1 The Usual Theory, and a Bit More Notation

{sec:the-usual-theor

For reference and to illustrate our new notation, we will now derive the Euler equation and other standard results for the problem described above. Since

$$\mathrm{v}_{\rightharpoondown}(a) := \mathrm{v}_{t\rightharpoondown}(a) := \beta \mathrm{v}_{\leftharpoonup(t+1)}(a) = \beta\,\mathbb{E}_{\leftharpoonup(t+1)}[\mathcal{G}_{t+1}^{1-\rho}\mathrm{v}_{t+1}(\overbrace{a(\mathsf{R}/\mathcal{G}_{t+1}) + \theta_{t+1}}^{m_{t+1}})], \tag{1}$$

given m_t, the first order condition for (12) with respect to a is

$$\begin{aligned} \mathrm{u}^c(m_t - a) = \mathrm{v}^a_{t\rightharpoondown}(a) &= \mathbb{E}_{\leftharpoonup(t+1)}[\beta \mathcal{R}_{t+1}\mathcal{G}_{t+1}^{1-\rho}\mathrm{v}^m_{t+1}(m_{t+1})] \\ &= \mathbb{E}_{\leftharpoonup(t+1)}[\beta \mathsf{R} \quad \mathcal{G}_{t+1}^{-\rho}\mathrm{v}^m_{t+1}(m_{t+1})] \end{aligned} \tag{2}$$ {eq:upceqEvtp1}

and because the Envelope theorem tells us that

$$\mathrm{v}^m_t(m_t) = \mathbb{E}_{\leftharpoonup(t+1)}[\beta \mathsf{R}\mathcal{G}_{t+1}^{-\rho}\mathrm{v}^m_{t+1}(m_{t+1})] \tag{3}$$ {eq:envelope}

we can substitute the LHS of (3) for the RHS of (2) to get

$$\mathrm{u}^c(c_t) = \mathrm{v}^m_t(m_t) \tag{4}$$ {eq:upcteqvtp}

and rolling forward one period,

$$\mathrm{u}^c(c_{t+1}) = \mathrm{v}^m_{t+1}(a_t\mathcal{R}_{t+1} + \theta_{t+1}) \tag{5}$$ {eq:upctp1EqVpxtp

so that substituting the LHS in equation (2) finally gives us the Euler equation for consumption:

$$\mathrm{u}^c(c_t) = \mathbb{E}_{t\rightharpoondown}[\beta \mathsf{R}\mathcal{G}_{t+1}^{-\rho}\mathrm{u}^c(c_{t+1})]. \tag{6}$$ {eq:cEuler}

We can now restate the problem (12) with our new within-stage notation:

$$\mathrm{v}(m) = \max_c \quad \mathrm{u}(c) + \mathrm{v}_{\rightharpoondown}(m-c) \tag{7}$$

whose first order condition with respect to c is

$$\mathrm{u}^c(c) = \mathrm{v}^a_{\rightharpoondown}(m-c) \tag{8}$$ {eq:upEqbetaOp}

which is mathematically equivalent to the usual Euler equation for consumption.

We will revert to this formulation when we reach section 6.8.