1 Introduction

The macroeconomics literature has expanded to allow for quantitative statements to be made about the relationship between inequality and the economy in recent recent years. One example of this is the finding that the distribution of wealth across households offers insight into how the economy as a whole responds to aggregate fiscal shocks.[1] Analysis of this sort has been accomplished in large part due to the widespread adoption of models that abandon the traditional representative agent assumption.

The first departure from the representative agent framework incorporates labor income risk. There is a precautionary savings motive in this setting. The availability of a riskless asset that partially insures against this income risk results in households choosing to hold different levels of market resources optimally. Krusell and Smith (1998)'s seminal work suggests that models assuming heterogeneity in individual income perform well in matching the aggregate capital stock but poorly in matching the distribution of wealth.

The next departure is to assuming heterogeneity among households beyond the ex-post realizations of the stochastic process for income. This will lead to more households to optimally hold lower levels of wealth. Carroll, Slacalek, Tokuoka, and White (2017) accomplish this by performing a structural estimation of *ex-ante* heterogeneity in time preferences which allows for the model's distribution of wealth to match the household wealth data much better.

Although the time preference factor (β) is a key parameter in determining a household's target level of market resources, it is not directly observable. The rate of return to, on the other hand, *is* directly observable and can be estimated. Recently, Fagereng, Guiso, Malacrino, and Pistaferri (2020) analyze 12 years of administrative tax records on capital income and wealth stock for all taxpayers in Norway from 2004-2015 to estimate these rates of return. This serves as motivation for the HA model of heterogeneous returns which I present in this paper.

2 Model

First, I describe labor income risk in this setting. Household income (y_t) can be expressed as the following:

$$y_t = p_t \xi_t W_t,$$

where the aggregate wage rate is (W_t), the permanent income component is (p_t), and the transitory shock component is (ξ_t). I assume that the level of permanent income for each household follows a geometric random walk:

[1] Parker, Schild, Erhard, and Johnson (2022) note that "In sum, while on average the [economic impact payments] EIPs appear to have gone to many households with incomes that were unharmed by the pandemic, some of the EIPs, mainly in the first round, did support short-term spending for some households, primarily those with low ex ante liquid wealth and those reliant on income that could not be earned by working from home."

$$p_t = p_{t-1}\psi_t,$$

where the white noise permanent shock to income with a mean of one is represented by ψ_t.

The probability of becoming unemployed is $\mho$; in this case, the agent will receive unemployment insurance payments of $\mu > 0$. With probability $1 - \mho$ the agent is employed and tax payments τ_t are collected as insurance for periods of unemployment. Altogether, the transitory component of income is given by:

$$\xi_t = \begin{cases} \mu & \text{with probability } \mho, \\ (1-\tau_t)l\theta_t & \text{with probability } 1-\mho, \end{cases}$$

where l is the time worked per agent and the parameter θ captures the white noise component of the transitory shock.

Next, I present a standard model of household behavior. The sequence of consumption functions $\{c_{t+n}\}_{n=0}^{\infty}$ associated with a household's optimal choice over a lifetime must satisfy[2]

$$\begin{aligned}
v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta\not{D}\mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t(m_t), \\
k_{t+1} &= \frac{a_t}{\not{D}\psi_{t+1}}, \\
m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}$$

2.1 Results

To solve and simulate the model, I follow the calibration scheme captured in table 1.

The solution of the model with no heterogeneity in returns (the R-point model) is the one which finds the value for the rate of return R which minimizes the distance between the simulated and empirical wealth shares at the 20th, 40th, 60th, and 80th percentiles. The empirical targets are computed using the 2004 Survey of Consumer Finances (SCF) data on household wealth. The estimation procedure finds this optimal value to be $\mathsf{R} = 1.0153$.

[2]Here, I denote a_t as assets, m_t as market resources (or cash-on-hand), k_t as capital, and $\daleth = (1-\delta)$ as the depreciation factor for capital. Each of the relevant variables have been normalized by the level of permanent income ($c_t = \frac{C_t}{p_t}$, and so on). This is the standard state-space reduction of the problem for numerical tractibility.

Description	Parameter	Value	Source
Time discount factor	β	0.99	Den Haan, Judd, and Juillard (2010)
CRRA	ρ	1	Den Haan, Judd, and Juillard (2010)
Capital share	α	0.36	Den Haan, Judd, and Juillard (2010)
Depreciation rate	δ	0.025	Den Haan, Judd, and Juillard (2010)
Time worked per employee	ℓ	1/.09	Den Haan, Judd, and Juillard (2010)
Capital/output ratio	$\frac{K}{Y}$	10.26	Den Haan, Judd, and Juillard (2010)
Effective interest rate	$r-\delta$	0.01	Den Haan, Judd, and Juillard (2010)
Wage rate	W	2.37	Den Haan, Judd, and Juillard (2010)
Unempl. insurance payment	μ	0.15	Den Haan, Judd, and Juillard (2010)
Probability of death	D	0.00625	Yields 40-year working life
Variance of $\log \theta_{t,i}$	σ_θ^2	0.010 x 4	Carroll (1992), Carroll, Slacalek, and Tokuoka (2015)
Variance of $\log \psi_{t,i}$	σ_ψ^2	0.010 x 4/11	Carroll (1992), Debacker, Heim, Panousi, Ramnath, and Vidangos (2013), Carroll, Slacalek, and Tokuoka (2015)
Unemployment rate	$\mho$	0.07	Mean in Den Haan, Judd, and Juillard (2010)

Table 1 Parameter values (quarterly frequency) for the perpetual youth (infinite horizon) model.

2.1.1 *Incorporating heterogeneous returns*

Recent studies by Fagereng, Guiso, Malacrino, and Pistaferri (2020) and Bach, Calvet, and Sodini (2018) have not only estimated the rate of return on asset holdings but have also uncovered significant heterogeneity across households. Given this motivation, the next estimation (the R-dist model) assumes the existence of multiple types of agents, each earning a distinct rate of return on their assets.

Specifically, I assume that different types of households have a time preference factor drawn uniformly from the interval $(\grave{\mathsf{R}}-\nabla, \grave{\mathsf{R}}+\nabla)$, where ∇ represents the level of dispersion. Afterward, the model is simulated to estimate the values of both $\grave{\mathsf{R}}$ and ∇ so that the model matches the inequality in the wealth distribution. To achieve this, the following minimization problem is solved:

$$\{\grave{\mathsf{R}}, \nabla\} = \arg\min_{\mathsf{R},\nabla} \left(\sum_{i=20,40,60,80} (w_i(\mathsf{R},\nabla) - \omega_i)^2 \right)^{\frac{1}{2}}$$

subject to the constraint that the aggregate capital-to-output ratio in this model matches that of the perfect foresight setting:

$$\frac{K}{Y} = \frac{K_{PF}}{Y_{PF}}.$$

Note that w_i and ω_i give the porportion of total aggregate net worth held by the top i percent in the model and in the data, respectively.

The estimation procedure finds this optimal values of $\mathsf{R} = 1.0106$ and $\nabla = 0.0112$.

The performance of the estimation of both the R-point and R-dist models, measured by their ability to match the SCF data, is compared in figure 1.

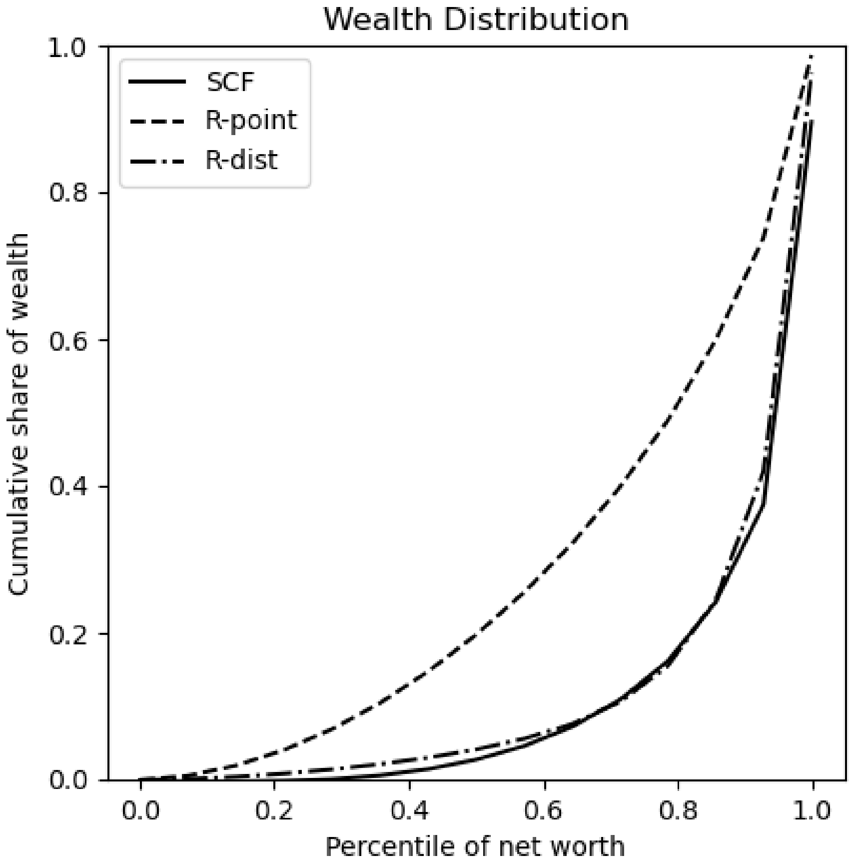

Figure 1 Perpetual youth lorenz curve v.s. data

3 Incorporating life cycle dynamics into the model

More realistic assumptions regarding the age and education level of households can have important implications for the income and mortality process of households. Here, I extend the model to incorporate these life cycle dynamics.

Households enter the economy at time t aged 24 years old and are endowed with an education level $e \in \{D, HS, C\}$, and initial permanent income level $\mathbf{p}_0$, and a capital stock k_0. The life cycle version of household income is given by:

$$y_t = \xi_t \mathbf{p}_t = (1-\tau)\theta_t \mathbf{p}_t,$$

where $\mathbf{p}_t = \psi_t \bar{\psi}_{es} \mathbf{p}_{t-1}$ and $\bar{\psi}_{es}$ captures the age-education-specific average growth factor. Households that have lived for s periods have permanent shocks drawn from a lognormal distribution with mean 1 and variance $\sigma^2_{\psi s}$ and transitory shocks drawn from a lognormal distribution with mean $\frac{1}{\cancel{\mho}}$ and variance $\sigma^2_{\theta s}$ with probability $\cancel{\mho} = (1 - \mho)$ and μ with probability $\mho$.

The normalized version of the age-education-specific consumption-saving problem for households is given by

$$
\begin{aligned}
v_{es}(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \not{D}_{es} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v_{es+1}(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t, \\
k_{t+1} &= \frac{a_t}{\psi_{t+1}}, \\
m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}
$$

3.1 Results

The additional parameters necessary to calibrate the life cycle version of the model are given in table 2.

Description	Parameter	Value
Population growth rate	N	0.0025
Technological growth rate	Γ	0.0037
Rate of high school dropouts	θ_D	0.11
Rate of high school graduates	θ_{HS}	0.55
Rate of college graduates	θ_C	.34
Average initial permanent income, dropout	$\bar{\mathbf{p}}_{D0}$	5000
Average initial permanent income, high school	$\bar{\mathbf{p}}_{HS0}$	7500
Average initial permanent income, college	$\bar{\mathbf{p}}_{C0}$	12000
Unempl. insurance payment	μ	0.15
Labor income tax rate	τ	0.0942

Table 2 Parameter values (quarterly frequency) for the life cycle model.

The estimation procedure finds this optimal value to be $\mathsf{R} = 1.0078$ for the R-point model in this setting. The estimation procedure for the R-dist model in the life cycle setting finds optimal values of $\mathsf{R} = 1.0005$ and $\nabla = 0.01836$. Notice the improved performance of the estimation in matching the data displayed in figure 2.

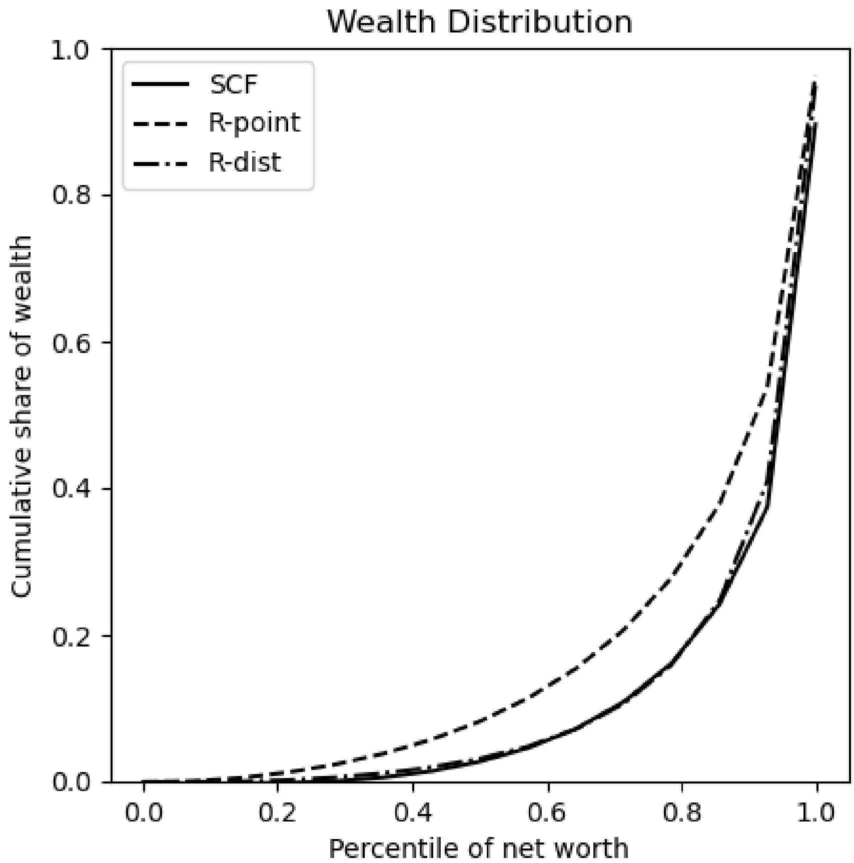

Figure 2 Life cycle lorenz curve v.s. data

4 Extensions

This concludes the preliminary results of this work. From here on, I discuss actionable extensions of the model which are of high priority to be completed.

4.1 Incorporating bequest motives

The model to this point completely explains the saving behavior of households through the precautionary saving motive present in this setting. The desire to leave bequests is thought to be an important reason for households to save, especially those at the top end of the wealth distribution. More generally, the following specification of additively separable wealth in the utility function[3] extends the model to accomodate these other reasons to accumulate assets:

[3]Alternative specifications, such as a non-separable utility function of consumption and wealth, may also be explored in this setting.

$$u(c_t, a_t) = \frac{c_t^{1-\rho}}{1-\rho} + \kappa\frac{(a_t - \underline{\text{a}})^{1-\Sigma}}{1-\Sigma}.$$

Straub (2019) provides calibration values for κ and $\underline{\text{a}}$ and estimation for the elasticity parameters.[4] However, I will need to make decisions about which additional features of the model should be estimated. This will require more empirical moments of the data to be matched by the simulated moments.

4.2 Incorporating portfolio choice

Portfolio choice is also an important feature of the consumption-saving problem of households not currently present in the model. Denoting the gross return on the risky asset as $\mathcal{R}_{t+1}$ and the proportion of the porfolio invested in the risky asset as ς_t, the revised maximization problem is

$$\begin{aligned}
v(m_t) &= \max_{c_t, \varsigma_t} u(c_t, a_t) + \beta \not{D} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\
&\quad \text{s.t.} \\
a_t &= m_t - c_t(m_t), \\
k_{t+1} &= \frac{a_t}{\not{D}\psi_{t+1}}, \\
\mathbb{R}_{t+1} &= \mathsf{R}_{t+1} = (\mathcal{R}_{t+1} - \mathsf{R}_{t+1})\varsigma_t \\
m_{t+1} &= (\mathbb{R} - \delta)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}$$

where $\mathbb{R}$ denotes the overall return on the portfolio across periods.[5]

There are at least two issues that must be resolved in this version of the model. First, once portfolio choice is incorporated, households may have different levels of risk aversion which determines their optimal portfilio share. This suggests that a distribution of risk preferences should be estimated as well, which would require more empirical moments for correct identification. Second, I must be careful to retain the notion of heterogeneous returns *conditional on the risky portfolio share* measured by Fagereng, Guiso, Malacrino, and Pistaferri (2020). With a single asset, the analogy is clear since there is no risky asset. I'd like to preserve this definition of heterogeneous returns, since it is the key novelty of the empirical motivation for this model.

[4]The paper also discusses the implications of assuming $\Sigma = \rho$ versus $\Sigma < \rho$. Incorporating the latter is a more difficult implementation in the existing code, but it is a highly desirable version of the model in order to match the empirical evidence on the saving behavior of households.

[5]The perpetual youth setting is provided for simplicity. It is straightforward to allow for portfolio choice in the life cycle setting.

4.3 Multiple SCF waves for empirical moments

A final exercise is to rerun the exercise for wealth data other than the 2004 SCF wave. This will constitute a sort of robustness check regarding the plausibility of the estimated heterogeneity in the parameters of interest required to match the SCF data.

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