Dissertation Preproposal:

Heterogeneous Rates of Return and the Distribution of Wealth

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Decory Edwards[1]

Inequality in the distribution of wealth is a notable topic of interest both in the mainstream and in the economics literature. Reliable measurements of the wealth holdings of households, obtained either through surveys or using capitalization methods on administrative tax data, suggest that the wealth distribution is both highly skewed and exhibit heavy (Pareto) upper tails. Benhabib and Bisin (2018) provide an excellent survey on both historical thought and theoretical explanations for stationary distributions of wealth with substantial skewness and heavy tails. Not surprisingly, the key arguments posited there also make their way into the standard consumption-saving problems which are a cornerstone of models prevalent in the heterogeneous agent (HA) macroeconomics literature. At the same time, recent evidence of heterogeneity in the rate of return for individuals provide motivation for an analogous assumption in the HA framework where a stationary, model distribution of wealth exists. In this way, one can test the theory regarding a meaningful relationship between stochastic returns and wealth inequality within a standard macroeconomic setting, where the direct effect of the returns to savings on the wealth accumulation process can be understood through its effect on the optimal behavior of households. A uniform distribution of the rate of return across households is estimated such that empirical moments of wealth (net worth) measured in the 2004 survey of consumer finances SCF are matched particularly well. In an effort to compare the structurally estimated distribution of returns from the model to the empirical distribution measured by Fagereng, Guiso, Malacrino, and Pistaferri (2020), a lognormal distribution across households is assumed. Not only does the lognormal assumption allow for the simulated moments to better fit the empirical moments for net worth, but the estimated distribution of returns is closer to its empirical counterpart as well.

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[1]Edwards: Department of Economics, Johns Hopkins University, dedwar65@jhu.edu,

1 Introduction

The unequal distribution of wealth, an extensively documented phenomenon in numerous countries, has not only endured over time but also intensified in recent years. This point is stressed in a recent article from the Institute for Policy Studies (IPS), which revealed that in 2018, the total wealth of the poorest half of Americans was eclipsed by the combined wealth of the three wealthiest men in the nation. Additionally, the IPS report notes that the combined wealth of the top five richest men on this list skyrocketed by a staggering 123% from March 2020 to October 2021[1].

Wealth inequality has also been a subject of considerable interest throughout history in various academic fields. The statistics literature, for instance, focused on linking the distribution of income to the observable skewness in wealth distribution. The economics literature went further by establishing microfoundations for individual wealth outcomes.

More recently, the macroeconomics literature on inequality has seen significant growth, with the distribution of wealth among households offering insight into how the economy as a whole responds to aggregate fiscal shocks. This has been accomplished in large part due to the widespread adoption of models that abandon the traditional representative agent assumption. Specifically, a model that studies the equilibrium outcomes of an economy composed of individual decision-makers using a single aggregate agent can only have one marginal propensity to consume (MPC). As a result, in response to an aggregate fiscal shock, all households would respond similarly to a one-time stimulus check, which does not align with what transpired during the pandemic[2].

The first departure from the representative agent framework entails positing an exogenously determined income process that generates a distribution of income among households. One can further assume that individuals face some level of potential unemployment in each period, creating a precautionary savings motive for consumers. The availability of a riskless asset that partially insures against this income risk results in households choosing to hold different levels of market resources optimally. Krusell and Smith (1998)'s seminal work suggests that models assuming heterogeneity in individual income perform well in matching the aggregate capital stock but poorly in matching the distribution of wealth.

The next step towards moving beyond the standard representative agent framework is to assume greater heterogeneity among households, leading more households to optimally hold lower levels of wealth. Carroll, Slacalek, Tokuoka, and White (2017) accomplish this by performing a structural estimation of *ex-ante* heterogeneity amongst households which allows for the model's distribution of wealth to closely match the 2004 Survey

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[1]See Inequality.org articles data November 21, 2022: "Wealth Inequality in the United States" and "Updates: Billionaire Wealth, U.S. Job Losses and Pandemic Profiteers" (date accessed: March 27, 2023)

[2]Parker, Schild, Erhard, and Johnson (2022) note that "In sum, while on average the [economic impact payments] EIPs appear to have gone to many households with incomes that were unharmed by the pandemic, some of the EIPs, mainly in the first round, did support short-term spending for some households, primarily those with low ex ante liquid wealth and those reliant on income that could not be earned by working from home."

of Consumer Finances (SCF) data on household wealth. Specifically, the authors assume different agents have different rates of time preference; this assumption of modest heterogeneity in time preferences is sufficient to match both the shape and skewness of the empirical distribution of wealth. Furthermore, while traditional representative agent models generate an aggregate marginal propensity to consume between 0.02 and 0.04, the β-dist model generates an aggregate MPC between 0.2 and 0.4. This range falls within the values estimated across households in the data.

It is worth noting that the time preference factor (β) is one of the key parameters that influences an individual's equilibrium target level of market resources, but it is not directly observable. On the other hand, estimating differences in the rate of return to financial assets across households is possible, as this variable *is* directly observable. Empirical research has been conducted to estimate such differences, with a recent example being the work of Fagereng, Guiso, Malacrino, and Pistaferri (2020). They analyzed 12 years of administrative tax records on capital income and wealth stock for all taxpayers in Norway from 2004-2015 to estimate these rates of return.

This paper aims to contribute the computational HA framework by performing a similar structural estimation exercise regarding ex-ante heterogeneity in the rate of return and comparing it to the recent, motivating empirical evidence on disparities in rates of return among households. Since the rate of return has a direct role in the wealth accumulation process, differences in returns to assets across households may be a compelling explanation for wealth inequality.

2 Model

2.1 Defining the stochastic income process

Each household's income (y_t) during a given period depends on three main factors. The first factor is the aggregate wage rate (W_t) that all households in the economy face. The second factor is the permanent income component (p_t), which represents an agent's present discounted value of human wealth. Lastly, the transitory shock component (ξ_t) reflects the potential risks that households may face in receiving their income payment during that period. Thus, household income can be expressed as the following:

$$y_t = p_t \xi_t W_t.$$

I assume that the level of permanent income for each household follows a geometric random walk:

$$p_t = p_{t-1} \psi_t,$$

The white noise permanent shock to income with a mean of one is represented by ψ_t, which is a significant component of household income. The probability of receiving income during a given period is determined by the transitory component, which is modeled to reflect the potential risks associated with becoming unemployed. Specifically,

if the probability of becoming unemployed is $\mho$, the agent will receive unemployment insurance payments of $\mu > 0$. On the other hand, if the agent is employed, which occurs with a probability of $1 - \mho$, the model allows for tax payments τ_t to be collected as insurance for periods of unemployment. The transitory component is then represented as:

$$
\xi_t = \begin{cases} \mu & \text{with probability } \mho, \\ (1 - \tau_t)l\theta_t & \text{with probability } 1 - \mho, \end{cases}
$$

where l is the time worked per agent and the parameter θ captures the white noise component of the transitory shock.

2.2 Baseline model for households

In the baseline version of the household's optimization problem for consumption-savings decisions each household aims to maximize its expected discounted utility of consumption $u(c) = \frac{c^{1-\rho}}{1-\rho}$ by solving the following:

$$
\max \mathbb{E}_t \sum_{n=0}^{\infty} (\not{D}\beta)^n u(c_{t+n}).
$$

The sequence of consumption functions $\{c_{t+n}\}_{n=0}^{\infty}$ associated with a household's optimal choice over a lifetime must satisfy[3]

$$
\begin{aligned}
v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \not{D} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t(m_t), \\
k_{t+1} &= \frac{a_t}{\not{D}\psi_{t+1}}, \\
m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}
$$

3 Results

To solve and simulate the model, I follow the calibration scheme captured in table 1.

3.1 The model without heterogeneity

The solution of the model with no heterogeneity in returns (referred to as the R-point model) is the one which finds the value for the rate of return R which minimizes the

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[3]Here, each of the relevant variables have been normalized by the level of permanent income ($c_t = \frac{C_t}{p_t}$, and so on). This is the standard state-space reduction of the problem for numerical tractibility.

Description	Parameter	Value	Source
Time discount factor	β	0.99	Den Haan, Judd, and Juillard (2010)
CRRA	ρ	1	Den Haan, Judd, and Juillard (2010)
Capital share	α	0.36	Den Haan, Judd, and Juillard (2010)
Depreciation rate	δ	0.025	Den Haan, Judd, and Juillard (2010)
Time worked per employee	ℓ	1/.09	Den Haan, Judd, and Juillard (2010)
Capital/output ratio	$\frac{K}{Y}$	10.26	Den Haan, Judd, and Juillard (2010)
Effective interest rate	$r - \delta$	0.01	Den Haan, Judd, and Juillard (2010)
Wage rate	W	2.37	Den Haan, Judd, and Juillard (2010)
Unempl. insurance payment	μ	0.15	Den Haan, Judd, and Juillard (2010)
Probability of death	D	0.00625	Yields 40-year working life
Variance of $\log \theta_{t,i}$	σ_θ^2	0.010 x 4	Carroll (1992), Carroll, Slacalek, and Tokuoka (2015)
Variance of $\log \psi_{t,i}$	σ_ψ^2	0.010 x 4/11	Carroll (1992), Debacker, Heim, Panousi, Ramnath, and Vidangos (2013), Carroll, Slacalek, and Tokuoka (2015)
Unemployment rate	$\mho$	0.07	Mean in Den Haan, Judd, and Juillard (2010)

Table 1   Parameter values (quarterly frequency) for the perpetual youth (infinite horizon) model.

distance between the simulated and empirical wealth shares at the 20th, 40th, 60th, and 80th percentiles of the corresponding wealth distribution. The estimation procedure finds this optimal value to be $\mathsf{R} = 1.0153$.

3.2 Incorporating heterogeneous returns

Recent studies by Fagereng, Guiso, Malacrino, and Pistaferri (2020) and Bach, Calvet, and Sodini (2018) have not only estimated the rate of return on asset holdings but have also uncovered significant heterogeneity across households. Given this motivation, the revised model assumes the existence of multiple types of agents, each earning a distinct rate of return on their assets.

Specifically, I assume that different types of households have a time preference factor drawn uniformly from the interval $(\grave{\mathsf{R}} - \nabla, \grave{\mathsf{R}} + \nabla)$, where ∇ represents the level of dispersion. Afterward, the model is simulated to estimate the values of both $\grave{\mathsf{R}}$ and ∇ so that the model matches the inequality in the wealth distribution. To achieve this, the following minimization problem is solved:

$$\{\grave{\mathsf{R}}, \nabla\} = \arg \min_{\mathsf{R}, \nabla} \left(\sum_{i=20,40,60,80} \left(w_i(\mathsf{R}, \nabla) - \omega_i \right)^2 \right)^{\frac{1}{2}}$$

subject to the constraint that the aggregate capital-to-output ratio in this model matches that of the perfect foresight setting:

$$\frac{K}{Y} = \frac{K_{PF}}{Y_{PF}}.$$

4

Note that w_i and ω_i give the porportion of total aggregate net worth held by the top i percent in the model and in the data, respectively.

The estimation procedure finds this optimal values of $\mathsf{R} = 1.0106$ and $\nabla = 0.0112$. The performance of the estimation of both the R-point and R-dist models, measured by their ability to match the 2004 SCF wealth data, is compared in figure 1.
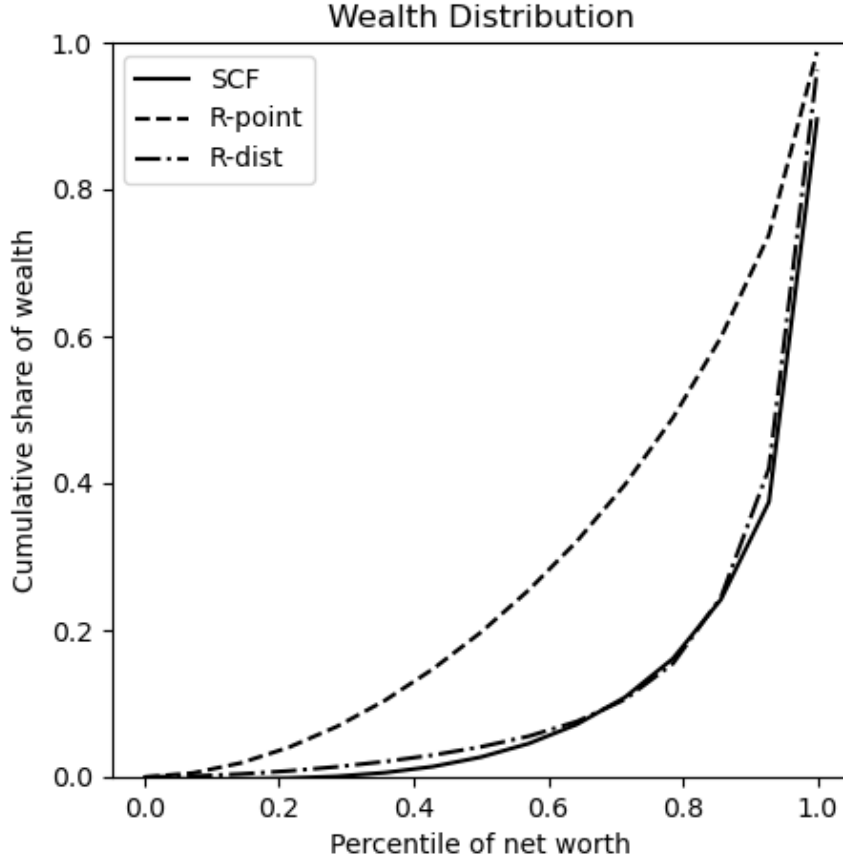
Figure 1 Perpetual youth lorenz curve v.s. data

4 Incorporating life cycle dynamics into the model

The case where households solve the infinite horizon version of the consumption-saving problem is an interesting limiting case. However, more realistic assumptions regarding the age and education level of households can have important implications for the income and mortality process of households. Here, we extend the model to incorporate these assumptions.

Households enter the economy at time t aged 24 years old and are endowed with an

education level $e \in \{D, HS, C\}$, and initial permanent income level $\mathbf{p}_0$, and a capital stock k_0. The life cycle version of household income can be expressed as the following:

$$y_t = \xi_t \mathbf{p}_t = (1 - \tau)\theta_t \mathbf{p}_t,$$

where $\mathbf{p}_t = \psi_t \bar{\psi}_{es} \mathbf{p}_{t-1}$ and $\bar{\psi}_{es}$ captures the age-education-specific average growth factor. Households that have lived for s periods have permanent shocks drawn from a lognormal distribution with mean 1 and variance $\sigma_{\psi s}^2$ and transitory shocks drawn from a lognormal distribution with mean $\frac{1}{\mathcal{B}}$ and variance $\sigma_{\theta s}^2$ with probability $\mathcal{B} = (1 - \mathcal{U})$ and μ with probability $\mathcal{U}$.

The normalized version of the age-education-specific consumption-saving problem for households is given by

$$
\begin{aligned}
v_{es}(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \mathcal{D}_{es}\mathbb{E}_t[\psi_{t+1}^{1-\rho}v_{es+1}(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t, \\
k_{t+1} &= \frac{a_t}{\psi_{t+1}}, \\
m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}
$$

5 Results for the life cycle model

The additional parameters necessary to calibrate the life cycle version of the model are given in table 2.

Description	Parameter	Value
Population growth rate	N	0.0025
Technological growth rate	Γ	0.0037
Rate of high school dropouts	θ_D	0.11
Rate of high school graduates	θ_{HS}	0.55
Rate of college graduates	θ_C	.34
Average initial permanent income, dropout	$\bar{\mathbf{p}}_{D0}$	5000
Average initial permanent income, high school	$\bar{\mathbf{p}}_{HS0}$	7500
Average initial permanent income, college	$\bar{\mathbf{p}}_{C0}$	12000
Unempl. insurance payment	μ	0.15
Labor income tax rate	τ	0.0942

Table 2 Parameter values (quarterly frequency) for the life cycle model.

The estimation procedure finds this optimal value to be $\mathsf{R} = 1.0078$ for the R-point model in this setting. The estimation procedure for the R-dist model in the life cycle setting finds optimal values of $\mathsf{R} = 1.0005$ and $\nabla = 0.01836$. Consider the improved

performance of the estimation in matching the 2004 SCF wealth data, which is compared in figure 2.
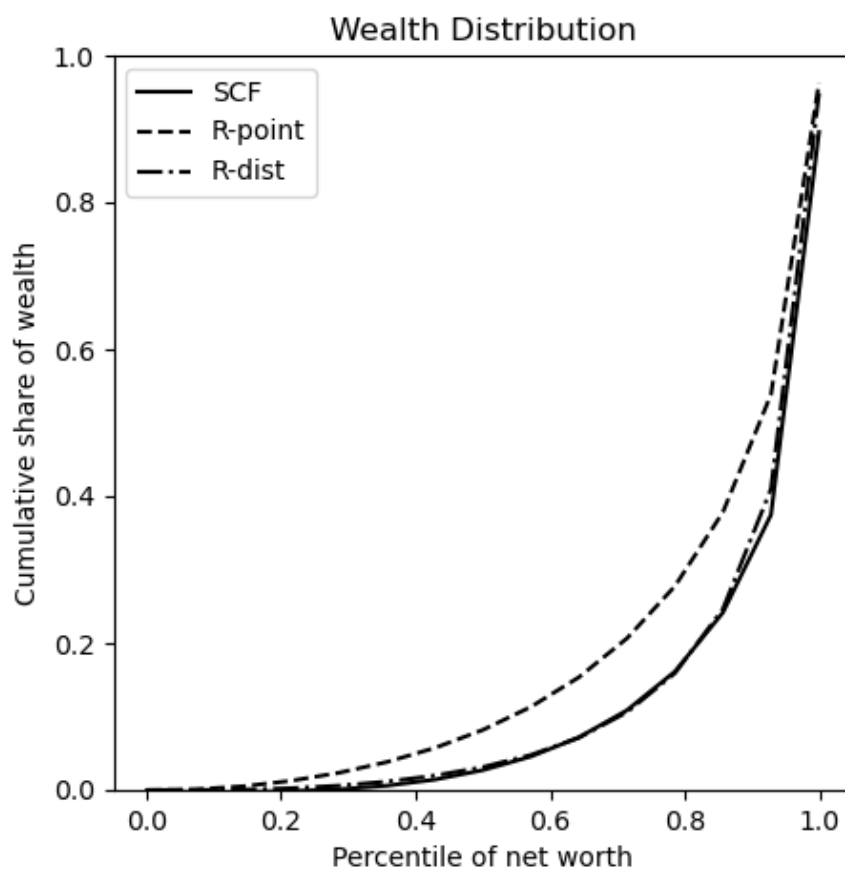
Figure 2   Life cycle lorenz curve v.s. data

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