Heterogeneous Returns and the Distribution of Wealth

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Why do macroeconomists care about inequality?

Empirical evidence shows that macroeconomic policies, as well as aggregate shocks, may have differential effects across households.

- Macro matters for inequality

Representative agent models have a difficult time matching empirical estimates of macroeconomic variables (MPC and the wealth distribution).

- Inequality matters for macro

Macro with heterogeneous agents

- Uninsurable, idiosyncratic risk to income and movements in aggregate productivity (**ks1998**)
- Ex-ante heterogeneity in the time preference of households (**cstw2017**)
- Classifying models with ex-ante and ex-post heterogeneity (**gkgv22**)
- Further surveys regarding heterogeneous agent macroeconomics (**Guvenen2011**) and (**Krueger2016**)

1. Comprehensive, administrative tax data in Norway from 2004 to 2015 (**aflgdmlp20**)

2. Asset holdings and income for Swedish residents from 1999 to 2007 (**lblcps18**)

3. Wealth held in equity accounts in India from 2002 to 2011 (**Campbell2019**)

4. DNB 2005 survey of dutch households regarding savings accounts and financial literacy (**Deuflhard2018**)

- Stochastic process for returns implying a stationary wealth distribution.
  (**Benhabib2011**), (**Benhabib2015**), (**Benhabib2016**)
- Stochastic process for returns which best fits the empirical distribution of wealth.
  (**Benhabib2019**)
- Endogenize heterogeneous returns through access to high return investment technology.
  (**Guler2022**)

1. Empirical evidence of heterogeneous returns
2. Model of saving with heterogeneous returns
3. Structural estimation of model to match wealth data

A closer look at **aflgdmlp20**

Following optimal portfolio choice theory from Merton (1969) and Samuelson (1969)

- Optimal share in the risky asset is given by

$$\alpha_{it}^m = \frac{\mathbb{E}(r_t^m - r_t^s)}{\gamma_i \sigma_t^2}.$$

- Individual *realized* return to financial assets can be written as
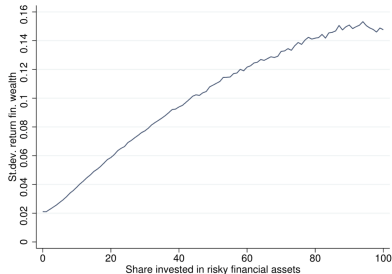
$$r_{it}^f = r_t^s + \alpha_{it}^m(r_t^m - r_t^s).$$

Figure: Heterogeneity in returns to financial wealth by share of risky assets from **aflgdmlp20**.

- Step 1: *linear panel data regression model* for the return to net worth

$$r_{it}^n = X_{it}^{'}\beta + u_{it}.$$

- Step 2: Add fixed effects

$$u_{it} = f_i + e_{it}.$$
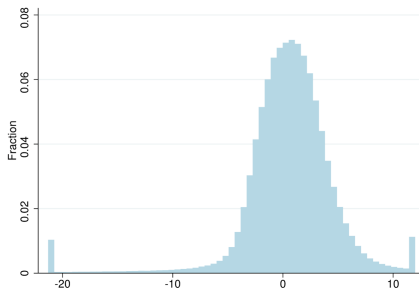
$\implies R^2$ goes from .33 to .5.

Figure: Distribution of fixed effects in the return to net worth from **aflgdmlp20**.

Labor income process

- Household income:

$$y_t = p_t \xi_t W_t$$

- Permanent component:

$$p_t = p_{t-1} \psi_t$$

- Transitory component:

$$\xi_t = \begin{cases} \mu & \text{with probability } \mho \\ (1 - \tau_t) l \theta_t & \text{with probability } 1 - \mho \end{cases}$$

(Normalized) Optimization problem for households: Choose profiles $\{c_{t_n}\}_{n=0}^{\infty}$ that satisfy

$$
\begin{aligned}
v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta\mathcal{D}\mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\
\text{s.t.} \\
a_t &= m_t - c_t(m_t), \\
k_{t+1} &= \frac{a_t}{\mathcal{D}\psi_{t+1}}, \\
m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}
$$

Production function

$$
Y = ZK^{\alpha}(\ell L)^{1-\alpha}
$$

Carroll2019bst provides the following *death-modified growth impatience condition* such that a unique target wealth-to-permanent income ratio exists for households:

$$\left(\frac{(R\beta)^{1/\rho}\mathbb{E}[\psi^{-1}]\cancel{D}}{\Gamma} \right) \quad < \quad 1.$$

Calibration

Standard calibration scheme used to simulate the model.

Description	Parameter	Value	Source
Time discount factor	β	0.99	Den Haan, Judd, and Juillard 2010
CRRA	ρ	1	Den Haan, Judd, and Juillard 2010
Capital share	α	0.36	Den Haan, Judd, and Juillard 2010
Depreciation rate	δ	0.025	Den Haan, Judd, and Juillard 2010
Time worked per employee	ℓ	1/.09	Den Haan, Judd, and Juillard 2010
Capital/output ratio	$\frac{K}{Y}$	10.26	Den Haan, Judd, and Juillard 2010
Effective interest rate	$r - \delta$	0.01	Den Haan, Judd, and Juillard 2010
Wage rate	W	2.37	Den Haan, Judd, and Juillard 2010
Unempl. insurance payment	μ	0.15	Den Haan, Judd, and Juillard 2010
Probability of death	D	0.00625	Yields 40-year working life
Variance of $\log \theta_{t,i}$	σ_θ^2	0.010 x 4	Carroll 1992,
			Carroll, Slacalek, and Tokuoka 2015
Variance of $\log \psi_{t,i}$	σ_ψ^2	0.010 x 4/11	Carroll 1992,
			Debacker et al. 2013,
			Carroll, Slacalek, and Tokuoka 2015
Unemployment rate	$\mho$	0.07	Mean in Den Haan, Judd, and Juillard 2010

Table 1: Parameter values (quarterly frequency) for the perpetual youth model with no aggregate uncertainty.

Estimation procedure

Simulated method of moments (SMM) estimation for R.

1. No ex-ante heterogeneity: *R-point* model
   R which matches the capital-to-output ($\frac{K}{Y}$) ratio of 10.26

2. Ex-ante heterogeneity: *R-dist* model
   Uniform distribution of R matching lorenz targets, given $\frac{K}{Y} = 10.26$
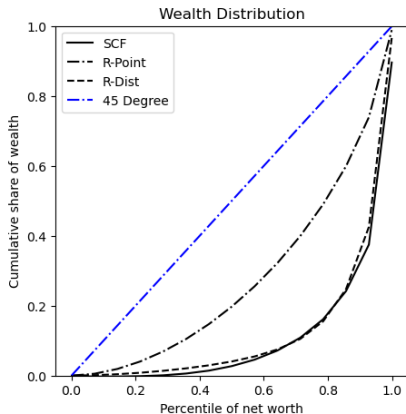
Estimation procedure with heterogeneity

Empirical lorenz targets using 2004 SCF data

Net worth percentile	Cumulative net worth
20th	-.18%
40th	.95%
60th	5.3%
80th	17.09%

Optimization problem for the *R-dist* model

$$\{\dot{R}, \nabla\} = \arg\min_{R,\nabla} \left(\sum_{i=20,40,60,80} (w_i(R, \nabla) - \omega_i)^2 \right)^{\frac{1}{2}}$$

s.t.

$$\frac{K}{Y} = 10.26.$$

How good is the match?

Wealth Distribution

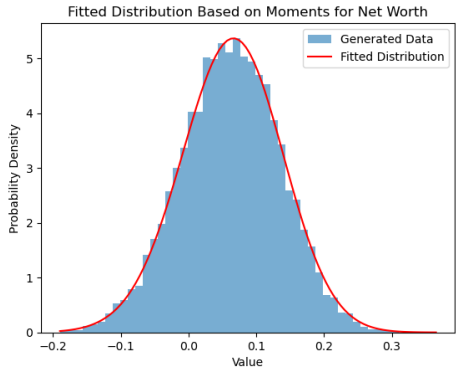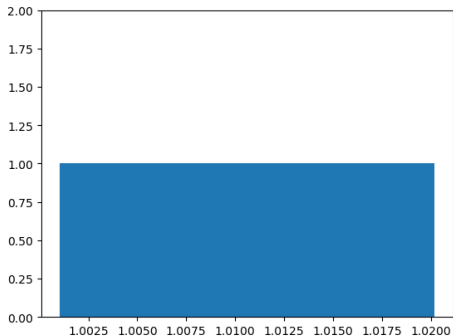|	*R*-point	*R*-dist
Center	1.015	1.0105
Spread	0	.011
Aggregate MPC	.095	.263

aflgdmlp20 describe the empirical distribution of returns.

TABLE 3
RETURNS TO WEALTH: SUMMARY STATISTICS[a]

Wealth Component	Mean	St. Dev.	Skewness	Kurtosis	P10	Median	P90
Net worth (before tax)	0.0379	0.0859	−0.79	47.75	−0.0308	0.0321	0.1109
Net worth (after tax)	0.0365	0.0781	−0.71	36.88	−0.0283	0.0316	0.1067
Net worth (before tax, unweighted)	0.0004	0.2205	−6.73	68.46	−0.0600	0.0230	0.1037
Net worth (after tax, unweighted)	0.0155	0.1546	−5.28	56.42	−0.0449	0.0247	0.1040
Financial wealth	0.0105	0.0596	−1.78	22.17	−0.0171	0.0084	0.0530
Safe fin. assets	0.0078	0.0188	4.38	53.52	−0.0106	0.0059	0.0268
Risky fin. assets	0.0425	0.2473	−0.08	6.22	−0.2443	0.0418	0.3037
Non-financial wealth	0.0511	0.0786	1.80	15.47	−0.0215	0.0429	0.1275
Housing	0.0485	0.0653	0.73	9.95	−0.0209	0.0441	0.1165
Private equity	0.1040	0.5169	18.01	836.79	−0.0531	0.0052	0.3616
Debt	0.0236	0.0216	2.51	29.50	0.0030	0.0215	0.0461
Long-term debt	0.0230	0.0209	3.54	56.92	0.0038	0.0209	0.0446
Consumer debt	0.0961	0.1086	4.60	82.60	−0.0124	0.0741	0.2119
Student debt	0.0078	0.0260	0.68	4.14	−0.0213	0.0074	0.0399

[a]The table reports summary statistics for various measures of real returns to wealth, pooling data for 2005–2015. Except when noted, all returns are value-weighted.

Fitted Distribution Based on Moments for Net Worth
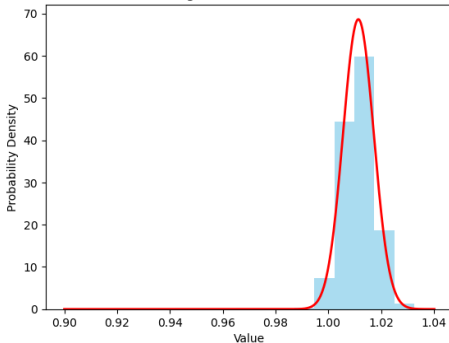
Matching the data with a lognormal distribution

Wealth Distribution
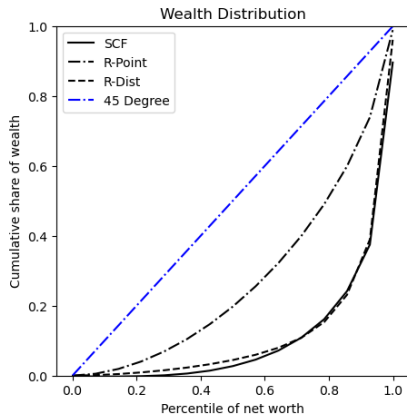
	R-point	R-dist
Center	1.015	1.0114
Spread	0	.006
Aggregate MPC	.095	.241

The estimated lognormal distribution
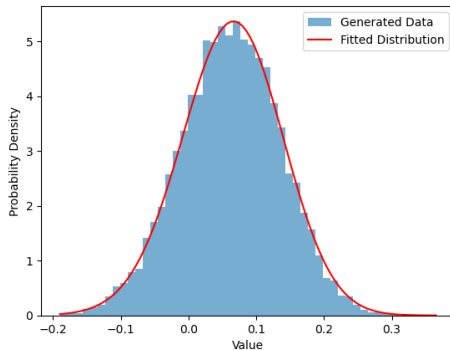
Work to be done

1. Potentially exclude retirement assets in the measure household net worth
   - Private pension wealth is unobservable in the dataset from **aflgdmlp20**
   - Are there significant differences between retirement asset holdings in the U.S. v.s. Norway?

2. Run the estimation for liquid wealth
   - How close is it to its empirical counterpart ("financial wealth")?

3. Life-cycle version of the model

4. More recent wealth data from the SCF

Future directions for this work

- Scale dependence of heterogeneous rates of return
- Endogenizing differences in the rate of return
- Implications of wealth v.s. capital income taxation
    - (**Benhabib2008**), (**Benhabib2011**), (**Guvenen2019**)