Exploring the Resolution of Delegations in Liquid Democracy with Fractional Delegation

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Structure

- Introduction
- Problem Statement
- Design
- Implementations
- Evaluation

Liquid Democracy: Introduction & Motivation

Liquid Democracy – Fractional Delegation

- Currently: one person, one delegation
- Fractional delegation: one person, many delegations

- Motivation:
  - Less concentration of voting power
  - Less loss of voting power
  - Empowers voters

- Drawbacks:*
  - Less intuitive
  - Not computationally trivial

Problem statement

- Given:
  - Voters who vote directly
  - Voters who delegate their vote
- Find:
  - Each voter‘s final voting power according to the delegations
  - Power must be conserved*

- Challenge: cyclic delegations & efficient computation

Design

- Definition: Well-formed delegation graph:
  - 1. Delegation graph with 2. no closed delegation cycles

1. $V = S \dot{\cup} D$, meaning that V is the union of the two *disjoint* sets of sinks and delegators.
2. Each edge $e \in E$ is a triple (u, v, w) denoting a delegation from node u to node v of weight w.
3. Each sink $s \in S$ has no outgoing edges.
4. Each delegator $d \in D$ has $n \in \mathbb{N}$ outgoing edges, each with a positive weight, such that the sum of all of its outgoing edge weights equals 1.

We define a **closed delegation cycle** $C \subseteq V$ in a delegation graph $G = (S \dot{\cup} D, E)$ as a cycle in G such that for every node $v \in C$, there exists no path from v to any sink node in S.

Design: Resolving Delegations

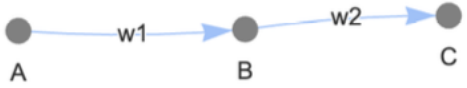
- For all nodes v:

$$p'_v = 1 + \sum_{(u,v,w)\in E} w p'_u$$

$$p_v = \begin{cases} p'_v & \text{if } v \in S \\ 0 & \text{if } v \in D \end{cases}$$

- System of linear equations

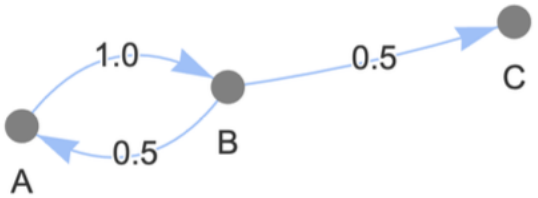
- A well-formed delegation graph:
  - Has exactly one solution,
  - which conserves power



$$\begin{aligned} p_C &= 1 + w_2 p'_B \\ &= 1 + w_2(1 + w_1 p'_A) \\ &= 1 + w_2(1 + w_1 \cdot 1) \end{aligned}$$

Implementations

- Linear Systems Solver
- Linear Programming Solver
- Iterative Solver
  - Provably similar to the other approaches

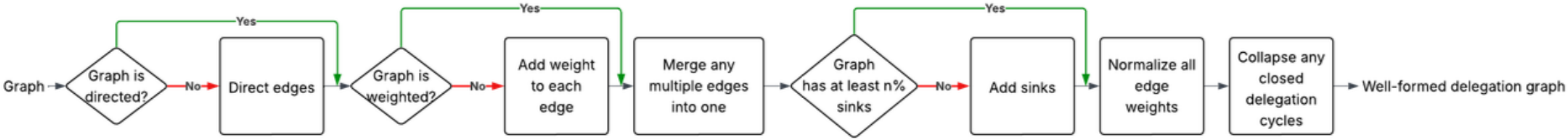


Implementations: Robustness

	Invalid Delegations	Closed Delegation Cycles
Linear Systems Solver	Invalid solution	Error
Linear Programming Solver	Invalid solution	Error
Iterative Solver	Depends	Won’t terminate

Evaluation: Method

- Preprocessing

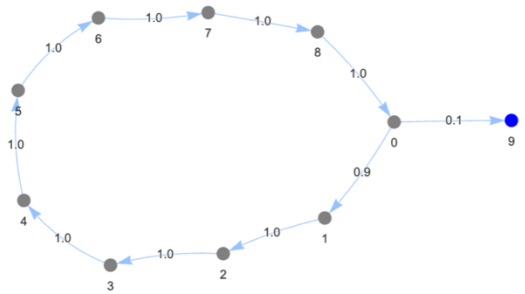


  - *The „Add sinks“ step is only for experimental purposes, to have the option to avoid graphs with no sinks for benchmarks*

- Measurement
  - of solving time only

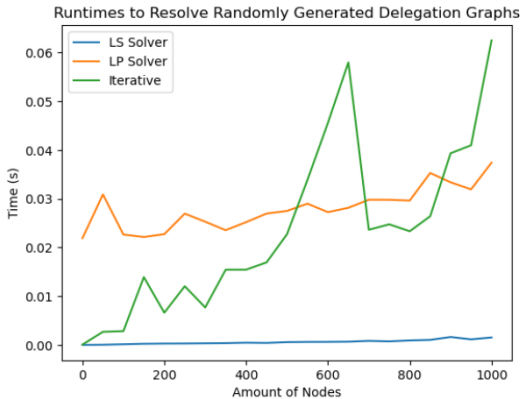
Evaluation: Overview

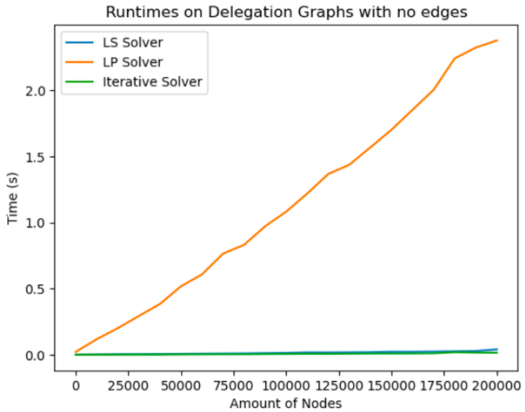
- Synthetic graphs
  - Small graphs
  - Large graphs
  - Dense graphs
  - Cycles which retain a lot of their power
  - No delegations
- Synthetic Social graphs
  - Small world graphs
  - R-Mat graphs
- Real-World Datasets
  - Bitcoin OTC trust network, Epinions, Slashdot Zoo

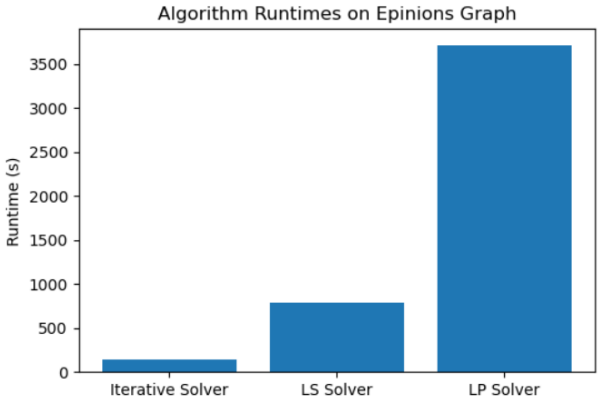


Evaluation: Key Insights

- Small graphs:
  - Linear Systems Solver
- Very sparse, very large graphs:
  - Iterative Solver (not perfectly precise!)
  - Linear Programming Solver‘s runtime grows slower, but it is slower



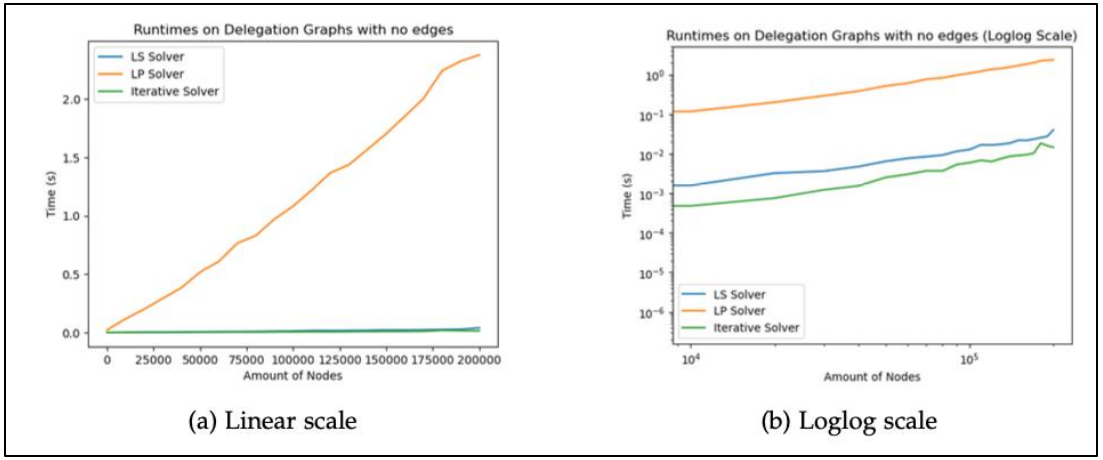




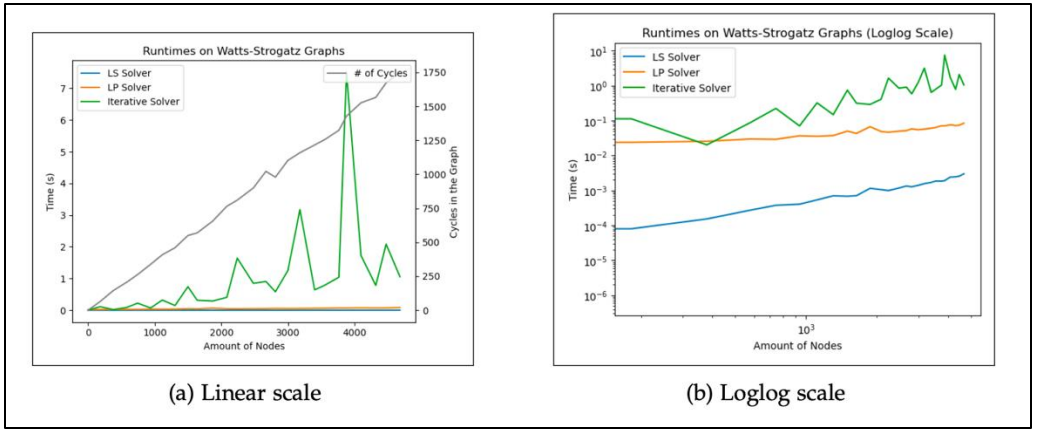
For the Future

- Does fractional delegation actually reduce vote power concentration?
- Benchmarking on real world data
- User study:
  - How easy is it to understand Liquid Democracy with fractional delegation?
  - Do / How do people delegate?

Thank you!



(a) Linear scale (b) Loglog scale



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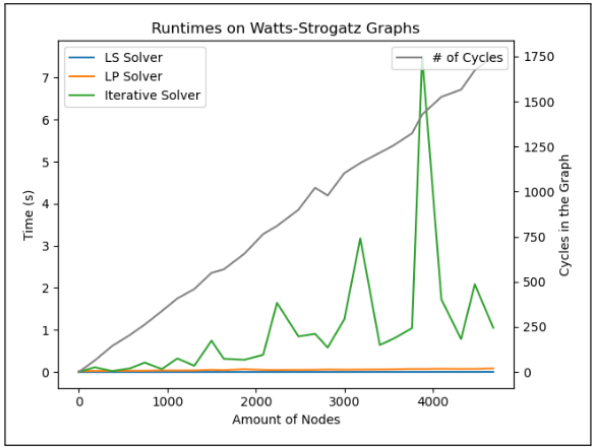
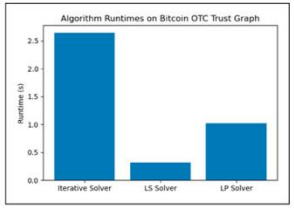
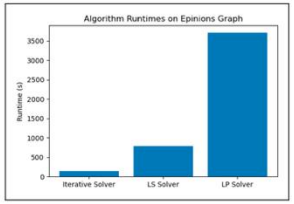


(a) Linear scale

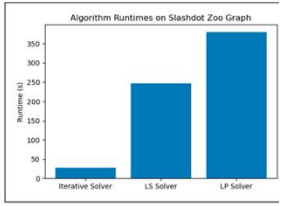
Thank you!



(a) Bitcoin OTC dataset

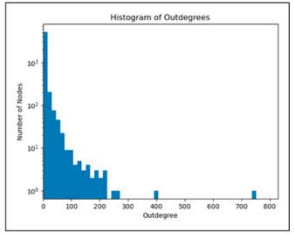


(b) Epinions dataset



(c) Slashdot Zoo dataset

Figure 5.17.: Runtimes



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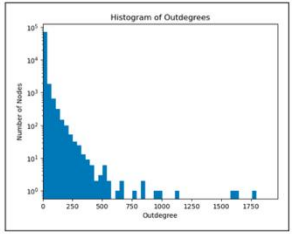

(b) Epinions dataset

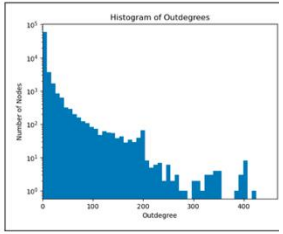

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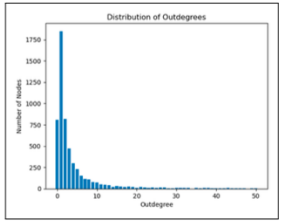
Figure 5.18.: Histogram of outdegrees



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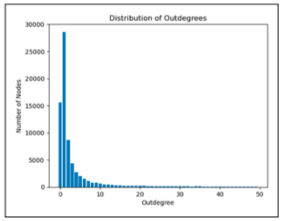
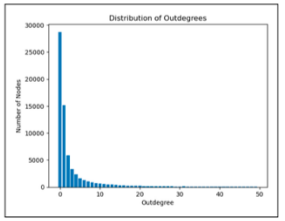
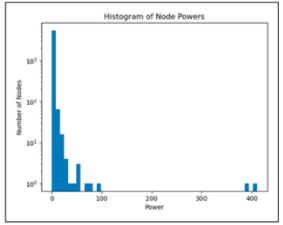


(b) Epinions dataset



(c) Slashdot Zoo dataset

Figure 5.19.: Distribution of outdegrees between 0 and 50



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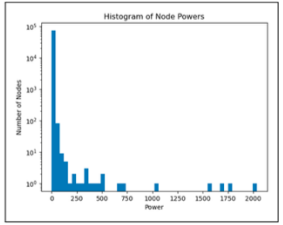
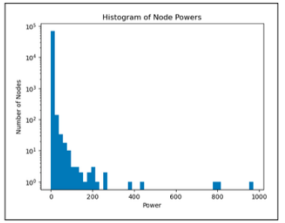


(b) Epinions dataset



(c) Slashdot Zoo dataset

Figure 5.20.: Distribution of powers