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Acquisition and Analysis of Neural Data, SS 2010
Tutorial Exercise Sheet 3

7 May 2010

Analytical exercises: Spike train AutoCorrelation and CrossCorrelation

The spike train autocorrelation function is defined as:

$$Q_{\rho\rho}(\tau) = \frac{1}{T}\int_0^T \langle \rho(t)\rho(t+\tau)\rangle \, \mathrm{d}t - \langle r\rangle^2 \quad .$$

It is useful for detecting patterns in spike trains, such as oscillations. The spike train crosscorrelation function is defined as:

$$Q_{\rho\rho'}(\tau) = \frac{1}{T}\int_0^T \langle \rho(t)\rho'(t+\tau)\rangle \, \mathrm{d}t - \langle r\rangle\langle r'\rangle \quad .$$

Similarly, the spike train crosscorrelation function is useful for determining relationships between two spike trains, such as whether they fire synchronously or in a phase-locked manner.

1. Poisson Autocorrelation histogram. Sketch the autocorrelation histogram N_m (defined in class) for a Poisson process with refractory period of 3 ms. Although it is only an approximate sketch, you must mark the relevant values on the appropriate axes. Take $\Delta t = 1$ ms and $T = 1$ s, and $\langle r\rangle = 100$ Hz. What would happen if the firing rate would exceed 300 Hz? And what if it would be lower than 10 Hz?

2. Analytical correlation. Calculate the (spike train) cross correlation function $Q_{\rho_1\rho_2}(\tau)$ for the spike trains whose firing rates are given by:

$$\begin{aligned} r_1(t) &= \big(1 + a_1 \sin(\omega\, t)\big)\, r_1 \\ r_2(t) &= \big(1 + a_2 \sin(\omega\, t + \phi)\big)\, r_2 \end{aligned} \qquad (1)$$

where a_1, a_2, ϕ, r_1 and r_2 are constants.

Hint 1: Remember that we can replace the trial-averaged neural response function with the firing rate within any "well-behaved" integral.
Hint 2: The following trigonometric identity might be useful:
$\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

3. Autocorrelation property

a) Prove the symmetry property of the spike train autocorrelation function, i.e., $Q_{\rho\rho}(\tau) = Q_{\rho\rho}(-\tau)$
Hint: Extend the limits of the defining integral to $\pm\infty$ and use a substitution.

b) How can you justify this extension?

Evaluation

Return your written solutions at the beginning of the next lecture (14 May 2010). Do not forget your name and student number! If you will not be able to attend, you will need to return the solutions to the tutors' office (ITB, room 2316) by Wednesday 12 May (since May 13 is a holiday). Moreover, we would also like you to specify how much time you needed to finish the exercise.

Numerical Exercises:
Homogeneous Poisson Process, Inhomogeneous Poisson Process and Absolute Refractoriness in a Homogeneous Poisson Process

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Download PoissonSpikeTimes.mat and, optionally, Hints3.m from the course website. The mat-file includes three different variables for spike times in milliseconds: SpikeTimes for homogeneous process, SpikeTimes_inh for inhomogeneous process and SpikeTimes_ref for homogeneous process with refractoriness. For homogeneous and inhomogeneous Poisson processes these are vectors. For the one with refractoriness, the variable is a matrix for which 6 different driving rates r have been used. These driving rates are given in the variable rates_ref. The file generatePoissonTrains.m explains how these Poisson processes are generated.

1. Homogeneous and inhomogeneous Poisson process. For spikes generated by both the homogeneous and the inhomogeneous Poisson process (vectors SpikeTimes and SpikeTimes_inh):

a) construct and plot the ISI histogram,

b) compute the CV and the Fano factor $F(T)$ (for $T = 100$ ms, non-overlapping bins), and

c) compute and plot the spike-train autocorrelation function (suggested temporal resolution: 0.1 ms).

d) Why could the CV and the Fano factor $F(T)$ deviate from the theoretical value (1)?

2. Poisson process with refractory period. For the Poisson spike trains with absolute refractoriness (matrix SpikeTimes_ref)

a) determine the resulting effective firing rate r_{eff} (spike count rate) and plot r_{eff} against r,

b) construct and plot the ISI histograms for all values of r,

c) determine the CV and the Fano factor $F(T)$ for all rates r, and plot these against r_{eff}.

d) Moreover, calculate and plot the spike train autocorrelation function for all values of r.

You might be able to recycle your code from problem 1 if you handle each driving rate separately.

Finished Early?

Extra: Gamma Process. You can earn extra points with this exercise. You can build your code on the one in the file generatePoissonTrains.m.

A Gamma process of order n (with a constant rate) can be generated from a homogeneous Poisson process by picking only each n_{th} spike.

a) Generate Gamma processes of order 2, 3, 5, and 10, each with a rate of $r = 100\,\text{Hz}$ (the rate of the underlying homogeneous Poisson process thus needs to be adjusted accordingly). For each Gamma process, generate 1000 spikes.

b) Construct the ISI histograms and calculate the CVs and compare your results to the analytical result

$$p_{\text{ISI}}(\tau) = \frac{r(r\tau)^k e^{-r\tau}}{k!}$$

where k is the order of the Gamma process.

c) Compile your results into a figure where the left column shows example spike trains of 100 ms length of each Gamma process, and the right column shows the corresponding ISI distributions, with the CV values next to them.

Evaluation

Send Paula your program code by e-mail until next Wednesday - each person their own code. In your program code, please use clear, descriptive variable names and comments, and include your name. If you cooperated with someone, mention it. Moreover, we would also like to ask you how much time you needed to finish the exercise.

You can use other programming languages than Matlab - in this case you will need to return both your (very clear) code and the plots specified in the exercises (in .jpg or .eps form). If you need the data in .dat-form please contact us.

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exercise webpage: http://itb.biologie.hu-berlin.de/~kuokkane/Teaching/2010_SS/index.htm