Log.Reg (1)

Logistic Regression

Perceptron Trick

x_0 add.

x_0	GPA	IQ	Placed
1	7.5	81	1
1	8.9	109	1
1	7.0	89	0

$$w_0 + w_1 x_1 + w_2 x_2 = 0$$

Epoch = 1000

$\eta = 0.01$

$$w_0 x_0 + w_1 x_1 + w_2 x_2 = 0$$

$$\sum_{i=0}^{n} w_i x_i$$

$$\boxed{\begin{array}{l} \text{if } x_i \in N \text{ and } \sum_{i=0}^{2} w_i x_i \geq 0 \\ \quad w_{new} = w_{old} - \eta x_i \\ \text{if } x_i \in P \text{ and } \sum_{i=1}^{2} w_i x_i < 0 \\ \quad w_{new} = w_{old} + \eta x_i \end{array}}$$

$$\boxed{w_{new} = w_{old} + \eta (y_i - \hat{y}) x_i}$$

y	$\hat{y}_i$	$y_i - \hat{y}$
1	1	0
0	0	0
1	0	1
0	1	-1

Logistic Regression

Sigmoid Function

$$\boxed{W_n = w_0 + (y - \hat{y}_i)\, x_i}$$

try to this $(y_i - \hat{y}_i) \neq 0$ (try to this).

because of Step funⁿ this happen

So use Sigmoid funⁿ

$$\sigma(z) = \frac{1}{1+e^{-z}} \qquad -\infty < z < \infty$$

$$0 < y < 1$$

give Now Sigmoid funⁿ.

$\sigma(z) = 0.5$

if 1 $\sigma(z)\ 0.5 >$

0 $\sigma(z)\ 0.5 <$

$$W_n = w_0 + \eta (y_i - \hat{y}_i)\, x_i$$

$$\hat{y}_i = \sigma(z) \qquad z = \Sigma\, w_i x_i$$

Still we will not get good Result.

Solved exp LgR-1

~~Question~~

Logistic Regression

$$\boxed{w_{new} = w_{old} + \eta (y_i - \hat{y}_i) x_i}$$

⇓ logistic fun

$$\sigma(\text{sigmoid}\ z) = \boxed{\frac{1}{1+e^{-z}}}$$

Where

$$\boxed{z = \sum w_i x_i}$$

	Add x_0	C.G.P.A (w)	I.Q	Placed
1	1	7	110	0
2	1	8.5	120	1
3	1	6	105	0
4	1	9	130	1
5	1	7.5	115	1

$$\boxed{XW = Y}$$

initial $\boxed{w = 1}$

Step-1 (1) $z = w_0 x_{01} + w_1 x_{11} + w_2 x_{21}$

w_0, w_1, w_2 all are 1

$z = 1*1 + 1*7 + 1*110 = 110$

$\sigma(z) = \frac{1}{1+e^{-110}} \approx 1$ (Pred).

Error = $(\underset{y_i - \hat{y}_i}{0 - 1} = -1)$.

update =

$w_{0\_new} = 1 + 0.01 * 1 * (0-1) = 0.99$

$w_{1\_new} = 1 + 0.01 * 7 * (0-1) = 0.93$

$w_{2\_new} = 1 + 0.01 * (0-1)\ 110 = -0.1$

Solved Exp Log Reg

Step-2 $(8.5, 120, 1)$

$z = w_0 x_{02} + w_1 x_{12} + w_2 x_{22}$.

$w_0 = 0.99, \; w_1 = 0.93, w_2 = -0.1$

$z = 0.99 * 1 + 0.93 * 8.5 + (-0.1) * 120 = -3.12$

$\sigma(z) = 0.0425$

$Error = (1 - 0.425)$.

$w_{0new} = 0.99 + 0.01 * (1 - 0.0425) * 1 = 0.996$

$w_{1new} = 0.93 + 0.01(1 - 0.0425) * 8.5 = 1.01$

$w_{2new} = -0.1 + 0.01(1 - 0.0425) * 120 \approx 1.04$

Step-3 $(6, 105, 0)$

$w_0 = 0.996, \; w_1 = 1.01 \; w_2 = 1.04$

$z = 0.996 * 1 + 1.01 * 6 + 1.04 * 105 = 117.20$

$\sigma(z) \approx 1$

$Error = (y_i - \hat{y}_i) = (0 - 1) = -1$

$w_{0new} = 0.996 + 0.01 * (-1) * 1 = 0.989$

$w_{1new} = 1.01 + 0.01 * (-1) * 6 = 0.95$

$w_{2new} = 1.04 + 0.01 * (-1) * 105 = 0.99$

Step-4 $(9, 103, 1)$ $w_0 = 0.989 \; w_1 = 0.95, \; w_2 = 0.99$

$z = 0.98 * 1 + 0.95 * 9 + 0.99 * 103 = 139.409$

$\sigma(z) = 1$

$Error = (y_i - \hat{y}_i) = (1 - 1) = 0$ (No. Change).

Same. weight

Step 5 $(7.5, 115, 1)$ $w_0 = 0.98, \; w_1 = 0.95, \; w_2 = 0.99$

$z = 0.98 * 1 + 0.95 * 7.5 + 0.99 * 115 = 122.99$

$\sigma(z) \approx 1$

$Error = 1 - 1 = 0$ (No change)

final weight = $\boxed{w_0 = 0.98 \quad w_1 = 0.95 \quad w_2 = 0.99}$

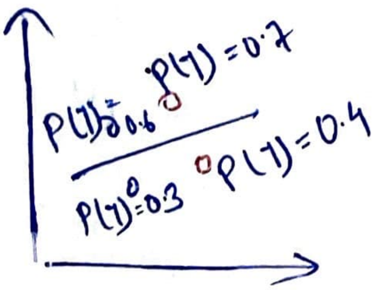
③ Binary Cross Entropy

Log Reg

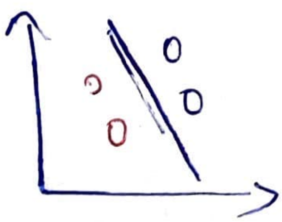
①

Find the Loss fun^n of logistic Regression.

① Perceptron Trick
② Sigmoid fun^n
③ Binary Cross Entropy (Max likelihood)
④ SoftMax



Model-1



Model-2

Maximum Likelihood

take Case $\boxed{\hat{y} = \sigma(z)}$ ← Sigmoid fun^n.

⇓

find each data Point based on Sigmoid fun^n of Predicted Value of y

M1 = $0.7 * 0.4 * 0.4 * 0.8$ = 0.089

M2 = $0.7 * 0.6 * 0.6 * 0.7$ = 0.176

$\boxed{\text{take Maxlikelihood } (M1, M2) = M2}$

Problem But answer is very small (Number)
So based on that I can do with the help of log.

$\log(M1) = -\log(0.7) - \log(0.4) - \log(0.4) - \log(0.8)$

⇓ this Number always (0 - 1) so log always be (-ve) so we convert into $\boxed{-\log(a)}$

$\boxed{\text{Cross Entropy}}$

Bing Cross entropy - (2) Log-Rg

so we want to reduce cross Entropy value

~~so we~~ because $(\log(0.1) > \log(0.4))$

so Cross Entropy we can write

$$\boxed{-\log(\hat{y}_1) - \log(\hat{y}_2) - \log(\hat{y}_3) - \log(\hat{y}_4)}$$

⇓

but It is Not correct

inplace of

$$\boxed{-\log(\hat{y}_i) = -y_i \log(\hat{y}_i) - (1-y_i)\log(1-\hat{y}_i)}$$

so Loss fun will

$$L = \sum_{i=1}^{n} -y_i \log(\hat{y}_i) - (1-y_i)\log(1-\hat{y}_i)$$

$$\boxed{L = -\frac{1}{n}\sum_{i=1}^{n} y_i \log(\hat{y}_i) + (1-y_i)\log(1-\hat{y}_i)}$$

⇓

* final log-loss fun
* Binary Cross Entropy

we have to minimize the Error

so we have to deferenciate above Equn.

Binary Cross Entropy ③

$$L = -\frac{1}{n}\sum y_i \log(\hat{y}_i) + (1-y_i)\log(1-\hat{y}_i)$$

Minimize the Loss

$\boxed{\sigma(W \cdot X) = \hat{Y}_i}$ → It is sigmoid fun

$$\sum_{i=1}^{n} y_i \log(\hat{y}_i) = Y_1\log(\hat{Y}_1) + Y_2\log(\hat{Y}_2) + Y_3\log(\hat{Y}_3) + Y_4\log(\hat{Y}_4)$$

$$\Downarrow$$

$$= [Y_1 \; Y_2 \; Y_3 \cdots Y_n] \begin{bmatrix} \log(\hat{Y}_1) \\ \log(\hat{Y}_2) \\ \vdots \\ \log(\hat{Y}_n) \end{bmatrix}$$

$$\sum_{i=1}^{n} Y_i \log(\hat{Y}_i) = Y \log \hat{Y}$$

$$= Y \log(\sigma(XW))$$

$$\sum_{i=1}^{n} (1-Y_i)\log(1-\hat{Y}_i) = (1-Y_i)\log(1-\hat{Y}_i)$$

So ⇓ in matrix $L = -\frac{1}{n}\left[Y \log \hat{Y} + (1-Y)\log(1-\hat{Y})\right]$

$\boxed{\text{where } \hat{Y} = \sigma(XW)}$

$$\Downarrow$$

Minimize the Loss

$$\Downarrow$$

Apply Gradient Decent

i.e = $\boxed{W_{new} = W_{old} - \eta \frac{\Delta L}{\Delta W}}$

Binary Cross Entropy - (4)

$\hat{y} = \sigma(XW)$

Find $\frac{\Delta L}{\Delta W} = \left[\frac{\partial L}{\partial w_0}, \frac{\partial L}{\partial w_1}, \frac{\partial L}{\partial w_2} \ldots\ldots\right]$

first - $\boxed{\frac{d}{\partial w} y \lg \hat{y}} \Rightarrow y \frac{d}{dw} \lg \hat{y}$

$\Rightarrow y * \frac{1}{\hat{y}} \frac{d}{dL}(\hat{y})$

$\Rightarrow \frac{y}{\hat{y}} \frac{d}{dL} \sigma(WX)$

$\Rightarrow \frac{y}{\hat{y}} \sigma(WX)[1-\sigma(WX)] \frac{d}{dw}(WX)$

$= \frac{y}{\cancel{\hat{y}}} \cancel{\hat{y}} [1-\hat{y}] * X$

$= \boxed{y(1-\hat{y}) \cdot X}$

same do with

$\frac{d}{dw}(1-y)\lg(1-\hat{y}) = \frac{(1-y)}{(1-\hat{y})} * \frac{d}{dw}(1-\hat{y})$

$= \frac{(1-y)}{(1-\hat{y})} \cancel{\frac{d}{dw}} \quad \boxed{-\hat{y}(1-y)X}$

so fine

$\frac{\partial L}{\partial w} = -\frac{1}{n}\left[y(1-\hat{y})X - \hat{y}(1-y)X\right]$

Binary Cross Entropy   Log Reg - 5

$$= -\frac{1}{n}\left[Y(1-\hat{Y}) - \hat{Y}(1-Y)\right]X$$

$$= -\frac{1}{n}\left[Y - \cancel{Y\hat{Y}} - \hat{Y} + \cancel{\hat{Y}Y}\right]X$$

$$\boxed{\frac{\partial L}{\partial W} = -\frac{1}{n}\left[Y - \hat{Y}\right]X}$$

$$\frac{\Delta L}{\Delta W}$$

$$\Downarrow$$

$$W_{new} = W_{old} - \eta \frac{\Delta L}{\Delta W}$$

So $\boxed{W_{new} = W_{old} + \eta \frac{1}{n}(Y_i - \hat{Y}_i)X}$ — Final weight

$$W = \begin{bmatrix} w_0 \\ \vdots \\ w_n \end{bmatrix} \qquad X = \begin{bmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & & \\ x_{n} & \cdots & x_{nm} \end{bmatrix}$$

$$Y = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} \qquad \hat{Y} = \begin{bmatrix} \hat{y}_1 \\ \vdots \\ \hat{y}_n \end{bmatrix}$$

$$\boxed{W_{new} = W_{old} + \eta \times \frac{1}{n}(Y - \hat{Y})X}$$

SoftMax Regression ①   Log Reg - ①

It will Use for MultiClass.

(Multinomial logistic Regression)

$$\sigma(z_i) = \frac{e^{z_i}}{\sum_{i=1}^{n} e^{z_i}}$$

is three Class ($\overset{1}{\text{Yes}}$, $\overset{0}{\text{No}}$, $\overset{2}{\text{Maybe}}$)

then $\sigma(\text{Yes}) = \frac{\text{Yes}}{\text{Yes} + \text{No} + \text{Maybe}}$

$$\sigma(z_1) = \frac{\sigma(z_i)}{\sigma(z_1) + \sigma(z_0) + \sigma(z_2)}$$

$$\sigma(z_0) = \frac{\sigma(z_0)}{\sigma(z_0) + \sigma(z_1) + \sigma(z_2)}$$

$$\sigma(z_2) = \frac{\sigma(z_2)}{\sigma(z_0) + \sigma(z_1) + \sigma(z_2)}$$

How to Solve

① Convert Output Column or Label data into OHC?

Y_{1i}	Y_{2i}	Y_{3i}
0 - OHC	1 - OHC	2 - OHC

D → D_1, D_2, D_3

D_1	D_2	D_3
m1 0	m2 1	m3 2

SoftMax-Regression ②

Catch Model here

Log Reg

M1 M2 M3

$z_1 = w_0^1 \; w_1^1 \; w_2^1$ $z_2 = w_0^2 \; w_1^2 \; w_2^2$ $z_3 = w_0^3 \; w_1^3 \; w_2^3$

find $\{ 7, 70 \}$

Yep $z_1 = w_0^1 + 7 w_1^1 + 70 w_2^1$

No $z_2 = w_0^2 + 7 w_1^2 + 70 w_2^2$

Maybe $z_3 = w_0^3 + 7 w_1^3 + 70 w_2^3$

$$\left.\begin{aligned} \sigma(\text{Yeps}) &= \frac{e^{z_1}}{e^{z_1} + e^{z_2} + e^{z_3}} \\ \sigma(\text{No}) &= \frac{e^{z_2}}{e^{z_1} + e^{z_2} + e^{z_3}} \\ \sigma(\text{Maybe}) &= \frac{e^{z_3}}{e^{z_1} + e^{z_2} + e^{z_3}} \end{aligned}\right\} \text{find Max Sigmoid-Value}$$

Logistic Regression

Polynomial Regression

$X_1 \quad X_2 \quad Y$

increase degree $= 3$

so

X_1 become $= (x_1^0 + x_1^1 + x_1^2 + x_1^3)$

X_2 become $= x_2^0 + x_2^1 + x_2^2 + x_2^3)$

so total Column will be

w_0 x_0	w_1 x_1^0	w_2 x_1^1	w_3 x_1^2	w_4 x_1^3	w_5 x_2^0	w_6 x_2^1	w_7 x_2^2	w_8 x_3^3	Y
|									
|									
|									

It will cross - **Overfitting Problem.**