Notes on Modeling Bequests

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Many consumption-saving models incorporate a probability of death in the given period as a way to ensure a stationary distribution of wealth. A common accompanying assumption of these models is that, upon death, assets are divided equally amongst all remaining survivors. This receipt of assets is a "surprise", since these assets were accumulated by a previous consumer due to their desire to finance future consumption via saving.

This rationale can be applied to incorporate bequests into the consumption-saving framework. Traditionally, this is done by extending the model to include (i) some overlapping generation component and/or (ii) some component to link current and future households (i.e. parent and descendants). [1]

With this in mind, there are three broadly defined classes of bequest models:

1. Accidental bequests - Households have some number of descendants who, upon death, receive their assets as bequests.

2. Dynastic (or altruistic) models - Households have some number of descendants *whose consumption they care about*. Upon death, those descendants receive assets of their predecessor as bequests.

3. Wealth in utility models - Households have some desire to hold assets *beyond* potentially fianancing their future consumption via saving.

Two surveys which provide a useful way of understanding the different types of bequests models are Laitner 2002 and Cagetti and De Nardi 2008. A further discussion on the implications of these different model assumptions is detailed in Carroll 1998.

1 Accidental Bequests

Not only are these models are natural extension of the consumption-saving framework after incorporating a probablility of death, they are also more consistent with the empirical literature on the nature of bequests. Indeed, there is little

[1] However, the most general specification dealing with bequests only requires this "surprise" event of death in a given period.

support for the idea that households accumulate resources for altrusitic considerations regarding their descendants. Gokhale et al. 2001 provides a thorough discussion of this counter-evidence and states that, in general, the empirical evidence suggests that bequests are "unintended or motivated by non-altruistic considerations".

1.1 Model from Abel 1985

$$\begin{aligned} \max \quad & U(c_1) + (1-p)\delta U(c_2) \\ \text{s.t.} \quad & W = B + Y - T - c_1 \end{aligned} \tag{1}$$

where B is determined by (i) R, W, (ii) the number of children, G, and most importantly (iii) the number of periods the household has survived *before* the terminal period [2] .

1.2 Model from Cagetti and De Nardi 2008

$$\begin{aligned} U = \quad & \mathbb{E}\left\{ \sum_{t=1}^{N} \beta^t \left(\Pi_{j=1}^{t} s_t \right) u(c_t) \right\} \\ V(x,t) = \max_{c,a'} \quad & \{u(c) + \beta s_{t+1} \mathbb{E}[v(x', t+1)|x]\} \\ \text{s.t.} \quad & W = B + Y - T - c_1 \end{aligned} \tag{2}$$

where $x = (a, z)$ and T are accidental bequests left by all deceased in a period, which are *assumed to be redistributed by the government to all people alive.*

2 Dynasitc or altruistic models

Models where current households care about the utility of their future descendants have the distinct implication that the entire dynasty's wealth should matter to the current household making the consumption-saving decision. Thus, there should be evidence of consumption smoothing across the dynasties. When compared to empirical evidence, this prediction does not hold up for most households (though it may be plausible for the extremely wealthy).

2.1 Model from Barro 1974

The utility function of the member of the ith generation has the form

$$U_i = U(c_i^y, c_i^o, U_{i+1}^*) \tag{3}$$

where U_{i+1}^* is the attainable utility of i's immediate descendant.

[2] in which case, they consume everything because they don't care about leaving bequests.

2.2 Model from Cagetti and De Nardi 2006

$$W_r(a) = \max_{c,a'} \quad \{u(c) + \beta\pi_0 \mathbb{E}W_r(a') + \eta\beta(1-\pi_0)\mathbb{E}V(a', y', \theta')\} \tag{4}$$

where η is the weight on the utility of the descendants. If $\eta = 1$, the household behaves as a dynasty.

3 Bequests and wealth in utility function

The most general class of models accomodating bequests are those that assume households receive direct utility from holding assets (i.e. other than the utility earned from being able to finance future consumption with saving today). This includes "warm glow" bequest models as a special case.

3.1 Model from Carroll 1998

$$\begin{aligned} \max_{c_t} \quad & u(c_T) + v(w_{T+1}) \\ \text{s.t.} \quad & w_{T+1} = w_T - c_T \end{aligned} \tag{5}$$

3.2 Model from Cagetti 2003

$$\begin{aligned} \max_{W_{t+1}, C_t} \quad & \mathbb{E}\left(\sum_{t=0}^{\tilde{T}} \beta^t e^{Z_t\theta} \frac{C_t^{1-\gamma}}{1-\gamma} + \beta^{\tilde{T}+1} e^{Z_{\tilde{T}}\theta} S(W_{\tilde{T}+1}) \right) \\ \text{s.t.} \quad & W_{t+1} = RW_t + Y_t - C_t + B_t \end{aligned} \tag{6}$$

where the utility from bequests is specified as

$$S(W) = \alpha \frac{W^{1-\gamma}}{1-\gamma}.$$

3.3 Model from Gourinchas and Parker 2002

$$\begin{aligned} V_\tau(X_\tau, P_\tau, Z_\tau) \quad & = \max_{c_\tau, \ldots, c_T} \left[\sum_{t=\tau}^{T} \beta^{t-\tau} v(Z_t) \frac{C_t^{1-\rho}}{1-\rho} + \beta^{T+1-\tau} \kappa v(Z_{T+1}) (X_{T+1} + hP_{T+1})^{1-\rho} \right] \\ \text{s.t.} \quad & X_{t+1} = R(X_t - C_t) + Y_{t+1} \end{aligned} \tag{7}$$

where X_{T+1} is cash on hand and H_{T+1} is illiquid wealth in the first year of retirement.

Now that we see how the wealth in the utility function enters the recursive formulation of the household's problem, form this point we include only the component of the utility function which captures the preference for wealth holdings beyond financing future consumption.

3.4 Model from De Nardi 2004

$$\phi(b) = \phi_1 \left(1 + \frac{b}{\phi_2}\right)^{1-\sigma} \tag{8}$$

where ϕ_1 reflects the parent's concern about leaving bequests and ϕ_2 measures the extent to which bequests are luxury goods.

3.5 Model from Dynan, Skinner, and Zeldes 2004

$$V(B_{is}) = \frac{\mu[(B_{is} + YL_{is}^c)^{1-\gamma} - 1]}{1-\gamma} \tag{9}$$

where μ is the trade-off parameter between own consumption and bequests B_{is} and YL_{is}^c is the value of the next generation's lifetime earnings.

3.6 Model from De Nardi and Yang 2016

$$\phi(b) = \phi_1 \left[(b + \phi_2)^{1-\gamma} - 1\right] \tag{10}$$

where ϕ_1 measures the strength of bequest motivess and ϕ_2 measures the extent to which bequests are luxury goods. When $\phi_2 > 0$, the marginal utility of small bequests is bounded.

3.7 Model from Saez and Stantcheva 2018

$$U_i(c_i, k_i, z_i) = c_i + a_i(k_i) - h_i(z_i) + \delta_i(k_i^{init} - k_i) \tag{11}$$

where utility for may be specified as

$$a_i(k) = \eta_i \frac{k_i^\gamma}{\gamma}$$

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3.8 Model from Straub et al. 2019

$$U(a) = \frac{(\frac{a}{o})^{1-\Sigma} - 1}{1-\Sigma} \tag{12}$$

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