Chp3Problems

September 21, 2023

0.1 Problems and applications

1. Using the neoclassical theory of distribution (of income),

- The labor force increases. Given that MPL is decreasing in L and $MPL = \frac{W}{P}$, this corresponds to a fall in the real wage. Since $F(K, L)$ is increasing in K, L, and increase in L increases the real rental rate of capital.
- The capital stock falls. Then $MPK \uparrow \implies \frac{R}{P} \uparrow$. Further, $F(K, L) \downarrow \implies MPL \downarrow \implies \frac{W}{P}$.
- Under Cobb-douglas, $Y = AK^{\alpha}L^{1-\alpha}$. MPL and MPK both increasing in A. So $A \uparrow \implies \frac{W}{P}, \frac{R}{P}$.

2. Consider the Cobb-douglas function $Y = AK^{.5}L^{.5}$, $\overline{K} = 100, \overline{L} = 100$.

- $\overline{Y} = F(\overline{K}, \overline{L}) = (100)^{\frac{1}{2}} * (100)^{\frac{1}{2}} = 10 * 10 = 100$ units of output.
- $MPL = (1-\alpha)\frac{Y}{L} = (.5)\frac{100}{100} = .5$

  $MPK = (\alpha)\frac{Y}{K} = (.5)\frac{100}{100} = .5$
- Labor income $= L * MPL = (1-\alpha) * Y = (.5)(100) = 50$.

  Then, the share of total output that labor receives is $\frac{50}{100} = .5$.
- Suppose that labor falls to $L' = 50$. Then, $Y' = F(\overline{K}, L') = (100)^{\frac{1}{2}} * (50)^{\frac{1}{2}} = 50\sqrt{2}$.
- $MPL = (1-\alpha)\frac{Y}{L} = (.5)\frac{50\sqrt{2}}{50} = (\frac{1}{2})\sqrt{2} \approx .71$

  $MPK = (\alpha)\frac{Y}{K} = (.5)\frac{50\sqrt{2}}{100} = (\frac{1}{2})\frac{\sqrt{2}}{2} \approx .35$
- $L * MPL = (1-\alpha) * Y = (.5)(50\sqrt{2}) =\approx 35.36$

  Then, the share of total output that labor receives is $\frac{\frac{1}{2}50\sqrt{2}}{50\sqrt{2}} = .5$.

4. When $\alpha = .3$, then $\overline{Y} = A\overline{K}^{.3}\overline{L}^{.7}$.

- $\frac{MPK*K}{Y} = \frac{\alpha*Y}{Y} = \alpha$

  $\frac{MPL*L}{Y} = \frac{(1-\alpha)*Y}{Y} = 1-\alpha$

  So, capital receives .3 or 30% of total income and labor receives 70%.
- Suppose labor increases by 10%. Then we may write $L' = (1.1)\overline{L}$. Then, the new level of output can be written as

$$Y' = F(\overline{K}, L') = A\overline{K}^{.3}((1.1)\overline{L})^{.7} = A\overline{K}^{.3}\overline{L}^{.7} * (1.1)^{.7}$$
$$= \overline{Y} * (1.069)$$

So, we can write the percentage change in output due to the 10% increase in labor as

$$\frac{Y' - \overline{Y}}{\overline{Y}} = \frac{(1.069) * \overline{Y} - \overline{Y}}{\overline{Y}} = \frac{(1.069 - 1) * \overline{Y}}{\overline{Y}} = .069 = 6.9\%.$$

Similarly, for the change in the wage rate,

$$\overline{MPL} = (1-\alpha)\frac{\overline{Y}}{\overline{L}}, MPL' = (1-\alpha)\frac{Y'}{L'}$$
$$\frac{MPL' - \overline{MPL}}{\overline{MPL}} = \frac{(1.069)(.7)\frac{\overline{Y}}{\overline{L}} - (.7)\frac{\overline{Y}}{\overline{L}}}{(.7)\frac{\overline{Y}}{\overline{L}}} = \frac{\frac{1.069}{L'} - \frac{1}{\overline{L}}}{\frac{1}{\overline{L}}} = \frac{1.069}{(1.1)\overline{L}}\left(\frac{\overline{L}}{1}\right) - 1 = .9718 - 1 = -.0282 = -2.82\%.$$

And the change in the rental price of capital,

$$\overline{MPK} = \alpha\frac{\overline{Y}}{\overline{K}}, MPK' = \alpha\frac{Y'}{K'}$$
$$\frac{MPK' - \overline{MPK}}{\overline{MPK}} = \frac{(1.069)(.3)\frac{\overline{Y}}{\overline{K}} - (.3)\frac{\overline{Y}}{\overline{K}}}{(.3)\frac{\overline{Y}}{\overline{K}}} = \frac{(1.069 - 1)}{1} = .069 = 6.9\%.$$

- Replicate the steps taken in the derivations above for an increase in capital of 10%.
- Suppose $\overline{A} \uparrow$ by 10%. Then $A' = (1.1)\overline{A}$.

$$Y' = (1.1)\overline{A}\overline{K}^{.3}\overline{L}^{.7} = (1.1)\overline{Y}$$

We can write the percentage change in output due to the 10% increase in labor as

$$\frac{Y' - \overline{Y}}{\overline{Y}} = \frac{(1.1) * \overline{Y} - \overline{Y}}{\overline{Y}} = \frac{(1.1 - 1) * \overline{Y}}{\overline{Y}} = .1 = 10\%.$$

For the wage rate,

$$\overline{MPL} = (.7)\frac{\overline{Y}}{\overline{L}}, MPL' = (.7)(1.1)\frac{\overline{Y}}{L'}$$

$$\frac{MPL' - \overline{MPL}}{\overline{MPL}} = \frac{(1.1)(.7)\frac{\overline{Y}}{\overline{L}} - (.7)\frac{\overline{Y}}{\overline{L}}}{(.7)\frac{\overline{Y}}{\overline{L}}} = \frac{1.1 - 1}{1} = .1 = 10\%.$$

The same holds for $\%\Delta MPK = 10\%$.

5. From the neoclassical theory $MPL = \frac{W}{P}$. From the assumptions of a Cobb-douglas production function $MPL = (1-\alpha)\frac{Y}{L}$.

   Thus, if labor productivity is measured by $\frac{Y}{L}$, it is reasonable that the wage rate tracks labor productivity.

   $MPL = \frac{W}{P}$ holds for the general specification of $F(K, L)$. With a different production function, such as increasing or decreasing returns to scale, this feature could still hold without the latter result which comes from the Cobb-douglas specification.

6. According to the neoclassical theory of distribution,

- $W_f, W_b \equiv$ nominal wages $\rightarrow$ dollars per hour $P_f, P_b \equiv$ prices $\rightarrow$ dollars per haircut/unit of food $A_f, A_b \equiv$ marginal productivity $\rightarrow$ output per additional barber/farmer
- $A_f = \frac{W_f}{P_f}$, so $A_f \uparrow \implies \frac{W_f}{P_f} \uparrow$. The real wage is measured in units of food per hour.
- $A_b = \frac{W_b}{P_b}$; if A_b is unchanged, $\frac{W_b}{P_b}$ is unchanged. The real wage is measured in number of haircuts per hour.
- This suggests that $W_f = W_b$ in the long run. In this way, there is only one "type" of worker.
- If $W_f = W_b$, $\frac{W_b}{P_b} = \frac{W_f}{P_b}$. If $P_f \neq P_b$ and $\frac{W_f}{P_f} \uparrow$, then compare $\frac{W_f}{P_f}$ and $\frac{W_f}{P_b}$. It must be that $P_f < P_b$. Thus, $1 < \frac{P_b}{P_f} \implies$ the price of haircuts is higher than the price of food.
- The marginal productivity of farmers increased. So, if I am a farmer, I receive the real wage of units of food per hour. From part e., $\frac{W_f}{P_b}$, we can express the real wage of units of food per hour for barbers as well. The farmer receives more units of food per hour.

  Similarly, $\frac{W_b}{P_f} > \frac{W_b}{P_b}$ since $P_f < P_b$ and $W_b = W_f$. Thus, the farmer benefits more from the higher technological progress since they have the same basket of consumption and farmers earn higher real wages.

7. Consider the production function $Y = K^{\frac{1}{3}}L^{\frac{1}{3}}H^{\frac{1}{3}}$.

- For the marginal product of labor,

$$\frac{\partial Y}{\partial L} = \frac{1}{3}K^{\frac{1}{3}}L^{\frac{-2}{3}}H^{\frac{1}{3}} = \frac{1}{3}K^{\frac{1}{3}}L^{\frac{1}{3}}L^{\frac{-3}{3}}H^{\frac{1}{3}} = \frac{1}{3}YL^{-1}$$

note, $H \uparrow \implies MPL \uparrow$.

- For the marginal product of human capital,

$$\frac{\partial Y}{\partial L} = K^{\frac{1}{3}} L^{\frac{1}{3}} \frac{1}{3} H^{\frac{-2}{3}} = \frac{1}{3} Y H^{-1}$$

note, $H \uparrow \implies H^{-1} \downarrow \implies MPH \downarrow$.

- Labor income $= MPL * L = \frac{1}{3}Y$. Human capital income $= MPH * H = \frac{1}{3}Y$. So each receives $\frac{1}{3}$ of the share of output.

  Firms can be thought of as renting both the labor and human capital of workers. These are inputs of the production process they don't own. They do own capital though. So, from the accounting profit equation, labor income and human capital income show up in the "economic profit" component.

- We can write unskilled wage as $W_u = MPL$ and skilled wages as $W_s = MPL + MPH$. Then,

$$\frac{W_s}{W_u} = \frac{MPL + MPH}{MPL} = 1 + \frac{MPH}{MPL} = 1 + \frac{\frac{1}{3}YH^{-1}}{\frac{1}{3}YL^{-1}} = 1 + \frac{L}{H}$$

as $H \uparrow$, there are diminishing marginal returns to skilled labor. So, $W_s \downarrow$ with $H \uparrow$, meaning $\frac{W_s}{W_u} \downarrow$ as well.

8. $T \uparrow$ by \$100 billion. Suppose $MPC = .6$. Then $C = C(Y - T) = .6(Y - T)$. Recall that national saving can be written in two ways:

$$S = Y - C - G$$
$$S = (Y - T - C) + (T - G)$$

- Public saving $\equiv T - G$, so $T \uparrow \implies (T - G) \uparrow$ by same amount \$100 billion.

- Private saving $\equiv Y - T - C = Y - T - C(Y - T) = Y - T - .6(Y - T)$.

  So, disposable income $(Y - T)$ falls by more than consumption $C(Y - T)$ does. Then private saving falls. $(Y - T)$ falls by \$100 billion, $C(Y - T)$ falls by (.6) * \$100 billion = \$60 billion. So private saving falls by \$40 billion.

- $S = Y - C - G = Y - .6(Y - T) - G$. Since $C(Y - T)$ falls by \$60 billion, S increases by \$60 billion since the term $C(Y - T)$ is negative. So national saving increases by \$60 billion.

- Since $S = I$, then investment increases by the same amount, \$60 billion. What facilitates this change in equilibrium is the interest rate.

9. $Y - C(Y - T) - G = I(r)$. If $C \uparrow$, then L.H.S. $\downarrow \implies$ R.H.S $\downarrow$. This occurs when the interest rate increases.

10. Given the expressions and values describing the economy, we can solve for the following:

- Public saving $= T - G = 2000 - 2500 = -500$

  Private saving $= Y - T - C = 8000 - 2000 - (1000 + \frac{2}{3}(8000 - 2000)) = 6000 - (1000 + 4000) = 1000$

  National saving $= Y - C - G = 8000 - (1000 + \frac{2}{3}(6000)) - 2500 = 8000 - 5000 - 2500 = 500$

- $S = I$ in equilibrium. Then, the equilibrium interest rate is given by $500 = 1200 - 100r \implies r = 7$.

- Suppose $G \downarrow$ by 500. So, $G' = 2000$.

  Public saving $= T - G' = 2000 - 2000 = 0$

  Private saving $= Y - T - C = 1000$, since nothing changed in this expression.

  National saving $= 8000 - 7000 = 1000$

- The new equlibrium interest rate is the one that solves $S = I \implies 1000 = 1200 - 100r \implies r = 2$.

11. Recall the equilibrium condition

$$
\begin{aligned}
S &= I \\
Y - C - G &= I \\
Y - C(Y - T) - G &= I
\end{aligned}
$$

So, if the MPC is equal to 1, then the balanced budget changes to taxes and government spending completely offset each other.

Since the MPC is generally strictly less than 1, then the increase in government spending has a larger effect on national saving thant he increase on taxes, which effect disposable income.

In this case, the LHS is smaller. So the RHS needs to be smaller, by adjustment of the interest rate. The interest rate rises then, so that the level of investment is smaller.

12. First, in the loanable funds market, note that $S = I \implies S = I_b + I_r$.

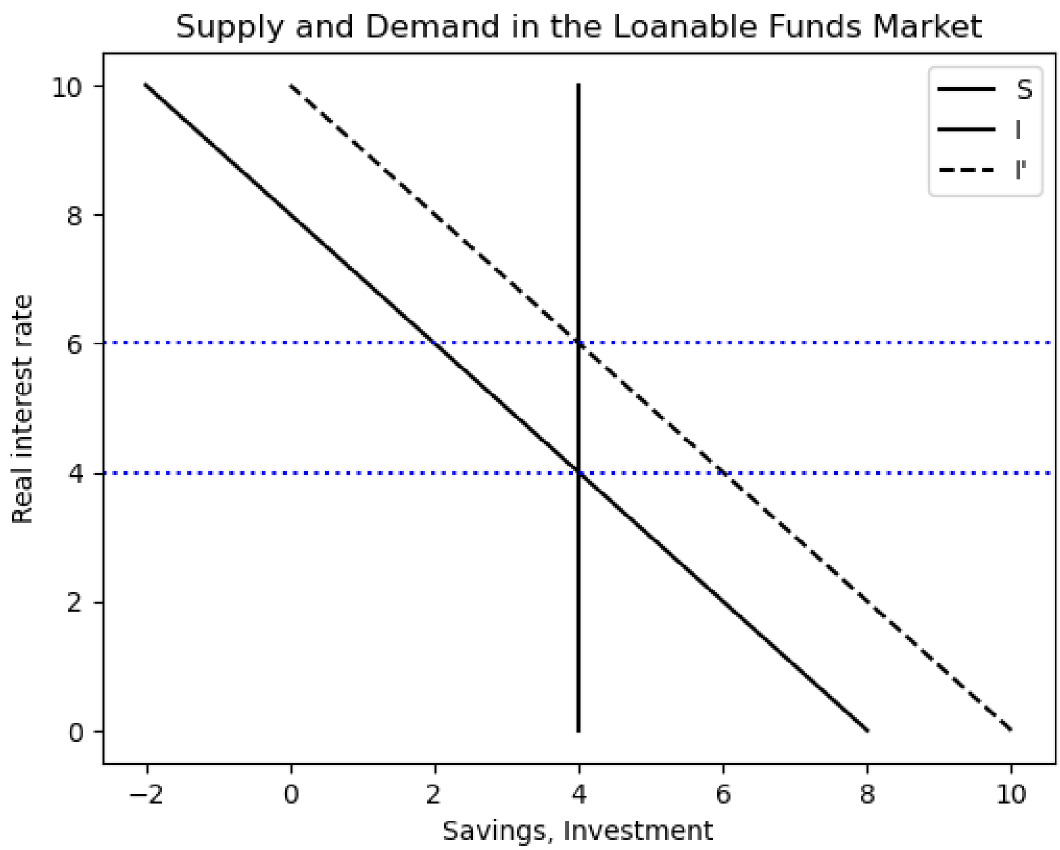
- Since $I = I_b + I_r$, $I_b \uparrow \implies I \uparrow$. An investment tax credit for business investment is akin to an upward shift in the demand for investment goods. This tax credit should have no effect on the demand for residential investment goods, under the assumption that I_b and I_r are neither substitutes nor complements. In either case, the demand for one of the goods will effect the demand for the other.

- Below is a graph capturing the fact that the supply of loanable funds will remain unchanged; however, the change in the demand for investment goods will lead to an increaase in the real interest rate.

- The quantity of investment is unchanged because the supply of funds is inelastic and fixed in the short run. Thus, the quantity of business and residential investment goods remain unchanged.

[66]:

```python
import matplotlib.pyplot as plt
import numpy as np

# Generate the price range for the curves
r = np.linspace(0, 10, 100)

# Define the supply curve (vertical line)
savings_quantity = np.full_like(r, 4)

# Define the demand curve (downward sloping line)
investment_quantity_old = 8 - r
investment_quantity_new = 10 - r

# Create the graph
plt.plot(savings_quantity, r, color="black", label='S')
plt.plot(investment_quantity_old, r, color="black", label='I')
plt.plot(investment_quantity_new, r, "--", color="black", label="I'")

# Add additional horizontal lines
plt.axhline(4, color='blue', linestyle=':')
plt.axhline(6, color='blue', linestyle=':')

# Add labels and title
plt.xlabel('Savings, Investment')
plt.ylabel('Real interest rate')
plt.title('Supply and Demand in the Loanable Funds Market')

# Add legend
plt.legend()

# Show the graph
plt.show()
```



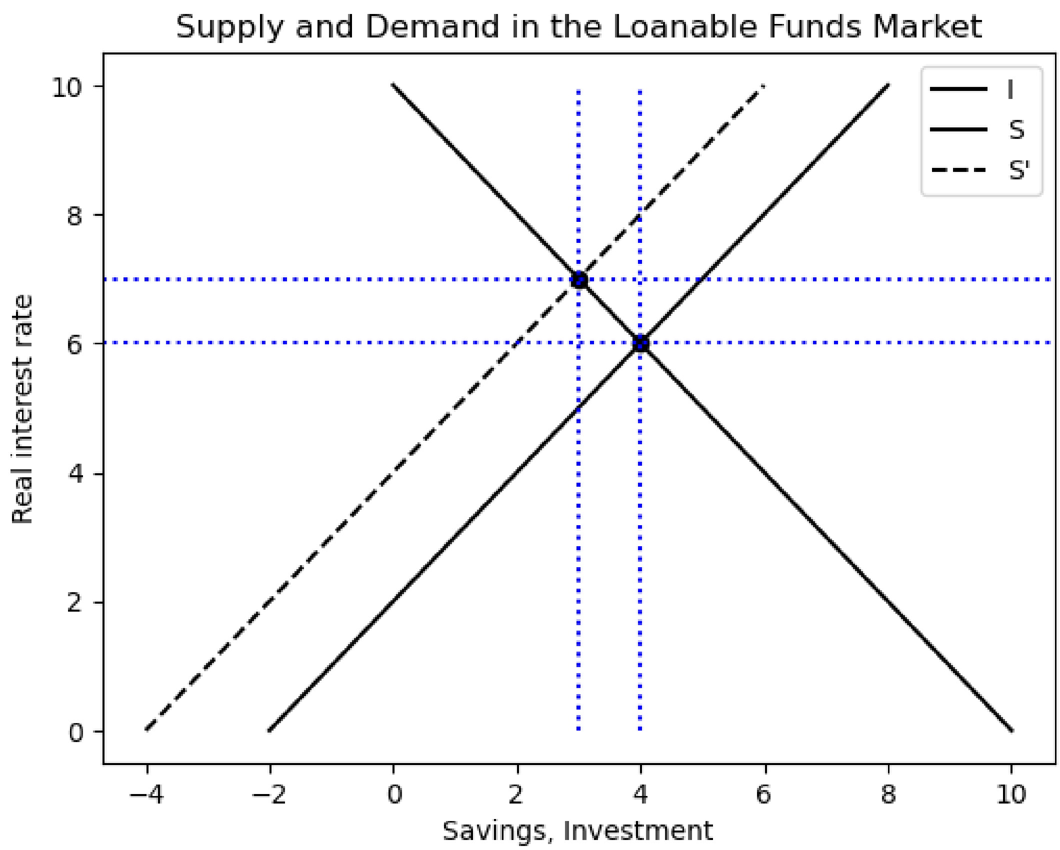
13. In this case, the supply of loanable goods is now upward sloping since $S = Y - C - G$. The graph is depicted below.

In this case, the quantity demanded of loanable goods will change in equilibrium when either curve shifts.

The graph depicts what happens now when governemnt purchases increase (and their effect on national savings outweighs the effect of an increase in taxes associated with a balanced budget). The quantity demanded of investment goods falls, and the interest rate increases.

[67]:
```
# Generate the price range for the curves
r = np.linspace(0, 10, 100)

# Define the supply curve (upward sloping line)
savings_quantity_old = r - 2
savings_quantity_new = r - 4

# Define the demand curve (downward sloping line)
investment_quantity = 10 - r

# Create the graph
```

```python
plt.plot(investment_quantity, r, color="black", label='I')
plt.plot(savings_quantity_old, r, color="black", label='S')
plt.plot(savings_quantity_new, r, "--", color="black", label="S'")

# Add additional horizontal lines
plt.axhline(7, color='blue', linestyle=':')
plt.axhline(6, color='blue', linestyle=':')

plt.plot(np.full_like(r, 4), r, color="blue", linestyle=":")
plt.plot(np.full_like(r, 3), r, color="blue", linestyle=":")

# Add dots for specific equilibrium points
plt.scatter([4, 3], [6, 7], color='black')

# Add labels and title
plt.xlabel('Savings, Investment')
plt.ylabel('Real interest rate')
plt.title('Supply and Demand in the Loanable Funds Market')

# Add legend
plt.legend()

# Show the graph
plt.show()
```



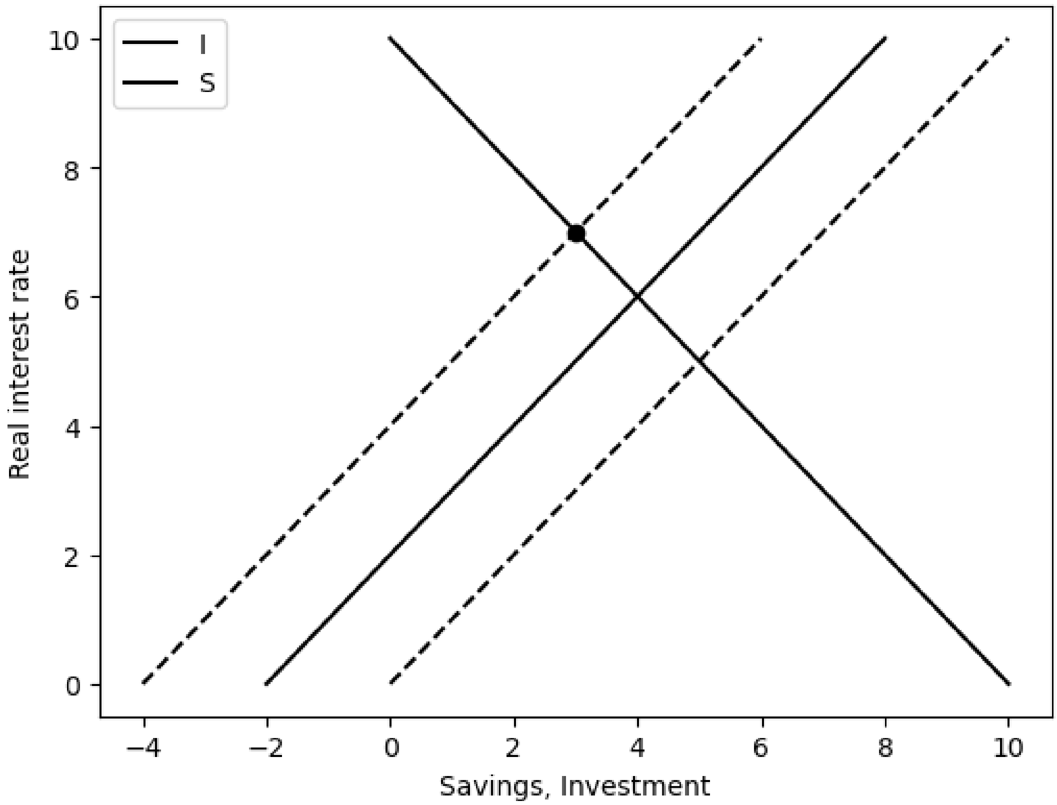
14. Below will be a series of graphs capturing each of the scenarios described below.

- In this case, you would see that the interest rate moves up or down, depending on whether or not supply moves left or right. Since investment is unchanged, it would look like interest rates are high when invetment is low.
- In this case, it would look like investment was high when the interest rate is high.
- In both of the previous cases, the interest rate and investment seem to be highly (perfectly) correlated. The correlation is either positive or negative. In this case, however, the relationship is less clear.

  For the same level of investment, the interest rate could be high or low in equilibrium. Similarly, for the same interest rate, the quantity demanded in equiliibrium could be high or low.
- The scenario described in part c. seems to be the most empirically realistic, since it is the only case where the relationship between interest rates and invesment seem unclear.

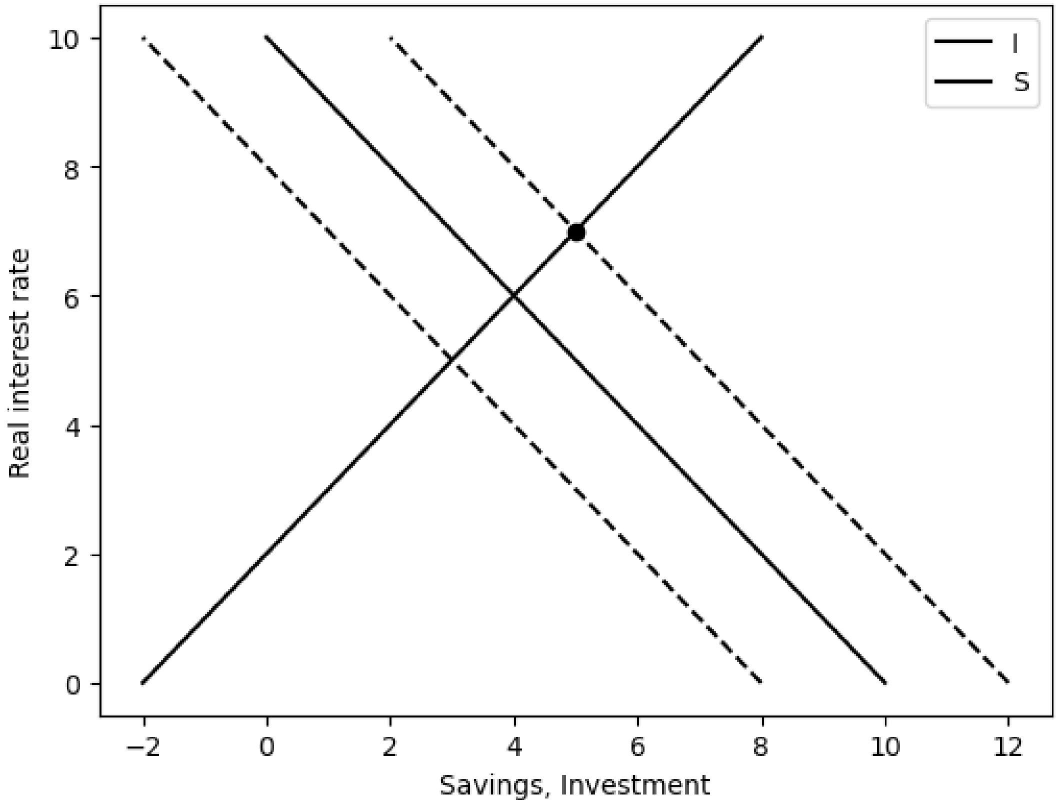
[68]:
```
# Generate the price range for the curves
r = np.linspace(0, 10, 100)

# Define the supply curve (upward sloping line)
```

```
savings_quantity = r - 2
savings_quantity_up = r
savings_quantity_down = r - 4

# Define the demand curve (downward sloping line)
investment_quantity = 10 - r

# Create the graph
plt.plot(investment_quantity, r, color="black", label='I')
plt.plot(savings_quantity, r, color="black", label='S')
plt.plot(savings_quantity_up, r, "--", color="black")
plt.plot(savings_quantity_down, r, "--", color="black")

# Add dots for specific equilibrium points
plt.scatter(3, 7, color='black')

# Add labels and title
plt.xlabel('Savings, Investment')
plt.ylabel('Real interest rate')

# Add legend
plt.legend()

# Show the graph
plt.show()
```



[69]:
```
# Generate the price range for the curves
r = np.linspace(0, 10, 100)

# Define the supply curve (upward sloping line)
savings_quantity = r - 2

# Define the demand curve (downward sloping line)
investment_quantity = 10 - r
investment_quantity_up = 12 - r
investment_quantity_down = 8 - r

# Create the graph
plt.plot(investment_quantity, r, color="black", label='I')
plt.plot(savings_quantity, r, color="black", label='S')
plt.plot(investment_quantity_up, r, "--", color="black")
plt.plot(investment_quantity_down, r, "--", color="black")

# Add dots for specific equilibrium points
plt.scatter(5, 7, color='black')

# Add labels and title
```

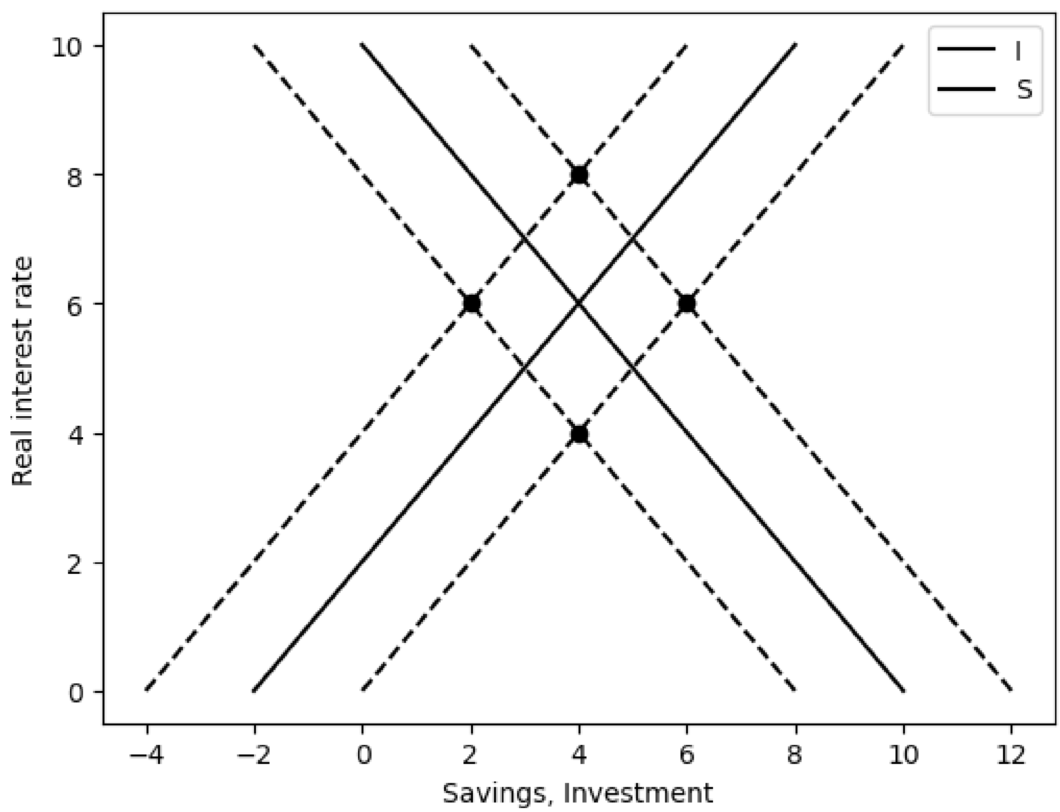
```python
plt.xlabel('Savings, Investment')
plt.ylabel('Real interest rate')

# Add legend
plt.legend()

# Show the graph
plt.show()
```



[70]:

```python
# Generate the price range for the curves
r = np.linspace(0, 10, 100)

# Define the supply curve (upward sloping line)
savings_quantity = r - 2
savings_quantity_up = r
savings_quantity_down = r - 4

# Define the demand curve (downward sloping line)
investment_quantity = 10 - r
investment_quantity_up = 12 - r
investment_quantity_down = 8 - r
```

```python
# Create the graph
plt.plot(investment_quantity, r, color="black", label='I')
plt.plot(investment_quantity_up, r, "--", color="black")
plt.plot(investment_quantity_down, r, "--", color="black")
plt.plot(savings_quantity, r, color="black", label='S')
plt.plot(savings_quantity_up, r, "--", color="black")
plt.plot(savings_quantity_down, r, "--", color="black")

# Add dots for specific equilibrium points
plt.scatter([2, 6], [6, 6], color='black')
plt.scatter([4, 4], [4, 8], color='black')

# Add labels and title
plt.xlabel('Savings, Investment')
plt.ylabel('Real interest rate')

# Add legend
plt.legend()

# Show the graph
plt.show()
```



```
[ ]:
```