# Heterogeneous Returns and the Distribution of Wealth

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# Why do macroeconomists care about inequality?

Empirical evidence shows that macroeconomic policies, as well as aggregate shocks, may have differential effects across households.

- Macro matters for inequality

Representative agent models have a difficult time matching empirical estimates of macroeconomic variables (MPC and the wealth distribution).

- Inequality matters for macro

# Macro with heterogeneous agents

- Uninsurable, idiosyncratic risk to income and movements in aggregate productivity (Krusell and Smith 1998)
- Ex-ante heterogeneity in the time preference of households (Carroll et al. 2017)
- Classifying models with ex-ante and ex-post heterogeneity (Kaplan and Violante 2022)
- Further surveys regarding heterogeneous agent macroeconomics (Guvenen 2011) and (Krueger, Mitman, and Perri 2016)

# Empirical estimates of returns

1. Comprehensive, administrative tax data in Norway from 2004 to 2015 (Fagereng et al. 2020)
2. Asset holdings and income for Swedish residents from 1999 to 2007 (Bach, Calvet, and Sodini 2018)
3. Wealth held in equity accounts in India from 2002 to 2011 (Campbell, Ramadorai, and Ranish 2019)
4. DNB 2005 survey of dutch households regarding savings accounts and financial literacy (Deuflhard, Georgarakos, and Inderst 2018)

# Related literature

- Stochastic process for returns implying a stationary wealth distribution. (Benhabib, Bisin, and Zhu 2011), (Benhabib, Bisin, and Zhu 2015), (Benhabib, Bisin, and Zhu 2016)
- Stochastic process for returns which best fits the empirical distribution of wealth.
  (Benhabib, Bisin, and Luo 2019)
- Endogenize heterogeneous returns through access to high return investment technology.
  (Guler, Kuruscu, and Robinson 2022)

# Outline

1. Empirical evidence of heterogeneous returns
2. Model of saving with heterogeneous returns
3. Structural estimation of model to match wealth data

# A closer look at Fagereng et al. 2020

Following optimal portfolio choice theory from Merton (1969) and Samuelson (1969)

- Optimal share in the risky asset is given by

$$\alpha_{it}^{m} = \frac{\mathbb{E}(r_t^m - r_t^s)}{\gamma_i \sigma_t^2}.$$

- Individual *realized* return to financial assets can be written as

$$r_{it}^{f} = r_t^s + \alpha_{it}^{m}(r_t^m - r_t^s).$$

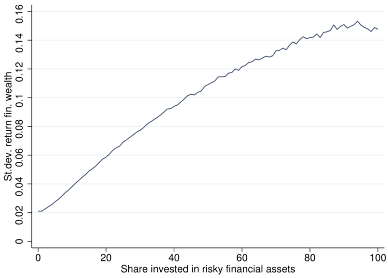



Figure: Heterogeneity in returns to financial wealth by share of risky assets from Fagereng et al. 2020.

- Step 1: *linear panel data regression model* for the return to net worth

$$r^n_{it} = X'_{it}\beta + u_{it}.$$

- Step 2: Add fixed effects

$$u_{it} = f_i + e_{it}.$$

$\Longrightarrow$ $R^2$ goes from .33 to .5.

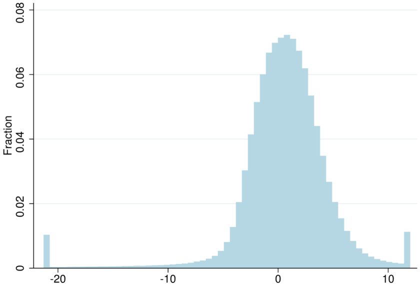



Figure: Distribution of fixed effects in the return to net worth from Fagereng et al. 2020.

# Labor income process

- Household income:

$$y_t = p_t \xi_t W_t$$

- Permanent component:

$$p_t = p_{t-1} \psi_t$$

- Transitory component:

$$\xi_t = \begin{cases} \mu & \text{with probability } \mho \\ (1-\tau_t)\ell\theta_t & \text{with probability } 1-\mho \end{cases}$$

# Heterogeneous agents in G.E.

(Normalized) Optimization problem for households: Choose profiles $\{c_{t_n}\}_{n=0}^{\infty}$ that satisfy

$$\begin{aligned}
v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \cancel{D} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t(m_t), \\
k_{t+1} &= \frac{a_t}{\cancel{D}\psi_{t+1}}, \\
m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}$$

Production function

$$Y = ZK^{\alpha}(\ell L)^{1-\alpha}$$

# Conditions for a stable wealth distribution

Carroll 2019 provides the following *death-modified growth impatience condition* such that a unique target wealth-to-permanent income ratio exists for households:

$$\left(\frac{(R\beta)^{1/\rho}\mathbb{E}[\psi^{-1}]\cancel{D}}{\Gamma}\right) \quad < \quad 1.$$

# Calibration

Standard calibration scheme used to simulate the model.

| Description | Parameter | Value | Source |
|---|---|---|---|
| Time discount factor | $\beta$ | 0.99 | Den Haan, Judd, and Juillard 2010 |
| CRRA | $\rho$ | 1 | Den Haan, Judd, and Juillard 2010 |
| Capital share | $\alpha$ | 0.36 | Den Haan, Judd, and Juillard 2010 |
| Depreciation rate | $\delta$ | 0.025 | Den Haan, Judd, and Juillard 2010 |
| Time worked per employee | $\ell$ | $1/.09$ | Den Haan, Judd, and Juillard 2010 |
| Capital/output ratio | $\frac{K}{Y}$ | 10.26 | Den Haan, Judd, and Juillard 2010 |
| Effective interest rate | $r-\delta$ | 0.01 | Den Haan, Judd, and Juillard 2010 |
| Wage rate | $W$ | 2.37 | Den Haan, Judd, and Juillard 2010 |
| Unempl. insurance payment | $\mu$ | 0.15 | Den Haan, Judd, and Juillard 2010 |
| Probability of death | $D$ | 0.00625 | Yields 40-year working life |
| Variance of $\log \theta_{t,i}$ | $\sigma_\theta^2$ | 0.010 x 4 | Carroll 1992, Carroll, Slacalek, and Tokuoka 2015 |
| Variance of $\log \psi_{t,i}$ | $\sigma_\psi^2$ | 0.010 x 4/11 | Carroll 1992, Debacker et al. 2013, Carroll, Slacalek, and Tokuoka 2015 |
| Unemployment rate | $\mho$ | 0.07 | Mean in Den Haan, Judd, and Juillard 2010 |

Table 1: Parameter values (quarterly frequency) for the perpetual youth model.

# Estimation procedure

Simulated method of moments (SMM) estimation for $R$.

1. No ex-ante heterogeneity: *R-point* model
   $R$ which matches the capital-to-output $(\frac{K}{Y})$ ratio of 10.26

2. Ex-ante heterogeneity: *R-dist* model
   **Uniform distribution** of $R$ matching lorenz targets, given $\frac{K}{Y} = 10.26$

# Estimation procedure with heterogeneity

Empirical lorenz targets using 2004 SCF data

| Net worth percentile | Cumulative net worth |
|---|---|
| 20th | -.18% |
| 40th | .95% |
| 60th | 5.3% |
| 80th | 17.09% |

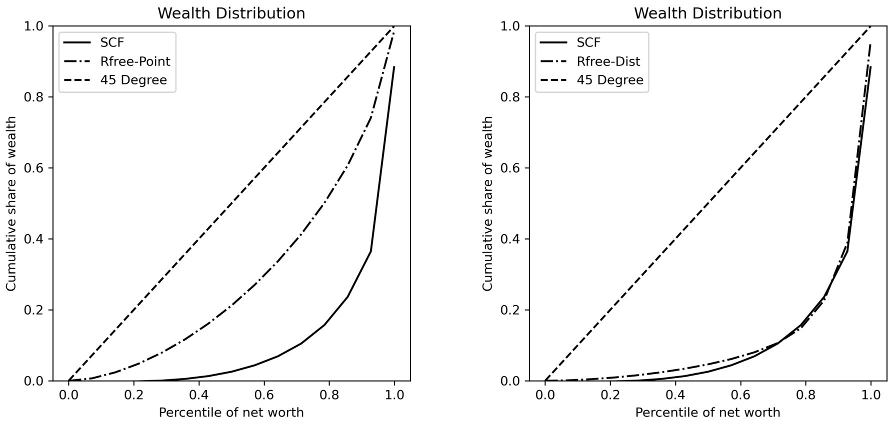
Optimization problem for the *R-dist* model

$$\{\grave{R}, \nabla\} = \arg\min_{R,\nabla} \left( \sum_{i=20,40,60,80} (w_i(R,\nabla) - \omega_i)^2 \right)^{\frac{1}{2}}$$

s.t.

$$\frac{K}{Y} = 10.26.$$

# How good is the fit?



# Lifecycle version of the model

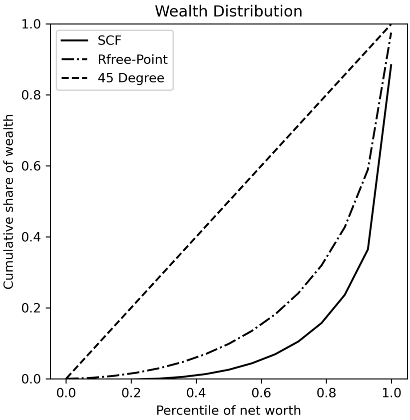
- Education cohort $e \in \{D, HS, C\}$
- Initial wealth $k_0$ and income $p_0$ levels
- Education-age dependent mortality rates (cite the paper)
- Modified labor income uncertainty $y_t = \xi_t \psi_t \overline{\psi}_{es} p_{t-1}$
    - Education-age dependent shock variances from (cite paper)

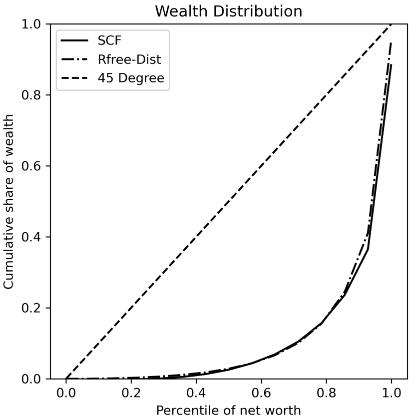
# Calibration

| Description | Parameter | Value |
|---|---|---|
| Population growth rate | $N$ | 0.0025 |
| Technological growth rate | $\Gamma$ | 0.0037 |
| Rate of high school dropouts | $\theta_D$ | 0.11 |
| Rate of high school graduates | $\theta_{HS}$ | 0.55 |
| Rate of college graduates | $\theta_C$ | 0.34 |
| Labor income tax rate | $\tau$ | 0.0942 |

Table 1: Parameter values (quarterly frequency) for the lifecycle model.

# How good is the fit?



# Revisiting the empirical evidence

Fagereng et al. 2020 describe the empirical distribution of returns.

TABLE 3
RETURNS TO WEALTH: SUMMARY STATISTICS[a]

| Wealth Component | Mean | St. Dev. | Skewness | Kurtosis | P10 | Median | P90 |
|---|---|---|---|---|---|---|---|
| Net worth (before tax) | 0.0379 | 0.0859 | −0.79 | 47.75 | −0.0308 | 0.0321 | 0.1109 |
| Net worth (after tax) | 0.0365 | 0.0781 | −0.71 | 36.88 | −0.0283 | 0.0316 | 0.1067 |
| Net worth (before tax, unweighted) | 0.0004 | 0.2205 | −6.73 | 68.46 | −0.0600 | 0.0230 | 0.1037 |
| Net worth (after tax, unweighted) | 0.0155 | 0.1546 | −5.28 | 56.42 | −0.0449 | 0.0247 | 0.1040 |
| Financial wealth | 0.0105 | 0.0596 | −1.78 | 22.17 | −0.0171 | 0.0084 | 0.0530 |
| Safe fin. assets | 0.0078 | 0.0188 | 4.38 | 53.52 | −0.0106 | 0.0059 | 0.0268 |
| Risky fin. assets | 0.0425 | 0.2473 | −0.08 | 6.22 | −0.2443 | 0.0418 | 0.3037 |
| Non-financial wealth | 0.0511 | 0.0786 | 1.80 | 15.47 | −0.0215 | 0.0429 | 0.1275 |
| Housing | 0.0485 | 0.0653 | 0.73 | 9.95 | −0.0209 | 0.0441 | 0.1165 |
| Private equity | 0.1040 | 0.5169 | 18.01 | 836.79 | −0.0531 | 0.0052 | 0.3616 |
| Debt | 0.0236 | 0.0216 | 2.51 | 29.50 | 0.0030 | 0.0215 | 0.0461 |
| Long-term debt | 0.0230 | 0.0209 | 3.54 | 56.92 | 0.0038 | 0.0209 | 0.0446 |
| Consumer debt | 0.0961 | 0.1086 | 4.60 | 82.60 | −0.0124 | 0.0741 | 0.2119 |
| Student debt | 0.0078 | 0.0260 | 0.68 | 4.14 | −0.0213 | 0.0074 | 0.0399 |

[a]The table reports summary statistics for various measures of real returns to wealth, pooling data for 2005–2015. Except when noted, all returns are value-weighted.

# Work to be done

1. Modify structural estimation to include bequest parameter
2. Incorporate choice of risky asset
3. Robustness checks
    - Wealth data from other waves of the SCF
    - Different measures of wealth (liquid or financial)

# Future directions for this work

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