Dissertation Preproposal:

Heterogeneous Rates of Return and the Distribution of Wealth

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Inequality in the distribution of wealth is a notable topic of interest both in the mainstream and in the economics literature. Reliable measurements of the wealth holdings of households, obtained either through surveys or using capitalization methods on administrative tax data, suggest that the wealth distribution is both highly skewed and exhibit heavy (Pareto) upper tails. **?** provide an excellent survey on both historical thought and theoretical explanations for stationary distributions of wealth with substantial skewness and heavy tails. Not surprisingly, the key arguments posited there also make their way into the standard consumption-saving problems which are a cornerstone of models prevalent in the heterogeneous agent (HA) macroeconomics literature. At the same time, recent evidence of heterogeneity in the rate of return for individuals provide motivation for an analogous assumption in the HA framework where a stationary, model distribution of wealth exists. In this way, one can test the theory regarding a meaningful relationship between stochastic returns and wealth inequality within a standard macroeconomic setting, where the direct effect of the returns to savings on the wealth accumulation process can be understood through its effect on the optimal behavior of households. A uniform distribution of the rate of return across households is estimated such that empirical moments of wealth (net worth) measured in the 2004 survey of consumer finances SCF are matched particularly well. In an effort to compare the structurally estimated distribution of returns from the model to the empirical distribution measured by **?**, a lognormal distribution across households is assumed. Not only does the lognormal assumption allow for the simulated moments to better fit the empirical moments for net worth, but the estimated distribution of returns is closer to its empirical counterpart as well.

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1 Introduction

The unequal distribution of wealth is an extensively documented phenomenon in numerous countries. Regrettably, this feature has not only endured over time but also intensified in recent years. This point is stressed in a recent article from the Institute for Policy Studies (IPS), which revealed that in 2018, the total wealth of the poorest half of Americans was eclipsed by the combined wealth of the three wealthiest men in the nation. The term "richest" denotes one's standing in Forbes magazine's list of the 400 richest individuals. Additionally, the IPS report notes that the combined wealth of the top five richest men on this list skyrocketed by a staggering 123% from March 2020 to October 2021[1].

The unequal distribution of wealth has also been a subject of considerable interest throughout history in various fields. The statistics literature, for instance, focused on linking the distribution of income to the observable skewness in wealth distribution. The economics literature went further by establishing microfoundations for individual wealth outcomes. Similarly, the macroeconomics literature on inequality has seen significant growth, with the distribution of wealth among households offering insight into how the economy as a whole responds to aggregate fiscal shocks. The recent stimulus checks issued during the pandemic serve as a timely example of this phenomenon.

The macroeconomics literature has undergone significant changes in recent years, with the widespread adoption of models that abandon the traditional representative agent assumption in their analysis. Specifically, a model that studies the equilibrium outcomes of an economy composed of individual decision-makers using a single aggregate agent can only have one marginal propensity to consume (MPC). As a result, in response to an aggregate fiscal shock, all households would respond similarly to a one-time stimulus check, which does not align with what transpired during the pandemic[2]. Heterogeneous agent models have emerged as a prominent alternative, offering a more accurate representation of the diversity of economic behaviors and outcomes among households.

The first departure from the representative agent framework entails positing an exogenously determined income process that generates a distribution of income among households. One common approach to incorporating heterogeneity is to adopt **?**'s description of a permanent and transitory component in the income process. To account for business cycle dynamics, one can further assume that individuals face some level of potential unemployment in each period, creating a precautionary savings motive for consumers. Given that such uncertainty cannot be fully insured against, the availability of a riskless asset that partially insures against income risk results in households choosing to hold different levels of market resources optimally.

[1]See Inequality.org articles data November 21, 2022: "Wealth Inequality in the United States" and "Updates: Billionaire Wealth, U.S. Job Losses and Pandemic Profiteers" (date accessed: March 27, 2023)

[2]**?** note that "In sum, while on average the [economic impact payments] EIPs appear to have gone to many households with incomes that were unharmed by the pandemic, some of the EIPs, mainly in the first round, did support short-term spending for some households, primarily those with low ex ante liquid wealth and those reliant on income that could not be earned by working from home."

?'s seminal work suggests that models assuming heterogeneity in individual income perform well in matching the aggregate capital stock but poorly in matching the distribution of wealth. The resulting optimal consumption function is concave in an individual's wealth holdings, meaning that the marginal propensity to consume out of income is increasingly higher at lower levels of wealth. Therefore, a model that places too many households in the middle of the wealth distribution relative to those at lower levels will struggle to match the average MPC estimated from household data. Since our focus is on the implications of fiscal policy for the entire economy, a macroeconomic model's failure to match the observed wealth distribution in its implied equilibrium is significant.

Moving beyond the standard representative agent framework, the next step is to assume greater heterogeneity among households, leading more households to optimally hold lower levels of wealth. **?**'s recent work provides a comprehensive survey of models that reject this assumption, instead utilizing heterogeneous agent, incomplete markets models featuring (i) uninsurable idiosyncratic income risk, (ii) a precautionary savings motive, and (iii) an endogenous wealth distribution.

? adopt this approach and further extend the baseline setting to allow for ex-ante heterogeneity amongst households. Specifically, they assume different agents have different rates of time preference, which reflects implicit characteristics of households relevant to their lifetime wealth accumulation. The authors find that this assumption of modest heterogeneity in time preferences is sufficient to match both the shape and skewness of the empirical distribution of wealth. Furthermore, while traditional representative agent models generate an aggregate marginal propensity to consume between 0.02 and 0.04, the β-dist model generates an aggregate MPC between 0.2 and 0.4. This range falls within the values estimated across households in the data.

The household's optimal consumption-savings problem contains additional elements that could contribute to disparities in wealth accumulation over the course of one's lifetime. It is worth noting that the time preference factor (β) is one of the key parameters that influences an individual's equilibrium target level of market resources, but it is not directly observable. Therefore, in order to estimate β, one would need to gather data through surveys or other methods that allow for the direct acquisition of information from households.

On the other hand, estimating differences in the rate of return to financial assets across households is possible, as this variable *is* directly observable. Empirical research has been conducted to estimate such differences, with a recent example being the work of **?**. They analyzed 12 years of administrative tax records on capital income and wealth stock for all taxpayers in Norway from 2004-2015 to estimate these rates of return.

This paper aims to enhance the computational, heterogeneous agent modelling framework by integrating recent empirical evidence on disparities in rates of return among households. The objective is to better align the observed wealth distribution with the model predictions, thereby generating more realistic estimates of the average marginal propensity to consume among households.

2 Model

2.1 Defining the stochastic income process

Each household's income (y_t) during a given period depends on three main factors. The first factor is the aggregate wage rate (W_t) that all households in the economy face. The second factor is the permanent income component (p_t), which represents an agent's present discounted value of human wealth. Lastly, the transitory shock component (ξ_t) reflects the potential risks that households may face in receiving their income payment during that period. Thus, household income can be expressed as the following:

$$y_t = p_t \xi_t W_t.$$

The level of permanent income for each household is subject to a stochastic process. In line with **?**'s description of the labor income process, we assume that this process follows a geometric random walk, which can be expressed as:

$$p_t = p_{t-1}\psi_t,$$

The white noise permanent shock to income with a mean of one is represented by ψ_t, which is a significant component of household income. The probability of receiving income during a given period is determined by the transitory component, which is modeled to reflect the potential risks associated with becoming unemployed. Specifically, if the probability of becoming unemployed is $\mho$, the agent will receive unemployment insurance payments of $\mu > 0$. On the other hand, if the agent is employed, which occurs with a probability of $1 - \mho$, the model allows for tax payments τ_t to be collected as insurance for periods of unemployment. The transitory component is then represented as:

$$\xi_t = \begin{cases} \mu & \text{with probability } \mho, \\ (1-\tau_t)l\theta_t & \text{with probability } 1-\mho, \end{cases}$$

where l is the time worked per agent and the parameter θ captures the white noise component of the transitory shock.

2.2 Baseline model for households

This paragraph presents the baseline version of the household's optimization problem for consumption-savings decisions, assuming no ex-ante heterogeneity. In this case, each household aims to maximize its expected discounted utility of consumption $u(c) = \frac{c^{1-\rho}}{1-\rho}$ by solving the following:

$$\max \mathbb{E}_t \sum_{n=0}^{\infty} (\not{D}\beta)^n u(c_{t+n}).$$

It's worth noting that the setting described here follows a perpetual youth model of buffer stock savings, similar to the seminal work of **?**. To solve this problem, we use the bellman equation, which means that the sequence of consumption functions $\{c_{t+n}\}_{n=0}^{\infty}$ associated with a household's optimal choice over a lifetime must satisfy[3]

$$\begin{aligned} v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \not{D} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\ &\text{s.t.} \\ a_t &= m_t - c_t(m_t), \\ k_{t+1} &= \frac{a_t}{\not{D}\psi_{t+1}}, \\ m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1}, \\ a_t &\geq 0. \end{aligned}$$

2.2.1 The analogy for rates of return

If we want to explore how different returns to assets can affect the endogenous wealth distribution, it's important to examine the following decomposition of a household's evolution of market resources over time:

1. Assets at the end of the period are equal to market resources minus consumption:

$$a_t = m_t - c_t.$$

2. Next period's capital is determined from this period's assets via

$$k_{t+1} = \frac{a_t}{\not{D}\psi_t}.$$

3. Finally, the transition from the beginning of period $t+1$ when capital has not yet been used to produce output, to the middle of that period when output has been produced and incorporated into resources but has not yet been consumed is:

$$m_{t+1} = (\daleth + r_t)K_{t+1} + \xi_{t+1}.$$

It's worth recalling that in this model, the rate of return to capital is represented as $(\daleth + r_t)$. This rate of return is directly related to the endogenous level of wealth, which is determined by the level of capital K_{t+1}. Therefore, if there are differences in the rate of return across households, this will result in further disparities in wealth holdings.

[3]Here, each of the relevant variables have been normalized by the level of permanent income ($c_t = \frac{C_t}{p_t}$, and so on). This is the standard state-space reduction of the problem for numerical tractibility.

3 Results

3.1 Matching observed inequality in the distribtution of wealth

3.1.1 *Adding ex-ante heterogeneity in time preferences*

In **?**'s baseline model, heterogeneity in the time preference factor is not accounted for. However, to address this, the model is extended to include this factor. This is done by assuming that different types of households have a time preference factor drawn uniformly from the interval $(\grave{\beta} - \nabla, \grave{\beta} + \nabla)$, where ∇ represents the level of dispersion. Afterward, the model is simulated to estimate the values of both $\grave{\beta}$ and ∇ so that the model matches the inequality in the wealth distribution. To achieve this, the following minimization problem is solved:

$$\{\grave{\beta}, \nabla\} = \arg\min_{\beta,\nabla} \left(\sum_{i=20,40,60,80} (w_i(\beta, \nabla) - \omega_i)^2 \right)^{\frac{1}{2}}$$

subject to the constraint that the aggregate capital-to-output ratio in this model matches that of the perfect foresight setting:

$$\frac{K}{Y} = \frac{K_{PF}}{Y_{PF}}.$$

Note that w_i and ω_i give the porportion of total aggregate net worth held by the top i percent in the model and in the data, respectively.

3.1.2 *The analogous exercise for ex-ante heterogenous rates of return*

The β-dist model proves to be useful in a setting where there are heterogeneous time preference factors since it captures an unobservable component of a household's decision-making process. While the microeconomics literature has put in considerable effort to estimate this parameter, there is currently no consensus on its value.

Recent studies by **?** and **?** have not only estimated the rate of return on asset holdings but have also uncovered significant heterogeneity across households. Given this motivation, the revised model assumes the existence of multiple types of agents, each earning a distinct rate of return on their assets. A calibration exercise akin to the one used in the β-dist model is then performed. This crucial step involves comparing the resulting endogenous distribution from simulating this calibrated model to its empirical counterpart to determine if there is an ex-ante distribution of rates of return that can match the observable inequality in the wealth distribution. If a distribution of returns to asset holdings satisfies this criterion, the final step involves reconciling this model heterogeneity with the observed differences in rates of return found in the aforementioned literature.

3.2 Policy implications

Following the analysis of the wealth distribution, the policy implications of heterogeneous rates of return will be examined by evaluating the impact of a one-time stimulus payment to all households on key macroeconomic variables (such as consumption, capital, and output).

My suspicion is that if the model provides a better match to the inequality in the distribution of wealth compared to the case where all households earn the same rate of return on their assets, there will be greater dispersion in the marginal propensity to consume across households. Consequently, this should result in different consumption and saving behavior among households in response to a one-time shock to income. As a result, the aggregate implications of the policy shock in this setting are expected to differ significantly from the setting where there is no heterogeneity in the rate of return.

I aim to compare the policy implications of this computational model with the actual consumption and saving behavior of households during the recent pandemic, which was characterized by the provision of several stimulus checks by the government. However, I am currently concerned about the feasibility of this final step in the project.