2 Baseline model: choice under risk

Here, I provide an explicit reformulation of the previous work done on proposing measures of income inequality using the choice under objective uncertainty literature.

2.1 Analytical framework

Denote the set of outcomes $Y = [0, \bar{y}]$. Then, the choice set is given by:

$$L = \left\{ p : 2^{[0,\bar{y}]} \to \mathbb{R} \mid p(\cdot) \text{ is an income frequency distribution} \right\}.$$

We may define a binary relation over both sets Y, L :

$$\succsim_y \subseteq [0, \bar{y}] \times [0, \bar{y}] \subseteq \mathbb{R} \times \mathbb{R}$$

$$\succsim L \times L.$$

Notice that $\succsim_y$ may be represented by a real-valued utility function. This is the social welfare $U(y)$ which is a key object of analysis in this paper.

2.2 Axiomatization

V 1 (Weak Order). *$\succsim$ is complete and transitive.*

V 2 (Continuity). *For every $p(\cdot)$, $p*(\cdot)$, $p'(\cdot) \in L$, if $p(\cdot) \succ p*(\cdot) \succ p'(\cdot)$, there exists $\alpha, \beta \in (0, 1)$ such that*

$$\alpha p + (1 - \alpha)p' \succ p* \succ \beta p + (1 - \beta)p'.$$

V 3 (Independence). *For every $p, p*, p' \in L$ and every $\alpha \in (0, 1)$ such that*

$$p \succsim p* \Longrightarrow \alpha p + (1 - \alpha)p' \succsim \alpha p^* + (1 - \alpha)p'.$$

2.3 Expected utility representation of the ranking over income distributions

The ranking over income distributions will be represented using the vNM-expected utility representation. That is,

$$p(y) \sim \int_0^{\bar{y}} U(y)p(y)dy \equiv W.$$

Formally, note the slight modification of the vNM-EU theorem in this setting of ranking income distributions:

Theorem 1. $\succsim \subseteq L \times L$ *satisfies* $(V1)$ ***weak order***, $(V2)$ ***continuity***, $(V3)$ ***independence*** *if and only if there exists* $U : Y \rightarrow \mathbb{R}$ *such that, for every* $p(y), p*(y) \in L$,

$$p(y) \succsim p*(y) \Longleftrightarrow \int_0^{\bar{y}} U(y)p(y)dy \geq \int_0^{\bar{y}} U(y)p*(y)dy.$$