

1 The Usual Theory, and a Bit More Notation

{sec:the-usual-theor

For reference and to illustrate our new notation, we will now derive the Euler equation and other standard results for the problem described above. Since

$$v_{\rightarrow}(a) \leftrightarrow v_{t\rightarrow}(a) = \beta v_{\leftarrow(t+1)}(a) = \beta \mathbb{E}_{\leftarrow(t+1)}[\mathcal{G}_{t+1}^{1-\rho} v_{t+1}(\overbrace{a(R/\mathcal{G}_{t+1}) + \theta_{t+1}}^{m_{t+1}})], \quad (1)$$

given m_t , the first order condition for (12) with respect to a is

$$\begin{aligned} u^c(m_t - a) = v_{t\rightarrow}^a(a) &= \mathbb{E}_{\leftarrow(t+1)}[\beta \mathcal{R}_{t+1} \mathcal{G}_{t+1}^{1-\rho} v_{t+1}^m(m_{t+1})] \\ &= \mathbb{E}_{\leftarrow(t+1)}[\beta \mathcal{R}_{t+1} \mathcal{G}_{t+1}^{-\rho} v_{t+1}^m(m_{t+1})] \end{aligned} \quad (2) \quad \{\text{eq:upceqEvtpt1}\}$$

and because the **Envelope** theorem tells us that

$$v_t^m(m_t) = \mathbb{E}_{\leftarrow(t+1)}[\beta \mathcal{R} \mathcal{G}_{t+1}^{-\rho} v_{t+1}^m(m_{t+1})] \quad (3) \quad \{\text{eq:envelope}\}$$

we can substitute the LHS of (3) for the RHS of (2) to get

$$u^c(c_t) = v_t^m(m_t) \quad (4) \quad \{\text{eq:upcteqvtp}\}$$

and rolling forward one period,

$$u^c(c_{t+1}) = v_{t+1}^m(a_t \mathcal{R}_{t+1} + \theta_{t+1}) \quad (5) \quad \{\text{eq:upctpt1EqVpxtpt}\}$$

so that substituting the LHS in equation (2) finally gives us the Euler equation for consumption:

$$u^c(c_t) = \mathbb{E}_{t\rightarrow}[\beta \mathcal{R} \mathcal{G}_{t+1}^{-\rho} u^c(c_{t+1})]. \quad (6) \quad \{\text{eq:cEuler}\}$$

We can now restate the problem (12) with our new within-stage notation:

$$v(m) = \max_c u(c) + v_{\rightarrow}(m - c) \quad (7)$$

whose first order condition with respect to c is

$$u^c(c) = v_{\rightarrow}^a(m - c) \quad (8) \quad \{\text{eq:FOCnew}\}$$

which is mathematically equivalent to the usual Euler equation for consumption.

We will revert to this formulation when we reach section 6.8.