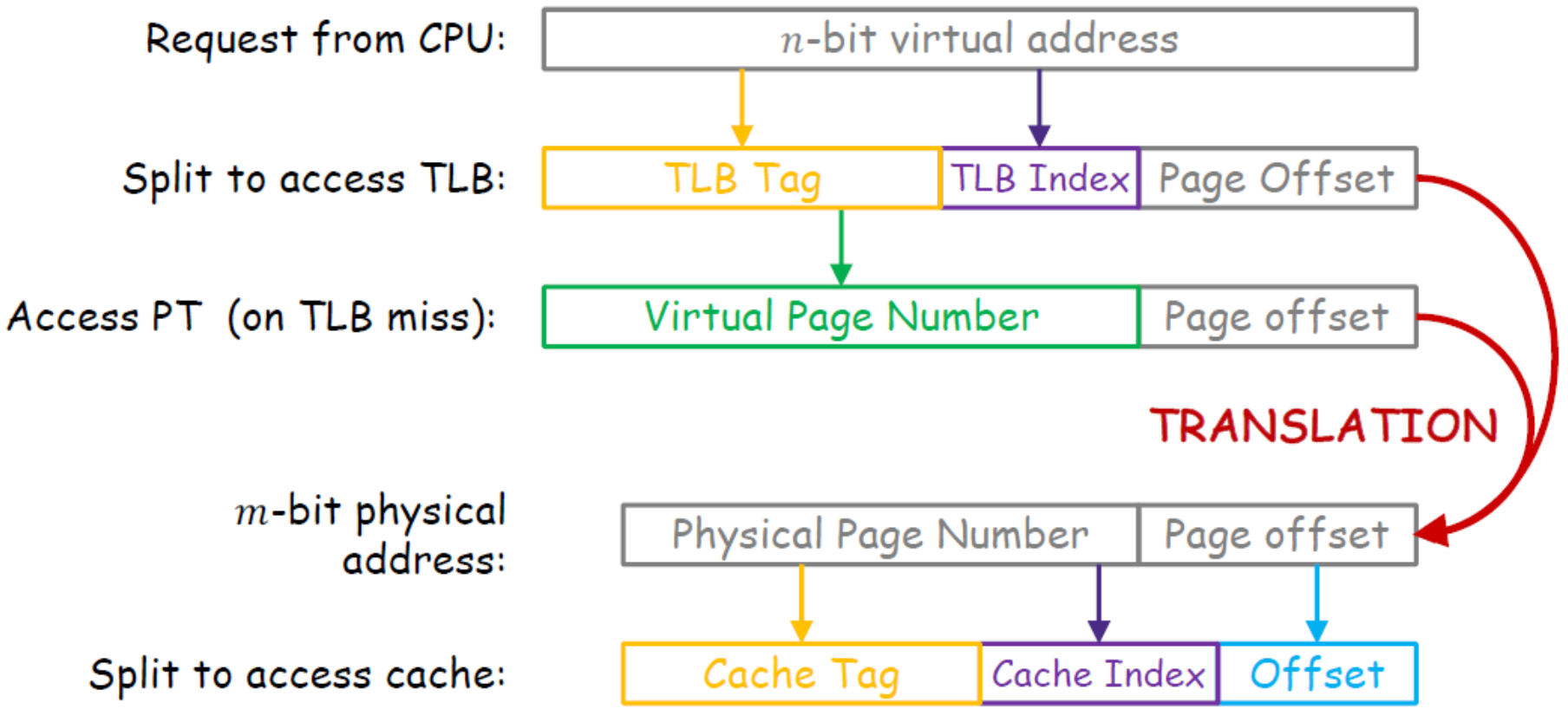
TLB hit eliminates a memory access!

TLB Miss



TLB miss incurs an additional memory/cache access (the PTE) - Fortunately, TLB misses are rare

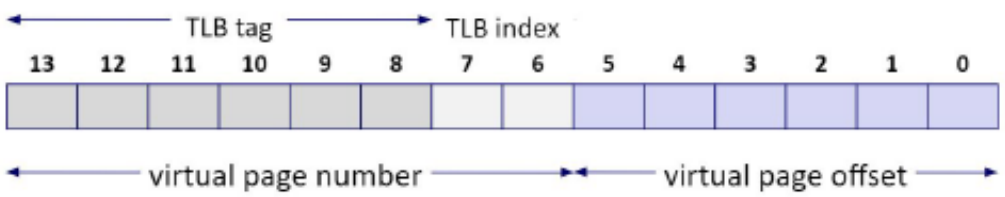
Address Manipulation

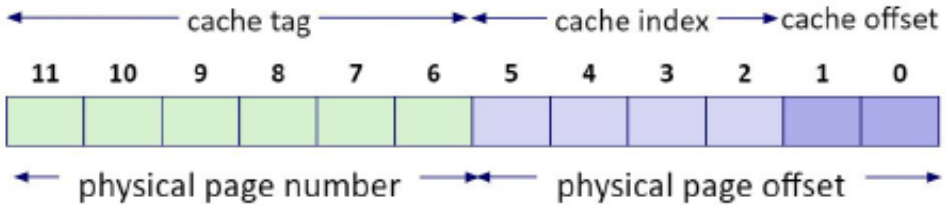


Cache Tutorial

A Small Example

Suppose we have a simple memory system with **14-bit** virtual addresses, **12-bit** physical addresses, and a page size of **64 bytes**. The TLB has **16 entries** in total and is **4-way** set associative. The cache is direct-mapped with **16 sets** and a block size of **4 bytes**.





Current State of Memory System:

TLB:

Set	Tag	PPN	V	Tag	PPN	V	Tag	PPN	V	Tag	PPN	V
0	03	–	0	09	0D	1	00	–	0	07	02	1
1	03	2D	1	02	–	0	04	–	0	0A	–	0
2	02	–	0	08	–	0	06	–	0	03	–	0
3	07	–	0	03	0D	1	0A	34	1	02	–	0

Page table (partial):

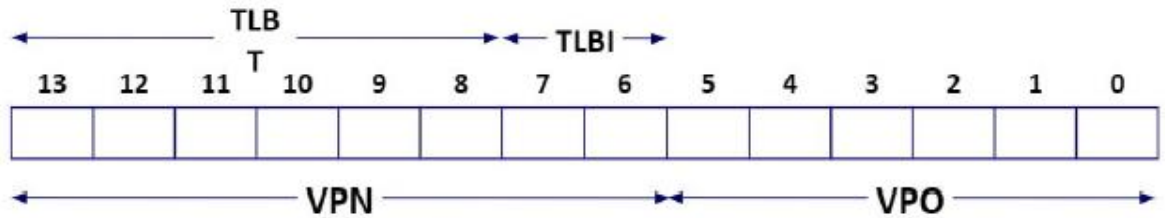
VPN	PPN	V	VPN	PPN	V
0	28	1	8	13	1
1	–	0	9	17	1
2	33	1	A	09	1
3	02	1	B	–	0
4	–	0	C	–	0
5	16	1	D	2D	1
6	–	0	E	–	0
7	–	0	F	0D	1

Cache:

Index	Tag	V	B0	B1	B2	B3	Index	Tag	V	B0	B1	B2	B3
0	19	1	99	11	23	11	8	24	1	3A	00	51	89
1	15	0	–	–	–	–	9	2D	0	–	–	–	–
2	1B	1	00	02	04	08	A	2D	1	93	15	DA	3B
3	36	0	–	–	–	–	B	0B	0	–	–	–	–
4	32	1	43	6D	8F	09	C	12	0	–	–	–	–
5	0D	1	36	72	F0	1D	D	16	1	04	96	34	15
6	31	0	–	–	–	–	E	13	1	83	77	1B	D3
7	16	1	11	C2	DF	03	F	14	0	–	–	–	–

Memory Requests

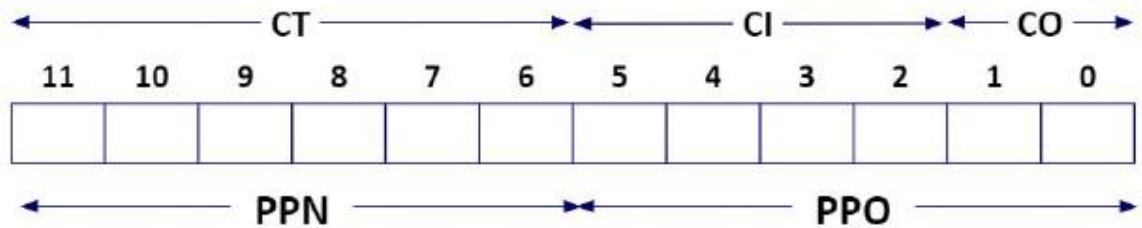
1. **Virtual Address: 0x0ACA**



VPN ____ TLBT ____ TLBI ____

TLB Hit? ____ Page Fault? ____ PPN ____

Physical Address:

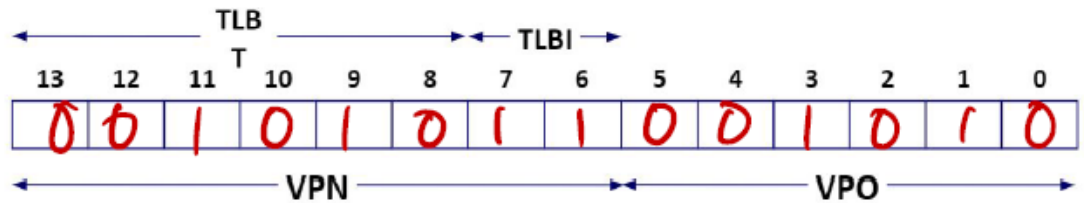


CT ____ CI ____ CO ____ Cache Hit? ____ Data (byte) ____

NOTE: It is just a coincidence that the PPN is the same width as the cache tag.

Memory Requests

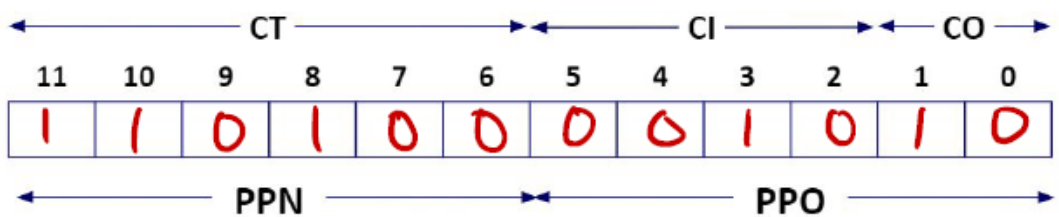
1. **Virtual Address: 0x0ACA**



VPN 0x2B TLBT 0x0A TLBI 0x3

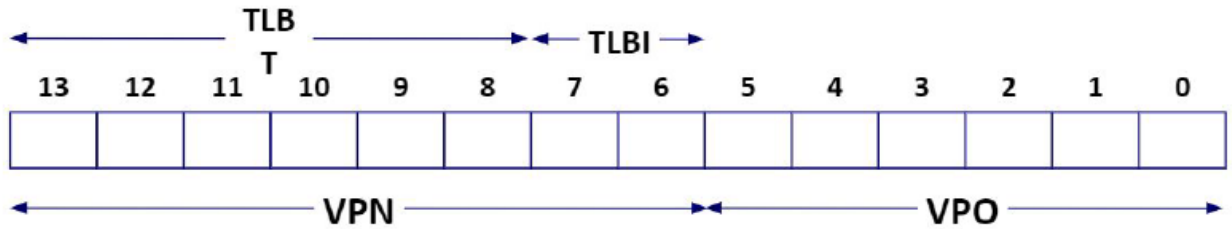
TLB Hit? Yes Page Fault? No PPN 0x34

Physical Address:



CT 0x34 CI 0x2 CO 0x2 Cache Hit? No Data (byte) 11

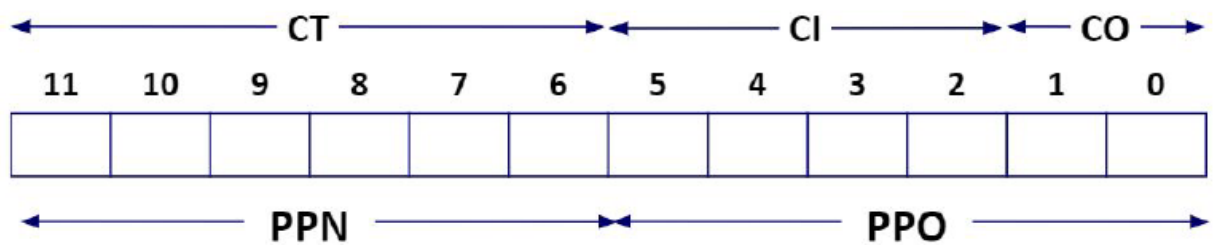
2. **Virtual Address: 0x015D**



VPN ____ TLBT ____ TLBI ____

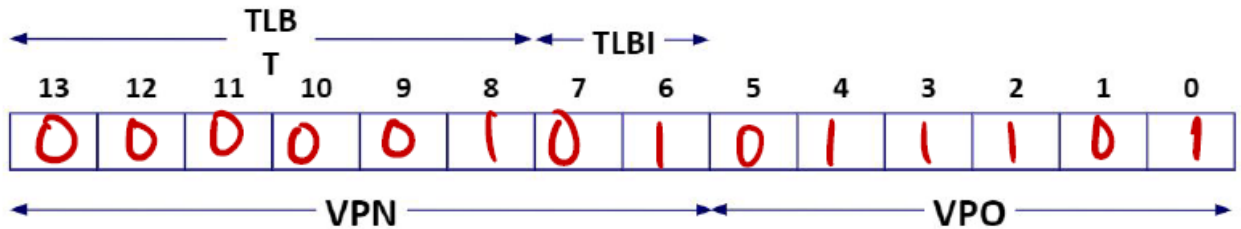
TLB Hit? ____ Page Fault? ____ PPN ____

Physical Address:



CT ____ CI ____CO ____ Cache Hit? ____ Data (byte) ____

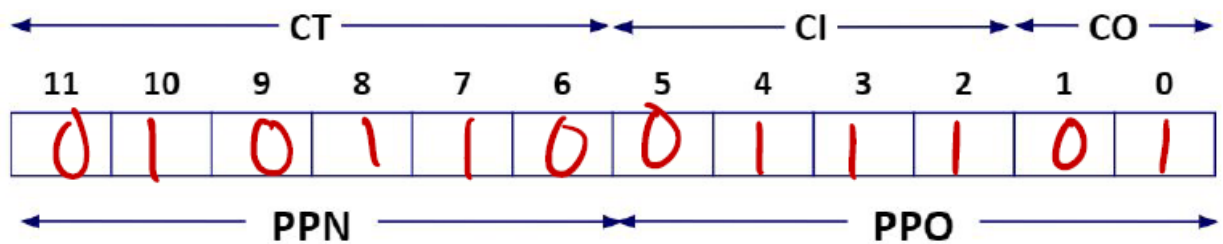
2. **Virtual Address: 0x015D**



VPN 0x05 TLBT 0x01 TLBI 0x1

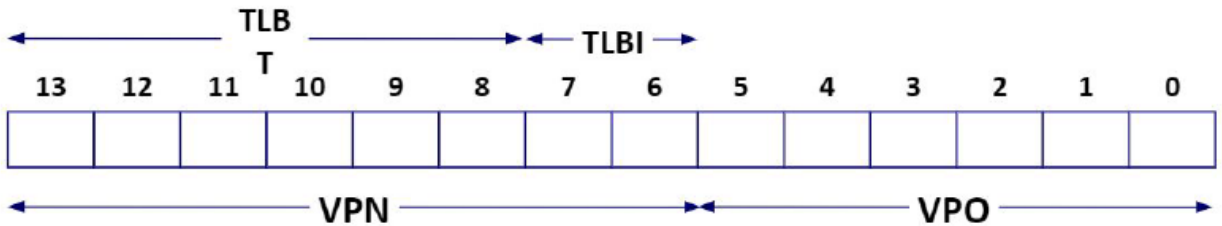
TLB Hit? No Page Fault? No PPN 0x16

Physical Address:



CT 0x16 CI 0x7 CO 0x1 Cache Hit? Yes Data (byte) 0xC2

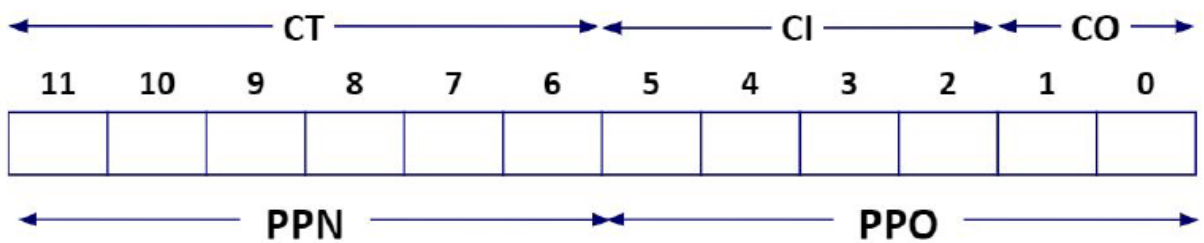
3. **Virtual Address: 0x0273**



VPN ____ TLBT ____ TLBI ___

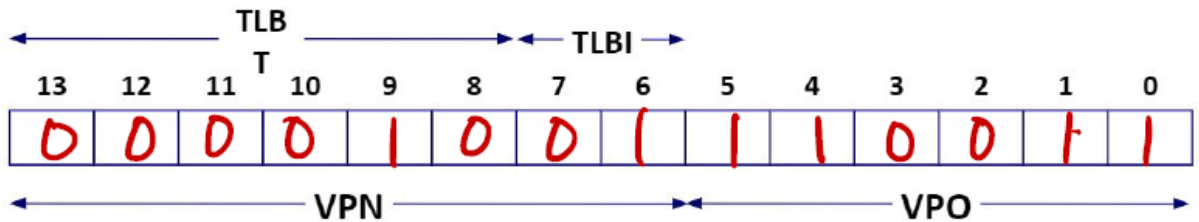
TLB Hit? ____ Page Fault? ____ PPN ____

Physical Address:



CT ____ CI ____CO ____ Cache Hit? ____ Data (byte) ____

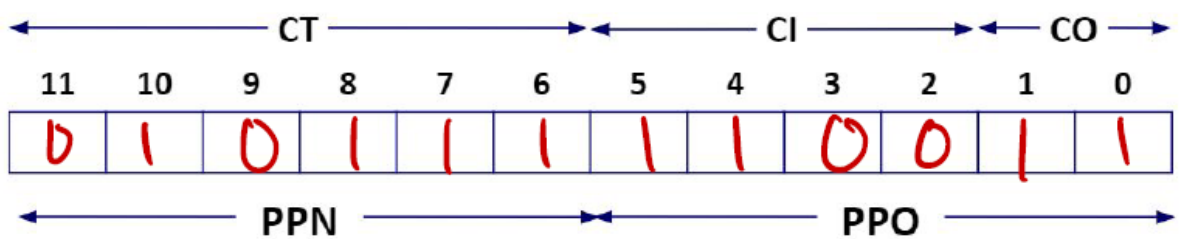
3. **Virtual Address: 0x0273**



VPN 0x09 TLBT 0x02 TLBI 0x01

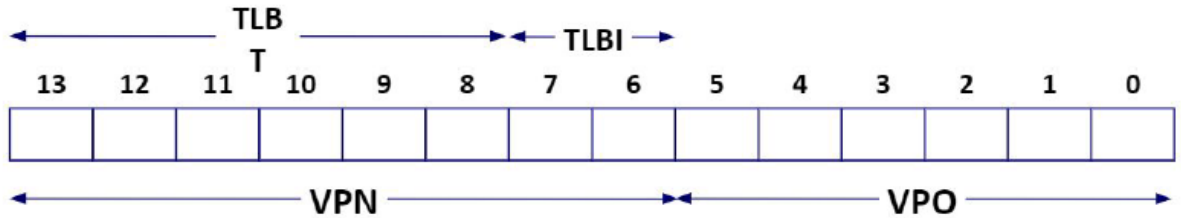
TLB Hit? No Page Fault? No PPN 0x17

Physical Address:



CT 0x17 CI 0xC CO 0x3 Cache Hit? No Data (byte) !!

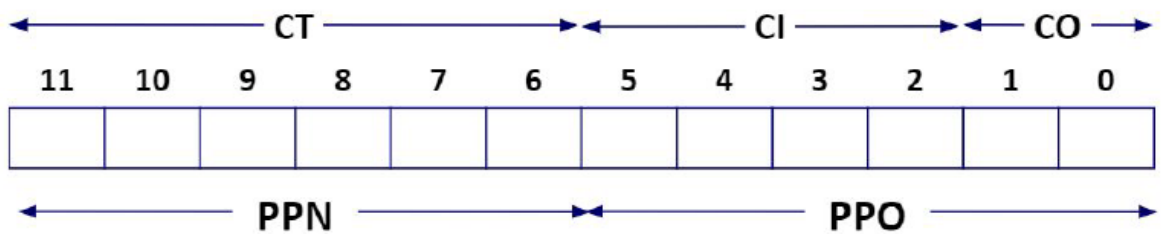
4. **Virtual Address: 0x036B**



VPN ____ TLBT ____ TLBI___

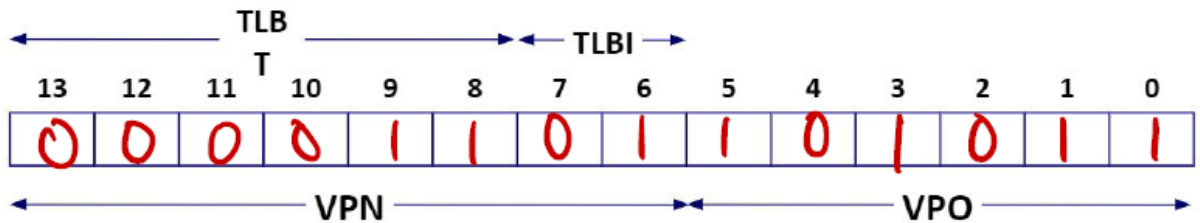
TLB Hit? ____ Page Fault? ____ PPN ____

Physical Address:



CT ____ CI ____CO ____ Cache Hit? ____ Data (byte) ____

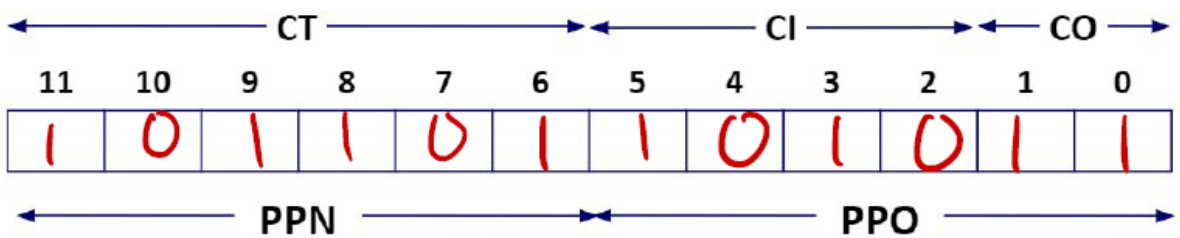
4. **Virtual Address: 0x036B**



VPN 0x0D TLBT 0x03 TLBI 0x1

TLB Hit? Yes Page Fault? No PPN 0x2D

Physical Address:



CT 0x2D CI 0xA CO 0x3 Cache Hit? Yes Data (byte) 0x3B

Virtual Memory Table

VA width (n)	PA width (m)	Page size (P)	VPN width	PPN width	Bits in PTE (assume V, D, R, W, X)
32	32	16 KiB			
32	26			13	
	32		21		22
		32 KiB	25		26
64			48		29

Virtual Memory Table

VA width (n)	PA width (m)	Page size (P)	VPN width	PPN width	Bits in PTE (assume V, D, R, W, X)
32	32	16 KiB	18	18	23
32	26	8 KiB	19	13	18
36	32	32 KiB	21	17	22
40	36	32 KiB	25	21	26
64	40	64 KiB	48	24	29

$p = \log_2 P$, VPN width $= n - p$, PPN width $= m - p$,

bits in PTE = PPN width + 5.

References

- https://www.youtube.com/watch?v=2i2N_Qo_FyM&list=PLEbnTDJUr_If_BnzJkkN_J0Tl3iXTL8vq
- https://www.geeksforgeeks.org/multilevel-paging-in-operating-system/
- https://ravindrababuravula.com/index.php