Heterogeneous Returns and the Distribution of Wealth

Decory Edwards

Johns Hopkins University

May 29, 2024

Why do macroeconomists care about inequality?

Empirical evidence shows that macroeconomic policies, as well as aggregate shocks, may have differential effects across households.

- Macro matters for inequality

Representative agent models have a difficult time matching empirical estimates of macroeconomic variables (MPC and the wealth distribution).

- Inequality matters for macro

Macro with heterogeneous agents

- Uninsurable, idiosyncratic risk to income and movements in aggregate productivity (**ks1998**)
- Ex-ante heterogeneity in the time preference of households (**cstw2017**)
- Classifying models with ex-ante and ex-post heterogeneity (**gkgv22**)
- Further surveys regarding heterogeneous agent macroeconomics (**Guvenen2011**) and (**Krueger2016**)

Empirical estimates of returns

1. Comprehensive, administrative tax data in Norway from 2004 to 2015 (**aflgdmlp20**)
2. Asset holdings and income for Swedish residents from 1999 to 2007 (**lblcps18**)
3. Wealth held in equity accounts in India from 2002 to 2011 (**Campbell2019**)
4. DNB 2005 survey of dutch households regarding savings accounts and financial literacy (**Deuflhard2018**)

Related literature

- Stochastic process for returns implying a stationary wealth distribution. (**Benhabib2011**), (**Benhabib2015**), (**Benhabib2016**)
- Stochastic process for returns which best fits the empirical distribution of wealth.
  (**Benhabib2019**)
- Endogenize heterogeneous returns through access to high return investment technology.
  (**Guler2022**)

Outline

1. Empirical evidence of heterogeneous returns
2. Model of saving with heterogeneous returns
3. Structural estimation of model to match wealth data

A closer look at **aflgdmlp20**

Following optimal portfolio choice theory from Merton (1969) and Samuelson (1969)

- Optimal share in the risky asset is given by

$$\alpha_{it}^{m} = \frac{\mathbb{E}(r_t^m - r_t^s)}{\gamma_i \sigma_t^2}.$$

- Individual *realized* return to financial assets can be written as

$$r_{it}^{f} = r_t^s + \alpha_{it}^{m}(r_t^m - r_t^s).$$

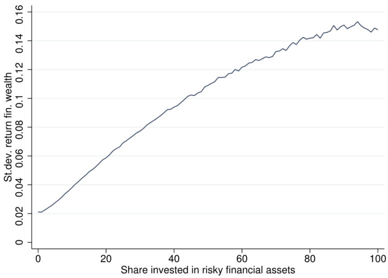

Figure: Heterogeneity in returns to financial wealth by share of risky assets from **aflgdmlp20**.

- Step 1: *linear panel data regression model* for the return to net worth

$$r^n_{it} = X^{'}_{it}\beta + u_{it}.$$

- Step 2: Add fixed effects

$$u_{it} = f_i + e_{it}.$$

$\implies$ R^2 goes from .33 to .5.

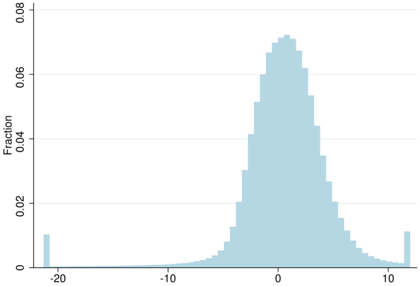

Figure: Distribution of fixed effects in the return to net worth from **aflgdmlp20**.

Labor income process

- Household income:

$$y_t = p_t \xi_t W_t$$

- Permanent component:

$$p_t = p_{t-1} \psi_t$$

- Transitory component:

$$\xi_t = \begin{cases} \mu & \text{with probability } \mho \\ (1-\tau_t) l \theta_t & \text{with probability } 1-\mho \end{cases}$$

Heterogeneous agents in G.E.

(Normalized) Optimization problem for households: Choose profiles $\{c_{t_n}\}_{n=0}^{\infty}$ that satisfy

$$
\begin{aligned}
v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \cancel{D} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t(m_t), \\
k_{t+1} &= \frac{a_t}{\cancel{D}\psi_{t+1}}, \\
m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}
$$

Production function

$$
Y = ZK^{\alpha}(\ell L)^{1-\alpha}
$$

Conditions for a stable wealth distribution

Carroll2019bst provides the following *death-modified growth impatience condition* such that a unique target wealth-to-permanent income ratio exists for households:

$$\left(\frac{(R\beta)^{1/\rho}\mathbb{E}[\psi^{-1}]\not{D}}{\Gamma}\right) < 1.$$

Calibration

Standard calibration scheme used to simulate the model.

Description	Parameter	Value	Source
Time discount factor	β	0.99	Den Haan, Judd, and Juillard 2010
CRRA	ρ	1	Den Haan, Judd, and Juillard 2010
Capital share	α	0.36	Den Haan, Judd, and Juillard 2010
Depreciation rate	δ	0.025	Den Haan, Judd, and Juillard 2010
Time worked per employee	ℓ	1/.09	Den Haan, Judd, and Juillard 2010
Capital/output ratio	$\frac{K}{Y}$	10.26	Den Haan, Judd, and Juillard 2010
Effective interest rate	$r-\delta$	0.01	Den Haan, Judd, and Juillard 2010
Wage rate	W	2.37	Den Haan, Judd, and Juillard 2010
Unempl. insurance payment	μ	0.15	Den Haan, Judd, and Juillard 2010
Probability of death	D	0.00625	Yields 40-year working life
Variance of $\log \theta_{t,i}$	σ^2_θ	0.010 x 4	Carroll 1992, Carroll, Slacalek, and Tokuoka 2015
Variance of $\log \psi_{t,i}$	σ^2_ψ	0.010 x 4/11	Carroll 1992, Debacker et al. 2013, Carroll, Slacalek, and Tokuoka 2015
Unemployment rate	$\mho$	0.07	Mean in Den Haan, Judd, and Juillard 2010

Table 1: Parameter values (quarterly frequency) for the perpetual youth model with no aggregate uncertainty.

Estimation procedure

Simulated method of moments (SMM) estimation for R.

1. No ex-ante heterogeneity: *R-point* model
   R which matches the capital-to-output ($\frac{K}{Y}$) ratio of 10.26

2. Ex-ante heterogeneity: *R-dist* model
   Uniform distribution of R matching lorenz targets, given $\frac{K}{Y} = 10.26$

Estimation procedure with heterogeneity

Empirical lorenz targets using 2004 SCF data

Net worth percentile	Cumulative net worth
20th	-.18%
40th	.95%
60th	5.3%
80th	17.09%

Optimization problem for the *R-dist* model

$$\{\grave{R}, \nabla\} = \arg\min_{R,\nabla} \left(\sum_{i=20,40,60,80} (w_i(R,\nabla) - \omega_i)^2 \right)^{\frac{1}{2}}$$

s.t.

$$\frac{K}{Y} = 10.26.$$

How good is the match?

Figures/hetreturns-PYunif.png

	R-point	R-dist
Center	1.015	1.0105
Spread	0	.011
Aggregate MPC	.095	.263

Revisiting the empirical evidence

aflgdmlp20 describe the empirical distribution of returns.

TABLE 3
RETURNS TO WEALTH: SUMMARY STATISTICS[a]

Wealth Component	Mean	St. Dev.	Skewness	Kurtosis	P10	Median	P90
Net worth (before tax)	0.0379	0.0859	−0.79	47.75	−0.0308	0.0321	0.1109
Net worth (after tax)	0.0365	0.0781	−0.71	36.88	−0.0283	0.0316	0.1067
Net worth (before tax, unweighted)	0.0004	0.2205	−6.73	68.46	−0.0600	0.0230	0.1037
Net worth (after tax, unweighted)	0.0155	0.1546	−5.28	56.42	−0.0449	0.0247	0.1040
Financial wealth	0.0105	0.0596	−1.78	22.17	−0.0171	0.0084	0.0530
Safe fin. assets	0.0078	0.0188	4.38	53.52	−0.0106	0.0059	0.0268
Risky fin. assets	0.0425	0.2473	−0.08	6.22	−0.2443	0.0418	0.3037
Non-financial wealth	0.0511	0.0786	1.80	15.47	−0.0215	0.0429	0.1275
Housing	0.0485	0.0653	0.73	9.95	−0.0209	0.0441	0.1165
Private equity	0.1040	0.5169	18.01	836.79	−0.0531	0.0052	0.3616
Debt	0.0236	0.0216	2.51	29.50	0.0030	0.0215	0.0461
Long-term debt	0.0230	0.0209	3.54	56.92	0.0038	0.0209	0.0446
Consumer debt	0.0961	0.1086	4.60	82.60	−0.0124	0.0741	0.2119
Student debt	0.0078	0.0260	0.68	4.14	−0.0213	0.0074	0.0399

[a]The table reports summary statistics for various measures of real returns to wealth, pooling data for 2005–2015. Except when noted, all returns are value-weighted.

Work to be done

1. Potentially exclude retirement assets in the measure household net worth
    - Private pension wealth is unobservable in the dataset from **aflgdmlp20**
    - Are there significant differences between retirement asset holdings in the U.S. v.s. Norway?
2. Run the estimation for liquid wealth
    - How close is it to its empirical counterpart ("financial wealth")?
3. Life-cycle version of the model
4. More recent wealth data from the SCF

Future directions for this work

- Scale dependence of heterogeneous rates of return
- Endogenizing differences in the rate of return
- Implications of wealth v.s. capital income taxation
  - (**Benhabib2008**), (**Benhabib2011**), (**Guvenen2019**)

References I