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Acquisition and Analysis of Neural Data, SS 2010
Tutorial Exercise Sheet 4
14 May 2010

Note: This exercise sheet covers two weeks' amount of work, so it is also going to be weighted as two sheets when summing up all your points for exercises.

Analytical exercises: Reverse correlation methods and spike train decoding

1. Firing rate estimate Reverse correlation methods can be used to construct an estimate $r_{est}(t)$ of the firing rate $r(t)$ evoked by a stimulus $s(t)$. The firing rate at any given time can be expressed as a weighted sum of the values taken by the stimulus at earlier times:

$$r_{est}(t) = r_0 + \int_0^\infty D(\tau)s(t-\tau)\,\mathrm{d}\tau$$

where r_0 accounts for background firing when $s = 0$, and $D(\tau)$ is a weighting function (also called *linear kernel*).

a) Write down two important differences between the above expression for the estimated firing rate and the "approximate" firing rate introduced in Chapter 1 ($r_{approx}(t) = \int_{-\infty}^{\infty} w(\tau)\rho(t-\tau)\,\mathrm{d}\tau$).

b) The above equation for the estimated firing rate (Eq. 2.1 in Dayan and Abbott) is unbounded. If we wanted to maximize this expression (as is done in the section 2.2 "The most effective stimulus" from Dayan and Abbott), we would need a constraint. The constraint used there is that the time integral of the square of the stimulus over the duration of the trial (simulus energy) is held fixed. (See Appendix B of chapter 2).
Calculate the energy for a sinusoidal stimulus of arbitrary amplitude A, frequency f and phase ϕ, over a trial of length $T \gg 1/f$.

2. Static Nonlinearities Sigmoidal functions represent a class of static nonlinearities and are very commonly used in science. For example, they are useful for compressing signals. Particularly in spike train statistics, they can be used to introduce saturation into estimates of neural responses.

a) Why is saturation useful/necessary?

b) An example of a sigmoidal function is Eq. 2.10 (Dayan and Abbott) which is described by

$$F(L) = \frac{r_{max}}{1 + e^{(g_1(L_{1/2} - L))}}$$

where r_{max} is the firing rate, $L_{1/2}$ is the value of L for which F achieves half of this maximal value, and g_1 determines how rapidly the firing rate increases as a function of L.
Show that this function indeed saturates, i.e, it does not grow with outbound.
Hint: Calculate an appropriate limit for $F(L)$.

c) Taking $g_1 = 1$ and $L_{1/2} = 10$, calculate the values of L for which $F(L)$ reaches 75% and 95% of r_{max}. Sketch $F(L)$.

3. Spike Train Decoding The stimulus estimate is constructed as a linear sum over all spikes. A spike occurring at time t_i contributes a kernel $K(t - t_i)$, and the total estimate is obtained by summing over all spikes,

$$s_{est}(t - \tau_0) = \sum_{i=1}^{n} K(t - t_i) - \langle r \rangle \int_{-\infty}^{\infty} K(\tau)\, \mathrm{d}\tau$$

Prove that the time average $s_{est}(t - \tau_0) = \frac{1}{T} \int_0^{\tau} s_{est}(t - \tau_0)\, \mathrm{d}t \overset{!}{=} 0$.

Hints: There is more than one way to prove this. Keep in mind: There is only one trial involved, so $\langle r \rangle = r$. Use the definition of r. Also remember that $\tau \ll T$, and in particular, the width of the kernel $K(\tau)$ is small compared to the length of the trial T.

Evaluation

Return your written solutions at the beginning of the next lecture (in **two** weeks, 28 May 2010). Do not forget your name and student number! If you will not be able to attend, you will need to return the solutions to the tutors' office (ITB, room 2316) by Thursday 27 May. Moreover, we would also like you to specify how much time you needed to finish the exercise.

Numerical Exercises: Simulating Responses of the H1 Neuron

Download the file c1p8.mat from the course webpage. It contains data from a fly H1 neuron responding to an approximate white-noise visual motion stimulus (data collected and provided by Rob de Ruyter van Steveninck). Data were collected for 20 minutes at a sampling rate of 500 Hz (time steps $\Delta t = 2\,\mathrm{ms}$). In the file, rho is a vector that gives the sequence of spiking events or nonevents at the sampled times (every 2 ms). When an element of rho is one, this indicates the presence of a spike at the corresponding time, whereas a zero value indicates no spike. The variable stim gives the sequence of stimulus values at the sampled times in units of degrees per second.

1. Spike-Triggered Average. Calculate and plot the spike-triggered average

$$C(\tau) = \frac{1}{n}\int_0^{\mathrm{T}} \rho(t)s(t-\tau)\,\mathrm{d}t$$

from these data for τ from 0 to 300 ms (150 time steps). Take care that your STA has the correct orientation, τ increasing from right to left.

2. Linear kernel D. Use $C(\tau)$ to construct a linear kernel $D(\tau)$,

$$D(\tau) = \frac{\langle r\rangle C(\tau)}{\sigma_{\mathrm{s}}^2},$$

where $\langle r\rangle$ is the mean firing rate of the neuron (spike count rate), and σ_{s}^2 the variance of the stimulus (see also equation 2.6 in the book). Note that when you calculate σ_{s}^2, you have to take the sampling interval into account. Thus, $\sigma_{\mathrm{s}}^2 = \mathrm{var(stim)}\cdot\Delta t$.

3. Linear estimate r_{est} of the firing rate. Convolve $D(\tau)$ with the stimulus to provide a linear estimate r_{est} of the firing rate of the H1 neuron:

$$r_{\mathrm{est}}(t) = r_0 + \int_0^{\infty} D(\tau)s(t-\tau)\mathrm{d}\tau$$

(see also equation 2.1 in the book). When perfoming the convolution in Matlab with the command conv, you again have to account for the sampling interval: $\int D(\tau)s(t-\tau)\mathrm{d}\tau = \Delta t\cdot\mathrm{conv(D,stim)}$. Choose r_0 so that the average firing rate r_{est} predicted by the model in response to the stimulus used for the data matches the actual average firing rate $\langle r\rangle$. Plot one second (from $t = 10\,\mathrm{s}$ to $t = 11\,\mathrm{s}$) of $r_{\mathrm{est}}(t)$ and of the neuron's actual firing rate $r(t)$ (determined with a sliding rectangular window of 20 ms width).

4. Comparison of the actual spike train with a Poisson spike train. Use a Poisson spike generator with $r_{\mathrm{est}}(t)$ to generate a synthetic spike train.

a) Plot examples of the actual and synthetic spike trains. Describe how are they similar and how do they differ.

b) Plot the autocorrelation function of the actual and the synthetic spike trains over the range 0 to 100 ms. Why is there a dip at a lag of 2 ms in the autocorrelation of the actual spike train? Is there a dip for the synthetic train too?

c) Plot the interspike interval histogram for both spike trains. Why is there a dip below 6 ms in the histogram for the actual spike train? What are the coefficients of variation for the two spike trains and why might they differ?

Note: These exercises are adapted from exercises provided on the webpage of Larry Abbott. They are all based on problems from Sebastian Seung.

Evaluation

Send Paula your program code by e-mail until Wednesday 27 May - each person their own code. In your program code, please use clear, descriptive variable names and comments, and include your name. If you cooperated with someone, mention it. Moreover, we would also like to ask you how much time you needed to finish the exercise.

You can use other programming languages than Matlab - in this case you will need to return both your (very clear) code and the plots specified in the exercises (in .jpg or .eps form). If you need the data in .dat-form please contact us.

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exercise webpage: http://itb.biologie.hu-berlin.de/∼kuokkane/Teaching/2010_SS/index.htm
or: http://sites.google.com/site/aand2010tutorial/