3 Model

First, I describe labor income risk in this setting. Household income (y_t) can be expressed as the following:

$$y_t = p_t \xi_t W_t,$$

where the aggregate wage rate is (W_t), the permanent income component is (p_t), and the transitory shock component is (ξ_t). I assume that the level of permanent income for each household follows a geometric random walk:

$$p_t = p_{t-1}\psi_t,$$

where the white noise permanent shock to income with a mean of one is represented by ψ_t.

The probability of becoming unemployed is $\mho$; in this case, the agent will receive unemployment insurance payments of $\mu > 0$. With probability $1 - \mho$ the agent is employed and tax payments τ_t are collected as insurance for periods of unemployment. Altogether, the transitory component of income is given by:

$$\xi_t = \begin{cases} \mu & \text{with probability } \mho, \\ (1-\tau_t)l\theta_t & \text{with probability } 1-\mho, \end{cases}$$

where l is the time worked per agent and the parameter θ captures the white noise component of the transitory shock.

Next, I present a standard model of household behavior. The sequence of consumption functions $\{c_{t+n}\}_{n=0}^{\infty}$ associated with a household's optimal choice over a lifetime must satisfy:

$$\begin{aligned} v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \not{D} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\ &\text{s.t.} \\ a_t &= m_t - c_t(m_t), \\ k_{t+1} &= \frac{a_t}{\not{D}\psi_{t+1}}, \\ m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1}, \\ a_t &\geq 0, \end{aligned}$$

where I denote a_t as assets, m_t as market resources, k_t as capital, and $\daleth = (1-\delta)$ as the depreciation factor for capital. [1]

[1] Each of the relevant variables have been normalized by the level of permanent income ($c_t = \frac{C_t}{p_t}$, and so on).

3.1 Results

To solve and simulate the model, I follow the calibration scheme captured in table 1.

Description	Parameter	Value	Source
Time discount factor	β	0.99	Den Haan, Judd, and Juillard (2010)
CRRA	ρ	1	Den Haan, Judd, and Juillard (2010)
Capital share	α	0.36	Den Haan, Judd, and Juillard (2010)
Depreciation rate	δ	0.025	Den Haan, Judd, and Juillard (2010)
Time worked per employee	ℓ	1/.09	Den Haan, Judd, and Juillard (2010)
Capital/output ratio	$\frac{K}{Y}$	10.26	Den Haan, Judd, and Juillard (2010)
Effective interest rate	$r-\delta$	0.01	Den Haan, Judd, and Juillard (2010)
Wage rate	W	2.37	Den Haan, Judd, and Juillard (2010)
Unempl. insurance payment	μ	0.15	Den Haan, Judd, and Juillard (2010)
Probability of death	D	0.00625	Yields 40-year working life
Variance of $\log \theta_{t,i}$	σ^2_θ	0.010 x 4	Carroll (1992), Carroll, Slacalek, and Tokuoka (2015)
Variance of $\log \psi_{t,i}$	σ^2_ψ	0.010 x 4/11	Carroll (1992), Debacker, Heim, Panousi, Ramnath, and Vidangos (2013), Carroll, Slacalek, and Tokuoka (2015)
Unemployment rate	$\mho$	0.07	Mean in Den Haan, Judd, and Juillard (2010)

Table 1 Parameter values (quarterly frequency) for the perpetual youth (infinite horizon) model.

The solution of the model with no heterogeneity in returns (the R-point model) is the one which finds the value for the rate of return R which minimizes the distance between the simulated and empirical wealth shares at the 20th, 40th, 60th, and 80th percentiles. The empirical targets are computed using the 2004 Survey of Consumer Finances (SCF) data on household wealth. The estimation procedure finds this optimal value to be $\mathsf{R} = 1.0153$.

3.1.1 *Incorporating heterogeneous returns*

Recent studies by Fagereng, Guiso, Malacrino, and Pistaferri (2020) and Bach, Calvet, and Sodini (2018) have not only estimated the rate of return on asset holdings but have also uncovered significant heterogeneity across households. Given this motivation, the next estimation (the R-dist model) assumes the existence of multiple types of agents, each earning a distinct rate of return on their assets.

Specifically, I assume that different types of households have a time preference factor drawn uniformly from the interval $(\grave{\mathsf{R}} - \nabla, \grave{\mathsf{R}} + \nabla)$, where ∇ represents the level of dispersion. Afterward, the model is simulated to estimate the values of both $\grave{\mathsf{R}}$ and ∇ so that the model matches the inequality in the wealth distribution. To achieve this, the following minimization problem is solved:

$$\{\grave{\mathsf{R}}, \nabla\} = \arg\min_{\mathsf{R},\nabla} \left(\sum_{i=20,40,60,80} (w_i(\mathsf{R}, \nabla) - \omega_i)^2 \right)^{\frac{1}{2}}$$

subject to the constraint that the aggregate capital-to-output ratio in this model matches that of the perfect foresight setting:

$$\frac{K}{Y} = \frac{K_{PF}}{Y_{PF}}.$$

Note that w_i and ω_i give the porportion of total aggregate net worth held by the top i percent in the model and in the data, respectively.

The estimation procedure finds this optimal values of $\mathsf{R} = 1.0106$ and $\nabla = 0.0112$. The performance of the estimation of both the R-point and R-dist models, measured by their ability to match the SCF data, is compared in figure 1.

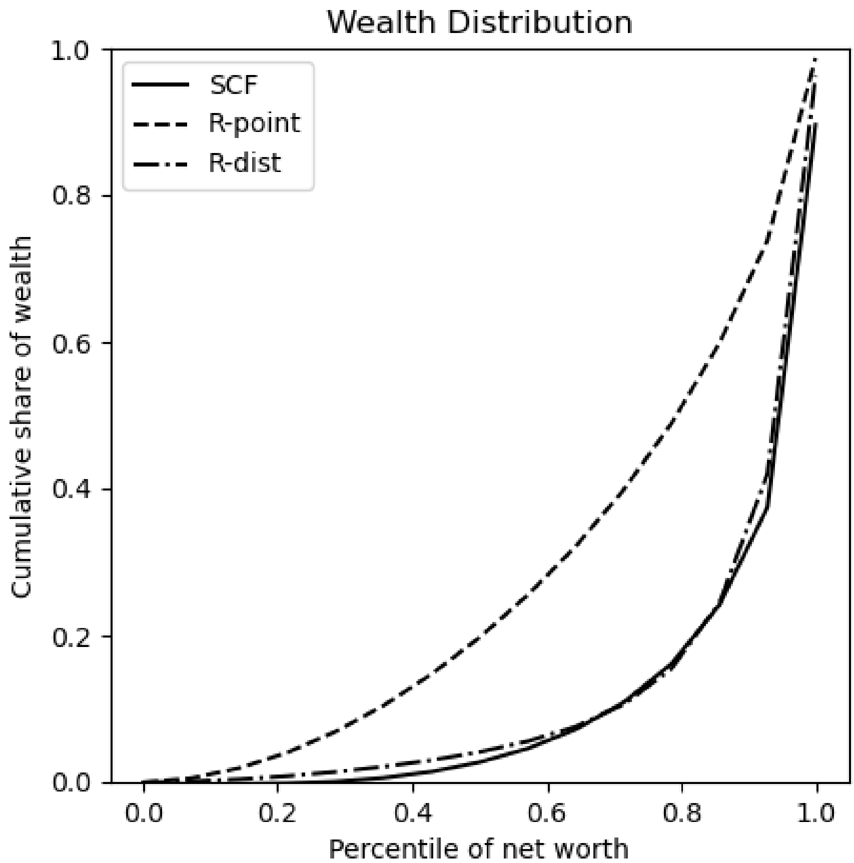

Figure 1 Perpetual youth lorenz curve v.s. data

References