1 The Usual Theory, and a Bit More Notation

{sec:the-usual-theor

For reference and to illustrate our new notation, we will now derive the Euler equation and other standard results for the problem described above. Since we can write value as of the end of the consumption stage as a function of a:

$$\mathrm{v}_{\rightharpoondown}(a) := \mathrm{v}_{t_{\rightharpoondown}}(a) := \beta \mathrm{v}_{\leftharpoondown(t+1)}(a) = \beta\,\mathbb{E}_{\leftharpoondown(t+1)}[\mathcal{G}_{t+1}^{1-\rho}\mathrm{v}_{t+1}(\overbrace{a(\mathsf{R}/\mathcal{G}_{t+1})+\theta_{t+1}}^{m_{t+1}})],$$

the first order condition for (12) with respect to a (given m_t) is

$$\begin{aligned} \mathrm{u}^c(m_t - a) = \mathrm{v}^a_{t_{\rightharpoondown}}(a) &= \mathbb{E}_{\leftharpoondown(t+1)}[\beta\mathcal{R}_{t+1}\mathcal{G}_{t+1}^{1-\rho}\mathrm{v}^m_{t+1}(m_{t+1})] \\ &= \mathbb{E}_{\leftharpoondown(t+1)}[\beta\mathsf{R} \quad \mathcal{G}_{t+1}^{-\rho}\mathrm{v}^m_{t+1}(m_{t+1})] \end{aligned} \tag{1}$$

{eq:upceqEvtp1}

and because the `Envelope` theorem tells us that

$$\mathrm{v}^m_t(m_t) = \mathbb{E}_{\leftharpoondown(t+1)}[\beta\mathsf{R}\mathcal{G}_{t+1}^{-\rho}\mathrm{v}^m_{t+1}(m_{t+1})] \tag{2}$$

{eq:envelope}

we can substitute the LHS of (3) for the RHS of (2) to get

$$\mathrm{u}^c(c_t) = \mathrm{v}^m_t(m_t) \tag{3}$$

{eq:upcteqvtp}

and rolling forward one period,

$$\mathrm{u}^c(c_{t+1}) = \mathrm{v}^m_{t+1}(a_t\mathcal{R}_{t+1} + \theta_{t+1}) \tag{4}$$

{eq:upctp1EqVpxtp

so that substituting the LHS in equation (2) finally gives us the Euler equation for consumption:

$$\mathrm{u}^c(c_t) = \mathbb{E}_{t_{\rightharpoondown}}[\beta\mathsf{R}\mathcal{G}_{t+1}^{-\rho}\mathrm{u}^c(c_{t+1})]. \tag{5}$$

{eq:cEuler}

We can now restate the problem (12) with our new within-stage notation:

$$\mathrm{v}(m) = \max_c \ \ \mathrm{u}(c) + \mathrm{v}_{\rightharpoondown}(m-c) \tag{6}$$

whose first order condition with respect to c is

$$\mathrm{u}^c(c) = \mathrm{v}^a_{\rightharpoondown}(m-c) \tag{7}$$

{eq:upEqbetaOp}

which is mathematically equivalent to the usual Euler equation for consumption.

We will revert to this formulation when we reach section 6.8.