Analysis of Electrical Circuits and Mechanical Systems with Fractional-order Elements: A Systems Theory Approach

Dissertation-I

Ву

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Outline

- Introduction.
- What is Fractional Calculus.
- Fractional Order (FO) Elements.
- Circuit With FO Elements.
- Fractional Order Transfer Function.
- Stability Of FO System.
- Problem Definition.

Introduction

- Integer Order System and its limitations.
- Fractional Order System and its approach.
- Basics of Fractional Order System.
- FO elements and its applications.
- Problem Definition.
- Conclusion.

Fractional Calculus

Real order generalization of fractional calculus

$$D^{\alpha} = \begin{cases} \frac{d^{\alpha}}{dt^{\alpha}} & \alpha > 0\\ 1 & \alpha = 0\\ \int_{a}^{t} (d\tau)^{\alpha} & \alpha < 0 \end{cases}$$
 (1)

with $\alpha \in \mathcal{R}$.

Riemann-Liouville:

Integral:

$$J_c^{\alpha} f(t) = \frac{1}{\Gamma \alpha} \int \frac{f(\tau)}{(t - \tau)^{1 - \alpha}} d\tau$$
 (2)

Derivative:

$$D^{\alpha}f(t) = \frac{d^{m}}{dt^{m}} \left[\frac{1}{\Gamma(m-\alpha)} \int_{a}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha+1-m}} d\tau\right], m \in \mathbb{Z}^{+}, m-1 < \alpha \leq m.$$
(3)

Grunwald-Letnikov:

Integral:

$$D^{-\alpha} = \lim_{h \to 0} h^{\alpha} \sum_{m=0}^{(t-a)/h} \frac{\Gamma(\alpha+m)}{m!\Gamma(\alpha)} f(t-mh), \tag{4}$$

Derivative:

$$D^{\alpha} = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{m=0}^{(t-a)/h} (-1)^m \frac{\Gamma(\alpha+1)}{m!\Gamma(\alpha-m+1)} f(t-mh), \tag{5}$$

Caputo:

$$D^{\alpha}f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^m(\tau)}{(t-\tau)^{\alpha+1-m}} d\tau, \tag{6}$$

- Inductor or lossy coil have hysteresis and eddy current losses.
- Capacitors produces dielectric losses.
- FO inductors or FO capacitors yield a more accurate description.

Schematic of FO Capacitor

The voltage-current relationships for an FO capacitor are

$$i_c(t) = C_\alpha \frac{d^\alpha}{dt^\alpha} v_c(t), \tag{7}$$

or

$$v_c(t) = \frac{1}{C_\alpha} \, _0 J_t{}^\alpha i_c(t), \tag{8}$$

where the constant $0<\alpha<1, \alpha\in\mathbf{R}$ is a measure of the losses in the capacitor.

$$j^{-\alpha} = \cos(\alpha \pi/2) - j\sin(\alpha \pi/2) = e^{-j\alpha \pi/2}$$
(9)

Phase Relationship Between V and I for FO capacitor

 ${\tt Phase Relation Btwn Vand If or FOC.pdf}$

Schematic of FO Inductor

For an FO inductor we have,

$$V_L(t) = L_\beta \frac{d^\beta}{dt^\beta} i_L(t) \tag{10}$$

where the fractional power β is related to the phenomenon of proximity effect. In frequency domain,

$$V_L(s) = s^{\beta} L_{\beta} I_L(s) \tag{11}$$

$$j = \cos(\pi/2) + j\sin(\pi/2) = e^{j\pi/2}$$
 (12)

$$j^{\beta} = \cos(\beta \pi/2) + j\sin(\beta \pi/2) = e^{j\beta \pi/2}$$
(13)

Phase Relationship Between V and I for FO inductor $\,$

 ${\tt PhasorDiagForFOL.pdf}$

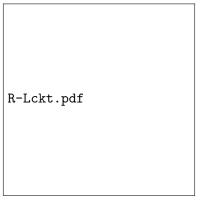
Stability of FO System

- Fractional Domain
- Mapping of S-plane to F-plane.
- ullet α increases the stable F-domain regions decreases.
- Stability decreases for $\alpha > 2$

Stability of FO

stability2.pdf

Problem Definition



Transfer Function of FO

$$\frac{I(s)}{V(s)} = \frac{\frac{1}{L_{\beta}}}{S^{\beta} + \frac{R}{L_{\beta}}}.$$
 (14)

Future Work

- Analysis of electrical circuits and mechanical systems with fractional-order elements.
- Development of various linear models.
- Stability analysis using Riemann sheet concept.
- Analysis of controllability, observability, reacheability, and solution of the state-equation.
- Design of linear compensators and controllers.
- Development of equivalent relationship between FO electrical and mechanical systems.



Thank You...