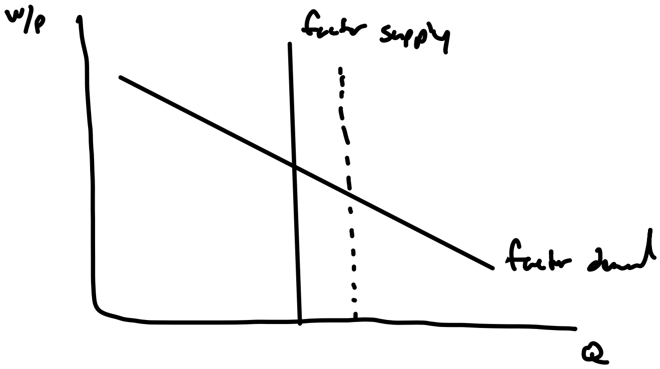
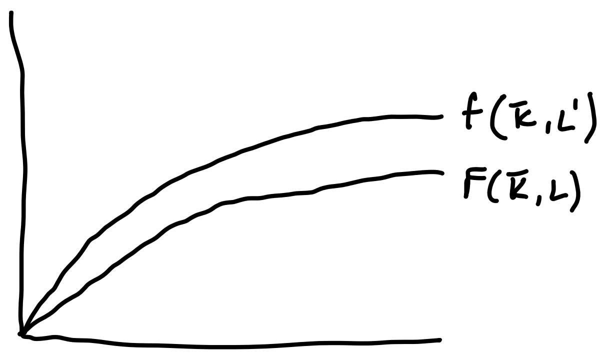
1.

a. $MPL = \frac{W}{P}$, MPL is decreasing in L ; so if $L \uparrow$, $MPL \downarrow \Rightarrow W/P \downarrow$

$F(K,L)$ is increasing in K, L. So $\uparrow L \Rightarrow MPK \uparrow \Rightarrow R/P \uparrow$





b. capital stock falls $\Rightarrow MPK \uparrow \Rightarrow \frac{R}{P} \uparrow$

$\Rightarrow F(K,L) \downarrow \Rightarrow MPL \downarrow \Rightarrow \frac{W}{P} \downarrow$

c. Under Cobb-douglas, $Y = AK^{\alpha}L^{1-\alpha}$; $MPL = (1-\alpha) A K^{\alpha} L^{-\alpha}$

$= (1-\alpha) Y/L$

$MPK = \alpha A K^{\alpha-1} L^{1-\alpha}$

$= \alpha Y/K$

So $\uparrow A \Rightarrow \uparrow MPL$ and $\uparrow MPK \Rightarrow W/P, R/P \uparrow$

d) Suppose $MPL = W/P$ and $MPK = R/P$. Then profit is

Profit $= P \cdot F(K,L) - WL - RK$; suppose price of factor inputs and outputs double.

But aren't the real wage and rental price the factor inputs? Then $MPL = \frac{2W}{2P}$ and $MPK = \frac{2R}{2P}$ so nothing changes.

2. $Y = K^{.5} L^{.5}$, $K = 100$, $L = 100$

a. $\bar{Y} = F(\bar{K}, \bar{L}) = (100)^{1/2} \cdot (100)^{1/2} = 10 \cdot 10 = 100$ units of output

b. $MPL = (1-\alpha)\, Y/L$

$= (1 - 1/2) \frac{100}{100}$

$= (.5)$

$MPK = \alpha\, Y/K$

$= (.5) \frac{100}{100}$

$= .5$

c. Labor income $= L \times MPL = (1-\alpha) \cdot Y = (.5)(100) = 50$

Share of total output $= \frac{50}{100} = .5$ (note that this is $1-\alpha$!)

d. $L' = 50$; $\bar{Y} = F(\bar{K}, L') = \sqrt{100} \cdot \sqrt{50} = 10 \cdot 5\sqrt{2} = 50\sqrt{2}$

e. $MPL = (1-\alpha)\, Y/L$

$= (.5) \frac{50\sqrt{2}}{50}$

$= \left(\frac{1}{2}\right)\sqrt{2}$

$= \frac{\sqrt{2}}{2} \approx .71$

$MPK = \alpha\, Y/K$

$= (.5) \frac{50\sqrt{2}}{100}$

$= \left(\frac{1}{2}\right) \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} \approx .35$

f. $L \times MPL = (1-\alpha) \cdot Y = \frac{1}{2} \cdot 50\sqrt{2} \approx 35.36$

share of total output $= \frac{\frac{1}{2} \cdot 50\sqrt{2}}{50\sqrt{2}} = 1/2$; remains $(1-\alpha)$, a feature of Cobb-Douglas production.

4. When $\alpha = .3$, then

$$\bar{Y} = A\bar{K}^{.3}\bar{L}^{.7}$$

a. $$\frac{MPK \times K}{Y} = \frac{\alpha Y}{Y} = \alpha \quad ; \quad \frac{MPL \times L}{Y} = \frac{(1-\alpha)Y}{Y} = (1-\alpha)$$

So, capital receives .3 or 30% of total income, and labor receives 70%.

b. $\bar{L} \uparrow$ by 10% $\Rightarrow L' = (1.1)\bar{L}$

$$Y' = F(\bar{K}, L') = A\bar{K}^{.3}[(1.1)\bar{L}]^{.7}$$

$$= A\bar{K}^{.3}\bar{L}^{.7} \cdot (1.1)^{.7} = F(\bar{K}, \bar{L}) \cdot (1.1)^{.7}$$

$$= \bar{Y} \cdot (1.069)$$

$$\frac{Y' - \bar{Y}}{\bar{Y}} = \frac{1.069 \cdot \bar{Y} - \bar{Y}}{\bar{Y}} = \frac{(1.069 - 1) \cdot \bar{Y}}{\bar{Y}} = .069$$ or 6.7% increase in output.

• For the change in the wage rate,

$$\overline{MPL} = (1-\alpha)\,\bar{Y}/\bar{L} \qquad MPL' = (1-\alpha)\,Y'/L'$$

$$= \frac{(.7) \cdot \bar{Y}}{\bar{L}} \qquad = \frac{(.7)(1.069) \cdot \bar{Y}}{L'}$$

$$\frac{MPL' - \overline{MPL}}{\overline{MPL}} = \frac{\frac{(.7)\bar{Y}(1.069)}{L'} - \frac{(.7)\bar{Y}}{\bar{L}}}{\frac{(.7)\bar{Y}}{\bar{L}}} = \frac{\frac{1.069}{L'} - \frac{1}{\bar{L}}}{\frac{1}{\bar{L}}} = \frac{\frac{1.069}{L'}}{\frac{1}{\bar{L}}} - 1$$

$$= \frac{1.069}{(1.1)\bar{L}}\left(\frac{\bar{L}}{1}\right) - 1 = \frac{1.069}{1.1} - 1 = .9718 - 1 = -.0282 \text{ or } -2.82\%$$

• For the change in the rental rate of capital,

$$\overline{MPK} = \alpha \, \overline{Y}/\overline{K} \qquad MPK' = \alpha \frac{Y'}{\overline{K}}$$

$$= .3 \left(\frac{\overline{Y}}{\overline{K}}\right) \qquad = \frac{(.3)(1.069)\overline{Y}}{\overline{K}}$$

$$\frac{MPK' - \overline{MPK}}{\overline{MPK}} = \frac{\frac{(.3)(1.069)\overline{Y}}{\overline{K}} - (.3)\frac{\overline{Y}}{\overline{K}}}{(.3)\frac{\overline{Y}}{\overline{K}}} = \frac{1.069 - 1}{1} = .069 \text{ or } 6.9\%$$

c. Omitted, since this is an analogy of part b.

d. Suppose $\bar{A} \uparrow$ by 10%. Then $A' = (1.1)\bar{A}$.

$$Y' = (1.1)\bar{A}\bar{K}^{.3}\bar{L}^{.7} = (1.1)\bar{Y}$$

Then, $\dfrac{Y' - \bar{Y}}{\bar{Y}} = \dfrac{(1.1)\bar{Y} - \bar{Y}}{\bar{Y}} = \dfrac{1.1 - 1}{1} = \dfrac{.1}{1} = .1$ or 10% increase in output

• For the wage rate,

$$\overline{MPL} = (1-\alpha)\,\overline{Y}/\overline{L} \qquad MPL' = (1-\alpha)\,Y'/\overline{L}$$

$$= (.7)\frac{\overline{Y}}{\overline{L}} \qquad = (.7)(1.1)\frac{\overline{Y}}{\overline{L}}$$

$$\frac{MPL' - \overline{MPL}}{\overline{MPL}} = \frac{(.7)(1.1)\frac{\overline{Y}}{\overline{L}} - (.7)\frac{\overline{Y}}{\overline{L}}}{(.7)\frac{\overline{Y}}{\overline{L}}} = \frac{1.1 - 1}{1} = .1 \text{ or } 10\%$$

The same holds for $\%\Delta MPK = 10\%$.

5. From neoclassical theory,

$$MPL = W/P$$

From the assumption of a Cobb-Douglas production function,

$$MPL = (1-\alpha)\, Y/L$$

Thus, if labor productivity is measured by Y/L, it is reasonable that the wage rate tracks labor productivity.

Yes, with a different production function, since $MPL = W/P$ for the general specification of $F(K,L)$. $F(\cdot)$ could exhibit increasing or decreasing returns to scale.

6.

a. $W_f, W_b \equiv$ nominal wages → dollars per hour

$P_f, P_b \equiv$ prices → dollars per haircut / unit of food

$A_f, A_b \equiv$ marginal productivity → output per additional farmer / barber

b. $A_f = \dfrac{W_f}{P_f}$; if $A_f \uparrow \Rightarrow \dfrac{W_f}{P_f} \uparrow$. The real wage is measured in units of food per hour.

c. $A_b = \dfrac{W_b}{P_b}$; if A_b is unchanged, $\dfrac{W_b}{P_b}$ is unchanged. The real wage is measured in number of haircuts per hour.

d. This suggests that $W_f = W_b$ in the long run. In this way, there is only one "type" of worker.

e. If $W_f = W_b$, then $\frac{W_b}{P_b} = \frac{W_f}{P_b}$.

If $P_f \neq P_b$ and $\frac{W_f}{P_f} \uparrow$, then compare $\frac{W_f}{P_f}$ and $\frac{W_f}{P_b}$. It must be

that $P_f < P_b$. $\Rightarrow 1 < \frac{P_b}{P_f} \Rightarrow$ the price of haircuts is higher than the price of food.

f. The marginal productivity of farmers increased. So, if I am a farmer, I receive the real wage of units of food per hour. From part e., $\frac{W_f}{P_b}$, we can express the real wage of units of food per hour for barbers as well. The farmer receives more units of food per hour.

Similarly, $\frac{W_b}{P_f} > \frac{W_b}{P_b}$ since $P_f < P_b$ and $W_b = W_f$. Thus, the farmers benefit more from the higher technological progress since they have the same basket of consumption and farmers earn higher real wages.

7. $Y = K^{1/3} L^{1/3} H^{1/3}$

a. $\frac{\partial Y}{\partial L} = \frac{1}{3} K^{1/3} L^{-2/3} H^{1/3} = \frac{1}{3} K^{1/3} L^{1/3} \cdot L^{-3/3} \cdot H^{1/3} = \frac{1}{3} Y \cdot L^{-1}$

note, $\uparrow H \Rightarrow MPL \uparrow$.

b. $\frac{\partial Y}{\partial H} = K^{1/3} L^{1/3} \frac{1}{3} H^{-2/3} = \frac{1}{3} Y \cdot H^{-1}$; note, as $H \uparrow$, $H^{-1} \downarrow \Rightarrow MPH \downarrow$ since it is scaled by H^{-1}.

c. Labor income $= MPL \cdot L = \frac{1}{3} \cdot Y$

human capital income $= MPH \cdot H = \frac{1}{3} \cdot Y$

$\Rightarrow$ each receives $\frac{\frac{1}{3} \cdot Y}{Y} = 1/3$ of the share of output.

Firms can be thought of as renting both the labor and human capital of workers. These are inputs of the production process they don't own. They do own capital though. From the accounting profit equation, we have

accounting profit = economic profit + $(MPK \cdot K)$

d. $W_u = MPL$, $W_s = MPL + MPH$

$$\Rightarrow \frac{W_s}{W_u} = \frac{MPL + MPH}{MPL} = 1 + \frac{MPH}{MPL} = 1 + \frac{\frac{1}{3} Y \cdot H^{-1}}{\frac{1}{3} Y \cdot L^{-1}}$$

$$= 1 + \frac{L}{H}$$

as $H \uparrow$, there are diminishing marginal returns to skilled labor. So, $W_s \downarrow$ with $H \uparrow$, meaning $\frac{W_s}{W_u} \downarrow$ as well.

8. $T \uparrow$ by \$100 billion. Suppose $MPC = .6$

$$C = C(Y-T) = .6(Y-T)$$

Recall national saving can be written in two ways

$$S = Y - C - G$$

$$S = (Y - T - C) + (T - G)$$

a) Public saving $\equiv T - G$, so $T\uparrow \Rightarrow (T-G)\uparrow$ by same amount, \$100 B.

b) Private saving $\equiv Y - T - C = Y - T - C(Y-T)$

$$= Y - T - [.6(Y-T)]$$

So, disposable income $(Y-T)$ falls by more than consumption $C(Y-T)$ does. Then private saving falls.

$(Y-T)$ falls by \$100 B, $C(Y-T)$ falls by $(.6)100\text{ B} = 60\text{ B}$. So private saving falls by \$40 B.

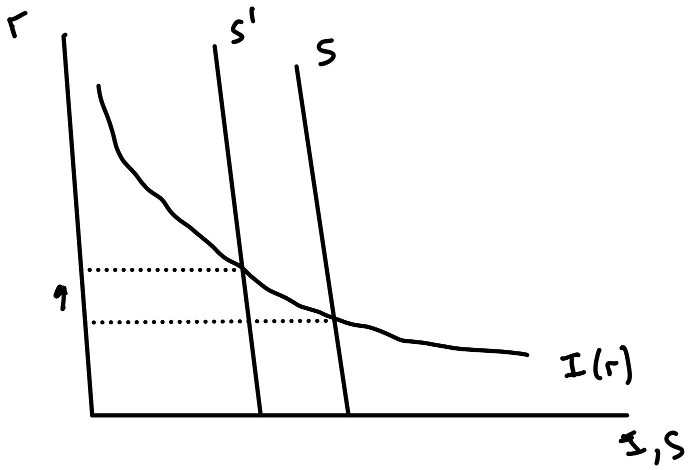
c) $S = Y - C - G = Y - .6(Y-T) - G$

Since $C(Y-T)$ falls by 60B, S increases by 60B (the term $C(Y-T)$ is negative). So national saving increases by 60B, consistent with a) and b).

d) Since $S = I$, then investment increases by the same amount, 60B. What facilitates this change in equilibrium is the interest rate.

9. $Y - C(Y-T) - G = I(r)$

If $C \uparrow$, then LHS $\downarrow$ $\Rightarrow$ RHS $\downarrow$. This occurs when the interest rate increases.



10.

a. Public saving $= T - G = 2000 - 2500 = -500$

Private saving $= Y - T - C = 8000 - 2000 - \left[1000 + \frac{2}{3}(8000 - 2000)\right]$

$= 6000 - \left[1000 + \frac{2}{3}(6000)\right]$

$= 6000 - [1000 + 4000] = 6000 - 5000$

$= 1000$

National saving $= Y - C - G = 8000 - \left[1000 + \frac{2}{3}(6000)\right] - 2500$

$= 8000 - (5000) - 2500 \quad = 8000 - 7500$

$= 500 \equiv S$

b) $S = I$ in equilibrium. Then, the equilibrium interest rate is given by

$500 = 1200 - 100r \Rightarrow 100r = 700 \Rightarrow r = 7$

c. Suppose $G \downarrow$ by 500. So, $G' = 2000$

Public saving $= T - G' = 2000 - 2000 = 0$

Private saving $= Y - T - C = 1000$, since nothing changed in this expression.

National saving $= 8000 - 7000 = 1000$

d. The new equilibrium interest rate is the one that solves

$$S = I$$

$$1000 = 1200 - 100r \Rightarrow 100r = 200$$

$$\Rightarrow r = 2$$

11. Recall the equilibrium condition

$$S = I$$

$$Y - C - G = I$$

$$Y - C(Y - T) - G = I$$

So, if the MPC is equal to 1, then the balanced budget changes to taxes and government completely offset each other.

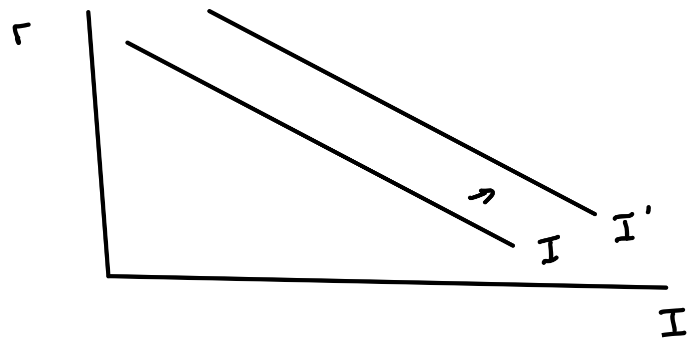
Since, the MPC is generally strictly less than 1, then the increase in government purchases has a larger effect on national saving than the increase on taxes, which effects disposable income.

In this case, the LHS is smaller. So the RHS needs to be smaller, by adjustment of the interest rate. The interest rate rises, so that the level of investment is smaller.

12. $S = I$

$S = I_b + I_r$

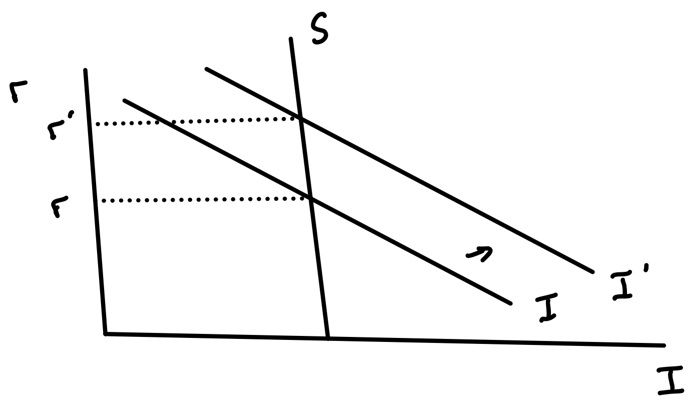
a. An investment tax credit for business investment is akin to an upward shift in the demand for investment goods.



Since $I = I_b + I_r$, $\uparrow I_b \Rightarrow \uparrow I$. However, this tax credit should have no effect on the demand for residential investment goods.

To be clear, this is under the assumption that I_b and I_r are neither substitutes nor complements. In either case, the demand for one of the goods will effect the demand for the other.

b.

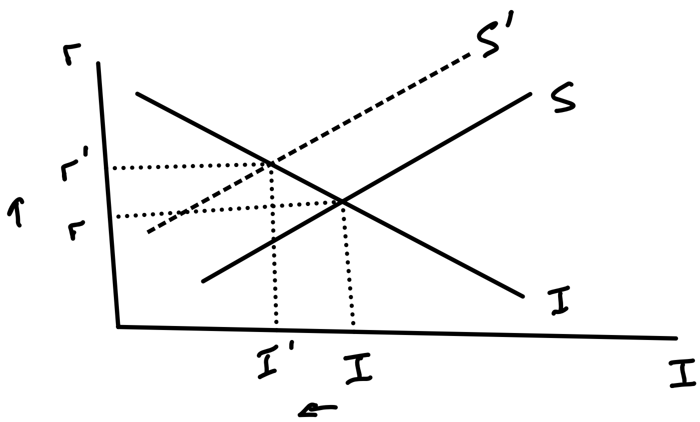


Since the supply of loanable funds is unchanged, then this change in the demand for loanable funds increases the interest rate.

c. The quantity of investment is unchanged. This is because the supply of funds is inelastic and fixed in the short run.

Likewise, the quantity of business and residential investment goods remain unchanged.

13. In this case, the supply of loanable goods is now an upward sloping line/curve, since $S = Y - C - G$:

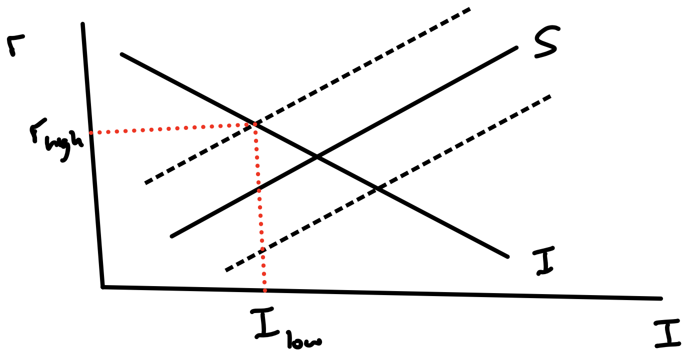


In this case, the quantity demanded of loanable goods will change in equilibrium when either curve shifts.

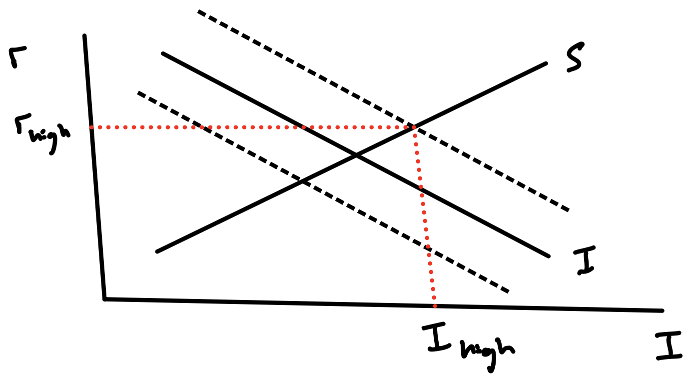
The graph above depicts what happens now when government purchases increase (and their effect outweighs the balanced budget increase in taxes). The quantity demanded of investment goods falls, and the interest rate increases.

14.

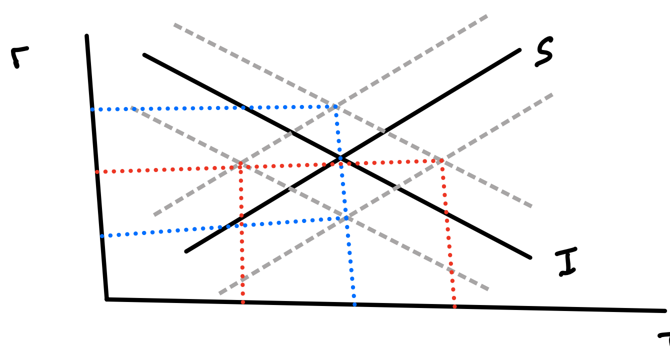
a. In this case, you would see that the interest rate moves up or down, depending on whether or not supply moves left or right. Since investment is unchanged, it would look like interest rates are high when investment is low.



b. In this case, it would look like investment was high when the interest rate is high.



c. In both of the previous cases, interest rates and investment seem to be highly (perfectly) correlated. The correlation is either positive or negative. In this case, the relationship is less clear. For the same level of investment, the interest rate could be high or low in equilibrium (blue lines). Similarly, for the same interest rate, the quantity demanded in equilibrium could be high or low.



d) The scenario described in c) seems to be the most empirically realistic, since it is the only case where the relationship between interest rates and investment seem unclear.