1 Introduction

The recent heterogeneous agent (HA) macro literature has managed to construct models that are microeconomically realistic in terms of household financial choices. Such models, either in an infinite horizon context or using a life cycle specification, are able to match measures of wealth inequality by assuming heterogeneity in time preference rates across agents (Carroll, Slacalek, Tokuoka, and White (2017)).

This literature makes the traditional assumption that all households earn the same rate of return for publicly available assets (like bank accounts or stock investments). But newly available estimates from Norwegian registry data find that, in fact, there are large differences across households in rates of return even within narrowly defined categories of assets (Fagereng, Guiso, Malacrino, and Pistaferri (2020)).

My research agenda is to understand both the consequences of these differences in rates of return, and their causes. Work that I've accomplished so far has found that when heterogeneity in rates of return consistent with the Norwegian data are substituted for the usual assumption of homogeneous rates of return, time preference heterogeneity is no longer necessary for such models to match the observed degree of inequality (in either the infinite horizon or the life cycle specification of the model).

2 Model

My model of labor income risk is standard: it has transitory and permanent shocks calibrated with standard datasets like the PSID. Specifically, household income (y_t) can be expressed as the following:

$$y_t = p_t \xi_t W_t,$$

where the aggregate wage rate is (W_t), the permanent income component is (p_t), and the transitory shock component is (ξ_t). I assume that the level of permanent income for each household follows a geometric random walk:

$$p_t = p_{t-1} \psi_t,$$

where the white noise permanent shock to income with a mean of one is represented by ψ_t.

The probability of becoming unemployed is $\mho$; in this case, the agent will receive unemployment insurance payments of $\mu > 0$. With probability $1 - \mho$ the agent is employed and tax payments τ_t are collected as insurance for periods of unemployment. Altogether, the transitory component of income is given by:

$$\xi_t = \begin{cases} \mu & \text{with probability } \mho, \\ (1 - \tau_t) l \theta_t & \text{with probability } 1 - \mho, \end{cases}$$

where l is the time worked per agent and the parameter θ captures the white noise component of the transitory shock.

Next, I present a standard model of household behavior. The sequence of consumption functions $\{c_{t+n}\}_{n=0}^{\infty}$ associated with a household's optimal choice over a lifetime must satisfy:

$$\begin{aligned}
v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \not{D} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t(m_t), \\
k_{t+1} &= \frac{a_t}{\not{D}\psi_{t+1}}, \\
m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0,
\end{aligned}$$

where I denote a_t as assets, m_t as market resources, k_t as capital, and $\daleth = (1-\delta)$ as the depreciation factor for capital (see Carroll (2019) for a theoretical exposition of consumption behavior in the infinite horizon setting). [1]

2.1 Results

To solve and simulate the model, I follow the calibration scheme captured in table 1.

Description	Parameter	Value	Source
Time discount factor	β	0.99	Den Haan, Judd, and Juillard (2010)
CRRA	ρ	1	Den Haan, Judd, and Juillard (2010)
Capital share	α	0.36	Den Haan, Judd, and Juillard (2010)
Depreciation rate	δ	0.025	Den Haan, Judd, and Juillard (2010)
Time worked per employee	ℓ	1/.09	Den Haan, Judd, and Juillard (2010)
Capital/output ratio	$\frac{K}{Y}$	10.26	Den Haan, Judd, and Juillard (2010)
Effective interest rate	$r-\delta$	0.01	Den Haan, Judd, and Juillard (2010)
Wage rate	W	2.37	Den Haan, Judd, and Juillard (2010)
Unempl. insurance payment	μ	0.15	Den Haan, Judd, and Juillard (2010)
Probability of death	D	0.00625	Yields 40-year working life
Variance of $\log \theta_{t,i}$	σ_θ^2	0.010 x 4	Carroll (1992), Carroll, Slacalek, and Tokuoka (2015)
Variance of $\log \psi_{t,i}$	σ_ψ^2	0.010 x 4/11	Carroll (1992), Debacker, Heim, Panousi, Ramnath, and Vidangos (2013), Carroll, Slacalek, and Tokuoka (2015)
Unemployment rate	$\mho$	0.07	Mean in Den Haan, Judd, and Juillard (2010)

Table 1 Parameter values (quarterly frequency) for the perpetual youth (infinite horizon) model.

The solution of the model with no heterogeneity in returns (the R-point model) is the one which finds the value for the rate of return R which minimizes the distance between

[1] Each of the relevant variables have been normalized by the level of permanent income ($c_t = \frac{C_t}{p_t}$, and so on).

the simulated and empirical wealth shares at the 20th, 40th, 60th, and 80th percentiles. The empirical targets are computed using the 2004 SCF data on household wealth. The estimation procedure finds this optimal value to be $\mathsf{R} = 1.0709$.

2.1.1 Incorporating heterogeneous returns

As noted above, recent studies by Fagereng, Guiso, Malacrino, and Pistaferri (2020) and Bach, Calvet, and Sodini (2018) have not only estimated the rate of return on asset holdings but have also uncovered significant heterogeneity across households. With this in mind, the next estimation (the R-dist model) assumes the existence of multiple types of agents, each earning a distinct rate of return on their assets.

I follow closely the procedure outlined by Carroll, Slacalek, Tokuoka, and White (2017). Specifically, I assume that different types of households have a time preference factor drawn from a uniform distribution on the interval $(\grave{\mathsf{R}} - \nabla, \grave{\mathsf{R}} + \nabla)$, where ∇ represents the level of dispersion. Afterward, the model is simulated to estimate the values of both $\grave{\mathsf{R}}$ and ∇ so that the model matches the inequality in the wealth distribution. To achieve this, the following minimization problem is solved:

$$\{\grave{\mathsf{R}}, \nabla\} = \arg\min_{\mathsf{R},\nabla} \left(\sum_{i=20,40,60,80} (w_i(\mathsf{R}, \nabla) - \omega_i)^2 \right)^{\frac{1}{2}}$$

subject to the constraint that the aggregate capital-to-output ratio in this model matches the calibrated value from the previous table of 10.26.

Note that w_i and ω_i give the porportion of total aggregate net worth held by the top i percent in the model and in the data, respectively.

The estimation procedure finds this optimal values of $\mathsf{R} = 1.0546$ and $\nabla = 0.0368$. The performance of the estimation of both the R-point and R-dist models, measured by their ability to match the SCF data, is compared in figure 1.

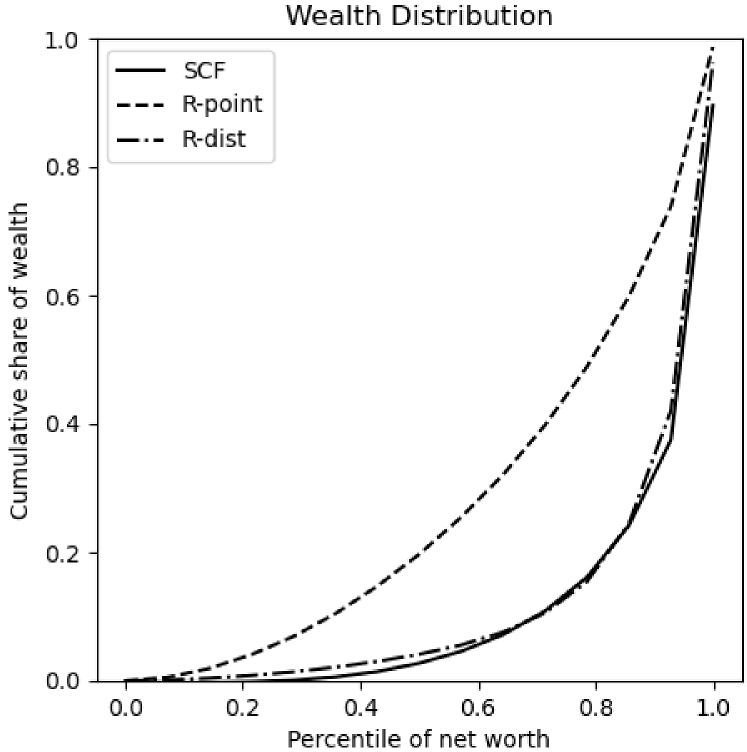

Figure 1 Perpetual youth lorenz curve v.s. data

3 Incorporating life cycle dynamics into the model

More realistic assumptions regarding the age and education level of households can have important implications for the income and mortality process of households. Here, I extend the model to incorporate these life cycle dynamics.

Households enter the economy at time t aged 24 years old and are endowed with an education level $e \in \{D, HS, C\}$, and initial permanent income level $\mathbf{p}_0$, and a capital stock k_0. The life cycle version of household income is given by:

$$y_t = \xi_t \mathbf{p}_t = (1 - \tau)\theta_t \mathbf{p}_t,$$

where $\mathbf{p}_t = \psi_t \bar{\psi}_{es} \mathbf{p}_{t-1}$ and $\bar{\psi}_{es}$ captures the age-education-specific average growth factor. Households that have lived for s periods have permanent shocks drawn from a lognormal distribution with mean 1 and variance $\sigma^2_{\psi s}$ and transitory shocks drawn from a lognormal distribution with mean $\frac{1}{\not\mho}$ and variance $\sigma^2_{\theta s}$ with probability $\not\mho = (1 - \mho)$ and μ with probability $\mho$.

The normalized version of the age-education-specific consumption-saving problem for households is given by

$$
\begin{aligned}
v_{es}(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \cancel{D}_{es} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v_{es+1}(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t, \\
k_{t+1} &= \frac{a_t}{\psi_{t+1}}, \\
m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}
$$

3.1 Results

The additional parameters necessary to calibrate the life cycle version of the model are given in table 2.

Description	Parameter	Value
Population growth rate	N	0.0025
Technological growth rate	Γ	0.0037
Rate of high school dropouts	θ_D	0.11
Rate of high school graduates	θ_{HS}	0.55
Rate of college graduates	θ_C	.34
Average initial permanent income, dropout	$\bar{\mathbf{p}}_{D0}$	5000
Average initial permanent income, high school	$\bar{\mathbf{p}}_{HS0}$	7500
Average initial permanent income, college	$\bar{\mathbf{p}}_{C0}$	12000
Unempl. insurance payment	μ	0.15
Labor income tax rate	τ	0.0942

Table 2 Parameter values (quarterly frequency) for the life cycle model.

The estimation procedure finds this optimal value to be $\mathsf{R} = 1.0626$ for the R-point model in this setting. The estimation procedure for the R-dist model in the life cycle setting finds optimal values of $\mathsf{R} = 1.0395$ and $\nabla = 0.0737$. Notice the improved performance of the estimation in matching the data displayed in figure 2.

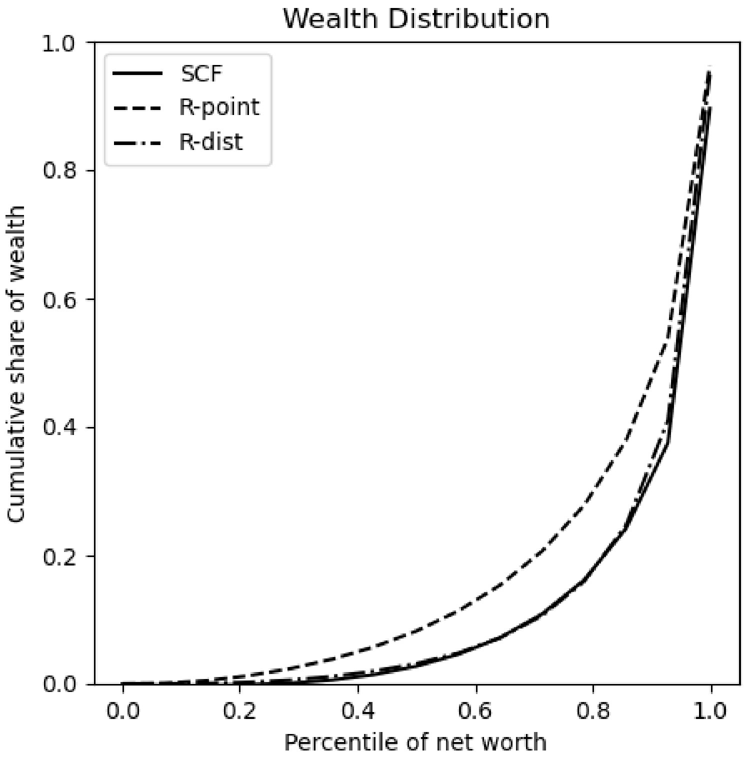

Figure 2 Life cycle lorenz curve v.s. data

4 Robustness Checks

The results in the previous section depend on the specified parameter values in each of the tables above. The next step for this project is to see how much my estimates for the optimal rate of return (with and without heterogeneity) changes for other plausible values for other parameters of interest to the consumption-saving problem. For example, empirical estimates of the time preference factor are known to vary substantially across methodologies used, but can typically be found in the range of 1% to 10%. Estimates of the CRRA factor in similar frameworks are generally between 1 and 4.

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