Unit-2

Curve Fitting

Q1) Fit a straight line to the following data:

x:	0	1	2	3	4
y:	1	1.8	3.3	4.5	6.3

$\boxed{y = a + bx}$ —(1) $\qquad \boxed{y = ax + b}$

Normal equations: $\qquad$ Eqⁿ ⇒ $\boxed{y = 0.72 + 1.33x}$

$$\Sigma y = a\,\Sigma 1 + b\,\Sigma x$$
$$\Sigma xy = a\,\Sigma x + b\,\Sigma x^2$$

$$\Sigma y = na + b\,\Sigma x \quad \text{—(2)}$$
$$\Sigma xy = a\,\Sigma x + b\,\Sigma x^2 \quad \text{—(3)}$$

x	y	x^2	xy
0	1	0	0
1	1.8	1	1.8
2	3.3	4	6.6
3	4.5	9	13.5
4	6.3	16	25.2
$\Sigma x = 10$	$\Sigma y = 16.0$	$\Sigma x^2 = 30$	47.1

$16 = 5a + b(10)$ $\qquad$ $47.1 = a(10) + 30b$

$\Rightarrow 5a + 10b = 16$

~~a = -0.18~~
~~b = 1.51~~

$a = 0.72$
$b = 1.33$

Q3 Fit $y = \frac{c_0}{x} + c_1\sqrt{x}$ — (1)

x:	0.1	0.2	0.4	0.5	1	2
y:	21	11	7	6	5	6

~~$\Sigma y = c_0 \Sigma\left(\frac{1}{x}\right) + c_1\Sigma(\sqrt{x})$~~

$$\Sigma \frac{y}{x} = c_0 \Sigma \frac{1}{x^2} + c_1 \Sigma\left(\frac{1}{\sqrt{x}}\right) \quad \text{— (2)}$$

$$\Sigma y\sqrt{x} = c_0 \Sigma \frac{1}{\sqrt{x}} + c_1 \cdot \Sigma x \quad \text{— (3)}$$

x	$\sqrt{x}$	y	$\frac{y}{x}$	$\frac{1}{x^2}$	$\frac{1}{\sqrt{x}}$	$y\sqrt{x}$
0.1	0.31	21	210	100	3.162	6.64
0.2	0.44	11	55	25	2.23	4.91
0.4	0.63	7	17.5	6.25	1.581	4.42
0.5	0.70	6	12	4	1.41	4.24
1	1	5	5	0.04	1	5
2	1.41	6	3	0.25	0.707	8.48
4.2	4.49	$\Sigma y = 56$	$\Sigma \frac{y}{x} = 92.5$	104	10.1008	33.71

$\Sigma x = 4.2$

$\Sigma y = 56$

$\Sigma \sqrt{x} = 4.49$

$\Sigma \frac{1}{x^2} = 104$

$\Sigma \frac{y}{x} = 92.5$

$\Sigma y\sqrt{x} = 33.71524342$

$\Sigma \frac{y}{x} = 302.5$

$\Sigma x^2 = 136.5$

$$302.5 = c_0(136.5) + c_1(10.1008)$$

$$33.714 = c_0(10.1008) + c_1(4.2)$$

$c_0 = 1.97329$

$c_1 = 3.28146$

Fit straight line $y = ax + b$ or $y = a + bx$

Fit a second degree parabola

$y = a + bx + cx^2$ $\qquad\qquad$ $y = ax^2 + bx + c$

Normal eq^n $\qquad\qquad$ Normal eq^n

$\Sigma y = an + b\Sigma x + c\Sigma x^2$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

$\Sigma x^2y = a\Sigma x^4 + (\Sigma x^3)b + c\Sigma$

Q2) Fit $y = c_0 x + \frac{c_1}{\sqrt{x}}$

x	y	xy	x^2	$\sqrt{x}$	$y/\sqrt{x}$	$1/\sqrt{x}$
0.2	16	3.2	0.04	0.447	35.77	2.23
0.3	14	4.2	0.09	0.54	25.56	1.825
0.5	11	5.5	0.25	0.70	15.55	1.41
1	6	6	1	1	6	1
2	3	6	4	1.41	2.12	0.707
4						

NE → $\Sigma xy = c_0 \Sigma x^2 + c_1 \Sigma \sqrt{x}$

$\Sigma \frac{y}{\sqrt{x}} = c_0 \Sigma \frac{1}{\sqrt{x}} + c_1 \Sigma x$

$\Sigma x = 4$
$\Sigma xy = 24.9$
$\Sigma x^2 = 5.38$
$\Sigma \sqrt{x} = 4.116256497$
$\Sigma y/\sqrt{x} = 85.01514319$
$\Sigma \frac{1}{\sqrt{x}} = 0.7071067281$

-13.45258725
23.63181572

$24.9 = c_0(5.38) + c_1(4.116256497)$

85.01514319 ~~0.7071067281~~ $= c_0(0.7071067811) + c_1(4)$

Q3) Fit a curve $y = ab^x$

x: 2 3 4 5 6
y: 144 172.8 207.4 248.8 298.5

$\log y = \log ab^x$

$\log y = \log a + x \log b \Rightarrow A + xB = Y$ — (1)

$\Sigma \log y = n \log a + (\Sigma x) \log b$ — (I)

$\Sigma x \cdot \log y = \Sigma x \log a + (\Sigma x^2) \log b$ — (II)

Normal Eq → $\Sigma Y = An + B\Sigma x$ — (I)

(II) $\Sigma xY = A\Sigma x + B\Sigma x^2$

x	y	$\log y$	x^2	xY
2	144	2.158362492	4	4.316724984
3	172.8	2.237543738	9	6.712631214
4	207.4	2.316808752	16	9.267235008
5	248.8	2.395850376	25	11.97925188
6	298.5	2.474944335	36	14.84966601
$\Sigma x = 20$		$\Sigma \log y = 11.58350961$	$\Sigma x^2 = 90$	$\Sigma xY = 47.1255011$

$5A + 20B = 11.58350961$
$20A + 90B = 47.1255011$

$A = 2.00011380$
$B = 0.07918147034$

$A = \log a$
$a = 100.0262073$

$B = \log b$
$b = 1.199905472 \approx 1.2$

$\boxed{y = (100.0262073)(1.2)^x}$

Q3) Fit $y = a x^b$ $\boxed{y = (7.173)\, x^{(1.952)}}$

x : 1 2 3 4 5

y : 7.1 27.8 62.1 110 161

$\log y = \log a + b \log x$

$Y = A + bX$ —(1)

$\Sigma XY = A\Sigma X + b\Sigma X^2$ —(2)

$\Sigma Y = An + b\Sigma X$ —(3)

x	y	log x	log y	XY
1	7.1	0	0.851	0
2	27.8	0.3010	1.444	0.434
3	62.1	0.477	1.793	0.855
4	110	0.602	2.041	1.229
5	161	0.698	2.004	1.542

$\Sigma X = 2.281685748$ $\Sigma Y = 8.134108804$

$A = 0.859227757$

$\boxed{\Sigma XY = 4.061768867}$ $\boxed{b = 1.882076518}$

$\boxed{a = 7.231491337}$

X^2
0
0.090
0.227
0.362
0.488

$\Sigma X^2 = 0.680739983$

$5A + b(2.281685748) = 8.134108804$

$(8.134108804)A + b(0.680739983) = 8.134108804$

Q3) Fit $PV^n = C$ to the following data

p: 0.5 1 1.5 2 2.5 3 — x

V: 1620 1000 750 620 520 460 — y

$PV^n = C$

$\log P + n \log V = \log C$

$X + nY = \log C$

$nY + X = \log C$

$nY = \log C - X$

$Y = \frac{1}{n} \log C - \frac{X}{n}$

$Y = A + Bx$

$A = \frac{\log C}{n}$ $B = -\frac{1}{n}$

$\Sigma Y = An + B\Sigma X$ —(1)

$\Sigma XY = A\Sigma X + B\Sigma X^2$ —(2)

$(17.25572914) = 6A + (1.051152522)B$

$(2.731971277) = (1.051152522)A + (0.598247719)B$

$\boxed{A = 2.99109987 \quad B = -0.702971326}$

$n = -\frac{1}{B} \Rightarrow n = 1.422533129$

$c = 10^{nA} = 18464.31275$

$\boxed{PV^{(1.422533129)} = 18464.31275}$

X = log P	Y = log V	XY	X^2
-0.301029995	3.209	-0.9661621	0.090
0	3	0	0
0.176	2.87	0.5062	0.031
0.301	2.79	0.840	0.0906
0.397	2.71	1.08080	0.158
0.477	2.66	1.270	0.227
$\Sigma X = 1.051152522$	$\Sigma Y = 17.25572914$	2.731971277	0.598247719

Q3 Fit $PV^{n} = C$

V:	50	60	70	90	100
P:	64.7	51.3	40.5	25.9	78

$\log P + n \log V = \log C$

$Y + n X = \log C$

$Y = \log C - nX$

$Y = A + BX$ — ①

$\boxed{A = \log C, \quad B = -n}$

$\boxed{Y = \log P, \quad X = \log V}$

$\Sigma Y = An + B\Sigma X$ — ②

$\Sigma XY = A\Sigma X + B\Sigma X^2$ — ③

$X = \log V$	$Y = \log P$	XY	X^2
1.69	1.81	3.07	2.88
1.77	1.71	3.04	3.16
1.845	1.60	2.96	3.40
1.95	1.41	2.76	3.81
2	1.89	3.78	4
$\Sigma X = 9.27046180$4	$\Sigma Y = 8.433871036$	15.62955118	$\Sigma X^2 = 17.271177151$

$(8.433871036) = 5A + (9.27046180\ldots)B$

$(15.62955118) = (9.27046180\ldots)A + (17.271177151)B$

$A = 2.224527239$

$B = -0.289848404$

$n = 0.289848404$

$C = 10^{A} = 167.979443$

$\boxed{PV^{0.28984\ldots} = 167.979\ldots}$

$\boxed{y = (1.49989)e^{0.50004x}}$

Q3 Fit $y = ae^{bx}$ $\Rightarrow$ $\boxed{y = (1.506278316)\, e^{(0.49997411\ldots)x}}$

x:	2	4	6	8	10
y:	4.077	11.084	30.128	81.897	222.62

$y = ae^{bx}$

$\log y = \log a + bx \log e$

$\log_e y = \log_e a + bx$ or $\log_{10} y = \log_{10} a + bx \log_{10} e$

$Y = A + bx$ $Y = A + Bx$

$A = \log_{10} a$

$A = \log_e a$ $\boxed{a = 1.506278316}$

$B = b \log_{10} e$

$\Sigma Y = An + b\Sigma x$ — ①

$\Sigma xY = A\Sigma x + b\Sigma x^2$ — ②

	Y	x	xY	x^2
	1.40	2	2.81	4
	2.40	4	9.62	16
3.40	3.40	6	20.43	36
4.40	5.90	8	35.24	64
5.40	2.347	10	54.054	100
$\Sigma Y =$	17.0274767	$\Sigma x =$ 30	122.1638249	220

$(17.0274767) = 5A + b(30)$

$(122.1638249) = 30A + b(220)$

$A = 0.405650635$

$b = 0.49997411\ldots$

Fit Secondary Parabola

Q) fit $f(t) = ae^{-3t} + be^{-2t}$

t : 0.1   0.2   0.3   0.4

$f(t)$: 0.76   0.58   0.44   0.35

$$y = ae^{-3t} + be^{-2t} \quad -(1)$$

$$\Sigma e^{-3t} y = a \Sigma e^{-6t} + b \Sigma e^{-5t} \quad -(2)$$

$$\Sigma e^{-2t} y = a \Sigma e^{-5t} + b \Sigma e^{-4t} \quad -(3)$$

t	$y = f(t)$	$y \cdot e^{-3t}$	ye^{-2t}	e^{-4t}	e^{-5t}	e^{-6t}
0.1	0.76	0.56	0.62	0.67	0.60	0.54
0.2	0.58	0.31	0.38	0.44	0.36	0.30
0.3	0.44	0.17	0.24	0.30	0.22	0.165
0.4	0.35	0.105	0.15	0.20	0.135	0.09
		1.16564122	1.40976325 6	1.62273974	1.33287554	1.106 0229

$$(1.1656412221) = a(1.10602269) + b(1.332875544)$$

$$(1.409763256) = a(1.332875544) + b(1.62273974)$$

$a = 0.68532813 7$
$b = 0.305844572$

⇒ Curve fitting correlation and Regression.

Correlation → Its coefficient between variable x and y is denoted by $r(x, y)$ or r_{xy}. $\boxed{-1 \le r \le +1}$

$$r = \frac{\Sigma (x - \bar{x})(y - \bar{y})}{\sqrt{\Sigma (x - \bar{x})^2} \sqrt{\Sigma (y - \bar{y})^2}}$$

$$= \frac{\frac{1}{n} \Sigma (x - \bar{x}) \cdot (y - \bar{y})}{\sqrt{\frac{\Sigma (x - \bar{x})^2}{n}} \sqrt{\frac{\Sigma (y - \bar{y})^2}{n}}}$$

covariance

$$\boxed{r = \frac{\mathrm{Cov}(x, y)}{\sigma_x \sigma_y}}$$

Working formula →

$$\boxed{r = \frac{n \Sigma xy - \Sigma x \Sigma y}{\sqrt{n \Sigma x^2 - (\Sigma x)^2} \sqrt{n \Sigma y^2 - (\Sigma y)^2}}}$$

Q⇒ Find the coefficient of correlation

x:	1	3	5	7	8	10
y:	8	12	15	17	18	20

$$r = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

x	y	xy	x^2	y^2
1	8	8	1	64
3	12	36	9	144
5	15	75	25	225
7	17	119	49	289
8	18	144	64	324
10	20	200	100	400
34	900	582	248	1446

$$r = \frac{6(582) - 34 \times 900}{\sqrt{6(248) - (1156)}\sqrt{6(1446) - (8100)}}$$

·688·5215938

·2857074·88

~~432~~

$= \cancel{432} \quad 3\cancel{1\ldots} \quad 432$

$= 1.29963$

~~...~~2

(*) Rank correlation coefficient

$$r = 1 - \frac{6\Sigma D^2}{n(n^2-1)}$$ if Ranks are non repeated.

$$r = 1 - \frac{6\left[\Sigma D^2 + \frac{1}{12}m_1(m_1^2-1) + \frac{1}{12}m_2(m_2^2-1) + \cdots\right]}{n(n^2-1)}$$

↳ if Rank are repeated

X	Y	R_x	R_y	$D = R_x - R_y$	D^2
7	8	2	1	1	1
9	2	1	3	-2	4
2	3	3	2	1	1
					$\Sigma D^2 = 6$

Q⇒ Calculate rank correlation coeff.

X	Y	R_x	R_y	D	D^2
68	62	4	5	-1	1
64	58	6	7	-1	1
75	68	2.5	3.5	-1	1
50	45	9	10	-1	1
64	81	6	1	5	25
80	60	1	6	-5	25
75	68	2.5	3.5	-1	1
40	48	10	9	1	1
55	50	8	8	0	0
64	70	6	2	4	16
					72

$\boxed{m_1 = 2,\ m_2 = 3,\ m_3 = 2}$

$$r = 1 - \frac{6\left[\Sigma D^2 + \frac{1}{12} m_1(m_1^2-1) + \frac{1}{12} m_2(m_2^2-1) + \frac{1}{12} m_3(m_3^2-1)\right]}{n(n^2-1)}$$

$$= 1 - \frac{6\left[72 + \frac{1}{12}(2\times(2^2-1)) + \frac{1}{12}(3\times(3^2-1)) + \frac{1}{12}(2\times(2^2-1))\right]}{10\times(100-1)}$$

(*) Obtain rank correlation coefficient

X	Y	R_x	R_y	$D = R_x - R_y$	D^2
15	50	7	3	4	16
20	30	5.5	5	0.5	0.25
27	55	4	2	2	4
13	30	8	5	3	9
45	25	3	7	-4	16
60	10	2	8	-6	36
20	30	5.5	5	0.5	0.25
75	70	1	1	0	0
				ΣD	81.5

$m_1 = 2$, $m_2 = 3$,

$$r = 1 - \frac{6\left[\Sigma D^2 + \frac{1}{12} m_1(m_1^2-1) + \frac{1}{12}(m_2)(m_2^2-1)\right]}{n(n^2-1)}$$

$$r = 1 - \frac{6\left[(81.5) + \frac{1}{12}\times 2(4-1) + \frac{1}{12}(3)(9-1)\right]}{8(64-1)}$$

$$= 1 - \frac{6\left[81.5 + \frac{1}{6}(3) + \frac{8}{4}\right]}{8[64-1]} = 1 - \frac{504}{8(63)}$$

$= 1 - 1$

$= 0$

$= \cancel{1 - \frac{(84)6}{8(63)}} \; \cancel{= 0.01190476}$

Q2 Calculate rank correlation

X	Y	R_x	R_y	$D = R_x - R_y$	D^2
10	30	9	9	0	0
15	42	5	3	2	4
12	45	8	2	6	36
17	46	3	1	2	4
13	33	7	8	-1	1
16	34	4	7	-3	9
24	40	1	4	-3	9
14	35	6	6	0	0
22	39	2	5	-3	9
					72

$$r = 1 - \frac{6\times 72}{9\times(81-1)} = 0.4$$

Correlation and Regression →

Regression line y on x: $(y-\bar{y}) = b_{yx}(x-\bar{x})$

$$\bar{x} = \frac{\Sigma x}{n} \qquad \bar{y} = \frac{\Sigma y}{n}$$

$$b_{yx} = \frac{n\Sigma y - \Sigma x\Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$b_{xy} = \frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

Regression line x on y: $(x-\bar{x}) = b_{xy}(y-\bar{y})$

Regression coefficient b_{xy} & b_{yx}

$$b_{xy} = \frac{r\sigma_x}{\sigma_y} \qquad b_{yx} = \frac{r\sigma_y}{\sigma_x}$$

$$\boxed{b_{xy} \times b_{yx} = r^2}$$

$$r = \sqrt{b_{xy} \times b_{yx}} \qquad -1 \le r \le +1$$

→ if b_{xy} and b_{yx} are positive r is +ve
→ if b_{xy} and b_{yx} are −ve r is −ve

$$x_1 = \underbrace{19.36}_{a} \underbrace{-1.3986}_{b} x_2 + \underbrace{0.2621}_{c} x_3$$

Linear regression, Non-linear regression, Multiple regression.

Find the multiple linear regression of x_1 on x_2 & x_3

x_1:	3	5	6	8	12	10
x_2:	10	10	5	7	5	2
x_3:	20	25	15	16	15	2

Fit $x_1 = a + bx_2 + cx_3$

~~a = 11.71935776~~
~~b = 11 8.4871033~~
~~c = −0.77985074~~

Normal equation →

$$\Sigma x_1 = an + b\Sigma x_2 + c\Sigma x_3 \quad (1)$$

$$\Sigma x_1 x_2 = a\Sigma x_2 + b(\Sigma x_2^2) + c\Sigma x_2 x_3 \quad (2)$$

$$\Sigma x_1 x_3 = a\Sigma x_3 + b(\Sigma x_2 x_3) + c(\Sigma x_3^2) \quad (3)$$

x_2x_3	x_1	x_2	x_3	x_1x_2	x_1x_3	x_2^2	x_3^2
200	3	10	20	30	60	100	400
250	5	10	25	50	125	100	625
75	6	5	15	30	90	25	225
112	8	7	16	56	128	49	256
75	12	5	15	60	180	25	225
4	10	2	2	20	20	4	4
716	44	39	93	246	603	303	1735

$$44 = 6a + b(39) + c(93)$$
$$246 = a(39) + b(303) + c(93)$$
$$603 = a(93) + b(731) + c(1735)$$

Q ⇒ [2022] Fit a parabolic curve of regression of y on x to the following data

x:	1	1.5	2	2.5	3	3.5	4
y:	1.1	1.3	1.6	2	2.7	3.4	4.1

$$y = a + bx + cx^2 \quad \text{---(1)}$$

Normal Equations

$$\Sigma y = an + b\Sigma x + c\Sigma x^2$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$
$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

x	y	x^2	x^3	x^4	xy	x^2y
1	1.1	1	1	1	1.1	1.1
1.5	1.3	2.25	3.37	5.06	1.95	2.92
2	1.6	4	8	16	3.2	6.4
2.5	2	6.25	15.625	39.06	5.0	12.5
3	2.7	9	27	81	8.1	24.3
3.5	3.4	12.25	42.875	150.06	11.9	41.65
4	4.1	16	64	256	16.4	65.6
17.5	16.2	50.75	161.875	548.1875	47.65	154.47

$$16.2 = 7a + 17.5b + 50.75c$$
$$47.65 = 17.5a + 50.75b + 161.875c$$
$$154.47 = 50.75a + 161.875b + 548.1875c$$

$a = 1.030714286$

$b = -0.1928$

$c = 0.241904761$

$$\boxed{y = (1.0357) - (0.1928)x + 0.2428x^2}$$

Ans

(*)

$$\tan\theta = \frac{1-r^2}{r}\left(\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\right)$$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$(y - \bar{y}) = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$$

$$(x - \bar{x}) = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$$

$$\boxed{(y - \bar{y}) = \frac{1}{r}\frac{\sigma_y}{\sigma_x}(x - \bar{x})}$$

$$\boxed{m_1 = r\frac{\sigma_y}{\sigma_x}} \qquad \boxed{m_2 = \frac{1}{r}\frac{\sigma_y}{\sigma_x}}$$

$$\frac{m_2 - m_1}{1 + m_1 m_2} = \frac{r\left(\frac{\sigma_y}{\sigma_x}\right) - \frac{1}{r}\left(\frac{\sigma_y}{\sigma_x}\right)}{1 + \cancel{\left(\frac{r\sigma_y}{\sigma_x}\right)\left(\frac{1}{r}\frac{\sigma_x}{\sigma_y}\right)}}$$

$$= \frac{\left(\frac{r^2 - 1}{r}\right)\frac{\sigma_y}{\sigma_x}}{2}$$

Q7 Obtain a regression plane by using multiple linear regression to fit the data given

x : 1 2 3 4
y : 0 1 2 3
z : 12 18 24 30

$y = a + bx + cz$

$\Sigma y = an + b\Sigma x + \Sigma z$ — (1)

$\Sigma xy = a\Sigma x + \Sigma xz + b\Sigma x$ — (2)

$\Sigma zy = a\Sigma z + b\Sigma xz + b\Sigma z^2$ — (3)

x	y	z	xz	xy	x^2	z^2
1	0	12	12	0	1	144
2	1	18	36	2	4	324
3	2	24	72	6	9	576
4	3	30	120	12	16	900

Q3 If the coff. of correlation between x and y is 0.5 & the acute angle between their lines of regression is $\tan^{-1}\left(\frac{3}{5}\right)$ show that $\sigma_x = \frac{\sigma_y}{2}$

$\tan\theta = \frac{1-r^2}{r}\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$

$\sigma_x = \frac{\sigma_y}{2}$

$\sigma_x = n\sigma_y$

$r = \frac{1}{2}$

$\frac{3}{5} = \frac{1-\frac{1}{4}}{\frac{1}{2}}\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$

$\frac{3}{5} = \frac{\frac{3}{4}}{\frac{1}{2}}\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$

$\boxed{2\sigma_x^2 + 2\sigma_y^2 = 5\sigma_x\sigma_y}$

$2\sigma_x^2 + 2\sigma_y^2 - 5\sigma_x\sigma_y = 0$

$2\sigma_x^2 - 4\sigma_x\sigma_y + 2\sigma_y^2 = 0$

$2(\sigma_x)(\sigma_x - 2\sigma_y) - \sigma_y(\sigma_x - 2\sigma_y) = 0$

$(\sigma_x - 2\sigma_y)(2\sigma_x - \sigma_y) = 0$

$2\sigma_x - \sigma_y = 0$

$\boxed{\sigma_x = \frac{\sigma_y}{2}}$

Q => Two random variable have two regression line with equation

$3x+2y=26$
$6x+y=31$

find the mean value of x & y correlation coeff. of x & y.

Mean value of x & y

$3x+2y=26$
$6x+y=31$

$3\bar{x}+2\bar{y}=26$ $\quad\bar{x}=4$
$6\bar{x}+\bar{y}=31$ $\quad\bar{y}=7$

$3x+2y-26=0$
$6x+y-31=0$

y on x $\qquad$ x on y
$y=31-6x$ $\qquad$ $x=\frac{26-2y}{3}$

$\boxed{b_{yx}=-6}$ $\qquad$ $\boxed{b_{xy}=-2}$

$r=\sqrt{-6\times(-2)}$
$=\sqrt{12}=0.5$
$\boxed{r=0.5}$

Q => Calculate the measure of skewness & kurtosis on the basis of moments

Marks	5-15	15-25	25-35	35-45	45-55
No. of students	1	3	5	7	4

Marks	No. of students (f)	x (mid)	fx
5-15	1	10	10
15-25	3	20	60
25-35	5	30	150
35-45	7	40	280
45-55	4	50	200
	20		700

~~$\bar{x} = \frac{175}{2}$~~ $\boxed{\bar{x}=7.5}$

Central moments (moments about) Mean:

$\bar{x}=\frac{700}{20}$

$\boxed{\bar{x}=35}$

$\mu_1=\frac{\Sigma f(x-\bar{x})}{\Sigma f}=0$

$\mu_2=\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}$ = Variance

$\mu_3=\frac{\Sigma f(x-\bar{x})^3}{\Sigma f}$ $\qquad \boxed{\mu_1=0}$ $\boxed{\mu_2=125}$

$\mu_4=\frac{\Sigma f(x-\bar{x})^4}{\Sigma f}$ $\qquad \boxed{\mu_3=-600}$ $\boxed{\mu_4=37625}$

$f(x-\bar{x})$	$f(x-\bar{x})^2$	$f(x-\bar{x})^3$	$f(x-\bar{x})^4$
-25	625	-15625	390625
-45	675	-10125	151875
-25	125	-625	3125
35	175	875	4375
60	900	13500	202500
0	2500	-12000	752500

Coeff. of skewness $= \pm\sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}} = -0.4293$ negatively skewed

Coeff. of kurtosis $= \beta_2 = \frac{\mu_4}{\mu_2^2} = 2.408$

↳ Platykurtosis

Moments about a point.

$\mu_1' = \frac{\Sigma f(x-A)}{\Sigma f}$ $\mu_2' = \frac{\Sigma f(x-A)^2}{\Sigma f}$ $\mu_3' = \frac{\Sigma f(x-A)^3}{\Sigma f}$

$\mu_4' = \frac{\Sigma f(x-A)^4}{\Sigma f}$

$f(x-A)$	$f(x-A)^2$	$f(x-A)^3$	$f(x-A)^4$
-20	900	-8000	160000
-30	300	-3000	30000
0	0	0	0
90	900	7000	70000
80	1600	32000	640000
100	3000	28000	900000

$\mu_1' = \frac{100}{20} = 5$ $\mu_3' = \frac{28000}{20} = 1400$

$\mu_2' = \frac{3000}{20} = 150$ $\mu_4' = \frac{900000}{20} = 45000$

Q1 Calculate the first four moments about mean and hence find β_1 & β_2.

x:	0	1	2	3	4	5	6	7	8
f:	1	8	28	56	70	56	28	8	1

x	f	xf	$f(x-\bar{x})$	$f(x-\bar{x})^2$	$f(x-\bar{x})^3$	$f(x-\bar{x})^4$
0	1	0	-4	16	-64	256
1	8	8	-24	72	-216	648
2	28	56	-56	112	-224	448
3	56	168	-56	56	-56	56
4	70	280	0	0	0	0
5	56	280	56	56	56	56
6	28	168	56	112	224	448
7	8	56	24	72	216	648
8	1	8	4	16	64	256
		1024	0	512	0	2816

$\bar{x} = \frac{1024}{256} = 4$

$\boxed{\mu_1 = \frac{0}{256} = 0}$

$\boxed{\mu_2 = \frac{512}{256} = 2}$

$\boxed{\mu_3 = \frac{0}{256} = 0}$

$\boxed{\mu_4 = \frac{2816}{256} = 11}$

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} =$

Q) ~~The first four moments about the number 4 are -0.20, 1.78, -2.31 & 10.87. Find the moment about the mean~~

Q) The first three moments of a distribution about the value 2 are 1, 16, and -40. Show that the mean is 3, variance = 15, $\mu_3 = -86$

$\boxed{A=2}$

$\mu_1' = \frac{\Sigma f(x-A)}{\Sigma f} = 1$

$\mu_2' = \frac{\Sigma f(x-2)^2}{\Sigma f} = 16$

$\mu_3' = \frac{\Sigma f(x-3)^3}{\Sigma f} = -40$

To prove → μ_2 = variance $= (SD)^2 = 15$

$\mu_3 = -86$

~~$\mu_1' = \bar{x} - A$~~

~~$1 = \bar{x} - 2$~~

~~$\boxed{\bar{x} = 3}$~~

$\mu_1' = \bar{x} - A$

$\boxed{\bar{x} = 3}$

~~$\mu_2' = \mu_2 - (\mu_1)^2$~~

~~$16 = \mu_2 - 1$~~

$\mu_2 = \mu_2' - (\mu_1')^2$

$= 16 - (1)^2$

$\boxed{\mu_2 = 15}$

$\mu_3 = \mu_3' - 3(\mu_2')(\mu_1') + 2(\mu_1')^3$

$= -40 - 3(16)(1) + 2(1)^3$

$= -40 - 48 + 2 = -86$

Unit - 3

Random Variable → Discrete R.V. / Continuous R.V.

Random Variable → Single R.V → Discrete 50% / Continuous; Double R.V → Discrete 50% / Continuous

Q) A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean and variance.

Success is getting 1 or 6

p (probability of getting success) $= \frac{2}{6} = \frac{1}{3}$

q (probability of not getting success) $= 1 - \frac{1}{3}$

$= \frac{2}{3}$

Probability distribution.

$X = 0$ P(no success) $= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$

$X = 1$ P(getting one success) $= {}^3C_1 \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{1}{}$

$X = 2$ P(getting two success) $= {}^3C_2 \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} =$

$x=3$ P(getting three green) $= {}^3C_3\left(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\right) = \frac{1}{27}$

x	0	1	2	3
P(x)	$\frac{8}{27}$	$\frac{12}{27}$	$\frac{6}{27}$	$\frac{1}{27}$

$\Sigma p(x) = \frac{8}{27} + \frac{12}{27} + \frac{6}{27} + \frac{1}{27}$

$\boxed{\Sigma p(x) = \frac{27}{27} = 1}$

$\boxed{P(X) \geq 0}$

$P(1 \leq x \leq 3) = p(1) + p(2) + p(3)$

$= \frac{12}{27} + \frac{6}{27} + \frac{1}{27}$

$\boxed{P(1 \leq x \leq 3) = \frac{19}{27}}$

Mean $= \Sigma p(x)\cdot x = 0 \times p(0) + 1 \times p(1) + 2\,p(x=2) + 3 \times p(x=3)$

$= 0 + 12/27 + 2(6/27) + 3(1/27) = 1$

Variance $= \Sigma p(x) x^2 - (\text{Mean})^2$

$= (0)^2 p(0) + (1)^2 p(1) + (2)^2 p(2) + (3)^2 p(3) - (\text{Mean})^2$

$= 0 + \frac{12}{27} + \frac{4\times 6}{27} + \frac{9 \times 1}{27} - 1$

$= \frac{12+24+9}{27} - 1 = \frac{45-27}{27} = \frac{18}{27} = \frac{6}{9} = \frac{2}{3}$

Q⇒ Random variable X has the following probability distribution

x:	0	1	2	3	4	5	6	7
p(x):	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$
	0	0.1	0.2	0.2	0.3	0.01	0.02	0.07+0.1

(a) Find K. (b) Evaluate $P(x<6)$, $P(x \geq 6)$ and $P(3 < x \leq 6)$ find minimum value of x so that $P(X \leq x) > 1/2$

$2K + 0 + K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$

$10K^2 + 7K - 1 = 0$

$K(10K + 7) = 0$

$\boxed{K = 0, \frac{-7}{10}}$

$K = \frac{-7 \pm \sqrt{49 - 4(10)(-1)}}{2(10)}$

$= \frac{-7 \pm \sqrt{49+40}}{20}$

$= \frac{-7 \pm \sqrt{89}}{20}$

$10K^2 + 9K - 1 = 0$

$10K^2 + 10K - K - 1 = 0$

$10K(K+1) - 1(K+1) = 0$

$(10K-1)(K+1) = 0$

$\boxed{K = \frac{1}{10}, -1}$

$p(x<6) = 1 - P(x \geq 6)$

$= 1 - [p(6) + p(7)]$

$= 1 - [0.02 + 0.07 + 0.1]$

$= 1 - [0.09 + 0.1]$

$= 1 - [0.19]$

$= \frac{0.81}{100}$

$P(x \geq 6) = 0.19$

$p(3 < x \leq 6) = P(4) + p(5) + P(6)$

$= 3K^2 + 3K = \frac{33}{108} = 0.33$

For $P(X \leq x) > \frac{1}{2}$

for $\boxed{X = 4}$

Q⇒

x:	0	1	2	3	4	5	6	7	8
$p(x)$:	a	3a	5a	7a	9a	11a	13a	15a	17a

i) Determine the value of a

ii) find $P(x<3)$, $P(x \geq 3)$, $P(2 \leq x < 5)$

iii) What is the smallest value of x $P(X \leq x) > 0.5$.

i) $a + 3a + 5a + 7a + 9a + 11a + 13 + 15 + 17a = 1$

$81a = 1$

$a = \frac{1}{81}$

ii) $P(x<3) = a + 3a + 5a$

$= 9a$

$= \frac{9}{81} = \frac{1}{9}$

$P(x \geq 3) = 1 - P(x<3)$

$= 1 - \frac{1}{9} = \frac{8}{9}$

iii) $P(2 \leq x < 5) = P(2) + P(3) + P(4)$

$= 5a + 7a + 9a$

$= 21a = \frac{21}{81} = \frac{7}{27}$

iii) $P(X \leq x) > 0.5$

$P(x=0) = a = \frac{1}{81}$

$P(x \leq 1) = a + 3a = \frac{4}{81}$

$P(x \leq 2) = 9a = \frac{1}{9}$

$P(x \leq 3) = 16a = \frac{16}{81}$

$P(x \leq 4) = 25a = \frac{25}{81}$

$P(x \leq 5) = 36a = \frac{36}{81} = \frac{4}{9}$

$P(x \leq 6) = 49a = \frac{49}{81}$

Probability density function → [PDF]

$$f(x) = \begin{cases} 0 & x < 0 \\ \frac{3x(2-x)}{4} & 0 \le x \le 2 \\ 0 & x > 2 \end{cases}$$

X is continuous.

$f(x) = \frac{x^2}{3}$ $x = 0, 1, 2$

Probability mass function.

$$\int_{-\infty}^{\infty} f(x) \cdot dx = 1$$

$$\Rightarrow \int_{-\infty}^{0} 0 \cdot dx + \int_{0}^{2} \frac{3x(2-x)}{4} dx + \int_{2}^{\infty} 0 \cdot dx$$

$$\Rightarrow \int_0^2 \frac{6x}{4} dx + \int_0^2 \left(-\frac{3x^2}{4}\right) dx$$

$$= \frac{3}{2}\left(\frac{x^2}{2}\right)_0^2 - \frac{3}{4}\left(\frac{x^3}{3}\right)_0^2$$

$$= \frac{3}{\cancel{2}}\left(\frac{4}{\cancel{2}}\right) - \frac{3}{\cancel{4}}\left(\frac{8}{\cancel{3}}\right)$$

$= 3 - 2$

$= 1$

$$P\left(\frac{1}{3} \le X \le \frac{1}{2}\right) = \int_{1/3}^{1/2} f(x) \cdot dx$$

Cumulative Distribution function → [ ]

or

Distribution function

$$F(x) = \int_{-\infty}^{x} f(x) \cdot dx = P(X \le x)$$

Distribution function.

1. $F'(x) = f(x)$
2. $F(-\infty) = 0$
3. $F(\infty) = 1$

Q ⇒ If $f(x) = cx^2$; $0 < x < 1$ is PDF determine c & find the probability that $\frac{1}{3} < x < \frac{1}{2}$ i.e. $P\left(\frac{1}{3} < x < \frac{1}{2}\right)$

$$P\left(\frac{1}{3} < x < \frac{1}{2}\right) = \int_{1/3}^{1/2} cx^2 dx = 1$$

$$= c\left[\frac{x^3}{3}\right]_{1/3}^{1/2} = 1$$

$$\Rightarrow \frac{c}{3}\left[\frac{1}{8} - \frac{1}{27}\right] = 1$$

$$c\left[\frac{27-8}{216}\right] = 3 \Rightarrow c\left[\frac{19}{216}\right] = 3$$

$$\int_{-\infty}^{\infty} cx^2 dx = 1$$

$$\Rightarrow \int_0^1 cx^2 dx = 1 \Rightarrow \boxed{c=3}$$

$$P\left(\tfrac{1}{3} < x < \tfrac{1}{2}\right) = \int_{1/3}^{1/2} 3x^2 dx = \frac{19}{216}$$

Q⇒ Let x be continuous random variable with PDF given by

$$f(x) = \begin{cases} Kx & 0 \le x < 1 \\ K & 1 \le x < 2 \\ -Kx+3K & 2 \le x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

Determine ① Constant K
② cdf

$$\int_{-\infty}^{\infty} f(x) = 1 \Rightarrow \int_0^1 Kx\,dx + \int_1^2 K\,dx + \int_2^3 (-Kx+3K)\,dx = 1$$

$$= K\left(\frac{x^2}{2}\right)_0^1 + K(x)_1^2 + \left(-K\left(\frac{x^2}{2}\right) + 3Kx\right)_2^3$$

$$= \frac{K}{2} + \frac{2K}{-K} + \left(9K - \frac{9K}{2} - 6K + 2K\right) = 1$$

$$= \frac{K}{2} + 2K + \left(\frac{9K}{2} + 5K\right) - K = 1$$

$$= 10K + 6K = 1$$

$$\cancel{16K = 1}$$

$$\cancel{K = 1}$$

$$\boxed{K = \frac{1}{2}}$$

$$F(x) = \begin{cases} 0 & -\infty < x < 0 \\ x^2/4 & 0 < x < 1 \\ \frac{2x-1}{4} & 1 < x < 2 \\ \left(-\frac{x^2}{4} + \frac{3x}{2} - \frac{5}{4}\right) & 2 < x < 3 \\ 1 & 3 < x < \infty \end{cases}$$

c.d.f ⇒

$-\infty < x < \infty$

$-\infty < x < 0$ $\quad \int_{-\infty}^{x} 0 \cdot dt = 0$

$0 < x < 1$ $\quad \int_{-\infty}^{0} 0 \cdot dt + \int_0^x \frac{1}{2} t\,dt = \frac{x^2}{4}$

$1 < x < 2$ $\quad \int_{-\infty}^{0} 0 \cdot dt + \int_0^1 \frac{1}{2} t\,dt + \int_1^x \frac{1}{2}\,dt = \frac{2x-1}{4}$

$2 \le x < 3$ $\quad \int_{-\infty}^{0} 0 \cdot dt + \int_0^1 \frac{1}{2} t\,dt + \int_1^2 \frac{1}{2}\,dt + \int_2^x \left(-\frac{1}{2}t + \frac{3}{2}\right) dt = \left\{-\frac{x^2}{4} + \frac{3x}{2} - \frac{5}{4}\right\}$

$3 < x < \infty$ $\quad \int_{-\infty}^{0} 0 \cdot dt + \int_0^1 \frac{1}{2} t\,dt + \int_1^2 \frac{1}{2}\,dt + \int_2^3 \left(-\frac{1}{2}t + \frac{3}{2}\right) dt + \int_3^x 0 \cdot dt = 1$

Q) The diameter say X of an electric cable, is assumed to be a continuous r.v. with p.d.f. $f(x) = 6x(-x+1)$ $0 < x < 1$

i) Check that above is a p.d.f.
ii) Obtain c.d.f.
iii) Compute $P(x \le 1/2 \mid 1/3 < x \le 2/3)$
iv) Determine k such that $P(x < k) = P(x > k)$

i) $\int_0^1 6x(-x+1)\,dx$

$= \left[-\left(\frac{6x^3}{3}\right) + \frac{6x^2}{2}\right]_0^1$

$= \left[-2x^3 + 3x^2\right]_0^1$

$= -2(1) + 3(1)$
$= -2 + 3$
$= 1$

ii) $-\infty < x < \infty$

$-\infty < x < 0 \rightarrow \int_{-\infty}^{0} 0 \cdot dt = 0$

$0 \le x \le 1 \rightarrow F(x) = \int_0^x 6t(1-t)\,dt + \int_{-\infty}^{0} 0 \cdot dt$

$= \left[\frac{6(t^2)}{2} - \frac{6(t^3)}{3}\right]_0^x$

$= (3t^2 - 2t^3)_0^x = 3x^2 - 2x^3$

$1 \le x < \infty$

$F(x) = 1$

$$F(x) = \begin{cases} 0 & -\infty < x < 0 \\ 3x^2 - 2x^3 & 0 \le x \le 1 \\ 1 & 1 \le x < \infty \end{cases} \rightarrow \boxed{\text{CDF}}$$

iii) $P(x \le 1/2 \mid 1/3 < x \le 2/3)$

$P(x \le 1/2) = \int_0^{1/2} 6x(-x+1)\,dx$

$= \left[-6\left(\frac{x^3}{3}\right) + 6\left(\frac{x^2}{2}\right)\right]_0^{1/2}$

$= \left[3x^2 - 2x^3\right]_0^{1/2}$

$= 3\left(\frac{1}{4}\right) - 2\left(\frac{1}{8}\right)$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{2}{4} = \frac{1}{2}$

$P(1/3 < x \le 2/3) = \int_{1/3}^{2/3} 6x(1-x)\,dx = \left[3x^2 - 2x^3\right]_{1/3}^{2/3}$

$= 3\left(\frac{4}{9}\right) - 2\left(\frac{8}{27}\right) - 3\left(\frac{1}{9}\right) + 2\left(\frac{1}{27}\right)$ $= \frac{12 \times 3}{27} - \frac{\ ?}{27} = \frac{3?}{27}$

$\frac{36 - 9 + 2}{27}$

$= \frac{36 - 9}{27}$

$= \frac{27}{27}$

$= 1$

(iv) $P(x<k) = P(x>k)$

$\Rightarrow \int_0^k 6x(1-x)\,dx = \int_k^1 6x(1-x)\,dx$

$\left[3x^2-2x^3\right]_0^k = \left[3x^2-2x^3\right]_k^1$

$3k^2-2k^3 = 3-2-3k^2+2k^3$

$\Rightarrow 4k^3-6k^2+1=0$

$\boxed{k=\frac{1}{2}}$

No complex value

(v) $P(A|B) = \frac{P(A\cap B)}{P(B)}$

$P(x\le 1/2 \mid 1/3\le x\le 2/3) = \frac{P(A\cap B)}{P(B)}$

$= \frac{P(1/3\le x\le 1/2)}{P(1/3\le x\le 2/3)}$

$= \frac{\int_{1/3}^{1/2} 6x(1-x)\,dx}{\frac{34}{27}}$

$= \left[3x^2-2x^3\right]_{1/3}^{1/2}$

$= 3\left(\frac{1}{4}\right)-2\left(\frac{1}{8}\right)-3\left(\frac{1}{9}\right)+2\left(\frac{1}{27}\right)$

$\frac{34}{27}$

$= \frac{\frac{3}{4}-\frac{1}{4}-\frac{3}{9}+\frac{2}{27}}{\frac{34}{27}}$

$= \frac{\frac{27\times 4-3\times 3+2}{27}}{\frac{34}{27}}$

$= \frac{\frac{108-9+2}{27}}{\frac{34}{27}}$

$= \frac{110-9}{34}$

Q3) Pdf:

$$f(x) = \begin{cases} \frac{x}{2} & 0 \le x < 1 \\ \frac{1}{2} & 1 \le x < 2 \\ -\frac{x}{2} + \frac{3}{2} & 2 \le x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

$$F(x) = \begin{cases} 0 & -\infty \le x < 0 \\ x^2/4 & 0 \le x < 1 \\ \frac{2x-1}{4} & 1 \le x < 2 \\ -\frac{x^2}{4} + \frac{3}{2}x - \frac{5}{4}, & 2 \le x < 3 \\ 1 & 3 \le x < \infty \end{cases}$$

iii) If x_1, x_2, x_3 are three independent observation from X what is the probability that exactly one of three numbers is larger than 1.5

$P(x > 1.5) = 1 - P(x < 1.5)$ $\quad \int_{-\infty}^{1.5} f(x)\,dx$

$= 1 - \left[\frac{1}{2}\right] = \frac{1}{2}$

P(exactly one of three larger than 1.5) $= 3C_1 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{8}$

29) (a) $\mu_1 = 0,\ \mu_2 = 2.3,\ \mu_3 = 0.9,\ \mu_4 = 15.65$

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(0.9)^2}{(2.3)^3} = 0.06657356$ $\quad \gamma_1 = 0.258016445$

(+ve skewness)

$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{15.65}{(2.3)^2} = 2.958412098$ $\quad \gamma_2 = \beta_2 - 3$

$= -0.041587102$

(Platy kurtosis)

(b) $\mu_1' = 1 \quad \mu_3' = 10$

$\mu_2' = 4 \quad \mu_4' = 45$

$\mu_1' = 1$

$\Rightarrow \bar{x} - A = 1$

$\bar{x} = 1 + A$

$\bar{x} = 1 + 4$

$\boxed{\bar{x} = 5}$

$\mu_1 = 0$

$\mu_2 = \mu_2' - (\mu_1')^2$

$= 4 - (1)^2$

$\boxed{\mu_2 = 3}$

$\mu_3 = \mu_3' - 3(\mu_2')(\mu_1') + 2(\mu_1')^3$

$= 10 - 3(4)(1) + 2(1)^3$

$= 10 - 12 + 2$

$\boxed{\mu_3 = 0}$

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0$

$\gamma_1 = \sqrt{\beta_1} \Rightarrow \gamma_1 = 0$

No skewness

$\mu_4 = \mu_4' - 4(\mu_3')(\mu_1') + 6(\mu_2')(\mu_1')^2 - 3(\mu_1')^4$

$= 45 - 4(10)(1) + 6(4)(1) - 3(1)^4$

$= 45 - 40 + 24 - 3$

$= 29 - 3$

$\boxed{\mu_4 = 26}$

$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{26}{(3)^2} = \frac{26}{9}$

$= 2.89$

$\gamma_2 = \beta_2 - 3 = 2.89 - 3$

$= -0.11$

$\gamma_2 < 0 \rightarrow$ Platy Kurtosis

(28)

CI	f	x (mid)	fx	$f(x-\bar{x})^2$
10-15	1	12.5	12.5	404.01
15-20	4	17.5	70	912.04
20-25	8	22.5	180	816.08
25-30	19	27.5	522.5	494.19
30-35	35	32.5	1137.5	0.35
35-40	20	37.5	750	480.2
40-45	7	42.5	297.5	686.07
45-50	5	47.5	237.5	1112.05
50-55	1	52.5	52.5	396.01
	100		3260	5299

$\bar{x} = \frac{3260}{100} = 32.6$

$\mu_2 = \frac{5299}{100}$

$\boxed{\mu_2 = 52.99}$

$f(x-\bar{x})^4$

16 3224.08

20795429

83298.32

12853.88

0.0035

11529.602

67241.72

246442.2

156823.92

949317.97

$\boxed{\mu_4 = 9493.1797}$

$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{9493.1797}{(52.39)^2}$

$\boxed{\beta = 3.458716016}$

$\gamma_2 = \beta_2 - 3 > 0.$

(26)

CI	f	cf
10-20	12	12
20-30	30	42
30-40	x	42+x
40-50	65	107+x
50-60	y	107+x+y
60-70	25	112+x+y
70-80	18	130+x+y
	$\Sigma f = 229$	130+x+y

$150 + x + y = 229$

$\boxed{x + y = 79}$

Median = 46

$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$ — (1)

$46 = 40 + \frac{10}{65}\left(\frac{229}{2} - (42+x)\right)$

$65 \times 6 = 10 \times 114.5 - 420 - 10x$

$10x = 1145 - 420 - 65 \times 6$

$\boxed{x = 33.5}$

$\boxed{y = 45.5}$

$N/4 = 249/4 = \boxed{62.25}$

(25)

CI	f	x	xf	cf
0-10	15	5	75	15
10-20	20	15	300	35
20-30	25	25	625	60
30-40	24	35	840	84
40-50	10	45	450	94
50-60	33	55	1815	127
60-70	71	65	4615	198
70-80	51	75	3825	249
	$\Sigma f = 249$		12545	

$\frac{N}{2} = \frac{249}{2} = 124.5$

$\boxed{\bar{x} = 50.381526}$

~~$Q_1 = 30 + \frac{(62.25 - 60)}{f} \times 10$~~

~~$= 30 + (0.25) \times 10$~~

~~$= 30 + 2.5$~~

~~$Q_1 = 32.5$~~

$Q_1 = 30 + (0.25) \times 10$

$= 30.9424$

$Q_3 = 30 + \frac{3(62.25 - 60)}{6} \times 10$

$= 30 +$

$\text{Median} = 50 + \frac{10}{33}(124.5 - 94)$

$\boxed{\text{Median} = 59.2424242}$

24

x	f	xf	$f(x-\bar{x})$	$f(x-\bar{x})^2$
59	0	0	0	0
61	2	122	-12.48	77.87
63	6	378	-25.44	107.8656
65	20	1300	-44.8	100.352
67	40	2680	-9.6	2.304
69	20	1380	35.2	61.952
71	8	568	30.08	113.1008
73	2	146	11.52	66.3552
75	2	150	15.52	120.4352
	160	6724	0	650.24

$\boxed{\bar{x} = 67.24}$ $\boxed{\mu_1 = 0}$ $\boxed{\mu_2 = 6.5024}$

$f(x-\bar{x})^3$	$f(x-\bar{x})^4$
0	0
-485.9	3032.27
-457.35	1939.16
-224.78	503.52
-0.55	0.132
109.03	131.072
425.25	1598.97
382.20	2201.50
934.57	7252.31
682.4448	16658.96776

$\boxed{\mu_3 = 6.824448}$ $\boxed{\mu_4 = 166.58}$

23

$\mu_1' = 1$ $A = 2$

$\mu_2' = 2.5$

$\mu_3' = 5.5$

$\mu_4' = 16$

$\boxed{\mu_1 = 0}$

$\mu_2 = \mu_2' - (\mu_1')^2$

$= (2.5) - (1)^2$

$\boxed{\mu_2 = 1.5}$

$\mu_3 = \mu_3' - 3(\mu_2')(\mu_1') + 2(\mu_1')^3$

$= 5.5 - 3(2.5)(1) + 2(1)^3$

$= 5.5 - 7.5 + 2$

$= 9.5 - 7.5$

$\boxed{\mu_3 = 0}$

$\mu_4 = \mu_4' - 4(\mu_3')(\mu_1') + 6(\mu_2')(\mu_1')^2 - 3(\mu_1')^4$

$= 16 - 4(5.5)(1) + 6(2.5)(1)^2 - 3(1)^4$

$= 16 - 22.0 + 15.0 - 3$

$= 31 - 25$

$\boxed{\mu_4 = 6}$

$\mu_1' = \bar{x} - A$

$1 = \bar{x} - 2$

$\boxed{\bar{x} = 3}$

$\boxed{V_1 = \bar{x} = 3}$

$V_2 = \mu_2 + (\bar{x})^2$

$= 1.5 + (3)^2$

$\boxed{V_2 = 1.5 + 9} = \boxed{V_2 = 10.5}$

$V_3 = \mu_3 + 3(\mu_2)(\bar{x}) + 2(\bar{x})^3$

$= 0 + 3(1.5)(3) + 2(3)^3$

$= 13.5 + 54$

$\boxed{V_3 = 67.5}$

$V_4 = \mu_4 + 4(\mu_3)(\bar{x}) + 6(\mu_2)(\bar{x}) + (\bar{x})^4$

$= 6 + 4(0)(3) + 6(1.5)(3) + (3)^4$

$= 6 + 27 + 81$

$= 6 + 108 = 114$

Q3

x	1	2	3	4
y	0	1	2	3
z	12	18	24	30

• Let $y = a + bx + cz$

$\Sigma y = an + b\Sigma x + c\Sigma z$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma zx$

$\Sigma yz = a\Sigma z + b\Sigma xz + c\Sigma z^2$

x	y	z	xy	yz	x^2	z^2	zx
1	0	12	0	0	1	144	0
2	1	18	2	18	4	324	36
3	2	24	6	48	9	576	72
4	3	30	12	90	16	900	120
10	6	84	20	156	30	1944	228

$6 = 4a + 10b + 84c$

$20 = 10a + 30b + 228c$

$156 = 84a + 228b + 1944c$

$a = -2.25$ $b = 0.625$ $z = 0.104$

$$\boxed{y = (-2.25) + (0.625)x + (0.104)z}$$

(47)

$y = a + bx + cx^2$

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

$\Sigma y = an + b\Sigma x + c\Sigma x^2$

$\Sigma yx = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

x	y	x^2	xy	x^3	x^4	x^2y
1	1	1	1	1	1	1
2	4	4	8	8	16	16
3	9	9	27	27	81	81
4	16	16	64	64	256	256
10	30	30	100	100	354	354

$354 = a(30) + b(100) + (354)c$

$30 = 4a + b(10) + c(30)$

$100 = a(10) + b(30) + c(100)$

$a = 0,\ b = 0,\ c = 1$

$$\boxed{y = x^2}$$

(48)

X	Y	Rx	Ry	D	D^2
50	55	5	5	0	0
60	65	3.5	3.5	0	0
60	65	3.5	3.5	0	0
70	70	2	2	0	0
80	90	1	1	0	0
					0

$$r = 1 - \frac{6\left(\Sigma D^2 + \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12}\right)}{n(n^2-1)}$$

$= 1 - \frac{6\left(0 + \frac{2\times3}{12} + \frac{2\times3}{12}\right)}{5\times24}$

$= 1 - \frac{6\left[0 + \frac{2(4-1)}{12} + \frac{2(4-1)}{12}\right]}{5(25-1)} = 1 - \frac{\left[\frac{2}{3} + \frac{2}{3}\right]}{4\times5}$

$= 1 - \frac{1}{20} = \frac{19}{20}$

$= \frac{20\times3 - 4}{20\times3} = 0.93$

(Q5) $y = ax^b$

$\log y = \log a + b \log x$

$\Rightarrow \boxed{Y = A + bX}$

$\Sigma Y = An + b\Sigma X$ — (1)

$\Sigma XY = A\Sigma X + b\Sigma X^2$ — (2)

x	y	X	Y	XY	X^2
2	4.5	0.30	0.65	0.19	0.09
3	9	0.47	0.95	0.45	0.22
4	16.5	0.60	1.21	0.73	0.36
5	25	0.69	1.39	0.97	0.48

~~0.0590619058~~

1.16929505

$\boxed{\Sigma X = 2.0791812460}$

$\boxed{\Sigma Y = 4.222878976}$

$\boxed{\Sigma XY = 2.36204245}$

$\boxed{a = 1.18947997574}$

$A = 1.5994 = \log_{10} a$

$A = 0.0753705 44$

$b = 1.886029323$

$(4.222878976) = 4A + b(2.0791812460)$

$(2.36204245) = A(2.07918124 6) + b(1.16929965)$

$\boxed{y = (1.18)(x)^{1.88}}$

Sessional - II → Unit - 3 → Complete
→ Unit - 4 → Expectation
Mgf.
Binomial Distribution
Poisson Distribution.

Q⇒ A random variable x has the following distribution

x:	1	2	3	4	8	9
pmf ← $p(x)$:	K	3K	5K	7K	9K	11K

} Probability Distribution

i) Find K
ii) find Mean
iii) $p(x \geq 3)$
iv) variance.

i) $\Sigma p(x) = 1$

$\Rightarrow K + 3K + 5K + 9K + 11K + 7K = 1$

$36K = 1$

$\boxed{K = \frac{1}{36}}$

iii) $p(x \geq 3) = p(3) + p(4) + p(8) + p(9)$
$= 1 - (p(x<3))$
$= 1 - (p(1) + p(2))$
$= 1 - (K + 3K)$
$= 1 - (4K)$
$= 1 - \frac{4}{36}$
$= \frac{32}{36} = 0.889$

ii) Mean $= \Sigma x p(x) = K + 6K + 15K + 28K + 72K + 99K$
$= \frac{221}{36} = 6.14$

iv) variance $= \Sigma x^2 p(x) - (\text{Mean})^2$
$= K + 12K + 45K + 112K + 576K + 891K$
$= \frac{1637}{36} - (6.14)^2 = 7.7772$

Q7) A random variable x has the following distribution:

x	0	1	2	3	4	5	6
$p(x)$	K	3K	5K	7K	9K	11K	13K

i) find K

$\Sigma p(x) = 49K = 1$

$\Rightarrow K = \frac{1}{49}$

ii) $P(x<4) = 1 - P(x \geq 4)$

$= 1 - (P(5) + P(6))$

$= 1 - (11K + 13K)$

$= 1 - (24 \times \frac{1}{49})$

$\boxed{P(x<4) = 0.5102}$

$P(x \geq 5) = P(5) + P(6)$

$= 11K + 13K$

$= \frac{24}{49}$

$\boxed{P(x \geq 5) = 0.4979}$

$P(3 < x \leq 6) = P(4) + P(5) + P(6)$

$= 9K + 11K + 13K$

$= 33K$

$= \frac{33}{49}$

$\boxed{P(3 < x \leq 6) = 0.67346}$

iv) What is the minimum value of x so that

$P(X \geq x) \geq 0.3$

$P(x \geq 0) = 1$

$P(x \geq 1) = 1 - P(0)$

$= 1 - \frac{1}{49}$

$= 0.97$

$P(x \geq 2) = 1 - (P(0) + P(1))$

$= 1 - (K + 3K)$

$= 1 - 4K$

$= 1 - \frac{4}{49}$

$= 0.91$

$P(x \geq 3) = 1 - (P(0) + P(1) + P(2))$

$= 1 - (K + 3K + 5K)$

$= 1 - 9K$

$= 1 - \frac{9}{36}$

$= 0.75$

$P(x \geq 4) = 9K + 11 + 13K$

$= 33K$

$= 0.67$

Q⇒

x :	-2	-1	0	1	2	3
$P(x)$:	0.1	K	0.2	$2K$	0.3	K

$$\Sigma p(x) = 0.1 + K + 0.2 + 2K + 0.3 + K = 1$$

$$\Rightarrow 0.6 + 4K = 1$$

$$4K = 0.4$$

$$\boxed{K = 0.1}$$

$$\text{Mean} = \Sigma x p(x) = -0.2 - 0.1 + 0 + 0.2 + 0.6 + 0.3$$

$$= 0.8$$

$$\text{variance} = \Sigma x^2 p(x) - (\text{Mean})^2$$

$$= 0.4 + 0.1 + 0 + 0.2 + 1.2 + 0.9 - (0.8)^2$$

$$= 2.8 - (0.8)^2$$

$$= 2.16$$

(*) Random variable → Single r.v.; → Double → Discrete; → Continuous.

Q⇒

$x \backslash y$	1	2	3	4	5	6	
0	0	0	1/32	2/32	2/32	3/32	8/32
1	1/16	1/16	1/8	1/8	1/8	1/8	10/16
2	1/32	1/32	1/64	1/64	0	2/64	8/64
	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{11}{64}$	$\frac{13}{64}$	$\frac{6}{32}$	$\frac{16}{64}$	

$$P(1,1) = \frac{1}{16}$$

(1) $P(x \le 1, y = 2) = P(0,2) + P(1,2)$

$$= 0 + \frac{1}{16}$$

$$= \frac{1}{16} \quad \text{Ans}$$

(2) $P(x \le 1) = P(0,1) + P(0,1) + P(0,2) + P(0,3) + P(0,4) + P(0,5) + P(1,1) + P(1,2) + P(1,3) + P(1,4) + P(1,5) + P(1,6)$

$$= 0 + 0 + \frac{1}{32} + \frac{2}{32} + \frac{2}{32} + \frac{3}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$= \frac{1+1+2+3+2+2+4+4+4+4}{32}$$

$$= \frac{28}{32} = \frac{7}{8}$$

(3) $P(y \le 3) = \frac{6}{32} + \frac{11}{64}$

$$= \frac{12+11}{64}$$

$$= \boxed{\frac{23}{64}} \Rightarrow P(y=1) + P(y=2) + P(y=3) = \frac{3}{32} + \frac{3}{32} +$$

(4) $P(X<3, Y\le 4) = P(0,1)+P(0,2)+P(0,3)+P(0,4)$
$+ P(1,1)+P(1,2)+P(1,3)+P(1,4)$
$+ P(2,1)+P(2,2)+P(2,3)+P(2,4)$

$= 0+0+\frac{1}{32}+\frac{2}{32}+\frac{1}{16}+\frac{1}{16}+\frac{1}{8}+\frac{1}{8}$
$+\frac{1}{32}+\frac{1}{32}+\frac{1}{64}+\frac{1}{64}$

$= \frac{1+2+4+2+2+4+4+1}{32}+\frac{2}{64}$

$= \frac{16\times 2+2}{64}$

$= \frac{34}{64} = \frac{9}{16}$

Marginal distribution of x and y:

$f_x(x) \to$ Marginal for x

$f_y(y) \to$ Marginal for y

$f_x(1) = \frac{10}{16}$, $f_x(0) = \frac{8}{32}$ marginal

$f_x(2) = \frac{8}{64}$

$f_y(1) = \frac{3}{32}$, $f_y(2) = \frac{3}{32}$, $f_y(3) = \frac{11}{64}$, $f_y(4) = \frac{13}{64}$

$f_y(5) = \frac{6}{32}$, $f_y(6) = \frac{16}{64}$

Compute marginal and conditional distribution.
find conditional for $Y=2$ given $X=1$

$f_{Y|X}(1,2) = \frac{f(1,2)}{f_x(1)} = \frac{1/16}{10/16} = \frac{1}{10} = 0.1$
given

$f_{X|Y}(2,1) = \frac{f(2,1)}{f_y(1)} = \frac{1/32}{3/32} = \frac{1}{3} = 0.34$

Q2 A two dimensional r.v. X and Y have a joint probability mass function $f(x,y) = \frac{1}{27}(2x+y)$ where x and y can assume only integer values 0, 1, 2. find marginal distribution of y and x and conditional distribution of y for $X=x$.

$y \backslash x$	0	1	2	$f_x(x)$
0	0	1/27	2/27	3/27
1	2/27	3/27	4/27	9/27
2	4/27	5/27	6/27	15/27
$f_y(y)$	6/27	9/27	12/27	

x	0	1	2
$f_x(x)$	3/27	9/27	15/27

y	0	1	2
$f_y(y)$	6/27	9/27	12/27

Conditional distribution on Y when X = x

Y \ X	0	1	2
0	$\frac{0}{3/27} = 0$	$\frac{1/27}{3/27} = \frac{1}{3}$	$\frac{2/27}{3/27} = \frac{2}{3}$
1	$\frac{2/27}{9/27} = \frac{2}{9}$	$\frac{3/27}{9/27} = \frac{3}{9}$	$\frac{4/27}{9/27} = \frac{4}{9}$
2	$\frac{4/27}{15/27} = \frac{4}{15}$	$\frac{5/27}{15/27} = \frac{5}{15}$	$\frac{6/27}{15/27} = \frac{6}{15}$

(2) For the joint probability distribution of two random variable X & Y

X \ Y	1	2	3	4
1	4/36	3/36	2/36	1/36
2	1/36	3/36	3/36	2/36
3	5/36	1/36	1/36	1/36
4	1/36	2/36	1/36	5/36

find (1) marginal distribution of X and Y

(2) conditional distribution of X given Y = 1

Conditional distribution of Y given X = 2

x	1	2	3	4
$f_X(x)$	10/36	9/36	8/36	9/36

y	1	2	3	4
$f_Y(y)$	11/36	9/36	7/36	9/36

(2) (a) $\frac{f_{XY}(x,1)}{f_Y(1)} =$

~~Y~~

~~$f_X(1) = 10/36$~~

X	1	2	3	4
$\frac{f_{XY}(x,1)}{f_Y(1)}$	$\frac{4/36}{11/36} = 4/11$	$\frac{1/36}{11/36} = 1/11$	$\frac{5/36}{11/36} = 5/11$	$\frac{1/36}{11/36} = 1/11$

X	1	2	3	4
$\frac{f_{XY}(y,2)}{f_X(2)}$	$\frac{1/36}{9/36} = 1/9$	$\frac{3/36}{9/36} = 3/9$	$\frac{3/36}{9/36} = 3/9$	$\frac{2/36}{9/36} = 2/9$

Double Random Variables:

Continuous R.V.

Q) The joint probability distribution function of a two dimensional r.v (X, Y) is given by

$$f(x,y) = \begin{cases} 2 & 0<x<1,\ 0<y<x \\ 0 & \text{elsewhere.} \end{cases}$$

1) Find the marginal and conditional density function.

2) Check whether X and Y are independent or not.

$$f_X(x) = \int_{y=-\infty}^{\infty} f(x,y)\,dy \qquad f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

marginal for x — marginal for y

$$f(x,y) = \begin{cases} 2 & 0<x<1,\ 0<y<x \\ 0 & \text{elsewhere.} \end{cases}$$

$$f_X(x) = \int_{y=0}^{x} 2\,dy = 2[y]_0^x = 2x$$

$$f_Y(y) = \int_{x=y}^{1} 2\,dx = 2[x]_y^1 = 2[1-y]$$

$$f_X(x) = 2x \quad \boxed{0<x<1} \qquad f_Y(y) = 2[1-y] \quad \boxed{0<y<1}$$

Conditional for X

$$f_{X|Y}(x,y) = \frac{f(x,y)}{f_Y(y)} = \frac{2}{2(1-y)} \quad \boxed{0<y<1}$$

Conditional for Y

$$f_{Y|X}(x,y) = \frac{f(x,y)}{f_X(x)} = \frac{2}{2x} = \frac{1}{x}$$

$\boxed{0<x<1}$

Check if X & Y are independent.

if $f(x,y) = f_X(x) \cdot f_Y(y)$

then X and Y are independent

$$2 \neq 2x \times \frac{1}{1-y}$$

X and Y are not independent

Q) $f(x,y) = \begin{cases} \frac{8}{9}xy & 1 \le x \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$

find marginal and conditional

$f_X(x) = \int_{y=-\infty}^{\infty} f(x,y)\,dy$

$= \int_{y=x}^{2} \frac{8}{9}xy\,dy$

$= \frac{8}{9}x\left[\frac{y^2}{2}\right]_x^2$

$= \frac{8}{9}x\left[\frac{-x^2+4}{2}\right]$

$= \frac{-8x^3}{9\times 2} + \frac{16x}{9}$

$= \frac{-4x^3}{9} + \frac{16x}{9}$

$1 \le x \le 2$

$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$

$= \int_1^y \frac{8xy}{9}\,dx$

$= \frac{8}{9}y\left[\frac{x^2}{2}\right]_1^y$

$= \frac{8}{9}y\left[\frac{y^2}{2} - \frac{1}{2}\right]$

$= \frac{8y^3}{9\times 2} - \frac{8y}{9\times 2}$

$= \frac{4}{9}(y^3 - y) = \frac{4y(y^2-1)}{9}$

$1 \le y \le 2$

Conditional for $x \Rightarrow \frac{f(x,y)}{f_X(x)} = \frac{\frac{8}{9}xy}{\frac{4}{9}\left[4x - x^3\right]}$

$= \frac{2y}{4-x^2}$

Conditional for $y = \frac{f(x,y)}{f_Y(y)}$

$= \frac{\frac{8}{9}xy}{\frac{4y}{9}(y^2-1)}$

$= \frac{2x}{y^2-1}$

$f(x,y) = f_X(x,y) \cdot f_Y(y)$

$\frac{8}{9}xy = \left(\frac{2x}{y^2-1}\right)\left(\frac{2y}{4-x^2}\right)$

$= \frac{4xy}{(y-1)(y+1)(2-x)(2+x)}$

Q7 Given $f(x,y) = e^{-(x+y)}$ $x \geq 0, y \geq 0$

Evaluate ① $P(X>1)$. ② $P(X<Y \mid X<2Y)$

$P(X+Y<1)$

① $P(X>1) = \int_{y=0}^{\infty}\int_{x=1}^{\infty} e^{-(x+y)}\,dx\,dy$

$= \int_{y=0}^{\infty} e^{-y}\left(\int_{1}^{\infty} e^{-x}\,dx\right) dy$

$= \int_{0}^{\infty} e^{-y}\left(-e^{-x}\right)_{1}^{\infty} dy$

$= \int_{0}^{\infty} e^{-y}\left(-(e^{-\infty} - e^{-1})\right) dy$

$= \int_{0}^{\infty} e^{-y}\left(+\frac{1}{e}\right) dy$

$= \frac{1}{e}\int_{0}^{\infty} e^{-y}\,dy$

$= \frac{1}{e}\left(-e^{-y}\right)_{0}^{\infty} dy$

$= \frac{1}{e}\left(-e^{-\infty} + e^{0}\right)$

$= \frac{1}{e} = e^{-1}$

② $P(X<Y \mid X<2Y)$

By using $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/2}{2/3} = \frac{3}{4}$

$= \frac{P(X<Y)}{P(X<2Y)}$

$= \int_{0}^{\infty}\int_{0}^{y} e^{-(x+y)}\,dx\,dy$

$= \int_{0}^{\infty} e^{-y}\left(-e^{-x}\right)_{0}^{y} dy$

$= \int_{0}^{\infty} e^{-y}\left(-e^{-y} + e^{-0}\right) dy$

$= \int_{0}^{\infty}\left(-e^{-2y} + e^{-y}\right) dy$

$= \left(+\frac{e^{-2y}}{2} - \frac{e^{-y}}{1}\right)_{0}^{\infty}$

$= \left(\frac{e^{-\infty}}{2} - e^{-\infty} - \frac{e^{0}}{2} + e^{0}\right)$

$= \frac{-1}{2} + 1$

$= \frac{1}{2}$

$P(X<2Y) = \frac{2}{3} = \int_{0}^{\infty}\int_{0}^{2y}\left(e^{-(x+y)}\right) dx\,dy = \frac{2}{3}$

$x < 1-y$

(3) $P(x+y<1) = \int_0^\infty \int_0^{1-y} e^{-(x+y)}\,dx\,dy$

$= \int_0^\infty e^{-y}\left(-e^{-x}\right)_0^{1-y} dy$

$= \int_0^\infty e^{-y}\left(-e^{y-1} + e^{0}\right) dy$

$= \int_0^\infty \left(-e^{-1} + e^{-y}\right) dy$

$= \left[-e^{-y} - e^{-1}y\right]_0^\infty$

$=$

$P(x+y<1) = \int_0^1 \int_0^{1-x} e^{-x} e^{-y}\,dy\,dx$

$= \int_0^1 e^{-x}\left(-e^{-y}\right)_0^{1-x} dx$

$= \int_0^1 e^{-x}\left(-e^{x-1} + e^{0}\right) dx$

$= \int_0^1 \left(-e^{-1} + e^{-x}\right) dx$

$= \left[(-e^{-1})x + (-e^{-x})\right]_0^1 = \left[(-e^{-1}) + (-e^{-1}) + 0 + e^{0}\right]$

$= \boxed{1 - 2e^{-1}}$

Q. $f(x) = \begin{cases} \frac{1}{8}(6-x-y) & 0 \le x < 2,\ 2 \le y < 4 \\ 0 & \text{otherwise} \end{cases}$

find. (1) $P(x<1 \cap y<3) = \int_0^1 \int_2^3 \frac{1}{8}(6-x-y)\,dx\,dy$

(2) $P(x+y<3)$

(3) $P(x<1 \mid y<3)$

(2) $P(x+y<3) = \int_0^1 \int_{y=2}^{3-x} \frac{1}{8}(6-x-y)\,dx\,dy$

$= \int_0^1 \left[\frac{1}{8}\left(6y - \frac{y^2}{2} - xy\right)\right]_2^{3-x} dy$

$= \int_0^1 \left[\frac{1}{8}\left(6y - \frac{(3-x)^2}{2} - xy - 2y\right)\right] dy$

~~$= \int_0^1 \left[\frac{y}{2} - \frac{xy}{8} - \frac{(3-x)^2}{16}\right] dy$~~

$= \int_0^1 \frac{1}{8}\left[\frac{3-x-2}{2} - \frac{x(3-x-2)}{8} - \frac{(3-x)^2}{6}\right]$

$= \int_0^1 \frac{1}{8}\left[\frac{1-x}{2} - \frac{x(1-x)}{8} - \frac{(3-x)^2}{6}\right]$

$= \int_0^1 \frac{1}{8}\left[\left(\frac{1-x}{2}\right) - \right.$

① $P(0<x<1 \cap 2<y<3) = \int_0^1 \int_2^3 \frac{1}{8}(6-x-y)\,dx\,dy$

$= \int_0^1 \frac{1}{8}\left[6y - xy - \frac{y^2}{2}\right]_2^3 dx$

$= \int_0^1 \frac{1}{8}\left[18 - 3x - \frac{9}{2} - 12 + 2x + \frac{4}{2}\right]$

$= \int_0^1 \frac{1}{8}\left[1 - x\right] dx$

$= \frac{1}{8}\left[x - \frac{x^2}{2}\right]_0^1$

$= \frac{1}{8}\left[1 - \frac{1}{2}\right]$

$= \frac{1}{16}$

Q2) $f(x,y) = 4xye^{-(x^2+y^2)}, \; x \geq 0, \; y \geq 0$

Test whether X and Y are independent find the conditional distribution of X given Y=y

$f(x,y) = f_X(x) \cdot f_Y(y)$

$f_X(x) = \int_{y=0}^{\infty} f(x,y)\,dy = 4xe^{-x^2}\int_0^{\infty} ye^{-y^2}dy$

Let $y^2 = t$

$dt = 2y\,dy$

$f_X(x) = 4xe^{-x^2}\int_0^{\infty}\left[\frac{e^{-t}\,dt}{2}\right]$

$= 4xe^{-x^2}\left[\frac{e^{-t}}{-2}\right]_0^{\infty}$

$= \frac{-4xe^{-x^2}}{2}\left[e^{-\infty} - e^{0}\right]$

$\boxed{f_X(x) = 2xe^{-x^2}}$

$f_Y(y) = \int_{x=0}^{\infty} f(x,y)\,dx = 4ye^{-y^2}\int_0^{\infty} xe^{-x^2}dx$

Let $p = x^2$

$= 4ye^{-y^2}\int_0^{\infty} \frac{e^{-p}\,dp}{2}$

$= 4ye^{-y^2}\left[\frac{e^{-\infty} - e^{0}}{(-2)}\right]$

$\boxed{f_Y(y) = 2ye^{-y^2}}$

$f_X(x) = 2xe^{-x^2}$

$f_Y(y) = 2ye^{-y^2}$

$f_X(x) \times f_Y(y) = 4xye^{-(x^2+y^2)}$

X & Y are independent

$$f_{X|Y}(x,y) = \frac{f(x,y)}{f_Y(y)} = \frac{4xye^{-x^2}e^{-y^2}}{2ye^{-y^2}}$$

$$= 2xe^{-x^2}$$

Q. $f(x,y) = \dfrac{9(1+x+y)}{2(1+x)^4(1+y)^4}$ $\quad 0<x<\infty$, $0<y<\infty$

find the marginal distribution of x & y

Conditional distribution of y given x

$$f_X(x) = \int_0^\infty \frac{9(1+x+y)}{2(1+x)^4(1+y)^4}\,dy = \frac{3}{4}\left(\frac{3+2x}{(1+x)^4}\right)$$

$$\frac{9}{2}\left[\int_0^\infty \frac{1}{(1+x)^4(1+y)^4}dy + \int_0^\infty \frac{x}{(1+x)^4(1+y)^4}dy + \int_0^\infty \frac{y}{(1+y)^4(1+x)^4}dy\right]$$

$$= \frac{9}{2}\left[\frac{1}{(1+x)^4}\int_0^\infty \frac{1}{(1+y)^4}dy + \frac{x}{(1+x)^4}\int_0^\infty \frac{1}{(1+y)^4}dy\right.$$

$$\left.\frac{1}{(1+x)^4}\int_0^\infty \frac{y}{(1+y)^4}dy\right]$$

$$= \frac{9}{2}\left[\frac{1}{(1+x)^4}\left(\frac{1}{-3(1+y)^3}\right)_0^\infty + \frac{x}{(1+x)^4}\left(\frac{1}{-3(1+y)^3}\right)_0^\infty\right.$$

$$+ \frac{1}{(1+x)^4}\Big($$

$$\frac{9}{2}\left(\frac{1}{(1+x)^4}\right)\int_0^\infty \left\{\frac{1}{(1+y)^3} + \frac{x}{(1+y)^4}\right\}dy$$

$$= \frac{9}{2}\left(\frac{1}{(1+x)^4}\right)\left[\frac{1}{-2(1+y)^2} + \frac{x}{-3(1+y)^3}\right]_0^\infty$$

$$= \frac{9}{2}\left(\frac{1}{(1+x)^4}\right)\left[\frac{1}{+2(1)} + \frac{x}{(+3)}\right]$$

$$= \frac{9}{2}\left(\frac{1}{(1+x)^4}\right)\left(\frac{3+2x}{6}\right)$$

$$f_X(x) = \frac{3}{2}\left(\frac{3+2x}{(1+x)^4}\right)$$

$$f_Y(y) = \frac{9}{2}\left(\frac{1}{(1+y)^4}\right)\int_0^\infty \left\{\frac{1}{(1+x)^3} + \frac{y}{(1+x)^4}\right\}dx$$

$$= \frac{9}{2}\left(\frac{1}{(1+y)^4}\right)\left[\frac{1}{2(1)} + \frac{y}{3}\right]$$

$$f_Y(y) = \frac{3}{2}\left(\frac{3+2y}{(1+y)^4}\right)$$

Q) $f(x) = \begin{cases} \frac{1}{4} & -2 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$

① $P(|x| > 1)$

② $P(2x + 3 > 5)$

$P(|x| > 1) = P(x < -1) + P(x > 1)$

$= \int_{-\infty}^{-1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx$

$= \int_{-2}^{-1} \frac{1}{4}\,dx + \int_{1}^{2} \frac{1}{4}\,dx$

$= \left[\frac{x}{4}\right]_{-2}^{-1} + \left[\frac{x}{4}\right]_{1}^{2}$

$= \left[\frac{-1+2}{4}\right] + \left[\frac{2-1}{4}\right]$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

$P(2x + 3 > 5) = P(x > 1)$

$2x > 2$

$\boxed{x > 1}$

$= \int_{1}^{\infty} f(x)\,dx$

$= \int_{1}^{2} \frac{1}{4}\,dx$

$= \left[\frac{x}{4}\right]_{1}^{2}$

$= \frac{2-1}{4}$

$= \frac{1}{4}$