Models for Higher Brain Function

Exercise Sheet 5: Observer Models in Signal Detection

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In signal detection theory, it is typically assumed that an observer performs a psychophysical task by monitoring an internal random variable X. The distribution of X is typically assumed to depend on the input stimulus S. In this exercise sheet, we will only consider detection experiments, in which S can only take one of two values — $S = 1$ implies that a signal was presented, $S = 0$ implies that no signal was persented.

In order to use signal detection theory to understand psychophysical data, it is necessary to define a rule that maps different realizations of X to different responses R.

1 High threshold model

Threshold models assume that the interanal random variable X can only take two values — $X = 1$, the "detect" state, or $X = 0$, the "non-detect" state. The characterizing assumption of the high threshold model is that a no signal stimulus can never trigger a detect state. In contrast, a signal stimulus can trigger a detect state with a certain probability p. Based on the value of X, an observer can also "guess". Guessing in this case means that the observer can choose to respond "yes" on some non-detect trials (being aware of the fact that $X = 0$ doesn't necessarily imply "no signal present").

- Write a function that simulates a signal detection experiment for an observer that follows the assumptions of the high threshold model.
- Use this function to trace out an ROC curve for the cases that $p = 0.3$, $p = 0.5$, and $p = 0.8$. Use 300 signal trials and 300 noise trials per simulated experiment and simulate 7 different experiments (i.e. guessing rates). How are these function related?

- How can you determine p from the ROC graph?

2 Low threshold model

The low threshold model is similar to the high threshold model: It also assumes $X \in \{0, 1\}$ and interpolates different trategies by guessing. However, the low threshold model also allows "detect" states to be triggered by no signal trials. Thus, there are two probabilities:

1. p_1 is the probability that a detect state was triggered by a signal stimulus,

2. p_0 is the probability that a nondetect state was triggered by a no-signal stimulus.

In addition, we have two guessing rates now:

1. γ_1 the probability to respond "yes" although $X = 0$,

2. γ_0 the probability to respond "no" although $X = 1$.

Note that any strategy in the low threshold model will have either $\gamma_1 = 0$ or $\gamma_0 = 0$ (Why?)!

Perform the same analysis as for the high threshold model. Instead of using $p = 0.3$, $p = 0.5$, $p = 0.8$, use $p := (p_1, p_0)$, with $p = (0.5, 0.3)$, $p = (0.6, 0.3)$, $p = (0.5, 0.2)$.

3 Equal variance gaussian model

A very prominent model assumes that X is a real random variable. Typically, X is given in terms of conditional distributions. If $S = 0$, $X \sim \mathcal{N}(0, 1)$ and if $S = 1$, $X \sim \mathcal{N}(d\prime, 1)$. There are two different strategies to map the values of X to responses:

1. A criterion value $c \in \mathbb{R}$ is selected and the observer responds "yes", whenever $X > c$.

2. The observer internally calculates a likelihood ratio

$$\ell = \frac{\mathbb{P}(X|S=1)}{\mathbb{P}(X|S=0)},$$

   and responds "yes" whenever $\ell > c$.

Perform the same steps as for the high threshold model for both response strategies. Use $d\prime = 0.5$, $d\prime = 1$, $d\prime = 1.5$. What happens if you rescale the axes of the ROC graph: Use $\Phi^{-1}(p)$ instead of p. The functions Φ^{-1} is the "percent point function" of the Normal distribution and can be found in python as

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scipy.stats.norm.ppf
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Can you see from the simulation results, why the two strategies are equivalent?

4 Unequal variance gaussian model

Here, we relax the assumptions of the equal variance gaussian model. We still assume that for $S = 0$, $X \sim \mathcal{N}(0, 1)$, but for $S = 1$, we assume $X \sim \mathcal{N}(d, \sigma)$.

Perform the same steps as in the previous exercise. In this case, the two strategies are actually different. Can you see why? For what values of c can you observe clearly different experimental results?