Heterogeneous Returns and the Distribution of Wealth

Decory Edwards

Johns Hopkins University

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Outline

1. Empirical evidence of heterogeneous returns
2. Model of saving with heterogeneous returns
3. Structural estimation of model to match wealth data

Why het. returns?

1. Observable feauture of household decision problem
2. Financial literacy and inequality

What are het. returns?

Following optimal portfolio choice theory from Merton (1969) and Samuelson (1969)

- Optimal share in the risky asset is given by

$$\alpha_{it}^m = \frac{\mathbb{E}(r_t^m - r_t^s)}{\gamma_i \sigma_t^2}.$$

- Individual *realized* return to financial assets can be written as

$$r_{it}^f = r_t^s + \alpha_{it}^m (r_t^m - r_t^s).$$

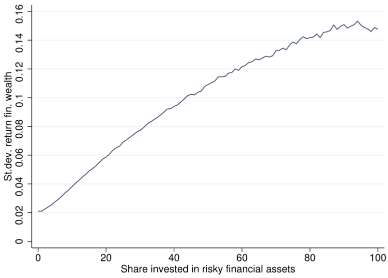


Figure: Heterogeneity in returns to financial wealth by share of risky assets from Fagereng et al. 2020.

Empirical estimate of heterogeneity

- Step 1: linear regression for the return to net worth using panel

$$r^n_{it} = X'_{it}\beta + u_{it}.$$

- Step 2: Add fixed effects

$$u_{it} = f_i + e_{it}.$$

$\implies$ R^2 goes from .33 to .5.

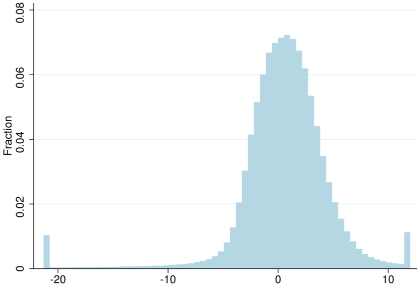


Figure: Distribution of fixed effects in the return to net worth from Fagereng et al. 2020.

Labor income process

- Household income:

$$y_t = p_t \xi_t W_t$$

- Permanent component:

$$p_t = p_{t-1} \psi_t$$

- Transitory component:

$$\xi_t = \begin{cases} \mu & \text{with probability } \mho \\ (1-\tau_t)\ell\theta_t & \text{with probability } 1-\mho \end{cases}$$

(Normalized) Optimization problem

Choose profiles $\{c_{t_n}\}_{n=0}^{\infty}$ that satisfy

$$
\begin{aligned}
v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \cancel{D} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t(m_t), \\
k_{t+1} &= \frac{a_t}{\cancel{D}\psi_{t+1}}, \\
m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}
$$

Production function

$$
Y = ZK^{\alpha}(\ell L)^{1-\alpha}
$$

Calibration

Standard calibration scheme used to simulate the model.

Description	Parameter	Value	Source
Time discount factor	β	0.99^4	Den Haan, Judd, and Juillard 2010
CRRA	ρ	1	Den Haan, Judd, and Juillard 2010
Capital share	α	0.36	Den Haan, Judd, and Juillard 2010
Depreciation rate	δ	0.025	Den Haan, Judd, and Juillard 2010
Time worked per employee	ℓ	$1/.09$	Den Haan, Judd, and Juillard 2010
Effective interest rate	$r-\delta$	0.01	Den Haan, Judd, and Juillard 2010
Wage rate	W	2.37	Den Haan, Judd, and Juillard 2010
Unempl. insurance payment	μ	0.15	Den Haan, Judd, and Juillard 2010
Probability of survival	$\not D$	$(1 - 0.00625)^4$	Yields 40-year working life
Std. dev of log $\theta_{t,i}$	σ_θ^2	0.010 x 4 x $\sqrt{4}$	Carroll 1992, Carroll, Slacalek, and Tokuoka 2015
Std. dev of log $\psi_{t,i}$	σ_ψ^2	0.010 x 4/11 x $\sqrt{4}$	Carroll 1992, Debacker et al. 2013, Carroll, Slacalek, and Tokuoka 2015
Unemployment rate	$\mho$	0.07	Mean in Den Haan, Judd, and Juillard 2010

Table 1: Parameter values (annual frequency) for the perpetual youth model.

Estimation procedure

Simulated method of moments (SMM) estimation for R using 2004 SCF wealth data.

1. No ex-ante heterogeneity: *R-point* model

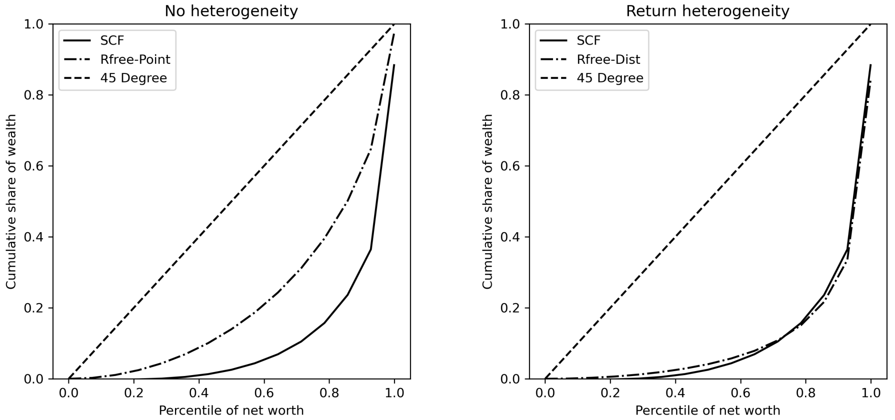
   Estimate a common rate of return across households by finding the $\grave{R}$ which matches the capital-to-output ratio ($\frac{K}{Y} = 7.47$).

2. Ex-ante heterogeneity: *R-dist* model

   Estimate a **Uniform distribution** of returns across households by finding the $\grave{R}, \nabla$ which match empirical Lorenz targets, given $\frac{K}{Y}$.

Net worth percentile	Cumulative net worth
20th	-.18%
40th	.95%
60th	5.3%
80th	17.09%

How good is the fit?



Lifecycle version of the model

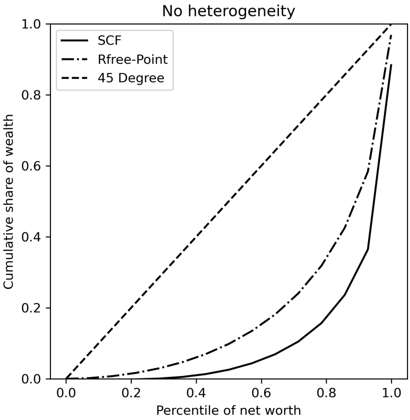
- Education cohort $e \in \{D, HS, C\}$
- Initial wealth-to-income k_0 and income p_0 levels
- Education-age dependent mortality rates (Brown, Liebman, and Pollet 2007)
- Modified labor income uncertainty $y_t = \xi_t \psi_t \overline{\psi}_{es} p_{t-1}$ (Cagetti 2003)
    - Education-age dependent shock variances (Sabelhaus and Song 2010)

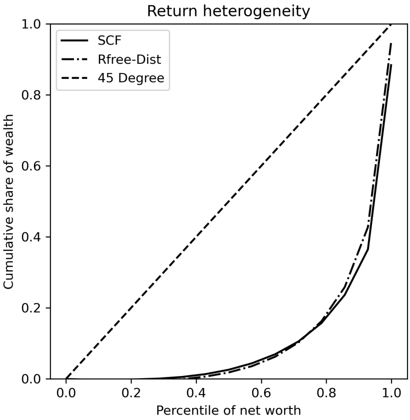
Calibration

Description	Parameter	Value
Population growth rate	N	0.0025
Technological growth rate	Γ	0.0037
Rate of high school dropouts	θ_D	0.11
Rate of high school graduates	θ_{HS}	0.55
Rate of college graduates	θ_C	0.34
Labor income tax rate	τ	0.0942

Table 2: Parameter values (annual frequency) for the lifecycle model.

How good is the fit?





Model performance: returns distribution

Empirical values from Fagereng et al. 2020

	Mean	St. Dev
Net worth (after tax)	0.0365	0.0781

Values from the structural estimation (uniform distribution for R)

	Mean	St. Dev
PY-Point	0.071	0.0
PY-Dist	0.055	0.006
LC-Point	0.063	0.0
LC-Dist	0.049	0.010

Model performance: untargeted moments

Empirical Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	-0.0723	-0.0657	-0.0266	0.1099
30-40	-0.008	0.0054	0.057	0.1813
40-50	-0.0001	0.0187	0.0776	0.2178
50-60	0.0018	0.0215	0.0766	0.2126
60-70	0.0011	0.0188	0.0726	0.2081

Simulated Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	-0.0002	0.0305	0.1005	0.2532
30-40	-0.0101	0.0122	0.0841	0.2609
40-50	-0.0052	0.0121	0.0787	0.2569
50-60	0.0011	0.0209	0.0866	0.2593
60-70	0.0016	0.0203	0.0811	0.2459

Work to be done

1. Robustness checks
   - Plausible parameter values for time preferences and risk aversion
   - Different measures of wealth (liquid and/or financial)
2. More untargeted moments
   - Wealth shares by education cohort

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