Problem 8.1 (3 points)
The entropy of a random vector $\mathbf{x}$ with probability density $p(\mathbf{x})$ is defined as

$$H(\mathbf{x}) = -E(log\, p(\mathbf{x}))$$

where E denotes the expectation. Consider an invertible transformation $\mathbf{y} = \mathbf{g}(\mathbf{x})$. Using the Jacobi determinant, find the relation between $H(\mathbf{y})$ and $H(\mathbf{x})$. Use this to show that the entropy is not scale invariant, i.e. $H(a\mathbf{x}) \neq H(\mathbf{x})$, for $a = const$.

Problem 8.2 (3 points)
Show that the entropy of a multivariate n-dimensional Gaussian random vector $\mathbf{x}$ with covariance matrix $\mathbf{\Sigma}$ has the form

$$H(\mathbf{x}_{Gauss}) = \frac{1}{2}\, log\, |det\, \mathbf{\Sigma}| + \frac{n}{2}\,(1 + log\, 2\pi)$$

Problem 8.3 (4 points)
The negentropy is defined as

$$J(\mathbf{x}) = H(\mathbf{x}_{Gauss}) - H(\mathbf{x})$$

where $\mathbf{x}_{Gauss}$ is a multivariate Gaussian with the same covariance matrix as $\mathbf{x}$. Show that the negentropy is invariant for invertible linear transformations $\mathbf{y} = \mathbf{A}\mathbf{x}$, i.e.

$$J(\mathbf{A}\mathbf{x}) = J(\mathbf{x})$$

from which it follows that the negentropy is scale-invariant.

Total points: 10

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