4 Extending the model: choice under ambiguity

Consider the excerpt from Gilboa (2009), which may serve as a precursor to the modeling choices that will be made to characterize a ranking over wealth distributions:

> To make sure that we understand the structure, observe that there are two sources of uncertainty: the choice of the state s, which is sometimes referred to as "subjective uncertainty", because no objective probabilities are given on it, and the choice of x, which is done with objective probabilities once you chose your act and Nature chose a state. Specifically, if you choose $f \in F$ and Nature chooses $s \in S$, a roulette wheel is spun, with distribution $f(s)$ over the outcomes X, so that your probability to get outcome x is $f(s)(x)$.

4.1 Analytical framework

Denote the set of outcomes $Y = [0, \bar{y}]$. Then, the choice set is given by:

$$L = \left\{ p : 2^{[0,\bar{y}]} \to \mathbb{R} \mid p(\cdot) \text{ is an income frequency distribution} \right\}.$$

We may define a binary relation over both sets Y, L :

$$\succsim_y \subseteq [0, \bar{y}] \times [0, \bar{y}] \subseteq \mathbb{R} \times \mathbb{R}$$

$$\succsim L \times L.$$

Notice that $\succsim_y$ may be represented by a real-valued utility function. This is the social welfare $U(y)$ which is a key object of analysis in this paper.

Preliminary remarks

We are concerned with the comparison of two frequency distributions $f(w)$ of an outcome w which we refer to as wealth. We seek to use the notion of *uncertainty aversion* as an analogy to the use of the notion of risk aversion in the characterization of a ranking over income distributions.

The presence of both objective and subjective uncertainty is at the heart of this analysis. Section 2 covered the analysis for objective uncertainty. Thus, the wealth frequency distribution $f(w)$ must be formalized in this abstract setting so that it explicitly captures both forms of uncertainty. Namely, each wealth distribution is associated with some relevant, underlying state space S (the source of subjective uncertainty), and its objective component $p(y) \in L$. WIth this in mind, we work under the following assumption on the functional form of wealth distributions for the remainder of this paper:

Assumption 1. *Each wealth distribution $f(w)$ can be written as $p(s)y$.*

4.2 Axiomatization

AA 1 (Weak Order). $\succsim$ *is complete and transitive.*

AA 2 (Continuity). *For every $f, g, h \in F$, if $f \succ g \succ h$, there exists $\alpha, \beta \in (0,1)$ such that*

$$\alpha f + (1-\alpha)h \succ g \succ \beta f + (1-\beta)h.$$

C-Independence (C-Independence). *For every $f, g \in F$, every constant $h \in F$ and every $\alpha \in (0,1)$,*

$$f \succsim g \Longleftrightarrow \alpha f + (1-\alpha)h \succsim \alpha g + (1-\alpha)h.$$

AA 3 (Monotonicity). *For every $f, g \in F$, $f(s) \geq g(s)$ for all $s \in S$ implies $f \geq g$.*

AA 4 (Non-trivality). *There exists $f, g \in X$ such that $f \succ g$.*

Uncertainty Aversion (Uncertainty Aversion). *For every $f, g \in F$, if $f \sim g$, then, for every $\alpha \in (0,1)$,*

$$\alpha f + (1-\alpha)g \succsim f.$$

4.3 Expected utility representation of the ranking over wealth distributions

Finally, we have the representation theorem by Gilboa and Schmeidler (1989).

Theorem 1. $\succsim$ *satisfies AA1, AA2, C-Independence, AA4, AA5, and Uncertainty aversion if and only if there exists a closed and convex set of probabilities on S, $C \subset \Delta(S)$, and a non-constant function $U : Y \to \mathbb{R}$ such that, for every $f, f^* \in F$,*

$$f \succsim f^* \Longleftrightarrow \min_{\lambda \in \Delta(S)} \int_S (\mathbb{E}_{p(s)} u) d\lambda \geq \min_{\lambda \in \Delta(S)} \int_S (\mathbb{E}_{p^*(s)} u) d\lambda.$$

From here on out, we assume that wealth distributions will be ranked according to:

$$W' \equiv \min_{\lambda \in \Delta(S)} \int_S (\mathbb{E}_{p(s)} u) d\lambda$$

$$= \min_{\lambda \in C} \int_S \int_0^{\bar{y}} p(s)(y) U(y) dy d\lambda.$$

References

Gilboa, Itzhak (2009): *Theory of Decision under Uncertainty*, Econometric Society Monographs. Cambridge University Press.

Gilboa, Itzhak, and David Schmeidler (1989): "Maxmin expected utility with non-unique prior," *Journal of Mathematical Economics*, 18(2), 141–153.