

# Matching Wealth Moments with Heterogeneous Returns

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## Abstract

Empirical evidence shows that individuals face widely varying returns on their assets. This heterogeneity has important implications for modeling wealth inequality. I incorporate heterogeneity in returns into a standard heterogeneous-agent model, and find the distribution required to match the empirical wealth distribution. In both infinite-horizon and life-cycle versions of the model, introducing even modest return heterogeneity yields a much better match to the top-heavy wealth distribution than assuming homogeneous returns. It also aligns more closely with untargeted patterns such as the distribution of marginal propensities to consume and age-wealth profiles. In addition, we compare a wealth tax and a capital income tax set to raise equal revenue. A capital income tax reduces wealth inequality more by targeting high-return households, but each policy entails different welfare trade-offs across the return distribution. I interpret the heterogeneity in returns required by the model as arising from banking-sector frictions (e.g., incomplete deposit rate pass-through) rather than differences in entrepreneurial talent or financial sophistication.

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# 1 Introduction

The unequal distribution of wealth is an extensively documented phenomenon in numerous countries. Regrettably, this feature has not only endured over time but also intensified in recent years. This point is stressed in a recent article from the Institute for Policy Studies (IPS), which revealed that in 2018, the total wealth of the poorest half of Americans was eclipsed by the combined wealth of the three wealthiest men in the nation. The term “richest” denotes one’s standing in Forbes magazine’s list of the 400 richest individuals. Additionally, the IPS report notes that the combined wealth of the top five richest men on this list skyrocketed by a staggering 123% from March 2020 to October 2021<sup>1</sup>.

The unequal distribution of wealth has also been a subject of considerable interest throughout history in various fields. The statistics literature, for instance, focused on linking the distribution of income to the observable skewness in wealth distribution. The economics literature went further by establishing microfoundations for wealth accumulation over the life cycle. To that end, the macroeconomics literature on inequality has seen significant growth, with the distribution of wealth among households offering insight into how the economy as a whole responds to aggregate fiscal shocks. The recent stimulus checks issued during the pandemic serve as a timely example of this phenomenon.

The macroeconomics literature has undergone significant changes in recent years, with the widespread adoption of models that abandon the traditional representative agent assumption in their analysis. As this setting will require that in equilibrium all agents hold the same level of wealth, it is not a desirable laboratory in terms of producing model objects, like the distribution of wealth, that can be compared to real world counterparts.

The first departure from the representative agent framework incorporates an exogenously determined income process that generates a distribution of income among households. One common approach to incorporating heterogeneity is to adopt Friedman 1957’s description of a permanent and transitory component in the income process. To account for business cycle dynamics, one can further assume that individuals face some level of potential unemployment in each period, creating a precautionary savings motive for consumers. Given that such uncertainty cannot be fully insured against, the availability of a riskless asset that partially insures against income risk results in households choosing to hold different levels of market resources optimally.

Krusell and Smith 1998’s seminal work suggests that models assuming heterogeneity in individual income perform well in matching the aggregate capital stock but poorly in matching the distribution of wealth. The next step is to assume there is some ex-ante heterogeneity among households, leading more households to optimally hold lower levels of wealth.<sup>2</sup> Carroll et al. 2017 adopt

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<sup>1</sup>See Inequality.org articles data November 21, 2022: “Wealth Inequality in the United States” and “Updates: Billionaire Wealth, U.S. Job Losses and Pandemic Profiteers” (date accessed: March 27, 2023)

<sup>2</sup>Kaplan and Violante 2022’s recent work provides a comprehensive survey of incomplete markets models with heterogeneous agents featuring (i) uninsurable idiosyncratic income risk,

this approach and assume that agents differ in their time preferences, which reflects implicit characteristics of households relevant to their lifetime wealth accumulation. The authors find that this assumption of modest heterogeneity in time preferences is sufficient to match both the shape and skewness of the empirical distribution of wealth.

It is worth noting that the time preference factor ( $\beta$ ) is one of the key parameters that influences an individual's equilibrium target level of market resources, but it is not directly observable. Therefore, in order to estimate  $\beta$ , one would need to gather data through surveys or other methods that allow for the direct acquisition of information from households. On the other hand, estimating differences in the rate of return to financial assets across households is possible, as this variable *is* directly observable.

In this paper I provide further evidence of the heterogeneous agent framework's ability to match wealth moments by adding a single source of heterogeneity across households beyond the realization of ex-post shocks to their income. I allow for households differ in the return earned on assets first in the infinite horizon setting. I further extend the model to allow for rich life cycle dynamics. From there, I compare the effects of revenue-equivalent wealth and capital income tax policies on wealth inequality and on welfare. Additionally, I interpret the heterogeneity in the returns on safe assets earned by households in the context of the transmissions channel of monetary policy and its variation across the banking sector. As a final step, I redo the entire exercise under the assumption that the returns across households are lognormally distributed.

## 2 Literature Review

### 2.1 Collecting Data on the Distribution of Wealth

Empirical estimates of the skewness in wealth holdings over time provide valuable insights for this paper. Surveys and the imputation of wealth levels using administrative income tax data (sometimes referred to as the *capitalization method*) are the standard ways of collecting household data on the distribution of wealth for empirical analysis.

Wolff 2004 provides an early analysis of measurements of wealth by the Survey of Consumer Finances (SCF)<sup>3</sup> by discussing both the concentration and composition of household wealth 1980s and 1990s. The author's analysis corroborates the story of significant and growing inequality in the distribution of wealth in the U.S. Specifically, although the wealth of the average household grew in the 1990s, most of the gains in wealth and income during this period were enjoyed by the upper 20 percent of the wealth distribution, and especially the top 1 percent. While from 1983 to 2003 the top 1 percent experienced 33

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(ii) a precautionary savings motive, and (iii) an endogenous wealth distribution.

<sup>3</sup>See Kennickell 2017a for an extensive description of the methodology for sampling the wealthiest households in the SCF and Kennickell 2017b for an analysis of the performance of the SCF at measuring the wealth of the top 1 percent.

percent of the total growth in net worth (89 percent for the top 20 percent), the average wealth of the poorest 40 percent of households fell by 44 percent during this same time period and had reached roughly \$2,900 by 2001.

Saez and Zucman 2014 employs the capitalization method on tax data from the Internal Revenue Service to estimate the distribution of wealth in the United States for a much longer time period of 1913 to 2012. The usefulness in the authors' approach is that they are able to decompose their measure of wealth and savings into fractiles (i.e. top 1 percent, top 10 percent, bottom 20 percent wealth shares), which allows them to analyze the evolution of wealth over time in a way that is standard in the existing literature on wealth inequality. The authors not only find that inequality in the U.S. wealth distribution is relatively high and has been growing significantly in the later periods of their dataset, but they also attribute this growth primarily to the wealthiest of households. Indeed, they cite that the wealth shares of the top .1 percent of the distribution grew from 7 percent in 1978 to 22 percent in 2012.

## 2.2 Explaining Inequality in the Distribution of Wealth

Benhabib and Bisin 2018 provide an insightful review of the literature on the documented skewness in the distribution of wealth. The survey begins with historical accounts of the origins of the shape of the wealth distribution, dating back as early to Pareto and Samuelson. The authors then provide the traditional theoretical explanations of this unequal distribution: (i) skewness in the (exogenous) distribution of earnings, (ii) stochastic returns to wealth and savings, and, importantly, (iii) microfoundations for the evolution of wealth resulting from the consumption and saving behavior of households<sup>4</sup>.

Gabaix et al. 2016 define a notion for the speed of convergence to provide an explanation for observed evolution of income inequality over time, specifically in the upper tail of the distribution in the past 40 years in the United States. Notably, the authors show that, in order to match the empirical dynamics of inequality, one needs to allow for more forms of heterogeneity in the income process for households that are not incorporated in the standard consumption and saving models.<sup>5</sup> The first form is *type dependence* in the income growth rate distribution, which models the case in which some households have a higher average income growth rate. The second form, *scale dependence*, captures the fact that higher income levels are more susceptible to shocks to their income growth. The authors find that former does a good job at explaining this fast rise in income inequality, and the latter can generate infinitely fast transitions in inequality.<sup>6</sup>

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<sup>4</sup>As explored in the next section, the emergence of heterogeneous agent models has been a significant development in investigating this issue. Bewley 1983, Aiyagari 1994, and Huggett 1993 are among the earliest examples.

<sup>5</sup>Note that, although this analysis is about the distribution of income, this literature notably asserts that the distribution of wealth inherits some of its skewness from the distribution of income

<sup>6</sup>As we will see, these notions of “type dependence” and “scale dependence” show up in the literature on household heterogeneity and the wealth and income distributions; most

De Nardi and Fella 2017 provide another survey of the literature, more focused on the microfoundations for the distribution of wealth. Specifically, the authors note a number of possible extensions of models of household consumption and saving behavior, inspired by observable differences and the demographics of households, which lead to differences in wealth accumulation over time. Earnings and rate of return risk, ex-ante heterogeneity in preferences, medical expenses, bequest motives, and entrepreneurship are all cited as potential avenues to better explain the shape of the distribution of wealth using the behavior of households.

### 2.3 Measurements of heterogeneous rates of return

The rationale behind incorporating heterogeneity in rates of return to asset holdings lies in the use of novel datasets in recent empirical research to quantify the differences in returns among individuals. Fagereng et al. 2020 document extensively the heterogeneity in realized returns using 12 years of data from Norway’s administrative tax records. The authors’ findings reveal substantial differences in the average returns to assets for individuals (*type dependence*), that this heterogeneity is found both within and across classes of assets with varying levels of risk, and that returns are positively correlated with wealth (*scale dependence*). Moreover, they further demonstrate that this discovery of heterogeneous returns exhibits significant persistence over time and are positively correlated across generations. Each of these findings provide not only motivation for the assumption of ex-ante heterogeneous rates of return in the buffer-stock savings model of households, but also provide a benchmark to compare the distribution of rates of return resulting from the estimation procedure aimed at best matching the empirical distribution of wealth, as in Carroll et al. 2017.

Bach, Calvet, and Sodini 2018 use administrative panel data on the balance sheets of Swedish residents to gauge historical and expected returns, as well as risks associated with asset holdings. Their analysis of portfolio performs supports the finding that heterogeneous returns play a considerable role in the levels and growth of top wealth shares over time.

Campbell, Ramadorai, and Ranish 2019 consider wealth held in equity accounts in India between 2002 and 2011 and find that heterogeneity in returns to investment, which can be achieved by both the inherent randomness associated with risky investment and differences in the investment strategies of investors, is a main contributor to the increase in inequality of wealth held in equity portfolios during the time period. Here, the authors attribute the scale dependence associated with the returns to equity portfolios to the finding that smaller accounts tend to be poorly diversified relative to their larger account counterparts.

Deuflhard, Georgarakos, and Inderst 2018 provide an important analysis for the heterogeneous agent, incomplete markets model with a precautionary saving motive and a *single asset* to partially insure against risk with by studying the

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importantly, in the discussion on heterogeneous rates of return to wealth.

performance of households' investments in savings accounts. Not only do they find substantial type dependence in the rate of return to these safe assets, they also attribute the heterogeneity in returns to differences in financial sophistication. Notably, providing an explanation for differences in returns to investments for households is a vital step in potentially endogenizing this form of ex-ante heterogeneity among households in future research.

Altmejd, Jansson, and Karabulut 2024 is a recent work which provides causal evidence of financial education leading to significant differences in portfolio returns. Using university application data from the Swedish National Archives and data from the Swedish Income and Wealth registry, they show that individuals marginally admitted to business or economics programs not only hold more money in stocks but earn a higher raw return on these holdings than their counterparts.

## 2.4 Recent HA models with heterogeneous rates of return

The paper Daminato and Pistaferri 2024 incorporates heterogeneous returns into the solution of a model of consumption-saving for households. There, they use data from the PSID to document heterogeneity in returns, which they state is comparable to that found in the Norwegian registry data used by Fagereng et al. 2020.

Benhabib, Bisin, and Luo 2019 proposes an overlapping generations model that incorporates intergenerational wealth transfers. There, agents face uncertainty regarding both labor and capital income. Benhabib, Bisin, and Luo 2017 undertake a similar exercise, where household preferences for bequests to the next generation are more explicitly defined. Both papers conclude that the distribution of earnings and differences in rates of savings and bequests are crucial in matching the characteristics of the observed wealth distribution's tail ends.

Guler, Kuruscu, and Robinson 2022 develop a life-cycle model that provides a comprehensive description of households' optimal decision-making to endogenize heterogeneity in the rate of return, as they consider optimal choices regarding housing and mortgage decisions. These modeling choices enable the authors to investigate the effects of aggregate fiscal shocks, including one-time stimulus payments and mortgage debt relief programs.

Menzio and Spinella 2025 introduce a model of the financial market with search frictions into the standard macroeconomic setting of an infinite-horizon decision problem for households and firms. A distribution of returns across households arises endogenously in their model, and they use the empirical findings from Fagereng et al. 2020 regarding the distribution of returns to net worth as notable targeted moments.

## 2.5 My contributions

After taking stock of the research focused on returns heterogeneity and the distribution of wealth, my paper is different from the literature in at least two ways. The first notable difference is that, my paper models the uncertainty

in labor income as a random walk, as opposed to an AR(1) process. The key implication of this modeling difference is that, the AR(1) specification leads to less uncertainty in earnings over the life cycle from the perspective of a household at the start of the horizon for which they are choosing an optimal consumption path. This will lead to less accumulation of wealth over the life cycle. In this way, the permanent income specification that I am using will attempt to describe *as much* of the dispersion of wealth across households as possible with labor income uncertainty. The remaining dispersion in wealth across households which cannot be explained by differences in earnings will be attributed to returns heterogeneity, ultimately leading to more modest estimates of differences in returns across households.

The second way in which my paper is different from the existing literature is that the life cycle version of my model is much richer in its calibration of earnings and mortality rates. Specifically, I use an earnings profile provided by Cagetti 2003, which distinguishes mean earnings not only by age but by education cohort. Furthermore, I use age-education dependent mortality rates provided by Brown, Liebman, and Pollet 2007, which is uncommon in this literature. Essentially, I distinguish between households demographically by more than just age by incorporating education levels. Doing so does provide another avenue to explain dispersion in wealth holdings across households. However, it is a limited role and some further ex-ante heterogeneity among households, such as time preferences or the rate of return, is still needed to match wealth moments precisely.

### 3 Model

#### 3.1 Defining the stochastic income process

Each household's income ( $y_t$ ) during a given period depends on three main factors. The first factor is the aggregate wage rate ( $W_t$ ) that all households in the economy face. The second factor is the permanent income component ( $p_t$ ), which represents an agent's present discounted value of human wealth. Lastly, the transitory shock component ( $\xi_t$ ) reflects the potential risks that households may face in receiving their income payment during that period. Thus, household income can be expressed as the following:

$$y_t = p_t \xi_t W_t.$$

The level of permanent income for each household is subject to a stochastic process. In line with Friedman 1957's description of the labor income process, we assume that this process follows a geometric random walk, which can be expressed as:

$$p_t = p_{t-1} \psi_t,$$

The white noise permanent shock to income with a mean of one is represented by  $\psi_t$ , which is a significant component of household income. The probability

of receiving income during a given period is determined by the transitory component, which is modeled to reflect the potential risks associated with becoming unemployed. Specifically, if the probability of becoming unemployed is  $\mathcal{U}$ , the agent will receive unemployment insurance payments of  $\mu > 0$ . On the other hand, if the agent is employed, which occurs with a probability of  $1 - \mathcal{U}$ , the model allows for tax payments  $\tau_t$  to be collected as insurance for periods of unemployment. The transitory component is then represented as:

$$\xi_t = \begin{cases} \mu & \text{with probability } \mathcal{U}, \\ (1 - \tau_t)l\theta_t & \text{with probability } 1 - \mathcal{U}, \end{cases}$$

where  $l$  is the time worked per agent and the parameter  $\theta$  captures the white noise component of the transitory shock.

### 3.2 Decision problem for households

This paragraph presents the baseline version of the household's optimization problem for consumption-savings decisions, assuming no ex-ante heterogeneity. In this case, each household aims to maximize its expected discounted utility of consumption  $u(c) = \frac{c^{1-\rho}}{1-\rho}$  by solving the following:

$$\max \mathbb{E}_t \sum_{n=0}^{\infty} (\beta \mathcal{D})^n u(c_{t+n}).$$

It's worth noting that the setting described here follows a perpetual youth model of buffer stock savings, similar to the seminal work of Krusell and Smith 1998. To solve this problem, we use the bellman equation, which means that the sequence of consumption functions  $\{c_{t+n}\}_{n=0}^{\infty}$  associated with a household's optimal choice over a lifetime must satisfy<sup>7</sup>

$$\begin{aligned} v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \mathcal{D} \mathbb{E}_t [\psi_{t+1}^{1-\rho} v(m_{t+1})] \\ \text{s.t.} & \\ a_t &= m_t - c_t(m_t), \\ k_{t+1} &= \frac{a_t}{\mathcal{D}\psi_{t+1}}, \\ m_{t+1} &= (1 + r_t)k_{t+1} + \xi_{t+1}, \\ a_t &\geq 0. \end{aligned}$$

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<sup>7</sup>Here, each of the relevant variables have been normalized by the level of permanent income ( $c_t = \frac{C_t}{p_t}$ , and so on). This is the standard state-space reduction of the problem for numerical tractability.



## 4 Results

### 4.1 The infinite horizon setting

#### 4.1.1 The model with no returns heterogeneity

To solve and simulate the model, I follow the calibration scheme captured in table 1.

Description	Parameter	Value	Source
Time discount factor	$\beta$	0.99 <sup>4</sup>	Den Haan, Judd, and Juillard 2010
CRRA	$\rho$	1	Den Haan, Judd, and Juillard 2010
Capital share	$\alpha$	0.36	Den Haan, Judd, and Juillard 2010
Depreciation rate	$\delta$	0.025	Den Haan, Judd, and Juillard 2010
Time worked per employee	$\ell$	1/.09	Den Haan, Judd, and Juillard 2010
Wage rate	$W$	2.37	Den Haan, Judd, and Juillard 2010
Unempl. insurance payment	$\mu$	0.15	Den Haan, Judd, and Juillard 2010
Probability of survival	$\mathcal{P}$	(1 - 0.00625) <sup>4</sup>	Yields 40-year working life
Std. dev of $\log \theta_{i,t}$	$\sigma_\theta^2$	0.010 x 4 x $\sqrt{4}$	Carroll 1992,
			Carroll, Slacalek, and Tokuoka 2015
Std. dev of $\log \psi_{i,t}$	$\sigma_\psi^2$	0.010 x 4/11 x $\sqrt{4}$	Carroll 1992,
			Debacker et al. 2013,
			Carroll, Slacalek, and Tokuoka 2015
Unemployment rate	$\mathcal{U}$	0.07	Mean in Den Haan, Judd, and Juillard 2010

**Table 1:** Parameter values (annual frequency) for the perpetual youth model.

The solution of the model with no heterogeneity in returns (the R-point model) is the one which finds the value for the rate of return  $R$  which minimizes the distance between the simulated and empirical wealth shares at the 20th, 40th, 60th, and 80th percentiles. The empirical targets are computed using the 2004 SCF data on household wealth. The estimation procedure finds this optimal value to be  $R = 1.0602$ .

#### 4.1.2 Incorporating heterogeneous returns

As noted above, recent studies by Fagereng et al. 2020 and Bach, Calvet, and Sodini 2018 have not only estimated the rate of return on asset holdings but have also uncovered significant heterogeneity across households. With this in mind, the next estimation (the R-dist model) assumes the existence of multiple types of agents, each earning a distinct rate of return on their assets.

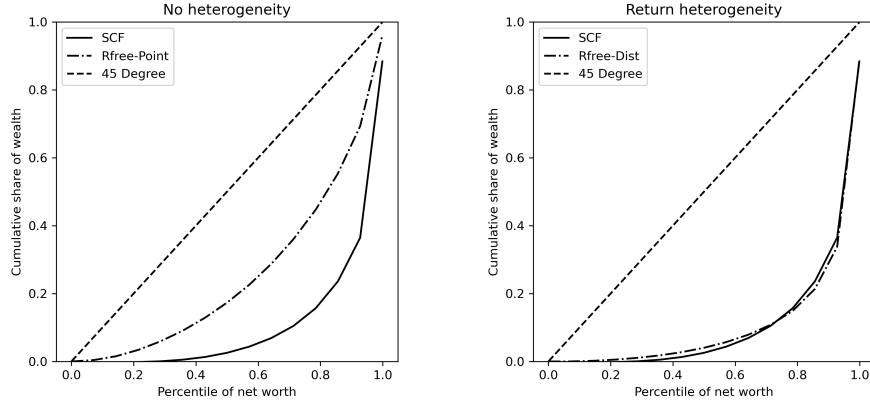
I follow closely the procedure outlined by Carroll et al. 2017. Specifically, I assume that different types of households have a time preference factor drawn from a uniform distribution on the interval  $(\bar{R} - \nabla, \bar{R} + \nabla)$ , where  $\nabla$  represents the level of dispersion. Afterward, the model is simulated to estimate the values of both  $\bar{R}$  and  $\nabla$  so that the model matches the inequality in the wealth distribution. To achieve this, the following minimization problem is solved:

$$\{\bar{R}, \nabla\} = \arg \min_{\bar{R}, \nabla} \left( \sum_{i=20,40,60,80} (w_i(\bar{R}, \nabla) - \omega_i)^2 \right)^{\frac{1}{2}}$$

subject to the constraint that the aggregate capital-to-output ratio in this model matches the calibrated value 5. This is towards the upper bound on plausibly calibrated values for the capital-to-output ratio, as much of the literature chooses values between 2 and 3.

Note that  $w_i$  and  $\omega_i$  give the porportion of total aggregate net worth held by the top  $i$  percent in the model and in the data, respectively.

The estimation procedure finds this optimal values of  $R = 1.0204$  and  $\nabla = 0.06833$ . These parameter values pin down the estimated uniform distribution. In the model, I've chosen to discretize that distribution to 7 chosen points. The performance of the estimation of both the R-point and R-dist models, measured by their ability to match the SCF data, is compared in figure 1.



**Figure 1:** Comparison of R-Point and R-Dist Models.

Additionally, here is a table compare the relevant moments of the empirical and simulated wealth distributions.

**Table 2:** Wealth Distribution (PY)

	0–20	20–40	40–60	60–80	80–95	Top 5%
<b>Wealth share (data)</b>	-0.002	0.011	0.044	0.118	0.255	0.574
<b>Wealth share (no het.)</b>	0.030	0.085	0.142	0.224	0.285	0.233
<b>Wealth share (het.)</b>	0.005	0.019	0.042	0.096	0.251	0.588

## 4.2 Incorporating life cycle dynamics into the model

More realistic assumptions regarding the age and education level of households can have important implications for the income and mortality process of households. Here, I extend the model to incorporate these life cycle dynamics.

Households enter the economy at time  $t$  aged 24 years old and are endowed with an education level  $e \in \{D, HS, C\}$ , and initial permanent income level  $\mathbf{p}_0$ , and a capital stock  $k_0$ . The life cycle version of household income is given by:

$$y_t = \xi_t \mathbf{p}_t = (1 - \tau) \theta_t \mathbf{p}_t,$$

where  $\mathbf{p}_t = \psi_t \bar{\psi}_{es} \mathbf{p}_{t-1}$  and  $\bar{\psi}_{es}$  captures the age-education-specific average growth factor. Households that have lived for  $s$  periods have permanent shocks drawn from a lognormal distribution with mean 1 and variance  $\sigma_{\psi_s}^2$  and transitory shocks drawn from a lognormal distribution with mean  $\frac{1}{\bar{\psi}}$  and variance  $\sigma_{\theta_s}^2$  with probability  $\mathcal{X} = (1 - \mathcal{U})$  and  $\mu$  with probability  $\mathcal{U}$ .

The normalized version of the age-education-specific consumption-saving problem for households is given by

$$\begin{aligned}
v_{es}(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \mathcal{D}_{es} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v_{es+1}(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t, \\
k_{t+1} &= \frac{a_t}{\psi_{t+1}}, \\
m_{t+1} &= (1 + r_t)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}$$

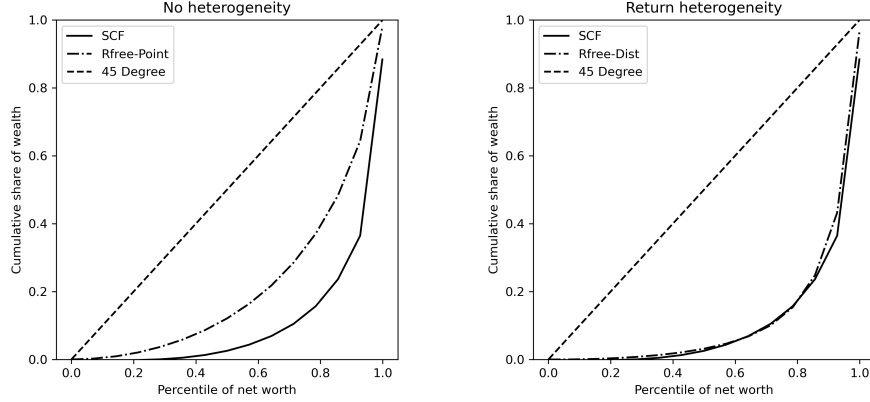
The additional parameters necessary to calibrate the life cycle version of the model are given in table 3. The age-education dependent mean income levels come from Cagetti 2003. The permanent and transitory shock variances come from Sabelhaus and Song 2010. The age-education dependent mortality rates come from Brown, Liebman, and Pollet 2007.

Description	Parameter	Value
Population growth rate	$N$	0.0025
Technological growth rate	$\Gamma$	0.0037
Rate of high school dropouts	$\theta_D$	0.11
Rate of high school graduates	$\theta_{HS}$	0.55
Rate of college graduates	$\theta_C$	0.34
Labor income tax rate	$\tau$	0.0942

**Table 3:** Parameter values (annual frequency) for the lifecycle model.

The estimation procedure finds this optimal value to be  $R = 1.0431$  for the R-point model in this setting. The estimation procedure for the R-dist model in the life cycle setting finds optimal values of  $R = 1.002$  and  $\nabla = 0.0953$ . Notice the improved performance of the estimation in matching the data displayed in figure 2.

Additionally, here is a table compare the relevant moments of the empirical and simulated wealth distributions.



**Figure 2:** Comparison of R-Point and R-Dist Models in the Life-Cycle Setting.

**Table 4:** Wealth Distribution (LC)

	0–20	20–40	40–60	60–80	80–95	Top 5%
Wealth share (data)	-0.002	0.011	0.044	0.118	0.255	0.574
Wealth share (no het.)	0.018	0.056	0.110	0.207	0.321	0.287
Wealth share (het.)	0.003	0.014	0.037	0.115	0.358	0.473

### 4.3 Untargeted moments

#### 4.3.1 The marginal propensity to consume

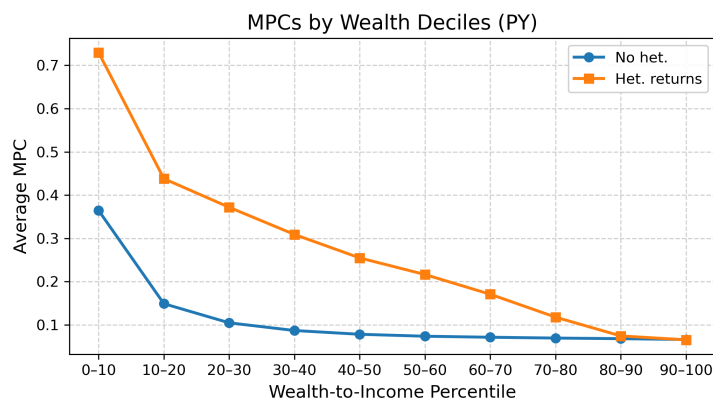
Naturally, adding a source of (ex-ante) heterogeneity beyond the addition of labor income uncertainty to the representative agent framework will allow the simulated distribution of wealth to match moments of the empirical wealth distribution particularly well. Although it is useful to see that the model with heterogeneous returns does a good job of matching the given lorenz targets, we need another way to assess the model’s performance.

It is well-documented in the literature on heterogeneous agent modeling that models with a precautionary savings motive will lead to a dispersion in the marginal propensity to consume (MPC) across households. This is a key feature in why the HA framework is able to produce estimates of the aggregate MPC which are close to empirical measurements, like those measured in Jappelli and Pistaferri 2014.<sup>8</sup>

In the infinite horizon setting, we already see that the model with and without heterogeneity will have different implications for the aggregate MPC and its dispersion across households. Namely, the aggregate MPC when households earn the same return on their savings is 11.3%, while it is 27.5% when hetero-

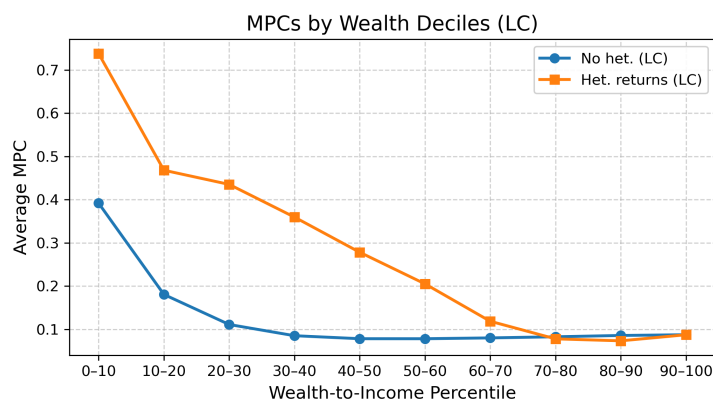
<sup>8</sup>There the authors estimate an MPC of about 48% and document significantly larger MPCs for households with low cash-on-hand.

geneous returns are present. We decompose this by looking at average MPCs by wealth decile in the figure below.



As you can see, MPCs are much higher in the case where households earn different returns. With heterogeneous returns, the model generates the observed extreme wealth concentration. Poor households are not just somewhat poorer—they're dramatically poorer, pushing them into the hand-to-mouth region where consumption tracks income almost one-for-one.

Since the life cycle setting generates a more realistic distribution of wealth (even without heterogeneous returns), it is reasonable that MPCs are slightly higher here: an aggregate MPC of 12.6% without heterogeneity and of 27.8% with heterogeneous returns. That said, the stark difference between the model with and without heterogeneity remains present here as well.



### 4.3.2 Wealth shares by age cohort

I also include age-dependent wealth moments from the same wave of the SCF to serve as another set of untargeted moments 7. These can be found in the following table.

**Empirical Lorenz Shares by Age (2004)**

age	20th	40th	60th	80th
25-30	-0.0723	-0.0657	-0.0266	0.1099
30-40	-0.008	0.0054	0.057	0.1813
40-50	-0.0001	0.0187	0.0776	0.2178
50-60	0.0018	0.0215	0.0766	0.2126
60-70	0.0011	0.0188	0.0726	0.2081

**Figure 3:** Empirical Lorenz Curve Targets from the 2004 SCF.

The next two tables present the simulated version of the untargeted moments for the model without heterogeneity 4, and then with heterogeneity 8. As you can see from the tables below, the age-dependent Lorenz targets that arise from the model again fit the data much better when returns heterogeneity is present versus when it is absent.

**Simulated Lorenz Shares by Age**

age	20th	40th	60th	80th
25-30	0.0449	0.1342	0.2773	0.5253
30-40	0.0421	0.1426	0.2984	0.5325
40-50	0.0465	0.1416	0.2875	0.5116
50-60	0.0506	0.1452	0.2878	0.5069
60-70	0.0466	0.1352	0.2706	0.4837

**Figure 4:** Simulated Untargeted Moments without Heterogeneity (R-point).

**Simulated Lorenz Shares by Age**

age	20th	40th	60th	80th
25-30	0.0229	0.0951	0.2203	0.4458
30-40	0.01	0.052	0.1423	0.3432
40-50	0.0057	0.0293	0.088	0.266
50-60	0.0044	0.0216	0.0695	0.2397
60-70	0.0039	0.0195	0.0662	0.2292

**Figure 5:** Simulated Untargeted Moments with Heterogeneity (R-dist).

## 5 Wealth v.s. Capital income tax

It is well known in the literature that the interesting setting to compare the effects of a tax on wealth versus one on capital income is when heterogeneous returns are present. Guvenen et al. 2023 documents that, when individuals earn different returns on their assets, a capital income tax will place a larger burden on those more efficient with their capital. On the other hand, the wealth tax will shift the burden towards those who are unproductive with their capital. This redistributive effect of the wealth tax leads to higher aggregate productivity, output, and ultimately larger welfare gains than the capital income tax. Following a similar line of reasoning, in this setting we can consider the quantitative effects of applying each of the tax schemes on distribution of wealth when heterogeneous returns are present.

Suppose we begin in an economy where the distribution of returns is the one needed to best match wealth moments measured in the data. The question then is, what happens to the distribution of wealth if a revenue equivalent tax rate is applied to every household? It is clear that a capital income tax will decrease wealth inequality, since households that earn any capital income will see a tax but those with zero or negative returns will see a capital tax of zero. This will lead to less wealthy households holding a higher share of the aggregate wealth than when the capital tax is not present. The effects of the wealth tax are less obvious, since all households hold some wealth and thus will be taxed accordingly.

In this setting, the wealth tax enters the household budget constraint in the following way:

$$m_{t+1} = (1 + r_t - \tau_w) k_{t+1} + \xi_{t+1}.$$

Similarly, for the capital income tax, we have

$$m_{t+1} = (1 + (1 - \tau_{ci}) r_t) k_{t+1} + \xi_{t+1}.$$

I compute aggregate income as GDP in this setting, and then find the wealth tax rate and the capital tax rate which would raise tax revenues which are equal to 1% of GDP. I do this for the estimated distribution of returns both for the infinite horizon and the life cycle setting.

### 5.1 Effects on the wealth distribution

From there, I apply the tax schemes to each households and find the resulting wealth distributions for both cases. The wealth moments can be easily compared before and after each of the policies were implemented. Below are the results of applying each of the tax schemes in the infinite horizon 5 version of the model.

	Lorenz points			
Tax scheme	20%	40%	60%	80%
None	.49%	2.4%	6.7%	16.3%
Wealth $\tau_w = .33\%$	.54%	2.8%	7.8%	18.7%
Capital income $\tau_{ci} = 4.96\%$	.59%	2.97%	7.8%	18.7%

**Table 5:** Tax policies in the infinite horizon setting.

Applying the tax schemes in both cases increases the share of the wealth held at each of the chosen percentiles. This makes sense, as such a tax should redistribute extreme levels of wealth towards lower rungs of the distribution. Although the effect is minimal in both cases, the capital income tax has a more pronounced effect on reducing wealth inequality than the flat wealth tax. This is in line with what the literature regarding the effects of these two policies when heterogeneous returns are present: if the capital income tax places a burden on the *most productive* households regarding use of capital, then it should lead to less wealth inequality since those households contribute most to the skewness in the distribution.

Next, I apply the tax schemes in the life cycle 6 version of the model. A similar result arises here, where the capital income tax has a larger effect on reducing wealth inequality than the wealth tax which raises the same level of tax revenue does. From the table, the effects in the life-cycle model are less pronounced than they are for the infinite horizon case.

	Lorenz points			
Tax scheme	20%	40%	60%	80%
None	.34%	1.8%	5.4%	16.98%
Wealth $\tau_w = .36\%$	.35%	1.8%	5.6%	17.1%
Capital income $\tau_{ci} = 7.1\%$	.37%	1.9%	5.9%	17.9%

**Table 6:** Tax policies in the life cycle setting.

## 5.2 Welfare effects of the tax policies

As a further comparison of the two tax policies when heterogeneous returns are present, I follow a method similar to Guvenen et al. 2023 and compare



newborns’ lifetime utility in the setting where they start with a returns value from the estimated distribution which best matched the SCF wealth data. I consider a consumption-equivalent measure of welfare by computing the percentage change in consumption that would make a newborn (who didn’t yet know their return type) indifferent from switching from the wealth tax to the capital income tax. Additionally, I decompose the effects of the policies by considering the consumption-equivalent measure of welfare for each of the estimated return types.

To do this, I retain my calibrated value of  $\rho = 1$  so that  $u(c) = \log c$ . In this case, the consumption-equivalent change  $\Delta$  between wealth tax (WT) and capital income tax (CIT) is defined by equating the lifetime utilities. So,

$$1 + \Delta = \exp\left(\frac{V^{WT} - V^{CIT}}{S}\right)$$

where  $S = \sum_{t=0}^{\infty} (\beta \cdot \mathcal{D})^t$ , and  $V^j$  is the lifetime value of a newborn under regime  $j \in \{WT, CIT\}$ . The parameter  $\Delta$  tell us *by what constant percentage change in consumption each period a newborn under the wealth tax regime would be as well off as in the capital income tax regime* (i.e. if  $\Delta > 0$ , then the newborn would be better off by switching to the capital income tax).

Below are the welfare effects of the tax policies for both the infinite horizon and life cycle settings. As a baseline for comparison, I also include the expected lifetime utility of the original regime with no tax policy present.

**Table 7:** Expected Welfare Gains from Tax Reform

	Infinite horizon	Life-cycle
WT vs CIT	0.20%	0.15%
WT vs Original	0.85%	0.35%
CIT vs Original	0.65%	0.20%

Notes: Policy A vs Policy B means: the expected welfare gain from switching from policy A to policy B. Entries are consumption-equivalent (CE) welfare gains,  $\Delta$ , expressed as percent changes. Positive values indicate the row’s left policy yields higher newborn lifetime welfare than the right policy.

As you can see from the table, the original regime is preferred to both tax policies. This is expected: my model is a partial equilibrium analysis, so we don’t get to see how households may benefit from the effect of the collected tax revenues on the economy. That said, notice that before households know their type, newborns would need to be compensated more under the wealth tax than under the capital income tax to be indifferent between the given tax regime and the regime with no tax! This point is reaffirmed by the fact that newborns under the wealth tax would also need to be compensated by .2% of their income to be indifferent between switching to the capital income tax regime. The point about this being a partial equilibrium analysis is subtle, but important. In particular, the Guvenen et al. 2023 paper cites that the wealth tax is the preferred policy by

their own welfare analysis. The key difference is that the channel through which taxes can effect aggregate output, productivity, and consumption is not present in my model. In a sense, you can consider this welfare analysis as identifying which tax regime is costlier in terms of expected lifetime utility.

Additionally, I decompose these welfare effects by considering which tax regime is preferred by each of the possible realized return types in the infinite horizon setting in the table below.

**Table 8:** Per-Type Welfare Gain and Baseline Return (WT vs CIT)

	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6	Type 7
Baseline $R$ (gross)	0.9618	0.9813	1.0009	1.0204	1.0399	1.0594	1.0790
CE $\Delta$ (WT vs CIT, %)	0.23%	0.27%	0.34%	0.32%	0.30%	0.24%	-0.31%

Notes: CE entries are consumption-equivalent welfare gains (pmv-weighted within type), expressed as percent. Positive values favor wealth taxation over capital income taxation for that return type. Baseline  $R$  are pre-tax gross returns by type (low  $\rightarrow$  high).

This decomposition explains why the expected welfare of the newborn was higher under the capital income tax than the wealth tax (and so the capital income tax was favored). Only the highest return type would have to pay some percentage of their consumption to be indifferent to switching to the capital income tax starting from the wealth tax! All other agents would require some compensation to reach this indifference. This is because, although many of the types receive some capital income, it is not enough for the capital income tax to be a large hit to their expected lifetime utility.

The life-cycle setting features a similar phenomenon and is provided below. Their we can decompose the effects further by comparing types within education groups.

**Table 9:** Per-Education Per-Type Welfare Gain and Baseline Return (WT vs CIT)

		Type 1	Type 2	Type 3	Type 4	Type 5	Type 6	Type 7
NoHS	Baseline $R$ (gross)	0.9200	0.9472	0.9744	1.0016	1.0289	1.0561	1.0833
	CE $\Delta$ (WT vs CIT, %)	0.118%	0.143%	0.182%	0.264%	0.340%	0.135%	-0.124%
HS	Baseline $R$ (gross)	0.9200	0.9472	0.9744	1.0016	1.0289	1.0561	1.0833
	CE $\Delta$ (WT vs CIT, %)	0.117%	0.142%	0.184%	0.271%	0.332%	0.115%	-0.108%
College	Baseline $R$ (gross)	0.9200	0.9472	0.9744	1.0016	1.0289	1.0561	1.0833
	CE $\Delta$ (WT vs CIT, %)	0.116%	0.142%	0.184%	0.267%	0.309%	0.098%	-0.106%

Notes: CE entries are consumption-equivalent welfare gains (pmv-weighted within type), expressed as percent. Positive values favor wealth taxation over capital income taxation for that return type. Baseline  $R$  are pre-tax gross returns by type (low  $\rightarrow$  high) at  $t = 0$ .

## 6 Mechanism for returns heterogeneity

There have been a number of potential explanations proposed regarding the persistent component of returns heterogeneity. Among the more common ones

is the idea that there are some business owners in the economy, and variability among their entrepreneurial talent leads to even more heterogeneity in labor income than the assumptions about labor income uncertainty that the standard HA models use. Cagetti and De Nardi 2006 and Cagetti and De Nardi 2009 are notable examples of explicitly modeling entrepreneurial talent as a component of the consumption-saving problem for households and assessing the ability of such models in matching empirical wealth moments.

Although differences in entrepreneurial talent modeled as variability in labor market productivity allows for the model to better match wealth moments at the upper tail of the distribution, it is an unsavory explanation for the mechanism I have in mind in this paper. Namely, this modeling choice will result in households (firms) with high levels of wealth (capital) earning lower rates of return, and vice versa for households (firms) with low levels of wealth (capital). As mentioned before, Fagereng et al. 2020 documents scale dependence regarding wealth heterogeneity: that returns, as well as the idiosyncratic, persistent component of returns, are positively correlated with wealth.

Another explanation from the literature which is closer to, but still not exactly the same as, the mechanism in this paper which allow two households with the same level of assets to earn a different return on them is financial literacy or sophistication. Lusardi and Mitchell 2014 offer a survey on models which explicitly allow for households to make a costly decision to build up a stock of financial literacy, in turn allowing them access to an investment technology offering a higher average return. Lusardi, Michaud, and Mitchell 2017 show that allowing for such endogenous financial accumulation in the standard consumption-saving framework again allows the model to match wealth moments particularly well, this time through the returns channel (as opposed to the labor income channel associated with entrepreneurial talent).

As I am particularly interested in a setting with a single, safe asset available, I investigate another cause for heterogeneity in the rate of return by following literature which documents substantial heterogeneity in the banking sector, specifically on rates offered to depositors.

## 6.1 Deposit rates and the sensitivity of deposit levels to changes in the market interest rate

As mentioned before, Deuffhard, Georgarakos, and Inderst 2018 and others have shown that there can be substantial heterogeneity in rates of return earned by depositors. The key step in incorporating this finding in the standard consumption-saving framework is to identify potential sources of this heterogeneity in banking.

Consider the balance sheet of banks within the financial sector and the problem they face in optimally choosing the rate to offer on deposits. In a simple setting, they accept deposits at an offered deposit rate, and then hold reserves within the central banking system which earn the market interest rate. This discrepancy between the offered deposit rate and the market interest rate leaves room for banks to earn profit.

It is well documented in the U.S. empirically that the level of deposits at a given bank will change due to exogenous changes in the Fed funds rate. Drechsler, Savov, and Schnabl 2017 propose a clear transmissions channel for monetary policy: changes in the Fed funds rate may lead banks to widen the interest spread they charge on deposits, which causes deposits to flee the bank. The authors find a strong, negative relationship between changes in the federal funds rate and the growth rate of deposits. Furthermore, they also note that a 100 basis point increase in the Fed funds rate leads to a higher deposit outflow in bank branches in more concentrated markets relative to those in less concentrated markets.<sup>9</sup>

With this in mind, variation in the strength of this transmission channel across banks can be viewed as a potential source of heterogeneity in the banking sector, as certain market characteristics would make the level of deposits offered by some banks less sensitive to changes in federal funds rate than other banks. For example, Sarkisyan and Viratyosin 2021 make a distinction between *local* and *globally integrated* banks, and show that “global banks lose much more deposits relative to local banks in response to unexpected changes in the federal funds rate”. Adrien d’Avernas et al. 2024 show a similar finding regarding heterogeneity in deposit rates, but for larger vs smaller banks.

With this in mind, the next step is to extend the standard HA model describing household consumption-saving decisions by explicitly modeling a banking sector. In this setting, banks will each solve a similar profit-maximization problem regarding accepting deposits at an offered deposit rate and holding reserves which earn the market interest rate. The key distinction between banks in the model will be how sensitive the level of deposits are to changes in the market interest rate, which is in line with the mentioned empirical evidence. From here, it will be clear how heterogeneity in returns may arise for households which are otherwise the same.

## 6.2 Model of heterogeneous deposit rates

Here I present a small, open economy with banks and households as the optimizing agents in the model. This will be a partial equilibrium analysis since the world interest rate is being taken as given. That said, I present a simple framework for describing the optimal behavior for banks setting deposit rates. By assuming that there is a cobb-douglas aggregate production function, I will find the marginal product of capital (less depreciation) that is consistent with the capital to output ratio from the model which matches its empirical counterpart. This effective interest rate will be considered the world or “market” interest rate and will be used along with the estimated distribution of heterogeneous returns to back out estimated values for elasticities of foreign deposits to the deposit rates for each of the banks in the model.

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<sup>9</sup>Branches in “more concentrated markets” refers to banks which operate in local deposit markets where a few banks hold large market shares.

### 6.2.1 Assumptions regarding the banking sector

There are a continuum of banks, identical in all respects other than the elasticity of the level of deposits to changes in the market interest rate.

The model is static in that, I assume that the bank chooses a deposit rate to offer to its clientele base at the start of the time horizon. Therefore, this decision depends solely on the market interest rate and the bank's given elasticity. Additionally, a given bank cannot take actions to increase the number of depositors at their given institution <sup>10</sup>.

Lastly, in this simple version of the model, I assume that households do not endogenously choose which banks to do business with. Clearly, this would lead us towards the literature on costly human capital acquisition and financial literacy. Instead, I assume that at the outset banks are assigned to a household at birth with some probability. The household is "stuck" with this bank assignment until death. In this way, the banking sector merely replaces the assignment of idiosyncratic rates of returns over the time horizon in the standard model.

### 6.2.2 Decision problem for banks accepting deposits

The sole distinction between banks in this model is the sensitivity of their level of deposits to changes in the market interest rate, which we will index by  $\varepsilon_i$ . This will be the source of returns heterogeneity in the model. To see this, I follow a similar, but simplified description of the decision problem for banks which can be found in Paul and Ulate 2024.

Let  $R^m$  be the market rate of return,  $R^d$  be the rate of return offered on deposits by a bank, and  $S(R^d, R^m)$  be the level of deposits held at a given bank.

Banks solve:

$$\max(R^m - R^d) \cdot S(R^d, R^m)$$

subject to:

$$S(R^d, R^m) = A \left( \frac{R^d}{R^m} \right)^\varepsilon$$

Importantly, the first order condition implies that the optimal deposit rate for the  $i$ -th bank is given by

$$R_i^d = \frac{\varepsilon_i}{1 + \varepsilon_i} R^m.$$

This is crucial for the model in that, so long as we calibrate the model for a particular value of  $R^m$ , estimating a uniform distribution of returns using the simulated method of moments will imply a corresponding distribution of elasticities (the one that minimizes the distance between simulated and lorenz wealth moments). In this way, the 7 discretized points capture 7 different deposit rates offered, which result in varying elasticities among 7 different bank types in the model. From the expression above, we see that banks with higher values of  $\varepsilon_i$  must set  $R_i^d$  closer to  $R^m$ .

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<sup>10</sup>For example, compare a bank in a suburb area of Montana (local) versus a bank near downtown Houston, Texas (globally integrated).

### 6.3 The implied distribution of bank heterogeneity

I’ve estimated the distribution of returns which matches the wealth moments. As mentioned earlier, I can use this and the assumptions regarding the bank’s decision problem to back out an implied distribution of  $\varepsilon$ . This will describe how the banks differ in the sensitivity of their deposit levels to changes in the market interest rate. Both in our simplified setting, and in the transmission channel empirically documented by Drechsler, Savov, and Schnabl 2017, differences in these sensitivities ultimately leads to differences in the deposit rates offered across banks.

First, I assume that the aggregate production function is Cobb-douglas, so that the marginal product of capital can be written as  $\alpha \frac{Y}{K}$ . With the calibrated values  $\delta = .025$ ,  $\alpha = .36$ , and the capital to output ratio 3, this setting has an effective interest rate of  $R^m = 1.095$  which can be used as the market interest rate.

Since the model with heterogeneity (i.e. the R-dist model) has 7 estimated points for the uniform distribution, the implied, estimated points for  $\varepsilon$  can be uniquely pinned down by the expression

$$\varepsilon_i = \frac{R_i^d}{R^m - R_i^d}.$$

For the infinte horizon and life-cycle settings, the estimated returns distribution which best matches 2004 SCF data on net worth and the corresponding implied elasticities are given in the table below.

**Table 10:** Estimated Returns and Implied Elasticities

PY		LC	
Estimated returns	Implied elasticities	Estimated returns	Implied elasticities
0.962	7.222	0.920	5.256
0.981	8.634	0.947	6.408
1.001	10.632	0.974	8.081
1.020	13.676	1.002	10.728
1.040	18.877	1.029	15.553
1.059	29.786	1.056	27.127
1.079	67.239	1.083	92.489

#### 6.3.1 Interpreting the implied distribution of elasticities

Another way to assess the model’s performance is to return to the literature on bank deposit sensitivities to changes in the federal funds rate. As we will see, the implied distribution of elasticities from my estimation method can be directly compared to those empirical estimates, and how well they match can be used to assess my model. Additionally, I include wealth shares by age cohort as an additional set of untargeted moments for a similar assessment.

The usefulness in choosing a functional form for the level of deposits at a given bank as  $S(\cdot) = A \left( \frac{R^d}{R^m} \right)^\varepsilon$  is that the parameter  $\varepsilon$  has a clear interpretation as the elasticity of deposits to changes in the market interest rate. It can be shown that:

$$-\varepsilon = \frac{\partial \ln S(\cdot)}{\partial \ln R^m}.$$

So, the elasticity parameter tells us how a percentage change in the market interest rate changes the level of deposit, in percent terms. This allows us to directly compare the implied elasticities following the SMM procedure to the empirical evidence on the transmission channel described by Drechsler, Savov, and Schnabl 2017 regarding the relationship between the Fed funds rate and the level of deposits at banks. For example, Genay and Halcomb 2004 finds that a 1% change in the Fed funds rate leads to about a 3% to 4% change in the level of deposits, depending on the size of the bank.

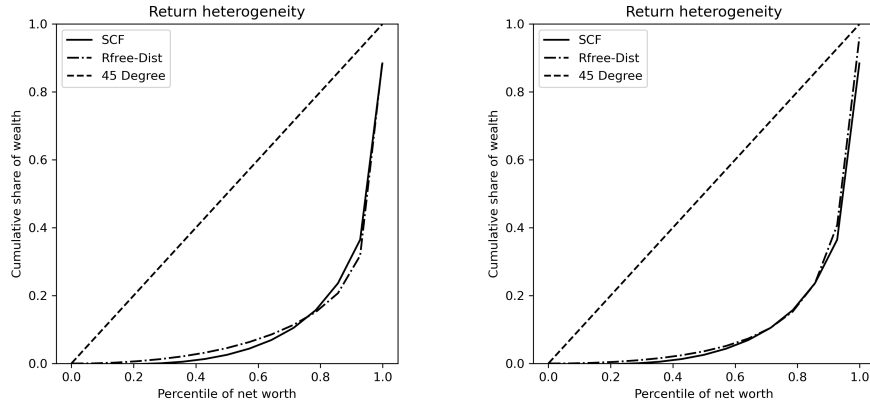
The amount of returns heterogeneity required to match wealth inequality using only safe assets (i.e. bank deposits) will lead to vastly overstated elasticities for the resulting banking sector. This is not surprising. If bank wish to attract depositors in the face of an increasing Fed funds rate, they will need to offer a higher rate on deposits, regardless of the size of the bank. This suggests that there will be less variation in the optimal deposit rates offered across the banking sector. The banks which do not offer competitive deposit rates will likely find that their depositors switch to other safe investment technologies like money market funds. Since my model doesn't match any moments regarding the number of banks in the economy, nor does it model returns heterogeneity by allowing for the choice between safe assets, it isn't too surprising that the elasticity of deposits to changes in the market interest rate is not well matched in this setting. That said, the ability of the model to back out a distribution of elasticities under the given assumptions is still useful.

## 7 Extension: Estimating a Lognormal distribution of returns

As I mentioned before, Fagereng et al. 2020 document a persistent component to individual returns using the Norwegian population data. In fact, they include a graphic of the distribution of the estimated fixed effect in the return to net worth for individuals, which is clearly not uniformly distributed. For this reason, I rerun the estimation of my model under the assumption that returns are instead lognormally distributed across households. As we will see, the takeaways from my paper are robust to this change.

### 7.0.1 Comparing the simulated wealth distributions

Since the model without heterogeneous returns is the same regardless of the distributional assumptions, below I include a figure of the results for the infinite horizon and life cycle models with heterogeneous returns, lognormally distributed across households.



**Figure 6:** Comparison of PY (left) vs LC (right) R-Dist Models.

It is clear from the figure that the same pattern appears here: the life cycle setting is better at matching the observed wealth distribution than the infinite horizon setting, but they both match wealth moments especially well. Again, I include the table of those wealth moments for each of the models and the data below.

### 7.1 Untargeted moments

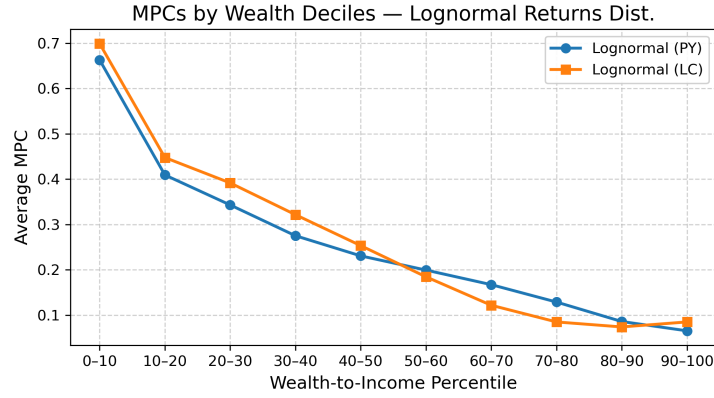
In the infinite horizon case, the aggregate MPC is 25.7%. The aggregate MPC in the life-cycle setting is essentially the same: 26%. Below is the breakdown of average MPCs by wealth deciles for both settings. Again, the MPC's are higher



**Table 11:** Wealth Distribution (Lognormal Returns)

	0–20	20–40	40–60	60–80	80–95	Top 5%
<b>Wealth share (data)</b>	-0.002	0.011	0.044	0.118	0.255	0.574
<b>Wealth share (PY)</b>	0.006	0.021	0.044	0.088	0.225	0.616
<b>Wealth share (LC)</b>	0.004	0.016	0.039	0.107	0.335	0.498

in general in the life-cycle setting since it is able to match lower moments of the wealth distribution better.



For the second set of untargeted moments, I include the cumulative wealth shares by age for this case of an estimated lognormal returns distribution. Again, these moments are closer to the empirical counterpart than the case with no heterogeneity. The best fits are for the wealth shares of older cohorts.

**Empirical Lorenz Shares by Age (2004)**

age	20th	40th	60th	80th
25-30	-0.0723	-0.0657	-0.0266	0.1099
30-40	-0.008	0.0054	0.057	0.1813
40-50	-0.0001	0.0187	0.0776	0.2178
50-60	0.0018	0.0215	0.0766	0.2126
60-70	0.0011	0.0188	0.0726	0.2081

**Figure 7:** Empirical Lorenz Curve Targets from the 2004 SCF.

## 7.2 Tax implications

I also consider the tax implications of the model in this setting. I begin with comparing the effects of each of the tax policies on wealth inequality for both the infinite horizon and the life-cycle cases. The same relationship holds here:

**Simulated Lorenz Shares by Age**

age	20th	40th	60th	80th
25-30	0.0261	0.1017	0.2285	0.4545
30-40	0.0126	0.0595	0.1531	0.3432
40-50	0.0074	0.0349	0.097	0.2547
50-60	0.0059	0.0272	0.0797	0.2296
60-70	0.0054	0.0254	0.076	0.2199

**Figure 8:** Simulated Untargeted Moments with Heterogeneity (R-dist).

each effect of the tax is small (since the policies are revenue-equivalent and only raise 1% of aggregate labor income), but the capital income tax reduces wealth inequality more.

	Lorenz points			
Tax scheme	20%	40%	60%	80%
None	0.60%	2.75%	7.27%	16.28%
Wealth $\tau_w = 0.34\%$	0.67%	3.14%	8.38%	18.86%
Capital income $\tau_{ci} = 5.08\%$	0.73%	3.37%	8.94%	20.02%

**Table 12:** Tax policies in the infinite horizon setting (lognormal returns across households).

	Lorenz points			
Tax scheme	20%	40%	60%	80%
None	0.42%	2.05%	5.98%	16.73%
Wealth $\tau_w = 0.33\%$	0.43%	2.12%	6.14%	16.86%
Capital income $\tau_{ci} = 4.97\%$	0.46%	2.23%	6.46%	17.72%

**Table 13:** Tax policies in the life-cycle setting (lognormal returns across households).

For the welfare effects in this setting, I begin with the expected welfare gains from starting with the wealth tax and switching to the capital income tax. Again, a newborn who didn't know their type would prefer the capital income tax over the wealth tax: the lifetime value of the newborn under the capital income regime is higher.

**Table 14:** Expected Welfare Gains from Tax Reform (Lognormal Returns)

	Infinite horizon	Life-cycle
WT vs CIT	0.22%	0.18%
WT vs Original	0.78%	0.36%
CIT vs Original	0.56%	0.18%

Notes: Entries are consumption-equivalent (CE) welfare gains,  $\Delta$ , expressed as percent changes under the assumption that individual returns follow a lognormal distribution across households.

A decomposition of these welfare effects can be done by return type for the infinite horizon case, and by education-return type in the life-cycle scenario. The pattern of results remains the same, where only the highest type in each case prefers the wealth tax over the capital income tax. A notable difference between the distributional assumptions is that, the estimated lognormal distribution requires less types with returns less than 1 than the uniform distribution did. Despite this, the welfare results remain robust.

**Table 15:** Per-Type Welfare Gain and Baseline Return (WT vs CIT, Lognormal Returns)

	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6	Type 7
Baseline $R$ (gross)	0.976	1.001	1.014	1.026	1.038	1.053	1.079
CE $\Delta$ (WT vs CIT, %)	0.267%	0.342%	0.329%	0.317%	0.308%	0.304%	-0.326%

Notes: CE entries are per-type consumption-equivalent welfare gains (pmv-weighted within type), expressed as percent. Positive values favor the wealth tax over the capital income tax for that return type. Baseline  $R$  values are pre-tax gross returns (low  $\rightarrow$  high) under the lognormal returns specification.

**Table 16:** Per-Education Per-Type Welfare Gain and Baseline Return (WT vs CIT, Lognormal Returns)

		Type 1	Type 2	Type 3	Type 4	Type 5	Type 6	Type 7
NoHS	Baseline $R$ (gross)	0.9363	0.9711	0.9910	1.0084	1.0260	1.0471	1.0865
	CE $\Delta$ (WT vs CIT, %)	0.132%	0.176%	0.225%	0.275%	0.335%	0.292%	-0.114%
HS	Baseline $R$ (gross)	0.9363	0.9711	0.9910	1.0084	1.0260	1.0471	1.0865
	CE $\Delta$ (WT vs CIT, %)	0.131%	0.177%	0.229%	0.283%	0.332%	0.250%	-0.098%
College	Baseline $R$ (gross)	0.9363	0.9711	0.9910	1.0084	1.0260	1.0471	1.0865
	CE $\Delta$ (WT vs CIT, %)	0.131%	0.177%	0.228%	0.276%	0.311%	0.218%	-0.096%

Notes: CE entries are consumption-equivalent welfare gains (pmv-weighted within type), expressed as percent. Positive values favor wealth taxation over capital income taxation for that return type. Baseline  $R$  are pre-tax gross returns by type (low  $\rightarrow$  high) at  $t = 0$  under the lognormal returns specification.

### 7.3 Mechanism for Bank Heterogeneity

To match each exercise that was done when the estimated distribution was Uniform, I include a table which captures the estimated returns distribution which best matches 2004 SCF data on net worth and the corresponding implied elasticities for both the infinite horizon and life-cycle settings here as well.

**Table 17:** Estimated Returns and Implied Elasticities (Lognormal)

PY		LC	
Estimated returns	Implied elasticities	Estimated returns	Implied elasticities
0.976	8.231	0.936	5.898
1.001	10.615	0.971	7.838
1.014	12.602	0.991	9.532
1.026	14.963	1.008	11.640
1.038	18.366	1.026	14.874
1.053	24.922	1.047	21.857
1.079	68.242	1.086	127.378

## 8 Conclusion

I find the ex-ante heterogeneity in rates of return needed to match wealth moments, which is a common practice in heterogeneous agent macroeconomics. The model sits well with other deviations from the representative agent framework in that it does a good job of producing a simulated distribution of wealth with significant skewness when compared to its empirical counterpart.

This paper is slightly different from the literature which intersects HA models and evidence of a persistent component to returns in that I focus less on common explanations for the latter (like entrepreneurial talent and financial sophistication). Instead, I focus more on heterogeneity in the banking sector regarding offered deposit rates. I incorporate that literature in the standard HA framework with a simple, but realistic story in that many households may be “stuck” with the bank in or around their neighborhood. That bank has complex financial decisions to make, which ultimately trickles down to the household through the channel of varying deposit rates offered.

Although I leave out the possibility of households switching to one bank or another, this story has a similar essence to the financial literacy story when attempting to explain how returns may be heterogeneous across individuals. However, it leaves out the risk associated with portfolio choice. This is a nice feature, since (i) untangling how much of the persistent component of returns comes from risk preferences and from financial sophistication is not so straightforward and (ii) there is significant heterogeneity in returns even when individuals hold no risky assets.

As an aside, this model is still a partial equilibrium analysis. The market interest rate is being taken as given. It is not determined by some market

clearing condition.

I view my model as the simplest implementation of a potential source of heterogeneity. With that in mind, in the simulation of the model and the resulting SMM estimation, I do not add banks as an agent type. Thus, the bank is not responding in every period to the level of deposits they receive after they set the optimal deposit rate based on the demand for foreign deposits that they face. This also means that I avoid choosing a particular scheme of allocating agents in the model to a particular bank. In this way, there are 7 types of banks just as there are 7 types of returns that an agent may receive.

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