3 Incorporating Life cycle dynamics into the model

The case where households solve the infinite horizon version of the consumption-saving problem is an interesting limiting case. However, more realistic assumptions regarding the age and education level of households can have important implications for the income and mortality process of households. Here, we extend the model to incorporate these assumptions.

Households enter the economy at time t aged 24 years old and are endowed with an education level $e \in \{D, HS, C\}$, and initial permanent income level $\mathbf{p}_0$, and a capital stock k_0. The life cycle version of household income can be expressed as the following:

$$y_t = \xi_t \mathbf{p}_t = (1-\tau)\theta_t \mathbf{p}_t,$$

where $\mathbf{p}_t = \psi_t \bar{\psi}_{es} \mathbf{p}_{t-1}$ and $\bar{\psi}_{es}$ captures the age-education-specific average growth factor. Households that have lived for s periods have permanent shocks drawn from a lognormal distribution with mean 1 and variance $\sigma^2_{\psi s}$ and transitory shocks drawn from a lognormal distribution with mean $\frac{1}{\cancel{\mho}}$ and variance $\sigma^2_{\theta s}$ with probability $\cancel{\mho} = (1-\mho)$ and μ with probability $\mho$.

The normalized version of the age-education-specific consumption-saving problem for households is given by

$$\begin{aligned}
v_{es}(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \cancel{D}_{es} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v_{es+1}(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t, \\
k_{t+1} &= \frac{a_t}{\psi_{t+1}}, \\
m_{t+1} &= (\daleth + r_t) k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}$$