Models of Higher Brain Functions: Analytic Tutorial

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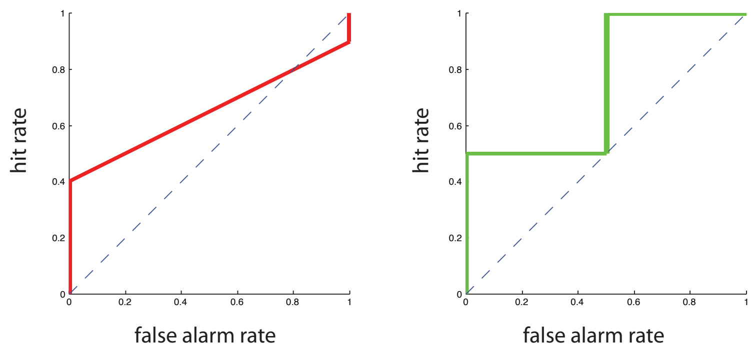
4 Signal Detection Theory

4.1 ROC Curves and PDFs

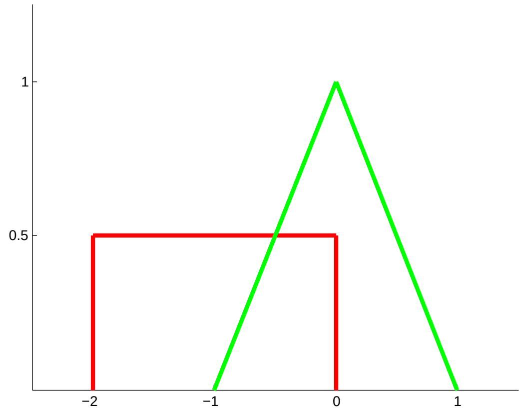
The graphs below show two ROC curves from a yes-no experiment in which the prior probability of a signal and noise event, $p_s = p_n = 0.5$. Furthermore, the costs associated with hits, false alarms, misses and correct rejections were equal.

For each ROC curve draw *two* sets of underlying densities $s(x)$ and $n(x)$ that would have resulted in the ROC curves shown—thus, in total, draw four pairs of possible $s(x)$ and $n(x)$ distributions. (The area under probability densities integrates to one, so try to ensure that the $s(x)$ and $n(x)$ pairs have roughly the same integral, please!) Explain why the ROC curve does not uniquely determine the densities $s(x)$ and $n(x)$. What is missing?



4.2 Hits, False Alarms and the ROC Curve

Given are the signal+noise density $s(x)$ in green as well as the noise-only density $n(x)$ in red as shown in the following graph:



In mathematical notation we can write:

$$n(x) = \begin{cases} 0 & \text{if } x < -2 \\ 0.5 & \text{if } -2 \leq x \leq 0 \\ 0 & \text{if } x > 0 \end{cases}$$

and

$$s(x) = \begin{cases} 0 & \text{if } x < -1 \\ x+1 & \text{if } -1 \leq x \leq 0 \\ 1-x & \text{if } 0 < x \leq 1 \\ 0 & \text{if } x > 1 \end{cases}$$

Calculate hits and false alarms for an observer placing her criterion λ at $\lambda = -3, -1, -0.5, \frac{\sqrt{2}-2}{2}, 0, \frac{2-\sqrt{2}}{2}, 0.5$ and 2.
Solve the exercise by first integrating $n(x)$ and $s(x)$ and then calculating the corresponding areas of $N(x)$ and $S(x)$ to the right of the criterion λ.
Draw the ROC curve for the observer.