Heterogeneous Returns and the Distribution of Wealth

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Brief history on wealth inequality

Benhabib and Bisin 2018 offers a useful survey of lit on modeling wealth inequality

1. Observable skewness in wealth holdings $\rightarrow$ assume distributional properties
2. Use distribution of income to explain distribution of wealth
3. Describe the process of accumulating wealth over the life cycle (i.e. dynamics of optimal consumption-saving behavior)

$\Longrightarrow$ an interest in wealth inequality and its determinants may naturally lead one towards heterogeneous agent macro modeling.

Outline

1. Empirical evidence of heterogeneous returns
2. Model of saving with heterogeneous returns
3. Structural estimation of model to match wealth data

Key finding: The life-cycle model with different returns for households generates a realistic amount of skewness in the distribution of wealth.

My contribution

- Why returns? $\rightarrow$ an observable feature of household's problem
- Labor income process: Random walk v.s. AR(1)
- Age-education dependent labor income process and mortality rates

What are het. returns?

Following optimal portfolio choice theory from Merton (1969) and Samuelson (1969)

- Optimal share in the risky asset is given by

$$\alpha_{it}^{m} = \frac{\mathbb{E}(r_t^m - r_t^s)}{\gamma_i \sigma_t^2}.$$

- Individual *realized* return to financial assets can be written as

$$r_{it}^{f} = r_t^s + \alpha_{it}^{m}(r_t^m - r_t^s).$$

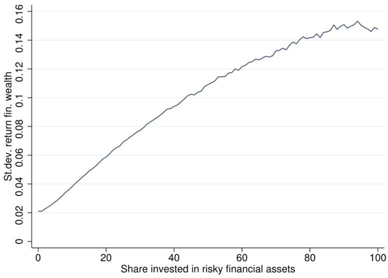


Figure: Heterogeneity in returns to financial wealth by share of risky assets from Fagereng et al. 2020.

Empirical estimate of heterogeneity

- Step 1: linear regression for the return to net worth using panel

$$r_{it}^{n} = X_{it}^{'}\beta + u_{it}.$$

- Step 2: Add fixed effects

$$u_{it} = f_i + e_{it}.$$

$\implies$ R^2 goes from .33 to .5.

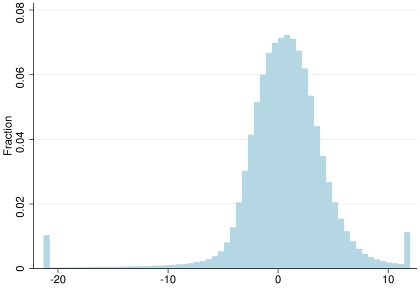


Figure: Distribution of fixed effects in the return to net worth from Fagereng et al. 2020.

Potential sources of return heterogeneity

- Entrpreneurship - “high levels of capital, low MPK”
- Financial literacy - closer, but generally aimed at risky assets

We know that there is much variation in the banking sector regarding rates offered on deposit accounts. Is there a mechanism we can exploit?

Mechanism

- "Transmission channel of monetary policy" by Drechsler, Savov, and Schnabl 2017
  - Sensitivity of bank deposits to market interest rate changes
- Δ in market rate $\rightarrow$ variation in Δ in deposits held at banks
  - Sarkisyan and Viratyosin 2021 - globally integrated vs local banks
  - Adrien d'Avernas, Andrea L. Eisfeldt, Can Huang, Richard Stanton, Nancy Wallace 2024 - small vs large banks

$\Longrightarrow$ variation in deposit rates offered across banks

A simple model of bank heterogeneity

Let R^m be the market rate of return, R^d be the rate of return offered on deposits by a bank, and $S(R^d, R^m)$ be the level of deposits held at a given bank.

Banks solve:

$$\max(R^m - R^d) \cdot S(R^d, R^m)$$

subject to:

$$S(R^d, R^m) = A \left(\frac{R^d}{R^m} \right)^{\varepsilon}$$

Show interpretation of ε

Interpreting the Elasticity Parameter ε

In this setting, the parameter ε has a clear interpretation as the elasticity of deposits to changes in the market interest rate. It can be shown that:

$$-\varepsilon = \frac{\partial S(\cdot)}{\partial R^m} \cdot \frac{R^m}{S(\cdot)}$$

Back to model

First order condition

Bank's optimal choice of R^d

The first order condition for the bank's optimization problem implies that:

$$R^d = \frac{\varepsilon}{1+\varepsilon} R^m$$

Back to model

Labor income process

- Household income:

$$y_t = p_t \xi_t W_t$$

- Permanent component:

$$p_t = p_{t-1} \psi_t$$

- Transitory component:

$$\xi_t = \begin{cases} \mu & \text{with probability } \mho \\ (1-\tau_t)\ell\theta_t & \text{with probability } 1-\mho \end{cases}$$

(Normalized) Optimization problem

Choose profiles $\{c_{t_n}\}_{n=0}^{\infty}$ that satisfy

$$\begin{aligned} v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \not{D} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\ \text{s.t.} & \\ a_t &= m_t - c_t(m_t), \\ k_{t+1} &= \frac{a_t}{\not{D}\psi_{t+1}}, \\ m_{t+1} &= (\daleth + r_t^d) k_{t+1} + \xi_{t+1}, \\ a_t &\geq 0. \end{aligned}$$

Production function

$$Y = ZK^{\alpha}(\ell L)^{1-\alpha}$$

Calibration

Description	Parameter	Value	Source
Time discount factor	β	0.99^4	Den Haan, Judd, and Juillard 2010
CRRA	ρ	1	Den Haan, Judd, and Juillard 2010
Capital share	α	0.36	Den Haan, Judd, and Juillard 2010
Depreciation rate	δ	0.025	Den Haan, Judd, and Juillard 2010
Time worked per employee	ℓ	1/.09	Den Haan, Judd, and Juillard 2010
Wage rate	W	2.37	Den Haan, Judd, and Juillard 2010
Unempl. insurance payment	μ	0.15	Den Haan, Judd, and Juillard 2010
Probability of survival	$\cancel{D}$	$(1 - 0.00625)^4$	Yields 40-year working life
Std. dev of $\log \theta_{t,i}$	σ_θ^2	$0.010 \times 4 \times \sqrt{4}$	Carroll 1992, Carroll, Slacalek, and Tokuoka 2015
Std. dev of $\log \psi_{t,i}$	σ_ψ^2	$0.010 \times 4/11 \times \sqrt{4}$	Carroll 1992, Debacker et al. 2013, Carroll, Slacalek, and Tokuoka 2015
Unemployment rate	$\mho$	0.07	Mean in Den Haan, Judd, and Juillard 2010

Table: Parameter values (annual frequency) for the perpetual youth model.

Estimation procedure

Simulated method of moments (SMM) estimation for R using 2004 SCF wealth data.

1. No ex-ante heterogeneity: *R-point* model

   Estimate a common rate of return across households by finding the $\grave{R}$ which matches the capital-to-output ratio ($\frac{K}{Y} = 3$).

2. Ex-ante heterogeneity: *R-dist* model

   Estimate a **Uniform distribution** of returns across households by finding the $\grave{R}, \nabla$ which match empirical Lorenz targets, given $\frac{K}{Y}$.

Net worth percentile	Cumulative net worth
20th	-.18%
40th	.95%
60th	5.3%
80th	17.09%

Estimation procedure

Simulated method of moments (SMM) estimation for R using 2004 SCF wealth data.

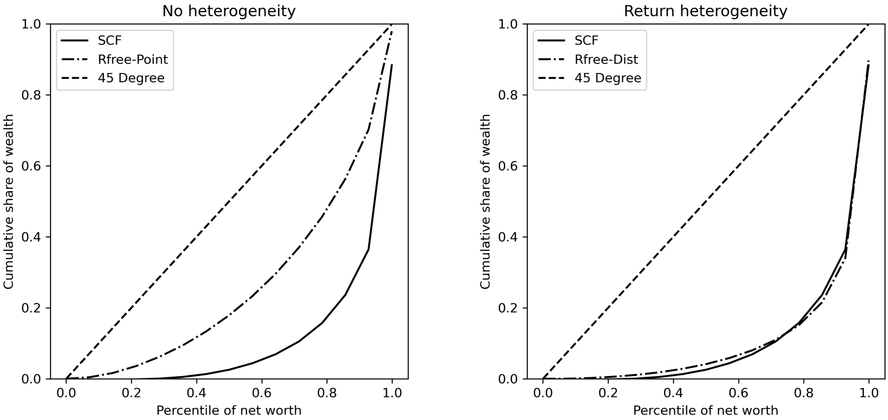
3. Implied distribution of elasticities ϵ

   The solution to the bank's optimization problem implies

$$\varepsilon = \frac{R^d}{R^m - R^d} \tag{1}$$

   Thus, so long as the market interest rate is given, the SMM procedure can be used to uniquely pin down a distribution of elasticites which describes banking heterogeneity.

How good is the fit?



Lifecycle version of the model

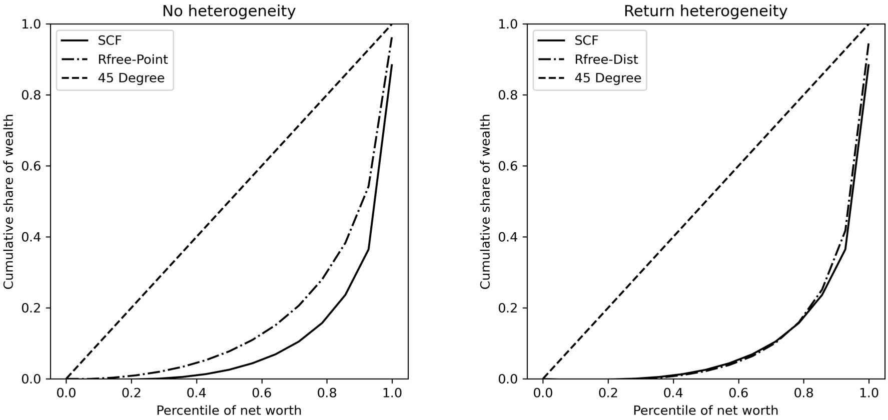
- Education cohort $e \in \{D, HS, C\}$
- Initial wealth-to-income k_0 and income p_0 levels
- Education-age dependent mortality rates (Brown, Liebman, and Pollet 2007)
- Modified labor income uncertainty $y_t = \xi_t \psi_t \overline{\psi}_{es} p_{t-1}$ (Cagetti 2003)
  - Education-age dependent shock variances (Sabelhaus and Song 2010)

Calibration

Description	Parameter	Value
Population growth rate	N	0.0025
Technological growth rate	Γ	0.0037
Rate of high school dropouts	θ_D	0.11
Rate of high school graduates	θ_{HS}	0.55
Rate of college graduates	θ_C	0.34
Labor income tax rate	τ	0.0942

Table: Parameter values (annual frequency) for the lifecycle model.

How good is the fit?



Model performance: returns distribution

Empirical values from Fagereng et al. 2020

	Mean	St. Dev
Net worth (after tax)	0.0365	0.0781

Values from the structural estimation (uniform distribution for R)

	Mean	St. Dev
PY-Point	0.060	0.0
PY-Dist	0.021	0.011
LC-Point	0.040	0.0
LC-Dist	0.023	0.009

Model performance: untargeted moments

Empirical Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	-0.0723	-0.0657	-0.0266	0.1099
30-40	-0.008	0.0054	0.057	0.1813
40-50	-0.0001	0.0187	0.0776	0.2178
50-60	0.0018	0.0215	0.0766	0.2126
60-70	0.0011	0.0188	0.0726	0.2081

Simulated Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	-0.0024	0.0242	0.0859	0.2242
30-40	-0.0124	0.0064	0.0662	0.2221
40-50	-0.0088	0.0046	0.0545	0.2077
50-60	-0.0006	0.0157	0.069	0.2234
60-70	0.0038	0.0239	0.0809	0.2341

Model performance: implied elasticites

PY		LC	
Estimated returns	Implied elasticities	Estimated returns	Implied elasticities
0.964	7.329	0.976	8.165
0.983	8.755	0.991	9.564
1.001	10.771	1.007	11.468
1.021	13.837	1.023	14.208
1.040	19.064	1.039	18.492
1.060	29.974	1.055	26.136
1.079	66.891	1.071	43.645

Genay and Halcomb 2004 - "A 1% increase in the fed funds rate over four quarters is associated with a 2.96% decline in the growth of core deposits at small banks and a 3.66% decline at large banks."

Conclusion

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