Analysis of Electrical Circuits and Mechanical Systems with Fractional-order Elements: A Systems Theory Approach

Dissertation-I

By

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Outline

Introduction

- Integer Order System and its limitations.
- Fractional Order System and its approach.
- Basics of Fractional Order System.
- FO elements and its applications.
- Problem Definition.
- Conclusion.

Fractional Calculus

Real order generalization of fractional calculus

$$D^{\alpha} = \begin{cases} \frac{d^{\alpha}}{dt^{\alpha}} & \alpha > 0 \\ 1 & \alpha = 0 \\ \int_a^t (d\tau)^{\alpha} & \alpha < 0 \end{cases} \tag{1}$$

with $\alpha \in \mathcal{R}$.

Riemann-Liouville:

Integral:

$$J_c{}^{\alpha} f(t) = \frac{1}{\Gamma\alpha} \int \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau \tag{2}$$

Derivative:

$$D^{\alpha} f(t) = \frac{d^m}{dt^m} [\frac{1}{\Gamma(m-\alpha)} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha+1-m}} d\tau], m \in Z^+, m-1 < \alpha \leq m. \tag{3}$$

Grunwald-Letnikov:

Integral:

$$D^{-\alpha} = \lim_{h \longrightarrow 0} h^{\alpha} \sum_{m=0}^{(t-a)/h} \frac{\Gamma(\alpha + m)}{m!\Gamma(\alpha)} f(t - mh), \tag{4}$$

Derivative:

$$D^{\alpha} = \lim_{h \longrightarrow 0} \frac{1}{h^{\alpha}} \sum_{m=0}^{(t-a)/h} (-1)^m \frac{\Gamma(\alpha + 1)}{m!\Gamma(\alpha - m + 1)} f(t - mh), \tag{5}$$

Caputo:

$$D^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^m(\tau)}{(t-\tau)^{\alpha+1-m}} d\tau, \tag{6}$$

- Inductor or lossy coil have hysteresis and eddy current losses.
- Capacitors produces dielectric losses.
- FO inductors or FO capacitors yield a more accurate description.

FO Elements

Schematic of FO Capacitor

FOmodelRpztnOfossyC.pdf

The voltage-current relationships for an FO capacitor are

$$i_c(t) = C_\alpha \frac{d^\alpha}{dt^\alpha} v_c(t), \tag{7}$$

or

$$v_c(t) = \frac{1}{C_\alpha}\ {}_0J_t{}^\alpha i_c(t), \tag{8}$$

where the constant $0 < \alpha < 1, \alpha \in \mathbf{R}$ is a measure of the losses in the capacitor.

$$j^{-\alpha} = \cos(\alpha\pi/2) - j\sin(\alpha\pi/2) = e^{-j\alpha\pi/2} \tag{9}$$

FO Elements

Phase Relationship Between V and I for FO capacitor

PhaseRelationBtwnVandIforFOC.pdf

FO Elements

Schematic of FO Inductor

FOmodelofL.pdf

For an FO inductor we have,

$$V_L(t) = L_\beta \frac{d^\beta}{dt^\beta} i_L(t) \tag{10}$$

where the fractional power β is related to the phenomenon of proximity effect. In frequency domain,

$$V_L(s) = s^\beta L_\beta I_L(s) \tag{11}$$

$$j = \cos(\pi/2) + j\sin(\pi/2) = e^{j\pi/2} \tag{12}$$

$$j^\beta = \cos(\beta\pi/2) + j\sin(\beta\pi/2) = e^{j\beta\pi/2} \tag{13}$$

FO Elements

Phase Relationship Between V and I for FO inductor

PhasorDiagForFOL.pdf

Stability of FO System

- Fractional Domain
- Mapping of S-plane to F-plane.
- α increases the stable F-domain regions decreases.
- Stability decreases for $\alpha > 2$

Stability of FO

stability2.pdf

Problem Definition

R-Lckt.pdf

Transfer Function of FO

$$\frac{I(s)}{V(s)} = \frac{\frac{1}{L_\beta}}{S^\beta + \frac{R}{L_\beta}}. \tag{14}$$

Future Work

- Analysis of electrical circuits and mechanical systems with fractional-order elements.
- Development of various linear models.
- Stability analysis using Riemann sheet concept.
- Analysis of controllability, observability, reacheability, and solution of the state-equation.
- Design of linear compensators and controllers.
- Development of equivalent relationship between FO electrical and mechanical systems.

Thank You...