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Quis Metode Numerik

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No. :

1. Selesaikan  $\ln 3 = 1,0986122$  dengan polinom Newton

$$\text{Jika } x_0 = 1 \rightarrow f(x_0) = 0$$

$$x_1 = 6 \rightarrow f(x_1) = 1,7917594$$

$$x_2 = 7 \rightarrow f(x_2) = 1,9459101$$

Jawab :

beda - terbagi hingga pertama :

$$f[x_1, x_0] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{1,7917594 - 0}{6 - 1}$$

$$= \frac{1,7917594}{5} = 0,35835188 \dots (b_1) //$$

$$f[x_2, x_1] = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{1,9459101 - 1,7917594}{7 - 6}$$

$$= \frac{0,1541507}{1} = 0,1541507 //$$

beda - terbagi hingga kedua :

$$f[x_2, x_1, x_0] = \frac{f[x_2, x_1] - f[x_1, x_0]}{x_2 - x_0}$$

$$= \frac{0,1541507 - 0,35835188}{7 - 1}$$

$$= -0,03403353 \dots (b_2) //$$

$$F_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$f_2(3) = 0 + 0,35835188(3 - 1) - 0,03403353(3 - 1)(3 - 6)$$

$$= 0 + 0,71670376 - 0,03403353(2)(-3)$$

$$= 0 + 0,71670376 + 0,20420118$$

$$= 0,92090494 //$$

No. :

$$\begin{aligned}\% \text{ galat relatif} &= \frac{1,0986122 - 0,92090494}{1,0986122} \times 100 \% \\ &= \frac{0,17770726}{1,0986122} \times 100 \% \\ &= 16,1756131964 \% \\ &= 16,2 \%\end{aligned}$$

2. Selesaikan  $\ln 4 = 1,3862943$  dengan polinom Lagrange

Jika :  $x_0 = 1 \rightarrow f(x_0) = 0$

$x_1 = 7 \rightarrow f(x_1) = 1,9459101$

$x_2 = 8 \rightarrow f(x_2) = 2,0794415$

$x_3 = 9 \rightarrow f(x_3) = 2,1972245$

Jawab :

Polinom orde pertama :

$$f_1(x) = \frac{x-x_1}{x_0-x_1} f(x_0) + \frac{x-x_0}{x_1-x_0} f(x_1)$$

$$f_1(4) = \frac{4-7}{1-7} f(0) + \frac{4-1}{7-1} 1,9459101$$

$$= 0,5 \times 0 + 0,5 \times 1,9459101$$

$$= 0,97295505$$

Polinom orde kedua

$$\begin{aligned}f_2(x) &= \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) \\ &\quad + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)\end{aligned}$$

$$f_2(4) = \frac{(4-7)(4-8)}{(1-7)(1-8)} 0 + \frac{(4-1)(4-8)}{(7-1)(7-8)} 1,9459101$$



No. :

$$+ \frac{(4-1)(4-7)}{(8-7)(8-1)} 2,0794415$$

$$= \frac{(-3)(-4)}{(-6)(-7)} 0 + \frac{(3)(-4)}{(6)(-1)} 1,9459101 + \frac{(3)(-3)}{(1)(7)} 2,0794415$$

$$= \frac{12}{42} 0 + \frac{(-12)}{-6} 1,9459101 + \frac{(-9)}{7} 2,0794415$$

$$= 0 + 3.8918202 + (-2.6735676429) = 1,2182525571 //$$

$$F_3(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1) + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

$$F_3(4) = \frac{(4-7)(4-8)(4-9)}{(1-7)(1-8)(1-9)} 0 + \frac{(4-1)(4-8)(4-9)}{(7-1)(7-8)(7-9)} 1,9459101$$

$$+ \frac{(4-1)(4-7)(4-9)}{(8-1)(8-7)(8-9)} 2,0794415 + \frac{(4-1)(4-7)(4-8)}{(9-1)(9-7)(9-8)} 2,1972245$$

$$= \frac{(-3)(-4)(-5)}{(-6)(-7)(-8)} 0 + \frac{(3)(-4)(-5)}{(6)(-1)(-2)} 1,9459101$$

$$+ \frac{(3)(-3)(-5)}{(7)(1)(-1)} 2,0794415 + \frac{(3)(-3)(-4)}{(8)(2)(1)} 2,1972245$$

$$= \frac{-60}{-336} 0 + \frac{60}{12} 1,9459101 + \frac{45}{-7} 2,0794415 + \frac{36}{16} 2,1972245$$

$$= 0 + 9,7295505 - 13,3678382143 + 4,943755125$$

$$= 1,3054674107 //$$

No.:

$$\begin{aligned}
 \% \text{ galat relatif} &= \frac{1,3862943 - 1,3054674107}{1,3862943} \times 100 \% \\
 &= \frac{0,0808268893}{1,3862943} \times 100 \% \\
 &= 5,8304278752 \% \\
 &= 5,8 \%
 \end{aligned}$$

3. Hitunglah nilai dari  $\int_2^6 5x^3 - 2x^2 + 10$  dengan  $h = \frac{1}{2}$  menggunakan metode trapesium dan Simpson  $\frac{1}{3}$  - metode trapezium.

$$h = \frac{1}{2} = 0,5 \quad h = \frac{b-a}{n} \quad \frac{1}{2} = \frac{6-2}{n} \quad n = 8$$

$$a = 2 \quad b = 6$$

maka banyak intervalnya adalah 8

$$f_0 = 5x^3 - 2x^2 + 10 = f(2) = 5(2)^3 - 2(2)^2 + 10 = 42$$

$$f_1 = 5x^3 - 2x^2 + 10 = f(2,5) = 5(2,5)^3 - 2(2,5)^2 + 10 = 75,625$$

$$f_2 = 5x^3 - 2x^2 + 10 = f(3) = 5(3)^3 - 2(3)^2 + 10 = 127$$

$$f_3 = 5x^3 - 2x^2 + 10 = f(3,5) = 5(3,5)^3 - 2(3,5)^2 + 10 = 199,875$$

$$f_4 = 5x^3 - 2x^2 + 10 = f(4) = 5(4)^3 - 2(4)^2 + 10 = 250$$

$$f_5 = 5x^3 - 2x^2 + 10 = f(4,5) = 5(4,5)^3 - 2(4,5)^2 + 10 = 425,125$$

$$f_6 = 5x^3 - 2x^2 + 10 = f(5) = 5(5)^3 - 2(5)^2 + 10 = 585$$

$$f_7 = 5x^3 - 2x^2 + 10 = f(5,5) = 5(5,5)^3 - 2(5,5)^2 + 10 = 781,375$$

$$f_8 = 5x^3 - 2x^2 + 10 = f(6) = 5(6)^3 - 2(6)^2 + 10 = 1018$$

$$I = \frac{h}{2} (f_0 + 2f_1 + 2f_2 + 2f_3 + 2f_4 + 2f_5 + 2f_6 + 2f_7 + f_8)$$

$$= \frac{0,5}{2} (42 + 2(75,625) + 2(127) + 2(199,875) + 2(250) + 2(425,125) + 2(585)$$

$$+ 2(781,375) + 1018)$$

$$= 0,25 (42 + 151,25 + 254 + 399,75 + 500 + 850,25 + 1170 + 1562,75 + 1018)$$

$$= 0,25 \cdot 5948 = 1487 //$$



Metode Simpson  $\frac{1}{3}$ .

$$I = \frac{h}{3} (f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + 4f_5 + 2f_6 + 4f_7 + f_8)$$

$$= \frac{0,5}{3} (42 + 4(75,625) + 2(127) + 4(199,875) + 2(250) + 4(425,125) +$$

$$2(585) + 4(781,375) + 1018)$$

$$= \frac{0,5}{3} (42 + 302,5 + 254 + 799,5 + 500 + 1700,5 + 1170 + 3125,5 + 1018)$$

$$= \frac{0,5}{3} \cdot 8912 = 1485,33$$

4. Selesaikan persamaan  $\frac{dy}{dx} = f(x, y) = 4x^2 + 2x - 10$  dengan  $y(0) = 1$

dari  $x = 0$  dan  $x = 3$  dengan panjang langkah  $\Delta x = \frac{3}{5}$

- Langkah 1 ( $i=0$ ) maka  $y_1$

$$y_1 = y_0 + f(x_0, y_0) \Delta x$$

$$y(0,6) = 1 + f(0; 1) \cdot 0,6$$

$$y(0,6) = 1 + (4(0)^2 + 2(0) - 10) \cdot 0,6$$

$$y(0,6) = 1 + (0 + 0 - 10) \cdot 0,6$$

$$y(0,6) = 1 + (-10) \cdot 0,6 = -5$$

$$y_1 = y(0,6) = -5$$

- Langkah 2 ( $i=1$ ) maka  $y_2$

$$y_2 = y_1 + f(x_1, y_1) \Delta x$$

$$y(1,2) = -5 + f(0,6; -5) \cdot 0,6$$

$$y(1,2) = -5 + (4(0,6)^2 + 2(0,6) - 10) \cdot 0,6$$

$$y(1,2) = -5 + (1,44 + 1,2 - 10) \cdot 0,6$$

$$y(1,2) = -5 + (-7,36) \cdot 0,6 = -9,416$$

$$y_2 = y(1,2) = -9,416$$

- Langkah 3 ( $i = 2$ ) maka  $y_3$

$$y_3 = y_2 + f(x_2, y_2) \Delta x$$

$$y(1,8) = -9,416 + f(1,2; -9,416) \cdot 0,6$$

$$y(1,8) = -9,416 + (4(1,2)^2 + 2(1,2) - 10) \cdot 0,6$$

$$y(1,8) = -9,416 + (5,76 + 2,4 - 10) \cdot 0,6$$

$$y(1,8) = -9,416 + (-1,84) \cdot 0,6 = -10,52$$

$$y_3 = y(1,8) = -10,52$$

- Langkah 4 ( $i = 3$ ) maka  $y_4$

$$y_4 = y_3 + f(x_3, y_3) \Delta x$$

$$y(2,4) = -10,52 + f(1,8; -10,52) \cdot 0,6$$

$$y(2,4) = -10,52 + (4(1,8)^2 + 2(1,8) - 10) \cdot 0,6$$

$$y(2,4) = -10,52 + (12,96 + 3,6 - 10) \cdot 0,6 = -6,584$$

$$y_4 = y(2,4) = -6,584$$

- Langkah 5 ( $i = 4$ ) maka  $y_5$

$$y_5 = y_4 + f(x_4, y_4) \Delta x$$

$$y(3) = -6,584 + f(2,4; -6,584) \cdot 0,6$$

$$y(3) = -6,584 + (4(2,4)^2 + 2(2,4) - 10) \cdot 0,6$$

$$y(3) = -6,584 + (23,04 + 4,8 - 10) \cdot 0,6$$

$$y(3) = -6,584 + (17,84) \cdot 0,6 = 4,12$$

$$y_5 = y(3) = 4,12$$

Nilai eksak dari titik  $x = 3$  :

$$y = \frac{4}{3}x^3 + x^2 - 10x + 1$$

$$= \frac{4}{3}(3)^3 + (3)^2 - 10(3) + 1$$

$$= 36 + 9 - 30 + 1 = 16$$

Maka kesalahan dengan menggunakan metode Euler adalah

$$E = \frac{16 - 4,12}{16} \times 100\% = 74,25\%$$