1 Introduction

A quantitative analysis of household finance can be accomplished using a standard framework of household decision making found in the computational heterogeneous agent (HA) macro literature. In particular, the setting is well equipped to study to the relationship between the consumption-saving behavior and income and wealth inequality.

One of the many important feautures of saving and borrowing behavior is the rate of return to assets. Reliable estimates of individual returns computed using population data in Norway by Fagereng, Guiso, Malacrino, and Pistaferri (2020) suggest that there is substantial heterogeneity in the rate of return. This is a notable finding for the management of consumer finances: it is well known that differences in returns to assets will lead to a skewed distribution of wealth[1].

The preliminary findings of this work provide some affirmation of this sentiment. A model which allows for heterogeneity in the rate of return produces a distribution of wealth which closely matches the inequality in measured wealth holdings.

However, what is less clear are (i) the determinants of these heterogeneous returns and (ii) the quantitative effects of these determinants on wealth inequality through this returns channel. Outside of incorporating other relevant features of the household consumption-saving problem, like bequest motives and portfolio choice, the next step in this work will be to provide answers to (i) and (ii). Two insights from the literature which I would like to explore to acomplish this are *financial literacy* and *trust in financial institutions*.

This proposal will outline the aforementioned preliminary results. Then I discuss the proposed extensions of the model in detail. The completion of these extensions will result in a finalized, deliverable version of this paper.

2 Model

First, I describe labor income risk in this setting. Household income (y_t) can be expressed as the following:

$$y_t = p_t \xi_t W_t,$$

where the aggregate wage rate is (W_t), the permanent income component is (p_t), and the transitory shock component is (ξ_t). I assume that the level of permanent income for each household follows a geometric random walk:

$$p_t = p_{t-1} \psi_t,$$

where the white noise permanent shock to income with a mean of one is represented by ψ_t.

[1] Benhabib and Bisin (2018) provide a useful survey of the wealth inequality literature.

The probability of becoming unemployed is $\mho$; in this case, the agent will receive unemployment insurance payments of $\mu > 0$. With probability $1 - \mho$ the agent is employed and tax payments τ_t are collected as insurance for periods of unemployment. Altogether, the transitory component of income is given by:

$$\xi_t = \begin{cases} \mu & \text{with probability } \mho, \\ (1-\tau_t)l\theta_t & \text{with probability } 1-\mho, \end{cases}$$

where l is the time worked per agent and the parameter θ captures the white noise component of the transitory shock.

Next, I present a standard model of household behavior. The sequence of consumption functions $\{c_{t+n}\}_{n=0}^{\infty}$ associated with a household's optimal choice over a lifetime must satisfy:

$$\begin{aligned} v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \not{D} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\ &\text{s.t.} \\ a_t &= m_t - c_t(m_t), \\ k_{t+1} &= \frac{a_t}{\not{D}\psi_{t+1}}, \\ m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1}, \\ a_t &\geq 0, \end{aligned}$$

where I denote a_t as assets, m_t as market resources, k_t as capital, and $\daleth = (1-\delta)$ as the depreciation factor for capital. [2]

2.1 Results

To solve and simulate the model, I follow the calibration scheme captured in table 1.

The solution of the model with no heterogeneity in returns (the R-point model) is the one which finds the value for the rate of return R which minimizes the distance between the simulated and empirical wealth shares at the 20th, 40th, 60th, and 80th percentiles. The empirical targets are computed using the 2004 Survey of Consumer Finances (SCF) data on household wealth. The estimation procedure finds this optimal value to be $\mathsf{R} = 1.0153$.

2.1.1 *Incorporating heterogeneous returns*

Recent studies by Fagereng, Guiso, Malacrino, and Pistaferri (2020) and Bach, Calvet, and Sodini (2018) have not only estimated the rate of return on asset holdings but have also uncovered significant heterogeneity across households. Given this motivation, the

[2] Each of the relevant variables have been normalized by the level of permanent income ($c_t = \frac{C_t}{p_t}$, and so on).

Description	Parameter	Value	Source
Time discount factor	β	0.99	Den Haan, Judd, and Juillard (2010)
CRRA	ρ	1	Den Haan, Judd, and Juillard (2010)
Capital share	α	0.36	Den Haan, Judd, and Juillard (2010)
Depreciation rate	δ	0.025	Den Haan, Judd, and Juillard (2010)
Time worked per employee	ℓ	1/.09	Den Haan, Judd, and Juillard (2010)
Capital/output ratio	$\frac{K}{Y}$	10.26	Den Haan, Judd, and Juillard (2010)
Effective interest rate	$r - \delta$	0.01	Den Haan, Judd, and Juillard (2010)
Wage rate	W	2.37	Den Haan, Judd, and Juillard (2010)
Unempl. insurance payment	μ	0.15	Den Haan, Judd, and Juillard (2010)
Probability of death	D	0.00625	Yields 40-year working life
Variance of $\log \theta_{t,i}$	σ_θ^2	0.010 x 4	Carroll (1992), Carroll, Slacalek, and Tokuoka (2015)
Variance of $\log \psi_{t,i}$	σ_ψ^2	0.010 x 4/11	Carroll (1992), Debacker, Heim, Panousi, Ramnath, and Vidangos (2013), Carroll, Slacalek, and Tokuoka (2015)
Unemployment rate	$\mho$	0.07	Mean in Den Haan, Judd, and Juillard (2010)

Table 1 Parameter values (quarterly frequency) for the perpetual youth (infinite horizon) model.

next estimation (the R-dist model) assumes the existence of multiple types of agents, each earning a distinct rate of return on their assets.

Specifically, I assume that different types of households have a time preference factor drawn uniformly from the interval $(\grave{\mathsf{R}} - \nabla, \grave{\mathsf{R}} + \nabla)$, where ∇ represents the level of dispersion. Afterward, the model is simulated to estimate the values of both $\grave{\mathsf{R}}$ and ∇ so that the model matches the inequality in the wealth distribution. To achieve this, the following minimization problem is solved:

$$\{\grave{\mathsf{R}}, \nabla\} = \arg\min_{\mathsf{R},\nabla} \left(\sum_{i=20,40,60,80} (w_i(\mathsf{R}, \nabla) - \omega_i)^2 \right)^{\frac{1}{2}}$$

subject to the constraint that the aggregate capital-to-output ratio in this model matches that of the perfect foresight setting:

$$\frac{K}{Y} = \frac{K_{PF}}{Y_{PF}}.$$

Note that w_i and ω_i give the porportion of total aggregate net worth held by the top i percent in the model and in the data, respectively.

The estimation procedure finds this optimal values of $\mathsf{R} = 1.0106$ and $\nabla = 0.0112$. The performance of the estimation of both the R-point and R-dist models, measured by their ability to match the SCF data, is compared in figure 1.

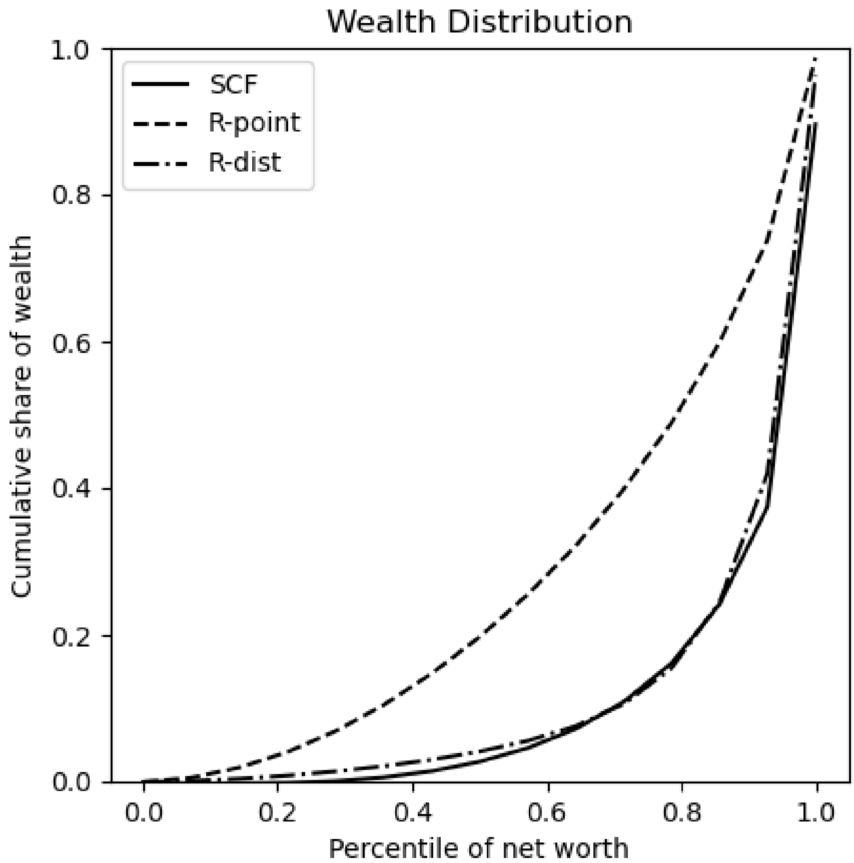

Figure 1 Perpetual youth lorenz curve v.s. data

3 Incorporating life cycle dynamics into the model

More realistic assumptions regarding the age and education level of households can have important implications for the income and mortality process of households. Here, I extend the model to incorporate these life cycle dynamics.

Households enter the economy at time t aged 24 years old and are endowed with an education level $e \in \{D, HS, C\}$, and initial permanent income level $\mathbf{p}_0$, and a capital stock k_0. The life cycle version of household income is given by:

$$y_t = \xi_t \mathbf{p}_t = (1-\tau)\theta_t \mathbf{p}_t,$$

where $\mathbf{p}_t = \psi_t \bar{\psi}_{es} \mathbf{p}_{t-1}$ and $\bar{\psi}_{es}$ captures the age-education-specific average growth factor. Households that have lived for s periods have permanent shocks drawn from a lognormal distribution with mean 1 and variance $\sigma^2_{\psi s}$ and transitory shocks drawn from a lognormal distribution with mean $\frac{1}{\not\mho}$ and variance $\sigma^2_{\theta s}$ with probability $\not\mho = (1 - \mho)$ and μ with probability $\mho$.

The normalized version of the age-education-specific consumption-saving problem for households is given by

$$
\begin{aligned}
v_{es}(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \cancel{D}_{es} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v_{es+1}(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t, \\
k_{t+1} &= \frac{a_t}{\psi_{t+1}}, \\
m_{t+1} &= (\daleth + r_t) k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}
$$

3.1 Results

The additional parameters necessary to calibrate the life cycle version of the model are given in table 2.

Description	Parameter	Value
Population growth rate	N	0.0025
Technological growth rate	Γ	0.0037
Rate of high school dropouts	θ_D	0.11
Rate of high school graduates	θ_{HS}	0.55
Rate of college graduates	θ_C	.34
Average initial permanent income, dropout	$\bar{\mathbf{p}}_{D0}$	5000
Average initial permanent income, high school	$\bar{\mathbf{p}}_{HS0}$	7500
Average initial permanent income, college	$\bar{\mathbf{p}}_{C0}$	12000
Unempl. insurance payment	μ	0.15
Labor income tax rate	τ	0.0942

Table 2 Parameter values (quarterly frequency) for the life cycle model.

The estimation procedure finds this optimal value to be $\mathsf{R} = 1.0078$ for the R-point model in this setting. The estimation procedure for the R-dist model in the life cycle setting finds optimal values of $\mathsf{R} = 1.0005$ and $\nabla = 0.01836$. Notice the improved performance of the estimation in matching the data displayed in figure 2.

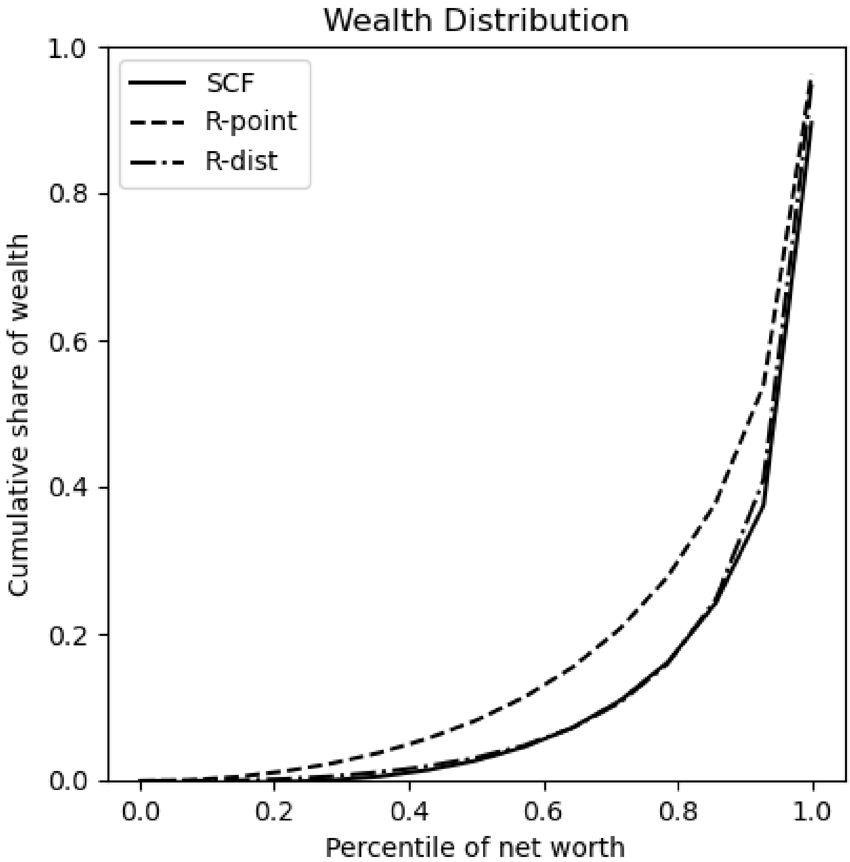

Figure 2 Life cycle lorenz curve v.s. data

4 Extensions

This concludes the preliminary results of this work. From here on, I discuss actionable extensions of the model which are of high priority to be completed.

4.1 Incorporating bequest motives

The desire to leave bequests is thought to be an important reason for households to save, especially those at the top end of the wealth distribution. More generally, the following specification of additively separable wealth in the utility function[3] extends the model to accomodate these other reasons to accumulate assets:

[3]Alternative specifications, such as a non-separable utility function of consumption and wealth, may also be explored in this setting.

$$u(c_t, a_t) = \frac{c_t^{1-\rho}}{1-\rho} + \kappa \frac{(a_t - \underline{\mathrm{a}})^{1-\Sigma}}{1-\Sigma}.$$

Straub (2019) provides calibration values for κ and $\underline{\mathrm{a}}$ and estimation for the elasticity parameters. However, I will need if additional parameters should be estimated and what corresponding empirical moments of the data will be needed for identification.

4.2 Incorporating portfolio choice

Portfolio choice is also an important feature of the consumption-saving problem of households not currently present in the model. Denoting the gross return on the risky asset as $\mathcal{R}_{t+1}$ and the proportion of the porfolio invested in the risky asset as ς_t, the revised maximization problem is

$$\begin{aligned}
v(m_t) &= \max_{c_t, \varsigma_t} u(c_t, a_t) + \beta \not{D} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t(m_t), \\
k_{t+1} &= \frac{a_t}{\not{D}\psi_{t+1}}, \\
\mathbb{R}_{t+1} &= \mathsf{R}_{t+1} = (\mathcal{R}_{t+1} - \mathsf{R}_{t+1})\varsigma_t \\
m_{t+1} &= (\mathbb{R} - \delta)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}$$

where $\mathbb{R}$ denotes the overall return on the portfolio across periods.[4]

4.3 Financial literacy and trust

Deuflhard, Georgarakos, and Inderst (2018) Lusardi and Mitchell (2014)
Guiso, Sapienza, and Zingales (2008) Butler, Giuliano, and Guiso (2016)

[4]The perpetual youth setting is provided for simplicity. It is straightforward to allow for portfolio choice in the life cycle setting.

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