1 Introduction

The unequal distribution of wealth, an extensively documented historical phenomenon, has intensified in recent years. This point is stressed in a recent article from the Institute for Policy Studies (IPS) which revealed that, in 2018, the total wealth of the poorest half of Americans was eclipsed by the combined wealth of the three wealthiest men in the nation. The IPS report further states that the combined wealth of the top five richest men on this list skyrocketed by a staggering 123% from March 2020 to October 2021[1].

Wealth inequality has also been a subject of considerable interest throughout history in various academic fields. The statistics literature, for instance, focused on linking the distribution of income to the observable skewness in wealth distribution. The economics literature went further by establishing microfoundations for individual wealth outcomes.

More recently, the macroeconomics literature on inequality has seen significant growth, with the distribution of wealth among households offering insight into how the economy as a whole responds to aggregate fiscal shocks.[2] This has been accomplished in large part due to the widespread adoption of models that abandon the traditional representative agent assumption.

The first departure from the representative agent framework entails positing an exogenously determined income process that generates a distribution of income among households. One can further assume that individuals face some level of potential unemployment in each period. In this revised setting, there is a precautionary savings motive for consumers. The availability of a riskless asset that partially insures against this income risk results in households choosing to hold different levels of market resources optimally. Krusell and Smith (1998)'s seminal work suggests that models assuming heterogeneity in individual income perform well in matching the aggregate capital stock but poorly in matching the distribution of wealth.

The next step towards moving beyond the standard representative agent framework is to assuming heterogeneity among households beyond the ex-post realizations of the stochastic process for income. This will lead to more households to optimally hold lower levels of wealth. Carroll, Slacalek, Tokuoka, and White (2017) accomplish this by performing a structural estimation of *ex-ante* heterogeneity amongst households which allows for the model's distribution of wealth to closely match the 2004 Survey of Consumer Finances (SCF) data on household wealth. The model with both ex-ante and ex-post heterogeneity does a much better job at matching observable inequality in the distribution of wealth.

---

[1]See Inequality.org articles data November 21, 2022: "Wealth Inequality in the United States" and "Updates: Billionaire Wealth, U.S. Job Losses and Pandemic Profiteers" (date accessed: March 27, 2023)

[2]Parker, Schild, Erhard, and Johnson (2022) note that "In sum, while on average the [economic impact payments] EIPs appear to have gone to many households with incomes that were unharmed by the pandemic, some of the EIPs, mainly in the first round, did support short-term spending for some households, primarily those with low ex ante liquid wealth and those reliant on income that could not be earned by working from home."

1.1 Contributions to the literature

It is worth noting that the time preference factor (β) is one of the key parameters that influences an individual's equilibrium target level of market resources, but it is not directly observable. Estimating differences in the rate of return to financial assets across households is possible, as this variable *is* directly observable. Empirical research has been conducted to estimate such differences, with a recent example being the work of Fagereng, Guiso, Malacrino, and Pistaferri (2020). They analyzed 12 years of administrative tax records on capital income and wealth stock for all taxpayers in Norway from 2004-2015 to estimate these rates of return.

Motivated by this recent evidence, this proposal provides (i) preliminary results and (ii) actionable extensions of a structural estimation exercise regarding ex-ante heterogeneity in the rate of return. Since the rate of return has a direct role in the wealth accumulation process, differences in returns to assets across households may be a compelling explanation for wealth inequality.

2 Model

2.1 Defining the stochastic income process

Each household's income (y_t) during a given period depends on three main factors. The first factor is the aggregate wage rate (W_t) that all households in the economy face. The second factor is the permanent income component (p_t), which represents an agent's present discounted value of human wealth. Lastly, the transitory shock component (ξ_t) reflects the potential risks that households may face in receiving their income payment during that period. Thus, household income can be expressed as the following:

$$y_t = p_t \xi_t W_t.$$

I assume that the level of permanent income for each household follows a geometric random walk:

$$p_t = p_{t-1} \psi_t,$$

The white noise permanent shock to income with a mean of one is represented by ψ_t, which is a significant component of household income. The probability of receiving income during a given period is determined by the transitory component, which is modeled to reflect the potential risks associated with becoming unemployed. Specifically, if the probability of becoming unemployed is $\mho$, the agent will receive unemployment insurance payments of $\mu > 0$. On the other hand, if the agent is employed, which occurs with a probability of $1 - \mho$, the model allows for tax payments τ_t to be collected as insurance for periods of unemployment. The transitory component is then represented as:

$$\xi_t = \begin{cases} \mu & \text{with probability } \mho, \\ (1-\tau_t)l\theta_t & \text{with probability } 1-\mho, \end{cases}$$

where l is the time worked per agent and the parameter θ captures the white noise component of the transitory shock.

2.2 Baseline model for households

In the baseline version of the household's optimization problem for consumption-savings decisions each household aims to maximize its expected discounted utility of consumption $u(c) = \frac{c^{1-\rho}}{1-\rho}$ by solving the following:

$$\max \mathbb{E}_t \sum_{n=0}^{\infty} (\not{D}\beta)^n u(c_{t+n}).$$

The sequence of consumption functions $\{c_{t+n}\}_{n=0}^{\infty}$ associated with a household's optimal choice over a lifetime must satisfy[3]

$$\begin{aligned} v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta\not{D}\mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\ &\text{s.t.} \\ a_t &= m_t - c_t(m_t), \\ k_{t+1} &= \frac{a_t}{\not{D}\psi_{t+1}}, \\ m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1}, \\ a_t &\geq 0. \end{aligned}$$

3 Results

To solve and simulate the model, I follow the calibration scheme captured in table 1.

3.1 The model without heterogeneity

The solution of the model with no heterogeneity in returns (referred to as the R-point model) is the one which finds the value for the rate of return R which minimizes the distance between the simulated and empirical wealth shares at the 20th, 40th, 60th, and 80th percentiles of the corresponding wealth distribution. The estimation procedure finds this optimal value to be $\mathsf{R} = 1.0153$.

[3]Here, I denote a_t as assets, m_t as market resources (or cash-on-hand), k_t as capital, and $\daleth = (1-\delta)$ as the depreciation factor for capital. Each of the relevant variables have been normalized by the level of permanent income ($c_t = \frac{C_t}{p_t}$, and so on). This is the standard state-space reduction of the problem for numerical tractibility.

Description	Parameter	Value	Source
Time discount factor	β	0.99	Den Haan, Judd, and Juillard (2010)
CRRA	ρ	1	Den Haan, Judd, and Juillard (2010)
Capital share	α	0.36	Den Haan, Judd, and Juillard (2010)
Depreciation rate	δ	0.025	Den Haan, Judd, and Juillard (2010)
Time worked per employee	ℓ	1/.09	Den Haan, Judd, and Juillard (2010)
Capital/output ratio	$\frac{K}{Y}$	10.26	Den Haan, Judd, and Juillard (2010)
Effective interest rate	$r-\delta$	0.01	Den Haan, Judd, and Juillard (2010)
Wage rate	W	2.37	Den Haan, Judd, and Juillard (2010)
Unempl. insurance payment	μ	0.15	Den Haan, Judd, and Juillard (2010)
Probability of death	D	0.00625	Yields 40-year working life
Variance of $\log \theta_{t,i}$	σ_θ^2	0.010 x 4	Carroll (1992), Carroll, Slacalek, and Tokuoka (2015)
Variance of $\log \psi_{t,i}$	σ_ψ^2	0.010 x 4/11	Carroll (1992), Debacker, Heim, Panousi, Ramnath, and Vidangos (2013), Carroll, Slacalek, and Tokuoka (2015)
Unemployment rate	$\mho$	0.07	Mean in Den Haan, Judd, and Juillard (2010)

Table 1 Parameter values (quarterly frequency) for the perpetual youth (infinite horizon) model.

3.2 Incorporating heterogeneous returns

Recent studies by Fagereng, Guiso, Malacrino, and Pistaferri (2020) and Bach, Calvet, and Sodini (2018) have not only estimated the rate of return on asset holdings but have also uncovered significant heterogeneity across households. Given this motivation, the revised model assumes the existence of multiple types of agents, each earning a distinct rate of return on their assets.

Specifically, I assume that different types of households have a time preference factor drawn uniformly from the interval $(\grave{\mathsf{R}}-\nabla, \grave{\mathsf{R}}+\nabla)$, where ∇ represents the level of dispersion. Afterward, the model is simulated to estimate the values of both $\grave{\mathsf{R}}$ and ∇ so that the model matches the inequality in the wealth distribution. To achieve this, the following minimization problem is solved:

$$\{\grave{\mathsf{R}}, \nabla\} = \arg\min_{\mathsf{R},\nabla} \left(\sum_{i=20,40,60,80} (w_i(\mathsf{R},\nabla) - \omega_i)^2 \right)^{\frac{1}{2}}$$

subject to the constraint that the aggregate capital-to-output ratio in this model matches that of the perfect foresight setting:

$$\frac{K}{Y} = \frac{K_{PF}}{Y_{PF}}.$$

Note that w_i and ω_i give the porportion of total aggregate net worth held by the top i percent in the model and in the data, respectively.

The estimation procedure finds this optimal values of $\mathsf{R} = 1.0106$ and $\nabla = 0.0112$.

The performance of the estimation of both the R-point and R-dist models, measured by their ability to match the 2004 SCF wealth data, is compared in figure 1.

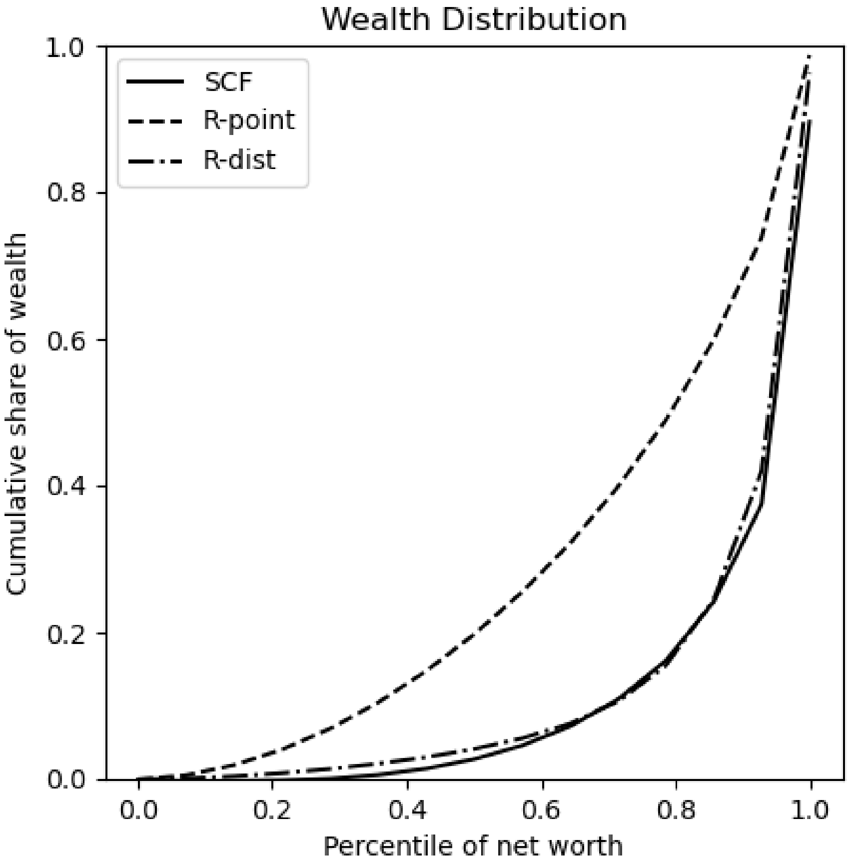

Figure 1 Perpetual youth lorenz curve v.s. data

4 Incorporating life cycle dynamics into the model

The case where households solve the infinite horizon version of the consumption-saving problem is an interesting limiting case. However, more realistic assumptions regarding the age and education level of households can have important implications for the income and mortality process of households. Here, we extend the model to incorporate these assumptions.

Households enter the economy at time t aged 24 years old and are endowed with an education level $e \in \{D, HS, C\}$, and initial permanent income level $\mathbf{p}_0$, and a capital stock k_0. The life cycle version of household income can be expressed as the following:

$$y_t = \xi_t \mathbf{p}_t = (1 - \tau)\theta_t \mathbf{p}_t,$$

where $\mathbf{p}_t = \psi_t \bar{\psi}_{es} \mathbf{p}_{t-1}$ and $\bar{\psi}_{es}$ captures the age-education-specific average growth factor. Households that have lived for s periods have permanent shocks drawn from a lognormal distribution with mean 1 and variance $\sigma^2_{\psi s}$ and transitory shocks drawn from a lognormal distribution with mean $\frac{1}{\cancel{\mho}}$ and variance $\sigma^2_{\theta s}$ with probability $\cancel{\mho} = (1 - \mho)$ and μ with probability $\mho$.

The normalized version of the age-education-specific consumption-saving problem for households is given by

$$
\begin{aligned}
v_{es}(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \cancel{D}_{es} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v_{es+1}(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t, \\
k_{t+1} &= \frac{a_t}{\psi_{t+1}}, \\
m_{t+1} &= (\daleth + r_t) k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}
$$

5 Results for the life cycle model

The additional parameters necessary to calibrate the life cycle version of the model are given in table 2.

Description	Parameter	Value
Population growth rate	N	0.0025
Technological growth rate	Γ	0.0037
Rate of high school dropouts	θ_D	0.11
Rate of high school graduates	θ_{HS}	0.55
Rate of college graduates	θ_C	.34
Average initial permanent income, dropout	$\bar{\mathbf{p}}_{D0}$	5000
Average initial permanent income, high school	$\bar{\mathbf{p}}_{HS0}$	7500
Average initial permanent income, college	$\bar{\mathbf{p}}_{C0}$	12000
Unempl. insurance payment	μ	0.15
Labor income tax rate	τ	0.0942

Table 2 Parameter values (quarterly frequency) for the life cycle model.

The estimation procedure finds this optimal value to be $\mathsf{R} = 1.0078$ for the R-point model in this setting. The estimation procedure for the R-dist model in the life cycle setting finds optimal values of $\mathsf{R} = 1.0005$ and $\nabla = 0.01836$. Consider the improved performance of the estimation in matching the 2004 SCF wealth data, which is compared in figure 2.

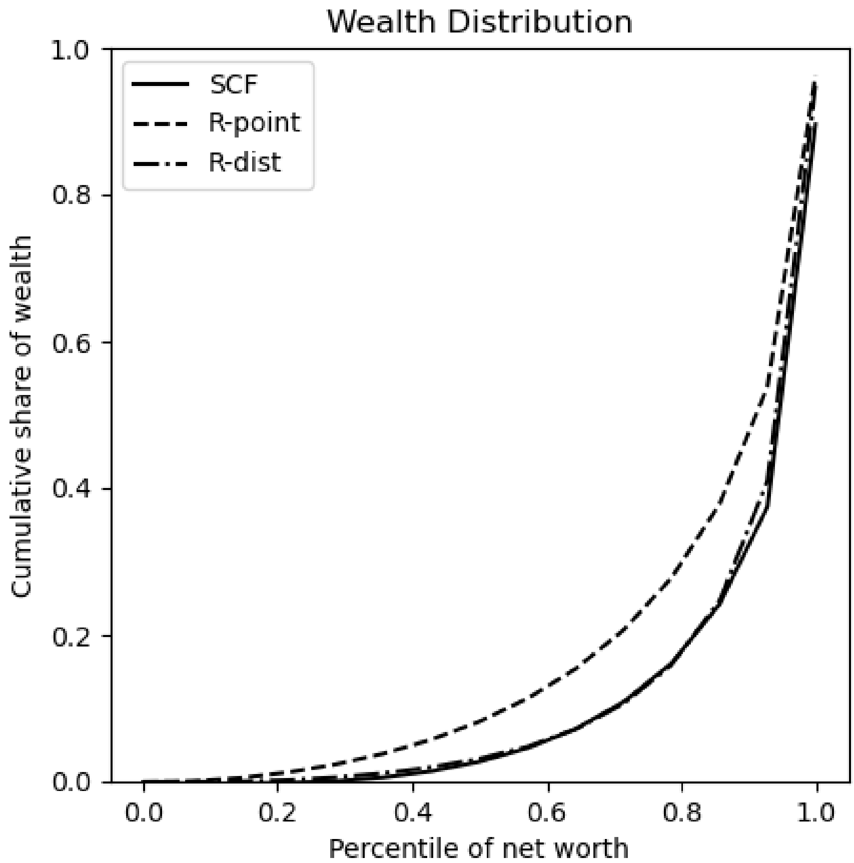

Figure 2 Life cycle lorenz curve v.s. data

6 Extensions

This concludes the preliminary results of this work. From here on, I discuss actionable extensions of the model which are of high priority to be completed.

6.1 Incorporating bequest motives

The model to this point completely explains the saving behavior of households through the precautionary saving motive present in this setting. The desire to leave bequests is thought to be an important reason for households to save, especially those at the top end of the wealth distribution. More generally, the following specification of additively separable wealth in the utility function extends the model to accomodate these other reasons to accumulate assets:

$$u(c_t, a_t) = \frac{c_t^{1-\rho}}{1-\rho} + \kappa \frac{(a_t - \underline{\mathrm{a}})^{1-\Sigma}}{1-\Sigma}.$$

It is straightforward to implement this in the existing code. Straub (2019) not only provides calibration values for κ and $\underline{a}$ and estimation for the elasticity parameters.[4] In my setting, I will need to make decisions about which features of the wealth component of the utility function should be estimated, and which should be calibrated. Alternative specifications, such as a non-separable utility function of consumption and wealth, may also be explored in this setting.

6.2 Incorporating portfolio choice

Portfolio choice is also an important feature of the consumption-saving problem of households not currently present in the model. Denoting the gross return on the risky asset as $\mathcal{R}_{t+1}$ and the proportion of the porfolio invested in the risky asset as ς, the revised maximization problem is

$$
\begin{aligned}
v(m_t) &= \max_{c_t, \varsigma_t} u(c_t, a_t) + \beta \cancel{D} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\
& \text{s.t.} \\
a_t &= m_t - c_t(m_t), \\
k_{t+1} &= \frac{a_t}{\cancel{D}\psi_{t+1}}, \\
\mathbb{R}_{t+1} &= \mathsf{R}_{t+1} = (\mathcal{R}_{t+1} - \mathsf{R}_{t+1})\varsigma_t \\
m_{t+1} &= (\mathbb{R} - \delta)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}
$$

where $\mathbb{R}$ denotes the overall return on the portfolio across periods.[5]

This full model will be accompanied by a revised structural estimation procedure. Here, I will assume that households earn the same rate of return on the safe asset R, but are allowed to be ex-ante heterogeneous in the rate of reutn on the risky asset. This will be captured by allowing each type of household in theestimation drawing from a lognormal distribution of $\mathcal{R}$ which has the same variance but different means. The goal of the procedure will be to find this distribution of returns to risky assets which allows the model to best match the empirical moments of the SCF wealth data.

Decisions will need to be made regarding what parameters are relevant to be estimated. The main issue is that, once portfolio choice is incorporated in a consumption-saving model, one should reasonable suspect that households have different levels of risk aversion. This suggests that a distribution of risk preferences, as well as returns, should be estimated. This will require more empirical moments so that the parameters can be correctly identified.

[4]The paper also discusses the implications of assuming $\Sigma = \rho$ versus $\Sigma < \rho$. Incorporating the latter is a more difficult implementation in the existing code, but it is a highly desirable version of the model in order to match the empirical evidence on the saving behavior of households.

[5]The perpetual youth setting is provided for simplicity. It is straightforward to allow for portfolio choice in the life cycle setting.

6.3 Multiple SCF waves for empirical moments

After the full model is calibrated and correctly specified, with empirical moments chosen to allow for accurate estimation of the paramters capturing ex-ante heterogeneity across households, I will rerun the exercise for wealth data other than the 2004 SCF wave. This will constitute a sort of robustness check regarding how plausible the resulting heterogeneity of returns and risk preferences are necessary to generate inequality in wealth which is closest to the observed inequality.

References

Bach, Laurent, Laurent E. Calvet, and Paolo Sodini (2018): "Rich Pickings? Risk, Return, and Skill in Household Wealth," *American Economic Review*, 110(9), 2703–47.

Carroll, Christopher, Jiri Slacalek, Kiichi Tokuoka, and Matthew N. White (2017): "The distribution of wealth and the marginal propensity to consume," *Quantitative Economics*, 8(3), 977–1020.

Carroll, Christopher D (1992): "The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence," *Brookings Pap. Econ. Act.*, 1992(2), 61–156.

Carroll, Christopher D, Jiri Slacalek, and Kiichi Tokuoka (2015): "Buffer-stock saving in a Krusell–Smith world," *Econ. Lett.*, 132, 97–100.

Debacker, Jason, Bradley Heim, Vasia Panousi, Shanthi Ramnath, and Ivan Vidangos (2013): "Rising Inequality: Transitory or Persistent? New Evidence from a Panel of U.S. Tax Returns," *Brookings Pap. Econ. Act.*, pp. 67–122.

Den Haan, Wouter J, Kenneth L Judd, and Michel Juillard (2010): "Computational suite of models with heterogeneous agents: Incomplete markets and aggregate uncertainty," *J. Econ. Dyn. Control*, 34(1), 1–3.

Fagereng, Andreas, Luigi Guiso, Davide Malacrino, and Luigi Pistaferri (2020): "Heterogeneity and Persistence in Returns to Wealth," *Econometrica*, 88(1), 115–170.

Krusell, Per, and Anthony Smith (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106(5), 867–896.

Parker, Jonathan A, Jake Schild, Laura Erhard, and David Johnson (2022): "Economic Impact Payments and Household Spending During the Pandemic," Discussion paper, National Bureau of Economic Research.

Straub, Ludwig (2019): "Consumption, Savings, and the Distribution of Permanent Income," Revise and resubmit at Econometrica.