# **BayesPy Documentation**

Release 0.4.1

**Jaakko Luttinen** 

November 02, 2015

# CONTENTS

1	Introduction	1
	1.1 Project information	. 1
	1.2 Similar projects	. 1
	1.3 Contributors	. 2
	1.4 Version history	. 2
2	User guide	7
	2.1 Installation	. 7
	2.2 Quick start guide	. 9
	2.3 Constructing the model	
	2.4 Performing inference	. 19
	2.5 Examining the results	
	2.6 Advanced topics	. 30
3	Examples	35
	3.1 Linear regression	
	3.2 Gaussian mixture model	
	3.3 Bernoulli mixture model	
	3.4 Hidden Markov model	
	3.5 Principal component analysis	
	3.6 Linear state-space model	
	3.7 Latent Dirichlet allocation	
4	Developer guide	71
	4.1 Workflow	. 71
	4.2 Variational message passing	
	4.3 Implementing inference engines	
	4.4 Implementing nodes	. 75
5	User API	81
	5.1 bayespy.nodes	. 81
	5.2 bayespy.inference	. 194
	5.3 bayespy.plot	
6	Developer API	211
	6.1 Developer nodes	
	6.2 Moments	
	6.3 Distributions	
	6.4 Utility functions	
Bi	liography	329

Python Module Index	331
Index	333

# INTRODUCTION

BayesPy provides tools for Bayesian inference with Python. The user constructs a model as a Bayesian network, observes data and runs posterior inference. The goal is to provide a tool which is efficient, flexible and extendable enough for expert use but also accessible for more casual users.

Currently, only variational Bayesian inference for conjugate-exponential family (variational message passing) has been implemented. Future work includes variational approximations for other types of distributions and possibly other approximate inference methods such as expectation propagation, Laplace approximations, Markov chain Monte Carlo (MCMC) and other methods. Contributions are welcome.

# 1.1 Project information

Copyright (C) 2011-2015 Jaakko Luttinen and other contributors (see below)

BayesPy including the documentation is licensed under the MIT License. See LICENSE file for a text of the license or visit http://opensource.org/licenses/MIT.

- Documentation:
  - http://bayespy.org
  - PDF file
  - RST format in doc directory
- · Repository: https://github.com/bayespy/bayespy.git
- Bug reports: https://github.com/bayespy/bayespy/issues
- Mailing list: bayespy@googlegroups.com
- IRC: #bayespy @ freenode
- Author: Jaakko Luttinen jaakko.luttinen@iki.fi
- Latest release:
- Build status:
- Unit test coverage:

# 1.2 Similar projects

VIBES (http://vibes.sourceforge.net/) allows variational inference to be performed automatically on a Bayesian network. It is implemented in Java and released under revised BSD license.

Bayes Blocks (http://research.ics.aalto.fi/bayes/software/) is a C++/Python implementation of the variational building block framework. The framework allows easy learning of a wide variety of models using variational Bayesian learning. It is available as free software under the GNU General Public License.

Infer.NET (http://research.microsoft.com/infernet/) is a .NET framework for machine learning. It provides message-passing algorithms and statistical routines for performing Bayesian inference. It is partly closed source and licensed for non-commercial use only.

PyMC (https://github.com/pymc-devs/pymc) provides MCMC methods in Python. It is released under the Academic Free License.

OpenBUGS (http://www.openbugs.info) is a software package for performing Bayesian inference using Gibbs sampling. It is released under the GNU General Public License.

Dimple (http://dimple.probprog.org/) provides Gibbs sampling, belief propagation and a few other inference algorithms for Matlab and Java. It is released under the Apache License.

Stan (http://mc-stan.org/) provides inference using MCMC with an interface for R and Python. It is released under the New BSD License.

PBNT - Python Bayesian Network Toolbox (http://pbnt.berlios.de/) is Bayesian network library in Python supporting static networks with discrete variables. There was no information about the license.

# 1.3 Contributors

The list of contributors:

- · Jaakko Luttinen
- · Hannu Hartikainen

Each file or the git log can be used for more detailed information.

# 1.4 Version history

### 1.4.1 Version 0.4.1 (2015-11-02)

• Define extra dependencies needed to build the documentation

### 1.4.2 Version 0.4.0 (2015-11-02)

- Implement Add node for Gaussian nodes
- Raise error if attempting to install on Python 2
- Return both relative and absolute errors from numerical gradient checking
- Add nose plugin to filter unit test warnings appropriately

### 1.4.3 Version 0.3.9 (2015-10-16)

• Fix Gaussian ARD node sampling

# 1.4.4 Version 0.3.8 (2015-10-16)

• Fix Gaussian node sampling

# 1.4.5 Version 0.3.7 (2015-09-23)

- Enable keyword arguments when plotting via the inference engine
- Add initial support for logging

# 1.4.6 Version 0.3.6 (2015-08-12)

- · Add maximum likelihood node for the shape parameter of Gamma
- Fix Hinton diagrams for 1-D and 0-D Gaussians
- · Fix autosave interval counter
- Fix bugs in constant nodes

# 1.4.7 Version 0.3.5 (2015-06-09)

- Fix indexing bug in VB optimization (not VB-EM)
- · Fix demos

# 1.4.8 Version 0.3.4 (2015-06-09)

- Fix computation of probability density of Dirichlet nodes
- Use unit tests for all code snippets in docstrings and documentation

### 1.4.9 Version 0.3.3 (2015-06-05)

- Change license to the MIT license
- Improve SumMultiply efficiency
- · Hinton diagrams for gamma variables
- Possible to load only nodes from HDF5 results

# 1.4.10 Version 0.3.2 (2015-03-16)

- · Concatenate node added
- · Unit tests for plotting fixed

1.4. Version history

# 1.4.11 Version 0.3.1 (2015-03-12)

- Gaussian mixture 2D plotting improvements
- Covariance matrix sampling improvements
- Minor documentation fixes

# 1.4.12 Version 0.3 (2015-03-05)

- Add gradient-based optimization methods (Riemannian/natural gradient or normal)
- · Add collapsed inference
- · Add the pattern search method
- · Add deterministic annealing
- · Add stochastic variational inference
- Add optional input signals to Gaussian Markov chains
- Add unit tests for plotting functions (by Hannu Hartikainen)
- Add printing support to nodes
- Drop Python 3.2 support

# 1.4.13 Version 0.2.3 (2014-12-03)

- Fix matplotlib compatibility broken by recent changes in matplotlib
- · Add random sampling for Binomial and Bernoulli nodes
- Fix minor bugs, for instance, in plot module

### 1.4.14 Version 0.2.2 (2014-11-01)

- Fix normalization of categorical Markov chain probabilities (fixes HMM demo)
- · Fix initialization from parameter values

# 1.4.15 Version 0.2.1 (2014-09-30)

- Add workaround for matplotlib 1.4.0 bug related to interactive mode which affected monitoring
- · Fix bugs in Hinton diagrams for Gaussian variables

# 1.4.16 Version 0.2 (2014-08-06)

- Added all remaining common distributions: Bernoulli, binomial, multinomial, Poisson, beta, exponential.
- Added Gaussian arrays (not just scalars or vectors).
- Added Gaussian Markov chains with time-varying or swithing dynamics.
- Added discrete Markov chains (enabling hidden Markov models).
- Added joint Gaussian-Wishart and Gaussian-gamma nodes.

- Added deterministic gating node.
- Added deterministic general sum-product node.
- Added parameter expansion for Gaussian arrays and time-varying/switching Gaussian Markov chains.
- Added new plotting functions: pdf, Hinton diagram.
- Added monitoring of posterior distributions during iteration.
- · Finished documentation and added API.

# 1.4.17 Version 0.1 (2013-07-25)

- Added variational message passing inference engine.
- · Added the following common distributions: Gaussian vector, gamma, Wishart, Dirichlet, categorical.
- · Added Gaussian Markov chain.
- Added parameter expansion for Gaussian vectors and Gaussian Markov chain.
- Added stochastic mixture node.
- Added deterministic dot product node.
- Created preliminary version of the documentation.

**CHAPTER** 

**TWO** 

# **USER GUIDE**

# 2.1 Installation

BayesPy is a Python 3 package and it can be installed from PyPI or the latest development version from GitHub. The instructions below explain how to set up the system by installing required packages, how to install BayesPy and how to compile this documentation yourself. However, if these instructions contain errors or some relevant details are missing, please file a bug report at https://github.com/bayespy/bayespy/issues.

# 2.1.1 Installing BayesPy

BayesPy can be installed easily by using Pip if the system has been properly set up. If you have problems with the following methods, see the following section for some help on installing the requirements. For instance, a bug in recent versions of h5py and pip may require you to install some of the requirements manually.

#### For users

First, you may want to set up a virtual environment. Using virtual environment is optional but recommended. To create and activate a new virtual environment, run (in the folder in which you want to create the environment):

```
virtualenv -p python3 --system-site-packages ENV source ENV/bin/activate
```

The latest release of BayesPy can be installed from PyPI simply as

```
pip install bayespy
```

If you want to install the latest development version of BayesPy, use GitHub instead:

```
pip install https://github.com/bayespy/bayespy/archive/master.zip
```

#### For developers

If you want to install the development version of BayesPy in such a way that you can easily edit the package, follow these instructions. Get the git repository:

```
git clone https://github.com/bayespy/bayespy.git
cd bayespy
```

Create and activate a new virtual environment (optional but recommended):

```
virtualenv -p python3 --system-site-packages ENV source ENV/bin/activate
```

### Install BayesPy in editable mode:

```
pip install -e .
```

### **Checking installation**

If you have problems installing BayesPy, read the next section for more details. It is recommended to run the unit tests in order to check that BayesPy is working properly. Thus, install Nose and run the unit tests:

```
pip install nose
nosetests bayespy
```

# 2.1.2 Installing requirements

BayesPy requires Python 3.3 (or later) and the following packages:

- NumPy (>=1.8.0),
- SciPy (>=0.13.0)
- matplotlib (>=1.2)
- h5py

Ideally, Pip should install the necessary requirements and a manual installation of these dependencies is not required. However, there are several reasons why the installation of these dependencies needs to be done manually in some cases. Thus, this section tries to give some details on how to set up your system. A proper installation of the dependencies for Python 3 can be a bit tricky and you may refer to http://www.scipy.org/install.html for more detailed instructions about the SciPy stack. Detailed instructions on installing recent SciPy stack for various platforms is out of the scope of these instructions, but we provide some general guidance here. There are basically three ways to install the dependencies:

- 1. Install a Python distribution which includes the packages. For Windows, Mac and Linux, there are several Python distributions which include all the necessary packages: http://www.scipy.org/install.html#scientific-python-distributions. For instance, you may try Anaconda or Enthought.
- 2. Install the packages using the system package manager. On Linux, the packages might be called something like python-scipy or scipy. However, it is possible that these system packages are not recent enough for BayesPy.
- 3. Install the packages using Pip:

```
pip install "distribute>=0.6.28"
pip install "numpy>=1.8.0" "scipy>=0.13.0" "matplotlib>=1.2" h5py
```

This also makes sure you have recent enough version of Distribute (required by Matplotlib). However, this installation method may require that the system has some libraries needed for compiling (e.g., C compiler, Python development files, BLAS/LAPACK). For instance, on Ubuntu (>= 12.10), you may install the required system libraries for each package as:

```
sudo apt-get build-dep python3-numpy
sudo apt-get build-dep python3-scipy
sudo apt-get build-dep python3-matplotlib
sudo apt-get build-dep python-h5py
```

Then installation using Pip should work.

# 2.1.3 Compiling documentation

This documentation can be found at http://bayespy.org/ in HTML and PDF formats. The documentation source files are also readable as such in reStructuredText format in doc/source/ directory. It is possible to compile the documentation into HTML or PDF yourself. In order to compile the documentation, Sphinx is required and a few extensions for it. Those can be installed as:

```
pip install "sphinx>=1.2.3" sphinxcontrib-tikz sphinxcontrib-bayesnet sphinxcontrib-bibtex "numpydoc
```

Or you can simply install BayesPy with doc extra, which will take care of installing the required dependencies:

```
pip install bayespy[doc]
```

In order to visualize graphical models in HTML, you need to have ImageMagick or Netpbm installed. The documentation can be compiled to HTML and PDF by running the following commands in the doc directory:

```
make html
make latexpdf
```

You can also run doctest to test code snippets in the documentation:

```
make doctest
```

or in the docstrings:

```
nosetests --with-doctest --doctest-options="+ELLIPSIS" bayespy
```

# 2.2 Quick start guide

This short guide shows the key steps in using BayesPy for variational Bayesian inference by applying BayesPy to a simple problem. The key steps in using BayesPy are the following:

- Construct the model
- Observe some of the variables by providing the data in a proper format
- · Run variational Bayesian inference
- Examine the resulting posterior approximation

To demonstrate BayesPy, we'll consider a very simple problem: we have a set of observations from a Gaussian distribution with unknown mean and variance, and we want to learn these parameters. In this case, we do not use any real-world data but generate some artificial data. The dataset consists of ten samples from a Gaussian distribution with mean 5 and standard deviation 10. This dataset can be generated with NumPy as follows:

```
>>> import numpy as np
>>> data = np.random.normal(5, 10, size=(10,))
```

# 2.2.1 Constructing the model

Now, given this data we would like to estimate the mean and the standard deviation as if we didn't know their values. The model can be defined as follows:

$$p(\mathbf{y}|\mu,\tau) = \prod_{n=0}^{9} \mathcal{N}(y_n|\mu,\tau)$$
$$p(\mu) = \mathcal{N}(\mu|0, 10^{-6})$$
$$p(\tau) = \mathcal{G}(\tau|10^{-6}, 10^{-6})$$

where  $\mathcal{N}$  is the Gaussian distribution parameterized by its mean and precision (i.e., inverse variance), and  $\mathcal{G}$  is the gamma distribution parameterized by its shape and rate parameters. Note that we have given quite uninformative priors for the variables  $\mu$  and  $\tau$ . This simple model can also be shown as a directed factor graph: This model can be

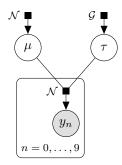


Fig. 2.1: Directed factor graph of the example model.

constructed in BayesPy as follows:

```
>>> from bayespy.nodes import GaussianARD, Gamma
>>> mu = GaussianARD(0, 1e-6)
>>> tau = Gamma(1e-6, 1e-6)
>>> y = GaussianARD(mu, tau, plates=(10,))
```

This is quite self-explanatory given the model definitions above. We have used two types of nodes GaussianARD and Gamma to represent Gaussian and gamma distributions, respectively. There are much more distributions in bayespy.nodes so you can construct quite complex conjugate exponential family models. The node y uses keyword argument plates to define the plates  $n=0,\ldots,9$ .

### 2.2.2 Performing inference

Now that we have created the model, we can provide our data by setting y as observed:

```
>>> y.observe(data)
```

Next we want to estimate the posterior distribution. In principle, we could use different inference engines (e.g., MCMC or EP) but currently only variational Bayesian (VB) engine is implemented. The engine is initialized by giving all the nodes of the model:

```
>>> from bayespy.inference import VB
>>> Q = VB(mu, tau, y)
```

The inference algorithm can be run as long as wanted (max. 20 iterations in this case):

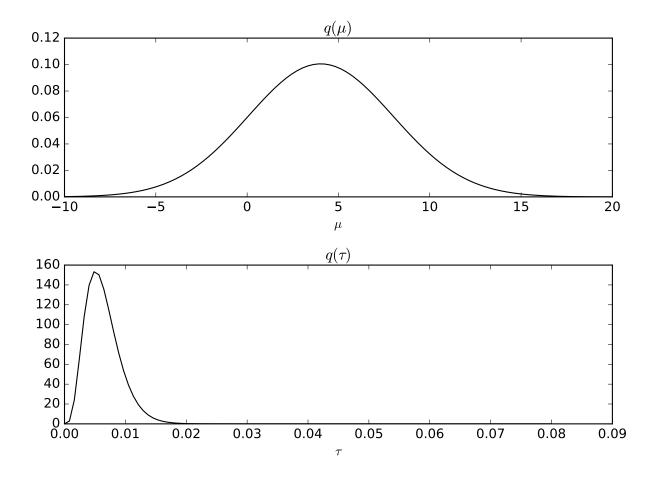
```
>>> Q.update(repeat=20)
Iteration 1: loglike=-6.020956e+01 (... seconds)
Iteration 2: loglike=-5.820527e+01 (... seconds)
Iteration 3: loglike=-5.820290e+01 (... seconds)
Iteration 4: loglike=-5.820288e+01 (... seconds)
Converged at iteration 4.
```

Now the algorithm converged after four iterations, before the requested 20 iterations. VB approximates the true posterior  $p(\mu, \tau | \mathbf{y})$  with a distribution which factorizes with respect to the nodes:  $q(\mu)q(\tau)$ .

# 2.2.3 Examining posterior approximation

The resulting approximate posterior distributions  $q(\mu)$  and  $q(\tau)$  can be examined, for instance, by plotting the marginal probability density functions:

```
>>> import bayespy.plot as bpplt
>>> bpplt.pyplot.subplot(2, 1, 1)
<matplotlib.axes...AxesSubplot object at 0x...>
>>> bpplt.pdf(mu, np.linspace(-10, 20, num=100), color='k', name=r'\mu')
[<matplotlib.lines.Line2D object at 0x...>]
>>> bpplt.pyplot.subplot(2, 1, 2)
<matplotlib.axes...AxesSubplot object at 0x...>
>>> bpplt.pdf(tau, np.linspace(1e-6, 0.08, num=100), color='k', name=r'\tau')
[<matplotlib.lines.Line2D object at 0x...>]
>>> bpplt.pyplot.tight_layout()
>>> bpplt.pyplot.show()
```



This example was a very simple introduction to using BayesPy. The model can be much more complex and each phase contains more options to give the user more control over the inference. The following sections give more details about the phases.

# 2.3 Constructing the model

In BayesPy, the model is constructed by creating nodes which form a directed network. There are two types of nodes: stochastic and deterministic. A stochastic node corresponds to a random variable (or a set of random variables) from a specific probability distribution. A deterministic node corresponds to a deterministic function of its parents. For a list of built-in nodes, see the *User API*.

# 2.3.1 Creating nodes

Creating a node is basically like writing the conditional prior distribution of the variable in Python. The node is constructed by giving the parent nodes, that is, the conditioning variables as arguments. The number of parents and their meaning depend on the node. For instance, a <code>Gaussian</code> node is created by giving the mean vector and the precision matrix. These parents can be constant numerical arrays if they are known:

```
>>> from bayespy.nodes import Gaussian
>>> X = Gaussian([2, 5], [[1.0, 0.3], [0.3, 1.0]])
```

or other nodes if they are unknown and given prior distributions:

```
>>> from bayespy.nodes import Gaussian, Wishart
>>> mu = Gaussian([0, 0], [[1e-6, 0],[0, 1e-6]])
>>> Lambda = Wishart(2, [[1, 0], [0, 1]])
>>> X = Gaussian(mu, Lambda)
```

Nodes can also be named by providing name keyword argument:

```
>>> X = Gaussian(mu, Lambda, name='x')
```

The name may be useful when referring to the node using an inference engine.

For the parent nodes, there are two main restrictions: non-constant parent nodes must be conjugate and the parent nodes must be mutually independent in the posterior approximation.

### Conjugacy of the parents

In Bayesian framework in general, one can give quite arbitrary probability distributions for variables. However, one often uses distributions that are easy to handle in practice. Quite often this means that the parents are given conjugate priors. This is also one of the limitations in BayesPy: only conjugate family prior distributions are accepted currently. Thus, although in principle one could give, for instance, gamma prior for the mean parameter mu, only Gaussian-family distributions are accepted because of the conjugacy. If the parent is not of a proper type, an error is raised. This conjugacy is checked automatically by BayesPy and NoConverterError is raised if a parent cannot be interpreted as being from a conjugate distribution.

### Independence of the parents

Another a bit rarely encountered limitation is that the parents must be mutually independent (in the posterior factorization). Thus, a node cannot have the same stochastic node as several parents without intermediate stochastic nodes. For instance, the following leads to an error:

```
>>> from bayespy.nodes import Dot
>>> Y = Dot(X, X)
Traceback (most recent call last):
...
ValueError: Parent nodes are not independent
```

The error is raised because X is given as two parents for Y, and obviously X is not independent of X in the posterior approximation. Even if X is not given several times directly but there are some intermediate deterministic nodes, an error is raised because the deterministic nodes depend on their parents and thus the parents of Y would not be independent. However, it is valid that a node is a parent of another node via several paths if all the paths or all except one path has intermediate stochastic nodes. This is valid because the intermediate stochastic nodes have independent posterior approximations. Thus, for instance, the following construction does not raise errors:

```
>>> from bayespy.nodes import Dot
>>> Z = Gaussian(X, [[1,0], [0,1]])
>>> Y = Dot(X, Z)
```

This works because there is now an intermediate stochastic node Z on the other path from X node to Y node.

### 2.3.2 Effects of the nodes on inference

When constructing the network with nodes, the stochastic nodes actually define three important aspects:

- 1. the prior probability distribution for the variables,
- 2. the factorization of the posterior approximation,

3. the functional form of the posterior approximation for the variables.

#### Prior probability distribution

First, the most intuitive feature of the nodes is that they define the prior distribution. In the previous example, mu was a stochastic GaussianARD node corresponding to  $\mu$  from the normal distribution, tau was a stochastic Gamma node corresponding to  $\tau$  from the gamma distribution, and y was a stochastic GaussianARD node corresponding to y from the normal distribution with mean  $\mu$  and precision  $\tau$ . If we denote the set of all stochastic nodes by  $\Omega$ , and by  $\pi_X$  the set of parents of a node X, the model is defined as

$$p(\Omega) = \prod_{X \in \Omega} p(X|\pi_X),$$

where nodes correspond to the terms  $p(X|\pi_X)$ .

#### Posterior factorization

Second, the nodes define the structure of the posterior approximation. The variational Bayesian approximation factorizes with respect to nodes, that is, each node corresponds to an independent probability distribution in the posterior approximation. In the previous example, mu and tau were separate nodes, thus the posterior approximation factorizes with respect to them:  $q(\mu)q(\tau)$ . Thus, the posterior approximation can be written as:

$$p(\tilde{\Omega}|\hat{\Omega}) \approx \prod_{X \in \tilde{\Omega}} q(X),$$

where  $\tilde{\Omega}$  is the set of latent stochastic nodes and  $\hat{\Omega}$  is the set of observed stochastic nodes. Sometimes one may want to avoid the factorization between some variables. For this purpose, there are some nodes which model several variables jointly without factorization. For instance, GaussianGammaISO is a joint node for  $\mu$  and  $\tau$  variables from the normal-gamma distribution and the posterior approximation does not factorize between  $\mu$  and  $\tau$ , that is, the posterior approximation is  $q(\mu, \tau)$ .

#### Functional form of the posterior

Last, the nodes define the functional form of the posterior approximation. Usually, the posterior approximation has the same or similar functional form as the prior. For instance, <code>Gamma</code> uses gamma distribution to also approximate the posterior distribution. Similarly, <code>GaussianARD</code> uses Gaussian distribution for the posterior. However, the posterior approximation of <code>GaussianARD</code> uses a full covariance matrix although the prior assumes a diagonal covariance matrix. Thus, there can be slight differences in the exact functional form of the posterior approximation but the rule of thumb is that the functional form of the posterior approximation is the same as or more general than the functional form of the prior.

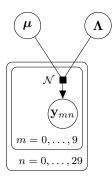
# 2.3.3 Using plate notation

#### **Defining plates**

Stochastic nodes take the optional parameter plates, which can be used to define plates of the variable. A plate defines the number of repetitions of a set of variables. For instance, a set of random variables  $y_{mn}$  could be defined as

$$\mathbf{y}_{mn} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Lambda}), \qquad m = 0, \dots, 9, \quad n = 0, \dots, 29.$$

This can also be visualized as a graphical model:



The variable has two plates: one for the index m and one for the index n. In BayesPy, this random variable can be constructed as:

```
>>> y = Gaussian(mu, Lambda, plates=(10,30))
```

**Note:** The plates are always given as a tuple of positive integers.

Plates also define indexing for the nodes, thus you can use simple NumPy-style slice indexing to obtain a subset of the plates:

```
>>> y_0 = y[0]
>>> y_0.plates
(30,)
>>> y_even = y[:,::2]
>>> y_even.plates
(10, 15)
>>> y_complex = y[:5, 10:20:5]
>>> y_complex.plates
(5, 2)
```

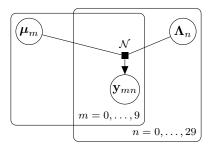
Note that this indexing is for the plates only, not for the random variable dimensions.

### Sharing and broadcasting plates

Instead of having a common mean and precision matrix for all  $y_{mn}$ , it is also possible to share plates with parents. For instance, the mean could be different for each index m and the precision for each index n:

$$\mathbf{y}_{mn} \sim \mathcal{N}(\boldsymbol{\mu}_m, \boldsymbol{\Lambda}_n), \qquad m = 0, \dots, 9, \quad n = 0, \dots, 29.$$

which has the following graphical representation:



This can be constructed in BayesPy, for instance, as:

```
>>> from bayespy.nodes import Gaussian, Wishart
>>> mu = Gaussian([0, 0], [[1e-6, 0], [0, 1e-6]], plates=(10,1))
>>> Lambda = Wishart(2, [[1, 0], [0, 1]], plates=(1,30))
>>> X = Gaussian(mu, Lambda)
```

There are a few things to notice here. First, the plates are defined similarly as shapes in NumPy, that is, they use similar broadcasting rules. For instance, the plates (10,1) and (1,30) broadcast to (10,30). In fact, one could use plates (10,1) and (30,) to get the broadcasted plates (10,30) because broadcasting compares the plates from right to left starting from the last axis. Second, X is not given plates keyword argument because the default plates are the plates broadcasted from the parents and that was what we wanted so it was not necessary to provide the keyword argument. If we wanted, for instance, plates (20,10,30) for X, then we would have needed to provide plates=(20,10,30).

The validity of the plates between a child and its parents is checked as follows. The plates are compared plate-wise starting from the last axis and working the way forward. A plate of the child is compatible with a plate of the parent if either of the following conditions is met:

- 1. The two plates have equal size
- 2. The parent has size 1 (or no plate)

Table below shows an example of compatible plates for a child node and its two parent nodes:

node	plates						
parent1		3	1	1	1	8	10
parent2			1	1	5	1	10
child	5	3	1	7	5	8	10

#### Plates in deterministic nodes

Note that plates can be defined explicitly only for stochastic nodes. For deterministic nodes, the plates are defined implicitly by the plate broadcasting rules from the parents. Deterministic nodes do not need more plates than this because there is no randomness. The deterministic node would just have the same value over the extra plates, but it is not necessary to do this explicitly because the child nodes of the deterministic node can utilize broadcasting anyway. Thus, there is no point in having extra plates in deterministic nodes, and for this reason, deterministic nodes do not use plates keyword argument.

#### Plates in constants

It is useful to understand how the plates and the shape of a random variable are connected. The shape of an array which contains all the plates of a random variable is the concatenation of the plates and the shape of the variable. For instance, consider a 2-dimensional Gaussian variable with plates (3,). If you want the value of the constant mean vector and constant precision matrix to vary between plates, they are given as (3,2)-shape and (3,2,2)-shape arrays, respectively:

```
>>> import numpy as np
\rightarrow \rightarrow mu = [[0,0], [1,1], [2,2]]
>>> Lambda = [ [[1.0, 0.0],
                  [0.0, 1.0]],
                 [[1.0, 0.9],
. . .
                  [0.9, 1.0]],
. . .
                 [[1.0, -0.3],
                  [-0.3, 1.0]]
. . .
>>> X = Gaussian(mu, Lambda)
>>> np.shape(mu)
(3, 2)
>>> np.shape(Lambda)
(3, 2, 2)
>>> X.plates
(3,)
```

Thus, the leading axes of an array are the plate axes and the trailing axes are the random variable axes. In the example above, the mean vector has plates (3,) and shape (2,2), and the precision matrix has plates (3,) and shape (2,2).

#### **Factorization of plates**

It is important to undestand the independency structure the plates induce for the model. First, the repetitions defined by a plate are independent a priori given the parents. Second, the repetitions are independent in the posterior approximation, that is, the posterior approximation factorizes with respect to plates. Thus, the plates also have an effect on the independence structure of the posterior approximation, not only prior. If dependencies between a set of variables need to be handled, that set must be handled as a some kind of multi-dimensional variable.

### Irregular plates

The handling of plates is not always as simple as described above. There are cases in which the plates of the parents do not map directly to the plates of the child node. The user API should mention such irregularities.

For instance, the parents of a mixture distribution have a plate which contains the different parameters for each cluster, but the variable from the mixture distribution does not have that plate:

```
>>> from bayespy.nodes import Gaussian, Wishart, Categorical, Mixture
>>> mu = Gaussian([[0], [0], [0]], [[[1]], [[1]], [[1]]])
>>> Lambda = Wishart(1, [ [[1]], [[1]]], [[1]]])
>>> Z = Categorical([1/3, 1/3, 1/3], plates=(100,))
>>> X = Mixture(Z, Gaussian, mu, Lambda)
>>> mu.plates
(3,)
>>> Lambda.plates
(3,)
>>> Z.plates
(100,)
>>> X.plates
(100,)
```

The plates (3,) and (100,) should not broadcast according to the rules mentioned above. However, when validating the plates, *Mixture* removes the plate which corresponds to the clusters in mu and Lambda. Thus, X has plates which are the result of broadcasting plates () and (100,) which equals (100,).

Also, sometimes the plates of the parents may be mapped to the variable axes. For instance, an automatic relevance determination (ARD) prior for a Gaussian variable is constructed by giving the diagonal elements of the precision matrix (or tensor). The Gaussian variable itself can be a scalar, a vector, a matrix or a tensor. A set of five  $4 \times 3$  -dimensional Gaussian matrices with ARD prior is constructed as:

```
>>> from bayespy.nodes import GaussianARD, Gamma
>>> tau = Gamma(1, 1, plates=(5,4,3))
>>> X = GaussianARD(0, tau, shape=(4,3))
>>> tau.plates
(5, 4, 3)
>>> X.plates
(5,)
```

Note how the last two plate axes of tau are mapped to the variable axes of X with shape (4,3) and the plates of X are obtained by taking the remaining leading plate axes of tau.

# 2.3.4 Example model: Principal component analysis

Now, we'll construct a bit more complex model which will be used in the following sections. The model is a probabilistic version of principal component analysis (PCA):

$$\mathbf{Y} = \mathbf{C}\mathbf{X}^T + \text{noise}$$

where  $\mathbf{Y}$  is  $M \times N$  data matrix,  $\mathbf{C}$  is  $M \times D$  loading matrix,  $\mathbf{X}$  is  $N \times D$  state matrix, and noise is isotropic Gaussian. The dimensionality D is usually assumed to be much smaller than M and N.

A probabilistic formulation can be written as:

$$p(\mathbf{Y}) = \prod_{m=0}^{M-1} \prod_{n=0}^{N-1} \mathcal{N}(y_{mn} | \mathbf{c}_m^T \mathbf{x}_n, \tau)$$

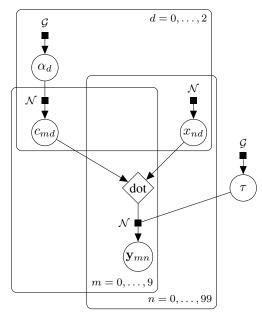
$$p(\mathbf{X}) = \prod_{n=0}^{N-1} \prod_{d=0}^{D-1} \mathcal{N}(x_{nd} | 0, 1)$$

$$p(\mathbf{C}) = \prod_{m=0}^{M-1} \prod_{d=0}^{D-1} \mathcal{N}(c_{md} | 0, \alpha_d)$$

$$p(\boldsymbol{\alpha}) = \prod_{d=0}^{D-1} \mathcal{G}(\alpha_d | 10^{-3}, 10^{-3})$$

$$p(\tau) = \mathcal{G}(\tau | 10^{-3}, 10^{-3})$$

where we have given automatic relevance determination (ARD) prior for C. This can be visualized as a graphical model:



Now, let us construct this model in BayesPy. First, we'll define the dimensionality of the latent space in our model:

```
>>> D = 3
```

Then the prior for the latent states **X**:

```
>>> X = GaussianARD(0, 1,
... shape=(D,),
... plates=(1,100),
... name='X')
```

Note that the shape of X is (D, ), although the latent dimensions are marked with a plate in the graphical model and they are conditionally independent in the prior. However, we want to (and need to) model the posterior dependency of the latent dimensions, thus we cannot factorize them, which would happen if we used plates=(1,100,D) and shape=(). The first plate axis with size 1 is given just for clarity.

The prior for the ARD parameters  $\alpha$  of the loading matrix:

```
>>> alpha = Gamma(1e-3, 1e-3, ... plates=(D,), ... name='alpha')
```

The prior for the loading matrix C:

```
>>> C = GaussianARD(0, alpha,
... shape=(D,),
... plates=(10,1),
... name='C')
```

Again, note that the shape is the same as for X for the same reason. Also, the plates of alpha, (D, ), are mapped to the full shape of the node C, (10, 1, D), using standard broadcasting rules.

The dot product is just a deterministic node:

```
>>> F = Dot(C, X)
```

However, note that Dot requires that the input Gaussian nodes have the same shape and that this shape has exactly one axis, that is, the variables are vectors. This the reason why we used shape (D, ) for X and C but from a bit different perspective. The node computes the inner product of D-dimensional vectors resulting in plates (10,100) broadcasted from the plates (1,100) and (10,1):

```
>>> F.plates (10, 100)
```

The prior for the observation noise  $\tau$ :

```
>>> tau = Gamma(1e-3, 1e-3, name='tau')
```

Finally, the observations are conditionally independent Gaussian scalars:

```
>>> Y = GaussianARD(F, tau, name='Y')
```

Now we have defined our model and the next step is to observe some data and to perform inference.

# 2.4 Performing inference

Approximation of the posterior distribution can be divided into several steps:

- Observe some nodes
- Choose the inference engine
- Initialize the posterior approximation
- Run the inference algorithm

In order to illustrate these steps, we'll be using the PCA model constructed in the previous section.

# 2.4.1 Observing nodes

First, let us generate some toy data:

```
>>> c = np.random.randn(10, 2)

>>> x = np.random.randn(2, 100)

>>> data = np.dot(c, x) + 0.1*np.random.randn(10, 100)
```

The data is provided by simply calling observe method of a stochastic node:

```
>>> Y.observe(data)
```

It is important that the shape of the data array matches the plates and shape of the node Y. For instance, if Y was Wishart node for  $3 \times 3$  matrices with plates (5, 1, 10), the full shape of Y would be (5, 1, 10, 3, 3). The data array should have this shape exactly, that is, no broadcasting rules are applied.

#### Missing values

It is possible to mark missing values by providing a mask which is a boolean array:

```
>>> Y.observe(data, mask=[[True], [False], [False], [True], [True], ... [False], [True], [True], [False]])
```

True means that the value is observed and False means that the value is missing. The shape of the above mask is (10,1), which broadcasts to the plates of Y, (10,100). Thus, the above mask means that the second, third, sixth and tenth rows of the  $10 \times 100$  data matrix are missing.

The mask is applied to the *plates*, not to the data array directly. This means that it is not possible to observe a random variable partially, each repetition defined by the plates is either fully observed or fully missing. Thus, the mask is applied to the plates. It is often possible to circumvent this seemingly tight restriction by adding an observable child node which factorizes more.

The shape of the mask is broadcasted to plates using standard NumPy broadcasting rules. So, if the variable has plates (5,1,10), the mask could have a shape (),(1,),(1,1),(1,1,1),(10,),(1,10),(1,10),(5,1,1) or (5,1,10). In order to speed up the inference, missing values are automatically integrated out if they are not needed as latent variables to child nodes. This leads to faster convergence and more accurate approximations.

# 2.4.2 Choosing the inference method

Inference methods can be found in <code>bayespy.inference</code> package. Currently, only variational Bayesian approximation is implemented (<code>bayespy.inference.VB</code>). The inference engine is constructed by giving the stochastic nodes of the model.

```
>>> from bayespy.inference import VB
>>> Q = VB(Y, C, X, alpha, tau)
```

There is no need to give any deterministic nodes. Currently, the inference engine does not automatically search for stochastic parents and children, thus it is important that all stochastic nodes of the model are given. This should be made more robust in future versions.

A node of the model can be obtained by using the name of the node as a key:

```
>>> Q['X'] <bayespy.inference.vmp.nodes.gaussian.GaussianARD object at 0x...>
```

Note that the returned object is the same as the node object itself:

```
>>> Q['X'] is X
True
```

Thus, one may use the object X when it is available. However, if the model and the inference engine are constructed in another function or module, the node object may not be available directly and this feature becomes useful.

# 2.4.3 Initializing the posterior approximation

The inference engines give some initialization to the stochastic nodes by default. However, the inference algorithms can be sensitive to the initialization, thus it is sometimes necessary to have better control over the initialization. For VB, the following initialization methods are available:

- initialize\_from\_prior: Use the current states of the parent nodes to update the node. This is the default initialization.
- initialize\_from\_parameters: Use the given parameter values for the distribution.
- initialize\_from\_value: Use the given value for the variable.
- initialize\_from\_random: Draw a random value for the variable. The random sample is drawn from the current state of the node's distribution.

Note that initialize\_from\_value and initialize\_from\_random initialize the distribution with a value of the variable instead of parameters of the distribution. Thus, the distribution is actually a delta distribution with a peak on the value after the initialization. This state of the distribution does not have proper natural parameter values nor normalization, thus the VB lower bound terms are np.nan for this initial state.

These initialization methods can be used to perform even a bit more complex initializations. For instance, a Gaussian distribution could be initialized with a random mean and variance 0.1. In our PCA model, this can be obtained by

```
>>> X.initialize_from_parameters(np.random.randn(1, 100, D), 10)
```

Note that the shape of the random mean is the sum of the plates (1, 100) and the variable shape (D,). In addition, instead of variance, GaussianARD uses precision as the second parameter, thus we initialized the variance to  $\frac{1}{10}$ . This random initialization is important in our PCA model because the default initialization gives C and X zero mean. If the mean of the other variable was zero when the other is updated, the other variable gets zero mean too. This would lead to an update algorithm where both means remain zeros and effectively no latent space is found. Thus, it is important to give non-zero random initialization for X if C is updated before X the first time. It is typical that at least some nodes need be initialized with some randomness.

By default, nodes are initialized with the method initialize\_from\_prior. The method is not very time consuming but if for any reason you want to avoid that default initialization computation, you can provide initialize=False when creating the stochastic node. However, the node does not have a proper state in that case, which leads to errors in VB learning unless the distribution is initialized using the above methods.

# 2.4.4 Running the inference algorithm

The approximation methods are based on iterative algorithms, which can be run using update method. By default, it takes one iteration step updating all nodes once:

```
>>> Q.update()
Iteration 1: loglike=-9.305259e+02 (... seconds)
```

The loglike tells the VB lower bound. The order in which the nodes are updated is the same as the order in which the nodes were given when creating Q. If you want to change the order or update only some of the nodes, you can give as arguments the nodes you want to update and they are updated in the given order:

```
>>> Q.update(C, X)
Iteration 2: loglike=-8.818976e+02 (... seconds)
```

It is also possible to give the same node several times:

```
>>> Q.update(C, X, C, tau)
Iteration 3: loglike=-8.071222e+02 (... seconds)
```

Note that each call to update is counted as one iteration step although not variables are necessarily updated. Instead of doing one iteration step, repeat keyword argument can be used to perform several iteration steps:

```
>>> Q.update(repeat=10)

Iteration 4: loglike=-7.167588e+02 (... seconds)

Iteration 5: loglike=-6.827873e+02 (... seconds)

Iteration 6: loglike=-6.259477e+02 (... seconds)

Iteration 7: loglike=-4.725400e+02 (... seconds)

Iteration 8: loglike=-3.270816e+02 (... seconds)

Iteration 9: loglike=-2.208865e+02 (... seconds)

Iteration 10: loglike=-1.658761e+02 (... seconds)

Iteration 11: loglike=-1.469468e+02 (... seconds)

Iteration 12: loglike=-1.420311e+02 (... seconds)

Iteration 13: loglike=-1.405139e+02 (... seconds)
```

The VB algorithm stops automatically if it converges, that is, the relative change in the lower bound is below some threshold:

```
>>> Q.update(repeat=1000)
Iteration 14: loglike=-1.396481e+02 (... seconds)
...
Iteration 488: loglike=-1.224106e+02 (... seconds)
Converged at iteration 488.
```

Now the algorithm stopped before taking 1000 iteration steps because it converged. The relative tolerance can be adjusted by providing tol keyword argument to the update method:

```
>>> Q.update(repeat=10000, tol=1e-6)
Iteration 489: loglike=-1.224094e+02 (... seconds)
...
Iteration 847: loglike=-1.222506e+02 (... seconds)
Converged at iteration 847.
```

Making the tolerance smaller, may improve the result but it may also significantly increase the iteration steps until convergence.

Instead of using update method of the inference engine VB, it is possible to use the update methods of the nodes directly as

```
>>> C.update()
or
```

```
>>> Q['C'].update()
```

However, this is not recommended, because the update method of the inference engine VB is a wrapper which, in addition to calling the nodes' update methods, checks for convergence and does a few other useful minor things. But if for any reason these direct update methods are needed, they can be used.

#### **Parameter expansion**

Sometimes the VB algorithm converges very slowly. This may happen when the variables are strongly coupled in the true posterior but factorized in the approximate posterior. This coupling leads to zigzagging of the variational parameters which progresses slowly. One solution to this problem is to use parameter expansion. The idea is to add an auxiliary variable which parameterizes the posterior approximation of several variables. Then optimizing this auxiliary variable actually optimizes several posterior approximations jointly leading to faster convergence.

The parameter expansion is model specific. Currently in BayesPy, only state-space models have built-in parameter expansions available. These state-space models contain a variable which is a dot product of two variables (plus some noise):

$$y = \mathbf{c}^T \mathbf{x} + \text{noise}$$

The parameter expansion can be motivated by noticing that we can add an auxiliary variable which rotates the variables c and x so that the dot product is unaffected:

$$y = \mathbf{c}^T \mathbf{x} + \text{noise} = \mathbf{c}^T \mathbf{R} \mathbf{R}^{-1} \mathbf{x} + \text{noise} = (\mathbf{R}^T \mathbf{c})^T (\mathbf{R}^{-1} \mathbf{x}) + \text{noise}$$

Now, applying this rotation to the posterior approximations  $q(\mathbf{c})$  and  $q(\mathbf{x})$ , and optimizing the VB lower bound with respect to the rotation leads to parameterized joint optimization of  $\mathbf{c}$  and  $\mathbf{x}$ .

The available parameter expansion methods are in module transformations:

```
>>> from bayespy.inference.vmp import transformations
```

First, you create the rotation transformations for the two variables:

```
>>> rotX = transformations.RotateGaussianARD(X)
>>> rotC = transformations.RotateGaussianARD(C, alpha)
```

Here, the rotation for C provides the ARD parameters alpha so they are updated simultaneously. In addition to RotateGaussianARD, there are a few other built-in rotations defined, for instance, RotateGaussian and RotateGaussianMarkovChain. It is extremely important that the model satisfies the assumptions made by the rotation class and the user is mostly responsible for this. The optimizer for the rotations is constructed by giving the two rotations and the dimensionality of the rotated space:

```
>>> R = transformations.RotationOptimizer(rotC, rotX, D)
```

Now, calling rotate method will find optimal rotation and update the relevant nodes (X, C and alpha) accordingly:

```
>>> R.rotate()
```

Let us see how our iteration would have gone if we had used this parameter expansion. First, let us re-initialize our nodes and VB algorithm:

```
>>> alpha.initialize_from_prior()
>>> C.initialize_from_prior()
>>> X.initialize_from_parameters(np.random.randn(1, 100, D), 10)
>>> tau.initialize_from_prior()
>>> Q = VB(Y, C, X, alpha, tau)
```

Then, the rotation is set to run after each iteration step:

```
>>> Q.callback = R.rotate
```

Now the iteration converges to the relative tolerance  $10^{-6}$  much faster:

```
>>> Q.update(repeat=1000, tol=1e-6)
Iteration 1: loglike=-9.363500e+02 (... seconds)
...
Iteration 18: loglike=-1.221354e+02 (... seconds)
Converged at iteration 18.
```

The convergence took 18 iterations with rotations and 488 or 847 iterations without the parameter expansion. In addition, the lower bound is improved slightly. One can compare the number of iteration steps in this case because the cost per iteration step with or without parameter expansion is approximately the same. Sometimes the parameter expansion can have the drawback that it converges to a bad local optimum. Usually, this can be solved by updating the nodes near the observations a few times before starting to update the hyperparameters and to use parameter expansion. In any case, the parameter expansion is practically necessary when using state-space models in order to converge to a proper solution in a reasonable time.

# 2.5 Examining the results

After the results have been obtained, it is important to be able to examine the results easily. The results can be examined either numerically by inspecting numerical arrays or visually by plotting distributions of the nodes. In addition, the posterior distributions can be visualized during the learning algorithm and the results can saved into a file.

# 2.5.1 Plotting the results

The module plot offers some plotting basic functionality:

```
>>> import bayespy.plot as bpplt
```

The module contains matplotlib.pyplot module if the user needs that. For instance, interactive plotting can be enabled as:

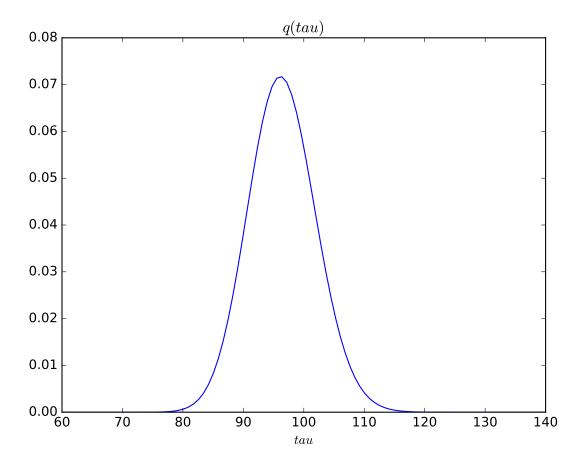
```
>>> bpplt.pyplot.ion()
```

The plot module contains some functions but it is not a very comprehensive collection, thus the user may need to write some problem- or model-specific plotting functions. The current collection is:

- pdf(): show probability density function of a scalar
- contour(): show probability density function of two-element vector
- hinton(): show the Hinton diagram
- plot (): show value as a function

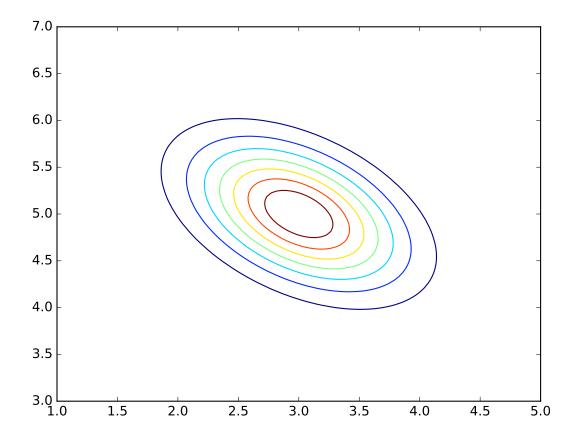
The probability density function of a scalar random variable can be plotted using the function pdf():

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.pdf(Q['tau'], np.linspace(60, 140, num=100))
[<matplotlib.lines.Line2D object at 0x...>]
```



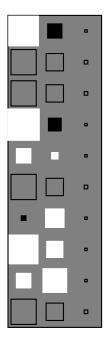
The variable tau models the inverse variance of the noise, for which the true value is  $0.1^{-2} = 100$ . Thus, the posterior captures the true value quite accurately. Similarly, the function contour() can be used to plot the probability density function of a 2-dimensional variable, for instance:

```
>>> V = Gaussian([3, 5], [[4, 2], [2, 5]])
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.contour(V, np.linspace(1, 5, num=100), np.linspace(3, 7, num=100))
<matplotlib.contour.QuadContourSet object at 0x...>
```



Both pdf() and contour() require that the user provides the grid on which the probability density function is computed. They also support several keyword arguments for modifying the output, similarly as plot and contour in matplotlib.pyplot. These functions can be used only for stochastic nodes. A few other plot types are also available as built-in functions. A Hinton diagram can be plotted as:

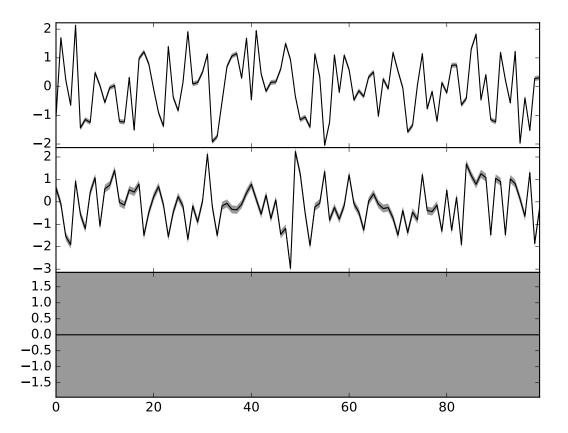
```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.hinton(C)
```



The diagram shows the elements of the matrix C. The size of the filled rectangle corresponds to the absolute value of the element mean, and white and black correspond to positive and negative values, respectively. The non-filled rectangle shows standard deviation. From this diagram it is clear that the third column of C has been pruned out and the rows that were missing in the data have zero mean and column-specific variance. The function hinton() is a simple wrapper for node-specific Hinton diagram plotters, such as  $gaussian_hinton()$  and  $dirichlet_hinton()$ . Thus, the keyword arguments depend on the node which is plotted.

Another plotting function is plot(), which just plots the values of the node over one axis as a function:

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.plot(X, axis=-2)
```



Now, the axis is the second last axis which corresponds to  $n=0,\ldots,N-1$ . As D=3, there are three subplots. For Gaussian variables, the function shows the mean and two standard deviations. The plot shows that the third component has been pruned out, thus the method has been able to recover the true dimensionality of the latent space. It also has similar keyword arguments to plot function in matplotlib.pyplot. Again, plot () is a simple wrapper over node-specific plotting functions, thus it supports only some node classes.

# 2.5.2 Monitoring during the inference algorithm

It is possible to plot the distribution of the nodes during the learning algorithm. This is useful when the user is interested to see how the distributions evolve during learning and what is happening to the distributions. In order to utilize monitoring, the user must set plotters for the nodes that he or she wishes to monitor. This can be done either when creating the node or later at any time.

The plotters are set by creating a plotter object and providing this object to the node. The plotter is a wrapper of one of the plotting functions mentioned above: PDFPlotter, ContourPlotter, HintonPlotter or FunctionPlotter. Thus, our example model could use the following plotters:

```
>>> tau.set_plotter(bpplt.PDFPlotter(np.linspace(60, 140, num=100)))
>>> C.set_plotter(bpplt.HintonPlotter())
>>> X.set_plotter(bpplt.FunctionPlotter(axis=-2))
```

These could have been given at node creation as a keyword argument plotter:

```
>>> V = Gaussian([3, 5], [[4, 2], [2, 5]],
... plotter=bpplt.ContourPlotter(np.linspace(1, 5, num=100),
... np.linspace(3, 7, num=100)))
```

When the plotter is set, one can use the plot method of the node to perform plotting:

```
>>> V.plot()
<matplotlib.contour.QuadContourSet object at 0x...>
```

Nodes can also be plotted using the plot method of the inference engine:

```
>>> Q.plot('C')
```

This method remembers the figure in which a node has been plotted and uses that every time it plots the same node. In order to monitor the nodes during learning, it is possible to use the keyword argument plot:

```
>>> Q.update(repeat=5, plot=True, tol=np.nan)
Iteration 19: loglike=-1.221354e+02 (... seconds)
Iteration 20: loglike=-1.221354e+02 (... seconds)
Iteration 21: loglike=-1.221354e+02 (... seconds)
Iteration 22: loglike=-1.221354e+02 (... seconds)
Iteration 23: loglike=-1.221354e+02 (... seconds)
```

Each node which has a plotter set will be plotted after it is updated. Note that this may slow down the inference significantly if the plotting operation is time consuming.

# 2.5.3 Posterior parameters and moments

If the built-in plotting functions are not sufficient, it is possible to use matplotlib.pyplot for custom plotting. Each node has get\_moments method which returns the moments and they can be used for plotting. Stochastic exponential family nodes have natural parameter vectors which can also be used. In addition to plotting, it is also possible to just print the moments or parameters in the console.

# 2.5.4 Saving and loading results

The results of the inference engine can be easily saved and loaded using VB. save () and VB. load () methods:

```
>>> Q.save(filename='tmp.hdf5')
>>> Q.load(filename='tmp.hdf5')
```

The results are stored in a HDF5 file. The user may set an autosave file in which the results are automatically saved regularly. Autosave filename can be set at creation time by autosave\_filename keyword argument or later using VB.set\_autosave() method. If autosave file has been set, the VB.save() and VB.load() methods use that file by default. In order for the saving to work, all stochastic nodes must have been given (unique) names.

However, note that these methods do *not* save nor load the node definitions. It means that the user must create the nodes and the inference engine and then use VB.load() to set the state of the nodes and the inference engine. If there are any differences in the model that was saved and the one which is tried to update using loading, then loading does not work. Thus, the user should keep the model construction unmodified in a Python file in order to be able to load the results later. Or if the user wishes to share the results, he or she must share the model construction Python file with the HDF5 results file.

# 2.6 Advanced topics

This section contains brief information on how to implement some advanced methods in BayesPy. These methods include Riemannian conjugate gradient methods, pattern search, simulated annealing, collapsed variational inference and stochastic variational inference. In order to use these methods properly, the user should understand them to some extent. They are also considered experimental, thus you may encounter bugs or unimplemented features. In any case, these methods may provide huge performance improvements easily compared to the standard VB-EM algorithm.

# 2.6.1 Gradient-based optimization

Variational Bayesian learning basically means that the parameters of the approximate posterior distributions are optimized to maximize the lower bound of the marginal log likelihood [3]. This optimization can be done by using gradient-based optimization methods. In order to improve the gradient-based methods, it is recommended to take into account the information geometry by using the Riemannian (a.k.a. natural) gradient. In fact, the standard VB-EM algorithm is equivalent to a gradient ascent method which uses the Riemannian gradient and step length 1. Thus, it is natural to try to improve this method by using non-linear conjugate gradient methods instead of gradient ascent. These optimization methods are especially useful when the VB-EM update equations are not available but one has to use fixed form approximation. But it is possible that the Riemannian conjugate gradient method improve performance even when the VB-EM update equations are available.

The optimization algorithm in VB. optimize() has a simple interface. Instead of using the default Riemannian geometry, one can use the Euclidean geometry by giving riemannian=False. It is also possible to choose the optimization method from gradient ascent (method='gradient') or conjugate gradient methods (only method='fletcher-reeves' implemented at the moment). For instance, we could optimize nodes C and X jointly using Euclidean gradient ascent as:

```
>>> Q = VB(Y, C, X, alpha, tau)
>>> Q.optimize(C, X, riemannian=False, method='gradient', maxiter=5)
Iteration ...
```

Note that this is very inefficient way of updating those nodes (bad geometry and not using conjugate gradients). Thus, one should understand the idea of these optimization methods, otherwise one may do something extremely inefficient. Most likely this method can be found useful in combination with the advanced tricks in the following sections.

**Note:** The Euclidean gradient has not been implemented for all nodes yet. The Euclidean gradient is required by the Euclidean geometry based optimization but also by the conjugate gradient methods in the Riemannian geometry. Thus, the Riemannian conjugate gradient may not yet work for all models.

It is possible to construct custom optimization algorithms with the tools provided by VB. For instance, VB.get\_parameters() and VB.set\_parameters() can be used to handle the parameters of nodes. VB.get\_gradients() is used for computing the gradients of nodes. The parameter and gradient objects are not numerical arrays but more complex nested lists not meant to be accessed by the user. Thus, for simple arithmetics with the parameter and gradient objects, use functions VB.add() and VB.dot(). Finally, VB.compute\_lowerbound() and VB.has\_converged() can be used to monitor the lower bound.

# 2.6.2 Collapsed inference

The optimization method can be used efficiently in such a way that some of the variables are collapsed, that is, marginalized out [1]. The collapsed variables must be conditionally independent given the observations and all other variables. Probably, one also wants that the size of the marginalized variables is large and the size of the optimized variables is small. For instance, in our PCA example, we could optimize as follows:

```
>>> Q.optimize(C, tau, maxiter=10, collapsed=[X, alpha])
Iteration ...
```

The collapsed variables are given as a list. This optimization does basically the following: It first computes the gradients for C and tau and takes an update step using the desired optimization method. Then, it updates the collapsed variables by using the standard VB-EM update equations. These two steps are taken in turns. Effectively, this corresponds to collapsing the variables X and alpha in a particular way. The point of this method is that the number of parameters in the optimization reduces significantly and the collapsed variables are updated optimally. For more details, see [1].

It is possible to use this method in such a way, that the collapsed variables are not conditionally independent given the observations and all other variables. However, in that case, the method does not anymore correspond to collapsing the variables but just using VB-EM updates after gradient-based updates. The method does not check for conditional independence, so the user is free to do this.

**Note:** Although the Riemannian conjugate gradient method has not yet been implemented for all nodes, it may be possible to collapse those nodes and optimize the other nodes for which the Euclidean gradient is already implemented.

#### 2.6.3 Pattern search

The pattern search method estimates the direction in which the approximate posterior distributions are updating and performs a line search in that direction [4]. The search direction is based on the difference in the VB parameters on successive updates (or several updates). The idea is that the VB-EM algorithm may be slow because it just zigzags and this can be fixed by moving to the direction in which the VB-EM is slowly moving.

BayesPy offers a simple built-in pattern search method *VB.pattern\_search()*. The method updates the nodes twice, measures the difference in the parameters and performs a line search with a small number of function evaluations:

```
>>> Q.pattern_search(C, X)
Iteration ...
```

Similarly to the collapsed optimization, it is possible to collapse some of the variables in the pattern search. The same rules of conditional independence apply as above. The collapsed variables are given as list:

```
>>> Q.pattern_search(C, tau, collapsed=[X, alpha])
Iteration ...
```

Also, a maximum number of iterations can be set by using maxiter keyword argument. It is not always obvious whether a pattern search will improve the rate of convergence or not but if it seems that the convergence is slow because of zigzagging, it may be worth a try. Note that the computational cost of the pattern search is quite high, thus it is not recommended to perform it after every VB-EM update but every now and then, for instance, after every 10 iterations. In addition, it is possible to write a more customized VB learning algorithm which uses pattern searches by using the different methods of VB discussed above.

# 2.6.4 Deterministic annealing

The standard VB-EM algorithm converges to a local optimum which can often be inferior to the global optimum and many other local optima. Deterministic annealing aims at finding a better local optimum, hopefully even the global optimum [5]. It does this by increasing the weight on the entropy of the posterior approximation in the VB lower bound. Effectively, the annealed lower bound becomes closer to a uniform function instead of the original multimodal lower bound. The weight on the entropy is recovered slowly and the optimization is much more robust to initialization.

In BayesPy, the annealing can be set by using  $VB.set\_annealing()$ . The given annealing should be in range (0, 1] but this is not validated in case the user wants to do something experimental. If annealing is set to 1, the original

VB lower bound is recovered. Annealing with 0 would lead to an improper uniform distribution, thus it will lead to errors. The entropy term is weighted by the inverse of this annealing term. An alternative view is that the model probability density functions are raised to the power of the annealing term.

Typically, the annealing is used in such a way that the annealing is small at the beginning and increased after every convergence of the VB algorithm until value 1 is reached. After the annealing value is increased, the algorithm continues from where it had just converged. The annealing can be used for instance as:

```
>>> beta = 0.1
>>> while beta < 1.0:
...     beta = min(beta*1.5, 1.0)
...     Q.set_annealing(beta)
...     Q.update(repeat=100, tol=1e-4)
Iteration ...</pre>
```

Here, the tol keyword argument is used to adjust the threshold for convergence. In this case, it is a bit larger than by default so the algorithm does not need to converge perfectly but a rougher convergence is sufficient for the next iteration with a new annealing value.

#### 2.6.5 Stochastic variational inference

In stochastic variational inference [2], the idea is to use mini-batches of large datasets to compute noisy gradients and learn the VB distributions by using stochastic gradient ascent. In order for it to be useful, the model must be such that it can be divided into "intermediate" and "global" variables. The number of intermediate variables increases with the data but the number of global variables remains fixed. The global variables are learnt in the stochastic optimization.

By denoting the data as  $Y = [Y_1, \dots, Y_N]$ , the intermediate variables as  $Z = [Z_1, \dots, Z_N]$  and the global variables as  $\theta$ , the model needs to have the following structure:

$$p(Y, Z, \theta) = p(\theta) \prod_{n=1}^{N} p(Y_n | Z_n, \theta) p(Z_n | \theta)$$

The algorithm consists of three steps which are iterated: 1) a random mini-batch of the data is selected, 2) the corresponding intermediate variables are updated by using normal VB update equations, and 3) the global variables are updated with (stochastic) gradient ascent as if there was as many replications of the mini-batch as needed to recover the original dataset size.

The learning rate for the gradient ascent must satisfy:

$$\sum_{i=1}^{\infty} \alpha_i = \infty \quad \text{and} \quad \sum_{i=1}^{\infty} \alpha^2 < \infty,$$

where i is the iteration number. An example of a valid learning parameter is  $\alpha_i = (\delta + i)^{-\gamma}$ , where  $\delta \ge 0$  is a delay and  $\gamma \in (0.5, 1]$  is a forgetting rate.

Stochastic variational inference is relatively easy to use in BayesPy. The idea is that the user creates a model for the size of a mini-batch and specifies a multiplier for those plate axes that are replicated. For the PCA example, the mini-batch model can be costructed as follows. We decide to use X as an intermediate variable and the other variables are global. The global variables alpha, C and tau are constructed identically as before. The intermediate variable X is constructed as:

```
>>> X = GaussianARD(0, 1,
... shape=(D,),
... plates=(1,5),
... plates_multiplier=(1,20),
... name='X')
```

Note that the plates are (1, 5) whereas they are (1, 100) in the full model. Thus, we need to provide a plates multiplier (1, 20) to define how the plates are replicated to get the full dataset. These multipliers do not need to be integers, in this case the latter plate axis is multiplied by 100/5 = 20. The remaining variables are defined as before:

```
>>> F = Dot(C, X)
>>> Y = GaussianARD(F, tau, name='Y')
```

Note that the plates of Y and F also correspond to the size of the mini-batch and they also deduce the plate multipliers from their parents, thus we do not need to specify the multiplier here explicitly (although it is ok to do so).

Let us construct the inference engine for the new mini-batch model:

```
>>> Q = VB(Y, C, X, alpha, tau)
```

Use random initialization for C to break the symmetry in C and X:

```
>>> C.initialize_from_random()
```

Then, stochastic variational inference algorithm could look as follows:

First, we ignore the bound checks because they are noisy. Then, the loop consists of three parts: 1) Draw a random mini-batch of the data (5 samples from 100). 2) Update the intermediate variable X. 3) Update global variables with gradient ascent using a proper learning rate.

## 2.6.6 Black-box variational inference

NOT YET IMPLEMENTED.

**CHAPTER** 

**THREE** 

# **EXAMPLES**

# 3.1 Linear regression

## 3.1.1 Data

The true parameters of the linear regression:

```
>>> k = 2 # slope
>>> c = 5 # bias
>>> s = 2 # noise standard deviation
```

#### Generate data:

```
>>> import numpy as np
>>> x = np.arange(10)
>>> y = k*x + c + s*np.random.randn(10)
```

# 3.1.2 **Model**

The regressors, that is, the input data:

```
>>> X = np.vstack([x, np.ones(len(x))]).T
```

Note that we added a column of ones to the regressor matrix for the bias term. We model the slope and the bias term in the same node so we do not factorize between them:

```
>>> from bayespy.nodes import GaussianARD
>>> B = GaussianARD(0, 1e-6, shape=(2,))
```

The first element is the slope which multiplies x and the second element is the bias term which multiplies the constant ones. Now we compute the dot product of X and B:

```
>>> from bayespy.nodes import SumMultiply
>>> F = SumMultiply('i,i', B, X)
```

The noise parameter:

```
>>> from bayespy.nodes import Gamma
>>> tau = Gamma(1e-3, 1e-3)
```

The noisy observations:

```
>>> Y = GaussianARD(F, tau)
```

# 3.1.3 Inference

Observe the data:

```
>>> Y.observe(y)
```

Construct the variational Bayesian (VB) inference engine by giving all stochastic nodes:

```
>>> from bayespy.inference import VB
>>> Q = VB(Y, B, tau)
```

Iterate until convergence:

```
>>> Q.update(repeat=1000)
Iteration 1: loglike=-4.595948e+01 (... seconds)
...
Iteration 5: loglike=-4.495017e+01 (... seconds)
Converged at iteration 5.
```

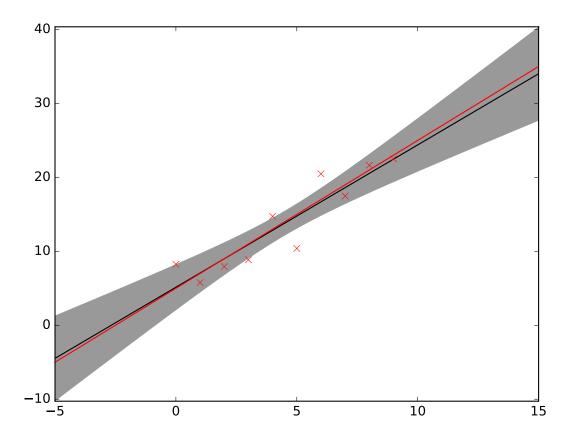
## 3.1.4 Results

Create a simple predictive model for new inputs:

```
>>> xh = np.linspace(-5, 15, 100)
>>> Xh = np.vstack([xh, np.ones(len(xh))]).T
>>> Fh = SumMultiply('i,i', B, Xh)
```

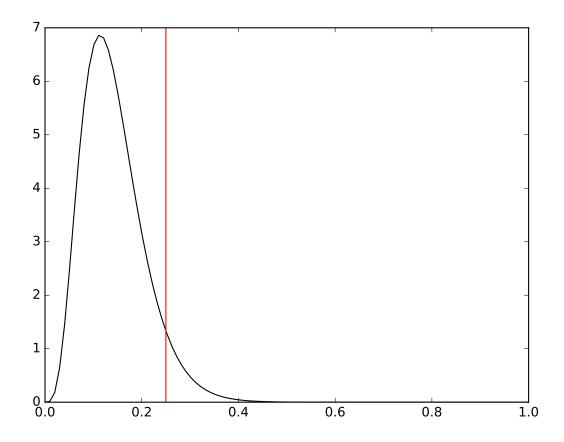
Note that we use the learned node B but create a new regressor array for predictions. Plot the predictive distribution of noiseless function values:

```
>>> import bayespy.plot as bpplt
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.plot(Fh, x=xh, scale=2)
>>> bpplt.plot(y, x=x, color='r', marker='x', linestyle='None')
>>> bpplt.plot(k*xh+c, x=xh, color='r');
```



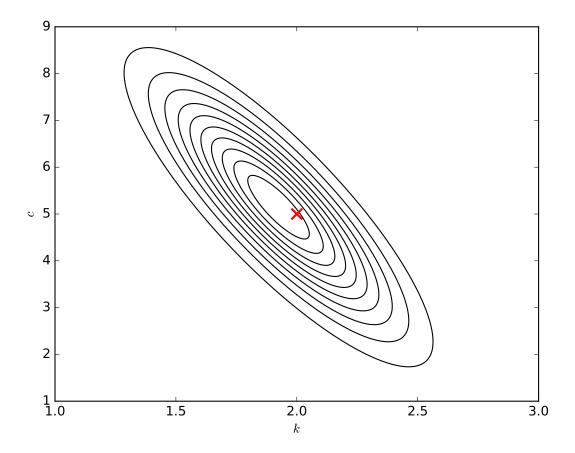
Note that the above plot shows two standard deviation of the posterior of the noiseless function, thus the data points may lie well outside this range. The red line shows the true linear function. Next, plot the distribution of the noise parameter and the true value,  $2^{-2} = 0.25$ :

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.pdf(tau, np.linspace(1e-6,1,100), color='k')
[<matplotlib.lines.Line2D object at 0x...>]
>>> bpplt.pyplot.axvline(s**(-2), color='r')
<matplotlib.lines.Line2D object at 0x...>
```



The noise level is captured quite well, although the posterior has more mass on larger noise levels (smaller precision parameter values). Finally, plot the distribution of the regression parameters and mark the true value:

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.contour(B, np.linspace(1,3,1000), np.linspace(1,9,1000),
... n=10, colors='k')
<matplotlib.contour.QuadContourSet object at 0x...>
>>> bpplt.plot(c, x=k, color='r', marker='x', linestyle='None',
... markersize=10, markeredgewidth=2)
>>> bpplt.pyplot.xlabel(r'$k$')
<matplotlib.text.Text object at 0x...>
>>> bpplt.pyplot.ylabel(r'$c$');
<matplotlib.text.Text object at 0x...>
```



In this case, the true parameters are captured well by the posterior distribution.

# 3.1.5 Improving accuracy

The model can be improved by not factorizing between B and tau but learning their joint posterior distribution. This requires a slight modification to the model by using <code>GaussianGammaISO</code> node:

```
>>> from bayespy.nodes import GaussianGammaISO
>>> B_tau = GaussianGammaISO(np.zeros(2), 1e-6*np.identity(2), 1e-3, 1e-3)
```

This node contains both the regression parameter vector and the noise parameter. We compute the dot product similarly as before:

```
>>> F_tau = SumMultiply('i,i', B_tau, X)
```

However, Y is constructed as follows:

```
>>> Y = GaussianARD(F_tau, 1)
```

Because the noise parameter is already in F\_tau we can give a constant one as the second argument. The total noise parameter for Y is the product of the noise parameter in F\_tau and one. Now, inference is run similarly as before:

```
>>> Y.observe(y)
>>> Q = VB(Y, B_tau)
>>> Q.update(repeat=1000)
```

```
Iteration 1: loglike=-4.678478e+01 (... seconds)
Iteration 2: loglike=-4.678478e+01 (... seconds)
Converged at iteration 2.
```

Note that the method converges immediately. This happens because there is only one unobserved stochastic node so there is no need for iteration and the result is actually the exact true posterior distribution, not an approximation. Currently, the main drawback of using this approach is that BayesPy does not yet contain any plotting utilities for nodes that contain both Gaussian and gamma variables jointly.

## 3.1.6 Further extensions

The approach discussed in this example can easily be extended to non-linear regression and multivariate regression. For non-linear regression, the inputs are first transformed by some known non-linear functions and then linear regression is applied to this transformed data. For multivariate regression, X and B are concatenated appropriately: If there are more regressors, add more columns to both X and B. If there are more output dimensions, add plates to B.

# 3.2 Gaussian mixture model

This example demonstrates the use of Gaussian mixture model for flexible density estimation, clustering or classification.

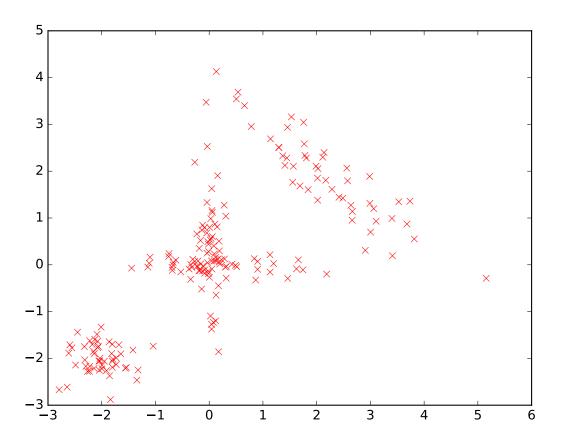
## 3.2.1 Data

First, let us generate some artificial data for the analysis. The data are two-dimensional vectors from one of the four different Gaussian distributions:

```
>>> import numpy as np
>>> y0 = np.random.multivariate_normal([0, 0], [[2, 0], [0, 0.1]], size=50)
>>> y1 = np.random.multivariate_normal([0, 0], [[0.1, 0], [0, 2]], size=50)
>>> y2 = np.random.multivariate_normal([2, 2], [[2, -1.5], [-1.5, 2]], size=50)
>>> y3 = np.random.multivariate_normal([-2, -2], [[0.5, 0], [0, 0.5]], size=50)
>>> y = np.vstack([y0, y1, y2, y3])
```

Thus, there are 200 data vectors in total. The data looks as follows:

```
>>> import bayespy.plot as bpplt
>>> bpplt.pyplot.plot(y[:,0], y[:,1], 'rx')
[<matplotlib.lines.Line2D object at 0x...>]
```



# 3.2.2 **Model**

For clarity, let us denote the number of the data vectors with  ${\tt N}$ 

```
>>> N = 200
```

and the dimensionality of the data vectors with D:

```
>>> D = 2
```

We will use a "large enough" number of Gaussian clusters in our model:

```
>>> K = 10
```

Cluster assignments Z and the prior for the cluster assignment probabilities alpha:

The mean vectors and the precision matrices of the clusters:

If either the mean or precision should be shared between clusters, then that node should not have plates, that is, plates=(). The data vectors are from a Gaussian mixture with cluster assignments Z and Gaussian component parameters mu and Lambda:

```
>>> from bayespy.nodes import Mixture
>>> Y = Mixture(Z, Gaussian, mu, Lambda,
... name='Y')
```

```
>>> Z.initialize_from_random()
```

```
>>> from bayespy.inference import VB
>>> Q = VB(Y, mu, Lambda, Z, alpha)
```

## 3.2.3 Inference

Before running the inference algorithm, we provide the data:

```
>>> Y.observe(y)
```

Then, run VB iteration until convergence:

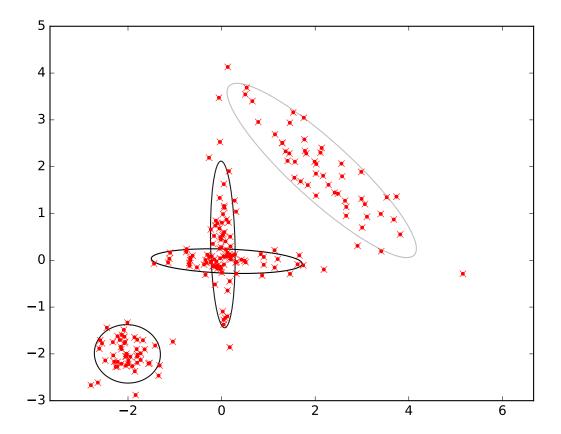
```
>>> Q.update(repeat=1000)
Iteration 1: loglike=-1.402345e+03 (... seconds)
...
Iteration 61: loglike=-8.888464e+02 (... seconds)
Converged at iteration 61.
```

The algorithm converges very quickly. Note that the default update order of the nodes was such that mu and Lambda were updated before Z, which is what we wanted because Z was initialized randomly.

## 3.2.4 Results

For two-dimensional Gaussian mixtures, the mixture components can be plotted using gaussian\_mixture\_2d():

>>> bpplt.gaussian\_mixture\_2d(Y, alpha=alpha, scale=2)



The function is called with scale=2 which means that each ellipse shows two standard deviations. From the ten cluster components, the model uses effectively the correct number of clusters (4). These clusters capture the true density accurately.

In addition to clustering and density estimation, this model could also be used for classification by setting the known class assignments as observed.

# 3.2.5 Advanced next steps

## Joint node for mean and precision

The next step for improving the results could be to use <code>GaussianWishart</code> node for modelling the mean vectors mu and precision matrices <code>Lambda</code> jointly without factorization. This should improve the accuracy of the posterior approximation and the speed of the VB estimation. However, the implementation is a bit more complex.

## Fast collapsed inference

# 3.3 Bernoulli mixture model

This example considers data generated from a Bernoulli mixture model. One simple example process could be a questionnaire for election candidates. We observe a set of binary vectors, where each vector represents a candidate in the election and each element in these vectors correspond to a candidate's answer to a yes-or-no question. The goal is to find groups of similar candidates and analyze the answer patterns of these groups.

## 3.3.1 Data

First, we generate artificial data to analyze. Let us assume that the questionnaire contains ten yes-or-no questions. We assume that there are three groups with similar opinions. These groups could represent parties. These groups have the following answering patterns, which are represented by vectors with probabilities of a candidate answering yes to the questions:

```
>>> p0 = [0.1, 0.9, 0.1, 0.9, 0.1, 0.9, 0.1, 0.9, 0.1, 0.9]

>>> p1 = [0.1, 0.1, 0.1, 0.1, 0.1, 0.9, 0.9, 0.9, 0.9, 0.9]

>>> p2 = [0.9, 0.9, 0.9, 0.9, 0.9, 0.1, 0.1, 0.1, 0.1, 0.1]
```

Thus, the candidates in the first group are likely to answer no to questions 1, 3, 5, 7 and 9, and yes to questions 2, 4, 6, 8, 10. The candidates in the second group are likely to answer yes to the last five questions, whereas the candidates in the third group are likely to answer yes to the first five questions. For convenience, we form a NumPy array of these vectors:

```
>>> import numpy as np
>>> p = np.array([p0, p1, p2])
```

Next, we generate a hundred candidates. First, we randomly select the group for each candidate:

```
>>> from bayespy.utils import random
>>> z = random.categorical([1/3, 1/3, 1/3], size=100)
```

Using the group patterns, we generate yes-or-no answers for the candidates:

```
>>> x = random.bernoulli(p[z])
```

This is our simulated data to be analyzed.

## 3.3.2 Model

Now, we construct a model for learning the structure in the data. We have a dataset of hundred 10-dimensional binary vectors:

```
>>> N = 100
>>> D = 10
```

We will create a Bernoulli mixture model. We assume that the true number of groups is unknown to us, so we use a large enough number of clusters:

```
>>> K = 10
```

We use the categorical distribution for the group assignments and give the group assignment probabilities an uninformative Dirichlet prior:

```
>>> from bayespy.nodes import Categorical, Dirichlet
>>> R = Dirichlet(K*[1e-5],
... name='R')
>>> Z = Categorical(R,
... plates=(N,1),
... name='Z')
```

Each group has a probability of a yes answer for each question. These probabilities are given beta priors:

```
>>> from bayespy.nodes import Beta
>>> P = Beta([0.5, 0.5],
... plates=(D,K),
... name='P')
```

The answers of the candidates are modelled with the Bernoulli distribution:

```
>>> from bayespy.nodes import Mixture, Bernoulli
>>> X = Mixture(Z, Bernoulli, P)
```

Here, Z defines the group assignments and P the answering probability patterns for each group. Note how the plates of the nodes are matched: Z has plates (N, 1) and P has plates (D, K), but in the mixture node the last plate axis of P is discarded and thus the node broadcasts plates (N, 1) and (D, 1) resulting in plates (N, D) for X.

#### 3.3.3 Inference

In order to infer the variables in our model, we construct a variational Bayesian inference engine:

```
>>> from bayespy.inference import VB
>>> Q = VB(Z, R, X, P)
```

This also gives the default update order of the nodes. In order to find different groups, they must be initialized differently, thus we use random initialization for the group probability patterns:

```
>>> P.initialize_from_random()
```

We provide our simulated data:

```
>>> X.observe(x)
```

Now, we can run inference:

```
>>> Q.update(repeat=1000)
Iteration 1: loglike=-6.872145e+02 (... seconds)
...
Iteration 17: loglike=-5.236921e+02 (... seconds)
Converged at iteration 17.
```

The algorithm converges in 17 iterations.

#### 3.3.4 Results

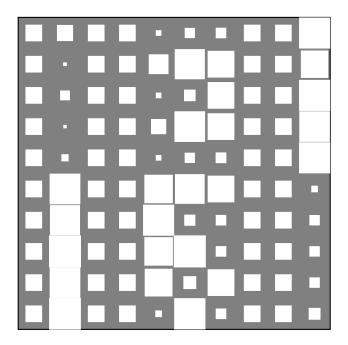
Now we can examine the approximate posterior distribution. First, let us plot the group assignment probabilities:

```
>>> import bayespy.plot as bpplt
>>> bpplt.hinton(R)
```



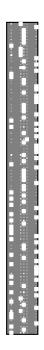
This plot shows that there are three dominant groups, which is equal to the true number of groups used to generate the data. However, there are still two smaller groups as the data does not give enough evidence to prune them out. The yes-or-no answer probability patterns for the groups can be plotted as:

>>> bpplt.hinton(P)



The three dominant groups have found the true patterns accurately. The patterns of the two minor groups some kind of mixtures of the three groups and they exist because the generated data happened to contain a few samples giving evidence for these groups. Finally, we can plot the group assignment probabilities for the candidates:

>>> bpplt.hinton(Z)



This plot shows the clustering of the candidates. It is possible to use HintonPlotter to enable monitoring during the VB iteration by providing plotter=HintonPlotter() for Z, P and R when creating the nodes.

# 3.4 Hidden Markov model

In this example, we will demonstrate the use of hidden Markov model in the case of known and unknown parameters. We will also use two different emission distributions to demonstrate the flexibility of the model construction.

# 3.4.1 Known parameters

This example follows the one presented in Wikipedia.

#### Model

Each day, the state of the weather is either 'rainy' or 'sunny'. The weather follows a first-order discrete Markov process. It has the following initial state probabilities

```
>>> a0 = [0.6, 0.4] # p(rainy)=0.6, p(sunny)=0.4
```

and state transition probabilities:

```
>>> A = [[0.7, 0.3], # p(rainy->rainy)=0.7, p(rainy->sunny)=0.3
... [0.4, 0.6]] # p(sunny->rainy)=0.4, p(sunny->sunny)=0.6
```

We will be observing one hundred samples:

```
>>> N = 100
```

The discrete first-order Markov chain is constructed as:

```
>>> from bayespy.nodes import CategoricalMarkovChain
>>> Z = CategoricalMarkovChain(a0, A, states=N)
```

However, instead of observing this process directly, we observe whether Bob is 'walking', 'shopping' or 'cleaning'. The probability of each activity depends on the current weather as follows:

```
>>> P = [[0.1, 0.4, 0.5], ... [0.6, 0.3, 0.1]]
```

where the first row contains activity probabilities on a rainy weather and the second row contains activity probabilities on a sunny weather. Using these emission probabilities, the observed process is constructed as:

```
>>> from bayespy.nodes import Categorical, Mixture
>>> Y = Mixture(Z, Categorical, P)
```

#### Data

In order to test our method, we'll generate artificial data from the model itself. First, draw realization of the weather process:

```
>>> weather = Z.random()
```

Then, using this weather, draw realizations of the activities:

```
>>> activity = Mixture(weather, Categorical, P).random()
```

#### Inference

Now, using this data, we set our variable Y to be observed:

```
>>> Y.observe(activity)
```

In order to run inference, we construct variational Bayesian inference engine:

```
>>> from bayespy.inference import VB
>>> Q = VB(Y, Z)
```

Note that we need to give all random variables to VB. In this case, the only random variables were Y and Z. Next we run the inference, that is, compute our posterior distribution:

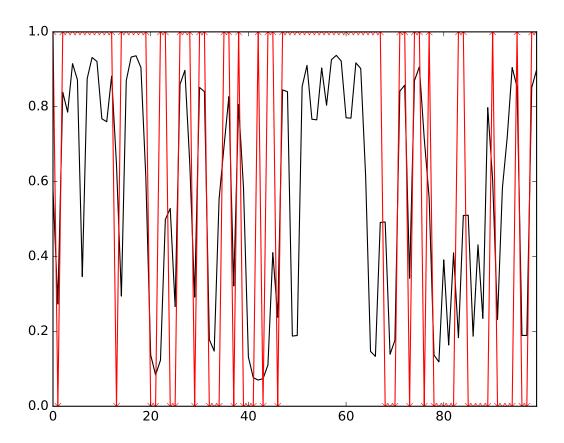
```
>>> Q.update()
Iteration 1: loglike=-1.095883e+02 (... seconds)
```

In this case, because there is only one unobserved random variable, we recover the exact posterior distribution and there is no need to iterate more than one step.

#### **Results**

One way to plot a 2-class categorical timeseries is to use the basic plot () function:

```
>>> import bayespy.plot as bpplt
>>> bpplt.plot(Z)
>>> bpplt.plot(1-weather, color='r', marker='x')
```



The black line shows the posterior probability of rain and the red line and crosses show the true state. Clearly, the method is not able to infer the weather very accurately in this case because the activies do not give that much information about the weather.

# 3.4.2 Unknown parameters

In this example, we consider unknown parameters for the Markov process and different emission distribution.

#### **Data**

We generate data from three 2-dimensional Gaussian distributions with different mean vectors and common standard deviation:

```
>>> import numpy as np
>>> mu = np.array([ [0,0], [3,4], [6,0] ])
>>> std = 2.0
```

Thus, the number of clusters is three:

```
>>> K = 3
```

And the number of samples is 200:

```
>>> N = 200
```

Each initial state is equally probable:

```
>>> p0 = np.ones(K) / K
```

State transition matrix is such that with probability 0.9 the process stays in the same state. The probability to move one of the other two states is 0.05 for both of those states.

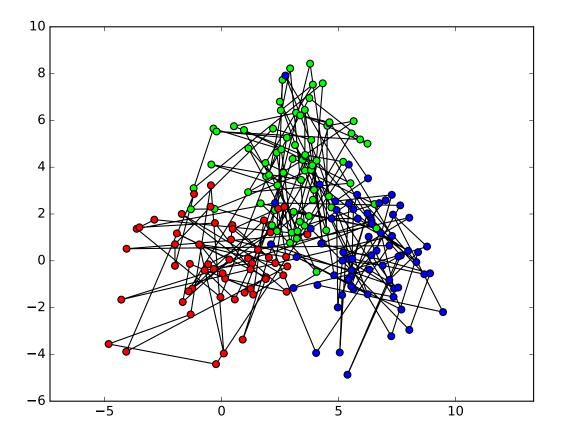
```
>>> q = 0.9
>>> r = (1-q) / (K-1)
>>> P = q*np.identity(K) + r*(np.ones((3,3))-np.identity(3))
```

Simulate the data:

```
>>> y = np.zeros((N,2))
>>> z = np.zeros(N)
>>> state = np.random.choice(K, p=p0)
>>> for n in range(N):
... z[n] = state
... y[n,:] = std*np.random.randn(2) + mu[state]
... state = np.random.choice(K, p=P[state])
```

Then, let us visualize the data:

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.pyplot.axis('equal')
(...)
>>> colors = [ [[1,0,0], [0,1,0], [0,0,1]][int(state)] for state in z ]
>>> bpplt.pyplot.plot(y[:,0], y[:,1], 'k-', zorder=-10)
[<matplotlib.lines.Line2D object at 0x...>]
>>> bpplt.pyplot.scatter(y[:,0], y[:,1], c=colors, s=40)
<matplotlib.collections.PathCollection object at 0x...>
```



Consecutive states are connected by a solid black line and the dot color shows the true class.

## Model

Now, assume that we do not know the parameters of the process (initial state probability and state transition probabilities). We give these parameters quite non-informative priors, but it is possible to provide more informative priors if such information is available:

```
>>> from bayespy.nodes import Dirichlet
>>> a0 = Dirichlet(1e-3*np.ones(K))
>>> A = Dirichlet(1e-3*np.ones((K,K)))
```

The discrete Markov chain is constructed as:

```
>>> Z = CategoricalMarkovChain(a0, A, states=N)
```

Now, instead of using categorical emission distribution as before, we'll use Gaussian distribution. For simplicity, we use the true parameters of the Gaussian distributions instead of giving priors and estimating them. The known standard deviation can be converted to a precision matrix as:

```
>>> Lambda = std**(-2) * np.identity(2)
```

Thus, the observed process is a Gaussian mixture with cluster assignments from the hidden Markov process 2:

```
>>> from bayespy.nodes import Gaussian
>>> Y = Mixture(Z, Gaussian, mu, Lambda)
```

Note that Lambda does not have cluster plate axis because it is shared between the clusters.

#### Inference

Let us use the simulated data:

```
>>> Y.observe(y)
```

Because VB takes all the random variables, we need to provide A and a0 also:

```
>>> Q = VB(Y, Z, A, a0)
```

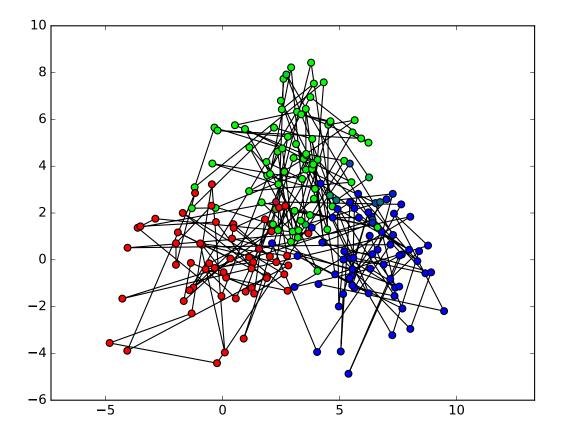
Then, run VB iteration until convergence:

```
>>> Q.update(repeat=1000)
Iteration 1: loglike=-9.963054e+02 (... seconds)
...
Iteration 8: loglike=-9.235053e+02 (... seconds)
Converged at iteration 8.
```

#### Results

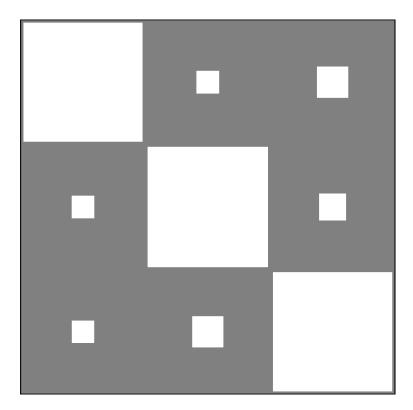
Plot the classification of the data similarly as the data:

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.pyplot.axis('equal')
(...)
>>> colors = Y.parents[0].get_moments()[0]
>>> bpplt.pyplot.plot(y[:,0], y[:,1], 'k-', zorder=-10)
[<matplotlib.lines.Line2D object at 0x...>]
>>> bpplt.pyplot.scatter(y[:,0], y[:,1], c=colors, s=40)
<matplotlib.collections.PathCollection object at 0x...>
```



The data has been classified quite correctly. Even samples that are more in the region of another cluster are classified correctly if the previous and next sample provide enough evidence for the correct class. We can also plot the state transition matrix:

>>> bpplt.hinton(A)



Clearly, the learned state transition matrix is close to the true matrix. The models described above could also be used for classification by providing the known class assignments as observed data to  $\mathbb Z$  and the unknown class assignments as missing data.

# 3.5 Principal component analysis

This example uses a simple principal component analysis to find a two-dimensional latent subspace in a higher dimensional dataset.

# 3.5.1 Data

Let us create a Gaussian dataset with latent space dimensionality two and some observation noise:

```
>>> M = 20
>>> N = 100

>>> import numpy as np
>>> x = np.random.randn(N, 2)
>>> w = np.random.randn(M, 2)
>>> f = np.einsum('ik, jk->ij', w, x)
```

>>> y = f + 0.1\*np.random.randn(M, N)

# 3.5.2 Model

We will use 10-dimensional latent space in our model and let it learn the true dimensionality:

```
>>> D = 10
```

Import relevant nodes:

```
>>> from bayespy.nodes import GaussianARD, Gamma, SumMultiply
```

The latent states:

```
>>> X = GaussianARD(0, 1, plates=(1,N), shape=(D,))
```

The loading matrix with automatic relevance determination (ARD) prior:

```
>>> alpha = Gamma(1e-5, 1e-5, plates=(D,))
>>> C = GaussianARD(0, alpha, plates=(M,1), shape=(D,))
```

Compute the dot product of the latent states and the loading matrix:

```
>>> F = SumMultiply('d,d->', X, C)
```

The observation noise:

```
>>> tau = Gamma(1e-5, 1e-5)
```

The observed variable:

```
>>> Y = GaussianARD(F, tau)
```

## 3.5.3 Inference

Observe the data:

```
>>> Y.observe(y)
```

We do not have missing data now, but they could be easily handled with mask keyword argument. Construct variational Bayesian (VB) inference engine:

```
>>> from bayespy.inference import VB
>>> Q = VB(Y, X, C, alpha, tau)
```

Initialize the latent subspace randomly, otherwise both  ${\tt X}$  and  ${\tt C}$  would converge to zero:

```
>>> C.initialize_from_random()
```

Now we could use VB. update() to run the inference. However, let us first create a parameter expansion to speed up the inference. The expansion is based on rotating the latent subspace optimally. This is optional but will usually improve the speed of the inference significantly, especially in high-dimensional problems:

```
>>> from bayespy.inference.vmp.transformations import RotateGaussianARD
>>> rot_X = RotateGaussianARD(X)
>>> rot_C = RotateGaussianARD(C, alpha)
```

By giving alpha for rot\_C, the rotation will also optimize alpha jointly with C. Now that we have defined the rotations for our variables, we need to construct an optimizer:

```
>>> from bayespy.inference.vmp.transformations import RotationOptimizer
>>> R = RotationOptimizer(rot_X, rot_C, D)
```

In order to use the rotations automatically, we need to set it as a callback function:

```
>>> Q.set_callback(R.rotate)
```

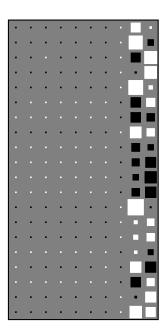
For more information about the rotation parameter expansion, see [7] and [6]. Now we can run the actual inference until convergence:

```
>>> Q.update(repeat=1000)
Iteration 1: loglike=-2.339710e+03 (... seconds)
...
Iteration 22: loglike=6.500...e+02 (... seconds)
Converged at iteration 2...
```

# 3.5.4 Results

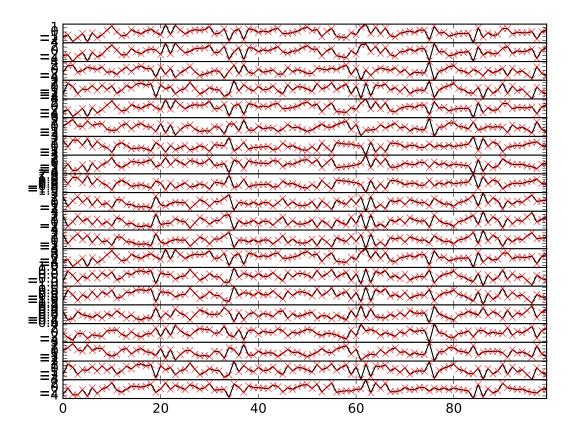
The results can be visualized, for instance, by plotting the Hinton diagram of the loading matrix:

```
>>> import bayespy.plot as bpplt
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.hinton(C)
```



The method has been able to prune out unnecessary latent dimensions and keep two components, which is the true number of components.

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.plot(F)
>>> bpplt.plot(f, color='r', marker='x', linestyle='None')
```



The reconstruction of the noiseless function values are practically perfect in this simple example. Larger noise variance, more latent space dimensions and missing values would make this problem more difficult. The model construction could also be improved by having, for instance, C and tau in the same node without factorizing between them in the posterior approximation. This can be achieved by using <code>GaussianGammaISO</code> node.

# 3.6 Linear state-space model

## 3.6.1 Model

In linear state-space models a sequence of M-dimensional observations  $\mathbf{Y} = (\mathbf{y}_1, \dots, \mathbf{y}_N)$  is assumed to be generated from latent D-dimensional states  $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_N)$  which follow a first-order Markov process:

$$\mathbf{x}_n = \mathbf{A}\mathbf{x}_{n-1} + \text{noise},$$
  
 $\mathbf{y}_n = \mathbf{C}\mathbf{x}_n + \text{noise},$ 

where the noise is Gaussian,  $\bf A$  is the  $D \times D$  state dynamics matrix and  $\bf C$  is the  $M \times D$  loading matrix. Usually, the latent space dimensionality D is assumed to be much smaller than the observation space dimensionality M in order to model the dependencies of high-dimensional observations efficiently.

In order to construct the model in BayesPy, first import relevant nodes:

```
>>> from bayespy.nodes import GaussianARD, GaussianMarkovChain, Gamma, Dot
```

The data vectors will be 30-dimensional:

```
>>> M = 30
```

There will be 400 data vectors:

```
>>> N = 400
```

Let us use 10-dimensional latent space:

```
>>> D = 10
```

The state dynamics matrix **A** has ARD prior:

```
>>> alpha = Gamma(1e-5,
... 1e-5,
... plates=(D,),
... name='alpha')
>>> A = GaussianARD(0,
... alpha,
... shape=(D,),
... plates=(D,),
... name='A')
```

Note that **A** is a  $D \times D$ -dimensional matrix. However, in BayesPy it is modelled as a collection (plates=(D,)) of D-dimensional vectors (shape=(D,)) because this is how the variables factorize in the posterior approximation of the state dynamics matrix in GaussianMarkovChain. The latent states are constructed as

where the first two arguments are the mean and precision matrix of the initial state, the third argument is the state dynamics matrix and the fourth argument is the diagonal elements of the precision matrix of the innovation noise. The node also needs the length of the chain given as the keyword argument  $n=\mathbb{N}$ . Thus, the shape of this node is  $(\mathbb{N}, \mathbb{D})$ .

The linear mapping from the latent space to the observation space is modelled with the loading matrix which has ARD prior:

```
>>> gamma = Gamma(1e-5,
... 1e-5,
... plates=(D,),
... name='gamma')
>>> C = GaussianARD(0,
... gamma,
... shape=(D,),
... plates=(M,1),
... name='C')
```

Note that the plates for C are (M, 1), thus the full shape of the node is (M, 1, D). The unit plate axis is added so that C broadcasts with X when computing the dot product:

```
>>> F = Dot(C,
... X,
... name='F')
```

This dot product is computed over the D-dimensional latent space, thus the result is a  $M \times N$ -dimensional matrix which is now represented with plates (M, N) in BayesPy:

```
>>> F.plates (30, 400)
```

We also need to use random initialization either for C or X in order to find non-zero latent space because by default both C and X are initialized to zero because of their prior distributions. We use random initialization for C and then we must update X the first time before updating C:

```
>>> C.initialize_from_random()
```

The precision of the observation noise is given gamma prior:

```
>>> tau = Gamma(1e-5,
... 1e-5,
... name='tau')
```

The observations are noisy versions of the dot products:

```
>>> Y = GaussianARD(F,
... tau,
... name='Y')
```

The variational Bayesian inference engine is then construced as:

```
>>> from bayespy.inference import VB
>>> Q = VB(X, C, gamma, A, alpha, tau, Y)
```

Note that X is given before C, thus X is updated before C by default.

#### 3.6.2 Data

Now, let us generate some toy data for our model. Our true latent space is four dimensional with two noisy oscillator components, one random walk component and one white noise component.

The true linear mapping is just random:

```
>>> c = np.random.randn(M,4)
```

Then, generate the latent states and the observations using the model equations:

We want to simulate missing values, thus we create a mask which randomly removes 80% of the data:

```
>>> from bayespy.utils import random
>>> mask = random.mask(M, N, p=0.2)
>>> Y.observe(y, mask=mask)
```

## 3.6.3 Inference

As we did not define plotters for our nodes when creating the model, it is done now for some of the nodes:

```
>>> import bayespy.plot as bpplt
>>> X.set_plotter(bpplt.FunctionPlotter(center=True, axis=-2))
>>> A.set_plotter(bpplt.HintonPlotter())
>>> C.set_plotter(bpplt.HintonPlotter())
>>> tau.set_plotter(bpplt.PDFPlotter(np.linspace(0.02, 0.5, num=1000)))
```

This enables plotting of the approximate posterior distributions during VB learning. The inference engine can be run using VB.update() method:

```
>>> Q.update(repeat=10)
Iteration 1: loglike=-1.439704e+05 (... seconds)
...
Iteration 10: loglike=-1.051441e+04 (... seconds)
```

The iteration progresses a bit slowly, thus we'll consider parameter expansion to speed it up.

### **Parameter expansion**

Section *Parameter expansion* discusses parameter expansion for state-space models to speed up inference. It is based on a rotating the latent space such that the posterior in the observation space is not affected:

$$\mathbf{y}_n = \mathbf{C}\mathbf{x}_n = (\mathbf{C}\mathbf{R}^{-1})(\mathbf{R}\mathbf{x}_n)$$
.

Thus, the transformation is  $C \to CR^{-1}$  and  $X \to RX$ . In order to keep the dynamics of the latent states unaffected by the transformation, the state dynamics matrix **A** must be transformed accordingly:

$$\mathbf{R}\mathbf{x}_n = \mathbf{R}\mathbf{A}\mathbf{R}^{-1}\mathbf{R}\mathbf{x}_{n-1}$$
,

resulting in a transformation  $A \to RAR^{-1}$ . For more details, refer to [6] and [7]. In BayesPy, the transformations are available in bayespy.inference.vmp.transformations:

```
>>> from bayespy.inference.vmp import transformations
```

The rotation of the loading matrix along with the ARD parameters is defined as:

```
>>> rotC = transformations.RotateGaussianARD(C, gamma)
```

For rotating X, we first need to define the rotation of the state dynamics matrix:

```
>>> rotA = transformations.RotateGaussianARD(A, alpha)
```

Now we can define the rotation of the latent states:

```
>>> rotX = transformations.RotateGaussianMarkovChain(X, rotA)
```

The optimal rotation for all these variables is found using rotation optimizer:

```
>>> R = transformations.RotationOptimizer(rotX, rotC, D)
```

Set the parameter expansion to be applied after each iteration:

```
>>> Q.callback = R.rotate
```

Now, run iterations until convergence:

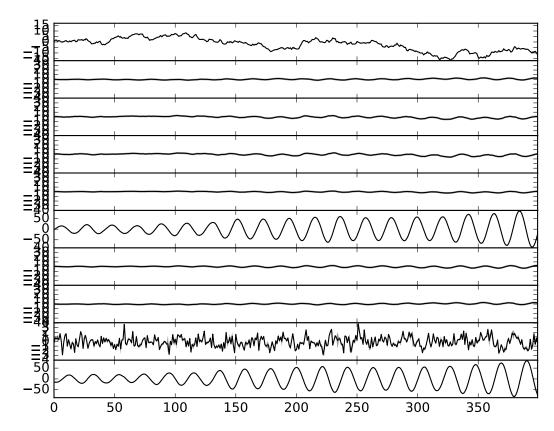
```
>>> Q.update(repeat=1000)
Iteration 11: loglike=-1.010806e+04 (... seconds)
...
Iteration 58: loglike=-8.906...e+03 (... seconds)
Converged at iteration ...
```

# 3.6.4 Results

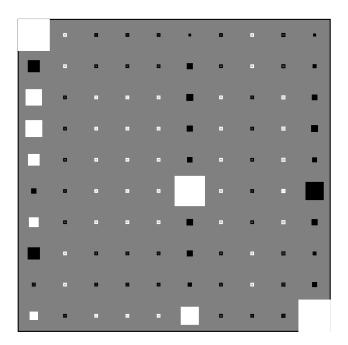
Because we have set the plotters, we can plot those nodes as:

```
>>> Q.plot(X, A, C, tau)
```

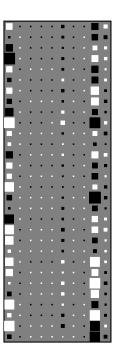


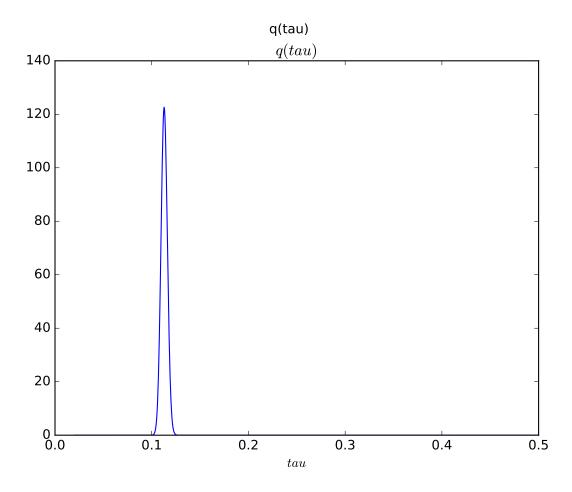


q(A)



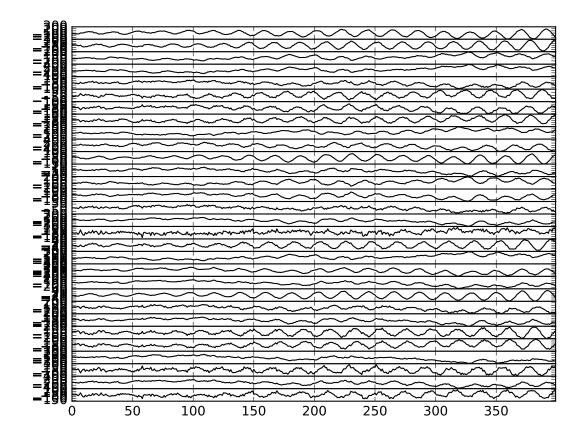
# q(C)





There are clearly four effective components in X: random walk (component number 1), random oscillation (7 and 10), and white noise (9). These dynamics are also visible in the state dynamics matrix Hinton diagram. Note that the white noise component does not have any dynamics. Also C shows only four effective components. The posterior of tau captures the true value  $3^{-2} \approx 0.111$  accurately. We can also plot predictions in the observation space:

```
>>> bpplt.plot(F, center=True)
```



We can also measure the performance numerically by computing root-mean-square error (RMSE) of the missing values:

```
>>> from bayespy.utils import misc
>>> misc.rmse(y[~mask], F.get_moments()[0][~mask])
5.18...
```

This is relatively close to the standard deviation of the noise (3), so the predictions are quite good considering that only 20% of the data was used.

# 3.7 Latent Dirichlet allocation

Latent Dirichlet allocation is a widely used topic model. The data is a collection of documents which contain words. The goal of the analysis is to find topics (distribution of words in topics) and document topics (distribution of topics in documents).

## 3.7.1 Data

The data consists of two vectors of equal length. The elements in these vectors correspond to the words in all documents combined. If there were M documents and each document had K words, the vectors contain  $M \cdot K$  elements. Let M be the number of documents in total. The first vector gives each word a document index  $i \in \{0, \dots, M-1\}$ 

defining to which document the word belongs. Let N be the size of the whole available vocabulary. The second vector gives each word a vocabulary index  $j \in \{0, ..., N-1\}$  defining which word it is from the vocabulary.

For this demo, we will just generate an artificial dataset for simplicity. We use the LDA model itself to generate the dataset. First, import relevant packages:

```
>>> import numpy as np
>>> from bayespy import nodes
```

Let us decide the number of documents and the number of words in those documents:

```
>>> n_documents = 10
>>> n_words = 10000
```

Randomly choose into which document each word belongs to:

Let us also decide the size of our vocabulary:

```
>>> n_vocabulary = 100
```

Also, let us decide the true number of topics:

```
>>> n_topics = 5
```

Generate some random distributions for the topics in each document:

```
>>> p_topic = nodes.Dirichlet(1e-1*np.ones(n_topics),
... plates=(n_documents,)).random()
```

Generate some random distributions for the words in each topic:

```
>>> p_word = nodes.Dirichlet(1e-1*np.ones(n_vocabulary),
... plates=(n_topics,)).random()
```

Sample topic assignments for each word in each document:

```
>>> topic = nodes.Categorical(p_topic[word_documents],
... plates=(n_words,)).random()
```

And finally, draw vocabulary indices for each word in all the documents:

Now, our dataset consists of word\_documents and corpus, which define the document and vocabulary indices for each word in our dataset.

#### Todo

Use some large real-world dataset, for instance, Wikipedia.

## 3.7.2 Model

Variable for learning the topic distribution for each document:

```
>>> p_topic = nodes.Dirichlet(np.ones(n_topics),
... plates=(n_documents,),
... name='p_topic')
```

Variable for learning the word distribution for each topic:

The document indices for each word in the corpus:

```
>>> from bayespy.inference.vmp.nodes.categorical import CategoricalMoments
>>> document_indices = nodes.Constant(CategoricalMoments(n_documents), word_documents,
... name='document_indices')
```

Variable for learning the topic assignments of each word in the corpus:

The vocabulary indices for each word in the corpus:

```
>>> words = nodes.Categorical(nodes.Gate(topics, p_word),
...
name='words')
```

### 3.7.3 Inference

Observe the corpus:

```
>>> words.observe(corpus)
```

Break symmetry by random initialization:

```
>>> p_topic.initialize_from_random()
>>> p_word.initialize_from_random()
```

Construct inference engine:

```
>>> from bayespy.inference import VB
>>> Q = VB(words, topics, p_word, p_topic, document_indices)
```

Run the VB learning algorithm:

```
>>> Q.update(repeat=1000)
Iteration ...
```

## 3.7.4 Results

Use bayespy.plot to plot the results:

```
>>> import bayespy.plot as bpplt
```

Plot the topic distributions for each document:

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.hinton(Q['p_topic'])
>>> bpplt.pyplot.title("Posterior topic distribution for each document")
<matplotlib.text.Text object at 0x...>
>>> bpplt.pyplot.xlabel("Topics")
<matplotlib.text.Text object at 0x...>
>>> bpplt.pyplot.ylabel("Documents")
<matplotlib.text.Text object at 0x...>
```

Plot the word distributions for each topic:

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.hinton(Q['p_word'])
>>> bpplt.pyplot.title("Posterior word distributions for each topic")
<matplotlib.text.Text object at 0x...>
>>> bpplt.pyplot.xlabel("Words")
<matplotlib.text.Text object at 0x...>
>>> bpplt.pyplot.ylabel("Topics")
<matplotlib.text.Text object at 0x...>
```

#### **Todo**

Create more illustrative plots.

# 3.7.5 Stochastic variational inference

LDA is a popular example for stochastic variational inference (SVI). Using SVI for LDA is quite simple in BayesPy. In SVI, only a subset of the dataset is used at each iteration step but this subset is "repeated" to get the same size as the original dataset. Let us define a size for the subset:

```
>>> subset_size = 1000
```

Thus, our subset will be repeat this many times:

```
>>> plates_multiplier = n_words / subset_size
```

Note that this multiplier doesn't need to be an integer.

Now, let us repeat the model construction with only one minor addition. The following variables are identical to previous:

The document indices vector is now a bit shorter, using only a subset:

Note that at this point, it doesn't matter which elements we chose for the subset. For the topic assignments of each word in the corpus we need to use plates\_multiplier because these topic assignments for the subset are "repeated" to recover the full dataset:

Finally, the vocabulary indices for each word in the corpus are constructed as before:

This node inherits the plates and multipliers from its parent topics, so there is no need to define them here. Again, break symmetry by random initialization:

```
>>> p_topic.initialize_from_random()
>>> p_word.initialize_from_random()
```

Construct inference engine:

```
>>> from bayespy.inference import VB
>>> Q = VB(words, topics, p_word, p_topic, document_indices)
```

In order to use SVI, we need to disable some lower bound checks, because the lower bound doesn't anymore necessarily increase at each iteration step:

```
>>> Q.ignore_bound_checks = True
```

For the stochastic gradient ascent, we'll define some learning parameters:

```
>>> delay = 1
>>> forgetting_rate = 0.7
```

Run the inference:

```
>>> for n in range(1000):
       # Observe a random mini-batch
        subset = np.random.choice(n_words, subset_size)
        Q['words'].observe(corpus[subset])
        Q['document_indices'].set_value(word_documents[subset])
. . .
        # Learn intermediate variables
. . .
        Q.update('topics')
. . .
       # Set step length
. . .
       step = (n + delay) ** (-forgetting_rate)
       # Stochastic gradient for the global variables
        Q.gradient_step('p_topic', 'p_word', scale=step)
Iteration 1: ...
```

If one is interested, the lower bound values during the SVI algorithm can be plotted as:

```
>>> bpplt.pyplot.figure()
<matplotlib.figure.Figure object at 0x...>
>>> bpplt.pyplot.plot(Q.L)
[<matplotlib.lines.Line2D object at 0x...>]
```

The other results can be plotted as before.

# **DEVELOPER GUIDE**

This chapter provides basic information for developers about contributing, the theoretical background and the core structure. It is assumed that the reader has read and is familiar with *User guide*.

# 4.1 Workflow

The main forum for BayesPy development is GitHub. Bugs and other issues can be reported at <a href="https://github.com/bayespy/bayespy/issues">https://github.com/bayespy/bayespy/bayespy/issues</a>. Contributions to the code and documentation are welcome and should be given as pull requests at <a href="https://github.com/bayespy/bayespy/pulls">https://github.com/bayespy/bayespy/pulls</a>. In order to create pull requests, it is recommended to fork the git repository, make local changes and submit these changes as a pull request. The style guide for writing docstrings follows the style guide of NumPy, available at <a href="https://github.com/numpy/numpy/blob/master/doc/HOWTO\_DOCUMENT.rst.txt">https://github.com/numpy/numpy/numpy/numpy/blob/master/doc/HOWTO\_DOCUMENT.rst.txt</a>. Detailed instructions on development workflow can be read from NumPy guide, available at <a href="https://docs.scipy.org/doc/numpy/dev/gitwash/development\_workflow.html">https://docs.scipy.org/doc/numpy/dev/gitwash/development\_workflow.html</a>. BayesPy uses the following acronyms to start the commit message:

- API: an (incompatible) API change
- BLD: change related to building numpy
- BUG: bug fix
- DEMO: modification in demo code
- DEP: deprecate something, or remove a deprecated object
- DEV: development tool or utility
- DOC: documentation
- · ENH: enhancement
- MAINT: maintenance commit (refactoring, typos, etc.)
- REV: revert an earlier commit
- STY: style fix (whitespace, PEP8)
- TST: addition or modification of tests
- REL: related to releasing

Since version 0.3.7, we have started following Vincent Driessen's branching model in how git is used.

# 4.1.1 Making releases

- Commit any current changes to git.
- Start a release branch: git flow release start x.y.z
- Edit version number in setup.py and commit.
- Add changes to CHANGELOG.rst and commit.
- Publish the release branch: git flow release publish x.y.z
- Finish the release: git flow release finish x.y.z. Write the following commit message: REL: Version x.y.z.
- Push to GitHub: git push && git push --tags
- Publish in PyPI: python setup.py release\_pypi
- Update the documentation web page: cd doc && make gh-pages
- Publish in mloss.org.
- Announcements to bayespy@googlegroups.com, scipy-user@scipy.org and numpy-discussion@scipy.org.

# 4.2 Variational message passing

This section briefly describes the variational message passing (VMP) framework, which is currently the only implemented inference engine in BayesPy. The variational Bayesian (VB) inference engine in BayesPy assumes that the posterior approximation factorizes with respect to nodes and plates. VMP is based on updating one node at a time (the plates in one node can be updated simultaneously) and iteratively updating all nodes in turns until convergence.

# 4.2.1 Standard update equation

The general update equation for the factorized approximation of node  $\theta$  is the following:

$$\log q(\boldsymbol{\theta}) = \langle \log p(\boldsymbol{\theta}|\operatorname{pa}(\boldsymbol{\theta})) \rangle + \sum_{\mathbf{x} \in \operatorname{ch}(\boldsymbol{\theta})} \langle \log p(\mathbf{x}|\operatorname{pa}(\mathbf{x})) \rangle + \operatorname{const}, \tag{4.1}$$

where  $pa(\theta)$  and  $ch(\theta)$  are the set of parents and children of  $\theta$ , respectively. Thus, the posterior approximation of a node is updated by taking a sum of the expectations of all log densities in which the node variable appears. The expectations are over the approximate distribution of all other variables than  $\theta$ . Actually, not all the variables are needed, because the non-constant part depends only on the Markov blanket of  $\theta$ . This leads to a local optimization scheme, which uses messages from neighbouring nodes.

The messages are simple for conjugate exponential family models. An exponential family distribution has the following log probability density function:

$$\log p(\mathbf{x}|\mathbf{\Theta}) = \mathbf{u}_{\mathbf{x}}(\mathbf{x})^{\mathrm{T}} \boldsymbol{\phi}_{\mathbf{x}}(\mathbf{\Theta}) + g_{\mathbf{x}}(\mathbf{\Theta}) + f_{\mathbf{x}}(\mathbf{x}), \tag{4.2}$$

where  $\Theta = \{\theta_j\}$  is the set of parents,  $\mathbf{u}$  is the sufficient statistic vector,  $\phi$  is the natural parameter vector, g is the negative log normalizer, and f is the log base function. Note that the log density is linear with respect to the terms that are functions of  $\mathbf{x}$ :  $\mathbf{u}$  and f. If a parent has a conjugate prior, (4.2) is also linear with respect to the parent's sufficient statistic vector. Thus, (4.2) can be re-organized with respect to a parent  $\theta_j$  as

$$\log p(\mathbf{x}|\boldsymbol{\Theta}) = \mathbf{u}_{\boldsymbol{\theta}_j}(\boldsymbol{\theta}_j)^{\mathrm{T}} \boldsymbol{\phi}_{\mathbf{x} \rightarrow \boldsymbol{\theta}_j}(\mathbf{x}, \{\boldsymbol{\theta}_k\}_{k \neq j}) + \mathrm{const},$$

where  $\mathbf{u}_{\theta_j}$  is the sufficient statistic vector of  $\theta_j$  and the constant part is constant with respect to  $\theta_j$ . Thus, the update equation (4.1) for  $\theta_j$  can be written as

$$\begin{split} \log q(\boldsymbol{\theta}_j) &= \mathbf{u}_{\boldsymbol{\theta}_j}(\boldsymbol{\theta}_j)^{\mathrm{T}} \langle \boldsymbol{\phi}_{\boldsymbol{\theta}_j} \rangle + f_{\boldsymbol{\theta}_j}(\boldsymbol{\theta}_j) + \mathbf{u}_{\boldsymbol{\theta}_j}(\boldsymbol{\theta}_j)^{\mathrm{T}} \sum_{\mathbf{x} \in \mathrm{ch}(\boldsymbol{\theta}_j)} \langle \boldsymbol{\phi}_{\mathbf{x} \to \boldsymbol{\theta}_j} \rangle + \mathrm{const}, \\ &= \mathbf{u}_{\boldsymbol{\theta}_j}(\boldsymbol{\theta}_j)^{\mathrm{T}} \left( \langle \boldsymbol{\phi}_{\boldsymbol{\theta}_j} \rangle + \sum_{\mathbf{x} \in \mathrm{ch}(\boldsymbol{\theta}_j)} \langle \boldsymbol{\phi}_{\mathbf{x} \to \boldsymbol{\theta}_j} \rangle \right) + f_{\boldsymbol{\theta}_j}(\boldsymbol{\theta}_j) + \mathrm{const}, \end{split}$$

where the summation is over all the child nodes of  $\theta_j$ . Because of the conjugacy,  $\langle \phi_{\theta_j} \rangle$  depends (multi)linearly on the parents' sufficient statistic vector. Similarly,  $\langle \phi_{\mathbf{x} \to \theta_j} \rangle$  depends (multi)linearly on the expectations of the children's and co-parents' sufficient statistics. This gives the following update equation for the natural parameter vector of the posterior approximation  $q(\phi_j)$ :

$$\tilde{\phi}_j = \langle \phi_{\theta_j} \rangle + \sum_{\mathbf{x} \in \text{ch}(\theta_j)} \langle \phi_{\mathbf{x} \to \theta_j} \rangle. \tag{4.3}$$

# 4.2.2 Variational messages

The update equation (4.3) leads to a message passing scheme: the term  $\langle \phi_{\theta_j} \rangle$  is a function of the parents' sufficient statistic vector and the term  $\langle \phi_{\mathbf{x} \to \theta_j} \rangle$  can be interpreted as a message from the child node  $\mathbf{x}$ . Thus, the message from the child node  $\mathbf{x}$  to the parent node  $\boldsymbol{\theta}$  is

$$\mathbf{m}_{\mathbf{x} \to \boldsymbol{\theta}} \equiv \langle \boldsymbol{\phi}_{\mathbf{x} \to \boldsymbol{\theta}} \rangle,$$

which can be computed as a function of the sufficient statistic vector of the co-parent nodes of  $\theta$  and the sufficient statistic vector of the child node  $\mathbf{x}$ . The message from the parent node  $\theta$  to the child node  $\mathbf{x}$  is simply the expectation of the sufficient statistic vector:

$$\mathbf{m}_{\boldsymbol{\theta} \to \mathbf{x}} \equiv \langle \mathbf{u}_{\boldsymbol{\theta}} \rangle.$$

In order to compute the expectation of the sufficient statistic vector we need to write  $q(\theta)$  as

$$\log q(\boldsymbol{\theta}) = \mathbf{u}(\boldsymbol{\theta})^{\mathrm{T}} \tilde{\boldsymbol{\phi}} + \tilde{g}(\tilde{\boldsymbol{\phi}}) + f(\boldsymbol{\theta}),$$

where  $\tilde{\phi}$  is the natural parameter vector of  $q(\theta)$ . Now, the expectation of the sufficient statistic vector is defined as

$$\langle \mathbf{u}_{\theta} \rangle = -\frac{\partial \tilde{g}}{\partial \tilde{\phi}_{\theta}} (\tilde{\phi}_{\theta}). \tag{4.4}$$

We call this expectation of the sufficient statistic vector as the moments vector.

### 4.2.3 Lower bound

Computing the VB lower bound is not necessary in order to find the posterior approximation, although it is extremely useful in monitoring convergence and possible bugs. The VB lower bound can be written as

$$\mathcal{L} = \langle \log p(\mathbf{Y}, \mathbf{X}) \rangle - \langle \log q(\mathbf{X}) \rangle,$$

where Y is the set of all observed variables and X is the set of all latent variables. It can also be written as

$$\mathcal{L} = \sum_{\mathbf{y} \in \mathbf{Y}} \langle \log p(\mathbf{y} | \text{pa}(\mathbf{y})) \rangle + \sum_{\mathbf{x} \in \mathbf{X}} \left[ \langle \log p(\mathbf{x} | \text{pa}(\mathbf{x})) \rangle - \langle \log q(\mathbf{x}) \right],$$

which shows that observed and latent variables contribute differently to the lower bound. These contributions have simple forms for exponential family nodes. Observed exponential family nodes contribute to the lower bound as follows:

$$\langle \log p(\mathbf{y}|pa(\mathbf{y})) \rangle = \mathbf{u}(\mathbf{y})^T \langle \boldsymbol{\phi} \rangle + \langle g \rangle + f(\mathbf{x}),$$

where y is the observed data. On the other hand, latent exponential family nodes contribute to the lower bound as follows:

$$\langle \log p(\mathbf{x}|\boldsymbol{\theta}) \rangle - \langle \log q(\mathbf{x}) \rangle = \langle \mathbf{u} \rangle^T (\langle \boldsymbol{\phi} \rangle - \tilde{\boldsymbol{\phi}}) + \langle g \rangle - \tilde{g}.$$

If a node is partially observed and partially unobserved, these formulas are applied plate-wise appropriately.

#### 4.2.4 Terms

To summarize, implementing VMP requires one to write for each stochastic exponential family node:

 $\langle \phi \rangle$  : the expectation of the prior natural parameter vector

Computed as a function of the messages from parents.

 $\tilde{\phi}$ : natural parameter vector of the posterior approximation

Computed as a sum of  $\langle \phi \rangle$  and the messages from children.

 $\langle \mathbf{u} \rangle$ : the posterior moments vector

Computed as a function of  $\tilde{\phi}$  as defined in (4.4).

 $\mathbf{u}(\mathbf{x})$ : the moments vector for given data

Computed as a function of of the observed data x.

 $\langle g \rangle$  : the expectation of the negative log normalizer of the prior

Computed as a function of parent moments.

 $\tilde{g}$ : the negative log normalizer of the posterior approximation

Computed as a function of  $\phi$ .

 $f(\mathbf{x})$ : the log base measure for given data

Computed as a function of the observed data x.

 $\langle \phi_{\mathbf{x} \to \boldsymbol{\theta}} \rangle$ : the message to parent  $\boldsymbol{\theta}$ 

Computed as a function of the moments of this node and the other parents.

Deterministic nodes require only the following terms:

 $\langle \mathbf{u} \rangle$  : the posterior moments vector

Computed as a function of the messages from the parents.

m: the message to a parent

Computed as a function of the messages from the other parents and all children.

# 4.3 Implementing inference engines

Currently, only variational Bayesian inference engine is implemented. This implementation is not very modular, that is, the inference engine is not well separated from the model construction. Thus, it is not straightforward to implement other inference engines at the moment. Improving the modularity of the inference engine and model construction is future work with high priority. In any case, BayesPy aims to be an efficient, simple and modular Bayesian package for variational inference at least.

# 4.4 Implementing nodes

The main goal of BayesPy is to provide a package which enables easy and flexible construction of simple and complex models with efficient inference. However, users may sometimes be unable to construct their models because the built-in nodes do not implement some specific features. Thus, one may need to implement new nodes in order to construct the model. BayesPy aims to make the implementation of new nodes both simple and fast. Probably, a large complex model can be constructed almost completely with the built-in nodes and the user needs to implement only a few nodes.

#### 4.4.1 Moments

In order to implement nodes, it is important to understand the messaging framework of the nodes. A node is a unit of calculation which communicates to its parent and child nodes using messages. These messages have types that need to match between nodes, that is, the child node needs to understand the messages its parents are sending and vice versa. Thus, a node defines which message type it requires from each of its parents, and only nodes that have that type of output message (i.e., the message to a child node) are valid parent nodes for that node.

The message type is defined by the moments of the parent node. The moments are a collection of expectations:  $\{\langle f_1(X)\rangle,\ldots,\langle f_N(X)\rangle\}$ . The functions  $f_1,\ldots,f_N$  (and the number of the functions) define the message type and they are the sufficient statistic as discussed in the previous section. Different message types are represented by *Moments* class hierarchy. For instance, GaussianMoments represents a message type with parent moments  $\{\langle \mathbf{x} \rangle, \langle \mathbf{x} \mathbf{x}^T \rangle\}$  and WishartMoments a message type with parent moments  $\{\langle \mathbf{A} \rangle, \langle \log |\mathbf{A}| \rangle\}$ .

Let us give an example: Gaussian node outputs GaussianMoments messages and Wishart node outputs WishartMoments messages. Gaussian node requires that it receives GaussianMoments messages from the mean parent node and WishartMoments messages from the precision parent node. Thus, Gaussian and Wishart are valid node classes as the mean and precision parent nodes of Gaussian node.

Note that several nodes may have the same output message type and some message types can be transformed to other message types using deterministic converter nodes. For instance, <code>Gaussian</code> and <code>GaussianARD</code> nodes both output <code>GaussianMoments</code> messages, deterministic <code>SumMultiply</code> also outputs <code>GaussianMoments</code> messages, and deterministic converter <code>MarkovChainToGaussian</code> converts <code>GaussianMarkovChainMoments</code> to <code>GaussianMoments</code>.

Each node specifies the message type requirements of its parents by Node.\_parent\_moments attribute which is a list of Moments sub-class instances. These moments objects have a few purpose when creating the node: 1) check that parents are sending proper messages; 2) if parents use different message type, try to add a converter which converts the messages to the correct type if possible; 3) if given parents are not nodes but numeric arrays, convert them to constant nodes with correct output message type.

When implementing a new node, it is not always necessary to implement a new moments class. If another node has the same sufficient statistic vector, thus the same moments, that moments class can be used. Otherwise, one must implement a simple moments class which has the following methods:

• Moments.compute\_fixed\_moments()

Computes the moments for a known value. This is used to compute the moments of constant numeric arrays and wrap them into constant nodes.

• Moments.compute\_dims\_from\_values()

Given a known value of the variable, return the shape of the variable dimensions in the moments. This is used to solve the shape of the moments array for constant nodes.

### 4.4.2 Distributions

In order to implement a stochastic exponential family node, one must first write down the log probability density function of the node and derive the terms discussed in section *Terms*. These terms are implemented and collected as a class which is a subclass of Distribution. The main reason to implement these methods in another class instead of the node class itself is that these methods can be used without creating a node, for instance, in Mixture class.

For exponential family distributions, the distribution class is a subclass of <code>ExponentialFamilyDistribution</code>, and the relation between the terms in section <code>Terms</code> and the methods is as follows:

• ExponentialFamilyDistribution.compute\_phi\_from\_parents()

Computes the expectation of the natural parameters  $\langle \phi \rangle$  in the prior distribution given the moments of the parents.

• ExponentialFamilyDistribution.compute\_cgf\_from\_parents()

Computes the expectation of the negative log normalizer  $\langle g \rangle$  of the prior distribution given the moments of the parents.

• ExponentialFamilyDistribution.compute\_moments\_and\_cqf()

Computes the moments  $\langle \mathbf{u} \rangle$  and the negative log normalizer  $\tilde{g}$  of the posterior distribution given the natural parameters  $\tilde{\phi}$ .

• ExponentialFamilyDistribution.compute\_message\_to\_parent()

Computes the message  $\langle \phi_{\mathbf{x} \to \boldsymbol{\theta}} \rangle$  from the node  $\mathbf{x}$  to its parent node  $\boldsymbol{\theta}$  given the moments of the node and the other parents.

• ExponentialFamilyDistribution.compute\_fixed\_moments\_and\_f()

Computes  $\mathbf{u}(\mathbf{x})$  and  $f(\mathbf{x})$  for given observed value  $\mathbf{x}$ . Without this method, variables from this distribution cannot be observed.

For each stochastic exponential family node, one must write a distribution class which implements these methods. After that, the node class is basically a simple wrapper and it also stores the moments and the natural parameters of the current posterior approximation. Note that the distribution classes do not store node-specific information, they are more like static collections of methods. However, sometimes the implementations depend on some information, such as the dimensionality of the variable, and this information must be provided, if needed, when constructing the distribution object.

In addition to the methods listed above, it is necessary to implement a few more methods in some cases. This happens when the plates of the parent do not map to the plates directly as discussed in section *Irregular plates*. Then, one must write methods that implement this plate mapping and apply the same mapping to the mask array:

• ExponentialFamilyDistribution.plates\_from\_parent()

Given the plates of the parent, return the resulting plates of the child.

• ExponentialFamilyDistribution.plates\_to\_parent()

Given the plates of the child, return the plates of the parent that would have resulted them.

• ExponentialFamilyDistribution.compute\_mask\_to\_parent()

Given the mask array of the child, apply the plate mapping.

It is important to understand when one must implement these methods, because the default implementations in the base class will lead to errors or weird results.

# 4.4.3 Stochastic exponential family nodes

After implementing the distribution class, the next task is to implement the node class. First, we need to explain a few important attributes before we can explain how to implement a node class.

Stochastic exponential family nodes have two attributes that store the state of the posterior distribution:

• phi

The natural parameter vector  $\tilde{\phi}$  of the posterior approximation.

• u

The moments  $\langle \mathbf{u} \rangle$  of the posterior approximation.

Instead of storing these two variables as vectors (as in the mathematical formulas), they are stored as lists of arrays with convenient shapes. For instance, Gaussian node stores the moments as a list consisting of a vector  $\langle \mathbf{x} \rangle$  and a matrix  $\langle \mathbf{x} \mathbf{x}^T \rangle$  instead of reshaping and concatenating these into a single vector. The same applies for the natural parameters phi because it has the same shape as u.

The shapes of the arrays in the lists u and phi consist of the shape caused by the plates and the shape caused by the variable itself. For instance, the moments of Gaussian node have shape (D,) and (D, D), where D is the dimensionality of the Gaussian vector. In addition, if the node has plates, they are added to these shapes. Thus, for instance, if the Gaussian node has plates (3, 7) and D is 5, the shape of u[0] and phi[0] would be (3, 7, 5) and the shape of u[1] and phi[1] would be (3, 7, 5, 5). This shape information is stored in the following attributes:

• plates: a tuple

The plates of the node. In our example, (3, 7).

• dims: a list of tuples

The shape of each of the moments arrays (or natural parameter arrays) without plates. In our example, [ (5, ), (5, 5) ].

Finally, three attributes define VMP for the node:

• \_moments: Moments sub-class instance

An object defining the moments of the node.

• \_parent\_moments: list of Moments sub-class instances

A list defining the moments requirements for each parent.

• \_distribution: Distribution sub-class instance

An object implementing the VMP formulas.

Basically, a node class is a collection of the above attributes. When a node is created, these attributes are defined. The base class for exponential family nodes, <code>ExponentialFamily</code>, provides a simple default constructor which does not need to be overwritten if <code>dims, \_moments</code>, <code>\_parent\_moments</code> and <code>\_distribution</code> can be provided as static class attributes. For instance, <code>Gamma</code> node defines these attributes statically. However, usually at least one of these attributes cannot be defined statically in the class. In that case, one must implement a class method which overloads <code>ExponentialFamily.\_constructor()</code>. The purpose of this method is to define all the attributes given the parent nodes. These are defined using a class method instead of <code>\_\_init\_</code>\_ method in order to be able to use

the class constructors statically, for instance, in Mixture class. This construction allows users to create mixtures of any exponential family distribution with simple syntax.

The parents of a node must be converted so that they have a correct message type, because the user may have provided numeric arrays or nodes with incorrect message type. Numeric arrays should be converted to constant nodes with correct message type. Incorrect message type nodes should be converted to correct message type nodes if possible. Thus, the constructor should use Node.\_ensure\_moments method to make sure the parent is a node with correct message type. Instead of calling this method for each parent node in the constructor, one can use ensureparents decorator to do this automatically. However, the decorator requires that \_parent\_moments attribute has already been defined statically. If this is not possible, the parent nodes must be converted manually in the constructor, because one should never assume that the parent nodes given to the constructor are nodes with correct message type or even nodes at all.

### 4.4.4 Deterministic nodes

Deterministic nodes are nodes that do not correspond to any probability distribution but rather a deterministic function. It does not have any moments or natural parameters to store. A deterministic node is implemented as a subclass of Deterministic base class. The new node class must implement the following methods:

• Deterministic.\_compute\_moments()

Computes the moments given the moments of the parents.

• Deterministic.\_compute\_message\_to\_parent()

Computes the message to a parent node given the message chilsome dren and the moments of the other parents. In cases. one may want Deterministic.\_compute\_message\_and\_mask\_to\_parent() implement Deterministic.\_message\_to\_parent() instead in order to gain more control over efficient computa-

Similarly as in Distribution class, if the node handles plates irregularly, it is important to implement the following methods:

• Deterministic.\_plates\_from\_parent()

Given the plates of the parent, return the resulting plates of the child.

• Deterministic.\_plates\_to\_parent()

Given the plates of the child, return the plates of the parent that would have resulted them.

• Deterministic.\_compute\_mask\_to\_parent()

Given the mask array, convert it to a plate mask of the parent.

### Converter nodes

Sometimes a node has incorrect message type but the message can be converted into a correct type. For instance, GaussianMarkovChain has GaussianMarkovChainMoments message type, which means moments  $\{\langle \mathbf{x}_n \rangle, \langle \mathbf{x}_n \mathbf{x}_n^T \rangle, \langle \mathbf{x}_n \mathbf{x}_{n-1}^T \rangle\}_{n=1}^N$ . These moments can be converted to GaussianMoments by ignoring the third element and considering the time axis as a plate axis. Thus, if a node requires GaussianMoments message from its parent, GaussianMarkovChain is a valid parent if its messages are modified properly. This conversion is implemented in MarkovChainToGaussian converter class. Converter nodes are simple deterministic nodes that have one parent node and they convert the messages to another message type.

For the user, it is not convenient if the exact message type has to be known and an explicit converter node needs to be created. Thus, the conversions are done automatically and the user will be unaware of them. In order to enable this automatization, when writing a converter node, one should register the converter to the moments class using

Moments.add\_converter(). For instance, a class X which converts moments A to moments B is registered as A.add\_conveter(B, X). After that, Node.\_ensure\_moments() and Node.\_convert() methods are used to perform the conversions automatically. The conversion can consist of several consecutive converter nodes, and the least number of conversions is used.

### **CHAPTER**

# **FIVE**

# **USER API**

bayespy.nodes	Package for nodes used to construct the model.
bayespy.inference	Package for Bayesian inference engines
bayespy.plot	Functions for plotting nodes.

# 5.1 bayespy.nodes

Package for nodes used to construct the model.

# 5.1.1 Stochastic nodes

Nodes for Gaussian variables:

Gaussian(mu, Lambda, **kwargs)	Node for Gaussian variables.
<pre>GaussianARD(mu, alpha[, ndim, shape])</pre>	Node for Gaussian variables with ARD prior.

# bayespy.nodes.Gaussian

class bayespy.nodes.Gaussian(mu, Lambda, \*\*kwargs)

Node for Gaussian variables.

The node represents a D-dimensional vector from the Gaussian distribution:

$$\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Lambda}),$$

where  $\mu$  is the mean vector and  $\Lambda$  is the precision matrix (i.e., inverse of the covariance matrix).

$$\mathbf{x}, \boldsymbol{\mu} \in \mathbb{R}^D$$
,  $\boldsymbol{\Lambda} \in \mathbb{R}^{D \times D}$ ,  $\boldsymbol{\Lambda}$  symmetric positive definite

**Parameters mu**: Gaussian-like node or GaussianGammaISO-like node or GaussianWishart-like node or array

Mean vector

Lambda: Wishart-like node or array

Precision matrix

#### See also:

Wishart, GaussianARD, GaussianWishart, GaussianGammaARD, GaussianGammaISO

\_\_init\_\_(mu, Lambda, \*\*kwargs)

Create Gaussian node

#### **Methods**

init(mu, Lambda, **kwargs)	Create Gaussian node
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
<pre>get_parameters()</pre>	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(mu, Lambda)	
<pre>initialize_from_prior()</pre>	
<pre>initialize_from_random()</pre>	Set the variable to a random sample from the current distribution.
<pre>initialize_from_value(x, *args)</pre>	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
lower_bound_contribution([gradient,])	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
rotate(R[, inv, logdet, Q])	
<pre>rotate_matrix(R1, R2[, inv1, logdet1, inv2,])</pre>	The vector is reshaped into a matrix by stacking the row vectors.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

# bayespy.nodes.Gaussian.\_\_init\_\_

Gaussian.\_\_init\_\_(mu, Lambda, \*\*kwargs)
Create Gaussian node

# bayespy.nodes.Gaussian.add\_plate\_axis

Gaussian.add\_plate\_axis(to\_plate)

# $bayes py. nodes. Gaussian. broadcasting\_multiplier$

Gaussian.broadcasting\_multiplier (plates, \*args)

82 Chapter 5. User API

#### bayespy.nodes.Gaussian.delete

```
Gaussian.delete()
```

Delete this node and the children

### bayespy.nodes.Gaussian.get\_gradient

```
Gaussian.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.Gaussian.get\_mask

```
Gaussian.get_mask()
```

#### bayespy.nodes.Gaussian.get\_moments

```
Gaussian.get_moments()
```

#### bayespy.nodes.Gaussian.get\_parameters

```
Gaussian.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

### bayespy.nodes.Gaussian.get\_riemannian\_gradient

```
Gaussian.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

#### bayespy.nodes.Gaussian.get\_shape

```
Gaussian.get_shape(ind)
```

### bayespy.nodes.Gaussian.has\_plotter

```
Gaussian.has_plotter()
```

Return True if the node has a plotter

# $bayes py. nodes. Gaussian. initialize\_from\_parameters$

 ${\tt Gaussian.initialize\_from\_parameters}~(\textit{mu}, \textit{Lambda})$ 

```
bayespy.nodes.Gaussian.initialize_from_prior
Gaussian.initialize_from_prior()
bayespy.nodes.Gaussian.initialize_from_random
Gaussian.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.Gaussian.initialize_from_value
Gaussian.initialize_from_value(x, *args)
bayespy.nodes.Gaussian.load
Gaussian.load(filename)
bayespy.nodes.Gaussian.logpdf
Gaussian.logpdf(X, mask=True)
    Compute the log probability density function Q(X) of this node.
bayespy.nodes.Gaussian.lower_bound_contribution
Gaussian.lower_bound_contribution (gradient=False, ignore_masked=True)
    Compute E[ log p(X|parents) - log q(X) ]
    If deterministic annealing is used, the term E[-\log q(X)] is divided by the anneling coefficient. That is,
    phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).
bayespy.nodes.Gaussian.lowerbound
Gaussian.lowerbound()
bayespy.nodes.Gaussian.move_plates
Gaussian.move_plates (from_plate, to_plate)
bayespy.nodes.Gaussian.observe
Gaussian.observe(x, *args, mask=True)
    Fix moments, compute f and propagate mask.
```

#### bayespy.nodes.Gaussian.pdf

```
Gaussian.pdf (X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.Gaussian.plot

```
Gaussian.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.Gaussian.random

```
Gaussian.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.Gaussian.rotate

```
Gaussian.rotate(R, inv=None, logdet=None, Q=None)
```

#### bayespy.nodes.Gaussian.rotate\_matrix

```
Gaussian.rotate_matrix(R1, R2, inv1=None, logdet1=None, inv2=None, logdet2=None, Q=None)
```

The vector is reshaped into a matrix by stacking the row vectors.

Computes R1\*X\*R2', which is identical to kron(R1,R2)\*x (??)

Note that this is slightly different from the standard Kronecker product definition because Numpy stacks row vectors instead of column vectors.

#### Parameters R1: ndarray

A matrix from the left

**R2** : ndarray

A matrix from the right

#### bayespy.nodes.Gaussian.save

```
Gaussian.save(filename)
```

#### bayespy.nodes.Gaussian.set\_parameters

```
Gaussian.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Gaussian.set\_plotter

Gaussian.set\_plotter(plotter)

#### bayespy.nodes.Gaussian.show

```
Gaussian.show()
```

Print the distribution using standard parameterization.

### bayespy.nodes.Gaussian.unobserve

Gaussian.unobserve()

#### bayespy.nodes.Gaussian.update

Gaussian.update(annealing=1.0)

#### **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

#### bayespy.nodes.Gaussian.dims

Gaussian.dims = None

# bayespy.nodes.Gaussian.plates

Gaussian.plates = None

### bayespy.nodes.Gaussian.plates\_multiplier

Gaussian.plates\_multiplier

Plate multiplier is applied to messages to parents

### bayespy.nodes.GaussianARD

**class** bayespy.nodes.**GaussianARD** (*mu*, *alpha*, *ndim=None*, *shape=None*, \*\*kwargs)
Node for Gaussian variables with ARD prior.

The node represents a *D*-dimensional vector from the Gaussian distribution:

$$\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \operatorname{diag}(\boldsymbol{\alpha})),$$

where  $\mu$  is the mean vector and  $\operatorname{diag}(\alpha)$  is the diagonal precision matrix (i.e., inverse of the covariance matrix).

$$\mathbf{x}, \boldsymbol{\mu} \in \mathbb{R}^D$$
,  $\alpha_d > 0$  for  $d = 0, \dots, D-1$ 

*Note:* The form of the posterior approximation is a Gaussian distribution with full covariance matrix instead of a diagonal matrix.

**Parameters mu**: Gaussian-like node or GaussianGammaISO-like node or GaussianGammaARD-like node or array

Mean vector

alpha: gamma-like node or array

Diagonal elements of the precision matrix

### See also:

Gamma, Gaussian, GaussianGammaARD, GaussianGammaISO, GaussianWishart

\_\_init\_\_ (mu, alpha, ndim=None, shape=None, \*\*kwargs)
Create GaussianARD node.

#### **Methods**

init(mu, alpha[, ndim, shape])	Create GaussianARD node.
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
<pre>get_parameters()</pre>	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_mean_and_covariance(mu, Cov)	
initialize_from_parameters(mu, alpha)	
initialize_from_prior()	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
lower_bound_contribution([gradient,])	Compute E[ log p(X parents) - log q(X) ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
rotate(R[, inv, logdet, axis, Q, subset])	
rotate_plates(Q[, plate_axis])	Approximate rotation of a plate axis.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

#### bayespy.nodes.GaussianARD.\_\_init\_\_

```
GaussianARD.__init__ (mu, alpha, ndim=None, shape=None, **kwargs)
Create GaussianARD node.
```

### bayespy.nodes.GaussianARD.add\_plate\_axis

```
GaussianARD.add_plate_axis(to_plate)
```

#### bayespy.nodes.GaussianARD.broadcasting\_multiplier

```
GaussianARD.broadcasting_multiplier(plates, *args)
```

#### bayespy.nodes.GaussianARD.delete

```
GaussianARD.delete()
```

Delete this node and the children

### bayespy.nodes.GaussianARD.get\_gradient

```
GaussianARD.get_gradient (rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.GaussianARD.get\_mask

```
GaussianARD.get_mask()
```

#### bayespy.nodes.GaussianARD.get\_moments

```
GaussianARD.get_moments()
```

### bayespy.nodes.GaussianARD.get\_parameters

```
GaussianARD.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# $bayes py. nodes. Gaussian ARD. get\_riemannian\_gradient$

```
GaussianARD.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

```
bayespy.nodes.GaussianARD.get_shape
GaussianARD.get_shape (ind)
bayespy.nodes.GaussianARD.has_plotter
GaussianARD.has_plotter()
    Return True if the node has a plotter
bayespy.nodes.GaussianARD.initialize_from_mean_and_covariance
GaussianARD.initialize_from_mean_and_covariance (mu, Cov)
bayespy.nodes.GaussianARD.initialize_from_parameters
GaussianARD.initialize_from_parameters (mu, alpha)
bayespy.nodes.GaussianARD.initialize_from_prior
GaussianARD.initialize_from_prior()
bayespy.nodes.GaussianARD.initialize_from_random
GaussianARD.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.GaussianARD.initialize_from_value
GaussianARD.initialize_from_value(x, *args)
bayespy.nodes.GaussianARD.load
GaussianARD.load(filename)
bayespy.nodes.GaussianARD.logpdf
GaussianARD.logpdf(X, mask=True)
    Compute the log probability density function Q(X) of this node.
bayespy.nodes.GaussianARD.lower_bound_contribution
GaussianARD.lower_bound_contribution(gradient=False, ignore_masked=True)
    Compute E[ \log p(X|parents) - \log q(X) ]
    If deterministic annealing is used, the term E[-\log q(X)] is divided by the anneling coefficient. That is,
    phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).
```

#### bayespy.nodes.GaussianARD.lowerbound

```
GaussianARD.lowerbound()
```

## bayespy.nodes.GaussianARD.move\_plates

```
GaussianARD.move_plates (from_plate, to_plate)
```

#### bayespy.nodes.GaussianARD.observe

```
GaussianARD.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

#### bayespy.nodes.GaussianARD.pdf

```
GaussianARD.pdf (X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.GaussianARD.plot

```
GaussianARD.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

# bayespy.nodes.GaussianARD.random

```
GaussianARD.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.GaussianARD.rotate

```
GaussianARD.rotate(R, inv=None, logdet=None, axis=-1, Q=None, subset=None)
```

### bayespy.nodes.GaussianARD.rotate\_plates

```
GaussianARD.rotate_plates(Q, plate_axis=-1)
```

Approximate rotation of a plate axis.

Mean is rotated exactly but covariance/precision matrix is rotated approximately.

#### bayespy.nodes.GaussianARD.save

```
GaussianARD.save (filename)
```

#### bayespy.nodes.GaussianARD.set\_parameters

```
GaussianARD.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.GaussianARD.set\_plotter

```
GaussianARD.set_plotter(plotter)
```

#### bayespy.nodes.GaussianARD.show

```
GaussianARD.show()
```

Print the distribution using standard parameterization.

### bayespy.nodes.GaussianARD.unobserve

```
GaussianARD.unobserve()
```

### bayespy.nodes.GaussianARD.update

GaussianARD.update(annealing=1.0)

### Attributes

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

#### bayespy.nodes.GaussianARD.dims

```
GaussianARD.dims = None
```

# bayespy.nodes.GaussianARD.plates

```
GaussianARD.plates = None
```

# $bayes py. nodes. Gaussian ARD. plates\_multiplier$

```
GaussianARD.plates_multiplier
```

Plate multiplier is applied to messages to parents

Nodes for precision and scale variables:

Gamma(a, b, **kwargs)	Node for gamma random variables.
Wishart(n, V, **kwargs)	Node for Wishart random variables.
Exponential(l, **kwargs)	Node for exponential random variables.

# bayespy.nodes.Gamma

 ${f class}$  bayespy.nodes. ${f Gamma}$  (a,b,\*\*kwargs)

Node for gamma random variables.

Parameters a: scalar or array

Shape parameter

 ${f b}$  : gamma-like node or scalar or array

Rate parameter

\_\_**init**\_\_ (*a*, *b*, \*\*kwargs)

Create gamma random variable node

### **Methods**

init(a, b, **kwargs)	Create gamma random variable node
add_plate_axis(to_plate)	
as_diagonal_wishart()	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
get_moments()	
get_parameters()	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	
<pre>initialize_from_prior()</pre>	
initialize_from_random()	Set the variable to a random sample from the current distribution.
<pre>initialize_from_value(x, *args)</pre>	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ log $p(X parents)$ - log $q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
	Continued on next page

### Table 5.8 – continued from previous page

unobserve()

update([annealing])

```
bayespy.nodes.Gamma.__init__
```

```
Gamma.\_\_init\_\_(a, b, **kwargs)
```

Create gamma random variable node

### bayespy.nodes.Gamma.add\_plate\_axis

```
Gamma.add_plate_axis (to_plate)
```

### bayespy.nodes.Gamma.as\_diagonal\_wishart

```
Gamma.as_diagonal_wishart()
```

#### bayespy.nodes.Gamma.broadcasting\_multiplier

```
Gamma.broadcasting_multiplier(plates, *args)
```

#### bayespy.nodes.Gamma.delete

```
Gamma.delete()
```

Delete this node and the children

#### bayespy.nodes.Gamma.get\_gradient

```
Gamma.get_gradient (rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

# bayespy.nodes.Gamma.get\_mask

```
Gamma.get_mask()
```

#### bayespy.nodes.Gamma.get\_moments

```
Gamma.get_moments()
```

### bayespy.nodes.Gamma.get\_parameters

```
\label{eq:Gamma.get_parameters} \begin{tabular}{ll} $\operatorname{Gamma.get\_parameters} (\ ) \\ & \operatorname{Return} \ parameters \ of \ the \ VB \ distribution. \end{tabular}
```

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Gamma.get\_riemannian\_gradient

```
Gamma.get_riemannian_gradient()
Computes the Riemannian/natural gradient.
```

#### bayespy.nodes.Gamma.get\_shape

```
Gamma.get_shape (ind)
```

#### bayespy.nodes.Gamma.has\_plotter

```
Gamma.has_plotter()
```

Return True if the node has a plotter

# bayespy.nodes.Gamma.initialize\_from\_parameters

```
Gamma.initialize_from_parameters(*args)
```

#### bayespy.nodes.Gamma.initialize\_from\_prior

```
Gamma.initialize_from_prior()
```

### bayespy.nodes.Gamma.initialize\_from\_random

```
Gamma.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

## bayespy.nodes.Gamma.initialize\_from\_value

```
Gamma.initialize_from_value(x, *args)
```

### bayespy.nodes.Gamma.load

```
Gamma.load(filename)
```

#### bayespy.nodes.Gamma.logpdf

```
Gamma.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.Gamma.lower\_bound\_contribution

```
Gamma.lower_bound_contribution(gradient=False, ignore_masked=True)
```

Compute E[ log p(X|parents) - log q(X) ]

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

#### bayespy.nodes.Gamma.lowerbound

```
Gamma.lowerbound()
```

#### bayespy.nodes.Gamma.move\_plates

```
Gamma.move_plates (from_plate, to_plate)
```

#### bayespy.nodes.Gamma.observe

```
Gamma.observe (x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

### bayespy.nodes.Gamma.pdf

```
Gamma.pdf (X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.Gamma.plot

```
Gamma.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

# bayespy.nodes.Gamma.random

```
Gamma.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.Gamma.save

```
Gamma.save(filename)
```

#### bayespy.nodes.Gamma.set\_parameters

```
Gamma.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Gamma.set\_plotter

```
Gamma.set_plotter(plotter)
```

### bayespy.nodes.Gamma.show

```
Gamma.show()
```

Print the distribution using standard parameterization.

#### bayespy.nodes.Gamma.unobserve

```
Gamma.unobserve()
```

# bayespy.nodes.Gamma.update

```
Gamma.update(annealing=1.0)
```

### **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

### bayespy.nodes.Gamma.dims

```
Gamma.dims = ((), ())
```

### bayespy.nodes.Gamma.plates

```
\texttt{Gamma.plates} = None
```

# $bayes py. nodes. Gamma. plates\_multiplier$

```
{\tt Gamma.plates.multiplier}
```

Plate multiplier is applied to messages to parents

# bayespy.nodes.Wishart

class bayespy.nodes.Wishart (n, V, \*\*kwargs)

Node for Wishart random variables.

The random variable  $\Lambda$  is a  $D \times D$  positive-definite symmetric matrix.

$$p(\mathbf{\Lambda}) = \text{Wishart}(\mathbf{\Lambda}|N, \mathbf{V})$$

Parameters n : scalar or array

N, degrees of freedom, N > D - 1.

**V**: Wishart-like node or (...,D,D)-array

V, scale matrix.

\_\_init\_\_ (*n*, *V*, \*\*kwargs)
Create Wishart node.

#### Methods

init(n, V, **kwargs)	Create Wishart node.
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
get_parameters()	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	
initialize_from_prior()	
<pre>initialize_from_random()</pre>	Set the variable to a random sample from the current distribution.
<pre>initialize_from_value(x, *args)</pre>	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $log p(X parents) - log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

```
bayespy.nodes.Wishart.__init__
```

```
Wishart.__init__(n, V, **kwargs)
Create Wishart node.
```

### bayespy.nodes.Wishart.add\_plate\_axis

```
Wishart.add_plate_axis (to_plate)
```

#### bayespy.nodes.Wishart.broadcasting\_multiplier

```
Wishart.broadcasting_multiplier(plates, *args)
```

#### bayespy.nodes.Wishart.delete

```
Wishart.delete()
```

Delete this node and the children

#### bayespy.nodes.Wishart.get\_gradient

```
Wishart.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.Wishart.get\_mask

```
Wishart.get_mask()
```

#### bayespy.nodes.Wishart.get\_moments

```
Wishart.get_moments()
```

#### bayespy.nodes.Wishart.get\_parameters

```
Wishart.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Wishart.get\_riemannian\_gradient

```
Wishart.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

```
bayespy.nodes.Wishart.get_shape
Wishart.get_shape (ind)
bayespy.nodes.Wishart.has_plotter
Wishart.has_plotter()
    Return True if the node has a plotter
bayespy.nodes.Wishart.initialize_from_parameters
Wishart.initialize_from_parameters(*args)
bayespy.nodes.Wishart.initialize_from_prior
Wishart.initialize_from_prior()
bayespy.nodes.Wishart.initialize_from_random
Wishart.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.Wishart.initialize_from_value
Wishart.initialize_from_value(x, *args)
bayespy.nodes.Wishart.load
Wishart.load(filename)
bayespy.nodes.Wishart.logpdf
Wishart.logpdf(X, mask=True)
    Compute the log probability density function Q(X) of this node.
bayespy.nodes.Wishart.lower_bound_contribution
Wishart.lower_bound_contribution(gradient=False, ignore_masked=True)
    Compute E[ log p(X|parents) - log q(X) ]
    If deterministic annealing is used, the term E[-\log q(X)] is divided by the anneling coefficient. That is,
    phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).
bayespy.nodes.Wishart.lowerbound
Wishart.lowerbound()
```

#### bayespy.nodes.Wishart.move\_plates

```
Wishart.move_plates (from_plate, to_plate)
```

#### bayespy.nodes.Wishart.observe

```
Wishart.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

#### bayespy.nodes.Wishart.pdf

```
Wishart.pdf(X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.Wishart.plot

```
Wishart.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.Wishart.random

```
Wishart.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.Wishart.save

```
Wishart.save (filename)
```

#### bayespy.nodes.Wishart.set\_parameters

```
Wishart.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.Wishart.set\_plotter

```
Wishart.set_plotter(plotter)
```

#### bayespy.nodes.Wishart.show

```
Wishart.show()
```

Print the distribution using standard parameterization.

#### bayespy.nodes.Wishart.unobserve

```
Wishart.unobserve()
```

### bayespy.nodes.Wishart.update

```
Wishart.update(annealing=1.0)
```

#### **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

#### bayespy.nodes.Wishart.dims

```
Wishart.dims = None
```

### bayespy.nodes.Wishart.plates

```
Wishart.plates = None
```

#### bayespy.nodes.Wishart.plates\_multiplier

```
Wishart.plates_multiplier
```

Plate multiplier is applied to messages to parents

#### bayespy.nodes.Exponential

```
{f class} bayespy.nodes. {f Exponential} (l,**kwargs)
```

Node for exponential random variables.

**Warning:** Use Gamma instead of this. Exponential(l) is equivalent to Gamma(1, l).

**Parameters 1**: gamma-like node or scalar or array

Rate parameter

# See also:

Gamma, Poisson

#### **Notes**

For simplicity, this is just a gamma node with the first parent fixed to one. Note that this is a bit inconsistent with the BayesPy philosophy which states that the node does not only define the form of the prior distribution but more importantly the form of the posterior approximation. Thus, one might expect that this node would have exponential posterior distribution approximation. However, it has a gamma distribution. Also, the moments are

gamma moments although only E[x] would be the moment of a exponential random variable. All this was done because: a) gamma was already implemented, so there was no need to implement anything, and b) people might easily use Exponential node as a prior definition and expect to get gamma posterior (which is what happens now). Maybe some day a pure Exponential node is implemented and the users are advised to use Gamma(1,b) if they want to use an exponential prior distribution but gamma posterior approximation.

\_\_init\_\_(*l*, \*\*kwargs)

#### **Methods**

init(l, **kwargs)	
add_plate_axis(to_plate)	
as_diagonal_wishart()	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
get_parameters()	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
<pre>get_shape(ind)</pre>	
has_plotter()	Return True if the node has a plotter
<pre>initialize_from_parameters(*args)</pre>	
<pre>initialize_from_prior()</pre>	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
lower_bound_contribution([gradient,])	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

### bayespy.nodes.Exponential.\_\_init\_\_

Exponential.\_\_init\_\_(l, \*\*kwargs)

### bayespy.nodes.Exponential.add\_plate\_axis

Exponential.add\_plate\_axis (to\_plate)

#### bayespy.nodes.Exponential.as\_diagonal\_wishart

```
Exponential.as_diagonal_wishart()
```

#### bayespy.nodes.Exponential.broadcasting\_multiplier

```
Exponential.broadcasting_multiplier(plates, *args)
```

#### bayespy.nodes.Exponential.delete

```
Exponential.delete()
```

Delete this node and the children

#### bayespy.nodes.Exponential.get\_gradient

```
Exponential.get_gradient (rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.Exponential.get\_mask

```
Exponential.get_mask()
```

### bayespy.nodes.Exponential.get\_moments

```
Exponential.get_moments()
```

#### bayespy.nodes.Exponential.get\_parameters

```
Exponential.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Exponential.get\_riemannian\_gradient

```
Exponential.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

#### bayespy.nodes.Exponential.get\_shape

```
Exponential.get_shape (ind)
```

```
bayespy.nodes.Exponential.has_plotter
Exponential.has_plotter()
    Return True if the node has a plotter
bayespy.nodes.Exponential.initialize_from_parameters
Exponential.initialize_from_parameters(*args)
bayespy.nodes.Exponential.initialize_from_prior
Exponential.initialize_from_prior()
bayespy.nodes.Exponential.initialize_from_random
Exponential.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.Exponential.initialize_from_value
Exponential.initialize_from_value(x, *args)
bayespy.nodes.Exponential.load
Exponential.load(filename)
bayespy.nodes.Exponential.logpdf
Exponential.logpdf(X, mask=True)
    Compute the log probability density function Q(X) of this node.
bayespy.nodes.Exponential.lower_bound_contribution
Exponential.lower_bound_contribution (gradient=False, ignore_masked=True)
    Compute E[ log p(X|parents) - log q(X) ]
    If deterministic annealing is used, the term E[-\log q(X)] is divided by the anneling coefficient. That is,
    phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).
bayespy.nodes.Exponential.lowerbound
Exponential.lowerbound()
bayespy.nodes.Exponential.move_plates
Exponential.move_plates (from_plate, to_plate)
```

#### bayespy.nodes.Exponential.observe

```
Exponential.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

## bayespy.nodes.Exponential.pdf

```
Exponential.pdf (X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.Exponential.plot

```
Exponential.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.Exponential.random

```
Exponential.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.Exponential.save

```
Exponential.save (filename)
```

## bayespy.nodes.Exponential.set\_parameters

```
Exponential.set_parameters (x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.Exponential.set\_plotter

```
Exponential.set_plotter(plotter)
```

## bayespy.nodes.Exponential.show

```
Exponential.show()
```

Print the distribution using standard parameterization.

## bayespy.nodes.Exponential.unobserve

```
Exponential.unobserve()
```

## bayespy.nodes.Exponential.update

Exponential.update(annealing=1.0)

## **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

#### bayespy.nodes.Exponential.dims

Exponential.dims = ((), ())

## bayespy.nodes.Exponential.plates

Exponential.plates = None

# bayespy.nodes.Exponential.plates\_multiplier

## Exponential.plates\_multiplier

Plate multiplier is applied to messages to parents

Nodes for modelling Gaussian and precision variables jointly (useful as prior for Gaussian nodes):

GaussianGammaISO(*args, **kwargs)	Node for Gaussian-gamma (isotropic) random variables.
GaussianGammaARD(mu, alpha, a, b, **kwargs)	Node for Gaussian and gamma random variables with ARD form.
GaussianWishart(*args, **kwargs)	Node for Gaussian-Wishart random variables.

## bayespy.nodes.GaussianGammalSO

class bayespy.nodes.GaussianGammaISO(\*args, \*\*kwargs)

Node for Gaussian-gamma (isotropic) random variables.

The prior:

$$\begin{aligned} p(x,\alpha|\mu,\Lambda,a,b) \\ p(x|\alpha,\mu,\Lambda) &= \mathcal{N}(x|\mu,\alpha Lambda) \\ p(\alpha|a,b) &= \mathcal{G}(\alpha|a,b) \end{aligned}$$

The posterior approximation  $q(x, \alpha)$  has the same Gaussian-gamma form.

Currently, supports only vector variables.

#### **Methods**

init(*args, **kwargs)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gaussian_mean_and_variance()	Return the mean and variance of the distribution
get_gradient(rg)	Computes gradient with respect to the natural parameters.
<pre>get_marginal_logpdf([gaussian, gamma])</pre>	Get the (marginal) log pdf of a subset of the variables
get_mask()	
<pre>get_moments()</pre>	
get_parameters()	Return parameters of the VB distribution.
get_riemannian_gradient()	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	
initialize_from_prior()	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
plotmatrix()	Creates a matrix of marginal plots.
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

# bayespy.nodes.GaussianGammalSO.\_\_init\_\_

GaussianGammaISO.\_\_init\_\_(\*args, \*\*kwargs)

# $bayes py. nodes. Gaussian Gammal SO. add\_plate\_axis$

GaussianGammaISO.add\_plate\_axis(to\_plate)

# $bayes py. nodes. Gaussian Gammal SO. broad casting\_multiplier$

GaussianGammaISO.broadcasting\_multiplier(plates, \*args)

#### bayespy.nodes.GaussianGammalSO.delete

```
GaussianGammaISO.delete()

Delete this node and the children
```

# $bayespy.nodes. Gaussian Gammal SO. get\_gaussian\_mean\_and\_variance$

```
GaussianGammaISO.get_gaussian_mean_and_variance()
Return the mean and variance of the distribution
```

## bayespy.nodes.GaussianGammalSO.get\_gradient

```
GaussianGammaISO.get_gradient (rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

## bayespy.nodes.GaussianGammalSO.get\_marginal\_logpdf

```
GaussianGammaISO.get_marginal_logpdf (gaussian=None, gamma=None)

Get the (marginal) log pdf of a subset of the variables
```

Parameters gaussian: list or None

Indices of the Gaussian variables to keep or None

gamma: bool or None

True if keep the gamma variable, otherwise False or None

**Returns** function

A function which computes log-pdf

## bayespy.nodes.GaussianGammalSO.get\_mask

```
GaussianGammaISO.get_mask()
```

## bayespy.nodes.GaussianGammalSO.get\_moments

```
GaussianGammaISO.get_moments()
```

#### bayespy.nodes.GaussianGammalSO.get\_parameters

```
{\tt GaussianGammaISO.get\_parameters()}
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.GaussianGammalSO.get\_riemannian\_gradient

```
GaussianGammaISO.get_riemannian_gradient()
Computes the Riemannian/natural gradient.
```

#### bayespy.nodes.GaussianGammalSO.get\_shape

```
GaussianGammaISO.get_shape (ind)
```

## bayespy.nodes.GaussianGammalSO.has\_plotter

```
GaussianGammaISO.has_plotter()
Return True if the node has a plotter
```

## bayespy.nodes.GaussianGammalSO.initialize\_from\_parameters

```
GaussianGammaISO.initialize_from_parameters(*args)
```

#### bayespy.nodes.GaussianGammalSO.initialize\_from\_prior

```
GaussianGammaISO.initialize_from_prior()
```

## bayespy.nodes.GaussianGammalSO.initialize\_from\_random

```
GaussianGammaISO.initialize_from_random()

Set the variable to a random sample from the current distribution.
```

## bayespy.nodes.GaussianGammalSO.initialize\_from\_value

```
GaussianGammaISO.initialize_from_value(x, *args)
```

#### bayespy.nodes.GaussianGammalSO.load

```
GaussianGammaISO.load(filename)
```

## bayespy.nodes.GaussianGammalSO.logpdf

```
GaussianGammaISO.logpdf (X, mask=True)
Compute the log probability density function Q(X) of this node.
```

#### bayespy.nodes.GaussianGammalSO.lower\_bound\_contribution

```
GaussianGammaISO.lower_bound_contribution (gradient=False, ignore\_masked=True)

Compute E[ log p(X|parents) - log q(X)]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

#### bayespy.nodes.GaussianGammalSO.lowerbound

```
GaussianGammaISO.lowerbound()
```

#### bayespy.nodes.GaussianGammalSO.move\_plates

```
GaussianGammaISO.move_plates (from_plate, to_plate)
```

## bayespy.nodes.GaussianGammalSO.observe

```
GaussianGammaISO.observe(x, *args, mask=True) Fix moments, compute f and propagate mask.
```

## bayespy.nodes.GaussianGammalSO.pdf

```
GaussianGammaISO.pdf (X, mask=True)
```

Compute the probability density function of this node.

## bayespy.nodes.GaussianGammalSO.plot

```
GaussianGammaISO.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.GaussianGammalSO.plotmatrix

```
GaussianGammaISO.plotmatrix()
```

Creates a matrix of marginal plots.

On diagonal, are marginal plots of each variable. Off-diagonal plot (i,j) shows the joint marginal density of  $x_i$  and  $x_j$ .

## bayespy.nodes.GaussianGammalSO.random

```
GaussianGammaISO.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.GaussianGammalSO.save

GaussianGammaISO.save (filename)

## bayespy.nodes.GaussianGammalSO.set\_parameters

GaussianGammaISO.set\_parameters(x)

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.GaussianGammalSO.set\_plotter

GaussianGammaISO.set\_plotter(plotter)

## bayespy.nodes.GaussianGammalSO.show

GaussianGammaISO.show()

Print the distribution using standard parameterization.

## bayespy.nodes.GaussianGammalSO.unobserve

GaussianGammaISO.unobserve()

## bayespy.nodes.GaussianGammalSO.update

GaussianGammaISO.update (annealing=1.0)

## **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

## bayespy.nodes.GaussianGammalSO.dims

GaussianGammaISO.dims = None

## bayespy.nodes.GaussianGammalSO.plates

 ${\tt GaussianGammaISO.plates = None}$ 

## bayespy.nodes.GaussianGammalSO.plates\_multiplier

GaussianGammaISO.plates\_multiplier

Plate multiplier is applied to messages to parents

## bayespy.nodes.GaussianGammaARD

**class** bayespy.nodes.**GaussianGammaARD** (*mu*, *alpha*, *a*, *b*, \*\**kwargs*)

Node for Gaussian and gamma random variables with ARD form.

The prior:

$$p(x,\tau|\mu,\alpha,a,b) = p(x|\tau,\mu,\alpha)p(\tau|a,b)$$
$$p(x|\alpha,\mu,\alpha) = \mathcal{N}(x|\mu,\mathrm{diag}(\alpha \tau))$$
$$p(\tau|a,b) = \mathcal{G}(\tau|a,b)$$

The posterior approximation  $q(x,\tau)$  has the same Gaussian-gamma form.

Warning: Not yet implemented.

## See also:

Gaussian, GaussianARD, Gamma, GaussianGammaISO, GaussianWishart

\_\_init\_\_ (mu, alpha, a, b, \*\*kwargs)

### **Methods**

init(mu, alpha, a, b, **kwargs)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
<pre>get_parameters()</pre>	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
<pre>initialize_from_parameters(*args)</pre>	
<pre>initialize_from_prior()</pre>	
initialize_from_random()	Set the variable to a random sample from the current distribution.
<pre>initialize_from_value(x, *args)</pre>	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
	Continued on next page

Table 5.17 – continued from previous page

plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

## bayespy.nodes.GaussianGammaARD.\_\_init\_\_

GaussianGammaARD.\_\_init\_\_(mu, alpha, a, b, \*\*kwargs)

## bayespy.nodes.GaussianGammaARD.add\_plate\_axis

GaussianGammaARD.add\_plate\_axis (to\_plate)

#### bayespy.nodes.GaussianGammaARD.broadcasting\_multiplier

GaussianGammaARD.broadcasting\_multiplier(plates, \*args)

## bayespy.nodes.GaussianGammaARD.delete

GaussianGammaARD.delete()

Delete this node and the children

## bayespy.nodes.GaussianGammaARD.get\_gradient

GaussianGammaARD.get\_gradient(rg)

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

## bayespy.nodes.GaussianGammaARD.get\_mask

GaussianGammaARD.get\_mask()

# $bayes py. nodes. Gaussian Gamma ARD. get\_moments$

GaussianGammaARD.get\_moments()

#### bayespy.nodes.GaussianGammaARD.get\_parameters

```
GaussianGammaARD.get_parameters()
Return parameters of the VB distribution.
```

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.GaussianGammaARD.get\_riemannian\_gradient

```
GaussianGammaARD.get_riemannian_gradient()
Computes the Riemannian/natural gradient.
```

## bayespy.nodes.GaussianGammaARD.get\_shape

```
GaussianGammaARD.get_shape (ind)
```

## bayespy.nodes.GaussianGammaARD.has\_plotter

```
GaussianGammaARD.has_plotter()
Return True if the node has a plotter
```

## bayespy.nodes.GaussianGammaARD.initialize\_from\_parameters

```
GaussianGammaARD.initialize_from_parameters(*args)
```

## bayespy.nodes.GaussianGammaARD.initialize\_from\_prior

```
GaussianGammaARD.initialize_from_prior()
```

## $bayes py. nodes. Gaussian Gamma ARD. initialize\_from\_random$

```
GaussianGammaARD.initialize_from_random()

Set the variable to a random sample from the current distribution.
```

## bayespy.nodes.GaussianGammaARD.initialize\_from\_value

```
GaussianGammaARD.initialize_from_value (x, *args)
```

## bayespy.nodes.GaussianGammaARD.load

```
GaussianGammaARD.load(filename)
```

#### bayespy.nodes.GaussianGammaARD.logpdf

```
GaussianGammaARD.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

## bayespy.nodes.GaussianGammaARD.lower\_bound\_contribution

```
GaussianGammaARD.lower_bound_contribution(gradient=False, ignore\_masked=True)
Compute E[log p(X|parents) - log q(X)]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

## bayespy.nodes.GaussianGammaARD.lowerbound

```
GaussianGammaARD.lowerbound()
```

## bayespy.nodes.GaussianGammaARD.move\_plates

GaussianGammaARD.move\_plates (from\_plate, to\_plate)

## bayespy.nodes.GaussianGammaARD.observe

```
GaussianGammaARD.observe(x, *args, mask=True) Fix moments, compute f and propagate mask.
```

## bayespy.nodes.GaussianGammaARD.pdf

```
GaussianGammaARD.pdf (X, mask=True)
```

Compute the probability density function of this node.

## bayespy.nodes.GaussianGammaARD.plot

```
GaussianGammaARD.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

# bayes py. nodes. Gaussian Gamma ARD. random

```
GaussianGammaARD.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.GaussianGammaARD.save

GaussianGammaARD.save (filename)

#### bayespy.nodes.GaussianGammaARD.set\_parameters

```
GaussianGammaARD.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.GaussianGammaARD.set\_plotter

GaussianGammaARD.set\_plotter(plotter)

## bayespy.nodes.GaussianGammaARD.show

```
GaussianGammaARD.show()
```

Print the distribution using standard parameterization.

## bayespy.nodes.GaussianGammaARD.unobserve

GaussianGammaARD.unobserve()

## bayespy.nodes.GaussianGammaARD.update

GaussianGammaARD.update(annealing=1.0)

### **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

## bayespy.nodes.GaussianGammaARD.dims

GaussianGammaARD.dims = None

## bayespy.nodes.GaussianGammaARD.plates

GaussianGammaARD.plates = None

## $bayes py. nodes. Gaussian Gamma ARD. plates\_multiplier$

GaussianGammaARD.plates\_multiplier

Plate multiplier is applied to messages to parents

# bayespy.nodes.GaussianWishart

 $\textbf{class} \; \texttt{bayespy.nodes.GaussianWishart} \; (*\textit{args}, **\textit{kwargs})$ 

Node for Gaussian-Wishart random variables.

The prior:

$$\begin{split} p(x,\Lambda|\mu,\alpha,V,n) \\ p(x|\Lambda,\mu,\alpha) &= (N)(x|\mu,\alpha^{-1}Lambda^{-1}) \\ p(\Lambda|V,n) &= (W)(\Lambda|n,V) \end{split}$$

The posterior approximation  $q(x,\Lambda)$  has the same Gaussian-Wishart form.

Currently, supports only vector variables.

## Methods

init(*args, **kwargs)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
get_parameters()	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	
initialize_from_prior()	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $log p(X parents) - log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

## bayespy.nodes.GaussianWishart.\_\_init\_\_

```
GaussianWishart.__init__(*args, **kwargs)
```

## bayespy.nodes.GaussianWishart.add\_plate\_axis

```
GaussianWishart.add_plate_axis(to_plate)
```

## bayespy.nodes.GaussianWishart.broadcasting\_multiplier

```
GaussianWishart.broadcasting_multiplier(plates, *args)
```

## bayespy.nodes.GaussianWishart.delete

```
GaussianWishart.delete()
```

Delete this node and the children

### bayespy.nodes.GaussianWishart.get\_gradient

```
GaussianWishart.get_gradient (rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

## bayespy.nodes.GaussianWishart.get\_mask

```
GaussianWishart.get_mask()
```

## bayespy.nodes.GaussianWishart.get\_moments

```
GaussianWishart.get_moments()
```

# bayespy.nodes.GaussianWishart.get\_parameters

```
GaussianWishart.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.GaussianWishart.get\_riemannian\_gradient

```
GaussianWishart.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

```
bayespy.nodes.GaussianWishart.get_shape
GaussianWishart.get_shape(ind)
bayespy.nodes.GaussianWishart.has_plotter
GaussianWishart.has_plotter()
    Return True if the node has a plotter
bayespy.nodes.GaussianWishart.initialize_from_parameters
GaussianWishart.initialize_from_parameters(*args)
bayespy.nodes.GaussianWishart.initialize_from_prior
GaussianWishart.initialize_from_prior()
bayespy.nodes.GaussianWishart.initialize_from_random
GaussianWishart.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.GaussianWishart.initialize_from_value
GaussianWishart.initialize_from_value(x, *args)
bayespy.nodes.GaussianWishart.load
GaussianWishart.load (filename)
bayespy.nodes.GaussianWishart.logpdf
GaussianWishart.logpdf(X, mask=True)
    Compute the log probability density function Q(X) of this node.
bayespy.nodes.GaussianWishart.lower_bound_contribution
GaussianWishart.lower_bound_contribution(gradient=False, ignore_masked=True)
    Compute E[ log p(X|parents) - log q(X) ]
    If deterministic annealing is used, the term E[-\log q(X)] is divided by the anneling coefficient. That is,
    phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).
bayespy.nodes.GaussianWishart.lowerbound
GaussianWishart.lowerbound()
```

#### bayespy.nodes.GaussianWishart.move\_plates

```
GaussianWishart.move_plates (from_plate, to_plate)
```

## bayespy.nodes.GaussianWishart.observe

```
GaussianWishart.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

## bayespy.nodes.GaussianWishart.pdf

```
GaussianWishart.pdf(X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.GaussianWishart.plot

```
GaussianWishart.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.GaussianWishart.random

```
GaussianWishart.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.GaussianWishart.save

```
GaussianWishart.save (filename)
```

#### bayespy.nodes.GaussianWishart.set\_parameters

```
GaussianWishart.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

### bayespy.nodes.GaussianWishart.set\_plotter

```
GaussianWishart.set_plotter(plotter)
```

## bayespy.nodes.GaussianWishart.show

```
GaussianWishart.show()
```

Print the distribution using standard parameterization.

## bayespy.nodes.GaussianWishart.unobserve

GaussianWishart.unobserve()

## bayespy.nodes.GaussianWishart.update

GaussianWishart.update (annealing=1.0)

#### **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

### bayespy.nodes.GaussianWishart.dims

GaussianWishart.dims = None

# bayespy.nodes.GaussianWishart.plates

GaussianWishart.plates = None

## bayespy.nodes.GaussianWishart.plates\_multiplier

GaussianWishart.plates\_multiplier

Plate multiplier is applied to messages to parents

Nodes for discrete count variables:

<pre>Bernoulli(p, **kwargs)</pre>	Node for Bernoulli random variables.
Binomial(n, p, **kwargs)	Node for binomial random variables.
Categorical(p, **kwargs)	Node for categorical random variables.
Multinomial(n, p, **kwargs)	Node for multinomial random variables.
Poisson(l, **kwargs)	Node for Poisson random variables.

## bayespy.nodes.Bernoulli

class bayespy.nodes.Bernoulli(p, \*\*kwargs)

Node for Bernoulli random variables.

The node models a binary random variable  $z \in \{0, 1\}$  with prior probability  $p \in [0, 1]$  for value one:

 $z \sim \text{Bernoulli}(p)$ .

**Parameters p**: beta-like node

Probability of a successful trial

## **Examples**

```
>>> import warnings
>>> warnings.filterwarnings('ignore', category=RuntimeWarning)
>>> from bayespy.nodes import Bernoulli, Beta
>>> p = Beta([1e-3, 1e-3])
>>> z = Bernoulli(p, plates=(10,))
>>> z.observe([0, 1, 1, 1, 0, 1, 1, 0, 1])
>>> p.update()
>>> import bayespy.plot as bpplt
>>> import numpy as np
>>> bpplt.pdf(p, np.linspace(0, 1, num=100))
[<matplotlib.lines.Line2D object at 0x...>]
```

\_\_init\_\_ (*p*, \*\*kwargs)
Create Bernoulli node.

## **Methods**

init(p, **kwargs)	Create Bernoulli node.
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
<pre>get_parameters()</pre>	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	
<pre>initialize_from_prior()</pre>	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $log p(X parents) - log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

122 Chapter 5. User API

#### bayespy.nodes.Bernoulli.\_\_init\_\_

```
Bernoulli.__init__(p, **kwargs)
Create Bernoulli node.
```

## bayespy.nodes.Bernoulli.add\_plate\_axis

```
Bernoulli.add_plate_axis(to_plate)
```

## bayespy.nodes.Bernoulli.broadcasting\_multiplier

```
Bernoulli.broadcasting_multiplier(plates, *args)
```

## bayespy.nodes.Bernoulli.delete

```
Bernoulli.delete()
```

Delete this node and the children

## bayespy.nodes.Bernoulli.get\_gradient

```
Bernoulli.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.Bernoulli.get\_mask

```
Bernoulli.get_mask()
```

#### bayespy.nodes.Bernoulli.get\_moments

```
Bernoulli.get_moments()
```

#### bayespy.nodes.Bernoulli.get\_parameters

```
Bernoulli.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.Bernoulli.get\_riemannian\_gradient

```
Bernoulli.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

```
bayespy.nodes.Bernoulli.get_shape
Bernoulli.get_shape (ind)
bayespy.nodes.Bernoulli.has_plotter
Bernoulli.has_plotter()
    Return True if the node has a plotter
bayespy.nodes.Bernoulli.initialize_from_parameters
Bernoulli.initialize_from_parameters(*args)
bayespy.nodes.Bernoulli.initialize_from_prior
Bernoulli.initialize_from_prior()
bayespy.nodes.Bernoulli.initialize_from_random
Bernoulli.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.Bernoulli.initialize_from_value
Bernoulli.initialize_from_value(x, *args)
bayespy.nodes.Bernoulli.load
Bernoulli.load(filename)
bayespy.nodes.Bernoulli.logpdf
Bernoulli.logpdf(X, mask=True)
    Compute the log probability density function Q(X) of this node.
bayespy.nodes.Bernoulli.lower_bound_contribution
Bernoulli.lower_bound_contribution(gradient=False, ignore_masked=True)
    Compute E[ \log p(X|parents) - \log q(X) ]
    If deterministic annealing is used, the term E[-\log q(X)] is divided by the anneling coefficient. That is,
    phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).
bayespy.nodes.Bernoulli.lowerbound
Bernoulli.lowerbound()
```

#### bayespy.nodes.Bernoulli.move\_plates

```
Bernoulli.move_plates (from_plate, to_plate)
```

## bayespy.nodes.Bernoulli.observe

```
Bernoulli.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

## bayespy.nodes.Bernoulli.pdf

```
Bernoulli.pdf(X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.Bernoulli.plot

```
Bernoulli.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.Bernoulli.random

```
Bernoulli.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.Bernoulli.save

```
Bernoulli.save (filename)
```

## bayespy.nodes.Bernoulli.set\_parameters

```
Bernoulli.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# bayespy.nodes.Bernoulli.set\_plotter

```
Bernoulli.set_plotter(plotter)
```

## bayespy.nodes.Bernoulli.show

```
Bernoulli.show()
```

Print the distribution using standard parameterization.

## bayespy.nodes.Bernoulli.unobserve

```
Bernoulli.unobserve()
```

## bayespy.nodes.Bernoulli.update

```
Bernoulli.update(annealing=1.0)
```

#### **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

## bayespy.nodes.Bernoulli.dims

```
Bernoulli.dims = None
```

# bayespy.nodes.Bernoulli.plates

```
Bernoulli.plates = None
```

## bayespy.nodes.Bernoulli.plates\_multiplier

```
Bernoulli.plates_multiplier
```

Plate multiplier is applied to messages to parents

## bayespy.nodes.Binomial

```
class bayespy.nodes.Binomial(n, p, **kwargs)
```

Node for binomial random variables.

The node models the number of successes  $x \in \{0, \dots, n\}$  in n trials with probability p for success:

 $x \sim \text{Binomial}(n, p)$ .

## **Parameters n**: scalar or array

Number of trials

p: beta-like node or scalar or array

Probability of a success in a trial

#### See also:

Bernoulli, Multinomial, Beta

## **Examples**

```
>>> import warnings
>>> warnings.filterwarnings('ignore', category=RuntimeWarning)
>>> from bayespy.nodes import Binomial, Beta
>>> p = Beta([1e-3, 1e-3])
>>> x = Binomial(10, p)
>>> x.observe(7)
>>> p.update()
>>> import bayespy.plot as bpplt
>>> import numpy as np
>>> bpplt.pdf(p, np.linspace(0, 1, num=100))
[<matplotlib.lines.Line2D object at 0x...>]
```

\_\_init\_\_ (*n*, *p*, \*\*kwargs)
Create binomial node

## **Methods**

init(n, p, **kwargs)	Create binomial node
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
get_parameters()	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	
<pre>initialize_from_prior()</pre>	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
lower_bound_contribution([gradient,])	Compute E[ $log p(X parents) - log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

#### bayespy.nodes.Binomial.\_\_init\_\_

```
Binomial.__init__(n, p, **kwargs)

Create binomial node
```

## bayespy.nodes.Binomial.add\_plate\_axis

```
Binomial.add_plate_axis(to_plate)
```

#### bayespy.nodes.Binomial.broadcasting\_multiplier

```
Binomial.broadcasting_multiplier(plates, *args)
```

## bayespy.nodes.Binomial.delete

```
Binomial.delete()
```

Delete this node and the children

## bayespy.nodes.Binomial.get\_gradient

```
Binomial.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.Binomial.get\_mask

```
Binomial.get_mask()
```

## bayespy.nodes.Binomial.get\_moments

```
Binomial.get_moments()
```

#### bayespy.nodes.Binomial.get\_parameters

```
Binomial.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.Binomial.get\_riemannian\_gradient

```
Binomial.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

```
bayespy.nodes.Binomial.get_shape
Binomial.get_shape(ind)
bayespy.nodes.Binomial.has_plotter
Binomial.has_plotter()
    Return True if the node has a plotter
bayespy.nodes.Binomial.initialize_from_parameters
Binomial.initialize_from_parameters(*args)
bayespy.nodes.Binomial.initialize_from_prior
Binomial.initialize_from_prior()
bayespy.nodes.Binomial.initialize_from_random
Binomial.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.Binomial.initialize_from_value
Binomial.initialize_from_value(x, *args)
bayespy.nodes.Binomial.load
Binomial.load(filename)
bayespy.nodes.Binomial.logpdf
Binomial.logpdf(X, mask=True)
    Compute the log probability density function Q(X) of this node.
bayespy.nodes.Binomial.lower_bound_contribution
Binomial.lower_bound_contribution(gradient=False, ignore_masked=True)
    Compute E[ log p(X|parents) - log q(X) ]
    If deterministic annealing is used, the term E[-\log q(X)] is divided by the anneling coefficient. That is,
    phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).
bayespy.nodes.Binomial.lowerbound
Binomial.lowerbound()
```

#### bayespy.nodes.Binomial.move\_plates

```
Binomial.move_plates (from_plate, to_plate)
```

## bayespy.nodes.Binomial.observe

```
Binomial.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

## bayespy.nodes.Binomial.pdf

```
Binomial.pdf (X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.Binomial.plot

```
Binomial.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.Binomial.random

```
Binomial.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.Binomial.save

```
Binomial.save(filename)
```

## bayespy.nodes.Binomial.set\_parameters

```
Binomial.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# $bayes py. nodes. Binomial. set\_plotter$

```
Binomial.set_plotter(plotter)
```

## bayespy.nodes.Binomial.show

```
Binomial.show()
```

Print the distribution using standard parameterization.

## bayespy.nodes.Binomial.unobserve

```
Binomial.unobserve()
```

## bayespy.nodes.Binomial.update

Binomial.update(annealing=1.0)

#### **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

# bayespy.nodes.Binomial.dims

```
Binomial.dims = None
```

## bayespy.nodes.Binomial.plates

Binomial.plates = None

## bayespy.nodes.Binomial.plates\_multiplier

## Binomial.plates\_multiplier

Plate multiplier is applied to messages to parents

## bayespy.nodes.Categorical

class bayespy.nodes.Categorical(p, \*\*kwargs)

Node for categorical random variables.

The node models a categorical random variable  $x \in \{0, \dots, K-1\}$  with prior probabilities  $\{p_0, \dots, p_{K-1}\}$  for each category:

$$p(x = k) = p_k$$
 for  $k \in \{0, \dots, K - 1\}$ .

Parameters p: Dirichlet-like node or (...,K)-array

Probabilities for each category

## See also:

Bernoulli, Multinomial, Dirichlet

\_\_**init**\_\_(p, \*\*kwargs)

Create Categorical node.

## **Methods**

init(p, **kwargs)	Create Categorical node.
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
get_moments()	
get_parameters()	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	
initialize_from_prior()	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $log p(X parents) - log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

# bayespy.nodes.Categorical.\_\_init\_\_

Categorical.\_\_init\_\_(p, \*\*kwargs)
Create Categorical node.

# $bayes py. nodes. Categorical. add\_plate\_axis$

Categorical.add\_plate\_axis(to\_plate)

# $bayes py. nodes. Categorical. broadcasting\_multiplier$

Categorical.broadcasting\_multiplier(plates, \*args)

# bayespy.nodes.Categorical.delete

Categorical.delete()

Delete this node and the children

#### bayespy.nodes.Categorical.get\_gradient

```
Categorical.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

## bayespy.nodes.Categorical.get\_mask

```
Categorical.get_mask()
```

### bayespy.nodes.Categorical.get\_moments

```
Categorical.get_moments()
```

## bayespy.nodes.Categorical.get\_parameters

```
Categorical.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

### bayespy.nodes.Categorical.get\_riemannian\_gradient

```
Categorical.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

#### bayespy.nodes.Categorical.get\_shape

```
Categorical.get_shape(ind)
```

#### bayespy.nodes.Categorical.has\_plotter

```
Categorical.has_plotter()
```

Return True if the node has a plotter

## bayespy.nodes.Categorical.initialize\_from\_parameters

```
Categorical.initialize_from_parameters(*args)
```

#### bayespy.nodes.Categorical.initialize\_from\_prior

```
Categorical.initialize_from_prior()
```

#### bayespy.nodes.Categorical.initialize\_from\_random

```
Categorical.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

#### bayespy.nodes.Categorical.initialize\_from\_value

```
Categorical.initialize_from_value(x, *args)
```

## bayespy.nodes.Categorical.load

```
Categorical.load(filename)
```

## bayespy.nodes.Categorical.logpdf

```
Categorical.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.Categorical.lower\_bound\_contribution

```
{\tt Categorical.lower\_bound\_contribution} \ (\textit{gradient=False}, \textit{ignore\_masked=True})
```

```
Compute E[ log p(X|parents) - log q(X) ]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

## bayespy.nodes.Categorical.lowerbound

```
Categorical.lowerbound()
```

## bayespy.nodes.Categorical.move\_plates

```
Categorical.move_plates (from_plate, to_plate)
```

# bayespy.nodes.Categorical.observe

```
Categorical.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

# bayespy.nodes.Categorical.pdf

```
Categorical.pdf(X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.Categorical.plot

```
Categorical.plot(fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.Categorical.random

```
Categorical.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.Categorical.save

```
Categorical.save (filename)
```

## bayespy.nodes.Categorical.set\_parameters

```
Categorical.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.Categorical.set\_plotter

```
Categorical.set_plotter(plotter)
```

## bayespy.nodes.Categorical.show

```
Categorical.show()
```

Print the distribution using standard parameterization.

#### bayespy.nodes.Categorical.unobserve

```
Categorical.unobserve()
```

# bayes py. nodes. Categorical. update

```
Categorical.update(annealing=1.0)
```

Continued on next page

## Table 5.27 – continued from previous page

## **Attributes**

plates
plates\_multiplier Plate multiplier is applied to messages to parents

## bayespy.nodes.Categorical.dims

Categorical.dims = None

## bayespy.nodes.Categorical.plates

Categorical.plates = None

#### bayespy.nodes.Categorical.plates\_multiplier

Categorical.plates\_multiplier

Plate multiplier is applied to messages to parents

## bayespy.nodes.Multinomial

class bayespy.nodes.Multinomial(n, p, \*\*kwargs)

Node for multinomial random variables.

Assume there are K categories and N trials each of which leads a success for exactly one of the categories. Given the probabilities  $p_0, \ldots, p_{K-1}$  for the categories, multinomial distribution is gives the probability of any combination of numbers of successes for the categories.

The node models the number of successes  $x_k \in \{0, ..., n\}$  in n trials with probability  $p_k$  for success in K categories.

$$\text{Multinomial}(\mathbf{x}|N,\mathbf{p}) = \frac{N!}{x_0! \cdots x_{K-1}!} p_0^{x_0} \cdots p_{K-1}^{x_{K-1}}$$

Parameters n : scalar or array

N, number of trials

**p** : Dirichlet-like node or (...,K)-array

p, probabilities of successes for the categories

## See also:

Dirichlet, Binomial, Categorical

\_\_**init**\_\_ (*n*, *p*, \*\**kwargs*)

Create Multinomial node.

## **Methods**

init(n, p, **kwargs)	Create Multinomial node.
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
get_parameters()	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
<pre>get_shape(ind)</pre>	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	
<pre>initialize_from_prior()</pre>	
initialize_from_random()	Set the variable to a random sample from the current distribution.
<pre>initialize_from_value(x, *args)</pre>	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $log p(X parents) - log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

# bayespy.nodes.Multinomial.\_\_init\_\_

Multinomial.\_\_init\_\_(n, p, \*\*kwargs)
Create Multinomial node.

# $bayespy.nodes. Multinomial. add\_plate\_axis$

Multinomial.add\_plate\_axis(to\_plate)

# $bayes py. nodes. Multinomial. broadcasting\_multiplier$

Multinomial.broadcasting.multiplier(plates, \*args)

# bayespy.nodes.Multinomial.delete

Multinomial.delete()

Delete this node and the children

#### bayespy.nodes.Multinomial.get\_gradient

```
Multinomial.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

## bayespy.nodes.Multinomial.get\_mask

```
Multinomial.get_mask()
```

### bayespy.nodes.Multinomial.get\_moments

```
Multinomial.get_moments()
```

#### bayespy.nodes.Multinomial.get\_parameters

```
Multinomial.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

### bayespy.nodes.Multinomial.get\_riemannian\_gradient

```
Multinomial.get_riemannian_gradient()
Computes the Riemannian/natural gradient.
```

#### bayespy.nodes.Multinomial.get\_shape

```
Multinomial.get_shape(ind)
```

#### bayespy.nodes.Multinomial.has\_plotter

```
Multinomial.has_plotter()
```

Return True if the node has a plotter

## bayespy.nodes.Multinomial.initialize\_from\_parameters

```
Multinomial.initialize_from_parameters(*args)
```

#### bayespy.nodes.Multinomial.initialize\_from\_prior

```
Multinomial.initialize_from_prior()
```

#### bayespy.nodes.Multinomial.initialize\_from\_random

```
Multinomial.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

#### bayespy.nodes.Multinomial.initialize\_from\_value

```
Multinomial.initialize_from_value(x, *args)
```

## bayespy.nodes.Multinomial.load

```
Multinomial.load(filename)
```

#### bayespy.nodes.Multinomial.logpdf

```
Multinomial.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.Multinomial.lower\_bound\_contribution

```
Multinomial.lower_bound_contribution(gradient=False, ignore_masked=True)
```

Compute E[ log p(X|parents) - log q(X) ]

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

## bayespy.nodes.Multinomial.lowerbound

```
Multinomial.lowerbound()
```

## bayespy.nodes.Multinomial.move\_plates

```
Multinomial.move_plates (from_plate, to_plate)
```

## bayespy.nodes.Multinomial.observe

```
Multinomial.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

# bayespy.nodes.Multinomial.pdf

```
Multinomial.pdf (X, mask=True)
```

Compute the probability density function of this node.

## bayespy.nodes.Multinomial.plot

```
Multinomial.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.Multinomial.random

```
Multinomial.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.Multinomial.save

```
Multinomial.save (filename)
```

## bayespy.nodes.Multinomial.set\_parameters

```
Multinomial.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

### bayespy.nodes.Multinomial.set\_plotter

```
Multinomial.set_plotter(plotter)
```

#### bayespy.nodes.Multinomial.show

```
Multinomial.show()
```

Print the distribution using standard parameterization.

#### bayespy.nodes.Multinomial.unobserve

```
Multinomial.unobserve()
```

## bayespy.nodes.Multinomial.update

```
Multinomial.update(annealing=1.0)
```

Continued on next page

# Table 5.29 – continued from previous page

# **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

# bayespy.nodes.Multinomial.dims

Multinomial.dims = None

## bayespy.nodes.Multinomial.plates

Multinomial.plates = None

# bayespy.nodes.Multinomial.plates\_multiplier

Multinomial.plates\_multiplier

Plate multiplier is applied to messages to parents

# bayespy.nodes.Poisson

class bayespy.nodes.Poisson(l, \*\*kwargs)

Node for Poisson random variables.

The node uses Poisson distribution:

$$p(x) = Poisson(x|\lambda)$$

where  $\lambda$  is the rate parameter.

Parameters 1: gamma-like node or scalar or array

 $\lambda$ , rate parameter

## See also:

Gamma, Exponential

\_\_init\_\_(*l*, \*\*kwargs)

Create Poisson random variable node

# Methods

init(l, **kwargs)	Create Poisson random variable node	;
add_plate_axis(to_plate)		
broadcasting_multiplier(plates, *args)		
delete()	Delete this node and the children	
		Continued on next page

Table 5.30 – continued from previous page

get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
<pre>get_parameters()</pre>	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	
initialize_from_prior()	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
lower_bound_contribution([gradient,])	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

# bayespy.nodes.Poisson.\_\_init\_\_

Poisson.\_\_init\_\_(l, \*\*kwargs)

Create Poisson random variable node

# bayespy.nodes.Poisson.add\_plate\_axis

Poisson.add\_plate\_axis(to\_plate)

# $bayespy. nodes. Poisson. broadcasting\_multiplier$

Poisson.broadcasting\_multiplier(plates, \*args)

# bayespy.nodes.Poisson.delete

Poisson.delete()

Delete this node and the children

#### bayespy.nodes.Poisson.get\_gradient

```
Poisson.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

## bayespy.nodes.Poisson.get\_mask

```
Poisson.get_mask()
```

### bayespy.nodes.Poisson.get\_moments

```
Poisson.get_moments()
```

# bayespy.nodes.Poisson.get\_parameters

```
Poisson.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.Poisson.get\_riemannian\_gradient

```
Poisson.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

#### bayespy.nodes.Poisson.get\_shape

```
Poisson.get_shape (ind)
```

#### bayespy.nodes.Poisson.has\_plotter

```
Poisson.has_plotter()
```

Return True if the node has a plotter

## bayespy.nodes.Poisson.initialize\_from\_parameters

```
Poisson.initialize_from_parameters(*args)
```

#### bayespy.nodes.Poisson.initialize\_from\_prior

```
Poisson.initialize_from_prior()
```

#### bayespy.nodes.Poisson.initialize\_from\_random

```
Poisson.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

## bayespy.nodes.Poisson.initialize\_from\_value

```
Poisson.initialize_from_value(x, *args)
```

## bayespy.nodes.Poisson.load

```
Poisson.load(filename)
```

## bayespy.nodes.Poisson.logpdf

```
Poisson.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.Poisson.lower\_bound\_contribution

```
Poisson.lower_bound_contribution(gradient=False, ignore_masked=True)
```

```
Compute E[ log p(X|parents) - log q(X) ]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

# bayespy.nodes.Poisson.lowerbound

```
Poisson.lowerbound()
```

# bayespy.nodes.Poisson.move\_plates

```
Poisson.move_plates (from_plate, to_plate)
```

# bayespy.nodes.Poisson.observe

```
Poisson.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

# bayespy.nodes.Poisson.pdf

```
Poisson.pdf (X, mask=True)
```

Compute the probability density function of this node.

# bayespy.nodes.Poisson.plot

```
Poisson.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

# bayespy.nodes.Poisson.random

```
Poisson.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.Poisson.save

```
Poisson.save (filename)
```

# bayespy.nodes.Poisson.set\_parameters

```
Poisson.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.Poisson.set\_plotter

```
Poisson.set_plotter(plotter)
```

# bayespy.nodes.Poisson.show

```
Poisson.show()
```

Print the distribution using standard parameterization.

#### bayespy.nodes.Poisson.unobserve

```
Poisson.unobserve()
```

## bayespy.nodes.Poisson.update

```
Poisson.update(annealing=1.0)
```

Continued on next page

# Table 5.31 – continued from previous page

## **Attributes**

```
dims
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

## bayespy.nodes.Poisson.dims

```
Poisson.dims = ((),)
```

## bayespy.nodes.Poisson.plates

```
Poisson.plates = None
```

## bayespy.nodes.Poisson.plates\_multiplier

## Poisson.plates\_multiplier

Plate multiplier is applied to messages to parents

Nodes for probabilities:

Beta(alpha, **kwargs)	Node for beta random variables.
Dirichlet(*args, **kwargs)	Node for Dirichlet random variables.

# bayespy.nodes.Beta

```
class bayespy.nodes.Beta(alpha, **kwargs)
```

Node for beta random variables.

The node models a probability variable  $p \in [0, 1]$  as

$$p \sim \text{Beta}(a, b)$$

where a and b are prior counts for success and failure, respectively.

**Parameters alpha**: (...,2)-shaped array

Two-element vector containing a and b

## **Examples**

```
>>> import warnings
>>> warnings.filterwarnings('ignore', category=RuntimeWarning)
>>> from bayespy.nodes import Bernoulli, Beta
>>> p = Beta([1e-3, 1e-3])
>>> z = Bernoulli(p, plates=(10,))
>>> z.observe([0, 1, 1, 1, 0, 1, 1, 0, 1])
>>> p.update()
```

```
>>> import bayespy.plot as bpplt
>>> import numpy as np
>>> bpplt.pdf(p, np.linspace(0, 1, num=100))
[<matplotlib.lines.Line2D object at 0x...>]
```

```
__init__ (alpha, **kwargs)
Create beta node
```

#### **Methods**

init(alpha, **kwargs)	Create beta node
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
get_parameters()	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	
<pre>initialize_from_prior()</pre>	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	
random() save(filename) set_parameters(x) set_plotter(plotter) show() unobserve()	Draw a random sample from the distribution.  Set the parameters of the VB distribution.  Print the distribution using standard parameterization.

# bayespy.nodes.Beta.\_\_init\_\_

```
Beta.__init__(alpha, **kwargs)
Create beta node
```

# bayespy.nodes.Beta.add\_plate\_axis

Beta.add\_plate\_axis (to\_plate)

#### bayespy.nodes.Beta.broadcasting\_multiplier

```
Beta.broadcasting_multiplier(plates, *args)
```

## bayespy.nodes.Beta.delete

```
Beta.delete()
```

Delete this node and the children

## bayespy.nodes.Beta.get\_gradient

```
Beta.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

## bayespy.nodes.Beta.get\_mask

```
Beta.get_mask()
```

## bayespy.nodes.Beta.get\_moments

```
Beta.get_moments()
```

# bayespy.nodes.Beta.get\_parameters

```
Beta.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# bayespy.nodes.Beta.get\_riemannian\_gradient

```
Beta.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

# bayespy.nodes.Beta.get\_shape

```
Beta.get_shape(ind)
```

## bayespy.nodes.Beta.has\_plotter

```
Beta.has_plotter()
```

Return True if the node has a plotter

```
bayespy.nodes.Beta.initialize_from_parameters
Beta.initialize_from_parameters(*args)
bayespy.nodes.Beta.initialize_from_prior
Beta.initialize_from_prior()
bayespy.nodes.Beta.initialize_from_random
Beta.initialize_from_random()
    Set the variable to a random sample from the current distribution.
bayespy.nodes.Beta.initialize_from_value
Beta.initialize_from_value(x, *args)
bayespy.nodes.Beta.load
Beta.load(filename)
bayespy.nodes.Beta.logpdf
Beta.logpdf(X, mask=True)
    Compute the log probability density function Q(X) of this node.
bayespy.nodes.Beta.lower_bound_contribution
Beta.lower_bound_contribution(gradient=False, ignore_masked=True)
    Compute E[ log p(X|parents) - log q(X) ]
    If deterministic annealing is used, the term E[-\log q(X)] is divided by the anneling coefficient. That is,
    phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).
bayespy.nodes.Beta.lowerbound
Beta.lowerbound()
bayespy.nodes.Beta.move_plates
Beta.move_plates (from_plate, to_plate)
bayespy.nodes.Beta.observe
Beta.observe(x, *args, mask=True)
    Fix moments, compute f and propagate mask.
```

# bayespy.nodes.Beta.pdf

```
Beta.pdf (X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.Beta.plot

```
Beta.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

# bayespy.nodes.Beta.random

```
Beta.random()
```

Draw a random sample from the distribution.

#### bayespy.nodes.Beta.save

```
Beta.save (filename)
```

## bayespy.nodes.Beta.set\_parameters

```
Beta.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Beta.set\_plotter

```
Beta.set_plotter(plotter)
```

#### bayespy.nodes.Beta.show

```
Beta.show()
```

Print the distribution using standard parameterization.

## bayespy.nodes.Beta.unobserve

```
Beta.unobserve()
```

#### bayespy.nodes.Beta.update

```
Beta.update(annealing=1.0)
```

## **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

# bayespy.nodes.Beta.dims

Beta.dims = None

#### bayespy.nodes.Beta.plates

Beta.plates = None

#### bayespy.nodes.Beta.plates\_multiplier

## Beta.plates\_multiplier

Plate multiplier is applied to messages to parents

# bayespy.nodes.Dirichlet

class bayespy.nodes.Dirichlet(\*args, \*\*kwargs)

Node for Dirichlet random variables.

The node models a set of probabilities  $\{\pi_0,\ldots,\pi_{K-1}\}$  which satisfy  $\sum_{k=0}^{K-1}\pi_k=1$  and  $\pi_k\in[0,1]$   $\forall k=0,\ldots,K-1$ .

$$p(\pi_0, \ldots, \pi_{K-1}) = Dirichlet(\alpha_0, \ldots, \alpha_{K-1})$$

where  $\alpha_k$  are concentration parameters.

The posterior approximation has the same functional form but with different concentration parameters.

Parameters alpha: (...,K)-shaped array

Prior counts  $\alpha_k$ 

#### See also:

Beta, Categorical, Multinomial, Categorical Markov Chain

#### Methods

init(*args, **kwargs)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
	Continued on next page

Table 5.35 – continued from previous page

get_mask()	
<pre>get_moments()</pre>	
<pre>get_parameters()</pre>	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	
initialize_from_prior()	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

# bayespy.nodes.Dirichlet.\_\_init\_\_

Dirichlet.\_\_init\_\_(\*args, \*\*kwargs)

# bayespy.nodes.Dirichlet.add\_plate\_axis

Dirichlet.add\_plate\_axis(to\_plate)

# bayespy.nodes.Dirichlet.broadcasting\_multiplier

Dirichlet.broadcasting\_multiplier(plates, \*args)

# bayespy.nodes.Dirichlet.delete

Dirichlet.delete()

Delete this node and the children

# bayespy.nodes.Dirichlet.get\_gradient

Dirichlet.get\_gradient(rg)

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.Dirichlet.get\_mask

```
Dirichlet.get_mask()
```

# bayespy.nodes.Dirichlet.get\_moments

```
Dirichlet.get_moments()
```

## bayespy.nodes.Dirichlet.get\_parameters

```
Dirichlet.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Dirichlet.get\_riemannian\_gradient

```
Dirichlet.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

#### bayespy.nodes.Dirichlet.get\_shape

```
Dirichlet.get_shape (ind)
```

# bayespy.nodes.Dirichlet.has\_plotter

```
Dirichlet.has_plotter()
```

Return True if the node has a plotter

# bayespy.nodes.Dirichlet.initialize\_from\_parameters

```
Dirichlet.initialize_from_parameters(*args)
```

# $bayes py. nodes. Dirichlet. in itialize\_from\_prior$

```
Dirichlet.initialize_from_prior()
```

## bayespy.nodes.Dirichlet.initialize\_from\_random

```
Dirichlet.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

#### bayespy.nodes.Dirichlet.initialize\_from\_value

```
Dirichlet.initialize_from_value(x, *args)
```

## bayespy.nodes.Dirichlet.load

```
Dirichlet.load(filename)
```

# bayespy.nodes.Dirichlet.logpdf

```
Dirichlet.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

## bayespy.nodes.Dirichlet.lower\_bound\_contribution

```
Dirichlet.lower_bound_contribution(gradient=False, ignore_masked=True)
```

Compute E[ log p(X|parents) - log q(X) ]

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

# bayespy.nodes.Dirichlet.lowerbound

```
Dirichlet.lowerbound()
```

# bayespy.nodes.Dirichlet.move\_plates

```
Dirichlet.move_plates (from_plate, to_plate)
```

# bayespy.nodes.Dirichlet.observe

```
Dirichlet.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

# bayespy.nodes.Dirichlet.pdf

```
Dirichlet.pdf(X, mask=True)
```

Compute the probability density function of this node.

# bayespy.nodes.Dirichlet.plot

```
Dirichlet.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.Dirichlet.random

```
Dirichlet.random()
```

Draw a random sample from the distribution.

## bayespy.nodes.Dirichlet.save

```
Dirichlet.save (filename)
```

# bayespy.nodes.Dirichlet.set\_parameters

```
Dirichlet.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# bayespy.nodes.Dirichlet.set\_plotter

```
Dirichlet.set_plotter(plotter)
```

# bayespy.nodes.Dirichlet.show

```
Dirichlet.show()
```

Print the distribution using standard parameterization.

# bayespy.nodes.Dirichlet.unobserve

```
Dirichlet.unobserve()
```

# bayespy.nodes.Dirichlet.update

```
Dirichlet.update(annealing=1.0)
```

# **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

# bayespy.nodes.Dirichlet.dims

```
Dirichlet.dims = None
```

#### bayespy.nodes.Dirichlet.plates

Dirichlet.plates = None

## bayespy.nodes.Dirichlet.plates\_multiplier

## Dirichlet.plates\_multiplier

Plate multiplier is applied to messages to parents

# Nodes for dynamic variables:

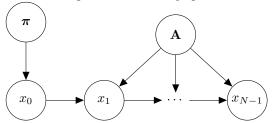
	CategoricalMarkovChain(pi, A[, states])	Node for categorical Markov chain random variables.
	GaussianMarkovChain(mu, Lambda, A, nu[, n,])	Node for Gaussian Markov chain random variables.
•	SwitchingGaussianMarkovChain(mu, Lambda, B,)	Node for Gaussian Markov chain random variables with switching d
•	VaryingGaussianMarkovChain(mu, Lambda, B, S, nu)	Node for Gaussian Markov chain random variables with time-varying

# bayespy.nodes.CategoricalMarkovChain

class bayespy.nodes.CategoricalMarkovChain (pi, A, states=None, \*\*kwargs)

Node for categorical Markov chain random variables.

The node models a Markov chain which has a discrete set of K possible states and the next state depends only on the previous state and the state transition probabilities. The graphical model is shown below:



where  $\pi$  contains the probabilities for the initial state and **A** is the state transition probability matrix. It is possible to have **A** varying in time.

$$p(x_0, \dots, x_{N-1}) = p(x_0) \prod_{n=1}^{N-1} p(x_n | x_{n-1}),$$

where

$$p(x_0 = k) = \pi_k, \quad \text{for } k \in \{0, \dots, K - 1\},$$

$$p(x_n = j | x_{n-1} = i) = a_{ij}^{(n-1)} \quad \text{for } n = 1, \dots, N - 1, \ i \in \{1, \dots, K - 1\}, \ j \in \{1, \dots, K - 1\}$$

$$a_{ij}^{(n)} = [\mathbf{A}_n]_{ij}$$

This node can be used to construct hidden Markov models by using Mixture for the emission distribution.

**Parameters pi**: Dirichlet-like node or (...,K)-array

 $\pi$ , probabilities for the first state. K-dimensional Dirichlet.

A: Dirichlet-like node or (K,K)-array or (...,1,K,K)-array or (...,N-1,K,K)-array

A, probabilities for state transitions. K-dimensional Dirichlet with plates (K,) or (...,1,K) or (...,N-1,K).

states: int, optional

N, the length of the chain.

## See also:

Categorical, Dirichlet, SwitchingGaussianMarkovChain

GaussianMarkovChain,

Mixture,

\_\_init\_\_ (pi, A, states=None, \*\*kwargs)
Create categorical Markov chain

# Methods

init(pi, A[, states])	Create categorical Markov chain
add_plate_axis(to_plate)	Create categorical ividixov cham
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
<pre>get_gradient(rg)</pre>	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
<pre>get_parameters()</pre>	Return parameters of the VB distribution.
get_riemannian_gradient()	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	•
<pre>initialize_from_prior()</pre>	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x,*args)	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
lower_bound_contribution([gradient,])	Compute E[ $log p(X parents) - log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

# bayespy.nodes.CategoricalMarkovChain.\_\_init\_\_

CategoricalMarkovChain.\_\_init\_\_(pi, A, states=None, \*\*kwargs)
Create categorical Markov chain

# bayespy.nodes.CategoricalMarkovChain.add\_plate\_axis

CategoricalMarkovChain.add\_plate\_axis (to\_plate)

#### bayespy.nodes.CategoricalMarkovChain.broadcasting\_multiplier

```
CategoricalMarkovChain.broadcasting_multiplier(plates, *args)
```

# bayespy.nodes.CategoricalMarkovChain.delete

```
CategoricalMarkovChain.delete()

Delete this node and the children
```

# bayespy.nodes.CategoricalMarkovChain.get\_gradient

```
CategoricalMarkovChain.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

# bayespy.nodes.CategoricalMarkovChain.get\_mask

```
CategoricalMarkovChain.get_mask()
```

#### bayespy.nodes.CategoricalMarkovChain.get\_moments

```
CategoricalMarkovChain.get_moments()
```

## bayespy.nodes.CategoricalMarkovChain.get\_parameters

```
CategoricalMarkovChain.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# $bayes py. nodes. Categorical Markov Chain. get\_riemannian\_gradient$

```
CategoricalMarkovChain.get_riemannian_gradient()
Computes the Riemannian/natural gradient.
```

## bayespy.nodes.CategoricalMarkovChain.get\_shape

```
CategoricalMarkovChain.get_shape(ind)
```

## bayespy.nodes.CategoricalMarkovChain.has\_plotter

```
CategoricalMarkovChain.has_plotter()
Return True if the node has a plotter
```

#### bayespy.nodes.CategoricalMarkovChain.initialize\_from\_parameters

CategoricalMarkovChain.initialize\_from\_parameters(\*args)

## bayespy.nodes.CategoricalMarkovChain.initialize\_from\_prior

CategoricalMarkovChain.initialize\_from\_prior()

## bayespy.nodes.CategoricalMarkovChain.initialize\_from\_random

CategoricalMarkovChain.initialize\_from\_random()

Set the variable to a random sample from the current distribution.

## bayespy.nodes.CategoricalMarkovChain.initialize\_from\_value

CategoricalMarkovChain.initialize\_from\_value(x, \*args)

## bayespy.nodes.CategoricalMarkovChain.load

CategoricalMarkovChain.load(filename)

## bayespy.nodes.CategoricalMarkovChain.logpdf

CategoricalMarkovChain.logpdf(X, mask=True)
Compute the log probability density function Q(X) of this node.

## bayespy.nodes.CategoricalMarkovChain.lower\_bound\_contribution

CategoricalMarkovChain.lower\_bound\_contribution(gradient=False, ig-nore\_masked=True)

Compute E[ log p(X|parents) - log q(X) ]

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

# bayespy.nodes.CategoricalMarkovChain.lowerbound

CategoricalMarkovChain.lowerbound()

## bayespy.nodes.CategoricalMarkovChain.move\_plates

CategoricalMarkovChain.move\_plates (from\_plate, to\_plate)

#### bayespy.nodes.CategoricalMarkovChain.observe

```
CategoricalMarkovChain.observe(x, *args, mask=True) Fix moments, compute f and propagate mask.
```

# bayespy.nodes.CategoricalMarkovChain.pdf

```
CategoricalMarkovChain.pdf(X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.CategoricalMarkovChain.plot

```
CategoricalMarkovChain.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

# bayespy.nodes.CategoricalMarkovChain.random

```
CategoricalMarkovChain.random()
```

Draw a random sample from the distribution.

# bayespy.nodes.CategoricalMarkovChain.save

```
CategoricalMarkovChain.save (filename)
```

# $bayes py. nodes. Categorical Markov Chain. set\_parameters$

```
CategoricalMarkovChain.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# bayespy.nodes.CategoricalMarkovChain.set\_plotter

```
CategoricalMarkovChain.set_plotter(plotter)
```

# bayespy.nodes.CategoricalMarkovChain.show

```
CategoricalMarkovChain.show()
```

Print the distribution using standard parameterization.

# bayespy.nodes.CategoricalMarkovChain.unobserve

```
{\tt Categorical Markov Chain.} \textbf{unobserve ()}
```

#### bayespy.nodes.CategoricalMarkovChain.update

CategoricalMarkovChain.update(annealing=1.0)

#### **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

#### bayespy.nodes.CategoricalMarkovChain.dims

CategoricalMarkovChain.dims = None

#### bayespy.nodes.CategoricalMarkovChain.plates

CategoricalMarkovChain.plates = None

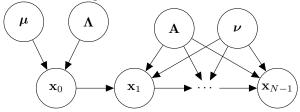
# bayespy.nodes.CategoricalMarkovChain.plates\_multiplier

CategoricalMarkovChain.plates\_multiplier
Plate multiplier is applied to messages to parents

# bayespy.nodes.GaussianMarkovChain

class bayespy.nodes.GaussianMarkovChain (mu, Lambda, A, nu, n=None, inputs=None, \*\*kwargs)
Node for Gaussian Markov chain random variables.

In a simple case, the graphical model can be presented as:



where  $\mu$  and  $\Lambda$  are the mean and the precision matrix of the initial state,  $\mathbf{A}$  is the state dynamics matrix and  $\nu$  is the precision of the innovation noise. It is possible that  $\mathbf{A}$  and/or  $\nu$  are different for each transition instead of being constant.

The probability distribution is

$$p(\mathbf{x}_0,\ldots,\mathbf{x}_{N-1}) = p(\mathbf{x}_0) \prod_{n=1}^{N-1} p(\mathbf{x}_n|\mathbf{x}_{n-1})$$

where

$$\begin{split} p(\mathbf{x}_0) &= \mathcal{N}(\mathbf{x}_0 | \boldsymbol{\mu}, \boldsymbol{\Lambda}) \\ p(\mathbf{x}_n | \mathbf{x}_{n-1}) &= \mathcal{N}(\mathbf{x}_n | \mathbf{A}_{n-1} \mathbf{x}_{n-1}, \operatorname{diag}(\boldsymbol{\nu}_{n-1})). \end{split}$$

```
Parameters mu: Gaussian-like node or (...,D)-array
```

 $\mu$ , mean of  $x_0$ , D-dimensional with plates (...)

Lambda: Wishart-like node or (...,D,D)-array

 $\Lambda$ , precision matrix of  $x_0$ ,  $D \times D$  -dimensional with plates (...)

A: Gaussian-like node or (D,D)-array or (...,1,D,D)-array or (...,N-1,D,D)-array

**A**, state dynamics matrix, *D*-dimensional with plates (D,) or (...,1,D) or (...,N-1,D)

nu: gamma-like node or (D,)-array or (...,1,D)-array or (...,N-1,D)-array

 $\nu$ , diagonal elements of the precision of the innovation process, plates (D,) or (...,1,D) or (...,N-1,D)

**n**: int, optional

N, the length of the chain. Must be given if A and  $\nu$  are constant over time.

## See also:

Gaussian, GaussianARD, Wishart, Gamma, SwitchingGaussianMarkovChain, VaryingGaussianMarkovChain, CategoricalMarkovChain

\_\_init\_\_ (mu, Lambda, A, nu, n=None, inputs=None, \*\*kwargs)
Create GaussianMarkovChain node.

#### **Methods**

init(mu, Lambda, A, nu[, n, inputs])	Create GaussianMarkovChain node.
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
<pre>get_parameters()</pre>	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
<pre>initialize_from_parameters(*args)</pre>	
<pre>initialize_from_prior()</pre>	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $log p(X parents) - log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random(*phi[, plates])	
rotate(R[, inv, logdet])	
	Continued on next page

Table 5.40 – continued from previous page

save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

#### bayespy.nodes.GaussianMarkovChain.\_\_init\_\_

GaussianMarkovChain.\_\_init\_\_ (mu, Lambda, A, nu, n=None, inputs=None, \*\*kwargs)
Create GaussianMarkovChain node.

# bayespy.nodes.GaussianMarkovChain.add\_plate\_axis

GaussianMarkovChain.add\_plate\_axis(to\_plate)

## bayespy.nodes.GaussianMarkovChain.broadcasting\_multiplier

GaussianMarkovChain.broadcasting\_multiplier(plates, \*args)

## bayespy.nodes.GaussianMarkovChain.delete

GaussianMarkovChain.delete()

Delete this node and the children

# bayespy.nodes.GaussianMarkovChain.get\_gradient

GaussianMarkovChain.get\_gradient(rg)

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

# $bayespy.nodes. Gaussian Markov Chain. get\_mask$

GaussianMarkovChain.get\_mask()

# bayespy.nodes.GaussianMarkovChain.get\_moments

GaussianMarkovChain.get\_moments()

#### bayespy.nodes.GaussianMarkovChain.get\_parameters

```
{\tt Gaussian Markov Chain.get\_parameters ()}
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.GaussianMarkovChain.get\_riemannian\_gradient

```
GaussianMarkovChain.get_riemannian_gradient()
Computes the Riemannian/natural gradient.
```

## bayespy.nodes.GaussianMarkovChain.get\_shape

```
GaussianMarkovChain.get_shape(ind)
```

## bayespy.nodes.GaussianMarkovChain.has\_plotter

```
GaussianMarkovChain.has_plotter()
Return True if the node has a plotter
```

## bayespy.nodes.GaussianMarkovChain.initialize\_from\_parameters

```
GaussianMarkovChain.initialize_from_parameters(*args)
```

# $bayes py. nodes. Gaussian Markov Chain. in itialize\_from\_prior$

```
GaussianMarkovChain.initialize_from_prior()
```

# bayespy.nodes.GaussianMarkovChain.initialize\_from\_random

```
GaussianMarkovChain.initialize_from_random()

Set the variable to a random sample from the current distribution.
```

# bayespy.nodes.GaussianMarkovChain.initialize\_from\_value

```
GaussianMarkovChain.initialize_from_value(x, *args)
```

# bayespy.nodes.GaussianMarkovChain.load

```
GaussianMarkovChain.load(filename)
```

#### bayespy.nodes.GaussianMarkovChain.logpdf

```
GaussianMarkovChain.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.GaussianMarkovChain.lower\_bound\_contribution

```
GaussianMarkovChain.lower_bound_contribution (gradient=False, ignore\_masked=True)

Compute E[log p(X|parents) - log q(X)]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

## bayespy.nodes.GaussianMarkovChain.lowerbound

```
GaussianMarkovChain.lowerbound()
```

## bayespy.nodes.GaussianMarkovChain.move\_plates

GaussianMarkovChain.move\_plates(from\_plate, to\_plate)

# bayespy.nodes.GaussianMarkovChain.observe

```
GaussianMarkovChain.observe(x, *args, mask=True) Fix moments, compute f and propagate mask.
```

# bayespy.nodes.GaussianMarkovChain.pdf

```
GaussianMarkovChain.pdf (X, mask=True)
```

Compute the probability density function of this node.

## bayespy.nodes.GaussianMarkovChain.plot

```
GaussianMarkovChain.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.GaussianMarkovChain.random

```
GaussianMarkovChain.random(*phi, plates=None)
```

#### bayespy.nodes.GaussianMarkovChain.rotate

GaussianMarkovChain.rotate(R, inv=None, logdet=None)

## bayespy.nodes.GaussianMarkovChain.save

GaussianMarkovChain.save (filename)

## bayespy.nodes.GaussianMarkovChain.set\_parameters

GaussianMarkovChain.set\_parameters(x)

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# bayespy.nodes.GaussianMarkovChain.set\_plotter

GaussianMarkovChain.set\_plotter(plotter)

# bayespy.nodes.GaussianMarkovChain.show

GaussianMarkovChain.show()

Print the distribution using standard parameterization.

# bayespy.nodes.GaussianMarkovChain.unobserve

GaussianMarkovChain.unobserve()

## bayespy.nodes.GaussianMarkovChain.update

GaussianMarkovChain.update(annealing=1.0)

# **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

# bayespy.nodes.GaussianMarkovChain.dims

GaussianMarkovChain.dims = None

# bayespy.nodes.GaussianMarkovChain.plates

GaussianMarkovChain.plates = None

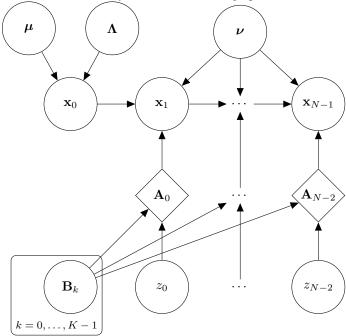
#### bayespy.nodes.GaussianMarkovChain.plates\_multiplier

GaussianMarkovChain.plates\_multiplier
Plate multiplier is applied to messages to parents

# bayespy.nodes.SwitchingGaussianMarkovChain

Node for Gaussian Markov chain random variables with switching dynamics.

The node models a sequence of Gaussian variables :math:  ${mathbf\{x\}\_0,ldots,mathbf\{x\}\_\{N-1\}}$ \$ with linear Markovian dynamics. The dynamics may change in time, which is obtained by having a set of matrices and at each time selecting one of them as the state dynamics matrix. The graphical model can be presented as:



where  $\mu$  and  $\Lambda$  are the mean and the precision matrix of the initial state,  $\nu$  is the precision of the innovation noise, and  $\mathbf{A}_n$  are the state dynamics matrix obtained by selecting one of the matrices  $\{\mathbf{B}_k\}_{k=0}^{K-1}$  at each time. The selections are provided by  $z_n \in \{0, \dots, K-1\}$ . The probability distribution is

$$p(\mathbf{x}_0, \dots, \mathbf{x}_{N-1}) = p(\mathbf{x}_0) \prod_{n=1}^{N-1} p(\mathbf{x}_n | \mathbf{x}_{n-1})$$

where

$$p(\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0 | \boldsymbol{\mu}, \boldsymbol{\Lambda})$$

$$p(\mathbf{x}_n | \mathbf{x}_{n-1}) = \mathcal{N}(\mathbf{x}_n | \mathbf{A}_{n-1} \mathbf{x}_{n-1}, \operatorname{diag}(\boldsymbol{\nu})), \quad \text{for } n = 1, \dots, N-1,$$

$$\mathbf{A}_n = \mathbf{B}_{z_n}, \quad \text{for } n = 0, \dots, N-2.$$

Parameters mu: Gaussian-like node or (...,D)-array

 $\mu$ , mean of  $x_0$ , D-dimensional with plates (...)

**Lambda**: Wishart-like node or (...,D,D)-array

 $\Lambda$ , precision matrix of  $x_0$ ,  $D \times D$  -dimensional with plates (...)

**B**: Gaussian-like node or (...,D,D,K)-array

 $\{\mathbf{B}_k\}_{k=0}^{K-1}$ , a set of state dynamics matrix,  $D \times K$ -dimensional with plates (...,D)

Z: categorical-like node or (...,N-1)-array

 $\{z_0, \dots, z_{N-2}\}$ , time-dependent selection, K-categorical with plates (...,N-1)

**nu**: gamma-like node or (...,D)-array

 $\nu$ , diagonal elements of the precision of the innovation process, plates (...,D)

n: int, optional

N, the length of the chain. Must be given if **Z** does not have plates over the time domain (which would not make sense).

#### See also:

Gaussian, GaussianARD, Wishart, Gamma, GaussianMarkovChain, VaryingGaussianMarkovChain, Categorical, CategoricalMarkovChain

#### **Notes**

Equivalent model block can be constructed with GaussianMarkovChain by explicitly using Gate to select the state dynamics matrix. However, that approach is not very efficient for large datasets because it does not utilize the structure of  $A_n$ , thus it explicitly computes huge moment arrays.

\_\_init\_\_ (mu, Lambda, B, Z, nu, n=None, \*\*kwargs)
Create SwitchingGaussianMarkovChain node.

# Methods

Create SwitchingGaussianMarkovChain node.
Delete this node and the children
Computes gradient with respect to the natural parameters.
Return parameters of the VB distribution.
Computes the Riemannian/natural gradient.
Return True if the node has a plotter
Set the variable to a random sample from the current distribution.
Compute the log probability density function $Q(X)$ of this node.
Compute E[ $log p(X parents) - log q(X)$ ]
Fix moments, compute f and propagate mask.
Compute the probability density function of this node.
Continued on next page

Table 5.42 – continued from previous page

plot([fig])	Plot the node distribution using the plotter of the node
random(*phi[, plates])	
rotate(R[, inv, logdet])	
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

### bayespy.nodes.SwitchingGaussianMarkovChain.\_init\_

SwitchingGaussianMarkovChain.\_\_init\_\_ (mu, Lambda, B, Z, nu, n=None, \*\*kwargs) Create SwitchingGaussianMarkovChain node.

## bayespy.nodes.SwitchingGaussianMarkovChain.add\_plate\_axis

SwitchingGaussianMarkovChain.add\_plate\_axis(to\_plate)

# bayespy.nodes.SwitchingGaussianMarkovChain.broadcasting\_multiplier

SwitchingGaussianMarkovChain.broadcasting\_multiplier(plates, \*args)

# bayespy.nodes.SwitchingGaussianMarkovChain.delete

SwitchingGaussianMarkovChain.delete()

Delete this node and the children

#### bayespy.nodes.SwitchingGaussianMarkovChain.get\_gradient

 ${\tt SwitchingGaussianMarkovChain.get\_gradient}\ (\textit{rg})$ 

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

# bayespy.nodes.SwitchingGaussianMarkovChain.get\_mask

SwitchingGaussianMarkovChain.get\_mask()

# $bayes py. nodes. Switching Gaussian Markov Chain. get\_moments$

SwitchingGaussianMarkovChain.get\_moments()

#### bayespy.nodes.SwitchingGaussianMarkovChain.get\_parameters

 ${\tt Switching Gaussian Markov Chain.} \textbf{get\_parameters} \ ()$ 

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.SwitchingGaussianMarkovChain.get\_riemannian\_gradient

SwitchingGaussianMarkovChain.get\_riemannian\_gradient()
Computes the Riemannian/natural gradient.

## bayespy.nodes.SwitchingGaussianMarkovChain.get\_shape

SwitchingGaussianMarkovChain.get\_shape(ind)

# bayespy.nodes.SwitchingGaussianMarkovChain.has\_plotter

SwitchingGaussianMarkovChain.has\_plotter()
Return True if the node has a plotter

## bayespy.nodes.SwitchingGaussianMarkovChain.initialize\_from\_parameters

SwitchingGaussianMarkovChain.initialize\_from\_parameters(\*args)

# bayespy.nodes.SwitchingGaussianMarkovChain.initialize\_from\_prior

SwitchingGaussianMarkovChain.initialize\_from\_prior()

# $bayes py. nodes. Switching Gaussian Markov Chain. initialize\_from\_random$

SwitchingGaussianMarkovChain.initialize\_from\_random()
Set the variable to a random sample from the current distribution.

## bayespy.nodes.SwitchingGaussianMarkovChain.initialize\_from\_value

SwitchingGaussianMarkovChain.initialize\_from\_value(x, \*args)

# bayes py. nodes. Switching Gaussian Markov Chain. load

SwitchingGaussianMarkovChain.load(filename)

#### bayespy.nodes.SwitchingGaussianMarkovChain.logpdf

```
SwitchingGaussianMarkovChain.logpdf (X, mask=True) Compute the log probability density function Q(X) of this node.
```

## bayespy.nodes.SwitchingGaussianMarkovChain.lower\_bound\_contribution

```
SwitchingGaussianMarkovChain.lower_bound_contribution(gradient=False, ig-nore_masked=True)
```

Compute E[  $\log p(X|parents) - \log q(X)$  ]

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

## bayespy.nodes.SwitchingGaussianMarkovChain.lowerbound

SwitchingGaussianMarkovChain.lowerbound()

#### bayespy.nodes.SwitchingGaussianMarkovChain.move\_plates

SwitchingGaussianMarkovChain.move\_plates(from\_plate, to\_plate)

# bayespy.nodes.SwitchingGaussianMarkovChain.observe

```
SwitchingGaussianMarkovChain.observe(x, *args, mask=True) Fix moments, compute f and propagate mask.
```

## bayespy.nodes.SwitchingGaussianMarkovChain.pdf

```
SwitchingGaussianMarkovChain.pdf (X, mask=True)
Compute the probability density function of this node.
```

#### bayespy.nodes.SwitchingGaussianMarkovChain.plot

```
{\tt SwitchingGaussianMarkovChain.plot}~(\textit{fig=None},~**kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.SwitchingGaussianMarkovChain.random

SwitchingGaussianMarkovChain.random(\*phi, plates=None)

#### bayespy.nodes.SwitchingGaussianMarkovChain.rotate

SwitchingGaussianMarkovChain.rotate(R, inv=None, logdet=None)

#### bayespy.nodes.SwitchingGaussianMarkovChain.save

SwitchingGaussianMarkovChain.save (filename)

## bayespy.nodes.SwitchingGaussianMarkovChain.set\_parameters

SwitchingGaussianMarkovChain.set\_parameters(x)

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# bayespy.nodes.SwitchingGaussianMarkovChain.set\_plotter

SwitchingGaussianMarkovChain.set\_plotter(plotter)

# bayespy.nodes.SwitchingGaussianMarkovChain.show

SwitchingGaussianMarkovChain.show()

Print the distribution using standard parameterization.

# bayes py. nodes. Switching Gaussian Markov Chain. unobserve

SwitchingGaussianMarkovChain.unobserve()

## bayespy.nodes.SwitchingGaussianMarkovChain.update

SwitchingGaussianMarkovChain.update(annealing=1.0)

# **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

# bayespy.nodes.SwitchingGaussianMarkovChain.dims

SwitchingGaussianMarkovChain.dims = None

# bayes py. nodes. Switching Gaussian Markov Chain. plates

SwitchingGaussianMarkovChain.plates = None

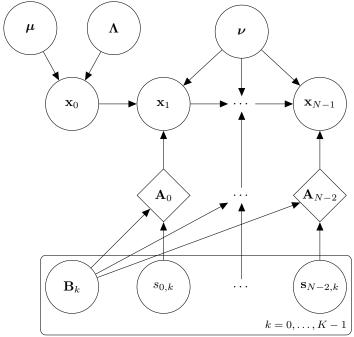
#### bayespy.nodes.SwitchingGaussianMarkovChain.plates\_multiplier

SwitchingGaussianMarkovChain.plates\_multiplier Plate multiplier is applied to messages to parents

# bayespy.nodes.VaryingGaussianMarkovChain

**class** bayespy.nodes.**VaryingGaussianMarkovChain** (*mu*, *Lambda*, *B*, *S*, *nu*, *n=None*, \*\*kwargs) Node for Gaussian Markov chain random variables with time-varying dynamics.

The node models a sequence of Gaussian variables  $\mathbf{x}_0, \dots, \mathbf{x}_{N-1}$  with linear Markovian dynamics. The time variability of the dynamics is obtained by modelling the state dynamics matrix as a linear combination of a set of matrices with time-varying linear combination weights. The graphical model can be presented as:



where  $\mu$  and  $\Lambda$  are the mean and the precision matrix of the initial state,  $\nu$  is the precision of the innovation noise, and  $A_n$  are the state dynamics matrix obtained by mixing matrices  $B_k$  with weights  $s_{n,k}$ .

The probability distribution is

$$p(\mathbf{x}_0, \dots, \mathbf{x}_{N-1}) = p(\mathbf{x}_0) \prod_{n=1}^{N-1} p(\mathbf{x}_n | \mathbf{x}_{n-1})$$

where

$$p(\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0 | \boldsymbol{\mu}, \boldsymbol{\Lambda})$$

$$p(\mathbf{x}_n | \mathbf{x}_{n-1}) = \mathcal{N}(\mathbf{x}_n | \mathbf{A}_{n-1} \mathbf{x}_{n-1}, \operatorname{diag}(\boldsymbol{\nu})), \quad \text{for } n = 1, \dots, N-1,$$

$$\mathbf{A}_n = \sum_{k=0}^{K-1} s_{n,k} \mathbf{B}_k, \quad \text{for } n = 0, \dots, N-2.$$

Parameters mu: Gaussian-like node or (...,D)-array

 $\mu$ , mean of  $x_0$ , D-dimensional with plates (...)

Lambda: Wishart-like node or (...,D,D)-array

 $\Lambda$ , precision matrix of  $x_0$ ,  $D \times D$  -dimensional with plates (...)

**B**: Gaussian-like node or (...,D,D,K)-array

 $\{\mathbf{B}_k\}_{k=0}^{K-1}$ , a set of state dynamics matrix,  $D \times K$ -dimensional with plates (...,D)

S: Gaussian-like node or (...,N-1,K)-array

 $\{s_0, \dots, s_{N-2}\}$ , time-varying weights of the linear combination, K-dimensional with plates (...,N-1)

nu: gamma-like node or (...,D)-array

 $\nu$ , diagonal elements of the precision of the innovation process, plates (...,D)

**n**: int, optional

N, the length of the chain. Must be given if **S** does not have plates over the time domain (which would not make sense).

#### See also:

Gaussian, GaussianARD, Wishart, Gamma, GaussianMarkovChain, SwitchingGaussianMarkovChain

#### **Notes**

Equivalent model block can be constructed with GaussianMarkovChain by explicitly using SumMultiply to compute the linear combination. However, that approach is not very efficient for large datasets because it does not utilize the structure of  $A_n$ , thus it explicitly computes huge moment arrays.

# References

[8]

\_\_init\_\_ (mu, Lambda, B, S, nu, n=None, \*\*kwargs)
Create VaryingGaussianMarkovChain node.

## **Methods**

Create VaryingGaussianMarkovChain node.
Delete this node and the children
Computes gradient with respect to the natural parameters.
Return parameters of the VB distribution.
Computes the Riemannian/natural gradient.
Return True if the node has a plotter
Set the variable to a random sample from the current distribution.
Continued on next page

Table 5.44 – continued from previous page

	and the state of t
initialize_from_value(x, *args)	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function Q(X) of this node.
lower_bound_contribution([gradient,])	Compute E[ log p(X parents) - log q(X) ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random(*phi[, plates])	
<pre>rotate(R[, inv, logdet])</pre>	
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

## bayespy.nodes.VaryingGaussianMarkovChain.\_\_init\_\_

VaryingGaussianMarkovChain.\_\_init\_\_ (mu, Lambda, B, S, nu, n=None, \*\*kwargs)
Create VaryingGaussianMarkovChain node.

# bayespy.nodes.VaryingGaussianMarkovChain.add\_plate\_axis

VaryingGaussianMarkovChain.add\_plate\_axis (to\_plate)

# $bayes py. nodes. Varying Gaussian Markov Chain. broadcasting\_multiplier$

VaryingGaussianMarkovChain.broadcasting\_multiplier(plates, \*args)

## bayespy.nodes.VaryingGaussianMarkovChain.delete

VaryingGaussianMarkovChain.delete()

Delete this node and the children

## bayespy.nodes.VaryingGaussianMarkovChain.get\_gradient

VaryingGaussianMarkovChain.get\_gradient (rg)

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.VaryingGaussianMarkovChain.get\_mask

VaryingGaussianMarkovChain.get\_mask()

## bayespy.nodes.VaryingGaussianMarkovChain.get\_moments

VaryingGaussianMarkovChain.get\_moments()

## bayespy.nodes.VaryingGaussianMarkovChain.get\_parameters

VaryingGaussianMarkovChain.get\_parameters()

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

# $bayes py. nodes. Varying Gaussian Markov Chain. get\_riemannian\_gradient$

VaryingGaussianMarkovChain.get\_riemannian\_gradient()
Computes the Riemannian/natural gradient.

# bayespy.nodes.VaryingGaussianMarkovChain.get\_shape

VaryingGaussianMarkovChain.get\_shape (ind)

## bayespy.nodes.VaryingGaussianMarkovChain.has\_plotter

VaryingGaussianMarkovChain.has\_plotter()
Return True if the node has a plotter

# $bayes py. nodes. Varying Gaussian Markov Chain. initialize\_from\_parameters$

VaryingGaussianMarkovChain.initialize\_from\_parameters(\*args)

# $bayes py. nodes. Varying Gaussian Markov Chain. in itialize\_from\_prior$

VaryingGaussianMarkovChain.initialize\_from\_prior()

# $bayes py. nodes. Varying Gaussian Markov Chain. initialize\_from\_random$

VaryingGaussianMarkovChain.initialize\_from\_random()
Set the variable to a random sample from the current distribution.

# bayespy.nodes.VaryingGaussianMarkovChain.initialize\_from\_value

VaryingGaussianMarkovChain.initialize\_from\_value(x, \*args)

#### bayespy.nodes.VaryingGaussianMarkovChain.load

VaryingGaussianMarkovChain.load(filename)

#### bayespy.nodes.VaryingGaussianMarkovChain.logpdf

VaryingGaussianMarkovChain.logpdf (*X*, *mask=True*)

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.VaryingGaussianMarkovChain.lower\_bound\_contribution

VaryingGaussianMarkovChain.lower\_bound\_contribution(gradient=False, ig-nore\_masked=True)

Compute E[ log p(X|parents) - log q(X) ]

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

#### bayespy.nodes.VaryingGaussianMarkovChain.lowerbound

VaryingGaussianMarkovChain.lowerbound()

#### bayespy.nodes.VaryingGaussianMarkovChain.move\_plates

VaryingGaussianMarkovChain.move\_plates (from\_plate, to\_plate)

#### bayespy.nodes.VaryingGaussianMarkovChain.observe

VaryingGaussianMarkovChain.**observe**(*x*, \**args*, *mask=True*) Fix moments, compute f and propagate mask.

#### bayespy.nodes.VaryingGaussianMarkovChain.pdf

VaryingGaussianMarkovChain.**pdf** (*X*, *mask=True*) Compute the probability density function of this node.

#### bayespy.nodes.VaryingGaussianMarkovChain.plot

VaryingGaussianMarkovChain.plot (fig=None, \*\*kwargs)

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.VaryingGaussianMarkovChain.random

VaryingGaussianMarkovChain.random(\*phi, plates=None)

#### bayespy.nodes.VaryingGaussianMarkovChain.rotate

VaryingGaussianMarkovChain.rotate(R, inv=None, logdet=None)

#### bayespy.nodes.VaryingGaussianMarkovChain.save

VaryingGaussianMarkovChain.save (filename)

#### bayespy.nodes.VaryingGaussianMarkovChain.set\_parameters

VaryingGaussianMarkovChain.set\_parameters(x)

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

## bayespy.nodes.VaryingGaussianMarkovChain.set\_plotter

VaryingGaussianMarkovChain.set\_plotter(plotter)

#### bayespy.nodes.VaryingGaussianMarkovChain.show

VaryingGaussianMarkovChain.show()

Print the distribution using standard parameterization.

#### bayespy.nodes.VaryingGaussianMarkovChain.unobserve

VaryingGaussianMarkovChain.unobserve()

#### bayespy.nodes.VaryingGaussianMarkovChain.update

VaryingGaussianMarkovChain.update(annealing=1.0)

## **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

## bayes py. nodes. Varying Gaussian Markov Chain. dims

VaryingGaussianMarkovChain.dims = None

#### bayespy.nodes.VaryingGaussianMarkovChain.plates

VaryingGaussianMarkovChain.plates = None

### bayespy.nodes.VaryingGaussianMarkovChain.plates\_multiplier

VaryingGaussianMarkovChain.plates\_multiplier

Plate multiplier is applied to messages to parents

Other stochastic nodes:

Mixture(z, node\_class, \*params[, cluster\_plate]) Node for exponential family mixture variables.

## bayespy.nodes.Mixture

**class** bayespy.nodes.**Mixture** (*z*, *node\_class*, \**params*, *cluster\_plate=-1*, \*\**kwargs*)

Node for exponential family mixture variables.

The node represents a random variable which is sampled from a mixture distribution. It is possible to mix any exponential family distribution. The probability density function is

$$p(x|z=k,\boldsymbol{\theta}_0,\ldots,\boldsymbol{\theta}_{K-1})=\phi(x|\boldsymbol{\theta}_k),$$

where  $\phi$  is the probability density function of the mixed exponential family distribution and  $\theta_0, \dots, \theta_{K-1}$  are the parameters of each cluster. For instance,  $\phi$  could be the Gaussian probability density function  $\mathcal{N}$  and  $\theta_k = \{\mu_k, \Lambda_k\}$  where  $\mu_k$  and  $\Lambda_k$  are the mean vector and precision matrix for cluster k.

Parameters z : categorical-like node or array

z, cluster assignment

node\_class : stochastic exponential family node class

Mixed distribution

params: types specified by the mixed distribution

Parameters of the mixed distribution. If some parameters should vary between clusters, those parameters' plate axis *cluster\_plate* should have a size which equals the number of clusters. For parameters with shared values, that plate axis should have length 1. At least one parameter should vary between clusters.

cluster\_plate : int, optional

Negative integer defining which plate axis is used for the clusters in the parameters. That plate axis is ignored from the parameters when considering the plates for this node. By default, mix over the last plate axis.

#### See also:

Categorical, Categorical Markov Chain

#### **Examples**

A simple 2-dimensional Gaussian mixture model with three clusters for 100 samples can be constructed, for instance, as:

\_\_init\_\_(z, node\_class, \*params, cluster\_plate=-1, \*\*kwargs)

#### **Methods**

init(z, node_class, *params[, cluster_plate])	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
get_parameters()	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	
initialize_from_prior()	
initialize_from_random()	Set the variable to a random sample from the current distribution.
<pre>initialize_from_value(x, *args)</pre>	
<pre>integrated_logpdf_from_parents(x, index)</pre>	Approximates the posterior predictive pdf int $p(x parents)$ q(parents) dparents
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_parameters(x)	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	
<u> </u>	

## bayespy.nodes.Mixture.\_\_init\_\_

Mixture.\_\_init\_\_(z, node\_class, \*params, cluster\_plate=-1, \*\*kwargs)

#### bayespy.nodes.Mixture.add\_plate\_axis

```
Mixture.add_plate_axis (to_plate)
```

#### bayespy.nodes.Mixture.broadcasting\_multiplier

```
Mixture.broadcasting_multiplier(plates, *args)
```

#### bayespy.nodes.Mixture.delete

```
Mixture.delete()
```

Delete this node and the children

#### bayespy.nodes.Mixture.get\_gradient

```
Mixture.get_gradient(rg)
```

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.nodes.Mixture.get\_mask

```
Mixture.get_mask()
```

## bayespy.nodes.Mixture.get\_moments

```
Mixture.get_moments()
```

#### bayespy.nodes.Mixture.get\_parameters

```
Mixture.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Mixture.get\_riemannian\_gradient

```
Mixture.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

#### bayespy.nodes.Mixture.get\_shape

```
Mixture.get_shape(ind)
```

## bayespy.nodes.Mixture.has\_plotter

```
Mixture.has_plotter()
```

Return True if the node has a plotter

#### bayespy.nodes.Mixture.initialize\_from\_parameters

```
Mixture.initialize_from_parameters(*args)
```

#### bayespy.nodes.Mixture.initialize\_from\_prior

```
Mixture.initialize_from_prior()
```

#### bayespy.nodes.Mixture.initialize\_from\_random

```
Mixture.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

#### bayespy.nodes.Mixture.initialize\_from\_value

```
Mixture.initialize_from_value(x, *args)
```

#### bayespy.nodes.Mixture.integrated\_logpdf\_from\_parents

```
Mixture.integrated_logpdf_from_parents(x, index)
```

Approximates the posterior predictive pdf int p(x|parents) q(parents) dparents in log-scale as int  $q(parents_i)$  exp( int  $q(parents_i)$  log  $p(x|parents_i)$  dparents\_i) dparents\_i.

## bayespy.nodes.Mixture.load

```
Mixture.load(filename)
```

#### bayespy.nodes.Mixture.logpdf

```
Mixture.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.nodes.Mixture.lower\_bound\_contribution

```
Mixture.lower_bound_contribution(gradient=False, ignore_masked=True)
```

```
Compute E[ log p(X|parents) - log q(X) ]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

#### bayespy.nodes.Mixture.lowerbound

```
Mixture.lowerbound()
```

#### bayespy.nodes.Mixture.move\_plates

```
Mixture.move_plates (from_plate, to_plate)
```

#### bayespy.nodes.Mixture.observe

```
Mixture.observe(x, *args, mask=True)
```

Fix moments, compute f and propagate mask.

## bayespy.nodes.Mixture.pdf

```
Mixture.pdf (X, mask=True)
```

Compute the probability density function of this node.

#### bayespy.nodes.Mixture.plot

```
Mixture.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

## bayespy.nodes.Mixture.random

```
Mixture.random()
```

Draw a random sample from the distribution.

### bayespy.nodes.Mixture.save

```
Mixture.save (filename)
```

## bayespy.nodes.Mixture.set\_parameters

```
Mixture.set_parameters(x)
```

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.nodes.Mixture.set\_plotter

```
Mixture.set_plotter(plotter)
```

#### bayespy.nodes.Mixture.show

```
Mixture.show()
```

Print the distribution using standard parameterization.

#### bayespy.nodes.Mixture.unobserve

```
Mixture.unobserve()
```

## bayespy.nodes.Mixture.update

Mixture.update(annealing=1.0)

#### **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

#### bayespy.nodes.Mixture.dims

```
Mixture.dims = None
```

## bayespy.nodes.Mixture.plates

Mixture.plates = None

## bayespy.nodes.Mixture.plates\_multiplier

```
Mixture.plates_multiplier
```

Plate multiplier is applied to messages to parents

## 5.1.2 Deterministic nodes

Dot(*args, **kwargs)	Node for computing inner product of several Gaussian vectors.	
SumMultiply(*args[, iterator_axis])	Node for computing general products and sums of Gaussian nodes.	
Add(*nodes, **kwargs) Node for computing sums of Gaussian nodes: $X + Y + Z$		
Gate(Z, X[, gated_plate, moments])	Deterministic gating of one node.	

## bayespy.nodes.Dot

```
bayespy.nodes.Dot(*args, **kwargs)
```

Node for computing inner product of several Gaussian vectors.

This is a simple wrapper of the much more general SumMultiply. For now, it is here for backward compatibility.

#### bayespy.nodes.SumMultiply

class bayespy.nodes.SumMultiply(\*args, iterator\_axis=None, \*\*kwargs)

Node for computing general products and sums of Gaussian nodes.

The node is similar to *numpy.einsum*, which is a very general function for computing dot products, sums, products and other sums of products of arrays.

For instance, consider the following arrays:

```
>>> import numpy as np
>>> X = np.random.randn(2, 3, 4)
>>> Y = np.random.randn(3, 5)
>>> Z = np.random.randn(4, 2)
```

Then, the Einstein summation can be used as:

```
>>> np.einsum('abc,bd,ca->da', X, Y, Z)
array([[...]])
```

SumMultiply node can be used similarly for Gaussian nodes. For instance, consider the following Gaussian nodes:

```
>>> from bayespy.nodes import GaussianARD
>>> X = GaussianARD(0, 1, shape=(2, 3, 4))
>>> Y = GaussianARD(0, 1, shape=(3, 5))
>>> Z = GaussianARD(0, 1, shape=(4, 2))
```

Then, similarly to *numpy.einsum*, SumMultiply could be used as:

```
>>> from bayespy.nodes import SumMultiply
>>> SumMultiply('abc,bd,ca->da', X, Y, Z)
<bayespy.inference.vmp.nodes.dot.SumMultiply object at 0x...>
```

or

```
>>> SumMultiply(X, [0,1,2], Y, [1,3], Z, [2,0], [3,0])
<bayespy.inference.vmp.nodes.dot.SumMultiply object at 0x...>
```

which is similar to the alternative syntax of numpy.einsum.

This node operates similarly as numpy.einsum. However, you must use all the elements of each node, that is, an operation like np.einsum('ii->i',X) is not allowed. Thus, for each node, each axis must be given unique id. The id identifies which axes correspond to which axes between the different nodes. Also, Ellipsis ('...') is not yet supported for simplicity. It would also have some problems with constant inputs (because how to determine ndim), so let us just forget it for now.

Each output axis must appear in the input mappings.

The keys must refer to variable dimension axes only, not plate axes.

The input nodes may be Gaussian-gamma (isotropic) nodes.

The output message is Gaussian-gamma (isotropic) if any of the input nodes is Gaussian-gamma.

#### Notes

This operation can be extremely slow if not used wisely. For large and complex operations, it is sometimes more efficient to split the operation into multiple nodes. For instance, the example above could probably be computed faster by

```
>>> XZ = SumMultiply(X, [0,1,2], Z, [2,0], [0,1])
>>> F = SumMultiply(XZ, [0,1], Y, [1,2], [2,0])
```

because the third axis ('c') could be summed out already in the first operation. This same effect applies also to numpy.einsum in general.

## **Examples**

```
Sum over the rows: 'ij->j'
```

Inner product of three vectors: 'i,i,i'

Matrix-vector product: 'ij,j->i'

Matrix-matrix product: 'ik,kj->ij'

Outer product: 'i,j->ij'

Vector-matrix-vector product: 'i,ij,j'

\_\_init\_\_ (Node1, map1, Node2, map2, ..., NodeN, mapN[, map\_out])

#### **Methods**

init(Node1, map1, Node2, map2,,)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_mask()	
<pre>get_moments()</pre>	
get_parameters()	
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
lower_bound_contribution([gradient])	
move_plates(from_plate, to_plate)	
plot([fig])	Plot the node distribution using the plotter of the node
set_plotter(plotter)	

## bayespy.nodes.SumMultiply.\_\_init\_\_

```
SumMultiply.__init__(Node1, map1, Node2, map2, ..., NodeN, mapN[, map_out])
```

#### bayespy.nodes.SumMultiply.add\_plate\_axis

```
SumMultiply.add_plate_axis(to_plate)
```

#### bayespy.nodes.SumMultiply.broadcasting\_multiplier

```
SumMultiply.broadcasting.multiplier(plates, *args)
```

#### bayespy.nodes.SumMultiply.delete

```
SumMultiply.delete()

Delete this node and the children
```

#### bayespy.nodes.SumMultiply.get\_mask

```
SumMultiply.get_mask()
```

### bayespy.nodes.SumMultiply.get\_moments

```
SumMultiply.get_moments()
```

#### bayespy.nodes.SumMultiply.get\_parameters

```
SumMultiply.get_parameters()
```

## bayespy.nodes.SumMultiply.get\_shape

```
SumMultiply.get_shape (ind)
```

#### bayespy.nodes.SumMultiply.has\_plotter

```
SumMultiply.has_plotter()

Return True if the node has a plotter
```

#### bayespy.nodes.SumMultiply.lower\_bound\_contribution

```
SumMultiply.lower_bound_contribution(gradient=False, **kwargs)
```

#### bayespy.nodes.SumMultiply.move\_plates

```
SumMultiply.move_plates (from_plate, to_plate)
```

#### bayespy.nodes.SumMultiply.plot

```
SumMultiply.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.SumMultiply.set\_plotter

```
SumMultiply.set_plotter(plotter)
```

#### **Attributes**

```
plates
plates_multiplier Plate multiplier is applied to messages to parents
```

### bayespy.nodes.SumMultiply.plates

```
SumMultiply.plates = None
```

#### bayespy.nodes.SumMultiply.plates\_multiplier

```
{\tt SumMultiply.plates\_multiplier}
```

Plate multiplier is applied to messages to parents

## bayespy.nodes.Add

```
class bayespy.nodes.Add (*nodes, **kwargs) Node for computing sums of Gaussian nodes: X+Y+Z.
```

#### See also:

```
Dot, SumMultiply
```

#### **Notes**

Shapes of the nodes must be identical. Plates are broadcasted.

This node sums nodes that are independent in the posterior approximation. However, summing variables puts a strong coupling among the variables, which is lost in this construction. Thus, it is usually better to use a single Gaussian node to represent the set of the summed variables and use SumMultiply node to compute the sum. In that way, the correlation between the variables is not lost. However, in some cases it is necessary or useful to use Add node.

### **Examples**

```
__init__ (X1, X2, ...)
```

Methods

init(X1, X2,)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_mask()	
<pre>get_moments()</pre>	
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
<pre>lower_bound_contribution([gradient])</pre>	
<pre>move_plates(from_plate, to_plate)</pre>	
plot([fig])	Plot the node distribution using the plotter of the node
set_plotter(plotter)	
(Protect)	

## bayespy.nodes.Add.\_\_init\_\_

Add.\_\_**init**\_\_(*X1*, *X2*,...)

## bayespy.nodes.Add.add\_plate\_axis

Add.add\_plate\_axis(to\_plate)

## bayespy.nodes.Add.broadcasting\_multiplier

Add.broadcasting\_multiplier(plates, \*args)

## bayespy.nodes.Add.delete

Add.delete()

Delete this node and the children

## $bayespy.nodes. Add. get\_mask$

Add.get\_mask()

## bayespy.nodes.Add.get\_moments

Add.get\_moments()

## $bayespy.nodes. Add. get\_shape$

Add.get\_shape(ind)

## bayespy.nodes.Add.has\_plotter

Add.has\_plotter()

Return True if the node has a plotter

#### bayespy.nodes.Add.lower\_bound\_contribution

Add.lower\_bound\_contribution(gradient=False, \*\*kwargs)

## bayespy.nodes.Add.move\_plates

Add.move\_plates (from\_plate, to\_plate)

#### bayespy.nodes.Add.plot

```
Add.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.nodes.Add.set\_plotter

Add.set\_plotter(plotter)

#### **Attributes**

plates	
plates_multiplier	Plate multiplier is applied to messages to parents

#### bayespy.nodes.Add.plates

Add.plates = None

## bayespy.nodes.Add.plates\_multiplier

## $\verb|Add.plates_multiplier||$

Plate multiplier is applied to messages to parents

## bayespy.nodes.Gate

**class** bayespy.nodes.**Gate** (*Z*, *X*, *gated\_plate=-1*, *moments=None*, \*\*kwargs)

Deterministic gating of one node.

Gating is performed over one plate axis.

Note: You should not use gating for several variables which parents of a same node if the gates use the same gate assignments. In such case, the results will be wrong. The reason is a general one: A stochastic node may not be a parent of another node via several paths unless at most one path has no other stochastic nodes between them.

```
__init__ (Z, X, gated_plate=-1, moments=None, **kwargs)
Constructor for the gating node.
```

#### Parameters Z: Categorical-like node

A variable which chooses the index along the gated plate axis

X: node

The node whose plate axis is gated

gated\_plate : int (optional)

The index of the plate axis to be gated (by default, -1, that is, the last axis).

#### Methods

init(Z, X[, gated_plate, moments])	Constructor for the gating node.
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_mask()	
<pre>get_moments()</pre>	
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
lower_bound_contribution([gradient])	
<pre>move_plates(from_plate, to_plate)</pre>	
plot([fig])	Plot the node distribution using the plotter of the node
set_plotter(plotter)	

#### bayespy.nodes.Gate.\_\_init\_\_

```
Gate.__init__(Z, X, gated_plate=-1, moments=None, **kwargs)
Constructor for the gating node.
```

## Parameters Z: Categorical-like node

A variable which chooses the index along the gated plate axis

 $\mathbf{X}$ : node

The node whose plate axis is gated

gated\_plate : int (optional)

The index of the plate axis to be gated (by default, -1, that is, the last axis).

## bayespy.nodes.Gate.add\_plate\_axis

```
Gate.add_plate_axis (to_plate)
```

## bayespy.nodes.Gate.broadcasting\_multiplier

```
Gate.broadcasting_multiplier(plates, *args)
```

```
bayespy.nodes.Gate.delete
Gate.delete()
    Delete this node and the children
bayespy.nodes.Gate.get_mask
Gate.get_mask()
bayespy.nodes.Gate.get_moments
Gate.get_moments()
bayespy.nodes.Gate.get_shape
Gate.get_shape (ind)
bayespy.nodes.Gate.has_plotter
Gate.has_plotter()
     Return True if the node has a plotter
bayespy.nodes.Gate.lower_bound_contribution
Gate.lower_bound_contribution(gradient=False, **kwargs)
bayespy.nodes.Gate.move_plates
Gate.move_plates (from_plate, to_plate)
bayespy.nodes.Gate.plot
Gate.plot (fig=None, **kwargs)
    Plot the node distribution using the plotter of the node
     Because the distributions are in general very difficult to plot, the user must specify some functions which
     performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is,
     functions that perform plotting for a node.
bayespy.nodes.Gate.set_plotter
Gate.set_plotter(plotter)
Attributes
```

plates	
plates_multiplier	Plate multiplier is applied to messages to parents

## bayespy.nodes.Gate.plates

Gate.plates = None

bayespy.nodes.Gate.plates\_multiplier

Gate.plates\_multiplier

Plate multiplier is applied to messages to parents

## 5.2 bayespy.inference

Package for Bayesian inference engines

## 5.2.1 Inference engines

VB(\*nodes[, tol, autosave\_filename, ...]) Variational Bayesian (VB) inference engine

## bayespy.inference.VB

class bayespy.inference.VB(\*nodes, tol=1e-05, autosave\_filename=None, autosave\_iterations=0, use\_logging=False, callback=None)

Variational Bayesian (VB) inference engine

Parameters nodes: nodes

Nodes that form the model. Must include all at least all stochastic nodes of the model.

tol: double, optional

Convergence criterion. Tolerance for the relative change in the VB lower bound.

**autosave\_filename**: string, optional Filename for automatic saving

autosave\_iterations : int, optional

Iteration interval between each automatic saving

callback: callable, optional

Function which is called after each update iteration step

\_\_init\_\_ (\*nodes, tol=1e-05, autosave\_filename=None, autosave\_iterations=0, use\_logging=False, call-back=None)

## Methods

init(*nodes[, tol, autosave_filename,])	
add(x1, x2[, scale])	Add two vectors (in parameter format)
compute_lowerbound([ignore_masked])	
compute_lowerbound_terms(*nodes)	
dot(x1, x2)	Computes dot products of given vectors (in parameter format)
<pre>get_gradients(*nodes[, euclidian])</pre>	Computes gradients (both Riemannian and normal)
<pre>get_iteration_by_nodes()</pre>	
get_parameters(*nodes)	Get parameters of the nodes
<pre>gradient_step(*nodes[, scale])</pre>	Update nodes by taking a gradient ascent step
has_converged([tol])	
load(*nodes[, filename, nodes_only])	
loglikelihood_lowerbound()	
<pre>optimize(*nodes[, maxiter, verbose, method,])</pre>	Optimize nodes using Riemannian conjugate gradient
<pre>pattern_search(*nodes[, collapsed, maxiter])</pre>	Perform simple pattern search [4].
plot(*nodes, **kwargs)	Plot the distribution of the given nodes (or all nodes)
<pre>plot_iteration_by_nodes([axes, diff])</pre>	Plot the cost function per node during the iteration.
save(*nodes[, filename])	
set_annealing(annealing)	Set deterministic annealing from range (0, 1].
set_autosave(filename[, iterations, nodes])	
set_callback(callback)	
set_parameters(x, *nodes)	Set parameters of the nodes
update(*nodes[, repeat, plot, tol, verbose])	
use_logging(use)	

## bayespy.inference.VB.\_\_init\_\_

VB.\_\_init\_\_ (\*nodes, tol=1e-05, autosave\_filename=None, autosave\_iterations=0, use\_logging=False, callback=None)

## bayespy.inference.VB.add

VB . add (x1, x2, scale=1)Add two vectors (in parameter format)

## bayespy.inference.VB.compute\_lowerbound

VB.compute\_lowerbound(ignore\_masked=True)

## bayespy.inference.VB.compute\_lowerbound\_terms

VB.compute\_lowerbound\_terms(\*nodes)

## bayespy.inference.VB.dot

VB.dot (x1, x2)

Computes dot products of given vectors (in parameter format)

```
bayespy.inference.VB.get_gradients
VB.get_gradients (*nodes, euclidian=False)
    Computes gradients (both Riemannian and normal)
bayespy.inference.VB.get_iteration_by_nodes
VB.get_iteration_by_nodes()
bayespy.inference.VB.get_parameters
VB.get_parameters(*nodes)
    Get parameters of the nodes
bayespy.inference.VB.gradient_step
VB.gradient_step(*nodes, scale=1.0)
    Update nodes by taking a gradient ascent step
bayespy.inference.VB.has_converged
VB.has_converged(tol=None)
bayespy.inference.VB.load
VB.load(*nodes, filename=None, nodes_only=False)
bayespy.inference.VB.loglikelihood_lowerbound
VB.loglikelihood_lowerbound()
bayespy.inference.VB.optimize
VB. optimize (*nodes, maxiter=10, verbose=True, method='fletcher-reeves', riemannian=True, col-
               lapsed=None, tol=None)
    Optimize nodes using Riemannian conjugate gradient
bayespy.inference.VB.pattern_search
VB.pattern_search (*nodes, collapsed=None, maxiter=3)
    Perform simple pattern search [4].
    Some of the variables can be collapsed.
```

#### bayespy.inference.VB.plot

```
VB.plot (*nodes, **kwargs)

Plot the distribution of the given nodes (or all nodes)
```

#### bayespy.inference.VB.plot\_iteration\_by\_nodes

```
VB.plot_iteration_by_nodes (axes=None, diff=False)
```

Plot the cost function per node during the iteration.

Handy tool for debugging.

## bayespy.inference.VB.save

```
VB.save(*nodes, filename=None)
```

#### bayespy.inference.VB.set\_annealing

## VB.set\_annealing(annealing)

Set deterministic annealing from range (0, 1].

With 1, no annealing, standard updates.

With smaller values, entropy has more weight and model probability equations less. With 0, one would obtain improper uniform distributions.

#### bayespy.inference.VB.set\_autosave

```
VB.set_autosave (filename, iterations=None, nodes=None)
```

#### bayespy.inference.VB.set\_callback

```
\verb|VB.set_callback| (callback)|
```

#### bayespy.inference.VB.set\_parameters

```
VB.set_parameters (x, *nodes)
Set parameters of the nodes
```

#### bayespy.inference.VB.update

```
VB.update(*nodes, repeat=1, plot=False, tol=None, verbose=True)
```

#### bayespy.inference.VB.use\_logging

```
VB.use_logging(use)
```

#### **Attributes**

ignore\_bound\_checks

## bayespy.inference.VB.ignore\_bound\_checks

VB.ignore\_bound\_checks

## 5.2.2 Parameter expansions

${\it vmp.transformations.RotationOptimizer}()$	Optimizer for rotation parameter expansion in state-spa
$ ext{vmp.transformations.RotateGaussian}(X)$	Rotation parameter expansion for bayespy.nodes.
vmp.transformations.RotateGaussianARD(X, *alpha)	Rotation parameter expansion for bayespy.nodes.
$ ext{vmp.transformations.RotateGaussianMarkovChain}(X,)$	Rotation parameter expansion for bayespy.nodes.
${\tt vmp.transformations.RotateSwitchingMarkovChain}(X,)$	Rotation for bayespy.nodes.VaryingGaussian
$ ext{vmp.transformations.RotateVaryingMarkovChain}(X,)$	Rotation for bayespy.nodes.SwitchingGauss.
vmp.transformations.RotateMultiple(*rotators)	Identical parameter expansion for several nodes simulta

## bayespy.inference.vmp.transformations.RotationOptimizer

 $\textbf{class} \, \texttt{bayespy.inference.vmp.transformations.RotationOptimizer} \, (block1, block2, D)$ 

Optimizer for rotation parameter expansion in state-space models

Rotates one model block with  $\mathbf{R}$  and one model block with  $\mathbf{R}^{-1}$ .

Parameters block1: rotator object

The first rotation parameter expansion object

block2: rotator object

The second rotation parameter expansion object

D: int

Dimensionality of the latent space

#### References

```
[7], [6]
__init__(block1, block2, D)
```

### Methods

198

```
__init__(block1, block2, D)

rotate([maxiter, check_gradient, verbose, ...]) Optimize the rotation of two separate model blocks jointly.
```

#### bayespy.inference.vmp.transformations.RotationOptimizer.\_\_init\_\_

```
RotationOptimizer.__init__(block1, block2, D)
```

#### bayespy.inference.vmp.transformations.RotationOptimizer.rotate

```
RotationOptimizer.rotate(maxiter=10, check_gradient=False, verbose=False, check_bound=False)
```

Optimize the rotation of two separate model blocks jointly.

If some variable is the dot product of two Gaussians, rotating the two Gaussians optimally can make the inference algorithm orders of magnitude faster.

First block is rotated with  $\mathbf{R}$  and the second with  $\mathbf{R}^{-T}$ .

Blocks must have methods: bound(U,s,V) and rotate(R).

## bayespy.inference.vmp.transformations.RotateGaussian

```
class bayespy.inference.vmp.transformations.RotateGaussian(X)
    Rotation parameter expansion for bayespy.nodes.Gaussian
    __init__(X)
```

#### **Methods**

init(X)	
bound(R[, logdet, inv])	
<pre>get_bound_terms(R[, logdet, inv])</pre>	
nodes()	
rotate(R[, inv, logdet])	
setup()	This method should be called just before optimization.

#### bayespy.inference.vmp.transformations.RotateGaussian...init\_

```
RotateGaussian.__init__(X)
```

#### bayespy.inference.vmp.transformations.RotateGaussian.bound

RotateGaussian.bound(*R*, *logdet=None*, *inv=None*)

### bayespy.inference.vmp.transformations.RotateGaussian.get\_bound\_terms

RotateGaussian.get\_bound\_terms(R, logdet=None, inv=None)

#### bayespy.inference.vmp.transformations.RotateGaussian.nodes

```
RotateGaussian.nodes()
```

#### bayespy.inference.vmp.transformations.RotateGaussian.rotate

```
RotateGaussian.rotate(R, inv=None, logdet=None)
```

#### bayespy.inference.vmp.transformations.RotateGaussian.setup

```
RotateGaussian.setup()
```

This method should be called just before optimization.

## bayespy.inference.vmp.transformations.RotateGaussianARD

```
 \textbf{class} \ \texttt{bayespy.inference.vmp.transformations.RotateGaussianARD} \ (X, \ *alpha, \ axis=-1, \\ precompute=False, \\ subset=None)
```

Rotation parameter expansion for bayespy.nodes.GaussianARD

The model:

```
alpha ~ N(a, b) X ~ N(mu, alpha)
```

X can be an array (e.g., GaussianARD).

Transform q(X) and q(alpha) by rotating X.

Requirements: \* X and alpha do not contain any observed values

```
__init__(X, *alpha, axis=-1, precompute=False, subset=None)
```

Precompute tells whether to compute some moments once in the setup function instead of every time in the bound function. However, they are computed a bit differently in the bound function so it can be useful too. Precomputation is probably beneficial only when there are large axes that are not rotated (by R nor Q) and they are not contained in the plates of alpha, and the dimensions for R and Q are quite small.

#### **Methods**

init(X, *alpha[, axis, precompute, subset])	Precompute tells whether to compute some moments once in the setup function ins
bound(R[, logdet, inv, Q])	
<pre>get_bound_terms(R[, logdet, inv, Q])</pre>	
nodes()	
rotate(R[, inv, logdet, Q])	
setup([plate_axis])	This method should be called just before optimization.

#### bayespy.inference.vmp.transformations.RotateGaussianARD.\_\_init\_\_

```
RotateGaussianARD.__init__(X, *alpha, axis=-1, precompute=False, subset=None)
```

Precompute tells whether to compute some moments once in the setup function instead of every time in the bound function. However, they are computed a bit differently in the bound function so it can be useful too. Precomputation is probably beneficial only when there are large axes that are not rotated (by R nor Q) and they are not contained in the plates of alpha, and the dimensions for R and Q are quite small.

#### bayespy.inference.vmp.transformations.RotateGaussianARD.bound

RotateGaussianARD.**bound** (R, logdet=None, inv=None, Q=None)

200 Chapter 5. User API

#### bayespy.inference.vmp.transformations.RotateGaussianARD.get\_bound\_terms

RotateGaussianARD.get\_bound\_terms(R, logdet=None, inv=None, Q=None)

#### bayespy.inference.vmp.transformations.RotateGaussianARD.nodes

RotateGaussianARD.nodes()

#### bayespy.inference.vmp.transformations.RotateGaussianARD.rotate

RotateGaussianARD.rotate(R, inv=None, logdet=None, Q=None)

#### bayespy.inference.vmp.transformations.RotateGaussianARD.setup

RotateGaussianARD.setup(plate\_axis=None)

This method should be called just before optimization.

For efficiency, sum over axes that are not in mu, alpha nor rotation.

If using Q, set rotate\_plates to True.

## bayespy.inference.vmp.transformations.RotateGaussianMarkovChain

class bayespy.inference.vmp.transformations.RotateGaussianMarkovChain(X,

\*args)

Rotation parameter expansion for bayespy.nodes.GaussianMarkovChain

Assume the following model.

Constant, unit isotropic innovation noise. Unit variance only?

Maybe: Assume innovation noise with unit variance? Would it help make this function more general with respect to A.

TODO: Allow constant A or not rotating A.

A may vary in time.

Shape of A: (N,D,D) Shape of AA: (N,D,D,D)

No plates for X.

\_\_init\_\_(X, \*args)

#### **Methods**

init(X, *args)	
bound(R[, logdet, inv])	
<pre>get_bound_terms(R[, logdet, inv])</pre>	
nodes()	
rotate(R[, inv, logdet])	
setup()	This method should be called just before optimization.

bayespy.inference.vmp.transformations.RotateGaussianMarkovChain...init\_

RotateGaussianMarkovChain.\_\_init\_\_(X, \*args)

bayes py. in ference. vmp. transformations. Rotate Gaussian Markov Chain. bound

RotateGaussianMarkovChain.bound(R, logdet=None, inv=None)

bayespy.inference.vmp.transformations.RotateGaussianMarkovChain.get\_bound\_terms

RotateGaussianMarkovChain.get\_bound\_terms(R, logdet=None, inv=None)

bayespy.inference.vmp.transformations.RotateGaussianMarkovChain.nodes

RotateGaussianMarkovChain.nodes()

bayes py. inference. vmp. transformations. Rotate Gaussian Markov Chain. rotate

RotateGaussianMarkovChain.rotate(*R*, *inv=None*, *logdet=None*)

bayespy.inference.vmp.transformations.RotateGaussianMarkovChain.setup

RotateGaussianMarkovChain.setup()

This method should be called just before optimization.

## bayespy.inference.vmp.transformations.RotateSwitchingMarkovChain

 $B_{-}rotator)$ 

Rotation for bayespy.nodes. VaryingGaussianMarkovChain

Assume the following model.

Constant, unit isotropic innovation noise.

 $A_n = B_{z_n}$ 

Gaussian B: (..., K, D) x (D) Categorical Z: (..., N-1) x (K) GaussianMarkovChain X: (...) x (N,D)

No plates for X.

\_\_**init**\_\_(*X*, *B*, *Z*, *B\_rotator*)

#### **Methods**

202

\_\_init\_\_(X, B, Z, B\_rotator)
bound(R[, logdet, inv])
get\_bound\_terms(R[, logdet, inv])
nodes()

Continued on next page

Table 5.64 – continued from previous page

rotate(R[, inv, logdet])	
setup()	This method should be called just before optimization.

bayespy.inference.vmp.transformations.RotateSwitchingMarkovChain.\_\_init\_\_

RotateSwitchingMarkovChain.\_\_init\_\_(X, B, Z, B\_rotator)

bayespy.inference.vmp.transformations.RotateSwitchingMarkovChain.bound

RotateSwitchingMarkovChain.bound(R, logdet=None, inv=None)

bayespy.inference.vmp.transformations.RotateSwitchingMarkovChain.get\_bound\_terms

RotateSwitchingMarkovChain.get\_bound\_terms (R, logdet=None, inv=None)

bayespy.inference.vmp.transformations.RotateSwitchingMarkovChain.nodes

RotateSwitchingMarkovChain.nodes()

bayespy.inference.vmp.transformations.RotateSwitchingMarkovChain.rotate

RotateSwitchingMarkovChain.rotate(R, inv=None, logdet=None)

bayes py. inference. vmp. transformations. Rotate Switching Markov Chain. setup

RotateSwitchingMarkovChain.setup()

This method should be called just before optimization.

## bayespy.inference.vmp.transformations.RotateVaryingMarkovChain

class bayespy.inference.vmp.transformations.RotateVaryingMarkovChain(X, B, S, B\_rotator)

Rotation for bayespy.nodes.SwitchingGaussianMarkovChain

Assume the following model.

Constant, unit isotropic innovation noise.

 $A_n = \sum_k B_k s_{kn}$ 

Gaussian B: (1,D) x (D,K) Gaussian S: (N,1) x (K) MC X: () x (N+1,D)

No plates for X.

\_\_**init**\_\_ (*X*, *B*, *S*, *B\_rotator*)

**Methods** 

init(X, B, S, B_rotator)	
bound(R[, logdet, inv])	
<pre>get_bound_terms(R[, logdet, inv])</pre>	
nodes()	
rotate(R[, inv, logdet])	
setup()	This method should be called just before optimization.

bayespy.inference.vmp.transformations.RotateVaryingMarkovChain.\_\_init\_\_

RotateVaryingMarkovChain.\_\_init\_\_(X, B, S, B\_rotator)

bayes py. inference. vmp. transformations. Rotate Varying Markov Chain. bound

RotateVaryingMarkovChain.**bound**(*R*, *logdet=None*, *inv=None*)

bayespy.inference.vmp.transformations.RotateVaryingMarkovChain.get\_bound\_terms

RotateVaryingMarkovChain.get\_bound\_terms(R, logdet=None, inv=None)

bayespy.inference.vmp.transformations.RotateVaryingMarkovChain.nodes

RotateVaryingMarkovChain.nodes()

bayespy.inference.vmp.transformations.RotateVaryingMarkovChain.rotate

RotateVaryingMarkovChain.rotate(*R*, *inv=None*, *logdet=None*)

bayespy.inference.vmp.transformations.RotateVaryingMarkovChain.setup

RotateVaryingMarkovChain.setup()

This method should be called just before optimization.

## bayespy.inference.vmp.transformations.RotateMultiple

class bayespy.inference.vmp.transformations.RotateMultiple(\*rotators)
 Identical parameter expansion for several nodes simultaneously

Performs the same rotation for multiple nodes and combines the cost effect.

\_\_init\_\_(\*rotators)

**Methods** 

\_\_init\_\_(\*rotators)
bound(R[, logdet, inv])
Continued on next page

204 Chapter 5. User API

Table 5.66 – continued from previous page

<pre>get_bound_terms(R[, logdet, inv])</pre>	
nodes()	
rotate(R[, inv, logdet])	
setup()	

bayespy.inference.vmp.transformations.RotateMultiple.\_\_init\_\_

RotateMultiple.\_\_init\_\_(\*rotators)

bayespy.inference.vmp.transformations.RotateMultiple.bound

RotateMultiple.bound(R, logdet=None, inv=None)

bayespy.inference.vmp.transformations.RotateMultiple.get\_bound\_terms

RotateMultiple.get\_bound\_terms (R, logdet=None, inv=None)

bayespy.inference.vmp.transformations.RotateMultiple.nodes

RotateMultiple.nodes()

bayes py. in ference. vmp. transformations. Rotate Multiple. rotate

RotateMultiple.rotate(R, inv=None, logdet=None)

bayespy.inference.vmp.transformations.RotateMultiple.setup

RotateMultiple.setup()

# 5.3 bayespy.plot

Functions for plotting nodes.

## 5.3.1 Functions

pdf(Z, x, *args[, name, axes, fig])	Plot probability density function of a scalar variable.
$contour(\mathbf{Z}, \mathbf{x}, \mathbf{y}[, \mathbf{n}, \mathbf{axes}, \mathbf{fig}])$	Plot 2-D probability density function of a 2-D variable.
plot(Y[, axis, scale, center])	Plot a variable or an array as 1-D function with errorbars
hinton(X, **kwargs)	Plot the Hinton diagram of a node
gaussian_mixture_2d(X[, alpha, scale, fill,])	Plot Gaussian mixture as ellipses in 2-D

5.3. bayespy.plot 205

## bayespy.plot.pdf

```
bayespy.plot.pdf (Z, x, *args, name=None, axes=None, fig=None, **kwargs)
Plot probability density function of a scalar variable.

Parameters Z: node or function

Stochastic node or log pdf function

x: array

Grid points
```

## bayespy.plot.contour

```
bayespy.plot.contour (Z, x, y, n=None, axes=None, fig=None, **kwargs)
Plot 2-D probability density function of a 2-D variable.

Parameters Z: node or function

Stochastic node or log pdf function

x: array

Grid points on x axis

y: array

Grid points on y axis
```

## bayespy.plot.plot

```
bayespy.plot.plot (Y, axis=-1, scale=2, center=False, **kwargs)
Plot a variable or an array as 1-D function with errorbars
```

## bayespy.plot.hinton

```
bayespy.plot.hinton(X, **kwargs)
Plot the Hinton diagram of a node
```

The keyword arguments depend on the node type. For some node types, the diagram also shows uncertainty with non-filled rectangles. Currently, beta-like, Gaussian-like and Dirichlet-like nodes are supported.

Parameters X : node

#### bayespy.plot.gaussian\_mixture\_2d

```
bayespy.plot.gaussian_mixture_2d(X, alpha=None, scale=2, fill=False, axes=None, **kwargs)
Plot Gaussian mixture as ellipses in 2-D

Parameters X: Mixture node

alpha: Dirichlet-like node (optional)

Probabilities for the clusters

scale: float (optional)

Scale for the covariance ellipses (by default, 2)
```

## 5.3.2 Plotters

Plotter(plotter, *args, **kwargs)	Wrapper for plotting functions and base class for node plotters
PDFPlotter(x_grid, **kwargs)	Plotter of probability density function of a scalar node
ContourPlotter(x1_grid, x2_grid, **kwargs)	Plotter of probability density function of a two-dimensional node
HintonPlotter(**kwargs)	Plotter of the Hinton diagram of a node
FunctionPlotter(**kwargs)	Plotter of a node as a 1-dimensional function
GaussianTimeseriesPlotter(**kwargs)	Plotter of a Gaussian node as a timeseries
CategoricalMarkovChainPlotter(**kwargs)	Plotter of a Categorical timeseries

## bayespy.plot.Plotter

```
class bayespy.plot.Plotter (plotter, *args, **kwargs)
```

Wrapper for plotting functions and base class for node plotters

The purpose of this class is to collect all the parameters needed by a plotting function and provide a callable interface which needs only the node as the input.

Plotter instances are callable objects that plot a given node using a specified plotting function.

Parameters plotter: function

Plotting function to use

args: defined by the plotting function

Additional inputs needed by the plotting function

kwargs: defined by the plotting function

Additional keyword arguments supported by the plotting function

#### **Examples**

First, create a gamma variable:

```
>>> import numpy as np
>>> from bayespy.nodes import Gamma
>>> x = Gamma(4, 5)
```

The probability density function can be plotted as:

```
>>> import bayespy.plot as bpplt
>>> bpplt.pdf(x, np.linspace(0.1, 10, num=100))
[<matplotlib.lines.Line2D object at 0x...>]
```

However, this can be problematic when one needs to provide a plotting function for the inference engine as the inference engine gives only the node as input. Thus, we need to create a simple plotter wrapper:

```
>>> p = bpplt.Plotter(bpplt.pdf, np.linspace(0.1, 10, num=100))
```

Now, this callable object p needs only the node as the input:

```
>>> p(x)
[<matplotlib.lines.Line2D object at 0x...>]
```

Thus, it can be given to the inference engine to use as a plotting function:

5.3. bayespy.plot 207

```
>>> x = Gamma(4, 5, plotter=p)
     >>> x.plot()
     [<matplotlib.lines.Line2D object at 0x...>]
     __init__(plotter, *args, **kwargs)
     Methods
                               __init__(plotter, *args, **kwargs)
     bayespy.plot.Plotter.__init__
     Plotter.__init__(plotter, *args, **kwargs)
bayespy.plot.PDFPlotter
class bayespy.plot.PDFPlotter(x_grid, **kwargs)
     Plotter of probability density function of a scalar node
          Parameters x_grid: array
                  Numerical grid on which the density function is computed and plotted
     See also:
     pdf
     __init__ (x_grid, **kwargs)
     Methods
                               __init__(x_grid, **kwargs)
     bayespy.plot.PDFPlotter.__init__
     PDFPlotter.__init__(x_grid, **kwargs)
bayespy.plot.ContourPlotter
class bayespy.plot.ContourPlotter(x1_grid, x2_grid, **kwargs)
     Plotter of probability density function of a two-dimensional node
          Parameters x1_grid : array
                  Grid for the first dimension
              x2_grid: array
                  Grid for the second dimension
     See also:
     contour
```

```
__init__ (x1_grid, x2_grid, **kwargs)
     Methods
                             __init__(x1_grid, x2_grid, **kwargs)
     bayespy.plot.ContourPlotter.__init__
     ContourPlotter.__init__(x1_grid, x2_grid, **kwargs)
bayespy.plot.HintonPlotter
class bayespy.plot.HintonPlotter(**kwargs)
     Plotter of the Hinton diagram of a node
     See also:
     hinton
     __init__(**kwargs)
     Methods
                              __init__(**kwargs)
     bayespy.plot.HintonPlotter.__init__
     HintonPlotter.__init__(**kwargs)
bayespy.plot.FunctionPlotter
class bayespy.plot.FunctionPlotter(**kwargs)
     Plotter of a node as a 1-dimensional function
     See also:
     plot
     __init__(**kwargs)
     Methods
                              __init__(**kwargs)
     bayespy.plot.FunctionPlotter.__init__
     FunctionPlotter.__init__(**kwargs)
```

5.3. bayespy.plot 209

## bayespy.plot.GaussianTimeseriesPlotter

```
class bayespy.plot.GaussianTimeseriesPlotter(**kwargs)
    Plotter of a Gaussian node as a timeseries
__init__(**kwargs)

Methods
__init__(**kwargs)
```

## $bayes py.plot. Gaussian Time series Plotter.\_\_init\_\_$

GaussianTimeseriesPlotter.\_\_init\_\_(\*\*kwargs)

## bayespy.plot.CategoricalMarkovChainPlotter

```
class bayespy.plot.CategoricalMarkovChainPlotter(**kwargs)
    Plotter of a Categorical timeseries
    __init__(**kwargs)

Methods
```

\_init\_\_(\*\*kwargs)

bayespy.plot.CategoricalMarkovChainPlotter.\_\_init\_\_

CategoricalMarkovChainPlotter.\_\_init\_\_(\*\*kwargs)

**CHAPTER** 

SIX

## **DEVELOPER API**

This chapter contains API specifications which are relevant to BayesPy developers and contributors.

# 6.1 Developer nodes

The following base classes are useful if writing new nodes:

node.Node(*parents, **kwargs)	Base class for all nodes.
stochastic.Stochastic(*args[, initialize, dims])	Base class for nodes that are stochastic.
<pre>expfamily.ExponentialFamily(*args, **kwargs)</pre>	A base class for nodes using natural parameterization <i>phi</i> .
deterministic.Deterministic(*args, **kwargs)	Base class for deterministic nodes.

# 6.1.1 bayespy.inference.vmp.nodes.node.Node

```
class bayespy.inference.vmp.nodes.node.Node(*parents, **kwargs)
    Base class for all nodes.

mask dims plates parents children name
Sub-classes must implement: 1. For computing the message to children:
    get_moments(self):

2.For computing the message to parents: _get_message_and_mask_to_parent(self, index)
Sub-classes may need to re-implement: 1. If they manipulate plates:
    _compute_mask_to_parent(index, mask) _plates_to_parent(self, index) _plates_from_parent(self, index)
__init__(*parents, **kwargs)
```

### Methods

init(*parents, **kwargs)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_mask()	
	Continued on next page

Table 6.2 – continued from previous page

<pre>get_moments()</pre>	
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
<pre>move_plates(from_plate, to_plate)</pre>	
plot([fig])	Plot the node distribution using the plotter of the node
set_plotter(plotter)	

bayespy.inference.vmp.nodes.node.Node.\_\_init\_\_

Node.\_\_init\_\_(\*parents, \*\*kwargs)

bayespy.inference.vmp.nodes.node.Node.add\_plate\_axis

Node.add\_plate\_axis(to\_plate)

 $bayes py. in ference. vmp. nodes. node. Node. broadcasting\_multiplier$ 

static Node.broadcasting\_multiplier (plates, \*args)

bayespy.inference.vmp.nodes.node.Node.delete

Node.delete()

Delete this node and the children

bayespy.inference.vmp.nodes.node.Node.get\_mask

Node.get\_mask()

bayespy.inference.vmp.nodes.node.Node.get\_moments

Node.get\_moments()

bayespy.inference.vmp.nodes.node.Node.get\_shape

Node.get\_shape(ind)

bayespy.inference.vmp.nodes.node.Node.has\_plotter

Node.has\_plotter()

Return True if the node has a plotter

bayespy.inference.vmp.nodes.node.Node.move\_plates

Node.move\_plates (from\_plate, to\_plate)

#### bayespy.inference.vmp.nodes.node.Node.plot

```
Node.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

### bayespy.inference.vmp.nodes.node.Node.set\_plotter

```
Node.set_plotter(plotter)
```

#### **Attributes**

plates	
plates_multiplier	Plate multiplier is applied to messages to parents

### bayespy.inference.vmp.nodes.node.Node.plates

```
Node.plates = None
```

### bayespy.inference.vmp.nodes.node.Node.plates\_multiplier

```
Node.plates_multiplier
```

Plate multiplier is applied to messages to parents

# 6.1.2 bayespy.inference.vmp.nodes.stochastic.Stochastic

Base class for nodes that are stochastic.

u observed

**Sub-classes must implement:** \_compute\_message\_to\_parent(parent, index, u\_self, \*u\_parents) \_update\_distribution\_and\_lowerbound(self, m, \*u) lowerbound(self) \_compute\_dims initialize\_from\_prior()

If you want to be able to observe the variable: \_compute\_fixed\_moments\_and\_f

Sub-classes may need to re-implement: 1. If they manipulate plates:

```
_compute_mask_to_parent(index, mask) _compute_plates_to_parent(self, index, plates) _compute_plates_from_parent(self, index, plates)
```

```
__init__ (*args, initialize=True, dims=None, **kwargs)
```

```
__init__(*args[, initialize, dims])

Continued on next page
```

Table 6.4 – continued from previous page

	and a manufacture based
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_mask()	
<pre>get_moments()</pre>	
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
load(filename)	
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x[, mask])	Fix moments, compute f and propagate mask.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

### bayespy.inference.vmp.nodes.stochastic.Stochastic.\_\_init\_\_

Stochastic.\_\_init\_\_(\*args, initialize=True, dims=None, \*\*kwargs)

### bayespy.inference.vmp.nodes.stochastic.Stochastic.add\_plate\_axis

Stochastic.add\_plate\_axis(to\_plate)

### bayespy.inference.vmp.nodes.stochastic.Stochastic.broadcasting\_multiplier

Stochastic.broadcasting\_multiplier(plates, \*args)

#### bayespy.inference.vmp.nodes.stochastic.Stochastic.delete

Stochastic.**delete**()

Delete this node and the children

#### bayespy.inference.vmp.nodes.stochastic.Stochastic.get\_mask

Stochastic.get\_mask()

# $bayes py. inference. vmp. nodes. stochastic. Stochastic. get\_moments$

Stochastic.get\_moments()

## bayespy.inference.vmp.nodes.stochastic.Stochastic.get\_shape

Stochastic.get\_shape(ind)

#### bayespy.inference.vmp.nodes.stochastic.Stochastic.has\_plotter

```
Stochastic.has_plotter()
Return True if the node has a plotter
```

#### bayespy.inference.vmp.nodes.stochastic.Stochastic.load

```
Stochastic.load(filename)
```

### bayespy.inference.vmp.nodes.stochastic.Stochastic.lowerbound

```
Stochastic.lowerbound()
```

## bayespy.inference.vmp.nodes.stochastic.Stochastic.move\_plates

```
Stochastic.move_plates (from_plate, to_plate)
```

### bayespy.inference.vmp.nodes.stochastic.Stochastic.observe

```
Stochastic.observe(x, mask=True)
```

Fix moments, compute f and propagate mask.

#### bayespy.inference.vmp.nodes.stochastic.Stochastic.plot

```
Stochastic.plot(fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

### bayespy.inference.vmp.nodes.stochastic.Stochastic.random

```
Stochastic.random()
```

Draw a random sample from the distribution.

### bayespy.inference.vmp.nodes.stochastic.Stochastic.save

```
Stochastic.save (filename)
```

#### bayespy.inference.vmp.nodes.stochastic.Stochastic.set\_plotter

```
Stochastic.set_plotter(plotter)
```

#### bayespy.inference.vmp.nodes.stochastic.Stochastic.show

```
Stochastic.show()
```

Print the distribution using standard parameterization.

#### bayespy.inference.vmp.nodes.stochastic.Stochastic.unobserve

```
Stochastic.unobserve()
```

#### bayespy.inference.vmp.nodes.stochastic.Stochastic.update

```
Stochastic.update(annealing=1.0)
```

#### **Attributes**

plates	
plates_multiplier	Plate multiplier is applied to messages to parents

### bayespy.inference.vmp.nodes.stochastic.Stochastic.plates

```
Stochastic.plates = None
```

#### bayespy.inference.vmp.nodes.stochastic.Stochastic.plates\_multiplier

```
Stochastic.plates_multiplier
```

Plate multiplier is applied to messages to parents

# 6.1.3 bayespy.inference.vmp.nodes.expfamily.ExponentialFamily

```
class bayespy.inference.vmp.nodes.expfamily.ExponentialFamily(*args, **kwargs)
    A base class for nodes using natural parameterization phi.
```

phi

**Sub-classes must implement the following static methods:** \_compute\_message\_to\_parent(index, u\_self, \*u\_parents) \_compute\_phi\_from\_parents(\*u\_parents, mask) \_compute\_moments\_and\_cgf(phi, mask) \_compute\_fixed\_moments\_and\_f(x, mask=True)

Sub-classes may need to re-implement: 1. If they manipulate plates:

```
_compute_mask_to_parent(index, mask) _compute_plates_to_parent(self, index, plates) _compute_plates_from_parent(self, index, plates)
```

```
__init__(*args, **kwargs)
```

init(*args, **kwargs)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_gradient(rg)	Computes gradient with respect to the natural parameters.
get_mask()	
<pre>get_moments()</pre>	
	Continued on next page

Table 6.6 – continued from previous page

get_parameters()	Return parameters of the VB distribution.
<pre>get_riemannian_gradient()</pre>	Computes the Riemannian/natural gradient.
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
initialize_from_parameters(*args)	
initialize_from_prior()	
initialize_from_random()	Set the variable to a random sample from the current distribution.
initialize_from_value(x, *args)	
load(filename)	
logpdf(X[, mask])	Compute the log probability density function $Q(X)$ of this node.
<pre>lower_bound_contribution([gradient,])</pre>	Compute E[ $\log p(X parents) - \log q(X)$ ]
lowerbound()	
<pre>move_plates(from_plate, to_plate)</pre>	
observe(x, *args[, mask])	Fix moments, compute f and propagate mask.
pdf(X[, mask])	Compute the probability density function of this node.
plot([fig])	Plot the node distribution using the plotter of the node
random()	Draw a random sample from the distribution.
save(filename)	
$set_parameters(x)$	Set the parameters of the VB distribution.
set_plotter(plotter)	
show()	Print the distribution using standard parameterization.
unobserve()	
update([annealing])	

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.\_\_init\_\_

ExponentialFamily.\_\_init\_\_(\*args, \*\*kwargs)

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.add\_plate\_axis

ExponentialFamily.add\_plate\_axis (to\_plate)

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.broadcasting\_multiplier

ExponentialFamily.broadcasting\_multiplier(plates, \*args)

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.delete

ExponentialFamily.delete()

Delete this node and the children

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.get\_gradient

ExponentialFamily.get\_gradient(rg)

Computes gradient with respect to the natural parameters.

The function takes the Riemannian gradient as an input. This is for three reasons: 1) You probably want to use the Riemannian gradient anyway so this helps avoiding accidental use of this function. 2) The gradient is computed by using the Riemannian gradient and chain rules. 3) Probably you need both Riemannian and normal gradients anyway so you can provide it to this function to avoid re-computing it.

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.get\_mask

```
ExponentialFamily.get_mask()
```

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.get\_moments

```
ExponentialFamily.get_moments()
```

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.get\_parameters

```
ExponentialFamily.get_parameters()
```

Return parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.get\_riemannian\_gradient

```
ExponentialFamily.get_riemannian_gradient()
```

Computes the Riemannian/natural gradient.

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.get\_shape

```
ExponentialFamily.get_shape (ind)
```

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.has\_plotter

```
ExponentialFamily.has_plotter()
```

Return True if the node has a plotter

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.initialize\_from\_parameters

```
ExponentialFamily.initialize_from_parameters(*args)
```

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.initialize\_from\_prior

```
ExponentialFamily.initialize_from_prior()
```

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.initialize\_from\_random

```
ExponentialFamily.initialize_from_random()
```

Set the variable to a random sample from the current distribution.

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.initialize\_from\_value

```
ExponentialFamily.initialize_from_value(x, *args)
```

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.load

ExponentialFamily.load(filename)

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.logpdf

```
ExponentialFamily.logpdf(X, mask=True)
```

Compute the log probability density function Q(X) of this node.

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.lower\_bound\_contribution

```
ExponentialFamily.lower_bound_contribution (gradient=False, ignore\_masked=True) Compute E[ log p(X|parents) - log q(X) ]
```

If deterministic annealing is used, the term  $E[-\log q(X)]$  is divided by the anneling coefficient. That is, phi and cgf of q are multiplied by the temperature (inverse annealing coefficient).

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.lowerbound

ExponentialFamily.lowerbound()

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.move\_plates

ExponentialFamily.move\_plates (from\_plate, to\_plate)

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.observe

```
ExponentialFamily.observe(x, *args, mask=True) Fix moments, compute f and propagate mask.
```

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.pdf

```
ExponentialFamily.pdf (X, mask=True)
```

Compute the probability density function of this node.

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.plot

```
ExponentialFamily.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.random

```
ExponentialFamily.random()
```

Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.save

ExponentialFamily.save (filename)

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.set\_parameters

ExponentialFamily.set\_parameters(x)

Set the parameters of the VB distribution.

The parameters should be such that they can be used for optimization, that is, use log transformation for positive parameters.

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.set\_plotter

ExponentialFamily.set\_plotter(plotter)

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.show

ExponentialFamily.show()

Print the distribution using standard parameterization.

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.unobserve

ExponentialFamily.unobserve()

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.update

ExponentialFamily.update(annealing=1.0)

#### **Attributes**

dims	
plates	
plates_multiplier	Plate multiplier is applied to messages to parents

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.dims

ExponentialFamily.dims = None

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.plates

ExponentialFamily.plates = None

### bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.plates\_multiplier

ExponentialFamily.plates\_multiplier

Plate multiplier is applied to messages to parents

# 6.1.4 bayespy.inference.vmp.nodes.deterministic.Deterministic

```
class bayespy.inference.vmp.nodes.deterministic.Deterministic(*args, **kwargs)
     Base class for deterministic nodes.
     Sub-classes must implement: 1. For implementing the deterministic function:
           _compute_moments(self, *u)
         2.One of the following options: a) Simple methods:
               _compute_message_to_parent(self, index, m, *u) not? _compute_mask_to_parent(self, index,
               mask)
            (a)More control with: _compute_message_and_mask_to_parent(self, index, m, *u)
     Sub-classes may need to re-implement: 1. If they manipulate plates:
                                                      _compute_plates_to_parent(self,
           _compute_mask_to_parent(index,
                                             mask)
                                                                                       index,
                                                                                                 plates)
           _compute_plates_from_parent(self, index, plates)
     __init__ (*args, **kwargs)
```

#### **Methods**

init(*args, **kwargs)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_mask()	
<pre>get_moments()</pre>	
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
lower_bound_contribution([gradient])	
<pre>move_plates(from_plate, to_plate)</pre>	
plot([fig])	Plot the node distribution using the plotter of the node
set_plotter(plotter)	

#### bayespy.inference.vmp.nodes.deterministic.Deterministic.\_\_init\_\_

```
Deterministic.__init__(*args, **kwargs)
```

#### bayespy.inference.vmp.nodes.deterministic.Deterministic.add\_plate\_axis

Deterministic.add\_plate\_axis (to\_plate)

#### bayespy.inference.vmp.nodes.deterministic.Deterministic.broadcasting\_multiplier

 ${\tt Deterministic.broadcasting\_multiplier}\ (plates,\ *args)$ 

#### bayespy.inference.vmp.nodes.deterministic.Deterministic.delete

```
Deterministic.delete()

Delete this node and the children
```

#### bayespy.inference.vmp.nodes.deterministic.Deterministic.get\_mask

```
Deterministic.get_mask()
```

### bayespy.inference.vmp.nodes.deterministic.Deterministic.get\_moments

```
Deterministic.get_moments()
```

## bayespy.inference.vmp.nodes.deterministic.Deterministic.get\_shape

```
Deterministic.get_shape (ind)
```

### bayespy.inference.vmp.nodes.deterministic.Deterministic.has\_plotter

```
Deterministic.has_plotter()

Return True if the node has a plotter
```

#### bayespy.inference.vmp.nodes.deterministic.Deterministic.lower\_bound\_contribution

```
Deterministic.lower_bound_contribution(gradient=False, **kwargs)
```

### bayespy.inference.vmp.nodes.deterministic.Deterministic.move\_plates

```
Deterministic.move_plates (from_plate, to_plate)
```

#### bayespy.inference.vmp.nodes.deterministic.Deterministic.plot

```
Deterministic.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.inference.vmp.nodes.deterministic.Deterministic.set\_plotter

```
Deterministic.set_plotter(plotter)
```

### **Attributes**

plates	
plates_multiplier	Plate multiplier is applied to messages to parents

### bayespy.inference.vmp.nodes.deterministic.Deterministic.plates

Deterministic.plates = None

#### bayespy.inference.vmp.nodes.deterministic.Deterministic.plates\_multiplier

Deterministic.plates\_multiplier

Plate multiplier is applied to messages to parents

The following nodes are examples of special nodes that remain hidden for the user although they are often implicitly used:

constant.Constant(moments, x, **kwargs)	Node for presenting constant values.
gaussian.GaussianToGaussianGammaISO(X, **kwargs)	Converter for Gaussian moments to Gaussian-gamma isotro
$ ext{gaussian.GaussianGammaISOToGaussianGammaARD}( ext{X},)$	Converter for Gaussian-gamma ISO moments to Gaussian-g
gaussian.GaussianGammaARDToGaussianWishart()	
gaussian.WrapToGaussianGammaISO(*parents,)	
gaussian.WrapToGaussianGammaARD(mu_alpha,)	
gaussian.WrapToGaussianWishart(X, Lambda,)	Wraps Gaussian and Wishart nodes into a Gaussian-Wishar

# 6.1.5 bayespy.inference.vmp.nodes.constant.Constant

class bayespy.inference.vmp.nodes.constant.Constant(moments, x, \*\*kwargs)
 Node for presenting constant values.

The node wraps arrays into proper node type.

\_\_init\_\_ (moments, x, \*\*kwargs)

#### Methods

init(moments, x, **kwargs)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_mask()	
<pre>get_moments()</pre>	
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
lower_bound_contribution([gradient])	
<pre>move_plates(from_plate, to_plate)</pre>	
plot([fig])	Plot the node distribution using the plotter of the node
set_plotter(plotter)	
set_value(x)	

### bayespy.inference.vmp.nodes.constant.Constant.\_\_init\_\_

Constant.\_\_init\_\_(moments, x, \*\*kwargs)

#### bayespy.inference.vmp.nodes.constant.Constant.add\_plate\_axis

```
Constant.add_plate_axis(to_plate)
```

#### bayespy.inference.vmp.nodes.constant.Constant.broadcasting\_multiplier

```
Constant.broadcasting_multiplier(plates, *args)
```

#### bayespy.inference.vmp.nodes.constant.Constant.delete

```
Constant.delete()
```

Delete this node and the children

### bayespy.inference.vmp.nodes.constant.Constant.get\_mask

```
Constant.get_mask()
```

#### bayespy.inference.vmp.nodes.constant.Constant.get\_moments

```
Constant.get_moments()
```

#### bayespy.inference.vmp.nodes.constant.Constant.get\_shape

```
Constant.get_shape(ind)
```

### bayespy.inference.vmp.nodes.constant.Constant.has\_plotter

```
Constant.has_plotter()
```

Return True if the node has a plotter

### bayespy.inference.vmp.nodes.constant.Constant.lower\_bound\_contribution

```
Constant.lower_bound_contribution(gradient=False, **kwargs)
```

#### bayespy.inference.vmp.nodes.constant.Constant.move\_plates

```
Constant.move_plates (from_plate, to_plate)
```

#### bayespy.inference.vmp.nodes.constant.Constant.plot

```
Constant.plot (fig=None, **kwargs)
```

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

### bayespy.inference.vmp.nodes.constant.Constant.set\_plotter

Constant.set\_plotter(plotter)

### bayespy.inference.vmp.nodes.constant.Constant.set\_value

Constant.set\_value(x)

#### **Attributes**

plates	
plates_multiplier	Plate multiplier is applied to messages to parents

# bayespy.inference.vmp.nodes.constant.Constant.plates

Constant.plates = None

### bayespy.inference.vmp.nodes.constant.Constant.plates\_multiplier

Constant.plates\_multiplier

Plate multiplier is applied to messages to parents

# 6.1.6 bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO

 ${\bf class} \; {\tt bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammaISO} \; (X,$ 

\*\*kwargs)

Converter for Gaussian moments to Gaussian-gamma isotropic moments

Combines the Gaussian moments with gamma moments for a fixed value 1.

\_\_**init**\_\_ (*X*, \*\**kwargs*)

init(X, **kwargs)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_mask()	
<pre>get_moments()</pre>	
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
lower_bound_contribution([gradient])	
<pre>move_plates(from_plate, to_plate)</pre>	
plot([fig])	Plot the node distribution using the plotter of the node
set_plotter(plotter)	

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.\_\_init\_\_

GaussianToGaussianGammaISO.\_\_init\_\_(X, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.add\_plate\_axis

GaussianToGaussianGammaISO.add\_plate\_axis (to\_plate)

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.broadcasting\_multiplier

GaussianToGaussianGammaISO.broadcasting\_multiplier(plates, \*args)

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.delete

GaussianToGaussianGammaISO.delete()

Delete this node and the children

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.get\_mask

GaussianToGaussianGammaISO.get\_mask()

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.get\_moments

GaussianToGaussianGammaISO.get\_moments()

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.get\_shape

GaussianToGaussianGammaISO.get\_shape (ind)

 $bayes py. inference. vmp. nodes. gaussian. Gaussian To Gaussian Gammal SO. has\_plotter$ 

GaussianToGaussianGammaISO.has\_plotter()
Return True if the node has a plotter

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.lower\_bound\_contribution

GaussianToGaussianGammaISO.lower\_bound\_contribution(gradient=False, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.move\_plates

GaussianToGaussianGammaISO.move\_plates (from\_plate, to\_plate)

#### bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.plot

GaussianToGaussianGammaISO.plot (fig=None, \*\*kwargs)

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

### bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.set\_plotter

GaussianToGaussianGammaISO.set\_plotter(plotter)

#### **Attributes**

plates	
plates_multiplier	Plate multiplier is applied to messages to parents

### bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.plates

GaussianToGaussianGammaISO.plates = None

### bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGammalSO.plates\_multiplier

GaussianToGaussianGammaISO.plates\_multiplier
Plate multiplier is applied to messages to parents

# 6.1.7 bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD

 ${f class}$  bayespy.inference.vmp.nodes.gaussian. ${f GaussianGammaISOToGaussianGammaARD}$  (X,

\*\*kwargs)

Converter for Gaussian-gamma ISO moments to Gaussian-gamma ARD moments

```
__init__ (X, **kwargs)
```

init(X, **kwargs)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_mask()	
<pre>get_moments()</pre>	
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
lower_bound_contribution([gradient])	
<pre>move_plates(from_plate, to_plate)</pre>	
	Continued on next page

#### Table 6.15 – continued from previous page

plot([fig])	Plot the node distribution using the plotter of the node
set_plotter(plotter)	

bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.\_\_init\_\_

GaussianGammaISOToGaussianGammaARD.\_\_init\_\_(X, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.add\_plate\_axis

GaussianGammaISOToGaussianGammaARD.add\_plate\_axis(to\_plate)

bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.broadcasting\_multiplier

GaussianGammaISOToGaussianGammaARD.broadcasting\_multiplier (plates, \*args)

bayes py. inference. vmp. nodes. gaussian. Gaussian Gammal SOTo Gaussian Gamma ARD. delete

GaussianGammaISOToGaussianGammaARD.**delete**()

Delete this node and the children

bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.get\_mask

GaussianGammaISOToGaussianGammaARD.get\_mask()

 $bayes py. inference. vmp. nodes. gaussian. Gaussian Gammal SOTo Gaussian Gamma ARD. get\_moments$ 

GaussianGammaISOToGaussianGammaARD.get\_moments()

bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.get\_shape

GaussianGammaISOToGaussianGammaARD.get\_shape (ind)

bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.has\_plotter

GaussianGammaISOToGaussianGammaARD.has\_plotter()
Return True if the node has a plotter

 $bayes py. inference. vmp. nodes. gaussian. Gaussian Gammal SOTo Gaussian Gamma ARD. lower\_bound\_contribution to the contribution of the contribu$ 

GaussianGammaISOToGaussianGammaARD.lower\_bound\_contribution(gradient=False, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.move\_plates

GaussianGammaISOToGaussianGammaARD.move\_plates (from\_plate, to\_plate)

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.plot

 $\texttt{GaussianGammaISOToGaussianGammaARD.plot} \ (\textit{fig=None}, \ **kwargs)$ 

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.set\_plotter

GaussianGammaISOToGaussianGammaARD.set\_plotter(plotter)

#### **Attributes**

plates	
plates_multiplier	Plate multiplier is applied to messages to parents

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.plates

GaussianGammaISOToGaussianGammaARD.plates = None

### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOToGaussianGammaARD.plates\_multiplier

GaussianGammaISOToGaussianGammaARD.plates\_multiplier
Plate multiplier is applied to messages to parents

# 6.1.8 bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart

\_\_init\_\_ (X\_alpha, \*\*kwargs)

init(X_alpha, **kwargs)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_mask()	
<pre>get_moments()</pre>	
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
lower_bound_contribution([gradient])	
<pre>move_plates(from_plate, to_plate)</pre>	
plot([fig])	Plot the node distribution using the plotter of the node
set_plotter(plotter)	

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.\_\_init\_\_

GaussianGammaARDToGaussianWishart.\_\_init\_\_(X\_alpha, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.add\_plate\_axis

GaussianGammaARDToGaussianWishart.add\_plate\_axis(to\_plate)

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.broadcasting\_multiplier

GaussianGammaARDToGaussianWishart.broadcasting\_multiplier(plates, \*args)

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.delete

GaussianGammaARDToGaussianWishart.delete()

Delete this node and the children

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.get\_mask

GaussianGammaARDToGaussianWishart.get\_mask()

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.get\_moments

GaussianGammaARDToGaussianWishart.get\_moments()

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.get\_shape

GaussianGammaARDToGaussianWishart.get\_shape (ind)

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.has\_plotter

GaussianGammaARDToGaussianWishart.has\_plotter()
Return True if the node has a plotter

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.lower\_bound\_contribution

GaussianGammaARDToGaussianWishart.lower\_bound\_contribution(gradient=False, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.move\_plates

GaussianGammaARDToGaussianWishart.move\_plates(from\_plate, to\_plate)

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.plot

GaussianGammaARDToGaussianWishart.plot (fig=None, \*\*kwargs)

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.set\_plotter

GaussianGammaARDToGaussianWishart.set\_plotter(plotter)

#### **Attributes**

plates	
plates_multiplier	Plate multiplier is applied to messages to parents

### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.plates

GaussianGammaARDToGaussianWishart.plates = None

### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGaussianWishart.plates\_multiplier

GaussianGammaARDToGaussianWishart.plates\_multiplier
Plate multiplier is applied to messages to parents

# 6.1.9 bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO

\_\_init\_\_(\*parents, \*\*kwargs)

init(*parents, **kwargs)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_mask()	
<pre>get_moments()</pre>	
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
<pre>lower_bound_contribution([gradient])</pre>	
<pre>move_plates(from_plate, to_plate)</pre>	
plot([fig])	Plot the node distribution using the plotter of the node
set_plotter(plotter)	

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.\_\_init\_\_

WrapToGaussianGammaISO.\_\_init\_\_(\*parents, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.add\_plate\_axis

WrapToGaussianGammaISO.add\_plate\_axis (to\_plate)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.broadcasting\_multiplier

WrapToGaussianGammaISO.broadcasting\_multiplier(plates, \*args)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.delete

WrapToGaussianGammaISO.delete()

Delete this node and the children

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.get\_mask

WrapToGaussianGammaISO.get\_mask()

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.get\_moments

WrapToGaussianGammaISO.get\_moments()

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.get\_shape

WrapToGaussianGammaISO.get\_shape (ind)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.has\_plotter

WrapToGaussianGammaISO.has\_plotter()
Return True if the node has a plotter

 $bayes py. in ference. vmp. nodes. gaussian. Wrap To Gaussian Gammal SO. lower\_bound\_contribution$ 

WrapToGaussianGammaISO.lower\_bound\_contribution(gradient=False, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.move\_plates

WrapToGaussianGammaISO.move\_plates (from\_plate, to\_plate)

#### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.plot

WrapToGaussianGammaISO.plot (fig=None, \*\*kwargs)

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.set\_plotter

WrapToGaussianGammaISO.set\_plotter(plotter)

#### **Attributes**

plates	
plates_multiplier	Plate multiplier is applied to messages to parents

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.plates

WrapToGaussianGammaISO.plates = None

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammalSO.plates\_multiplier

WrapToGaussianGammaISO.plates multiplier
Plate multiplier is applied to messages to parents

6.1.10 bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD

\_\_init\_\_ (mu\_alpha, tau, \*\*kwargs)

init(mu_alpha, tau, **kwargs)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_mask()	
<pre>get_moments()</pre>	
get_shape(ind)	
has_plotter()	Return True if the node has a plotter
<pre>lower_bound_contribution([gradient])</pre>	
<pre>move_plates(from_plate, to_plate)</pre>	
plot([fig])	Plot the node distribution using the plotter of the node
set_plotter(plotter)	

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD...init\_

WrapToGaussianGammaARD.\_\_init\_\_(mu\_alpha, tau, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.add\_plate\_axis

WrapToGaussianGammaARD.add\_plate\_axis(to\_plate)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.broadcasting\_multiplier

WrapToGaussianGammaARD.broadcasting\_multiplier(plates, \*args)

bayes py. inference. vmp. nodes. gaussian. Wrap To Gaussian Gamma ARD. delete

WrapToGaussianGammaARD.delete()

Delete this node and the children

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.get\_mask

WrapToGaussianGammaARD.get\_mask()

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.get\_moments

WrapToGaussianGammaARD.get\_moments()

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.get\_shape

WrapToGaussianGammaARD.get\_shape (ind)

 $bayes py. inference. vmp. nodes. gaussian. Wrap To Gaussian Gamma ARD. has\_plotter$ 

WrapToGaussianGammaARD.has\_plotter()
Return True if the node has a plotter

 $bayes py. inference. vmp. nodes. gaussian. Wrap To Gaussian Gamma ARD. lower\_bound\_contribution$ 

WrapToGaussianGammaARD.lower\_bound\_contribution(gradient=False, \*\*kwargs)

bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.move\_plates

WrapToGaussianGammaARD.move\_plates (from\_plate, to\_plate)

#### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.plot

WrapToGaussianGammaARD.plot (fig=None, \*\*kwargs)

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.set\_plotter

WrapToGaussianGammaARD.set\_plotter(plotter)

#### **Attributes**

plates	
plates_multiplier	Plate multiplier is applied to messages to parents

#### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.plates

WrapToGaussianGammaARD.plates = None

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaARD.plates\_multiplier

WrapToGaussianGammaARD.plates\_multiplier

Plate multiplier is applied to messages to parents

# 6.1.11 bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart

 $\textbf{class} \ \texttt{bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart} \ (X, & \textit{Lambda}, \\ **kwargs)$ 

Wraps Gaussian and Wishart nodes into a Gaussian-Wishart node.

### The following node combinations can be wrapped:

- · Gaussian and Wishart
- · Gaussian-gamma and Wishart
- · Gaussian-Wishart and gamma

\_\_init\_\_(X, Lambda, \*\*kwargs)

init(X, Lambda, **kwargs)	
add_plate_axis(to_plate)	
broadcasting_multiplier(plates, *args)	
delete()	Delete this node and the children
get_mask()	
	Continued on next page

Table 6.23 – continued from previous page

Return True if the node has a plotter
Plot the node distribution using the plotter of the node

## bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.\_\_init\_\_

WrapToGaussianWishart.\_\_init\_\_(X, Lambda, \*\*kwargs)

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.add\_plate\_axis

WrapToGaussianWishart.add\_plate\_axis(to\_plate)

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.broadcasting\_multiplier

WrapToGaussianWishart.broadcasting\_multiplier(plates, \*args)

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.delete

WrapToGaussianWishart.delete()

Delete this node and the children

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.get\_mask

WrapToGaussianWishart.get\_mask()

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.get\_moments

WrapToGaussianWishart.get\_moments()

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.get\_shape

WrapToGaussianWishart.get\_shape(ind)

#### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.has\_plotter

WrapToGaussianWishart.has\_plotter()
Return True if the node has a plotter

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.lower\_bound\_contribution

WrapToGaussianWishart.lower\_bound\_contribution (gradient=False, \*\*kwargs)

#### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.move\_plates

WrapToGaussianWishart.move\_plates (from\_plate, to\_plate)

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.plot

WrapToGaussianWishart.plot(fig=None, \*\*kwargs)

Plot the node distribution using the plotter of the node

Because the distributions are in general very difficult to plot, the user must specify some functions which performs the plotting as wanted. See, for instance, bayespy.plot.plotting for available plotters, that is, functions that perform plotting for a node.

#### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.set\_plotter

WrapToGaussianWishart.set\_plotter(plotter)

#### **Attributes**

plates	
plates_multiplier	Plate multiplier is applied to messages to parents

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.plates

WrapToGaussianWishart.plates = None

### bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart.plates\_multiplier

WrapToGaussianWishart.plates\_multiplier
Plate multiplier is applied to messages to parents

## 6.2 Moments

node.Moments	Base class for defining the expectation of the su
gaussian.GaussianMoments(ndim)	Class for the moments of Gaussian variables.
gaussian_markov_chain.GaussianMarkovChainMoments	
gaussian.GaussianGammaISOMoments(ndim)	Class for the moments of Gaussian-gamma-ISC
gaussian.GaussianGammaARDMoments(ndim)	Class for the moments of Gaussian-gamma-AR
gaussian.GaussianWishartMoments	Class for the moments of Gaussian-Wishart var
gamma.GammaMoments	Class for the moments of gamma variables.
wishart.WishartMoments	
beta.BetaMoments	Class for the moments of beta variables.
dirichlet.DirichletMoments	Class for the moments of Dirichlet variables.
bernoulli.BernoulliMoments()	Class for the moments of Bernoulli variables.
binomial.BinomialMoments( $f N$ )	Class for the moments of binomial variables
categorical.CategoricalMoments(categories)	Class for the moments of categorical variables.
	Continue

6.2. Moments 237

### Table 6.25 – continued from previous page

categorical_markov_chain.CategoricalMarkovChainMoments()	Class for the moments of categorical Markov ch
multinomial.MultinomialMoments	Class for the moments of multinomial variables
poisson.PoissonMoments	Class for the moments of Poisson variables

# 6.2.1 bayespy.inference.vmp.nodes.node.Moments

class bayespy.inference.vmp.nodes.node.Moments

Base class for defining the expectation of the sufficient statistics.

The benefits:

- •Write statistic-specific features in one place only. For instance, covariance from Gaussian message.
- •Different nodes may have identically defined statistic so you need to implement related features only once. For instance, Gaussian and GaussianARD differ on the prior but the moments are the same.
- •General processing nodes which do not change the type of the moments may "inherit" the features from the parent node. For instance, slicing operator.
- •Conversions can be done easily in both of the above cases if the message conversion is defined in the moments class. For instance, GaussianMarkovChain to Gaussian and VaryingGaussianMarkovChain to Gaussian.

### \_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

add_converter(moments_to, converter)	
compute_dims_from_values(x)	
$compute\_fixed\_moments(x)$	
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

### bayespy.inference.vmp.nodes.node.Moments.add\_converter

classmethod Moments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.node.Moments.compute\_dims\_from\_values

Moments.compute\_dims\_from\_values(x)

#### bayespy.inference.vmp.nodes.node.Moments.compute\_fixed\_moments

 $Moments.compute\_fixed\_moments(x)$ 

## bayespy.inference.vmp.nodes.node.Moments.get\_converter

Moments.get\_converter (moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.2 bayespy.inference.vmp.nodes.gaussian.GaussianMoments

class bayespy.inference.vmp.nodes.gaussian.GaussianMoments (ndim)
 Class for the moments of Gaussian variables.

\_\_init\_\_(ndim)

#### **Methods**

init(ndim)	
<pre>add_converter(moments_to, converter)</pre>	
compute_dims_from_values(x)	Return the shape of the moments for a fixed value.
compute_fixed_moments(x)	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

### bayespy.inference.vmp.nodes.gaussian.GaussianMoments.\_\_init\_\_

GaussianMoments.\_\_init\_\_(ndim)

#### bayespy.inference.vmp.nodes.gaussian.GaussianMoments.add\_converter

GaussianMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.gaussian.GaussianMoments.compute\_dims\_from\_values

GaussianMoments.compute\_dims\_from\_values(x)

Return the shape of the moments for a fixed value.

#### bayespy.inference.vmp.nodes.gaussian.GaussianMoments.compute\_fixed\_moments

 ${\tt Gaussian Moments.compute\_fixed\_moments}\,(x)$ 

Compute the moments for a fixed value

#### bayespy.inference.vmp.nodes.gaussian.GaussianMoments.get\_converter

GaussianMoments.get\_converter(moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

6.2. Moments 239

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.3 bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainMoments

class bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainMoments

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### Methods

<pre>add_converter(moments_to, converter)</pre>	
compute_dims_from_values(x)	
compute_fixed_moments(x)	
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainMoments.add\_converter

GaussianMarkovChainMoments.add\_converter(moments.to, converter)

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainMoments.compute\_dims\_from\_v

GaussianMarkovChainMoments.compute\_dims\_from\_values(x)

 $bayes py. inference. vmp. nodes. gaussian\_markov\_chain. Gaussian Markov Chain Moments. compute\_fixed\_moments. co$ 

GaussianMarkovChainMoments.compute\_fixed\_moments(x)

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainMoments.get\_converter

GaussianMarkovChainMoments.get\_converter(moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

### 6.2.4 bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOMoments

class bayespy.inference.vmp.nodes.gaussian.GaussianGammaISOMoments(ndim)
 Class for the moments of Gaussian-gamma-ISO variables.

```
__init__(ndim)
```

Create moments object for Gaussian-gamma isotropic variables

ndim=0: scalar ndim=1: vector ndim=2: matrix ...

#### Methods

init(ndim)	Create moments object for Gaussian-gamma isotropic variables
<pre>add_converter(moments_to, converter)</pre>	
compute_dims_from_values(x, alpha)	Return the shape of the moments for a fixed value.
compute_fixed_moments(x, alpha)	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOMoments.\_init\_

```
GaussianGammaISOMoments.__init__(ndim)
```

Create moments object for Gaussian-gamma isotropic variables

ndim=0: scalar ndim=1: vector ndim=2: matrix ...

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOMoments.add\_converter

GaussianGammaISOMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOMoments.compute\_dims\_from\_values

GaussianGammaISOMoments.compute\_dims\_from\_values (x, alpha)

Return the shape of the moments for a fixed value.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOMoments.compute\_fixed\_moments

GaussianGammaISOMoments.compute\_fixed\_moments(x, alpha)

Compute the moments for a fixed value

x is a mean vector. alpha is a precision scale

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSOMoments.get\_converter

GaussianGammaISOMoments.get\_converter(moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

6.2. Moments 241

# 6.2.5 bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMoments

class bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMoments(ndim)
 Class for the moments of Gaussian-gamma-ARD variables.

\_\_init\_\_(ndim)

Create moments object for Gaussian-gamma isotropic variables

ndim=0: scalar ndim=1: vector ndim=2: matrix ...

#### **Methods**

init(ndim)	Create moments object for Gaussian-gamma isotropic variables
<pre>add_converter(moments_to, converter)</pre>	
compute_dims_from_values(x, alpha)	Return the shape of the moments for a fixed value.
compute_fixed_moments(x, alpha)	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMoments.\_init\_

GaussianGammaARDMoments.\_\_init\_\_(ndim)

Create moments object for Gaussian-gamma isotropic variables

ndim=0: scalar ndim=1: vector ndim=2: matrix ...

### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMoments.add\_converter

GaussianGammaARDMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMoments.compute\_dims\_from\_values

GaussianGammaARDMoments.compute\_dims\_from\_values(x, alpha)

Return the shape of the moments for a fixed value.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMoments.compute\_fixed\_moments

GaussianGammaARDMoments.compute\_fixed\_moments(x, alpha)

Compute the moments for a fixed value

x is a mean vector. alpha is a precision scale

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMoments.get\_converter

 ${\tt GaussianGammaARDMoments.get\_converter}~(\textit{moments\_to})$ 

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their

converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.6 bayespy.inference.vmp.nodes.gaussian.GaussianWishartMoments

class bayespy.inference.vmp.nodes.gaussian.GaussianWishartMoments
 Class for the moments of Gaussian-Wishart variables.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### Methods

add_converter(moments_to, converter)	
compute_dims_from_values(x, Lambda)	Return the shape of the moments for a fixed value.
compute_fixed_moments(x, Lambda)	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

### bayespy.inference.vmp.nodes.gaussian.GaussianWishartMoments.add\_converter

GaussianWishartMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.gaussian.GaussianWishartMoments.compute\_dims\_from\_values

GaussianWishartMoments.compute\_dims\_from\_values (x, Lambda)
Return the shape of the moments for a fixed value.

#### bayespy.inference.vmp.nodes.gaussian.GaussianWishartMoments.compute\_fixed\_moments

GaussianWishartMoments.compute\_fixed\_moments(x, Lambda)

Compute the moments for a fixed value

x is a vector. Lambda is a precision matrix

### bayespy.inference.vmp.nodes.gaussian.GaussianWishartMoments.get\_converter

GaussianWishartMoments.get\_converter (moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

6.2. Moments 243

# 6.2.7 bayespy.inference.vmp.nodes.gamma.GammaMoments

class bayespy.inference.vmp.nodes.gamma.GammaMoments
 Class for the moments of gamma variables.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

add_converter(moments_to, converter)	
compute_dims_from_values(x)	Return the shape of the moments for a fixed value.
compute_fixed_moments(x)	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

### bayespy.inference.vmp.nodes.gamma.GammaMoments.add\_converter

GammaMoments.add\_converter (moments\_to, converter)

### bayespy.inference.vmp.nodes.gamma.GammaMoments.compute\_dims\_from\_values

GammaMoments.compute\_dims\_from\_values (x)

Return the shape of the moments for a fixed value.

#### bayespy.inference.vmp.nodes.gamma.GammaMoments.compute\_fixed\_moments

GammaMoments.compute\_fixed\_moments(x)

Compute the moments for a fixed value

#### bayespy.inference.vmp.nodes.gamma.GammaMoments.get\_converter

GammaMoments.get\_converter (moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.8 bayespy.inference.vmp.nodes.wishart.WishartMoments

class bayespy.inference.vmp.nodes.wishart.WishartMoments

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

add_converter(moments_to, converter)	
compute_dims_from_values(x)	Compute the dimensions of phi and u.
compute_fixed_moments(Lambda)	Compute moments for fixed x.
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.wishart.WishartMoments.add\_converter

WishartMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.wishart.WishartMoments.compute\_dims\_from\_values

WishartMoments.compute\_dims\_from\_values(x)
Compute the dimensions of phi and u.

#### bayespy.inference.vmp.nodes.wishart.WishartMoments.compute\_fixed\_moments

WishartMoments.compute\_fixed\_moments (Lambda)
Compute moments for fixed x.

#### bayespy.inference.vmp.nodes.wishart.WishartMoments.get\_converter

WishartMoments.get\_converter (moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

### 6.2.9 bayespy.inference.vmp.nodes.beta.BetaMoments

class bayespy.inference.vmp.nodes.beta.BetaMoments
 Class for the moments of beta variables.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### Methods

<pre>add_converter(moments_to, converter)</pre>	
compute_dims_from_values(p)	Return the shape of the moments for a fixed value.
compute_fixed_moments(p)	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

6.2. Moments 245

#### bayespy.inference.vmp.nodes.beta.BetaMoments.add\_converter

BetaMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.beta.BetaMoments.compute\_dims\_from\_values

BetaMoments.compute\_dims\_from\_values(p)

Return the shape of the moments for a fixed value.

#### bayespy.inference.vmp.nodes.beta.BetaMoments.compute\_fixed\_moments

BetaMoments.compute\_fixed\_moments(p)

Compute the moments for a fixed value

#### bayespy.inference.vmp.nodes.beta.BetaMoments.get\_converter

BetaMoments.get\_converter (moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

### 6.2.10 bayespy.inference.vmp.nodes.dirichlet.DirichletMoments

class bayespy.inference.vmp.nodes.dirichlet.DirichletMoments
 Class for the moments of Dirichlet variables.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### Methods

<pre>add_converter(moments_to, converter)</pre>	
$compute\_dims\_from\_values(x)$	Return the shape of the moments for a fixed value.
compute_fixed_moments(p)	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.dirichlet.DirichletMoments.add\_converter

DirichletMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.dirichlet.DirichletMoments.compute\_dims\_from\_values

DirichletMoments.compute\_dims\_from\_values(x)

Return the shape of the moments for a fixed value.

#### bayespy.inference.vmp.nodes.dirichlet.DirichletMoments.compute\_fixed\_moments

DirichletMoments.compute\_fixed\_moments(p)

Compute the moments for a fixed value

### bayespy.inference.vmp.nodes.dirichlet.DirichletMoments.get\_converter

DirichletMoments.get\_converter(moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.11 bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments

class bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments
 Class for the moments of Bernoulli variables.

\_\_init\_\_()

#### Methods

init()	
<pre>add_converter(moments_to, converter)</pre>	
compute_dims_from_values(x)	Return the shape of the moments for a fixed value.
$compute\_fixed\_moments(x)$	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments.\_\_init\_\_

BernoulliMoments.\_\_init\_\_()

### bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments.add\_converter

BernoulliMoments.add\_converter (moments\_to, converter)

6.2. Moments 247

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments.compute\_dims\_from\_values

BernoulliMoments.compute\_dims\_from\_values(x)

Return the shape of the moments for a fixed value.

The realizations are scalars, thus the shape of the moment is ().

### bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments.compute\_fixed\_moments

 ${\tt BernoulliMoments.compute\_fixed\_moments}\,(x)$ 

Compute the moments for a fixed value

### bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments.get\_converter

BernoulliMoments.get\_converter(moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.12 bayespy.inference.vmp.nodes.binomial.BinomialMoments

 ${f class}$  bayespy.inference.vmp.nodes.binomial.BinomialMoments (N)

Class for the moments of binomial variables

\_\_init\_\_(N)

#### **Methods**

init( <b>N</b> )	
<pre>add_converter(moments_to, converter)</pre>	
compute_dims_from_values(x)	Return the shape of the moments for a fixed value.
compute_fixed_moments(x)	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.binomial.BinomialMoments.\_\_init\_\_

BinomialMoments.\_\_init\_(N)

#### bayespy.inference.vmp.nodes.binomial.BinomialMoments.add\_converter

BinomialMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.binomial.BinomialMoments.compute\_dims\_from\_values

BinomialMoments.compute\_dims\_from\_values(x)

Return the shape of the moments for a fixed value.

The realizations are scalars, thus the shape of the moment is ().

## bayespy.inference.vmp.nodes.binomial.BinomialMoments.compute\_fixed\_moments

BinomialMoments.compute\_fixed\_moments(x)

Compute the moments for a fixed value

## bayespy.inference.vmp.nodes.binomial.BinomialMoments.get\_converter

BinomialMoments.get\_converter(moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.13 bayespy.inference.vmp.nodes.categorical.CategoricalMoments

\_\_init\_\_(categories)

Create moments object for categorical variables

#### Methods

init(categories)	Create moments object for categorical variables	
<pre>add_converter(moments_to, converter)</pre>		
compute_dims_from_values(x)	Return the shape of the moments for a fixed value.	
compute_fixed_moments(x)	Compute the moments for a fixed value	
get_converter(moments_to)	Finds conversion to another moments type if possible.	

#### bayespy.inference.vmp.nodes.categorical.CategoricalMoments.\_\_init\_\_

Categorical Moments .\_\_init\_\_ (categories)

Create moments object for categorical variables

## bayespy.inference.vmp.nodes.categorical.CategoricalMoments.add\_converter

Categorical Moments.add\_converter (moments\_to, converter)

6.2. Moments 249

#### bayespy.inference.vmp.nodes.categorical.CategoricalMoments.compute\_dims\_from\_values

CategoricalMoments.compute\_dims\_from\_values(x)

Return the shape of the moments for a fixed value.

The observations are scalar.

## bayespy.inference.vmp.nodes.categorical.CategoricalMoments.compute\_fixed\_moments

CategoricalMoments.compute\_fixed\_moments(x)

Compute the moments for a fixed value

## bayespy.inference.vmp.nodes.categorical.CategoricalMoments.get\_converter

CategoricalMoments.get\_converter(moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.14 bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainMomer

class bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainMoments(categorical Class for the moments of categorical Markov chain variables.

**\_\_init**\_\_(categories)

Create moments object for categorical Markov chain variables.

#### **Methods**

init(categories)	Create moments object for categorical Markov chain variables.
<pre>add_converter(moments_to, converter)</pre>	
compute_dims_from_values(x)	Return the shape of the moments for a fixed value.
compute_fixed_moments(x)	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainMoments.\_\_init\_\_

CategoricalMarkovChainMoments.\_\_init\_\_(categories)

Create moments object for categorical Markov chain variables.

## bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainMoments.add\_converter

CategoricalMarkovChainMoments.add\_converter (moments\_to, converter)

## bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainMoments.compute\_dims\_fro

```
CategoricalMarkovChainMoments.compute_dims_from_values (x) Return the shape of the moments for a fixed value.
```

# bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainMoments.compute\_fixed\_mo

```
CategoricalMarkovChainMoments.compute_fixed_moments(x)
Compute the moments for a fixed value
```

## bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainMoments.get\_converter

```
CategoricalMarkovChainMoments.get_converter(moments_to)
```

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.15 bayespy.inference.vmp.nodes.multinomial.MultinomialMoments

class bayespy.inference.vmp.nodes.multinomial.MultinomialMoments
 Class for the moments of multinomial variables.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

add_converter(moments_to, converter)	
compute_dims_from_values(x)	Return the shape of the moments for a fixed value.
$compute\_fixed\_moments(x)$	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.multinomial.MultinomialMoments.add\_converter

MultinomialMoments.add\_converter (moments\_to, converter)

## bayespy.inference.vmp.nodes.multinomial.MultinomialMoments.compute\_dims\_from\_values

 $\texttt{MultinomialMoments.compute\_dims\_from\_values}\left(x\right)$ 

Return the shape of the moments for a fixed value.

6.2. Moments 251

#### bayespy.inference.vmp.nodes.multinomial.MultinomialMoments.compute\_fixed\_moments

MultinomialMoments.compute\_fixed\_moments(x)
Compute the moments for a fixed value

x must be a vector of counts.

## bayespy.inference.vmp.nodes.multinomial.MultinomialMoments.get\_converter

MultinomialMoments.get\_converter(moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.2.16 bayespy.inference.vmp.nodes.poisson.PoissonMoments

class bayespy.inference.vmp.nodes.poisson.PoissonMoments
 Class for the moments of Poisson variables

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

<pre>add_converter(moments_to, converter)</pre>	
compute_dims_from_values(x)	Return the shape of the moments for a fixed value.
$compute\_fixed\_moments(x)$	Compute the moments for a fixed value
<pre>get_converter(moments_to)</pre>	Finds conversion to another moments type if possible.

#### bayespy.inference.vmp.nodes.poisson.PoissonMoments.add\_converter

PoissonMoments.add\_converter (moments\_to, converter)

#### bayespy.inference.vmp.nodes.poisson.PoissonMoments.compute\_dims\_from\_values

PoissonMoments.compute\_dims\_from\_values(x)

Return the shape of the moments for a fixed value.

The realizations are scalars, thus the shape of the moment is ().

#### bayespy.inference.vmp.nodes.poisson.PoissonMoments.compute\_fixed\_moments

PoissonMoments.compute\_fixed\_moments(x)

Compute the moments for a fixed value

#### bayespy.inference.vmp.nodes.poisson.PoissonMoments.get\_converter

PoissonMoments.get\_converter (moments\_to)

Finds conversion to another moments type if possible.

Note that a conversion from moments A to moments B may require intermediate conversions. For instance: A->C->D->B. This method finds the path which uses the least amount of conversions and returns that path as a single conversion. If no conversion path is available, an error is raised.

The search algorithm starts from the original moments class and applies all possible converters to get a new list of moments classes. This list is extended by adding recursively all parent classes because their converters are applicable. Then, all possible converters are applied to this list to get a new list of current moments classes. This is iterated until the algorithm hits the target moments class or its subclass.

# 6.3 Distributions

stochastic.Distribution	A base class for the VMP form
expfamily.ExponentialFamilyDistribution	Sub-classes implement distrib
gaussian.GaussianDistribution	Class for the VMP formulas of
gaussian.GaussianARDDistribution(shape, ndim_mu)	
gaussian.GaussianGammaISODistribution	Class for the VMP formulas of
gaussian.GaussianGammaARDDistribution()	
gaussian.GaussianWishartDistribution	Class for the VMP formulas of
${\it gaussian\_markov\_chain.GaussianMarkovChainDistribution}(N,D)$	Sub-classes implement distrib
${\it gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribution} (N,D,K)$	Sub-classes implement distrib
${\it gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution} (N,D)$	Sub-classes implement distrib
gamma.GammaDistribution	Class for the VMP formulas of
wishart.WishartDistribution	Sub-classes implement distrib
beta.BetaDistribution	Class for the VMP formulas of
dirichlet.DirichletDistribution	Class for the VMP formulas of
bernoulli.BernoulliDistribution()	Class for the VMP formulas of
binomial.BinomialDistribution (N)	Class for the VMP formulas of
categorical.CategoricalDistribution(categories)	Class for the VMP formulas of
categorical_markov_chain.CategoricalMarkovChainDistribution()	Class for the VMP formulas of
multinomial.MultinomialDistribution(trials)	Class for the VMP formulas of
poisson.PoissonDistribution	Class for the VMP formulas of

# 6.3.1 bayespy.inference.vmp.nodes.stochastic.Distribution

```
class bayespy.inference.vmp.nodes.stochastic.Distribution
   A base class for the VMP formulas of variables.
```

Sub-classes implement distribution specific computations.

If a sub-class maps the plates differently, it needs to overload the following methods:

```
•compute_mask_to_parent
•plates_to_parent
•plates_from_parent
__init__()
Initialize self. See help(type(self)) for accurate signature.
```

#### **Methods**

<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index,)</pre>	Compute the message to a parent node.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
<pre>plates_to_parent(index, plates)</pre>	Resolves the plate mapping to a parent.
random(*params[, plates])	Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.stochastic.Distribution.compute\_mask\_to\_parent

Distribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

## bayespy.inference.vmp.nodes.stochastic.Distribution.compute\_message\_to\_parent

Distribution.compute\_message\_to\_parent (parent, index, u\_self, \*u\_parents)

Compute the message to a parent node.

## bayespy.inference.vmp.nodes.stochastic.Distribution.plates\_from\_parent

Distribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

## bayespy.inference.vmp.nodes.stochastic.Distribution.plates\_to\_parent

Distribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.stochastic.Distribution.random

Distribution.random(\*params, plates=None)
Draw a random sample from the distribution.

# 6.3.2 bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution

class bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution
 Sub-classes implement distribution specific computations.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

compute_cgf_from_parents(*u_parents)	
compute_fixed_moments_and_f(x[, mask])	
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
compute_mask_to_parent(index, mask)	Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index,)	
<pre>compute_moments_and_cgf(phi[, mask])</pre>	
<pre>compute_phi_from_parents(*u_parents[, mask])</pre>	
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*params[, plates])	Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_cgf\_from\_parents

ExponentialFamilyDistribution.compute\_cgf\_from\_parents(\*u\_parents)

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_fixed\_moments\_and\_f

ExponentialFamilyDistribution.compute\_fixed\_moments\_and\_f(x, mask=True)

## bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_gradient

ExponentialFamilyDistribution.compute\_gradient (g, u, phi)Compute the standard gradient with respect to the natural parameters.

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_logpdf

ExponentialFamilyDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute  $E[\log p(X)]$  given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_mask\_to\_parent

ExponentialFamilyDistribution.compute\_mask\_to\_parent(index, mask) Maps the mask to the plates of a parent.

## bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_message\_to\_parent

ExponentialFamilyDistribution.compute\_message\_to\_parent (parent, index, u\_self, \*u\_parents)

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_moments\_and\_cgf

ExponentialFamilyDistribution.compute\_moments\_and\_cgf (phi, mask=True)

## bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.compute\_phi\_from\_parents

ExponentialFamilyDistribution.compute\_phi\_from\_parents (\*u\_parents, mask=True)

#### bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.plates\_from\_parent

ExponentialFamilyDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

## bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.plates\_to\_parent

ExponentialFamilyDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

## bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistribution.random

ExponentialFamilyDistribution.random(\*params, plates=None)

Draw a random sample from the distribution.

# 6.3.3 bayespy.inference.vmp.nodes.gaussian.GaussianDistribution

class bayespy.inference.vmp.nodes.gaussian.GaussianDistribution
 Class for the VMP formulas of Gaussian variables.

Currently, supports only vector variables.

#### **Notes**

Message passing equations:

$$\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Lambda}),$$

 $\mathbf{x}, \boldsymbol{\mu} \in \mathbb{R}^D$ ,  $\boldsymbol{\Lambda} \in \mathbb{R}^{D \times D}$ ,  $\boldsymbol{\Lambda}$  symmetric positive definite

$$\log \mathcal{N}(\mathbf{x}|\boldsymbol{\mu},\boldsymbol{\Lambda}) = -\frac{1}{2}\mathbf{x}^{\mathrm{T}}\boldsymbol{\Lambda}\mathbf{x} + \mathbf{x}^{\mathrm{T}}\boldsymbol{\Lambda}\boldsymbol{\mu} - \frac{1}{2}\boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{\Lambda}\boldsymbol{\mu} + \frac{1}{2}\log|\boldsymbol{\Lambda}| - \frac{D}{2}\log(2\pi)$$

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### Methods

compute_cgf_from_parents(u_mu_Lambda)	Compute $\mathrm{E}_{q(p)}[g(p)]$
$compute\_fixed\_moments\_and\_f(x[, mask])$	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
compute_logpdf(u, phi, g, f, ndims)	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
	Continued on next page

Table 6.45 – continued from previous page

compute_mask_to_parent(index, mask)	Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index, u,)	Compute the message to a parent node.
<pre>compute_moments_and_cgf(phi[, mask])</pre>	Compute the moments and $g(\phi)$ .
compute_phi_from_parents(u_mu_Lambda[, mask])	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
<pre>plates_to_parent(index, plates)</pre>	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.compute\_cgf\_from\_parents

 $\label{lem:compute_cgf_from_parents} \textbf{GaussianDistribution.compute\_cgf\_from\_parents} \ (\textit{u\_mu\_Lambda}) \\ \textbf{Compute} \ \mathbf{E}_{q(p)}[g(p)]$ 

$$g(\boldsymbol{\mu}, \boldsymbol{\Lambda}) = -\frac{1}{2}\operatorname{tr}(\boldsymbol{\mu}\boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{\Lambda}) + \frac{1}{2}\log|\boldsymbol{\Lambda}|$$

## bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.compute\_fixed\_moments\_and\_f

GaussianDistribution.compute\_fixed\_moments\_and\_f (x, mask=True)Compute the moments and f(x) for a fixed value.

$$\begin{aligned} \mathbf{u}(\mathbf{x}) &= \begin{bmatrix} \mathbf{x} \\ \mathbf{x} \mathbf{x}^T \end{bmatrix} \\ f(\mathbf{x}) &= -\frac{D}{2} \log(2\pi) \end{aligned}$$

#### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.compute\_gradient

GaussianDistribution.compute\_gradient (g, u, phi)

Compute the standard gradient with respect to the natural parameters.

Gradient of the moments:

$$\begin{split} \mathrm{d}\overline{\mathbf{u}} &= \begin{bmatrix} \frac{1}{2}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1}\phi_1 - \frac{1}{2}\phi_2^{-1}\mathrm{d}\phi_1 \\ -\frac{1}{4}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1}\phi_1\phi_1^\mathrm{T}\phi_2^{-1} - \frac{1}{4}\phi_2^{-1}\phi_1\phi_1^\mathrm{T}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1} + \frac{1}{2}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1} + \frac{1}{4}\phi_2^{-1}\mathrm{d}\phi_1\phi_1^\mathrm{T}\phi_2^{-1} + \frac{1}{4}\phi_2^{-1}\phi_1\mathrm{d}\phi_1^\mathrm{T}\phi_2^{-1} \end{bmatrix} \\ &= \begin{bmatrix} 2(\overline{u}_2 - \overline{u}_1\overline{u}_1^\mathrm{T})\mathrm{d}\phi_2\overline{u}_1 + (\overline{u}_2 - \overline{u}_1\overline{u}_1^\mathrm{T})\mathrm{d}\phi_1 \\ u_2d\phi_2u_2 - 2u_1u_1^\mathrm{T}d\phi_2u_1u_1^\mathrm{T} + 2(u_2 - u_1u_1^\mathrm{T})d\phi_1u_1^\mathrm{T} \end{bmatrix} \end{split}$$

Standard gradient given the gradient with respect to the moments, that is, given the Riemannian gradient  $\tilde{\nabla}$ :

$$\nabla = \begin{bmatrix} (\overline{u}_2 - \overline{u}_1 \overline{u}_1^T) \tilde{\nabla}_1 + 2(u_2 - u_1 u_1^T) \tilde{\nabla}_2 u_1 \\ (u_2 - u_1 u_1^T) \tilde{\nabla}_1 u_1^T + u_1 \tilde{\nabla}_1^T (u_2 - u_1 u_1^T) + 2u_2 \tilde{\nabla}_2 u_2 - 2u_1 u_1^T \tilde{\nabla}_2 u_1 u_1^T \end{bmatrix}$$

# bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.compute\_logpdf

GaussianDistribution.compute\_logpdf (u, phi, g, f, ndims)

Compute  $E[\log p(X)]$  given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

#### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.compute\_mask\_to\_parent

GaussianDistribution.compute\_mask\_to\_parent(index, mask)

Maps the mask to the plates of a parent.

### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.compute\_message\_to\_parent

GaussianDistribution.compute\_message\_to\_parent (parent, index, u, u\_mu\_Lambda) Compute the message to a parent node.

$$\begin{split} \phi_{\boldsymbol{\mu}}(\mathbf{x}, \boldsymbol{\Lambda}) &= \begin{bmatrix} \boldsymbol{\Lambda} \mathbf{x} \\ -\frac{1}{2} \boldsymbol{\Lambda} \end{bmatrix} \\ \phi_{\boldsymbol{\Lambda}}(\mathbf{x}, \boldsymbol{\mu}) &= \begin{bmatrix} -\frac{1}{2} \mathbf{x} \mathbf{x}^{\mathrm{T}} + \frac{1}{2} \mathbf{x} \boldsymbol{\mu}^{\mathrm{T}} + \frac{1}{2} \boldsymbol{\mu} \mathbf{x}^{\mathrm{T}} - \frac{1}{2} \boldsymbol{\mu} \boldsymbol{\mu}^{\mathrm{T}} \\ \frac{1}{2} \end{bmatrix} \end{split}$$

#### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.compute\_moments\_and\_cgf

GaussianDistribution.compute\_moments\_and\_cgf (phi, mask=True)

Compute the moments and  $g(\phi)$ .

$$\begin{aligned} \overline{\mathbf{u}}(\phi) &= \begin{bmatrix} -\frac{1}{2}\phi_2^{-1}\phi_1 \\ \frac{1}{4}\phi_2^{-1}\phi_1\phi_1^{\mathrm{T}}\phi_2^{-1} - \frac{1}{2}\phi_2^{-1} \end{bmatrix} \\ g_{\phi}(\phi) &= \frac{1}{4}\phi_1^{\mathrm{T}}\phi_2^{-1}\phi_1 + \frac{1}{2}\log|-2\phi_2| \end{aligned}$$

#### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.compute\_phi\_from\_parents

GaussianDistribution.compute\_phi\_from\_parents (u\_mu\_Lambda, mask=True)

Compute the natural parameter vector given parent moments.

$$\phi(oldsymbol{\mu}, oldsymbol{\Lambda}) = egin{bmatrix} oldsymbol{\Lambda} oldsymbol{\mu} \ -rac{1}{2}oldsymbol{\Lambda} \end{bmatrix}$$

#### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.plates\_from\_parent

GaussianDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

#### bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.plates\_to\_parent

GaussianDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

## bayespy.inference.vmp.nodes.gaussian.GaussianDistribution.random

GaussianDistribution.random(\*phi, plates=None)

Draw a random sample from the distribution.

# 6.3.4 bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution

Log probability density function:

$$\log p(x|\mu,\alpha) = -\frac{1}{2}x^T \operatorname{diag}(\alpha)x + x^T \operatorname{diag}(\alpha)\mu - \frac{1}{2}\mu^T \operatorname{diag}(\alpha)\mu + \frac{1}{2}\sum_i \log \alpha_i - \frac{D}{2}\log(2\pi)$$

Parent has moments:

$$\begin{bmatrix} \alpha \circ \mu \\ \alpha \circ \mu \circ \mu \\ \alpha \\ \log(\alpha) \end{bmatrix}$$

\_\_init\_\_ (shape, ndim\_mu)

#### **Methods**

init(shape, ndim_mu)	
compute_cgf_from_parents(u_mu_alpha)	Compute the value of the cumulant generating function.
<pre>compute_fixed_moments_and_f(x[, mask])</pre>	Compute $u(x)$ and $f(x)$ for given $x$ .
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute E[log p(X)] given E[u], E[phi], E[g] and E[f].
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index, u,)</pre>	
<pre>compute_moments_and_cgf(phi[, mask])</pre>	
<pre>compute_phi_from_parents(u_mu_alpha[, mask])</pre>	
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the Gaussian distribution.

#### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.\_\_init\_\_

 ${\tt Gaussian ARDDistribution.\_init\_(\it shape, ndim\_mu)}$ 

## bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_cgf\_from\_parents

GaussianARDDistribution.compute\_cgf\_from\_parents(u\_mu\_alpha)
Compute the value of the cumulant generating function.

## bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_fixed\_moments\_and\_f

GaussianARDDistribution.compute\_fixed\_moments\_and\_f (x, mask=True)Compute u(x) and f(x) for given x.

#### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_gradient

GaussianARDDistribution.compute\_gradient(g, u, phi)

Compute the standard gradient with respect to the natural parameters.

Gradient of the moments:

$$\begin{split} \mathrm{d}\overline{\mathbf{u}} &= \begin{bmatrix} \frac{1}{2}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1}\phi_1 - \frac{1}{2}\phi_2^{-1}\mathrm{d}\phi_1 \\ -\frac{1}{4}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1}\phi_1\phi_1^T\phi_2^{-1} - \frac{1}{4}\phi_2^{-1}\phi_1\phi_1^T\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1} + \frac{1}{2}\phi_2^{-1}\mathrm{d}\phi_2\phi_2^{-1} + \frac{1}{4}\phi_2^{-1}\mathrm{d}\phi_1\phi_1^T\phi_2^{-1} + \frac{1}{4}\phi_2^{-1}\phi_1\mathrm{d}\phi_1^T\phi_2^{-1} \end{bmatrix} \\ &= \begin{bmatrix} 2(\overline{u}_2 - \overline{u}_1\overline{u}_1^T)\mathrm{d}\phi_2\overline{u}_1 + (\overline{u}_2 - \overline{u}_1\overline{u}_1^T)\mathrm{d}\phi_1 \\ u_2d\phi_2u_2 - 2u_1u_1^Td\phi_2u_1u_1^T + 2(u_2 - u_1u_1^T)d\phi_1u_1^T \end{bmatrix} \end{split}$$

Standard gradient given the gradient with respect to the moments, that is, given the Riemannian gradient  $\tilde{\nabla}$ :

$$\nabla = \begin{bmatrix} (\overline{u}_2 - \overline{u}_1 \overline{u}_1^T) \tilde{\nabla}_1 + 2(u_2 - u_1 u_1^T) \tilde{\nabla}_2 u_1 \\ (u_2 - u_1 u_1^T) \tilde{\nabla}_1 u_1^T + u_1 \tilde{\nabla}_1^T (u_2 - u_1 u_1^T) + 2u_2 \tilde{\nabla}_2 u_2 - 2u_1 u_1^T \tilde{\nabla}_2 u_1 u_1^T \end{bmatrix}$$

## bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_logpdf

GaussianARDDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

## bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_mask\_to\_parent

GaussianARDDistribution.compute\_mask\_to\_parent(index, mask)

Maps the mask to the plates of a parent.

## $bayes py. inference. vmp. nodes. gaussian. Gaussian ARD Distribution. compute\_message\_to\_parent$

GaussianARDDistribution.compute\_message\_to\_parent(parent, index, u, u\_mu\_alpha)

$$m = \begin{bmatrix} x \\ [-\frac{1}{2}, \dots, -\frac{1}{2}] \\ -\frac{1}{2} \operatorname{diag}(xx^T) \\ [\frac{1}{2}, \dots, \frac{1}{2}] \end{bmatrix}$$

#### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_moments\_and\_cgf

GaussianARDDistribution.compute\_moments\_and\_cgf (phi, mask=True)

#### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.compute\_phi\_from\_parents

GaussianARDDistribution.compute\_phi\_from\_parents(u\_mu\_alpha, mask=True)

## bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.plates\_from\_parent

GaussianARDDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

### bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.plates\_to\_parent

GaussianARDDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

## bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution.random

GaussianARDDistribution.random(\*phi, plates=None)

Draw a random sample from the Gaussian distribution.

# 6.3.5 bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution

class bayespy.inference.vmp.nodes.gaussian.GaussianGammaISODistribution
 Class for the VMP formulas of Gaussian-Gamma-ISO variables.

Currently, supports only vector variables.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### Methods

compute_cgf_from_parents(u_mu_Lambda, u_a, u_b)	Compute $E_{q(p)}[g(p)]$
<pre>compute_fixed_moments_and_f(x, alpha[, mask])</pre>	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute E[log p(X)] given E[u], E[phi], E[g] and E[f].
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index, u,)	Compute the message to a parent node.
compute_moments_and_cgf(phi[, mask])	Compute the moments and $g(\phi)$ .
compute_phi_from_parents(u_mu_Lambda, u_a, u_b)	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
<pre>plates_to_parent(index, plates)</pre>	Resolves the plate mapping to a parent.
random(*params[, plates])	Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_cgf\_from\_parents

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_fixed\_moments\_and\_f

GaussianGammaISODistribution.compute\_fixed\_moments\_and\_f (x, alpha, mask=True)Compute the moments and f(x) for a fixed value.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_gradient

GaussianGammaISODistribution.compute\_gradient (g, u, phi)Compute the standard gradient with respect to the natural parameters.

## bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_logpdf

GaussianGammaISODistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_mask\_to\_parent

GaussianGammaISODistribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

## bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_message\_to\_parent

```
GaussianGammaISODistribution.compute_message_to_parent (parent, index, u, u_mu_Lambda, u_a, u_b)
```

Compute the message to a parent node.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_moments\_and\_cgf

GaussianGammaISODistribution.compute\_moments\_and\_cgf (phi, mask=True)

Compute the moments and  $g(\phi)$ .

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.compute\_phi\_from\_parents

```
GaussianGammaISODistribution.compute_phi_from_parents(u_mu_Lambda, u_a, u_b, mask=True)

Compute the natural parameter vector given parent moments.
```

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.plates\_from\_parent

```
GaussianGammaISODistribution.plates_from_parent (index, plates)
Resolve the plate mapping from a parent.
```

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

## bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.plates\_to\_parent

GaussianGammaISODistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

## bayespy.inference.vmp.nodes.gaussian.GaussianGammalSODistribution.random

GaussianGammaISODistribution.random(\*params, plates=None)
Draw a random sample from the distribution.

## 6.3.6 bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution

class bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution

\_\_init\_\_()

#### Methods

Compute the standard gradient with respect to the natural parameters.
Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
Maps the mask to the plates of a parent.
Resolve the plate mapping from a parent.
Resolves the plate mapping to a parent.
Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.\_\_init\_\_

GaussianGammaARDDistribution.\_\_init\_\_()

# $bayes py. inference. vmp. nodes. gaussian. Gaussian Gamma ARD Distribution. compute \verb|\_cgf| from \verb|\_parents| and the properties of the p$

 ${\tt GaussianGammaARDDistribution.} \textbf{compute\_cgf\_from\_parents} \ (*\textit{u\_parents})$ 

## bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.compute\_fixed\_moments\_and\_f

GaussianGammaARDDistribution.compute\_fixed\_moments\_and\_f(x, mask=True)

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.compute\_gradient

GaussianGammaARDDistribution.compute\_gradient (g, u, phi)Compute the standard gradient with respect to the natural parameters.

### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.compute\_logpdf

GaussianGammaARDDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

## bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.compute\_mask\_to\_parent

GaussianGammaARDDistribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.compute\_message\_to\_parent

GaussianGammaARDDistribution.compute\_message\_to\_parent (parent, index, u\_self, \*u\_parents)

## bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.compute\_moments\_and\_cgf

GaussianGammaARDDistribution.compute\_moments\_and\_cgf (phi, mask=True)

## bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.compute\_phi\_from\_parents

GaussianGammaARDDistribution.compute\_phi\_from\_parents(\*u\_parents, mask=True)

## bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.plates\_from\_parent

GaussianGammaARDDistribution.plates\_from\_parent (index, plates)
Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

## bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.plates\_to\_parent

GaussianGammaARDDistribution.plates\_to\_parent (index, plates)
Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistribution.random

GaussianGammaARDDistribution.random(\*params, plates=None)
Draw a random sample from the distribution.

# 6.3.7 bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution

class bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution
 Class for the VMP formulas of Gaussian-Wishart variables.

Currently, supports only vector variables.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

compute_cgf_from_parents(u_mu_alpha, u_V, u_n)	Compute $E_{q(p)}[g(p)]$
<pre>compute_fixed_moments_and_f(x, Lambda[, mask])</pre>	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index, u,)	Compute the message to a parent node.
<pre>compute_moments_and_cgf(phi[, mask])</pre>	Compute the moments and $g(\phi)$ .
compute_phi_from_parents(u_mu_alpha, u_V, u_n)	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
<pre>plates_to_parent(index, plates)</pre>	Resolves the plate mapping to a parent.
random(*params[, plates])	Draw a random sample from the distribution.

## bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_cgf\_from\_parents

#### bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_fixed\_moments\_and\_f

```
GaussianWishartDistribution.compute_fixed_moments_and_f(x, Lambda, mask=True)

Compute the moments and f(x) for a fixed value.
```

## bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_gradient

```
GaussianWishartDistribution.compute_gradient (g, u, phi)
Compute the standard gradient with respect to the natural parameters.
```

## bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_logpdf

```
GaussianWishartDistribution.compute_logpdf (u, phi, g, f, ndims)
Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.
```

## bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_mask\_to\_parent

```
GaussianWishartDistribution.compute_mask_to_parent(index, mask)

Maps the mask to the plates of a parent.
```

## bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_message\_to\_parent

GaussianWishartDistribution.compute\_message\_to\_parent (parent, index, u, u\_mu\_alpha, u\_V, u\_n)

Compute the message to a parent node.

## bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_moments\_and\_cgf

GaussianWishartDistribution.compute\_moments\_and\_cgf (phi, mask=True)

Compute the moments and  $g(\phi)$ .

#### bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.compute\_phi\_from\_parents

GaussianWishartDistribution.compute\_phi\_from\_parents( $u\_mu\_alpha$ ,  $u\_V$ ,  $u\_n\_mask=True$ )

Compute the natural parameter vector given parent moments.

## bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.plates\_from\_parent

GaussianWishartDistribution.plates\_from\_parent (index, plates)
Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

## bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.plates\_to\_parent

GaussianWishartDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.random

GaussianWishartDistribution.random(\*params, plates=None)
Draw a random sample from the distribution.

# 6.3.8 bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution

 ${\bf class} \ {\tt bayespy.inference.vmp.nodes.gaussian\_markov\_chain.{\bf GaussianMarkovChainDistribution}\ (N, \\ D)$ 

Sub-classes implement distribution specific computations.

 $\_$ init $\_(N, D)$ 

Continued on next page

## Table 6.50 - continued from previous page

#### Methods

init(N, D)	
compute_cgf_from_parents(u_mu, u_Lambda,)	Compute CGF using the moments of the parents.
<pre>compute_fixed_moments_and_f(x[, mask])</pre>	Compute $u(x)$ and $f(x)$ for given $x$ .
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute E[log p(X)] given E[u], E[phi], E[g] and E[f].
<pre>compute_mask_to_parent(index, mask)</pre>	
compute_message_to_parent(parent, index, u,)	Compute a message to a parent.
<pre>compute_moments_and_cgf(phi[, mask])</pre>	Compute the moments and the cumulant-generating function.
compute_phi_from_parents(u_mu, u_Lambda,)	Compute the natural parameters using parents' moments.
<pre>plates_from_parent(index, plates)</pre>	Compute the plates using information of a parent node.
plates_to_parent(index, plates)	Computes the plates of this node with respect to a parent.
random(*params[, plates])	Draw a random sample from the distribution.

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.\_\_init\_\_

GaussianMarkovChainDistribution.\_\_init\_(N, D)

 $bayespy. inference. vmp. nodes. gaussian\_markov\_chain. Gaussian MarkovChain Distribution. compute\_cgf\_from\_lambda from the cgf\_from\_lambda from\_lambda from\_la$ 

GaussianMarkovChainDistribution.compute\_cgf\_from\_parents( $u\_mu$ ,  $u\_Lambda$ ,  $u\_A$ ,  $u\_v$ , \* $u\_inputs$ )

Compute CGF using the moments of the parents.

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.compute\_fixed\_mon

GaussianMarkovChainDistribution.compute\_fixed\_moments\_and\_f (x, mask=True)Compute u(x) and f(x) for given x.

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.compute\_gradient

GaussianMarkovChainDistribution.compute\_gradient (g, u, phi)Compute the standard gradient with respect to the natural parameters.

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.compute\_logpdf

GaussianMarkovChainDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.compute\_mask\_to\_p

GaussianMarkovChainDistribution.compute\_mask\_to\_parent(index, mask)

## bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.compute\_message\_

```
GaussianMarkovChainDistribution.compute_message_to_parent (parent, index, u, u_mu, u_Lambda, u_A, u_v, *u_inputs)
```

Compute a message to a parent.

Parameters index: int

Index of the parent requesting the message.

**u**: list of ndarrays

Moments of this node.

u\_mu: list of ndarrays

Moments of parent mu.

u\_Lambda: list of ndarrays

Moments of parent Lambda.

u\_A: list of ndarrays

Moments of parent A.

u\_v: list of ndarrays

Moments of parent v.

**u\_inputs**: list of ndarrays

Moments of input signals.

# $bayes py. in ference. vmp. nodes. gaussian\_markov\_chain. Gaussian Markov Chain Distribution. compute\_moments and the compute\_moments are compute\_moments and the compute\_mom$

```
{\tt Gaussian MarkovChain Distribution.} \textbf{compute\_moments\_and\_cgf} \ (\textit{phi}, \textit{mask=True})
```

Compute the moments and the cumulant-generating function.

This basically performs the filtering and smoothing for the variable.

Parameters phi

Returns u

g

# $bayespy. inference. vmp. nodes. gaussian\_markov\_chain. Gaussian Markov Chain Distribution. compute\_phi\_from\_lambda from the comput$

```
GaussianMarkovChainDistribution.compute_phi_from_parents(u\_mu, u\_Lambda, u\_A, u\_v, *u\_inputs, mask=True)
```

Compute the natural parameters using parents' moments.

Parameters u\_parents: list of list of arrays

List of parents' lists of moments.

Returns phi: list of arrays

Natural parameters.

dims: tuple

Shape of the variable part of phi.

# $bayespy. inference. vmp. nodes. gaussian\_markov\_chain. Gaussian Markov Chain Distribution. plates\_from\_parent and the contraction of the contrac$

GaussianMarkovChainDistribution.plates\_from\_parent (index, plates)

Compute the plates using information of a parent node.

If the plates of the parents are: mu: (...) Lambda: (...) A: (...,N-1,D) v: (...,N-1,D) N: ()

the resulting plates of this node are (...)

Parameters index: int

Index of the parent to use.

## bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.plates\_to\_parent

 ${\tt Gaussian Markov Chain Distribution. {\tt plates\_to\_parent}~(\it index, plates)}$ 

Computes the plates of this node with respect to a parent.

If this node has plates (...), the latent dimensionality is D and the number of time instances is N, the plates with respect to the parents are:

Parameters index: int

The index of the parent node to use.

#### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.GaussianMarkovChainDistribution.random

GaussianMarkovChainDistribution.random(\*params, plates=None)
Draw a random sample from the distribution.

# 6.3.9 bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDi

class bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribut:

Sub-classes implement distribution specific computations.

```
\_init\_(N, D, K)
```

#### **Methods**

init(N, D, K)	
compute_cgf_from_parents(u_mu, u_Lambda,)	Compute CGF using the moments of the parents.
<pre>compute_fixed_moments_and_f(x[, mask])</pre>	Compute $u(x)$ and $f(x)$ for given $x$ .
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
compute_logpdf(u, phi, g, f, ndims)	Compute E[log p(X)] given E[u], E[phi], E[g] and E[f].
compute_mask_to_parent(index, mask)	
compute_message_to_parent(parent, index, u,)	Compute a message to a parent.
	Continued on next page

## Table 6.51 – continued from previous page

compute_moments_and_cgf(phi[, mask])	Compute the moments and the cumulant-generating function.
compute_phi_from_parents(u_mu, u_Lambda,)	Compute the natural parameters using parents' moments.
<pre>plates_from_parent(index, plates)</pre>	Compute the plates using information of a parent node.
<pre>plates_to_parent(index, plates)</pre>	Computes the plates of this node with respect to a parent.
random(*params[, plates])	Draw a random sample from the distribution.

## bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribution.\_\_init\_\_

SwitchingGaussianMarkovChainDistribution.\_\_init\_(N, D, K)

# $bayespy. in ference. vmp. nodes. gaussian\_markov\_chain. Switching Gaussian Markov Chain Distribution. compute\_chain. Compute\_chain.$

SwitchingGaussianMarkovChainDistribution.compute\_cgf\_from\_parents(u\_mu,

u\_Lambda, u\_B.

uZ,

*u\_v*)

Compute CGF using the moments of the parents.

# $bayes py. inference. vmp. nodes. gaussian\_markov\_chain. Switching Gaussian Markov Chain Distribution. compute a property of the computed propert$

SwitchingGaussianMarkovChainDistribution.compute\_fixed\_moments\_and\_f(x, mask=True)

Compute u(x) and f(x) for given x.

# $bayes py. inference. vmp. nodes. gaussian\_markov\_chain. Switching Gaussian Markov Chain Distribution. compute\_markov\_chain. Switching Gaussian Markov Chain Distribution. Switching Gaussian Marko$

SwitchingGaussianMarkovChainDistribution.compute\_gradient (g, u, phi)Compute the standard gradient with respect to the natural parameters.

# bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribution.compute\_

SwitchingGaussianMarkovChainDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute  $E[\log p(X)]$  given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

# $bayes py. in ference. vmp. nodes. gaussian\_markov\_chain. Switching Gaussian Markov Chain Distribution. compute\_chain. Co$

SwitchingGaussianMarkovChainDistribution.compute\_mask\_to\_parent(index, mask)

## bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribution.compute\_

```
SwitchingGaussianMarkovChainDistribution.compute_message_to_parent (parent,
```

index

и,

 $u\_mu$ ,

u\_Lambda,

 $u\_B$ ,

mask=True)

*u*\_*Z*, *u*\_*v*)

Compute a message to a parent.

Parameters index: int

Index of the parent requesting the message.

u: list of ndarrays

Moments of this node.

**u**\_**mu**: list of ndarrays

Moments of parent mu.

u\_Lambda: list of ndarrays

Moments of parent Lambda.

**u\_B**: list of ndarrays

Moments of parent B.

**u\_Z**: list of ndarrays

Moments of parent Z.

 $u\_v$ : list of ndarrays

Moments of parent v.

# bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribution.compute\_

```
{\tt Switching Gaussian Markov Chain Distribution.} \textbf{compute\_moments\_and\_cgf} \ (\textit{phi}, \textit{phi}, \textit{ph
```

Compute the moments and the cumulant-generating function.

This basically performs the filtering and smoothing for the variable.

Parameters phi

Returns u

g

## bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribution.compute\_

SwitchingGaussianMarkovChainDistribution.compute\_phi\_from\_parents ( $u\_mu$ ,  $u\_Lambda$ ,  $u\_B$ ,  $u\_Z$ ,  $u\_v$ , mask=True)

Compute the natural parameters using parents' moments.

Parameters u\_parents: list of list of arrays

List of parents' lists of moments.

Returns phi: list of arrays

Natural parameters.

dims: tuple

Shape of the variable part of phi.

# bayespy.inference.vmp.nodes.gaussian\_markov\_chain.SwitchingGaussianMarkovChainDistribution.plates\_fro

SwitchingGaussianMarkovChainDistribution.plates\_from\_parent (index, plates)

Compute the plates using information of a parent node.

**If the plates of the parents are:** mu: (...) Lambda: (...) B: (...,D) S: (...,N-1) v: (...,N-1,D) N: ()

the resulting plates of this node are (...)

Parameters index: int

Index of the parent to use.

# 

SwitchingGaussianMarkovChainDistribution.plates\_to\_parent (index, plates)
Computes the plates of this node with respect to a parent.

If this node has plates (...), the latent dimensionality is D and the number of time instances is N, the plates with respect to the parents are:

```
mu: (...) Lambda: (...) A: (...,N-1,D) v: (...,N-1,D)
```

Parameters index: int

The index of the parent node to use.

## $bayes py. inference. vmp. nodes. gaussian\_markov\_chain. Switching Gaussian Markov Chain Distribution. random and the contraction of the contract$

SwitchingGaussianMarkovChainDistribution.random(\*params, plates=None)

Draw a random sample from the distribution.

# 6.3.10 bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDis

 ${\bf class} \ {\bf bayespy.inference.vmp.nodes.gaussian\_markov\_chain.Varying Gaussian Markov Chain Distribution Chain Chai$ 

Sub-classes implement distribution specific computations.

 $\_$ init $\_(N, D)$ 

#### **Methods**

init(N, D)	
compute_cgf_from_parents(u_mu, u_Lambda,)	Compute CGF using the moments of the parents.
<pre>compute_fixed_moments_and_f(x[, mask])</pre>	Compute $u(x)$ and $f(x)$ for given x.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
compute_logpdf(u, phi, g, f, ndims)	Compute E[log p(X)] given E[u], E[phi], E[g] and E[f].
compute_mask_to_parent(index, mask)	
compute_message_to_parent(parent, index, u,)	Compute a message to a parent.
compute_moments_and_cgf(phi[, mask])	Compute the moments and the cumulant-generating function.
compute_phi_from_parents(u_mu, u_Lambda,)	Compute the natural parameters using parents' moments.
plates_from_parent(index, plates)	Compute the plates using information of a parent node.
plates_to_parent(index, plates)	Computes the plates of this node with respect to a parent.
random(*params[, plates])	Draw a random sample from the distribution.

 $bayespy. inference. vmp. nodes. gaussian\_markov\_chain. Varying Gaussian Markov Chain Distribution.\_\_init\_\_$ 

 $VaryingGaussianMarkovChainDistribution.__init_(N, D)$ 

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.compute\_cg

VaryingGaussianMarkovChainDistribution.compute\_cgf\_from\_parents(u\_mu,

u\_Lambda, u\_B, u\_S.

 $u_{-}v$ )

Compute CGF using the moments of the parents.

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.compute\_fix

 $VaryingGaussianMarkovChainDistribution.compute_fixed_moments_and_f(x,$ 

mask=True)

Compute u(x) and f(x) for given x.

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.compute\_gr

bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.compute\_log

VaryingGaussianMarkovChainDistribution.compute\_gradient (g, u, phi)Compute the standard gradient with respect to the natural parameters.

VaryingGaussianMarkovChainDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute  $E[\log p(X)]$  given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

## bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.compute\_markov\_chainDistribution.compute\_ma

VaryingGaussianMarkovChainDistribution.compute\_mask\_to\_parent(index, mask)

# $bayes py. inference. vmp. nodes. gaussian\_markov\_chain. Varying Gaussian Markov Chain Distribution. compute\_model and the compute for the co$

VaryingGaussianMarkovChainDistribution.compute\_message\_to\_parent (parent,

index, u, u\_mu, u\_Lambda, u\_B, u\_S, u\_v)

Compute a message to a parent.

Parameters index: int

Index of the parent requesting the message.

**u**: list of ndarrays

Moments of this node.

u\_mu: list of ndarrays

Moments of parent mu.

u\_Lambda: list of ndarrays

Moments of parent Lambda.

u\_B: list of ndarrays

Moments of parent B.

u\_S: list of ndarrays

Moments of parent S.

u\_v: list of ndarrays

Moments of parent v.

# bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.compute\_me

VaryingGaussianMarkovChainDistribution.compute\_moments\_and\_cgf (phi,

mask=True)

Compute the moments and the cumulant-generating function.

This basically performs the filtering and smoothing for the variable.

Parameters phi

Returns u

g

## bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.compute\_ph

VaryingGaussianMarkovChainDistribution.compute\_phi\_from\_parents(u\_mu,

u\_Lambda, u\_B, u\_S, u\_v, mask=True)

Compute the natural parameters using parents' moments.

Parameters u\_parents: list of list of arrays

List of parents' lists of moments.

Returns phi: list of arrays

Natural parameters.

dims: tuple

Shape of the variable part of phi.

# bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.plates\_from

 ${\tt Varying Gaussian Markov Chain Distribution. \textbf{plates\_from\_parent} \ (\textit{index}, \textit{plates})}$ 

Compute the plates using information of a parent node.

If the plates of the parents are: mu: (...) Lambda: (...) B: (...,D) S: (...,N-1) v: (...,N-1,D) N: ()

the resulting plates of this node are (...)

Parameters index: int

Index of the parent to use.

# $bayes py. inference. vmp. nodes. gaussian\_markov\_chain. Varying Gaussian Markov Chain Distribution. plates\_to\_particle for the property of t$

VaryingGaussianMarkovChainDistribution.plates\_to\_parent (index, plates)

Computes the plates of this node with respect to a parent.

If this node has plates (...), the latent dimensionality is D and the number of time instances is N, the plates with respect to the parents are:

```
mu: (...) Lambda: (...) A: (...,N-1,D) v: (...,N-1,D)
```

Parameters index: int

The index of the parent node to use.

#### bayespy.inference.vmp.nodes.gaussian\_markov\_chain.VaryingGaussianMarkovChainDistribution.random

VaryingGaussianMarkovChainDistribution.random(\*params, plates=None)

Draw a random sample from the distribution.

# 6.3.11 bayespy.inference.vmp.nodes.gamma.GammaDistribution

class bayespy.inference.vmp.nodes.gamma.GammaDistribution
 Class for the VMP formulas of gamma variables.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

compute_cgf_from_parents(*u_parents)	Compute $E_{q(p)}[g(p)]$
<pre>compute_fixed_moments_and_f(x[, mask])</pre>	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the moments and $g(\phi)$ .
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
compute_mask_to_parent(index, mask)	Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index,)	Compute the message to a parent node.
<pre>compute_moments_and_cgf(phi[, mask])</pre>	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(*u_parents[, mask])</pre>	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
<pre>plates_to_parent(index, plates)</pre>	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

## bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_cgf\_from\_parents

 $\label{eq:compute_cgf_from_parents} \textbf{GammaDistribution.compute\_cgf\_from\_parents} \ (*u\_parents) \\ \textbf{Compute} \ \textbf{E}_{g(p)}[g(p)]$ 

#### bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_fixed\_moments\_and\_f

GammaDistribution.compute\_fixed\_moments\_and\_f (x, mask=True)Compute the moments and f(x) for a fixed value.

#### bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_gradient

GammaDistribution.compute\_gradient (g, u, phi)Compute the moments and  $g(\phi)$ .

$$d\overline{\mathbf{u}} = \begin{bmatrix} -\frac{d\phi_2}{phi_1} + \frac{\phi_2}{\phi_1^2} d\phi_1\\ \psi^{(1)}(\phi_2) d\phi_2 - \frac{1}{\phi_1} d\phi_1 \end{bmatrix}$$

Standard gradient given the gradient with respect to the moments, that is, given the Riemannian gradient  $\tilde{\nabla}$ :

$$\nabla = \begin{bmatrix} \nabla_1 \frac{\phi_2}{\phi_1^2} - \nabla_2 \frac{1}{\phi_1} \\ \nabla_2 \psi^{(1)}(\phi_2) - \nabla_1 \frac{1}{\phi_1} \end{bmatrix}$$

#### bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_logpdf

GammaDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

## bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_mask\_to\_parent

GammaDistribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

#### bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_message\_to\_parent

GammaDistribution.compute\_message\_to\_parent (parent, index,  $u\_self$ ,  $u\_a$ ,  $u\_b$ )

Compute the message to a parent node.

## bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_moments\_and\_cgf

GammaDistribution.compute\_moments\_and\_cgf (phi, mask=True)

Compute the moments and  $g(\phi)$ .

$$\overline{\mathbf{u}}(\phi) = \begin{bmatrix} -\frac{\phi_2}{\phi_1} \\ \psi(\phi_2) - \log(-\phi_1) \end{bmatrix} \\
q_{\phi}(\phi) = TODO$$

#### bayespy.inference.vmp.nodes.gamma.GammaDistribution.compute\_phi\_from\_parents

GammaDistribution.compute\_phi\_from\_parents (\*u\_parents, mask=True)

Compute the natural parameter vector given parent moments.

## bayespy.inference.vmp.nodes.gamma.GammaDistribution.plates\_from\_parent

GammaDistribution.plates\_from\_parent(index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

#### bayespy.inference.vmp.nodes.gamma.GammaDistribution.plates\_to\_parent

GammaDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.gamma.GammaDistribution.random

GammaDistribution.random(\*phi, plates=None)
Draw a random sample from the distribution.

# 6.3.12 bayespy.inference.vmp.nodes.wishart.WishartDistribution

class bayespy.inference.vmp.nodes.wishart.WishartDistribution
 Sub-classes implement distribution specific computations.

Distribution for kimesk symmetric positive definite matrix.

$$\Lambda \sim \mathcal{W}(n, V)$$

Note: V is inverse scale matrix.

$$p(\Lambda|n, V) = ...$$

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

compute_cgf_from_parents(u_n, u_V)	CGF from parents
<pre>compute_fixed_moments_and_f(Lambda[, mask])</pre>	Compute $u(x)$ and $f(x)$ for given $x$ .
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute E[log p(X)] given E[u], E[phi], E[g] and E[f].
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index,)	
compute_moments_and_cgf(phi[, mask])	Return moments and cgf for given natural parameters
compute_phi_from_parents(u_n, u_V[, mask])	Compute natural parameters
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*params[, plates])	Draw a random sample from the distribution.

## bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_cgf\_from\_parents

WishartDistribution.compute\_cgf\_from\_parents  $(u_.n, u_.V)$  CGF from parents

$$g(n, V) = \frac{n}{2} \log |V| - \frac{nk}{2} \log 2 - \log \Gamma_k(\frac{n}{2})$$

## bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_fixed\_moments\_and\_f

WishartDistribution.compute\_fixed\_moments\_and\_f(Lambda, mask=True)
Compute u(x) and f(x) for given x.

## bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_gradient

WishartDistribution.compute\_gradient (g, u, phi)

Compute the standard gradient with respect to the natural parameters.

## bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_logpdf

WishartDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

### bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_mask\_to\_parent

WishartDistribution.compute\_mask\_to\_parent (index, mask)
Maps the mask to the plates of a parent.

#### bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_message\_to\_parent

WishartDistribution.compute\_message\_to\_parent (parent, index,  $u\_self$ ,  $u\_n$ ,  $u\_V$ )

## bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_moments\_and\_cgf

WishartDistribution.compute\_moments\_and\_cgf (phi, mask=True)
Return moments and cgf for given natural parameters

$$\langle u \rangle = \begin{bmatrix} \phi_2(-\phi_1)^{-1} \\ -\log|-\phi_1| + \psi_k(\phi_2) \end{bmatrix}$$

$$a(\phi) = \phi_2 \log|-\phi_1| - \log \Gamma_k(\phi_2)$$

## bayespy.inference.vmp.nodes.wishart.WishartDistribution.compute\_phi\_from\_parents

WishartDistribution.compute\_phi\_from\_parents(u\_n, u\_V, mask=True)
Compute natural parameters

$$\phi(n,V) = \begin{bmatrix} -\frac{1}{2}V\\ \frac{1}{2}n \end{bmatrix}$$

#### bayespy.inference.vmp.nodes.wishart.WishartDistribution.plates\_from\_parent

WishartDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

#### bayespy.inference.vmp.nodes.wishart.WishartDistribution.plates\_to\_parent

WishartDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.wishart.WishartDistribution.random

WishartDistribution.random(\*params, plates=None)
Draw a random sample from the distribution.

# 6.3.13 bayespy.inference.vmp.nodes.beta.BetaDistribution

class bayespy.inference.vmp.nodes.beta.BetaDistribution

Class for the VMP formulas of beta variables.

Although the realizations are scalars (probability p), the moments is a two-dimensional vector: [log(p), log(1-p)].

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

compute_cgf_from_parents(u_alpha)	Compute $E_{q(p)}[g(p)]$
compute_fixed_moments_and_f(p[, mask])	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the moments and $g(\phi)$ .
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index,)</pre>	Compute the message to a parent node.
compute_moments_and_cgf(phi[, mask])	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(u_alpha[, mask])</pre>	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_cqf\_from\_parents

BetaDistribution.compute\_cgf\_from\_parents (u\_alpha) Compute  $\mathrm{E}_{q(p)}[g(p)]$ 

#### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_fixed\_moments\_and\_f

BetaDistribution.compute\_fixed\_moments\_and\_f (p, mask=True)Compute the moments and f(x) for a fixed value.

#### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_gradient

 ${\tt BetaDistribution.compute\_gradient}\ (\textit{g},\textit{u},\textit{phi})$ 

Compute the moments and  $q(\phi)$ .

 $psi(phi_1) - psi(sum_d phi_{1,d})$ 

Standard gradient given the gradient with respect to the moments, that is, given the Riemannian gradient  $\tilde{\nabla}$ :

$$\nabla = \left[ (\psi^{(1)}(\phi_1) - \psi^{(1)}(\sum_d \phi_{1,d}) \nabla_1 \right]$$

#### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_logpdf

BetaDistribution.compute\_logpdf(u, phi, g, f, ndims)

Compute  $E[\log p(X)]$  given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

## bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_mask\_to\_parent

```
BetaDistribution.compute_mask_to_parent(index, mask)

Maps the mask to the plates of a parent.
```

#### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_message\_to\_parent

```
BetaDistribution.compute_message_to_parent (parent, index, u_self, u_alpha) Compute the message to a parent node.
```

#### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_moments\_and\_cgf

```
BetaDistribution.compute_moments_and_cgf (phi, mask=True)
Compute the moments and g(\phi).
```

#### bayespy.inference.vmp.nodes.beta.BetaDistribution.compute\_phi\_from\_parents

```
BetaDistribution.compute_phi_from_parents (u_alpha, mask=True)

Compute the natural parameter vector given parent moments.
```

## bayespy.inference.vmp.nodes.beta.BetaDistribution.plates\_from\_parent

```
BetaDistribution.plates_from_parent (index, plates)
Resolve the plate mapping from a parent.
```

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

## bayespy.inference.vmp.nodes.beta.BetaDistribution.plates\_to\_parent

```
BetaDistribution.plates_to_parent (index, plates)
Resolves the plate mapping to a parent.
```

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.beta.BetaDistribution.random

```
BetaDistribution.random(*phi, plates=None)
Draw a random sample from the distribution.
```

# 6.3.14 bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution

#### **Methods**

compute_cgf_from_parents(u_alpha)	Compute $E_{q(p)}[g(p)]$
compute_fixed_moments_and_f(p[, mask])	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the moments and $g(\phi)$ .
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index,)</pre>	Compute the message to a parent node.
compute_moments_and_cgf(phi[, mask])	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(u_alpha[, mask])</pre>	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_cgf\_from\_parents

 $\label{eq:compute_cgf_from_parents} \textbf{Compute} \cdot \textbf{Compute} \cdot \textbf{Compute} \cdot \textbf{E}_{q(p)}[g(p)]$ 

## bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_fixed\_moments\_and\_f

DirichletDistribution.compute\_fixed\_moments\_and\_f (p, mask=True)Compute the moments and f(x) for a fixed value.

$$u(p) = \begin{bmatrix} \log(p_1) \\ \vdots \\ \log(p_D) \end{bmatrix}$$

$$f(p) = -\sum_{d} \log(p_d)$$

#### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_gradient

DirichletDistribution.compute\_gradient(g, u, phi)

Compute the moments and  $g(\phi)$ .

$$psi(phi_1) - psi(sum_d phi_{1,d})$$

Standard gradient given the gradient with respect to the moments, that is, given the Riemannian gradient  $\tilde{\nabla}$ :

$$\nabla = \left[ (\psi^{(1)}(\phi_1) - \psi^{(1)}(\sum_d \phi_{1,d}) \nabla_1 \right]$$

## bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_logpdf

DirichletDistribution.compute\_logpdf(u, phi, g, f, ndims)

Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

## bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_mask\_to\_parent

DirichletDistribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

#### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_message\_to\_parent

DirichletDistribution.compute\_message\_to\_parent (parent, index, u\_self, u\_alpha)

Compute the message to a parent node.

## bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_moments\_and\_cgf

DirichletDistribution.compute\_moments\_and\_cgf (phi, mask=True)

Compute the moments and  $g(\phi)$ .

$$\overline{\mathbf{u}}(\phi) = \left[ \psi(\phi_1) - \psi(\sum_d \phi_{1,d}) \right]$$
$$g_{\phi}(\phi) = TODO$$

#### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.compute\_phi\_from\_parents

DirichletDistribution.compute\_phi\_from\_parents (u\_alpha, mask=True)

Compute the natural parameter vector given parent moments.

## bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.plates\_from\_parent

DirichletDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

#### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.plates\_to\_parent

DirichletDistribution.plates\_to\_parent (index, plates)
Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution.random

DirichletDistribution.random(\*phi, plates=None)

Draw a random sample from the distribution.

# 6.3.15 bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution

class bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution
 Class for the VMP formulas of Bernoulli variables.

\_\_init\_\_()

#### Methods

init()	
compute_cgf_from_parents(u_p)	Compute $E_{q(p)}[g(p)]$
$compute\_fixed\_moments\_and\_f(x[, mask])$	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
compute_logpdf(u, phi, g, f, ndims)	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index,)</pre>	Compute the message to a parent node.
<pre>compute_moments_and_cgf(phi[, mask])</pre>	Compute the moments and $g(\phi)$ .
compute_phi_from_parents(u_p[, mask])	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

## bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.\_\_init\_\_

BernoulliDistribution.\_\_init\_\_()

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_cqf\_from\_parents

BernoulliDistribution.compute\_cgf\_from\_parents (u-p) Compute  $\mathbf{E}_{q(p)}[g(p)]$ 

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_fixed\_moments\_and\_f

BernoulliDistribution.compute\_fixed\_moments\_and\_f (x, mask=True)Compute the moments and f(x) for a fixed value.

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_gradient

BernoulliDistribution.compute\_gradient (g, u, phi)Compute the standard gradient with respect to the natural parameters.

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_logpdf

BernoulliDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

## bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_mask\_to\_parent

BernoulliDistribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_message\_to\_parent

BernoulliDistribution.compute\_message\_to\_parent (parent, index, u\_self, u\_p) Compute the message to a parent node.

### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_moments\_and\_cgf

BernoulliDistribution.compute\_moments\_and\_cgf (phi, mask=True)

Compute the moments and  $g(\phi)$ .

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.compute\_phi\_from\_parents

BernoulliDistribution.compute\_phi\_from\_parents (u\_p, mask=True)
Compute the natural parameter vector given parent moments.

## bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.plates\_from\_parent

BernoulliDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.plates\_to\_parent

BernoulliDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution.random

BernoulliDistribution.random(\*phi, plates=None)

Draw a random sample from the distribution.

# 6.3.16 bayespy.inference.vmp.nodes.binomial.BinomialDistribution

class bayespy.inference.vmp.nodes.binomial.BinomialDistribution (N) Class for the VMP formulas of binomial variables.

\_\_init\_\_(N)

#### **Methods**

init(N)	
$compute\_cgf\_from\_parents(u\_p)$	Compute $E_{q(p)}[g(p)]$
$compute\_fixed\_moments\_and\_f(x[, mask])$	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute $E[\log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index,)	Compute the message to a parent node.
compute_moments_and_cgf(phi[, mask])	Compute the moments and $g(\phi)$ .
compute_phi_from_parents(u_p[, mask])	Compute the natural parameter vector given parent moments.
	Continued on next page

6.3. Distributions 285

### Table 6.58 – continued from previous page

<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
<pre>plates_to_parent(index, plates)</pre>	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.\_\_init\_\_

BinomialDistribution.\_\_**init**\_\_(N)

### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_cgf\_from\_parents

```
BinomialDistribution.compute_cgf_from_parents (u_p) Compute \mathrm{E}_{q(p)}[g(p)]
```

#### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_fixed\_moments\_and\_f

```
BinomialDistribution.compute_fixed_moments_and_f (x, mask=True)
Compute the moments and f(x) for a fixed value.
```

#### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_gradient

```
BinomialDistribution.compute_gradient (g, u, phi)
Compute the standard gradient with respect to the natural parameters.
```

#### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_logpdf

```
BinomialDistribution.compute_logpdf (u, phi, g, f, ndims)
Compute E[\log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.
```

#### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_mask\_to\_parent

```
BinomialDistribution.compute_mask_to_parent(index, mask)

Maps the mask to the plates of a parent.
```

#### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_message\_to\_parent

```
BinomialDistribution.compute_message_to_parent (parent, index, u\_self, u\_p)

Compute the message to a parent node.
```

### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_moments\_and\_cgf

```
BinomialDistribution.compute_moments_and_cgf (phi, mask=True)

Compute the moments and g(\phi).
```

#### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.compute\_phi\_from\_parents

```
BinomialDistribution.compute_phi_from_parents (u_p, mask=True)

Compute the natural parameter vector given parent moments.
```

### bayespy.inference.vmp.nodes.binomial.BinomialDistribution.plates\_from\_parent

BinomialDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

# bayespy.inference.vmp.nodes.binomial.BinomialDistribution.plates\_to\_parent

BinomialDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

# bayespy.inference.vmp.nodes.binomial.BinomialDistribution.random

BinomialDistribution.random(\*phi, plates=None)

Draw a random sample from the distribution.

# 6.3.17 bayespy.inference.vmp.nodes.categorical.CategoricalDistribution

\_\_init\_\_ (categories)

Create VMP formula node for a categorical variable

categories is the total number of categories.

#### Methods

init(categories)	Create VMP formula node for a categorical variable
compute_cgf_from_parents(u_p)	Compute $E_{q(p)}[g(p)]$
<pre>compute_fixed_moments_and_f(x[, mask])</pre>	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
compute_logpdf(u, phi, g, f, ndims)	Compute $E[log p(X)]$ given $E[u]$ , $E[phi]$ , $E[g]$ and $E[f]$ .
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
<pre>compute_message_to_parent(parent, index, u, u_p)</pre>	Compute the message to a parent node.
<pre>compute_moments_and_cgf(phi[, mask])</pre>	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(u_p[, mask])</pre>	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
<pre>plates_to_parent(index, plates)</pre>	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

# bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.\_\_init\_\_

CategoricalDistribution.\_\_init\_\_(categories)

Create VMP formula node for a categorical variable

categories is the total number of categories.

6.3. Distributions 287

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_cgf\_from\_parents

```
CategoricalDistribution.compute_cgf_from_parents (u_p) Compute \mathrm{E}_{q(p)}[g(p)]
```

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_fixed\_moments\_and\_f

```
CategoricalDistribution.compute_fixed_moments_and_f (x, mask=True)
Compute the moments and f(x) for a fixed value.
```

## bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_gradient

```
CategoricalDistribution.compute_gradient (g, u, phi)
Compute the standard gradient with respect to the natural parameters.
```

#### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_logpdf

```
CategoricalDistribution.compute_logpdf (u, phi, g, f, ndims)
Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.
```

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_mask\_to\_parent

```
CategoricalDistribution.compute_mask_to_parent (index, mask)

Maps the mask to the plates of a parent.
```

## bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_message\_to\_parent

```
CategoricalDistribution.compute_message_to_parent (parent, index, u, u_p)

Compute the message to a parent node.
```

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_moments\_and\_cgf

```
CategoricalDistribution.compute_moments_and_cgf (phi, mask=True) Compute the moments and g(\phi).
```

#### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.compute\_phi\_from\_parents

```
CategoricalDistribution.compute_phi_from_parents (u_p, mask=True)
Compute the natural parameter vector given parent moments.
```

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.plates\_from\_parent

```
CategoricalDistribution.plates_from_parent (index, plates)
Resolve the plate mapping from a parent.
```

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.plates\_to\_parent

CategoricalDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

### bayespy.inference.vmp.nodes.categorical.CategoricalDistribution.random

CategoricalDistribution.random(\*phi, plates=None)

Draw a random sample from the distribution.

# 6.3.18 bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution

class bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution

Class for the VMP formulas of categorical Markov chain variables.

\_\_init\_\_ (categories, states)

Create VMP formula node for a categorical variable

categories is the total number of categories. states is the length of the chain.

#### **Methods**

init(categories, states)	Create VMP formula node for a categorical variable
compute_cgf_from_parents(u_p0, u_P)	Compute $E_{q(p)}[g(p)]$
<pre>compute_fixed_moments_and_f(x[, mask])</pre>	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute E[log p(X)] given E[u], E[phi], E[g] and E[f].
<pre>compute_mask_to_parent(index, mask)</pre>	Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index, u,)	Compute the message to a parent node.
compute_moments_and_cgf(phi[, mask])	Compute the moments and $g(\phi)$ .
compute_phi_from_parents(u_p0, u_P[, mask])	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
<pre>plates_to_parent(index, plates)</pre>	Resolves the plate mapping to a parent.
random(*phi[, plates])	Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.\_\_init\_\_

CategoricalMarkovChainDistribution.\_\_init\_\_(categories, states)

Create VMP formula node for a categorical variable

categories is the total number of categories. states is the length of the chain.

### bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_cgf\_free

CategoricalMarkovChainDistribution.compute\_cgf\_from\_parents  $(u\_p0, u\_P)$  Compute  $\mathrm{E}_{q(p)}[g(p)]$ 

6.3. Distributions 289

bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_fixed\_

 $\texttt{CategoricalMarkovChainDistribution.} \textbf{compute\_fixed\_moments\_and\_f} \ (x, \\ \textit{mask=True})$ 

Compute the moments and f(x) for a fixed value.

bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_gradie

CategoricalMarkovChainDistribution.compute\_gradient (g, u, phi)Compute the standard gradient with respect to the natural parameters.

bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_logpd

CategoricalMarkovChainDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_mask

CategoricalMarkovChainDistribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_messatespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_messatespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_messatespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_messatespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_messatespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_messatespy.inference.vmp.nodes.categorical\_markov\_chain.Categorical\_markov\_chain

 $\label{lem:compute_message_to_parent} \ (\textit{parent}, \textit{index}, \textit{u}, \\ \textit{u\_p0}, \textit{u\_P})$ 

Compute the message to a parent node.

bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_mome

CategoricalMarkovChainDistribution.compute\_moments\_and\_cgf (phi, mask=True)

Compute the moments and  $g(\phi)$ .

bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.compute\_phi\_fre

CategoricalMarkovChainDistribution.compute\_phi\_from\_parents  $(u\_p0, u\_P, mask=True)$ 

Compute the natural parameter vector given parent moments.

bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.plates\_from\_par

CategoricalMarkovChainDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

## bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.plates\_to\_paren

CategoricalMarkovChainDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

### bayespy.inference.vmp.nodes.categorical\_markov\_chain.CategoricalMarkovChainDistribution.random

CategoricalMarkovChainDistribution.random(\*phi, plates=None)
Draw a random sample from the distribution.

# 6.3.19 bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution

class bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution(trials)
 Class for the VMP formulas of multinomial variables.

\_\_init\_\_(trials)

Create VMP formula node for a multinomial variable

trials is the total number of trials.

#### **Methods**

init(trials)	Create VMP formula node for a multinomial variable
compute_cgf_from_parents(u_p)	Compute $E_{q(p)}[g(p)]$
<pre>compute_fixed_moments_and_f(x[, mask])</pre>	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
compute_logpdf(u, phi, g, f, ndims)	Compute E[log p(X)] given E[u], E[phi], E[g] and E[f].
compute_mask_to_parent(index, mask)	Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index, u, u_p)	Compute the message to a parent node.
compute_moments_and_cgf(phi[, mask])	Compute the moments and $g(\phi)$ .
compute_phi_from_parents(u_p[, mask])	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
plates_to_parent(index, plates)	Resolves the plate mapping to a parent.
random(*phi)	Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.\_\_init\_\_

MultinomialDistribution.\_\_init\_\_(trials)

Create VMP formula node for a multinomial variable

trials is the total number of trials.

## bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_cgf\_from\_parents

```
\label{eq:multinomialDistribution.compute_cgf_from_parents} \textit{(u\_p)} \\ \textit{Compute } \mathbf{E}_{q(p)}[g(p)]
```

6.3. Distributions 291

#### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_fixed\_moments\_and\_f

MultinomialDistribution.compute\_fixed\_moments\_and\_f (x, mask=True)Compute the moments and f(x) for a fixed value.

#### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_gradient

MultinomialDistribution.compute\_gradient (g, u, phi)Compute the standard gradient with respect to the natural parameters.

#### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_logpdf

MultinomialDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

## bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_mask\_to\_parent

MultinomialDistribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

#### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_message\_to\_parent

MultinomialDistribution.compute\_message\_to\_parent (parent, index, u, u\_p)

Compute the message to a parent node.

#### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_moments\_and\_cgf

 $\label{local_model} \textit{MultinomialDistribution.compute\_moments\_and\_cgf} \ (phi, mask=True) \\ \textit{Compute the moments and } g(\phi).$ 

#### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.compute\_phi\_from\_parents

MultinomialDistribution.compute\_phi\_from\_parents (u\_p, mask=True)

Compute the natural parameter vector given parent moments.

#### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.plates\_from\_parent

MultinomialDistribution.plates\_from\_parent (index, plates)

Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.plates\_to\_parent

MultinomialDistribution.plates\_to\_parent (index, plates)

Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

### bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution.random

MultinomialDistribution.random(\*phi)

Draw a random sample from the distribution.

# 6.3.20 bayespy.inference.vmp.nodes.poisson.PoissonDistribution

class bayespy.inference.vmp.nodes.poisson.PoissonDistribution
 Class for the VMP formulas of Poisson variables.

\_\_init\_\_()

Initialize self. See help(type(self)) for accurate signature.

#### **Methods**

compute_cgf_from_parents(u_lambda)	Compute $E_{q(p)}[g(p)]$
compute_fixed_moments_and_f(x[, mask])	Compute the moments and $f(x)$ for a fixed value.
compute_gradient(g, u, phi)	Compute the standard gradient with respect to the natural parameters.
<pre>compute_logpdf(u, phi, g, f, ndims)</pre>	Compute E[log p(X)] given E[u], E[phi], E[g] and E[f].
compute_mask_to_parent(index, mask)	Maps the mask to the plates of a parent.
compute_message_to_parent(parent, index, u,)	Compute the message to a parent node.
compute_moments_and_cgf(phi[, mask])	Compute the moments and $g(\phi)$ .
<pre>compute_phi_from_parents(u_lambda[, mask])</pre>	Compute the natural parameter vector given parent moments.
<pre>plates_from_parent(index, plates)</pre>	Resolve the plate mapping from a parent.
<pre>plates_to_parent(index, plates)</pre>	Resolves the plate mapping to a parent.
random(*phi)	Draw a random sample from the distribution.

#### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_cgf\_from\_parents

PoissonDistribution.compute\_cgf\_from\_parents ( $u\_lambda$ ) Compute  $\mathbf{E}_{q(p)}[g(p)]$ 

### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_fixed\_moments\_and\_f

PoissonDistribution.compute\_fixed\_moments\_and\_f (x, mask=True)Compute the moments and f(x) for a fixed value.

#### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_gradient

PoissonDistribution.compute\_gradient (g, u, phi)Compute the standard gradient with respect to the natural parameters.

### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_logpdf

PoissonDistribution.compute\_logpdf (u, phi, g, f, ndims)Compute E[log p(X)] given E[u], E[phi], E[g] and E[f]. Does not sum over plates.

6.3. Distributions 293

#### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_mask\_to\_parent

PoissonDistribution.compute\_mask\_to\_parent (index, mask)

Maps the mask to the plates of a parent.

#### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_message\_to\_parent

PoissonDistribution.compute\_message\_to\_parent (parent, index, u, u\_lambda) Compute the message to a parent node.

## bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_moments\_and\_cgf

PoissonDistribution.compute\_moments\_and\_cgf (phi, mask=True)
Compute the moments and  $g(\phi)$ .

#### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.compute\_phi\_from\_parents

PoissonDistribution.compute\_phi\_from\_parents (u\_lambda, mask=True)
Compute the natural parameter vector given parent moments.

### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.plates\_from\_parent

PoissonDistribution.plates\_from\_parent (index, plates)
Resolve the plate mapping from a parent.

Given the plates of a parent's moments, this method returns the plates that the moments has for this distribution.

### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.plates\_to\_parent

PoissonDistribution.plates\_to\_parent (index, plates)
Resolves the plate mapping to a parent.

Given the plates of the node's moments, this method returns the plates that the message to a parent has for the parent's distribution.

#### bayespy.inference.vmp.nodes.poisson.PoissonDistribution.random

PoissonDistribution.random(\*phi)

Draw a random sample from the distribution.

# 6.4 Utility functions

linalg	General numerical functions and methods.	
random	General functions random sampling and distributions.	
optimize		
misc	General numerical functions and methods.	

# 6.4.1 bayespy.utils.linalg

General numerical functions and methods.

#### **Functions**

block_banded_solve(A, B, y)	Invert symmetric, banded, positive-definite matrix.
chol(C)	· · · · · · · · · · · · · · · · · · ·
chol_inv(U)	
chol_logdet(U)	
chol_solve(U, b[, out, matrix])	
dot(*arrays)	Compute matrix-matrix product.
<pre>inner(*args[, ndim])</pre>	Compute inner product.
inv(A[, ndim])	General array inversion.
logdet_chol(U)	
logdet_cov(C)	
logdet_tri(R)	Logarithm of the absolute value of the determinant of a triangular matrix.
$m\_dot(A, b)$	
mmdot(A, B[, ndim])	Compute matrix-matrix product.
mvdot(A, b[, ndim])	Compute matrix-vector product.
outer(A, B[, ndim])	Computes outer product over the last axes of A and B.
solve_triangular(U, B, **kwargs)	
tracedot(A, B)	Computes trace(A*B).
transpose(X[, ndim])	Transpose the matrix.

# bayespy.utils.linalg.block\_banded\_solve

bayespy.utils.linalg.block\_banded\_solve(A, B, y)

Invert symmetric, banded, positive-definite matrix.

A contains the diagonal blocks.

B contains the superdiagonal blocks (their transposes are the subdiagonal blocks).

Shapes: A: (..., N, D, D) B: (..., N-1, D, D) y: (..., N, D)

The algorithm is basically LU decomposition.

Computes only the diagonal and super-diagonal blocks of the inverse. The true inverse is dense, in general.

Assume each block has the same size.

Return: \* inverse blocks \* solution to the system \* log-determinant

# bayespy.utils.linalg.chol

bayespy.utils.linalg.chol(C)

## bayespy.utils.linalg.chol\_inv

bayespy.utils.linalg.chol\_inv(U)

### bayespy.utils.linalg.chol\_logdet

```
bayespy.utils.linalg.chol_logdet(U)
```

#### bayespy.utils.linalg.chol\_solve

```
bayespy.utils.linalg.chol_solve(U, b, out=None, matrix=False)
```

#### bayespy.utils.linalg.dot

```
bayespy.utils.linalg.dot(*arrays)
```

Compute matrix-matrix product.

You can give multiple arrays, the dot product is computed from left to right: A1\*A2\*A3\*...\*AN. The dot product is computed over the last two axes of each arrays. All other axes must be broadcastable.

## bayespy.utils.linalg.inner

```
bayespy.utils.linalg.inner(*args, ndim=1)
Compute inner product.
```

The number of arrays is arbitrary. The number of dimensions is arbitrary.

#### bayespy.utils.linalg.inv

```
bayespy.utils.linalg.inv (A, ndim=1)
General array inversion.
```

Supports broadcasting and inversion of multidimensional arrays. For instance, an array with shape (4,3,2,3,2) could mean that there are four (3\*2) x (3\*2) matrices to be inverted. This can be done by inv(A, ndim=2). For inverting scalars, ndim=0. For inverting matrices, ndim=1.

#### bayespy.utils.linalg.logdet\_chol

```
bayespy.utils.linalg.logdet_chol(U)
```

#### bayespy.utils.linalg.logdet\_cov

```
bayespy.utils.linalg.logdet_cov(C)
```

### bayespy.utils.linalg.logdet\_tri

```
bayespy.utils.linalg.logdet_tri(R)
```

Logarithm of the absolute value of the determinant of a triangular matrix.

# bayespy.utils.linalg.m\_dot

```
bayespy.utils.linalg.m_dot(A, b)
```

### bayespy.utils.linalg.mmdot

```
bayespy.utils.linalg.mmdot(A, B, ndim=1)
Compute matrix-matrix product.

Applies broadcasting.
```

# bayespy.utils.linalg.mvdot

```
bayespy.utils.linalg.mvdot(A, b, ndim=1)
Compute matrix-vector product.
Applies broadcasting.
```

#### bayespy.utils.linalg.outer

```
bayespy.utils.linalg.outer (A, B, ndim=1)
Computes outer product over the last axes of A and B.
```

The other axes are broadcasted. Thus, if A has shape (..., N) and B has shape (..., M), then the result has shape (..., N, M).

Using the argument *ndim* it is possible to change that how many axes trailing axes are used for the outer product. For instance, if ndim=3, A and B have shapes (...,N1,N2,N3) and (...,M1,M2,M3), the result has shape (...,N1,M1,N2,M2,N3,M3).

### bayespy.utils.linalg.solve\_triangular

```
bayespy.utils.linalg.solve_triangular(U, B, **kwargs)
```

### bayespy.utils.linalg.tracedot

```
bayespy.utils.linalg.tracedot (A, B)
Computes trace(A*B).
```

## bayespy.utils.linalg.transpose

```
bayespy.utils.linalg.transpose(X, ndim=1)
Transpose the matrix.
```

# 6.4.2 bayespy.utils.random

General functions random sampling and distributions.

#### **Functions**

alpha_beta_recursion(logp0, logP)	Compute alpha-beta recursion for Markov chain
bernoulli(p[, size])	Draw random samples from the Bernoulli distribution.
<pre>categorical(p[, size])</pre>	Draw random samples from a categorical distribution.
	Continued on next page

Table 6.65 – continued from previous page

	1 1 9
correlation(D)	Draw a random correlation matrix.
covariance(D[, size, nu])	Draw a random covariance matrix.
dirichlet(alpha[, size])	Draw random samples from the Dirichlet distribution.
gamma_entropy(a, log_b, gammaln_a, psi_a,)	Entropy of $\mathcal{G}(a,b)$ .
gamma_logpdf(bx, logx, a_logx, a_logb, gammaln_a)	Log-density of $\mathcal{G}(x a,b)$ .
gaussian_entropy(logdet_V, D)	Compute the entropy of a Gaussian distribution.
<pre>gaussian_gamma_to_t(mu, Cov, a, b[, ndim])</pre>	Integrates gamma distribution to obtain parameters of t distribution
gaussian_logpdf(yVy, yVmu, muVmu, logdet_V, D)	Log-density of a Gaussian distribution.
<pre>intervals(N, length[, amount, gap])</pre>	Return random non-overlapping parts of a sequence.
invwishart_rand(nu, V)	
$logodds\_to\_probability(x)$	Solves p from $log(p/(1-p))$
<pre>mask(*shape[, p])</pre>	Return a boolean array of the given shape.
orth(D)	Draw random orthogonal matrix.
sphere([N])	Draw random points uniformly on a unit sphere.
svd(s)	Draw a random matrix given its singular values.
t_logpdf(z2, logdet_cov, nu, D)	
wishart(nu, V)	Draw a random sample from the Wishart distribution.
wishart_rand(nu, V)	Draw a random sample from the Wishart distribution.

#### bayespy.utils.random.alpha\_beta\_recursion

bayespy.utils.random.alpha\_beta\_recursion(logp0, logP)

Compute alpha-beta recursion for Markov chain

Initial state log-probabilities are in p0 and state transition log-probabilities are in P. The probabilities do not need to be scaled to sum to one, but they are interpreted as below:

 $logp0 = log P(z_0) + log P(y_0|z_0) logP[...,n,;;] = log P(z_{n+1}|z_n) + log P(y_{n+1}|z_n+1)$ 

# bayespy.utils.random.bernoulli

bayespy.utils.random.bernoulli(p, size=None)

Draw random samples from the Bernoulli distribution.

### bayespy.utils.random.categorical

bayespy.utils.random.categorical(p, size=None)

Draw random samples from a categorical distribution.

### bayespy.utils.random.correlation

 $\verb|bayespy.utils.random.correlation|(D)|$ 

Draw a random correlation matrix.

#### bayespy.utils.random.covariance

bayespy.utils.random.covariance(D, size=(), nu=None)

Draw a random covariance matrix.

Draws from inverse-Wishart distribution. The distribution of each element is independent of the dimensionality of the matrix.

```
C \sim Inv-W(I, D)
```

#### Parameters D: int

Dimensionality of the covariance matrix.

### bayespy.utils.random.dirichlet

```
bayespy.utils.random.dirichlet (alpha, size=None)
Draw random samples from the Dirichlet distribution.
```

# bayespy.utils.random.gamma\_entropy

```
bayespy.utils.random.gamma_entropy (a, log\_b, gammaln\_a, psi\_a, a\_psi\_a)
Entropy of \mathcal{G}(a, b).
```

If you want to get the gradient, just let each parameter be a gradient of that term.

```
Parameters \mathbf{a}: ndarray a \log_{-}\mathbf{b}: ndarray \log(b) \mathbf{gammaln\_a}: ndarray \log\Gamma(a) \mathbf{psi\_a}: ndarray \psi(a) \mathbf{a\_psi\_a}: ndarray a\psi(a)
```

#### bayespy.utils.random.gamma\_logpdf

```
bayespy.utils.random.gamma_logpdf (bx, logx, a\_logx, a\_logb, gammaln\_a) Log-density of \mathcal{G}(x|a,b).
```

If you want to get the gradient, just let each parameter be a gradient of that term.

```
Parameters bx : ndarray bx logx : ndarray log(x) a\_logx : ndarray a log(x) a\_logb : ndarray a log(b) gammaln\_a : ndarray log \Gamma(a)
```

### bayespy.utils.random.gaussian\_entropy

bayespy.utils.random.gaussian\_entropy(logdet\_V, D)

Compute the entropy of a Gaussian distribution.

If you want to get the gradient, just let each parameter be a gradient of that term.

Parameters logdet\_V : ndarray or double

The log-determinant of the precision matrix.

D: int

The dimensionality of the distribution.

### bayespy.utils.random.gaussian\_gamma\_to\_t

bayespy.utils.random.gaussian\_gamma\_to\_t (mu, Cov, a, b, ndim=1)
Integrates gamma distribution to obtain parameters of t distribution

### bayespy.utils.random.gaussian\_logpdf

bayespy.utils.random.gaussian\_logpdf(yVy, yVmu, muVmu, logdet\_V, D)

Log-density of a Gaussian distribution.

$$\mathcal{G}(\mathbf{y}|\boldsymbol{\mu}, \mathbf{V}^{-1})$$

Parameters yVy: ndarray or double

$$\mathbf{y}^T \mathbf{V} \mathbf{y}$$

yVmu: ndarray or double

$$\mathbf{v}^T \mathbf{V} \boldsymbol{\mu}$$

muVmu: ndarray or double

$$\mu^T \mathbf{V} \mu$$

logdet\_V: ndarray or double

Log-determinant of the precision matrix,  $\log |\mathbf{V}|$ .

D: int

Dimensionality of the distribution.

#### bayespy.utils.random.intervals

bayespy.utils.random.intervals(N, length, amount=1, gap=0)

Return random non-overlapping parts of a sequence.

For instance, N=16, length=2 and amount=4: [0, |1, 2|, 3, 4, 5, |6, 7|, 8, 9, |10, 11|, |12, 13|, 14, 15] that is, [1,2,6,7,10,11,12,13]

However, the function returns only the indices of the beginning of the sequences, that is, in the example: [1,6,10,12]

### bayespy.utils.random.invwishart\_rand

```
bayespy.utils.random.invwishart_rand(nu, V)
```

#### bayespy.utils.random.logodds\_to\_probability

```
bayespy.utils.random.logodds_to_probability(x)
Solves p from log(p/(1-p))
```

### bayespy.utils.random.mask

```
bayespy.utils.random.mask (*shape, p=0.5)
Return a boolean array of the given shape.
```

Parameters d0, d1, ..., dn: int

Shape of the output.

**p** : value in range [0,1]

A probability that the elements are *True*.

### bayespy.utils.random.orth

```
bayespy.utils.random.orth (D) Draw random orthogonal matrix.
```

# bayespy.utils.random.sphere

```
\label{eq:continuous} \mbox{Draw random points uniformly on a unit sphere.}
```

Returns (latitude,longitude) in degrees.

### bayespy.utils.random.svd

```
bayespy.utils.random.svd(s)

Draw a random matrix given its singular values.
```

### bayespy.utils.random.t\_logpdf

```
bayespy.utils.random.t_logpdf(z2, logdet_cov, nu, D)
```

# bayespy.utils.random.wishart

```
bayespy.utils.random.wishart (nu, V)
Draw a random sample from the Wishart distribution.
```

Parameters nu: int

## bayespy.utils.random.wishart\_rand

bayespy.utils.random.wishart\_rand (nu, V)Draw a random sample from the Wishart distribution.

Parameters nu: int

# 6.4.3 bayespy.utils.optimize

#### **Functions**

<pre>check_gradient(f, x0[, verbose, epsilon,])</pre>	Simple wrapper for SciPy's gradient checker.
minimize(f, x0[, maxiter, verbose])	Simple wrapper for SciPy's optimize.

### bayespy.utils.optimize.check\_gradient

bayespy.utils.optimize.check\_gradient (f, x0, verbose=True, epsilon=1.4901161193847656e-08, return\_abserr=False)

Simple wrapper for SciPy's gradient checker.

The given function must return a tuple: (value, gradient).

Returns absolute and relative errors

# bayespy.utils.optimize.minimize

bayespy.utils.optimize.minimize (f, x0, maxiter=None, verbose=False) Simple wrapper for SciPy's optimize.

The given function must return a tuple: (value, gradient).

# 6.4.4 bayespy.utils.misc

General numerical functions and methods.

### **Functions**

T(X)	Transpose the matrix.
add_axes(X[, num, axis])	
add_leading_axes(x, n)	
add_trailing_axes(x, n)	
array_to_scalar(x)	
$atleast_nd(X, d)$	
<pre>axes_to_collapse(shape_x, shape_to)</pre>	
block_banded(D, B)	Construct a symmetric block-banded matrix.
broadcasted_shape(*shapes)	Computes the resulting broadcasted shape for a given set of shapes.
broadcasted_shape_from_arrays(*arrays)	Computes the resulting broadcasted shape for a given set of arrays.
broadcasting_multiplier(plates, *args)	Compute the plate multiplier for given shapes.
ceildiv(a, b)	Compute a divided by b and rounded up.
	Continued on next page

Table 6.67 – continued from previous page

	nded from previous page
chol(C)	
$chol_{inv}(U)$	
chol_logdet(U)	
$chol\_solve(U, b)$	
cholesky(K)	
composite_function(function_list)	Construct a function composition from a list of functions.
diag(X[, ndim])	Create a diagonal array given the diagonal elements.
diagonal(A)	
dist_haversine(c1, c2[, radius])	
find_set_index(index, set_lengths)	Given set sizes and an index, returns the index of the set
first(L)	
gaussian_logpdf(y_invcov_y, y_invcov_mu,)	
get_diag(X[, ndim])	Get the diagonal of an array.
grid(x1, x2)	Returns meshgrid as a (M*N,2)-shape array.
identity(*shape)	
invgamma(x)	Inverse gamma function.
invpsi(x)	Inverse digamma (psi) function.
is_callable(f)	
is_numeric(a)	
is_shape_subset(sub_shape, full_shape)	
is_string(s)	
isinteger(x)	
kalman_filter(y, U, A, V, mu0, Cov0[, out])	Perform Kalman filtering to obtain filtered mean and covariance.
logdet_chol(U)	
logsumexp(X[, axis, keepdims])	Compute log(sum(exp(X)) in a numerically stable way
m_digamma(a, d)	Returns the derivative of the log of multivariate gamma.
m_dot(A, b)	
$m_{\text{outer}}(A, B)$	
m_solve_triangular(U, B, **kwargs)	
make_equal_length(*shapes)	Make tuples equal length.
make_equal_ndim(*arrays)	Add trailing unit axes so that arrays have equal ndim
mean(X[, axis, keepdims])	Compute the mean, ignoring NaNs.
moveaxis(A, axis_from, axis_to)	Move the axis <i>axis_from</i> to position <i>axis_to</i> .
multidigamma(a, d)	Returns the derivative of the log of multivariate gamma.
multiply_shapes(*shapes)	Compute element-wise product of lists/tuples.
nans([size])	1
nested_iterator(max_inds)	
parse_command_line_arguments(mandatory_args,)	Parse command line arguments of style "-parameter=value".
remove_whitespace(s)	<u> </u>
repeat_to_shape(A, s)	
rmse(y1, y2[, axis])	
rts_smoother(mu, Cov, A, V[, removethis])	Perform Rauch-Tung-Striebel smoothing to obtain the posterior.
safe_indices(inds, shape)	Makes sure that indices are valid for given shape.
squeeze(X)	Remove leading axes that have unit length.
squeeze_to_dim(X, dim)	
sum_multiply(*args[, axis, sumaxis, keepdims])	
sum_multiply_to_plates(*arrays[, to_plates,])	Compute the product of the arguments and sum to the target shape.
sum_product(*args[, axes_to_keep,])	2 and product of the arguments and sum to the target shape.
sum_to_dim(A, dim)	Sum leading axes of A such that A has dim dimensions.
$\frac{\text{sum_to_shape}(X, s)}{\text{sum_to_shape}(X, s)}$	Sum axes of the array such that the resulting shape is as given.
	Continued on next page
	Continued on next page

Table 6.67 – continued from previous page

symm(X)	Make X symmetric.
tempfile([prefix, suffix])	
trues(shape)	
unique(l)	Remove duplicate items from a list while preserving order.
write_to_hdf5(group, data, name)	Writes the given array into the HDF5 file.
zipper_merge(*lists)	Combines lists by alternating elements from them.

### bayespy.utils.misc.T

bayespy.utils.misc. $\mathbf{T}(X)$ Transpose the matrix.

## bayespy.utils.misc.add\_axes

bayespy.utils.misc.add\_axes(X, num=1, axis=0)

### bayespy.utils.misc.add\_leading\_axes

bayespy.utils.misc.add\_leading\_axes (x, n)

### bayespy.utils.misc.add\_trailing\_axes

bayespy.utils.misc.add\_trailing\_axes(x, n)

#### bayespy.utils.misc.array\_to\_scalar

bayespy.utils.misc.array\_to\_scalar(x)

## bayespy.utils.misc.atleast\_nd

bayespy.utils.misc.atleast\_nd(X, d)

# bayespy.utils.misc.axes\_to\_collapse

bayespy.utils.misc.axes\_to\_collapse(shape\_x, shape\_to)

#### bayespy.utils.misc.block\_banded

bayespy.utils.misc.block\_banded(D, B)

Construct a symmetric block-banded matrix.

D contains square diagonal blocks. B contains super-diagonal blocks.

The resulting matrix is:

#### bayespy.utils.misc.broadcasted\_shape

```
bayespy.utils.misc.broadcasted_shape(*shapes)
```

Computes the resulting broadcasted shape for a given set of shapes.

Uses the broadcasting rules of NumPy. Raises an exception if the shapes do not broadcast.

### bayespy.utils.misc.broadcasted\_shape\_from\_arrays

```
bayespy.utils.misc.broadcasted_shape_from_arrays(*arrays)
```

Computes the resulting broadcasted shape for a given set of arrays.

Raises an exception if the shapes do not broadcast.

#### bayespy.utils.misc.broadcasting\_multiplier

```
bayespy.utils.misc.broadcasting_multiplier(plates, *args)
```

Compute the plate multiplier for given shapes.

The first shape is compared to all other shapes (using NumPy broadcasting rules). All the elements which are non-unit in the first shape but 1 in all other shapes are multiplied together.

This method is used, for instance, for computing a correction factor for messages to parents: If this node has non-unit plates that are unit plates in the parent, those plates are summed. However, if the message has unit axis for that plate, it should be first broadcasted to the plates of this node and then summed to the plates of the parent. In order to avoid this broadcasting and summing, it is more efficient to just multiply by the correct factor. This method computes that factor. The first argument is the full plate shape of this node (with respect to the parent). The other arguments are the shape of the message array and the plates of the parent (with respect to this node).

#### bayespy.utils.misc.ceildiv

```
bayespy.utils.misc.ceildiv (a, b)
Compute a divided by b and rounded up.
```

#### bayespy.utils.misc.chol

```
bayespy.utils.misc.chol(C)
```

#### bavespv.utils.misc.chol\_inv

```
bayespy.utils.misc.chol_inv(U)
```

# bayespy.utils.misc.chol\_logdet

```
bayespy.utils.misc.chol_logdet(U)
```

#### bayespy.utils.misc.chol\_solve

```
bayespy.utils.misc.chol_solve(U,b)
```

### bayespy.utils.misc.cholesky

```
bayespy.utils.misc.cholesky(K)
```

#### bayespy.utils.misc.composite\_function

```
bayespy.utils.misc.composite_function(function_list)
```

Construct a function composition from a list of functions.

Given a list of functions [f,g,h], constructs a function  $h \circ g \circ f$ . That is, returns a function z, for which z(x) = h(g(f(x))).

#### bayespy.utils.misc.diag

```
bayespy.utils.misc.diag(X, ndim=1)
```

Create a diagonal array given the diagonal elements.

The diagonal array can be multi-dimensional. By default, the last axis is transformed to two axes (diagonal matrix) but this can be changed using ndim keyword. For instance, an array with shape (K,L,M,N) can be transformed to a set of diagonal 4-D tensors with shape (K,L,M,N,M,N) by giving ndim=2. If ndim=3, the result has shape (K,L,M,N,L,M,N), and so on.

Diagonality means that for the resulting array Y holds:  $Y[...,i_1,i_2,...,i_ndim,j_1,j_2,...,j_ndim]$  is zero if  $i_n!=j_n$  for any n.

# bayespy.utils.misc.diagonal

```
bayespy.utils.misc.diagonal(A)
```

#### bayespy.utils.misc.dist\_haversine

```
bayespy.utils.misc.dist_haversine(c1, c2, radius=6372795)
```

### bayespy.utils.misc.find\_set\_index

```
bayespy.utils.misc.find_set_index(index, set_lengths)
```

Given set sizes and an index, returns the index of the set

The given index is for the concatenated list of the sets.

### bayespy.utils.misc.first

```
bayespy.utils.misc.first(L)
```

## bayespy.utils.misc.gaussian\_logpdf

bayespy.utils.misc.gaussian\_logpdf(y\_invcov\_y, y\_invcov\_mu, mu\_invcov\_mu, logdetcov, D)

#### bayespy.utils.misc.get\_diag

```
bayespy.utils.misc.get_diag(X, ndim=1)
Get the diagonal of an array.
```

If ndim>1, take the diagonal of the last 2\*ndim axes.

# bayespy.utils.misc.grid

```
bayespy.utils.misc.grid (xI, x2)
Returns meshgrid as a (M*N,2)-shape array.
```

#### bayespy.utils.misc.identity

```
bayespy.utils.misc.identity(*shape)
```

## bayespy.utils.misc.invgamma

```
bayespy.utils.misc.invgamma (x) Inverse gamma function.
```

See: http://mathoverflow.net/a/28977

# bayespy.utils.misc.invpsi

```
bayespy.utils.misc.invpsi(x)
Inverse digamma (psi) function.
```

The digamma function is the derivative of the log gamma function. This calculates the value Y > 0 for a value X such that digamma(Y) = X.

See: http://www4.ncsu.edu/~pfackler/

#### bayespy.utils.misc.is\_callable

```
bayespy.utils.misc.is_callable(f)
```

#### bayespy.utils.misc.is\_numeric

```
bayespy.utils.misc.is_numeric(a)
```

#### bayespy.utils.misc.is\_shape\_subset

```
bayespy.utils.misc.is_shape_subset (sub_shape, full_shape)
```

### bayespy.utils.misc.is\_string

```
bayespy.utils.misc.is_string(s)
```

# bayespy.utils.misc.isinteger

```
bayespy.utils.misc.isinteger(x)
```

#### bayespy.utils.misc.kalman\_filter

```
bayespy.utils.misc.kalman_filter(y, U, A, V, mu0, Cov0, out=None)
```

Perform Kalman filtering to obtain filtered mean and covariance.

The parameters of the process may vary in time, thus they are given as iterators instead of fixed values.

```
Parameters y : (N,D) array
```

"Normalized" noisy observations of the states, that is, the observations multiplied by the precision matrix U (and possibly other transformation matrices).

U: (N,D,D) array or N-list of (D,D) arrays

Precision matrix (i.e., inverse covariance matrix) of the observation noise for each time instance.

A: (N-1,D,D) array or (N-1)-list of (D,D) arrays

Dynamic matrix for each time instance.

V: (N-1,D,D) array or (N-1)-list of (D,D) arrays

Covariance matrix of the innovation noise for each time instance.

#### Returns mu: array

Filtered mean of the states.

Cov: array

Filtered covariance of the states.

#### See also:

rts\_smoother

#### bayespy.utils.misc.logdet\_chol

```
bayespy.utils.misc.logdet_chol(U)
```

#### bayespy.utils.misc.logsumexp

```
bayespy.utils.misc.logsumexp(X, axis=None, keepdims=False)
Compute log(sum(exp(X))) in a numerically stable way
```

# bayespy.utils.misc.m\_digamma

```
bayespy.utils.misc.m_digamma(a,d)
```

Returns the derivative of the log of multivariate gamma.

### bayespy.utils.misc.m\_dot

```
bayespy.utils.misc.m_dot(A, b)
```

### bayespy.utils.misc.m\_outer

```
bayespy.utils.misc.m_outer (A, B)
```

### bayespy.utils.misc.m\_solve\_triangular

```
bayespy.utils.misc.m_solve_triangular(U, B, **kwargs)
```

#### bayespy.utils.misc.make\_equal\_length

```
bayespy.utils.misc.make_equal_length(*shapes)

Make tuples equal length.
```

Add leading 1s to shorter tuples.

#### bayespy.utils.misc.make\_equal\_ndim

```
bayespy.utils.misc.make_equal_ndim(*arrays)

Add trailing unit axes so that arrays have equal ndim
```

# bayespy.utils.misc.mean

```
bayespy.utils.misc.mean (X, axis=None, keepdims=False)
Compute the mean, ignoring NaNs.
```

### bayespy.utils.misc.moveaxis

```
bayespy.utils.misc.moveaxis(A, axis_from, axis_to)
Move the axis axis_from to position axis_to.
```

#### bayespy.utils.misc.multidigamma

```
bayespy.utils.misc.multidigamma (a, d)
Returns the derivative of the log of multivariate gamma.
```

### bayespy.utils.misc.multiply\_shapes

```
bayespy.utils.misc.multiply_shapes(*shapes)
Compute element-wise product of lists/tuples.
```

Shorter lists are concatenated with leading 1s in order to get lists with the same length.

#### bayespy.utils.misc.nans

```
bayespy.utils.misc.nans(size=())
```

#### bayespy.utils.misc.nested\_iterator

```
bayespy.utils.misc.nested_iterator(max_inds)
```

#### bayespy.utils.misc.parse\_command\_line\_arguments

bayespy.utils.misc.parse\_command\_line\_arguments (mandatory\_args, \*optional\_args\_list, argv=None) \*optional\_args\_list,

Parse command line arguments of style "-parameter=value".

Parameter specification is tuple: (name, converter, description).

Some special handling:

- •If converter is None, the command line does not accept any value for it, but instead use either "-option" to enable or "-no-option" to disable.
- •If argument name contains hyphens, those are converted to underscores in the keys of the returned dictionaries.

#### Parameters mandatory\_args: list of tuples

Specs for mandatory arguments

optional\_args\_list : list of lists of tuples

Specs for each optional arguments set

argv: list of strings (optional)

The command line arguments. By default, read sys.argv.

Returns args: dictionary

The parsed mandatory arguments

**kwargs**: dictionary

The parsed optional arguments

#### **Examples**

```
>>> from pprint import pprint as print
>>> (args, kwargs) = parse_command_line_arguments(
        # Mandatory arguments
            ('name', str, "full name...
''age', int, "Age (years)"),
            ('employed', None, "Working"),
        # Optional arguments
                        str, "Phone number"),
            ('phone',
            ('favorite-color', str, "Favorite color")
        ],
        arqv=['--name=John Doe',
              '--age=42',
              '--no-employed',
. . .
              '--favorite-color=pink']
. . . )
>>> print (args)
{'age': 42, 'employed': False, 'name': 'John Doe'}
>>> print (kwargs)
{'favorite_color': 'pink'}
```

It is possible to have several optional argument sets:

```
>>> (args, kw_info, kw_fav) = parse_command_line_arguments(
        # Mandatory arguments
            ('name',
                        str, "Full name"),
        # Optional arguments (contact information)
            ('phone', str, "Phone number"),
            ('email', str, "E-mail address")
        ],
        # Optional arguments (preferences)
            ('favorite-color', str, "Favorite color"),
. . .
            ('favorite-food', str, "Favorite food")
        ],
        argv=['--name=John Doe',
              '--favorite-color=pink',
              '--email=john.doe@email.com',
              '--favorite-food=spaghetti']
...)
>>> print (args)
{'name': 'John Doe'}
>>> print (kw_info)
{ 'email': 'john.doe@email.com'}
>>> print(kw_fav)
{'favorite_color': 'pink', 'favorite_food': 'spaghetti'}
```

### bayespy.utils.misc.remove\_whitespace

```
bayespy.utils.misc.remove_whitespace(s)
```

#### bayespy.utils.misc.repeat\_to\_shape

```
bayespy.utils.misc.repeat_to_shape (A, s)
```

#### bayespy.utils.misc.rmse

```
bayespy.utils.misc.rmse(y1, y2, axis=None)
```

### bayespy.utils.misc.rts\_smoother

```
bayespy.utils.misc.rts_smoother(mu, Cov, A, V, removethis=None)
```

Perform Rauch-Tung-Striebel smoothing to obtain the posterior.

The function returns the posterior mean and covariance of each state. The parameters of the process may vary in time, thus they are given as iterators instead of fixed values.

Parameters mu: (N,D) array

Mean of the states from Kalman filter.

Cov: (N,D,D) array

Covariance of the states from Kalman filter.

A: (N-1,D,D) array or (N-1)-list of (D,D) arrays

Dynamic matrix for each time instance.

V: (N-1,D,D) array or (N-1)-list of (D,D) arrays

Covariance matrix of the innovation noise for each time instance.

Returns mu: array

Posterior mean of the states.

Cov: array

Posterior covariance of the states.

#### See also:

kalman\_filter

# bayespy.utils.misc.safe\_indices

```
bayespy.utils.misc.safe_indices(inds, shape)
```

Makes sure that indices are valid for given shape.

The shorter shape determines the length.

For instance,

```
>>> safe_indices( (3, 4, 5), (1, 6) ) (0, 5)
```

#### bayespy.utils.misc.squeeze

```
bayespy.utils.misc.squeeze(X)
```

Remove leading axes that have unit length.

For instance, a shape (1,1,4,1,3) will be reshaped to (4,1,3).

#### bayespy.utils.misc.squeeze\_to\_dim

```
bayespy.utils.misc.squeeze_to_dim(X, dim)
```

# bayespy.utils.misc.sum\_multiply

bayespy.utils.misc.sum\_multiply(\*args, axis=None, sumaxis=True, keepdims=False)

# bayespy.utils.misc.sum\_multiply\_to\_plates

```
bayespy.utils.misc.sum_multiply_to_plates(*arrays, to_plates=(), from_plates=None, ndim=0)
```

Compute the product of the arguments and sum to the target shape.

### bayespy.utils.misc.sum\_product

```
bayespy.utils.misc.sum_product(*args, axes_to_keep=None, axes_to_sum=None, keep-dims=False)
```

# bayespy.utils.misc.sum\_to\_dim

```
bayespy.utils.misc.sum_to_dim(A, dim)
Sum leading axes of A such that A has dim dimensions.
```

## bayespy.utils.misc.sum\_to\_shape

```
bayespy.utils.misc.sum_to_shape (X, s)
```

Sum axes of the array such that the resulting shape is as given.

Thus, the shape of the result will be s or an error is raised.

# bayespy.utils.misc.symm

```
bayespy.utils.misc.symm(X)

Make X symmetric.
```

# bayespy.utils.misc.tempfile

```
bayespy.utils.misc.tempfile(prefix='', suffix='')
```

#### bayespy.utils.misc.trues

```
bayespy.utils.misc.trues(shape)
```

#### bayespy.utils.misc.unique

```
bayespy.utils.misc.unique(l)
```

Remove duplicate items from a list while preserving order.

#### bayespy.utils.misc.write\_to\_hdf5

```
bayespy.utils.misc.write_to_hdf5 (group, data, name)
Writes the given array into the HDF5 file.
```

#### bayespy.utils.misc.zipper\_merge

```
bayespy.utils.misc.zipper_merge(*lists)
```

Combines lists by alternating elements from them.

Combining lists [1,2,3], ['a','b','c'] and [42,666,99] results in [1,'a',42,2,'b',666,3,'c',99]

The lists should have equal length or they are assumed to have the length of the shortest list.

This is known as alternating merge or zipper merge.

#### Classes

CholeskyDense(K)	
CholeskySparse(K)	
TestCase([methodName])	Simple base class for unit testing.

### bayespy.utils.misc.CholeskyDense

 ${f class}$  bayespy.utils.misc.CholeskyDense(K)

 $\_\mathtt{init}_{-}(K)$ 

#### **Methods**

init( <b>K</b> )	
logdet()	
solve(b)	
trace_solve_gradient(dK)	

bayespy.utils.misc.CholeskyDense.\_\_init\_\_

CholeskyDense.\_\_**init**\_\_(K)

bayespy.utils.misc.CholeskyDense.logdet

CholeskyDense.logdet()

bayespy.utils.misc.CholeskyDense.solve

CholeskyDense.solve(b)

 $bayespy.utils.misc. Cholesky Dense.trace\_solve\_gradient$ 

CholeskyDense.trace\_solve\_gradient(dK)

# bayespy.utils.misc.CholeskySparse

class bayespy.utils.misc.CholeskySparse(K)

 $\_$ init $\_$ (K)

Methods

init( <b>K</b> )	
logdet()	
solve(b)	
trace_solve_gradient(dK)	

# bayespy.utils.misc.CholeskySparse.\_\_init\_\_

CholeskySparse.\_\_init\_\_(K)

# bayespy.utils.misc.CholeskySparse.logdet

CholeskySparse.logdet()

# bayespy.utils.misc.CholeskySparse.solve

CholeskySparse.solve(b)

### bayespy.utils.misc.CholeskySparse.trace\_solve\_gradient

CholeskySparse.trace\_solve\_gradient(dK)

### bayespy.utils.misc.TestCase

class bayespy.utils.misc.TestCase (methodName='runTest')

Simple base class for unit testing.

Adds NumPy's features to Python's unittest.

\_\_init\_\_ (methodName='runTest')

Create an instance of the class that will use the named test method when executed. Raises a ValueError if the instance does not have a method with the specified name.

#### **Methods**

init([methodName])	Create an instance of the class that will use the named test method when e
addCleanup(function, *args, **kwargs)	Add a function, with arguments, to be called when the test is completed.
addTypeEqualityFunc(typeobj, function)	Add a type specific assertEqual style function to compare a type.
assertAllClose(A, B[, msg, rtol, atol])	
<pre>assertAlmostEqual(first, second[, places,])</pre>	Fail if the two objects are unequal as determined by their difference round
assertAlmostEquals(*args, **kwargs)	
assertArrayEqual(A, B[, msg])	
assertCountEqual(first, second[, msg])	An unordered sequence comparison asserting that the same elements, reg-
assertDictContainsSubset(subset, dictionary)	Checks whether dictionary is a superset of subset.
assertDictEqual(d1, d2[, msg])	
assertEqual(first, second[, msg])	Fail if the two objects are unequal as determined by the '==' operator.
assertEquals(*args, **kwargs)	
assertFalse(expr[, msg])	Check that the expression is false.

	Table 6.71-
assertGreater(a, b[, msg])	Just like self.assertTrue( $a > b$ ), but with a nicer default message.
assertGreaterEqual(a, b[, msg])	Just like self.assertTrue( $a \ge b$ ), but with a nicer default message.
<pre>assertIn(member, container[, msg])</pre>	Just like self.assertTrue(a in b), but with a nicer default message.
<pre>assertIs(expr1, expr2[, msg])</pre>	Just like self.assertTrue(a is b), but with a nicer default message.
assertIsInstance(obj, cls[, msg])	Same as self.assertTrue(isinstance(obj, cls)), with a nicer default message
assertIsNone(obj[, msg])	Same as self.assertTrue(obj is None), with a nicer default message.
<pre>assertIsNot(expr1, expr2[, msg])</pre>	Just like self.assertTrue(a is not b), but with a nicer default message.
assertIsNotNone(obj[, msg])	Included for symmetry with assertIsNone.
assertLess(a, b[, msg])	Just like self.assertTrue(a < b), but with a nicer default message.
assertLessEqual(a, b[, msg])	Just like self.assertTrue(a <= b), but with a nicer default message.
assertListEqual(list1, list2[, msg])	A list-specific equality assertion.
assertLogs([logger, level])	Fail unless a log message of level level or higher is emitted on logger_nan
assertMessage(M1, M2)	
assert Message To Child (X, u)	
assertMultiLineEqual(first, second[, msg])	Assert that two multi-line strings are equal.
assertNotAlmostEqual(first, second[,])	Fail if the two objects are equal as determined by their difference rounded
assertNotAlmostEquals(*args, **kwargs)	
assertNotEqual(first, second[, msg])	Fail if the two objects are equal as determined by the '!=' operator.
assertNotEquals(*args, **kwargs)	
<pre>assertNotIn(member, container[, msg])</pre>	Just like self.assertTrue(a not in b), but with a nicer default message.
assertNotIsInstance(obj, cls[, msg])	Included for symmetry with assertIsInstance.
<pre>assertNotRegex(text, unexpected_regex[, msg])</pre>	Fail the test if the text matches the regular expression.
assertRaises(expected_exception, *args, **kwargs)	Fail unless an exception of class expected_exception is raised by the callal
assertRaisesRegex(expected_exception,)	Asserts that the message in a raised exception matches a regex.
assertRaisesRegexp(*args, **kwargs)	
<pre>assertRegex(text, expected_regex[, msg])</pre>	Fail the test unless the text matches the regular expression.
assertRegexpMatches(*args, **kwargs)	
<pre>assertSequenceEqual(seq1, seq2[, msg, seq_type])</pre>	An equality assertion for ordered sequences (like lists and tuples).
<pre>assertSetEqual(set1, set2[, msg])</pre>	A set-specific equality assertion.
assertTrue(expr[, msg])	Check that the expression is true.
<pre>assertTupleEqual(tuple1, tuple2[, msg])</pre>	A tuple-specific equality assertion.
assertWarns(expected_warning, *args, **kwargs)	Fail unless a warning of class warnClass is triggered by the callable when
<pre>assertWarnsRegex(expected_warning,)</pre>	Asserts that the message in a triggered warning matches a regexp.
assert_(*args, **kwargs)	
countTestCases()	
debug()	Run the test without collecting errors in a TestResult
defaultTestResult()	
doCleanups()	Execute all cleanup functions.
<pre>fail([msg])</pre>	Fail immediately, with the given message.
failIf(*args, **kwargs)	
failIfAlmostEqual(*args, **kwargs)	
failIfEqual(*args, **kwargs)	
failUnless(*args, **kwargs)	
<pre>failUnlessAlmostEqual(*args, **kwargs)</pre>	
<pre>failUnlessEqual(*args, **kwargs)</pre>	
failUnlessRaises(*args, **kwargs)	
<i>id</i> ()	
run([result])	
setUp()	Hook method for setting up the test fixture before exercising it.
setUpClass()	Hook method for setting up class fixture before running tests in the class.

#### Table 6.71 -

shortDescription()	Returns a one-line description of the test, or None if no description has be
skipTest(reason)	Skip this test.
subTest([msg])	Return a context manager that will return the enclosed block of code in a
tearDown()	Hook method for deconstructing the test fixture after testing it.
tearDownClass()	Hook method for deconstructing the class fixture after running all tests in

#### bayespy.utils.misc.TestCase.\_\_init\_\_

TestCase.\_\_init\_\_(methodName='runTest')

Create an instance of the class that will use the named test method when executed. Raises a ValueError if the instance does not have a method with the specified name.

#### bayespy.utils.misc.TestCase.addCleanup

TestCase.addCleanup (function, \*args, \*\*kwargs)

Add a function, with arguments, to be called when the test is completed. Functions added are called on a LIFO basis and are called after tearDown on test failure or success.

Cleanup items are called even if setUp fails (unlike tearDown).

#### bayespy.utils.misc.TestCase.addTypeEqualityFunc

TestCase.addTypeEqualityFunc (typeobj, function)

Add a type specific assertEqual style function to compare a type.

This method is for use by TestCase subclasses that need to register their own type equality functions to provide nicer error messages.

Args:

**typeobj:** The data type to call this function on when both values are of the same type in assertE-qual().

**function:** The callable taking two arguments and an optional msg= argument that raises self.failureException with a useful error message when the two arguments are not equal.

#### bayespy.utils.misc.TestCase.assertAllClose

TestCase.assertAllClose (A, B, msg='Arrays not almost equal', rtol=0.0001, atol=0)

#### bayespy.utils.misc.TestCase.assertAlmostEqual

 $\texttt{TestCase.assertAlmostEqual} \ (\textit{first, second, places=None, msg=None, delta=None})$ 

Fail if the two objects are unequal as determined by their difference rounded to the given number of decimal places (default 7) and comparing to zero, or by comparing that the between the two objects is more than the given delta.

Note that decimal places (from zero) are usually not the same as significant digits (measured from the most significant digit).

If the two objects compare equal then they will automatically compare almost equal.

#### bayespy.utils.misc.TestCase.assertAlmostEquals

TestCase.assertAlmostEquals(\*args, \*\*kwargs)

#### bayespy.utils.misc.TestCase.assertArrayEqual

TestCase.assertArrayEqual (A, B, msg='Arrays not equal')

#### bayespy.utils.misc.TestCase.assertCountEqual

```
TestCase.assertCountEqual (first, second, msg=None)
```

An unordered sequence comparison asserting that the same elements, regardless of order. If the same element occurs more than once, it verifies that the elements occur the same number of times.

#### self.assertEqual(Counter(list(first)), Counter(list(second)))

#### **Example:**

- [0, 1, 1] and [1, 0, 1] compare equal.
- [0, 0, 1] and [0, 1] compare unequal.

### bayespy.utils.misc.TestCase.assertDictContainsSubset

TestCase.assertDictContainsSubset (subset, dictionary, msg=None)

Checks whether dictionary is a superset of subset.

#### bayespy.utils.misc.TestCase.assertDictEqual

```
TestCase.assertDictEqual(d1, d2, msg=None)
```

#### bayespy.utils.misc.TestCase.assertEqual

```
TestCase.assertEqual (first, second, msg=None)
```

Fail if the two objects are unequal as determined by the '==' operator.

#### bayespy.utils.misc.TestCase.assertEquals

```
TestCase.assertEquals(*args, **kwargs)
```

### bayespy.utils.misc.TestCase.assertFalse

```
TestCase.assertFalse(expr, msg=None)
```

Check that the expression is false.

#### bayespy.utils.misc.TestCase.assertGreater

```
TestCase.assertGreater(a, b, msg=None)
```

Just like self.assertTrue(a > b), but with a nicer default message.

#### bayespy.utils.misc.TestCase.assertGreaterEqual

```
TestCase.assertGreaterEqual (a, b, msg=None)
```

Just like self.assertTrue( $a \ge b$ ), but with a nicer default message.

#### bayespy.utils.misc.TestCase.assertIn

```
TestCase.assertIn (member, container, msg=None)
```

Just like self.assertTrue(a in b), but with a nicer default message.

### bayespy.utils.misc.TestCase.assertIs

```
TestCase.assertIs(expr1, expr2, msg=None)
```

Just like self.assertTrue(a is b), but with a nicer default message.

#### bayespy.utils.misc.TestCase.assertIsInstance

```
TestCase.assertIsInstance(obj, cls, msg=None)
```

Same as self.assertTrue(isinstance(obj, cls)), with a nicer default message.

#### bayespy.utils.misc.TestCase.assertIsNone

```
TestCase.assertIsNone(obj, msg=None)
```

Same as self.assertTrue(obj is None), with a nicer default message.

### bayespy.utils.misc.TestCase.assertIsNot

```
TestCase.assertIsNot (expr1, expr2, msg=None)
```

Just like self.assertTrue(a is not b), but with a nicer default message.

#### bayespy.utils.misc.TestCase.assertIsNotNone

```
TestCase.assertIsNotNone(obj, msg=None)
```

Included for symmetry with assertIsNone.

#### bayespy.utils.misc.TestCase.assertLess

```
TestCase.assertLess(a, b, msg=None)
```

Just like self.assertTrue(a < b), but with a nicer default message.

#### bayespy.utils.misc.TestCase.assertLessEqual

```
TestCase.assertLessEqual(a, b, msg=None)
```

Just like self.assertTrue( $a \le b$ ), but with a nicer default message.

#### bayespy.utils.misc.TestCase.assertListEqual

```
TestCase.assertListEqual(list1, list2, msg=None)
```

A list-specific equality assertion.

**Args:** list1: The first list to compare. list2: The second list to compare. msg: Optional message to use on failure instead of a list of

differences.

#### bayespy.utils.misc.TestCase.assertLogs

```
TestCase.assertLogs(logger=None, level=None)
```

Fail unless a log message of level *level* or higher is emitted on *logger\_name* or its children. If omitted, *level* defaults to INFO and *logger* defaults to the root logger.

This method must be used as a context manager, and will yield a recording object with two attributes: *output* and *records*. At the end of the context manager, the *output* attribute will be a list of the matching formatted log messages and the *records* attribute will be a list of the corresponding LogRecord objects.

#### Example:

#### bayespy.utils.misc.TestCase.assertMessage

```
TestCase.assertMessage (M1, M2)
```

#### bayespy.utils.misc.TestCase.assertMessageToChild

```
\texttt{TestCase.assertMessageToChild}\left(X,u\right)
```

#### bayespy.utils.misc.TestCase.assertMultiLineEqual

```
TestCase.assertMultiLineEqual (first, second, msg=None)
```

Assert that two multi-line strings are equal.

#### bayespy.utils.misc.TestCase.assertNotAlmostEqual

```
TestCase.assertNotAlmostEqual (first, second, places=None, msg=None, delta=None)
```

Fail if the two objects are equal as determined by their difference rounded to the given number of decimal

places (default 7) and comparing to zero, or by comparing that the between the two objects is less than the given delta.

Note that decimal places (from zero) are usually not the same as significant digits (measured from the most significant digit).

Objects that are equal automatically fail.

### bayespy.utils.misc.TestCase.assertNotAlmostEquals

```
TestCase.assertNotAlmostEquals(*args, **kwargs)
```

### bayespy.utils.misc.TestCase.assertNotEqual

```
TestCase.assertNotEqual (first, second, msg=None)
```

Fail if the two objects are equal as determined by the '!=' operator.

#### bayespy.utils.misc.TestCase.assertNotEquals

```
TestCase.assertNotEquals(*args, **kwargs)
```

### bayespy.utils.misc.TestCase.assertNotIn

```
TestCase.assertNotIn (member, container, msg=None)
```

Just like self.assertTrue(a not in b), but with a nicer default message.

## bayespy.utils.misc.TestCase.assertNotIsInstance

```
TestCase.assertNotIsInstance(obj, cls, msg=None)
```

Included for symmetry with assertIsInstance.

## bayespy.utils.misc.TestCase.assertNotRegex

```
TestCase.assertNotRegex (text, unexpected_regex, msg=None)
```

Fail the test if the text matches the regular expression.

#### bayespy.utils.misc.TestCase.assertRaises

```
TestCase.assertRaises (expected_exception, *args, **kwargs)
```

Fail unless an exception of class expected\_exception is raised by the callable when invoked with specified positional and keyword arguments. If a different type of exception is raised, it will not be caught, and the test case will be deemed to have suffered an error, exactly as for an unexpected exception.

If called with the callable and arguments omitted, will return a context object used like this:

```
with self.assertRaises(SomeException):
    do_something()
```

An optional keyword argument 'msg' can be provided when assertRaises is used as a context object.

The context manager keeps a reference to the exception as the 'exception' attribute. This allows you to inspect the exception after the assertion:

```
with self.assertRaises(SomeException) as cm:
    do_something()
    the_exception = cm.exception
    self.assertEqual(the_exception.error_code, 3)
```

#### bayespy.utils.misc.TestCase.assertRaisesRegex

TestCase.assertRaisesRegex (expected\_exception, expected\_regex, \*args, \*\*kwargs)

Asserts that the message in a raised exception matches a regex.

**Args:** expected\_exception: Exception class expected to be raised. expected\_regex: Regex (re pattern object or string) expected

to be found in error message.

args: Function to be called and extra positional args. kwargs: Extra kwargs. msg: Optional message used in case of failure. Can only be used

when assertRaisesRegex is used as a context manager.

### bayespy.utils.misc.TestCase.assertRaisesRegexp

```
TestCase.assertRaisesRegexp(*args, **kwargs)
```

#### bayespy.utils.misc.TestCase.assertRegex

TestCase.assertRegex (text, expected\_regex, msg=None)

Fail the test unless the text matches the regular expression.

#### bayespy.utils.misc.TestCase.assertRegexpMatches

```
TestCase.assertRegexpMatches(*args, **kwargs)
```

### bayespy.utils.misc.TestCase.assertSequenceEqual

```
TestCase.assertSequenceEqual (seq1, seq2, msg=None, seq_type=None)
```

An equality assertion for ordered sequences (like lists and tuples).

For the purposes of this function, a valid ordered sequence type is one which can be indexed, has a length, and has an equality operator.

**Args:** seq1: The first sequence to compare. seq2: The second sequence to compare. seq\_type: The expected datatype of the sequences, or None if no

datatype should be enforced.

msg: Optional message to use on failure instead of a list of differences.

#### bayespy.utils.misc.TestCase.assertSetEqual

```
TestCase.assertSetEqual (set1, set2, msg=None)
```

A set-specific equality assertion.

**Args:** set1: The first set to compare. set2: The second set to compare. msg: Optional message to use on failure instead of a list of

differences.

assertSetEqual uses ducktyping to support different types of sets, and is optimized for sets specifically (parameters must support a difference method).

#### bayespy.utils.misc.TestCase.assertTrue

```
TestCase.assertTrue(expr, msg=None)
```

Check that the expression is true.

## bayespy.utils.misc.TestCase.assertTupleEqual

```
TestCase.assertTupleEqual (tuple1, tuple2, msg=None)
```

A tuple-specific equality assertion.

**Args:** tuple1: The first tuple to compare. tuple2: The second tuple to compare. msg: Optional message to use on failure instead of a list of

differences.

## bayespy.utils.misc.TestCase.assertWarns

```
TestCase.assertWarns (expected_warning, *args, **kwargs)
```

Fail unless a warning of class warnClass is triggered by the callable when invoked with specified positional and keyword arguments. If a different type of warning is triggered, it will not be handled: depending on the other warning filtering rules in effect, it might be silenced, printed out, or raised as an exception.

If called with the callable and arguments omitted, will return a context object used like this:

```
with self.assertWarns(SomeWarning):
    do_something()
```

An optional keyword argument 'msg' can be provided when assertWarns is used as a context object.

The context manager keeps a reference to the first matching warning as the 'warning' attribute; similarly, the 'filename' and 'lineno' attributes give you information about the line of Python code from which the warning was triggered. This allows you to inspect the warning after the assertion:

```
with self.assertWarns(SomeWarning) as cm:
    do_something()
the_warning = cm.warning
self.assertEqual(the_warning.some_attribute, 147)
```

#### bayespy.utils.misc.TestCase.assertWarnsRegex

```
TestCase.assertWarnsRegex(expected_warning, expected_regex, *args, **kwargs)
```

Asserts that the message in a triggered warning matches a regexp. Basic functioning is similar to assertWarns() with the addition that only warnings whose messages also match the regular expression are considered successful matches.

**Args:** expected\_warning: Warning class expected to be triggered. expected\_regex: Regex (re pattern object or string) expected

to be found in error message.

args: Function to be called and extra positional args. kwargs: Extra kwargs. msg: Optional message used in case of failure. Can only be used

when assertWarnsRegex is used as a context manager.

### bayespy.utils.misc.TestCase.assert

```
TestCase.assert_(*args, **kwargs)
```

## bayespy.utils.misc.TestCase.countTestCases

```
TestCase.countTestCases()
```

## bayespy.utils.misc.TestCase.debug

```
TestCase.debug()
```

Run the test without collecting errors in a TestResult

## bayespy.utils.misc.TestCase.defaultTestResult

```
TestCase.defaultTestResult()
```

## bayespy.utils.misc.TestCase.doCleanups

```
TestCase.doCleanups()
```

Execute all cleanup functions. Normally called for you after tearDown.

## bayespy.utils.misc.TestCase.fail

```
TestCase.fail (msg=None)
```

Fail immediately, with the given message.

## bayespy.utils.misc.TestCase.faillf

```
TestCase.failIf(*args, **kwargs)
```

```
TestCase.failIfAlmostEqual(*args, **kwargs)
bayespy.utils.misc.TestCase.failIfEqual
TestCase.failIfEqual (*args, **kwargs)
bayespy.utils.misc.TestCase.failUnless
TestCase.failUnless(*args, **kwargs)
bayespy.utils.misc.TestCase.failUnlessAlmostEqual
TestCase.failUnlessAlmostEqual(*args, **kwargs)
bayespy.utils.misc.TestCase.failUnlessEqual
TestCase.failUnlessEqual(*args, **kwargs)
bayespy.utils.misc.TestCase.failUnlessRaises
TestCase.failUnlessRaises(*args, **kwargs)
bayespy.utils.misc.TestCase.id
TestCase.id()
bayespy.utils.misc.TestCase.run
TestCase.run (result=None)
bayespy.utils.misc.TestCase.setUp
TestCase.setUp()
    Hook method for setting up the test fixture before exercising it.
bayespy.utils.misc.TestCase.setUpClass
TestCase.setUpClass()
    Hook method for setting up class fixture before running tests in the class.
```

bayespy.utils.misc.TestCase.failIfAlmostEqual

#### bayespy.utils.misc.TestCase.shortDescription

```
TestCase.shortDescription()
```

Returns a one-line description of the test, or None if no description has been provided.

The default implementation of this method returns the first line of the specified test method's docstring.

### bayespy.utils.misc.TestCase.skipTest

```
TestCase.skipTest (reason)
Skip this test.
```

#### bayespy.utils.misc.TestCase.subTest

```
TestCase.subTest (msg=None, **params)
```

Return a context manager that will return the enclosed block of code in a subtest identified by the optional message and keyword parameters. A failure in the subtest marks the test case as failed but resumes execution at the end of the enclosed block, allowing further test code to be executed.

#### bayespy.utils.misc.TestCase.tearDown

```
TestCase.tearDown()
```

Hook method for deconstructing the test fixture after testing it.

## bayespy.utils.misc.TestCase.tearDownClass

```
TestCase.tearDownClass()
```

Hook method for deconstructing the class fixture after running all tests in the class.

#### **Attributes**

longMessage	
maxDiff	

#### bayespy.utils.misc.TestCase.longMessage

```
TestCase.longMessage = True
```

## bayes py. utils. misc. Test Case. max Diff

```
TestCase.maxDiff = 640
```

### **Bibliography**

- · Bibliography
- · genindex

- modindex
- search

- [1] James Hensman, Magnus Rattray, and Neil D. Lawrence. Fast variational inference in the conjugate exponential family. In F. Pereira, C.J.C. Burges, L. Bottou, and K.Q. Weinberger, editors, *Advances in Neural Information Processing Systems* 25, 2888–2896. Lake Tahoe, Nevada, USA, 2012.
- [2] Matthew D. Hoffman, David M. Blei, Chong Wang, and John Paisley. Stochastic variational inference. *Journal of Machine Learning Research*, 14:1303–47, 2013.
- [3] Antti Honkela, Tapani Raiko, Mikael Kuusela, Matti Tornio, and Juha Karhunen. Approximate Riemannian conjugate gradient learning for fixed-form variational Bayes. *Journal of Machine Learning Research*, 11:3235–3268, 2010.
- [4] Antti Honkela, Harri Valpola, and Juha Karhunen. Accelerating cyclic update algorithms for parameter estimation by pattern searches. *Neural Processing Letters*, 17(2):191–203, 2003. doi:10.1023/A:1023655202546.
- [5] K. Katahira, K. Watanabe, and M. Okada. Deterministic annealing variant of variational Bayes method. *Journal of Physics: Conference Series*, 2008.
- [6] Jaakko Luttinen. Fast variational Bayesian linear state-space model. In Hendrik Blockeel, Kristian Kersting, Siegfried Nijssen, and Filip Železný, editors, *Machine Learning and Knowledge Discovery in Databases*, volume 8188 of Lecture Notes in Computer Science, pages 305–320. Springer, 2013. doi:10.1007/978-3-642-40988-2\_20.
- [7] Jaakko Luttinen and Alexander Ilin. Transformations in variational Bayesian factor analysis to speed up learning. *Neurocomputing*, 73:1093–1102, 2010. doi:10.1016/j.neucom.2009.11.018.
- [8] Jaakko Luttinen, Tapani Raiko, and Alexander Ilin. Linear state-space model with time-varying dynamics. In Toon Calders, Floriana Esposito, Eyke Hüllermeier, and Rosa Meo, editors, *Machine Learning and Knowledge Discovery in Databases*, volume 8725 of Lecture Notes in Computer Science, pages 338–353. Springer, 2014. doi:10.1007/978-3-662-44851-9\_22.

330 Bibliography

## PYTHON MODULE INDEX

# b

bayespy.inference, 194
bayespy.nodes, 81
bayespy.plot, 205
bayespy.utils.linalg, 295
bayespy.utils.misc, 302
bayespy.utils.optimize, 302
bayespy.utils.random, 297

332 Python Module Index

Symbols	init()	(bayes py. inference. vmp. nodes. gaussian. Gaussian Gamma ARD Discovery and the property of
init() (bayespy.inference.VB method), 194, 195		method), 263
init() (bayespy.inference.vmp.nodes.bernoulli.BernoulliD		(hayespy.inference.vmp.nodes.gaussian.GaussianGammaARDMomethod), 242
init() (bayespy.inference.vmp.nodes.bernoulli.Bernoulli.M		(bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToC method), 229, 230
init() (bayespy.inference.vmp.nodes.beta.BetaDistribution		(bayespy.inference.vmp.nodes.gaussian.GaussianGammaISODistrmethod), 261
init() (bayespy.inference.vmp.nodes.beta.BetaMoments		(bayespy.inference.vmp.nodes.gaussian.GaussianGammaISOMommethod), 240, 241
init() (bayespy.inference.vmp.nodes.binomial.BinomialD		(hayespy.inference.vmp.nodes.gaussian.GaussianGammaISOToGamethod), 227, 228
init() (bayespy.inference.vmp.nodes.binomial.BinomialM		memod), 239
init() (bayespy.inference.vmp.nodes.categorical.Categoric		(bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGamimethod), 225, 226
init() (bayespy.inference.vmp.nodes.categorical.Categoric method), 249	calinton	(hayespy.inference.vmp.nodes.gaussian.GaussianWishartDistributionsthod), 265
init() (bayespy.inference.vmp.nodes.categorical_markov_method), 289	chaih.C	(bayespayinference) xmp nodes gaussian. Gaussian Wishart Moments method), 243
init() (bayespy.inference.vmp.nodes.categorical_markov_method), 250	chaih.C	(bayespy)inference ympynodest gaussian. WrapToGaussianGamma/method), 233, 234
init() (bayespy.inference.vmp.nodes.constant.Constant		(bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGammaI method), 231, 232
init() (bayespy.inference.vmp.nodes.deterministic.Determ		(bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart method), 235, 236
init() (bayespy.inference.vmp.nodes.dirichlet.DirichletDr		(bayespy.inference.vmp.nodes.gaussian_markov_chain.GaussianM method), 266, 267
init() (bayespy.inference.vmp.nodes.dirichlet.DirichletMo		(bayespy.inference.vmp.nodes.gaussian_markov_chain.GaussianM method), 240
init() (bayespy.inference.vmp.nodes.expfamily.Exponenti		(bayespy.inference.vmp.nodes.gaussian_markov_chain.SwitchingCmethod), 269, 270
init() (bayespy.inference.vmp.nodes.expfamily.Exponenti		(bayespyrinference.vmp.nodes.gaussian_markov_chain.VaryingGaumethod), 273
init() (bayespy.inference.vmp.nodes.gamma.GammaDistr method), 276	ribution)	(bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution method), 291
init() (bayespy.inference.vmp.nodes.gamma.GammaMon method), 244	nentt()	(bayespy.inference.vmp.nodes.multinomial.MultinomialMoments method), 251
init() (bayespy.inference.vmp.nodes.gaussian.GaussianAimethod), 259	RÜÜistr	ibution (bayespy.inference.vmp.nodes.node.Moments method), 238
init() (bayespy.inference.vmp.nodes.gaussian.GaussianDemethod), 256	istPibuti)	on (bayespy.inference.vmp.nodes.node.Node method), 211, 212

```
__init__() (bayespy.inference.vmp.nodes.poisson.PoissonDistribution method), 174, 175
         method), 293
                                                         __init__() (bayespy.nodes.Wishart method), 97, 98
__init__() (bayespy.inference.vmp.nodes.poisson.PoissonMoments_()
                                                                    (bayespy.plot.CategoricalMarkovChainPlotter
         method), 252
                                                                   method), 210
_init_() (bayespy.inference.vmp.nodes.stochastic.Distributioninit_() (bayespy.plot.ContourPlotter method), 208, 209
         method), 253
                                                         __init__() (bayespy.plot.FunctionPlotter method), 209
__init__() (bayespy.inference.vmp.nodes.stochastic.Stochastic_init__()
                                                                         (bayespy.plot.GaussianTimeseriesPlotter
         method), 213, 214
                                                                   method), 210
_init_() (bayespy.inference.vmp.nodes.wishart.WishartDistribution() (bayespy.plot.HintonPlotter method), 209
         method), 278
                                                         __init__() (bayespy.plot.PDFPlotter method), 208
_init_() (bayespy.inference.vmp.nodes.wishart.WishartMomants_() (bayespy.plot.Plotter method), 208
         method), 244
                                                         __init__() (bayespy.utils.misc.CholeskyDense method),
__init__() (bayespy.inference.vmp.transformations.RotateGaussian
         method), 199
                                                         __init__() (bayespy.utils.misc.CholeskySparse method),
__init__() (bayespy.inference.vmp.transformations.RotateGaussianARD)14, 315
         method), 200
                                                         __init__() (bayespy.utils.misc.TestCase method), 315, 317
__init__() (bayespy.inference.vmp.transformations.RotateGaussianMarkovChain
         method), 201, 202
_init__() (bayespy.inference.vmp.transformations.RotateMultiple(class in bayespy.nodes), 188
         method), 204, 205
                                                         add() (bayespy.inference.VB method), 195
_init__() (bayespy.inference.vmp.transformations.RotateSwitchingMarkon Chainle bayespy.utils.misc), 304
         method), 202, 203
                                                         add_converter() (bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments
__init__() (bayespy.inference.vmp.transformations.RotateVaryingMarkoviChain, 247
         method), 203, 204
                                                         add_converter() (bayespy.inference.vmp.nodes.beta.BetaMoments
__init__() (bayespy.inference.vmp.transformations.RotationOptimizer method), 246
         method), 198, 199
                                                         add_converter() (bayespy.inference.vmp.nodes.binomial.BinomialMoments
__init__() (bayespy.nodes.Add method), 188, 190
                                                                   method), 248
__init__() (bayespy.nodes.Bernoulli method), 122, 123
                                                         add_converter() (bayespy.inference.vmp.nodes.categorical.CategoricalMom
__init__() (bayespy.nodes.Beta method), 147
                                                                   method), 249
__init__() (bayespy.nodes.Binomial method), 127, 128
                                                         add_converter() (bayespy.inference.vmp.nodes.categorical_markov_chain.Ca
__init__() (bayespy.nodes.Categorical method), 131, 132
                                                                   method), 250
__init__()
               (bayespy.nodes.CategoricalMarkovChain
                                                         add_converter() (bayespy.inference.vmp.nodes.dirichlet.DirichletMoments
         method), 157
                                                                   method), 246
__init__() (bayespy.nodes.Dirichlet method), 151, 152
                                                         add\_converter()\ (bayespy.inference.vmp.nodes.gamma.GammaMoments
__init__() (bayespy.nodes.Exponential method), 102
                                                                   method), 244
__init__() (bayespy.nodes.Gamma method), 92, 93
                                                         add_converter() (bayespy.inference.vmp.nodes.gaussian.GaussianGammaAl
__init__() (bayespy.nodes.Gate method), 191, 192
                                                                   method), 242
__init__() (bayespy.nodes.Gaussian method), 81, 82
                                                         add_converter() (bayespy.inference.vmp.nodes.gaussian.GaussianGammaIS
__init__() (bayespy.nodes.GaussianARD method), 87, 88
                                                                   method), 241
__init__() (bayespy.nodes.GaussianGammaARD method),
                                                         add_converter() (bayespy.inference.vmp.nodes.gaussian.GaussianMoments
                                                                   method), 239
__init__() (bayespy.nodes.GaussianGammaISO method),
                                                         add_converter() (bayespy.inference.vmp.nodes.gaussian.GaussianWishartM
         106, 107
                                                                   method), 243
__init__() (bayespy.nodes.GaussianMarkovChain method),
                                                         add_converter() (bayespy.inference.vmp.nodes.gaussian_markov_chain.Gaus
                                                                   method), 240
__init__() (bayespy.nodes.GaussianWishart method), 117,
                                                         add_converter() (bayespy.inference.vmp.nodes.multinomial.MultinomialMo
         118
                                                                   method), 251
__init__() (bayespy.nodes.Mixture method), 180
                                                         add\_converter()\ (bayespy.inference.vmp.nodes.node.Moments
__init__() (bayespy.nodes.Multinomial method), 136, 137
                                                                   class method), 238
__init__() (bayespy.nodes.Poisson method), 141, 142
                                                         add_converter() (bayespy.inference.vmp.nodes.poisson.PoissonMoments
__init__() (bayespy.nodes.SumMultiply method), 186
                                                                   method), 252
__init__() (bayespy.nodes.SwitchingGaussianMarkovChain
                                                         add_converter() (bayespy.inference.vmp.nodes.wishart.WishartMoments
         method), 168, 169
                                                                   method), 245
__init__() (bayespy.nodes.VaryingGaussianMarkovChain
                                                         add_leading_axes() (in module bayespy.utils.misc), 304
```

```
add_plate_axis() (bayespy.inference.vmp.nodes.constant.Consdahplate_axis() (bayespy.nodes.VaryingGaussianMarkovChain
              method), 224
                                                                                                   method), 175
add_plate_axis() (bayespy.inference.vmp.nodes.deterministicalDeterministics() (bayespy.nodes.Wishart method), 98
              method), 221
                                                                                     add_trailing_axes() (in module bayespy.utils.misc), 304
add_plate_axis() (bayespy.inference.vmp.nodes.expfamily.Exploid@wialFa(inf)bayespy.utils.misc.TestCase method), 317
              method), 217
                                                                                     addTypeEqualityFunc()
                                                                                                                              (bayespy.utils.misc.TestCase
add_plate_axis() (bayespy.inference.vmp.nodes.gaussian.GaussianGammatAcRID_ToGaussianWishart
                                                                                     alpha_beta_recursion() (in module bayespy.utils.random),
              method), 230
add_plate_axis() (bayespy.inference.vmp.nodes.gaussian.GaussianGammaISOToGaussianGammaARD
              method), 228
                                                                                     array_to_scalar() (in module bayespy.utils.misc), 304
add\_plate\_axis() \ (bayespy.inference.vmp.nodes.gaussian. \textbf{Gausssiain} \textbf{\textbf{FoGalussialmaGt}} (in maISO(bayespy.nodes. Exponential and add_plate\_axis)) \ (bayespy.nodes. Exponential add_plate\_axis) \ (bay
              method), 226
                                                                                                   method), 103
add_plate_axis() (bayespy.inference.vmp.nodes.gaussian.Wraps TdiGenous alaw Gshantu (a ARAD) respy.nodes.Gamma method),
              method), 234
                                                                                                    93
add_plate_axis() (bayespy.inference.vmp.nodes.gaussian.Wrap3cnGán(sbayeSpy.unial$Sti)sc.TestCase method), 324
                                                                                     assertAllClose() (bayespy.utils.misc.TestCase method),
              method), 232
add_plate_axis() (bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart
              method), 236
                                                                                     assertAlmostEqual()
                                                                                                                              (bayespy.utils.misc.TestCase
add_plate_axis() (bayespy.inference.vmp.nodes.node.Node
                                                                                                   method), 317
              method), 212
                                                                                     assertAlmostEquals()
                                                                                                                              (bayespy.utils.misc.TestCase
add_plate_axis() (bayespy.inference.vmp.nodes.stochastic.Stochastic method), 318
              method), 214
                                                                                     assertArrayEqual()
                                                                                                                              (bayespy.utils.misc.TestCase
add_plate_axis() (bayespy.nodes.Add method), 190
                                                                                                   method), 318
add_plate_axis() (bayespy.nodes.Bernoulli method), 123
                                                                                     assertCountEqual()
                                                                                                                              (bayespy.utils.misc.TestCase
add_plate_axis() (bayespy.nodes.Beta method), 147
                                                                                                   method), 318
add_plate_axis() (bayespy.nodes.Binomial method), 128
                                                                                     assertDictContainsSubset() (bayespy.utils.misc.TestCase
add_plate_axis() (bayespy.nodes.Categorical method),
                                                                                                   method), 318
                                                                                     assertDictEqual() (bayespy.utils.misc.TestCase method),
              132
add_plate_axis() (bayespy.nodes.CategoricalMarkovChain
                                                                                                   318
              method), 157
                                                                                     assertEqual() (bayespy.utils.misc.TestCase method), 318
add_plate_axis() (bayespy.nodes.Dirichlet method), 152
                                                                                     assertEquals() (bayespy.utils.misc.TestCase method), 318
add_plate_axis() (bayespy.nodes.Exponential method),
                                                                                     assertFalse() (bayespy.utils.misc.TestCase method), 318
              102
                                                                                     assertGreater() (bayespy.utils.misc.TestCase method),
add_plate_axis() (bayespy.nodes.Gamma method), 93
                                                                                                   319
add_plate_axis() (bayespy.nodes.Gate method), 192
                                                                                     assertGreaterEqual()
                                                                                                                              (bayespy.utils.misc.TestCase
add_plate_axis() (bayespy.nodes.Gaussian method), 82
                                                                                                   method), 319
add_plate_axis() (bayespy.nodes.GaussianARD method),
                                                                                     assertIn() (bayespy.utils.misc.TestCase method), 319
                                                                                     assertIs() (bayespy.utils.misc.TestCase method), 319
                                                                                     assertIsInstance() (bayespy.utils.misc.TestCase method),
add_plate_axis()
                           (bayespy.nodes.GaussianGammaARD
              method), 113
                                                                                                    319
add_plate_axis()
                            (bayespy.nodes.GaussianGammaISO
                                                                                     assertIsNone()
                                                                                                             (bayespy.utils.misc.TestCase method),
              method), 107
                                                                                                   319
add_plate_axis()
                         (bayespy.nodes.GaussianMarkovChain
                                                                                     assertIsNot() (bayespy.utils.misc.TestCase method), 319
              method), 163
                                                                                     assertIsNotNone() (bayespy.utils.misc.TestCase method),
add_plate_axis()
                                   (bayespy.nodes.GaussianWishart
              method), 118
                                                                                     assertLess() (bayespy.utils.misc.TestCase method), 319
add_plate_axis() (bayespy.nodes.Mixture method), 181
                                                                                     assertLessEqual() (bayespy.utils.misc.TestCase method),
add_plate_axis() (bayespy.nodes.Multinomial method),
                                                                                     assertListEqual() (bayespy.utils.misc.TestCase method),
              137
add_plate_axis() (bayespy.nodes.Poisson method), 142
                                                                                                   320
add_plate_axis() (bayespy.nodes.SumMultiply method),
                                                                                     assertLogs() (bayespy.utils.misc.TestCase method), 320
                                                                                     assertMessage() (bayespy.utils.misc.TestCase method),
add_plate_axis() (bayespy.nodes.SwitchingGaussianMarkovChain
                                                                                                    320
              method), 169
                                                                                     assertMessageToChild()
                                                                                                                              (bayespy.utils.misc.TestCase
```

method), 320		BetaDistribution	(class	in
assertMultiLineEqual()	(bayespy.utils.misc.TestCase	bayespy.inferen	ice.vmp.nodes.beta), 280	
method), 320		BetaMoments	(class	in
assertNotAlmostEqual()	(bayespy.utils.misc.TestCase	bayespy.inferen	ice.vmp.nodes.beta), 245	
method), 320		Binomial (class in bayesp	y.nodes), 126	
assertNotAlmostEquals()	(bayespy.utils.misc.TestCase	BinomialDistribution	(class	in
method), 321		bayespy.inferen	ce.vmp.nodes.binomial),	
assertNotEqual() (bayespy.	utils.misc.TestCase method)			
321		BinomialMoments	(class	in
assertNotEquals() (bayespy 321	.utils.misc.TestCase method)	bayespy.inferen 248	ce.vmp.nodes.binomial),	
assertNotIn() (bavespy.utils	.misc.TestCase method), 321	block_banded() (in modul	le bavespy.utils.misc), 30	4
assertNotIsInstance()	(bayespy.utils.misc.TestCase			
method), 321	(- nj Fj	295		
, · · · · · · · · · · · · · · · · · · ·	utils.misc.TestCase method)		ce.vmp.transformations.R	RotateGaussian
321		method), 199	_	
	s.misc.TestCase method), 321	bound() (bayespy.inference	ce.vmp.transformations.R	RotateGaussianARD
assertRaisesRegex()	(bayespy.utils.misc.TestCase			
method), 322		bound() (bayespy.inference	ce.vmp.transformations.R	RotateGaussianMarkovCh
assertRaisesRegexp()	(bayespy.utils.misc.TestCase			
method), 322		bound() (bayespy.inference	ce.vmp.transformations.R	RotateMultiple
	.misc.TestCase method), 322	method), 205		
assertRegexpMatches()	(bayespy.utils.misc.TestCase	V	ce.vmp.transformations.R	RotateSwitchingMarkovC
method), 322		method), 203		
assertSequenceEqual()	(bayespy.utils.misc.TestCase		ce.vmp.transformations.R	RotateVaryingMarkovCha
method), 322		method), 204		
	utils.misc.TestCase method)			
323		broadcasted_shape_from_a	•	module
assertTrue() (bayespy.utils.r		bayespy.utils.m		
assertTupleEqual()	(bayespy.utils.misc.TestCase			
method), 323			nce.vmp.nodes.constant.	Constant
	.misc.TestCase method), 323	method), 224		
assertWarnsRegex()	(bayespy.utils.misc.TestCase			
method), 324			nce.vmp.nodes.determini	stic.Deterministic
atleast_nd() (in module baye		method), 221		
axes_to_collapse() (in modu	le bayespy.utils.misc), 304	broadcasting_multiplier()		
Р			nce.vmp.nodes.expfamily	y.ExponentialFamily
В		method), 217		
bayespy.inference (module)	, 194	broadcasting_multiplier()		
bayespy.nodes (module), 81			nce.vmp.nodes.gaussian.	GaussianGammaARDTo
bayespy.plot (module), 205		method), 230		
bayespy.utils.linalg (module	e), 295	broadcasting_multiplier()		
bayespy.utils.misc (module)	, 302		nce.vmp.nodes.gaussian.	GaussianGammaISOToG
bayespy.utils.optimize (mod	lule), 302	method), 228		
bayespy.utils.random (modu	ıle), 297	broadcasting_multiplier()		
Bernoulli (class in bayespy.	nodes), 121		nce.vmp.nodes.gaussian.	GaussianToGaussianGam
bernoulli() (in module bayes	spy.utils.random), 298	method), 226		
BernoulliDistribution	(class in			
bayespy.inference	.vmp.nodes.bernoulli),		nce.vmp.nodes.gaussian.	WrapToGaussianGamma
283		method), 234		
BernoulliMoments	(class in		_	
bayespy.inference	vmp nodes bernoulli)	(bayespy.infere	nce.vmp.nodes.gaussian.	WrapToGaussianGamma
	.viiip.iiodes.oeiiiodiii),	4 4 000		
247 Beta (class in bayespy.node		method), 232 broadcasting_multiplier()		

	nce.vmp.nodes.gaussian.WrapTo	GaussianWi(shaytespy.nodes.SwitchingGaussianMarkovChain method), 169
method), 236		
broadcasting_multiplier()	1 1 37 1	broadcasting_multiplier()
	nce.vmp.nodes.node.Node	(bayespy.nodes.VaryingGaussianMarkovChain
static method),	212	method), 175
broadcasting_multiplier()		broadcasting_multiplier() (bayespy.nodes.Wishart
	nce.vmp.nodes.stochastic.Stochas	
method), 214		broadcasting_multiplier() (in module bayespy.utils.misc),
broadcasting_multiplier() 190	(bayespy.nodes.Add method),	305
<pre>broadcasting_multiplier()</pre>	(bayespy.nodes.Bernoulli	C
method), 123		Categorical (class in bayespy.nodes), 131
broadcasting_multiplier()	(bayespy.nodes.Beta method),	categorical() (in module bayespy.utils.random), 298
148		CategoricalDistribution (class in
<pre>broadcasting_multiplier()</pre>	(bayespy.nodes.Binomial	bayespy.inference.vmp.nodes.categorical),
method), 128		287
<pre>broadcasting_multiplier()</pre>	(bayespy.nodes.Categorical	CategoricalMarkovChain (class in bayespy.nodes), 156
method), 132	(11)	CategoricalMarkovChainDistribution (class in
broadcasting_multiplier()		bayespy.inference.vmp.nodes.categorical_markov_chain),
	CategoricalMarkovChain	289
method), 158	eategoriean vario venam	CategoricalMarkovChainMoments (class in
broadcasting_multiplier()	(bayespy.nodes.Dirichlet	bayespy.inference.vmp.nodes.categorical_markov_chain),
method), 152	(bayespy.nodes.Diricinet	250
broadcasting_multiplier()	(hayasny nodes Evnonantial	,
	(bayespy.nodes.Exponential	CategoricalMarkovChainPlotter (class in bayespy.plot),
method), 103	(havaanu nadaa Camma	210
broadcasting_multiplier()	(bayespy.nodes.Gamma	Categorical Moments (class in
method), 93	(harrage day Cata mathad)	bayespy.inference.vmp.nodes.categorical),
	(bayespy.nodes.Gate method),	249
192	4 1 6 :	ceildiv() (in module bayespy.utils.misc), 305
broadcasting_multiplier()	(bayespy.nodes.Gaussian	check_gradient() (in module bayespy.utils.optimize), 302
method), 82		chol() (in module bayespy.utils.linalg), 295
broadcasting_multiplier()	(bayespy.nodes.GaussianARD	chol() (in module bayespy.utils.misc), 305
method), 88		chol_inv() (in module bayespy.utils.linalg), 295
$broadcasting\_multiplier()$		chol_inv() (in module bayespy.utils.misc), 305
	GaussianGammaARD	chol_logdet() (in module bayespy.utils.linalg), 296
method), 113		chol_logdet() (in module bayespy.utils.misc), 305
broadcasting_multiplier()		chol_solve() (in module bayespy.utils.linalg), 296
(bayespy.nodes	GaussianGammaISO method),	chol_solve() (in module bayespy.utils.misc), 305
107		cholesky() (in module bayespy.utils.misc), 306
broadcasting_multiplier()		CholeskyDense (class in bayespy.utils.misc), 314
(bayespy.nodes	GaussianMarkovChain	CholeskySparse (class in bayespy.utils.misc), 314
method), 163		composite_function() (in module bayespy.utils.misc), 306
broadcasting_multiplier()		compute_cgf_from_parents()
	GaussianWishart method),	(bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution
118		method), 284
<pre>broadcasting_multiplier()</pre>	(bayespy.nodes.Mixture	compute_cgf_from_parents()
method), 181	` ' 1'	(bayespy.inference.vmp.nodes.beta.BetaDistribution
<pre>broadcasting_multiplier()</pre>	(bayespy.nodes.Multinomial	method), 280
method), 137	(cu) espymouesuvianium	compute_cgf_from_parents()
broadcasting_multiplier()	(bayespy.nodes.Poisson	(bayespy.inference.vmp.nodes.binomial.BinomialDistribution
method), 142	(ou) copy.modes.i oisson	method), 286
broadcasting_multiplier()	(bayespy.nodes.SumMultiply	compute_cgf_from_parents()
method), 186	(Sujespj.modes.Summunipry	(bayespy.inference.vmp.nodes.categorical.CategoricalDistribution
broadcasting multiplier()		method) 288

```
compute_cgf_from_parents()
                                                                                                                                                      compute_dims_from_values()
                         (bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.Wharkov.ChuimDinotlbsutionegorical.CategoricalMoments
                         method), 289
                                                                                                                                                                               method), 250
compute_cgf_from_parents()
                                                                                                                                                      compute_dims_from_values()
                         (bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution(bayespy.inference.vmp.nodes.categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categor
                         method), 282
                                                                                                                                                                               method), 251
compute_cgf_from_parents()
                                                                                                                                                      compute_dims_from_values()
                         (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.Daytrippytinference.vmp.nodes.dirichlet.DirichletMoments
                         method), 255
                                                                                                                                                                               method), 247
compute_cgf_from_parents()
                                                                                                                                                      compute_dims_from_values()
                         (bayespy.inference.vmp.nodes.gamma.GammaDistribution (bayespy.inference.vmp.nodes.gamma.GammaMoments
                         method), 276
                                                                                                                                                                               method), 244
compute_cgf_from_parents()
                                                                                                                                                      compute_dims_from_values()
                         (bayespy.inference.vmp.nodes.gaussian.GaussianARDDistrithativespy.inference.vmp.nodes.gaussian.GaussianGammaARDMo
                         method), 259
                                                                                                                                                                               method), 242
compute_cgf_from_parents()
                                                                                                                                                      compute_dims_from_values()
                         (bayespy, inference, vmp, nodes, gaussian, Gaussian Distributio (bayespy, inference, vmp, nodes, gaussian, Gaussian Gamma ISO Mor
                         method), 257
                                                                                                                                                                               method), 241
compute_cgf_from_parents()
                                                                                                                                                      compute_dims_from_values()
                         (bayespy, inference, vmp.nodes, gaussian, Gaussian Gamma ARDD) istpibutforence, vmp.nodes, gaussian, Gaussian Moments
                         method), 263
                                                                                                                                                                               method), 239
compute_cgf_from_parents()
                                                                                                                                                      compute_dims_from_values()
                         (bayespy.inference.vmp.nodes.gaussian.GaussianGammaIS@Daistsibntitference.vmp.nodes.gaussian.GaussianWishartMoment
                         method), 261
                                                                                                                                                                               method), 243
compute_cgf_from_parents()
                                                                                                                                                      compute_dims_from_values()
                         (bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.inference.vmp.nodes.gaussian_markov_chain.GaussianN
                         method), 265
                                                                                                                                                                               method), 240
compute_cgf_from_parents()
                                                                                                                                                      compute_dims_from_values()
                         (bayespy.inference.vmp.nodes.gaussian_markov_chain.Gaus@bary&spykin/@hariodDistriphutoides.multinomial.MultinomialMoments
                         method), 267
                                                                                                                                                                               method), 251
compute_cgf_from_parents()
                                                                                                                                                      compute_dims_from_values()
                         (bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@bayespy.in
                         method), 270
                                                                                                                                                                               method), 238
compute_cgf_from_parents()
                                                                                                                                                      compute_dims_from_values()
                         (bayespy,inference,vmp,nodes,gaussian_markov_chain.Varyinta@espy,imfMencev@hain@lestrimissom.PoissonMoments
                         method), 273
                                                                                                                                                                               method), 252
compute_cgf_from_parents()
                                                                                                                                                      compute_dims_from_values()
                         (bayespy.inference.vmp.nodes.multinomial.MultinomialDist(bbytispy.inference.vmp.nodes.wishart.WishartMoments
                         method), 291
                                                                                                                                                                                method), 245
compute_cgf_from_parents()
                                                                                                                                                      compute_fixed_moments()
                         (bayespy,inference.vmp,nodes,poisson,PoissonDistribution (bayespy,inference.vmp,nodes,bernoulli,BernoulliMoments
                         method), 293
                                                                                                                                                                               method), 248
compute_cgf_from_parents()
                                                                                                                                                      compute_fixed_moments()
                         (bayespy,inference.vmp.nodes.wishart.WishartDistribution (bayespy,inference.vmp.nodes.beta.BetaMoments
                         method), 278
                                                                                                                                                                               method), 246
compute_dims_from_values()
                                                                                                                                                      compute_fixed_moments()
                         (bayespy, inference, vmp. nodes, bernoulli, Bernoulli Moments (bayespy, inference, vmp. nodes, binomial, Binomial Moments
                                                                                                                                                                               method), 249
                         method), 248
compute_dims_from_values()
                                                                                                                                                      compute_fixed_moments()
                         (bayespy.inference.vmp.nodes.beta.BetaMoments
                                                                                                                                                                               (bayespy.inference.vmp.nodes.categorical.CategoricalMoments
                         method), 246
                                                                                                                                                                               method), 250
compute_dims_from_values()
                                                                                                                                                      compute_fixed_moments()
                         (bayespy,inference.vmp.nodes.binomial.BinomialMoments (bayespy,inference.vmp.nodes.categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_m
                         method), 249
                                                                                                                                                                               method), 251
```

```
compute_fixed_moments()
                                                        compute_fixed_moments_and_f()
         (bayespy.inference.vmp.nodes.dirichlet.DirichletMoments (bayespy.inference.vmp.nodes.gamma.GammaDistribution
         method), 247
                                                                 method), 276
compute_fixed_moments()
                                                        compute_fixed_moments_and_f()
         (bayespy.inference.vmp.nodes.gamma.GammaMoments
                                                                 (bayespy.inference.vmp.nodes.gaussian.GaussianARDDistributio
         method), 244
                                                                 method), 260
compute_fixed_moments()
                                                        compute_fixed_moments_and_f()
         (bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDM/cspy.inference.vmp.nodes.gaussian.GaussianDistribution
         method), 242
                                                                 method), 257
compute_fixed_moments()
                                                        compute_fixed_moments_and_f()
         (bayespy, inference, vmp, nodes, gaussian, Gaussian Gamma IS (DM) versputin ference, vmp, nodes, gaussian, Gaussian Gamma ARDDis
         method), 241
                                                                 method), 263
compute_fixed_moments()
                                                        compute_fixed_moments_and_f()
         (bayespy.inference.vmp.nodes.gaussian.GaussianMoments (bayespy.inference.vmp.nodes.gaussian.GaussianGammaISODist
         method), 239
                                                                 method), 262
compute_fixed_moments()
                                                        compute_fixed_moments_and_f()
         (bayespy.inference.vmp.nodes.gaussian.GaussianWishartMobartspy.inference.vmp.nodes.gaussian.GaussianWishartDistribu
         method), 243
                                                                 method), 265
compute_fixed_moments()
                                                        compute_fixed_moments_and_f()
         (bayespy,inference.vmp.nodes.gaussian_markov_chain.Gaus&bary&sprkinf@hraio&Mompentxdes.gaussian_markov_chain.GaussianN
         method), 240
                                                                 method), 267
compute_fixed_moments()
                                                        compute_fixed_moments_and_f()
         (bayespy.inference.vmp.nodes.multinomial.MultinomialMo(harytespy.inference.vmp.nodes.gaussian_markov_chain.Switching
         method), 252
                                                                 method), 270
compute_fixed_moments()
                                                        compute_fixed_moments_and_f()
         (bayespy.inference.vmp.nodes.node.Moments
                                                                  (bayespy.inference.vmp.nodes.gaussian_markov_chain.VaryingGa
         method), 238
                                                                 method), 273
compute_fixed_moments()
                                                        compute_fixed_moments_and_f()
         (bayespy.inference.vmp.nodes.poisson.PoissonMoments
                                                                 (bayespy.inference.vmp.nodes.multinomial.MultinomialDistribut
         method), 252
                                                                 method), 292
compute_fixed_moments()
                                                        compute_fixed_moments_and_f()
         (bayespy.inference.vmp.nodes.wishart.WishartMoments
                                                                 (bayespy.inference.vmp.nodes.poisson. Poisson Distribution\\
         method), 245
                                                                 method), 293
compute_fixed_moments_and_f()
                                                        compute_fixed_moments_and_f()
         (bayespy, inference, vmp, nodes, bernoulli, Bernoulli Distribution havespy, inference, vmp, nodes, wishart. Wishart Distribution
         method), 284
                                                                 method), 278
compute_fixed_moments_and_f()
                                                        compute_gradient() (bayespy.inference.vmp.nodes.bernoulli.BernoulliDistri
         (bayespy.inference.vmp.nodes.beta.BetaDistribution
                                                                 method), 284
         method), 280
                                                        compute_gradient() (bayespy.inference.vmp.nodes.beta.BetaDistribution
compute_fixed_moments_and_f()
                                                                 method), 280
         (bayespy.inference.vmp.nodes.binomial.BinomialDistributionadient() (bayespy.inference.vmp.nodes.binomial.BinomialDistri
                                                                 method), 286
         method), 286
compute_fixed_moments_and_f()
                                                        compute_gradient() (bayespy.inference.vmp.nodes.categorical.CategoricalD
         (bayespy.inference.vmp.nodes.categorical.CategoricalDistribution), 288
         method), 288
                                                        compute_gradient() (bayespy.inference.vmp.nodes.categorical_markov_chai
compute_fixed_moments_and_f()
                                                                 method), 290
         (bayespy.inference.vmp.nodes.categorical_markovcohapnt@ateagdicints()Mlankev@hinfidDistribvutinpunodes.dirichlet.DirichletDistrib
                                                                 method), 282
         method), 290
compute_fixed_moments_and_f()
                                                        compute_gradient() (bayespy.inference.vmp.nodes.expfamily.ExponentialFa
         (bayespy.inference.vmp.nodes.dirichlet.DirichletDistributionmethod), 255
         method), 282
                                                        compute\_gradient() \ (bayespy.inference.vmp.nodes.gamma.GammaDistribut) \\
```

compute\_fixed\_moments\_and\_f()

method), 255

method), 276

method), 260

(bayespy, inference, vmp, nodes, expfamily, Exponential Fautilg Distribution was presented by the control of th

```
compute_gradient() (bayespy.inference.vmp.nodes.gaussian.Compute_Britth()t(bayespy.inference.vmp.nodes.poisson.PoissonDistribution
                  method), 257
                                                                                                                                  method), 293
compute_gradient() (bayespy.inference.vmp.nodes.gaussian.@mpute@pdf@ARDDistribute@mce.vmp.nodes.wishart.WishartDistribute
                  method), 264
                                                                                                                                  method), 279
compute_gradient() (bayespy.inference.vmp.nodes.gaussian.Compute_gradient() (bayespy.inference.VB method),
                  method), 262
compute_gradient() (bayespy.inference.vmp.nodes.gaussian.ComsaitenWischabtDixdriteutins(i)
                                                                                                                                                                                (bayespy.inference.VB
                                                                                                                                  method), 195
                  method), 265
compute_gradient() (bayespy.inference.vmp.nodes.gaussian_compute_chaink@aussianMarkovChainDistribution
                                                                                                                                  (bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribution
                  method), 267
compute_gradient() (bayespy.inference.vmp.nodes.gaussian_markov_chainDistribution
                  method), 270
                                                                                                               compute_mask_to_parent()
compute_gradient() (bayespy.inference.vmp.nodes.gaussian_markov_chlainy&pry.inferencesiunfylantwdexChlatin Distribution
                  method), 273
                                                                                                                                  method), 281
compute_gradient() (bayespy.inference.vmp.nodes.multinomiahlyfutleimounkalDiptaibut()) n
                  method), 292
                                                                                                                                  (bayespy.inference.vmp.nodes.binomial.BinomialDistribution
compute_gradient() (bayespy.inference.vmp.nodes.poisson.PoissonDistmithtid), 286
                  method), 293
                                                                                                               compute_mask_to_parent()
compute_gradient() (bayespy.inference.vmp.nodes.wishart.WishartDis(tibytespy.inference.vmp.nodes.categorical.CategoricalDistribution
                                                                                                                                  method), 288
                  method), 278
compute_logpdf() (bayespy.inference.vmp.nodes.bernoulli.BeompulteDisasibutticparent()
                  method), 284
                                                                                                                                  (bayespy.inference.vmp.nodes.categorical_markov_chain.Categor
compute_logpdf() (bayespy.inference.vmp.nodes.beta.BetaDistributiormethod), 290
                  method), 280
                                                                                                               compute_mask_to_parent()
compute_logpdf() (bayespy.inference.vmp.nodes.binomial.BinomialDistribution) (bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution)
                  method), 286
                                                                                                                                  method), 283
compute_logpdf() (bayespy.inference.vmp.nodes.categoricalcontentalSkistrihationt()
                  method), 288
                                                                                                                                  (bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistr
compute_logpdf() (bayespy.inference.vmp.nodes.categorical_markov_ahath@te@fricalMarkovChainDistribution
                  method), 290
                                                                                                               compute_mask_to_parent()
compute_logpdf() (bayespy.inference.vmp.nodes.dirichlet.DirichletDis(bibutspy.inference.vmp.nodes.gamma.GammaDistribution
                  method), 282
                                                                                                                                  method), 277
compute_logpdf() (bayespy.inference.vmp.nodes.expfamily. Express time 
                                                                                                                                  (bayespy.inference.vmp.nodes.gaussian.GaussianARDDistributio
                  method), 255
compute_logpdf() (bayespy.inference.vmp.nodes.gamma.GammaDistributhoul), 260
                  method), 276
                                                                                                               compute_mask_to_parent()
compute_logpdf() (bayespy.inference.vmp.nodes.gaussian.GaussianARDDestpibutforence.vmp.nodes.gaussian.GaussianDistribution
                  method), 260
                                                                                                                                  method), 258
compute_logpdf() (bayespy.inference.vmp.nodes.gaussian.GausspartDistaisbutionparent()
                  method), 257
                                                                                                                                  (bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDis
compute_logpdf() (bayespy.inference.vmp.nodes.gaussian.GaussianGamethoAlR.DD4stribution
                  method), 264
                                                                                                               compute_mask_to_parent()
compute_logpdf() (bayespy.inference.vmp.nodes.gaussian.GaussianGa(bayes)) (bayespy.inference.vmp.nodes.gaussian.GaussianGammaISODist
                  method), 262
                                                                                                                                  method), 262
compute_logpdf() (bayespy.inference.vmp.nodes.gaussian.GaussparteVinshaktDispribartiOn
                  method), 265
                                                                                                                                  (bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribu
compute_logpdf() (bayespy.inference.vmp.nodes.gaussian_markov_chainetiand)si2nfMarkovChainDistribution
                  method), 267
                                                                                                               compute_mask_to_parent()
compute_logpdf() (bayespy.inference.vmp.nodes.gaussian_markov_cha/brassian/MacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.GaussianMacklessGhaissDistributkion_chain.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gauss
                  method), 270
                                                                                                                                  method), 267
compute_logpdf() (bayespy.inference.vmp.nodes.gaussian_markoutchainsKatyipgtGatt(scianMarkovChainDistribution
                  method), 273
                                                                                                                                  (bayespy.inference.vmp.nodes.gaussian_markov_chain.Switching
compute_logpdf() (bayespy.inference.vmp.nodes.multinomial.MultinomialDittribution
                  method), 292
                                                                                                               compute_mask_to_parent()
```

```
(bayespy.inference.vmp.nodes.gaussian_markov_chain.Varyithg@assysianf\textsfankov\ChainDestributisoian_markov_chain.GaussianN
                             method), 274
                                                                                                                                                                                                                  method), 268
compute_mask_to_parent()
                                                                                                                                                                                   compute_message_to_parent()
                              (bayespy.inference.vmp.nodes.multinomial.MultinomialDistibytispy.inference.vmp.nodes.gaussian_markov_chain.Switching
                                                                                                                                                                                                                  method), 271
                             method), 292
compute_mask_to_parent()
                                                                                                                                                                                   compute_message_to_parent()
                             (bayespy,inference.vmp.nodes.poisson.PoissonDistribution (bayespy,inference.vmp.nodes.gaussian_markov_chain.VaryingGa
                             method), 294
                                                                                                                                                                                                                  method), 274
compute_mask_to_parent()
                                                                                                                                                                                   compute_message_to_parent()
                                                                                                                                                                                                                  (bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution)
                             (bayespy.inference.vmp.nodes.stochastic.Distribution
                              method), 254
                                                                                                                                                                                                                  method), 292
compute_mask_to_parent()
                                                                                                                                                                                   compute_message_to_parent()
                              (bayespy.inference.vmp.nodes.wishart.WishartDistribution (bayespy.inference.vmp.nodes.poisson.PoissonDistribution
                              method), 279
                                                                                                                                                                                                                  method), 294
compute_message_to_parent()
                                                                                                                                                                                   compute_message_to_parent()
                              (bayespy,inference,vmp,nodes,bernoulli,BernoulliDistributi@hayespy,inference,vmp,nodes,stochastic,Distribution
                             method), 284
                                                                                                                                                                                                                 method), 254
compute_message_to_parent()
                                                                                                                                                                                    compute_message_to_parent()
                             (bayespy.inference.vmp.nodes.beta.BetaDistribution
                                                                                                                                                                                                                  (bayespy.inference.vmp.nodes.wishart.WishartDistribution
                             method), 281
                                                                                                                                                                                                                  method), 279
compute_message_to_parent()
                                                                                                                                                                                   compute_moments_and_cgf()
                             (bayespy.inference.vmp.nodes.binomial. Binomial Distributio \verb|| \verb| bayespy.inference.vmp.nodes.bernoulli. Bernoulli Distribution || bayespy.inference.vmp.nodes.bernoulli. Bernoulli Distribution || bayespy.inference.vmp.nodes.bernoulli. Bernoulli Distribution || bayespy.inference.vmp.nodes.bernoulli Distribution || 
                             method), 286
                                                                                                                                                                                                                  method), 285
                                                                                                                                                                                   compute_moments_and_cgf()
compute_message_to_parent()
                             (bayespy.inference.vmp.nodes.categorical.CategoricalDistribbayespy.inference.vmp.nodes.beta.BetaDistribution
                             method), 288
                                                                                                                                                                                                                 method), 281
compute_message_to_parent()
                                                                                                                                                                                   compute_moments_and_cgf()
                             (bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.WmfckonChuinDistribution) (bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.Ca(bayespy.inference.vmp.nodes.categorical_markov_chain.ca(bayespy.inference.vmp.nodes.categorical_markov_chain.ca(bayespy.inference.vmp.nodes.categorical_markov_chain.ca(bayespy.inference.vmp.nodes.ca(bayespy.inference.v
                             method), 290
                                                                                                                                                                                                                 method), 286
compute_message_to_parent()
                                                                                                                                                                                    compute_moments_and_cgf()
                              (bayespy,inference.vmp.nodes.dirichlet.DirichletDistribution(bayespy,inference.vmp.nodes.categorical.CategoricalDistribution
                             method), 283
                                                                                                                                                                                                                  method), 288
compute_message_to_parent()
                                                                                                                                                                                   compute_moments_and_cgf()
                              (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily,Dayterspytinference.vmp.nodes.categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categorical_markov_chain.Categoric
                             method), 255
                                                                                                                                                                                                                 method), 290
compute_message_to_parent()
                                                                                                                                                                                   compute_moments_and_cgf()
                             (bayespy.inference.vmp.nodes.gamma.GammaDistribution (bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution
                             method), 277
                                                                                                                                                                                                                  method), 283
compute_message_to_parent()
                                                                                                                                                                                   compute_moments_and_cgf()
                             (bayespy.inference.vmp.nodes.gaussian.GaussianARDDistrithativespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistrithativespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistrithativespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistrithativespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistrithativespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistrithativespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistrithativespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.nodes.expfamilyDistrithativespy.inference.vmp.
                             method), 260
                                                                                                                                                                                                                  method), 255
compute_message_to_parent()
                                                                                                                                                                                   compute_moments_and_cgf()
                              (bayespy, inference, vmp.nodes, gaussian, Gaussian Distributio (bayespy, inference, vmp.nodes, gamma, Gamma Distribution)
                             method), 258
                                                                                                                                                                                                                 method), 277
compute_message_to_parent()
                                                                                                                                                                                   compute_moments_and_cgf()
                              (bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDistributionence.vmp.nodes.gaussian.GaussianARDDistributio
                             method), 264
                                                                                                                                                                                                                  method), 260
compute_message_to_parent()
                                                                                                                                                                                   compute_moments_and_cgf()
                              (bayespy, inference, vmp.nodes, gaussian, Gaussian Gamma IS (Chaises in but inference, vmp.nodes, gaussian, Gaussian Distribution
                             method), 262
                                                                                                                                                                                                                 method), 258
                                                                                                                                                                                   compute_moments_and_cgf()
compute_message_to_parent()
                             (bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution.inference.vmp.nodes.gaussian.GaussianGammaARDDis
                             method), 266
                                                                                                                                                                                                                  method), 264
compute_message_to_parent()
                                                                                                                                                                                   compute_moments_and_cgf()
```

```
(bayespy.inference.vmp.nodes.gaussian.GaussianGammaIS@Baisesibratinference.vmp.nodes.gaussian.GaussianGammaARDDis
              method), 262
                                                                                                       method), 264
compute_moments_and_cgf()
                                                                                        compute_phi_from_parents()
               (bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribution inference.vmp.nodes.gaussian.GaussianGammaISODist
              method), 266
                                                                                                       method), 262
compute_moments_and_cgf()
                                                                                        compute_phi_from_parents()
               (bayespy.inference.vmp.nodes.gaussian_markov_chain.Gaussbary&sprkin/EhraioeDistributides.gaussian.GaussianWishartDistribu
              method), 268
                                                                                                       method), 266
compute_moments_and_cgf()
                                                                                        compute_phi_from_parents()
              (bayespy.inference.vmp.nodes.gaussian_markov_chain.Swit@biye@gaysinfam@Aackov@haidDistaibstion.markov_chain.GaussianM
               method), 271
                                                                                                       method), 268
compute_moments_and_cgf()
                                                                                        compute_phi_from_parents()
               (bayespy,inference,vmp,nodes,gaussian_markov_chain,Varyithg@aspxiianf\deltarkev@hpin\deltaributsoian_markov_chain.Switching
               method), 274
                                                                                                       method), 272
compute_moments_and_cgf()
                                                                                        compute_phi_from_parents()
               (bayespy,inference.vmp.nodes.multinomial.MultinomialDistibutispy,inference.vmp.nodes.gaussian_markov_chain.VaryingGa
              method), 292
                                                                                                       method), 275
compute_moments_and_cgf()
                                                                                        compute_phi_from_parents()
               (bayespy, inference, vmp, nodes, poisson, Poisson Distribution (bayespy, inference, vmp, nodes, multinomial, Multinomial Distribution)
              method), 294
                                                                                                       method), 292
compute_moments_and_cgf()
                                                                                        compute_phi_from_parents()
               (bayespy,inference,vmp,nodes,wishart,WishartDistribution (bayespy,inference,vmp,nodes,poisson,PoissonDistribution
               method), 279
                                                                                                       method), 294
compute_phi_from_parents()
                                                                                        compute_phi_from_parents()
               (bayespy,inference,vmp,nodes,bernoulli,BernoulliDistributi@hayespy,inference,vmp,nodes,wishart,WishartDistribution
              method), 285
                                                                                                       method), 279
compute_phi_from_parents()
                                                                                        Constant (class in bayespy.inference.vmp.nodes.constant),
               (bayespy.inference.vmp.nodes.beta.BetaDistribution
              method), 281
                                                                                        contour() (in module bayespy.plot), 206
compute_phi_from_parents()
                                                                                        ContourPlotter (class in bayespy.plot), 208
               (bayespy.inference.vmp.nodes.binomial.Binomial.Distrillatition() (in module bayespy.utils.random), 298
               method), 286
                                                                                        countTestCases() (bayespy.utils.misc.TestCase method),
compute_phi_from_parents()
               (bayespy.inference.vmp.nodes.categorical.Categorical#Disstrict@n module bayespy.utils.random), 298
              method), 288
compute_phi_from_parents()
               (bayespy.inference.vmp.nodes.categorical_markovachaig: Categorical Markov Chain Distribution), 324
               method), 290
                                                                                        defaultTestResult()
                                                                                                                                   (bayespy.utils.misc.TestCase
compute_phi_from_parents()
                                                                                                       method), 324
              (bayespy.inference.vmp.nodes.dirichlet.DirichletDistributionayespy.inference.vmp.nodes.constant.Constant
              method), 283
                                                                                                       method), 224
compute_phi_from_parents()
                                                                                        delete() (bayespy.inference.vmp.nodes.deterministic.Deterministic
               (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily.Pistribution)
               method), 255
                                                                                        delete() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily
compute_phi_from_parents()
                                                                                                       method), 217
              (bayespy.inference.vmp.nodes.gamma.GammaDistributionbayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGa
               method), 277
                                                                                                       method), 230
compute_phi_from_parents()
                                                                                        delete() (bayespy.inference.vmp.nodes.gaussian.GaussianGammaISOToGau
               (bayespy.inference.vmp.nodes.gaussian.GaussianARDDistributiond), 228
              method), 260
                                                                                        delete()\ (bayespy.inference.vmp.nodes.gaussian.Gaussian To Gaussian Gamman Gaussian Gaussi
compute_phi_from_parents()
                                                                                                       method), 226
              (bayespy.inference.vmp.nodes.gaussian.Gaussian Distribution yespy.inference.vmp.nodes.gaussian.Wrap To Gaussian Gamma A
               method), 258
                                                                                                       method), 234
compute_phi_from_parents()
```

$delete() \ (bayes py. inference. vmp. nodes. gaussian. Wrap To Gaussian and Gauss$	adsiran@ayuspaySi@des.GaussianARD attribute), 91
method), 232	dims (bayespy.nodes.GaussianGammaARD attribute),
delete() (bayespy.inference.vmp.nodes.gaussian.WrapToGamethod), 236	dims (bayespy.nodes.GaussianGammaISO attribute), 111
delete() (bayespy.inference.vmp.nodes.node.Node	dims (bayespy.nodes.GaussianMarkovChain attribute),
method), 212	166
delete() (bayespy.inference.vmp.nodes.stochastic.Stochastic	
method), 214	dims (bayespy.nodes.Mixture attribute), 184
delete() (bayespy.nodes.Add method), 190	dims (bayespy.nodes.Multinomial attribute), 141
delete() (bayespy.nodes.Bernoulli method), 123	dims (bayespy.nodes.Poisson attribute), 146
delete() (bayespy.nodes.Beta method), 148	dims (bayespy.nodes.SwitchingGaussianMarkovChain
delete() (bayespy.nodes.Binomial method), 128	attribute), 172
delete() (bayespy.nodes.Categorical method), 128 delete() (bayespy.nodes.Categorical method), 132	dims (bayespy.nodes.VaryingGaussianMarkovChain at-
	tribute), 178
method), 158	dims (bayespy.nodes.Wishart attribute), 101
delete() (bayespy.nodes.Dirichlet method), 152	Dirichlet (class in bayespy.nodes), 151
delete() (bayespy.nodes.Exponential method), 103	dirichlet() (in module bayespy.utils.random), 299
delete() (bayespy.nodes.Gamma method), 93	DirichletDistribution (class in
delete() (bayespy.nodes.Gate method), 193	bayespy.inference.vmp.nodes.dirichlet), 281
delete() (bayespy.nodes.Gaussian method), 83	DirichletMoments (class in
delete() (bayespy.nodes.GaussianARD method), 88	bayespy.inference.vmp.nodes.dirichlet), 246
delete() (bayespy.nodes.GaussianGammaARD method),	dist_haversine() (in module bayespy.utils.misc), 306
113	Distribution (class in bayespy.inference.vmp.nodes.stochastic),
delete() (bayespy.nodes.GaussianGammaISO method), 108	253 doCleanups() (bayespy.utils.misc.TestCase method), 324
delete() (bayespy.nodes.GaussianMarkovChain method),	dot() (bayespy.inference.VB method), 195
163	Dot() (in module bayespy.nodes), 184
delete() (bayespy.nodes.GaussianWishart method), 118 delete() (bayespy.nodes.Mixture method), 181	dot() (in module bayespy.utils.linalg), 296
delete() (bayespy.nodes.Multinomial method), 137	E
delete() (bayespy.nodes.Poisson method), 142	
delete() (bayespy.nodes.SumMultiply method), 187	Exponential (class in bayespy.nodes), 101
delete() (bayespy.nodes.SwitchingGaussianMarkovChain	ExponentialFamily (class in
method), 169	bayespy.inference.vmp.nodes.expfamily), 216
delete() (bayespy.nodes.VaryingGaussianMarkovChain	ExponentialFamilyDistribution (class in
method), 175	bayespy.inference.vmp.nodes.expfamily),
delete() (bayespy.nodes.Wishart method), 98	254
Deterministic (class in	Г
bayespy.inference.vmp.nodes.deterministic),	F
221	fail() (bayespy.utils.misc.TestCase method), 324
diag() (in module bayespy.utils.misc), 306	failIf() (bayespy.utils.misc.TestCase method), 324
diagonal() (in module bayespy.utils.misc), 306	failIfAlmostEqual() (bayespy.utils.misc.TestCase
dims (bayespy.inference.vmp.nodes.expfamily.Exponential	Family method), 325
attribute), 220	failIfEqual() (bayespy.utils.misc.TestCase method), 325
dims (bayespy.nodes.Bernoulli attribute), 126	failUnless() (bayespy.utils.misc.TestCase method), 325
dims (bayespy.nodes.Beta attribute), 151	failUnlessAlmostEqual() (bayespy.utils.misc.TestCase
dims (bayespy.nodes.Binomial attribute), 131	method), 325
dims (bayespy.nodes.Categorical attribute), 136	failUnlessEqual() (bayespy.utils.misc.TestCase method),
dims (bayespy.nodes.CategoricalMarkovChain attribute),	325
161	failUnlessRaises() (bayespy.utils.misc.TestCase method),
dims (bayespy.nodes.Dirichlet attribute), 155	325
dims (bayespy.nodes.Exponential attribute), 106	find_set_index() (in module bayespy.utils.misc), 306
dims (bayespy.nodes.Gamma attribute), 96	first() (in module bayespy.utils.misc), 306
dims (bayespy.nodes.Gaussian attribute), 86	FunctionPlotter (class in bayespy plot) 209

G	get_bound_terms() (bayespy.inference.vmp.transformations.RotateGaussian
Gamma (class in bayespy.nodes), 92	method), 201
gamma_entropy() (in module bayespy.utils.random), 299	get_bound_terms() (bayespy.inference.vmp.transformations.RotateGaussian method), 202
gamma_logpdf() (in module bayespy.utils.random), 299	get_bound_terms() (bayespy.inference.vmp.transformations.RotateMultiple
GammaDistribution (class in	method), 205
bayespy.inference.vmp.nodes.gamma), 276	get_bound_terms() (bayespy.inference.vmp.transformations.RotateSwitchir
GammaMoments (class in	method), 203
bayespy.inference.vmp.nodes.gamma), 244	get_bound_terms() (bayespy.inference.vmp.transformations.RotateVaryingl
Gate (class in bayespy.nodes), 191	method), 204
Gaussian (class in bayespy.nodes), 81	get_converter() (bayespy.inference.vmp.nodes.bernoulli.BernoulliMoments
gaussian_entropy() (in module bayespy.utils.random), 300	method), 248
gaussian_gamma_to_t() (in module bayespy.utils.random),	get_converter() (bayespy.inference.vmp.nodes.beta.BetaMoments
300	method), 246
gaussian_logpdf() (in module bayespy.utils.misc), 306	get_converter() (bayespy.inference.vmp.nodes.binomial.BinomialMoments
gaussian_logpdf() (in module bayespy.utils.random), 300	
gaussian_mixture_2d() (in module bayespy.plot), 206	method), 249
GaussianARD (class in bayespy.nodes), 86	get_converter() (bayespy.inference.vmp.nodes.categorical.CategoricalMom
GaussianARDDistribution (class in	method), 250
bayespy.inference.vmp.nodes.gaussian), 259	get_converter() (bayespy.inference.vmp.nodes.categorical_markov_chain.Co
GaussianDistribution (class in	method), 251
bayespy.inference.vmp.nodes.gaussian), 256	get_converter() (bayespy.inference.vmp.nodes.dirichlet.DirichletMoments
GaussianGammaARD (class in bayespy.nodes), 112	method), 247
GaussianGammaARDDistribution (class in	get_converter() (bayespy.inference.vmp.nodes.gamma.GammaMoments
bayespy.inference.vmp.nodes.gaussian), 263	method), 244
GaussianGammaARDMoments (class in	get_converter() (bayespy.inference.vmp.nodes.gaussian.GaussianGammaA
bayespy.inference.vmp.nodes.gaussian), 242	method), 242
GaussianGammaARDToGaussianWishart (class in	get_converter() (bayespy.inference.vmp.nodes.gaussian.GaussianGammaIS
bayespy.inference.vmp.nodes.gaussian), 229	method), 241
GaussianGammaISO (class in bayespy.nodes), 106	$get\_converter() \ (bayes py.inference.vmp.nodes.gaussian. Gaussian Moments$
GaussianGammaISODistribution (class in	method), 239
bayespy.inference.vmp.nodes.gaussian), 261	$get\_converter() \ (bayespy.inference.vmp.nodes.gaussian. Gaussian Wishart Mattheward (bayespy.inference.vmp.nodes.gaussian. Gaussian. Gaussia$
GaussianGammaISOMoments (class in	method), 243
bayespy.inference.vmp.nodes.gaussian), 240	get_converter() (bayespy.inference.vmp.nodes.gaussian_markov_chain.Gau
GaussianGammaISOToGaussianGammaARD (class in	method), 240
bayespy.inference.vmp.nodes.gaussian), 227	get_converter() (bayespy.inference.vmp.nodes.multinomial.MultinomialMo
GaussianMarkovChain (class in bayespy.nodes), 161	method), 252
GaussianMarkovChainDistribution (class in	get_converter() (bayespy.inference.vmp.nodes.node.Moments
bayespy.inference.vmp.nodes.gaussian_markov_o	chain), method), 238
266	get_converter() (bayespy.inference.vmp.nodes.poisson.PoissonMoments
GaussianMarkovChainMoments (class in	method), 253
bayespy.inference.vmp.nodes.gaussian_markov_c	chert converter() (bayespy.inference.vmp.nodes.wishart.WishartMoments
240	method), 245
GaussianMoments (class in	get_diag() (in module bayespy.utils.misc), 307
bayespy.inference.vmp.nodes.gaussian), 239	get_gaussian_mean_and_variance()
GaussianTimeseriesPlotter (class in bayespy.plot), 210	(bayespy.nodes.GaussianGammaISO method),
GaussianToGaussianGammaISO (class in	108
bayespy.inference.vmp.nodes.gaussian), 225	get_gradient() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily
GaussianWishart (class in bayespy.nodes), 117	method), 217
GaussianWishartDistribution (class in	get_gradient() (bayespy.nodes.Bernoulli method), 123
bayespy.inference.vmp.nodes.gaussian), 265	get_gradient() (bayespy.nodes.Beta method), 148
GaussianWishartMoments (class in	get_gradient() (bayespy.nodes.Binomial method), 128
boyagny informacy ump nodes goussian) 243	get_gradient() (bayespy.nodes.Categorical method), 133
get hound terms() (havesny inference ymp transformations	Ret gradient() (bayespy.nodes.CategoricalMarkovChain
method), 199	method), 158

```
get_gradient() (bayespy.nodes.Dirichlet method), 152
                                                                                                                                                          method), 158
get_gradient() (bayespy.nodes.Exponential method), 103
                                                                                                                                    get_mask() (bayespy.nodes.Dirichlet method), 153
get_gradient() (bayespy.nodes.Gamma method), 93
                                                                                                                                    get_mask() (bayespy.nodes.Exponential method), 103
get_gradient() (bayespy.nodes.Gaussian method), 83
                                                                                                                                    get_mask() (bayespy.nodes.Gamma method), 93
get_gradient() (bayespy.nodes.GaussianARD method), 88
                                                                                                                                    get_mask() (bayespy.nodes.Gate method), 193
get_gradient()
                                         (bayespy.nodes.GaussianGammaARD
                                                                                                                                    get_mask() (bayespy.nodes.Gaussian method), 83
                      method), 113
                                                                                                                                    get_mask() (bayespy.nodes.GaussianARD method), 88
                                                                                                                                                                              (bayespy.nodes.GaussianGammaARD
get_gradient()
                                             (bayespy.nodes.GaussianGammaISO
                                                                                                                                    get_mask()
                      method), 108
                                                                                                                                                          method), 113
get_gradient()
                                         (bayespy.nodes.GaussianMarkovChain
                                                                                                                                                                                 (bayespy.nodes.GaussianGammaISO
                                                                                                                                    get_mask()
                      method), 163
                                                                                                                                                          method), 108
get_gradient() (bayespy.nodes.GaussianWishart method),
                                                                                                                                                                             (bayespy.nodes.GaussianMarkovChain
                                                                                                                                    get_mask()
                                                                                                                                                          method), 163
get_gradient() (bayespy.nodes.Mixture method), 181
                                                                                                                                    get_mask() (bayespy.nodes.GaussianWishart method),
get_gradient() (bayespy.nodes.Multinomial method), 138
get_gradient() (bayespy.nodes.Poisson method), 143
                                                                                                                                    get_mask() (bayespy.nodes.Mixture method), 181
get_gradient() (bayespy.nodes.SwitchingGaussianMarkovChazin_mask() (bayespy.nodes.Multinomial method), 138
                      method), 169
                                                                                                                                    get_mask() (bayespy.nodes.Poisson method), 143
get_gradient() (bayespy.nodes.VaryingGaussianMarkovChaiget_mask() (bayespy.nodes.SumMultiply method), 187
                                                                                                                                    get\_mask() \ (bayespy.nodes. Switching Gaussian Markov Chain
                      method), 175
get_gradient() (bayespy.nodes.Wishart method), 98
                                                                                                                                                          method), 169
get_gradients() (bayespy.inference.VB method), 196
                                                                                                                                    get_mask() (bayespy.nodes.VaryingGaussianMarkovChain
get_iteration_by_nodes() (bayespy.inference.VB method),
                                                                                                                                                          method), 176
                                                                                                                                    get_mask() (bayespy.nodes.Wishart method), 98
get_marginal_logpdf() (bayespy.nodes.GaussianGammaISO get_moments() (bayespy.inference.vmp.nodes.constant.Constant
                      method), 108
                                                                                                                                                          method), 224
get_mask() (bayespy.inference.vmp.nodes.constant.Constantget_moments() (bayespy.inference.vmp.nodes.deterministic.Deterministic
                      method), 224
                                                                                                                                                          method), 222
get_mask() (bayespy.inference.vmp.nodes.deterministic.Detgetnimistivents() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily
                      method), 222
                                                                                                                                                          method), 218
get_mask() (bayespy.inference.vmp.nodes.expfamily.ExponentiallFamnehts() (bayespy.inference.vmp.nodes.gaussian.GaussianGammaAF
                      method), 218
                                                                                                                                                          method), 230
get_mask() (bayespy.inference.vmp.nodes.gaussian.Gaussian@aussian@aussian.GaussianGammaISO
                      method), 230
                                                                                                                                                          method), 228
get_mask() (bayespy.inference.vmp.nodes.gaussian.Gaussian@ammonakstosto Charvessian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.
                      method), 228
                                                                                                                                                          method), 226
get_mask() (bayespy.inference.vmp.nodes.gaussian.GaussiangEbt(havakst).inference.vmp.nodes.gaussian.WrapToGaussianGa
                      method), 226
                                                                                                                                                          method), 234
get_mask() (bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGastusniamGaussianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastusnianGastu
                      method), 234
                                                                                                                                                          method), 232
get_mask() (bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGaussianWrapToGau
                      method), 232
                                                                                                                                                          method), 236
get_mask() (bayespy.inference.vmp.nodes.gaussian.WrapTo@atussiamWitst@attbayespy.inference.vmp.nodes.node.Node
                      method), 236
                                                                                                                                                          method), 212
                                  (bayespy.inference.vmp.nodes.node.Node get_moments() (bayespy.inference.vmp.nodes.stochastic.Stochastic
get_mask()
                      method), 212
                                                                                                                                                          method), 214
get_mask() (bayespy.inference.vmp.nodes.stochastic.Stochastet_moments() (bayespy.nodes.Add method), 190
                      method), 214
                                                                                                                                    get_moments() (bayespy.nodes.Bernoulli method), 123
                                                                                                                                    get_moments() (bayespy.nodes.Beta method), 148
get_mask() (bayespy.nodes.Add method), 190
get_mask() (bayespy.nodes.Bernoulli method), 123
                                                                                                                                    get_moments() (bayespy.nodes.Binomial method), 128
get_mask() (bayespy.nodes.Beta method), 148
                                                                                                                                    get_moments() (bayespy.nodes.Categorical method), 133
get_mask() (bayespy.nodes.Binomial method), 128
                                                                                                                                    get_moments() (bayespy.nodes.CategoricalMarkovChain
get_mask() (bayespy.nodes.Categorical method), 133
                                                                                                                                                          method), 158
get_mask()
                                   (bayespy.nodes.CategoricalMarkovChain
                                                                                                                                  get_moments() (bayespy.nodes.Dirichlet method), 153
```

get_moments() (bayespy.nodes.Exponential method), 103	187
get_moments() (bayespy.nodes.Gamma method), 93	get_parameters() (bayespy.nodes.SwitchingGaussianMarkovChain
get_moments() (bayespy.nodes.Gate method), 193	method), 170
get_moments() (bayespy.nodes.Gaussian method), 83	get_parameters() (bayespy.nodes.VaryingGaussianMarkovChain
get_moments() (bayespy.nodes.GaussianARD method),	method), 176
88	get_parameters() (bayespy.nodes.Wishart method), 98
get_moments() (bayespy.nodes.GaussianGammaARD	get_riemannian_gradient()
method), 113	(bayespy.inference.vmp.nodes.expfamily.ExponentialFamily
get_moments() (bayespy.nodes.GaussianGammaISO	method), 218
method), 108	get_riemannian_gradient() (bayespy.nodes.Bernoulli
get_moments() (bayespy.nodes.GaussianMarkovChain	method), 123
method), 163	get_riemannian_gradient() (bayespy.nodes.Beta method),
**	get_Hemailian_gradient() (bayespy.nodes.beta method),
get_moments() (bayespy.nodes.GaussianWishart	
method), 118	get_riemannian_gradient() (bayespy.nodes.Binomial
get_moments() (bayespy.nodes.Mixture method), 181	method), 128
get_moments() (bayespy.nodes.Multinomial method), 138	get_riemannian_gradient() (bayespy.nodes.Categorical
get_moments() (bayespy.nodes.Poisson method), 143	method), 133
get_moments() (bayespy.nodes.SumMultiply method),	<pre>get_riemannian_gradient()</pre>
187	(bayespy.nodes.CategoricalMarkovChain
get_moments() (bayespy.nodes.SwitchingGaussianMarkovC	
method), 169	get_riemannian_gradient() (bayespy.nodes.Dirichlet
get_moments() (bayespy.nodes.VaryingGaussianMarkovCh	
method), 176	get_riemannian_gradient() (bayespy.nodes.Exponential
get_moments() (bayespy.nodes.Wishart method), 98	method), 103
get_parameters() (bayespy.inference.VB method), 196	get_riemannian_gradient() (bayespy.nodes.Gamma
get_parameters() (bayespy.inference.vmp.nodes.expfamily.I	Exponential Fiethids), 94
method), 218	get_riemannian_gradient() (bayespy.nodes.Gaussian
get_parameters() (bayespy.nodes.Bernoulli method), 123	method), 83
get_parameters() (bayespy.nodes.Beta method), 148	get_riemannian_gradient() (bayespy.nodes.GaussianARD
get_parameters() (bayespy.nodes.Binomial method), 128	method), 88
get_parameters() (bayespy.nodes.Categorical method),	get_riemannian_gradient()
133	(bayespy.nodes.GaussianGammaARD
$get\_parameters()  (bayes py. nodes. Categorical Markov Chain$	method), 114
method), 158	get_riemannian_gradient()
get_parameters() (bayespy.nodes.Dirichlet method), 153	(bayespy.nodes.GaussianGammaISO method),
<pre>get_parameters() (bayespy.nodes.Exponential method),</pre>	109
103	get_riemannian_gradient()
get_parameters() (bayespy.nodes.Gamma method), 94	(bayespy.nodes.GaussianMarkovChain
get_parameters() (bayespy.nodes.Gaussian method), 83	method), 164
get_parameters() (bayespy.nodes.GaussianARD method),	get_riemannian_gradient()
88	(bayespy.nodes.GaussianWishart method),
get_parameters() (bayespy.nodes.GaussianGammaARD	118
method), 114	get_riemannian_gradient() (bayespy.nodes.Mixture
get_parameters() (bayespy.nodes.GaussianGammaISO	method), 181
method), 108	get_riemannian_gradient() (bayespy.nodes.Multinomial
get_parameters() (bayespy.nodes.GaussianMarkovChain	method), 138
method), 164	get_riemannian_gradient() (bayespy.nodes.Poisson
get_parameters() (bayespy.nodes.GaussianWishart	method), 143
method), 118	get_riemannian_gradient()
get_parameters() (bayespy.nodes.Mixture method), 181	(bayespy.nodes.SwitchingGaussianMarkovChain
get_parameters() (bayespy.nodes.Multinomial method),	method), 170
138	get_riemannian_gradient()
get_parameters() (bayespy.nodes.Poisson method), 143	(bayespy.nodes.VaryingGaussianMarkovChain
<pre>get_parameters() (bayespy.nodes.SumMultiply method),</pre>	method), 176

```
get_riemannian_gradient()
                                              (bayespy.nodes.Wishart gradient_step() (bayespy.inference.VB method), 196
                                                                                  grid() (in module bayespy.utils.misc), 307
             method), 98
get_shape() (bayespy.inference.vmp.nodes.constant.Constant
              method), 224
                                                                                  Н
get_shape() (bayespy.inference.vmp.nodes.deterministic.Deterministicged() (bayespy.inference.VB method), 196
             method), 222
                                                                                  has_plotter() (bayespy.inference.vmp.nodes.constant.Constant
get_shape() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyethod). 224
              method), 218
                                                                                  has_plotter() (bayespy.inference.vmp.nodes.deterministic.Deterministic
get_shape() (bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDTaGaussianWishart
              method), 230
                                                                                  has_plotter() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily
get_shape() (bayespy.inference.vmp.nodes.gaussian.GaussianGammaISQTQGaussianGammaARD
              method), 228
                                                                                  has_plotter() (bayespy.inference.vmp.nodes.gaussian.GaussianGammaARD
get_shape() (bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGappmatSO
              method), 226
                                                                                  has\_plotter()\ (bayespy.inference.vmp.nodes.gaussian. Gaussian Gamma ISOT)
get_shape() (bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussianGaussi
              method), 234
                                                                                  has_plotter() (bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianG
get_shape() (bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGappmalSO26
              method), 232
                                                                                  has_plotter() (bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGam
get_shape() (bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishatti). 234
              method), 236
                                                                                  has_plotter() (bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGam
get_shape()
                      (bayespy.inference.vmp.nodes.node.Node
                                                                                                method), 232
              method), 212
                                                                                  has_plotter() (bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWish
get_shape() (bayespy.inference.vmp.nodes.stochastic.Stochastic
                                                                                                method), 236
              method), 214
                                                                                  has_plotter()
                                                                                                       (bayespy.inference.vmp.nodes.node.Node
get_shape() (bayespy.nodes.Add method), 190
                                                                                                method), 212
get_shape() (bayespy.nodes.Bernoulli method), 124
                                                                                  has_plotter() (bayespy.inference.vmp.nodes.stochastic.Stochastic
get_shape() (bayespy.nodes.Beta method), 148
                                                                                                method), 215
get_shape() (bayespy.nodes.Binomial method), 129
                                                                                  has_plotter() (bayespy.nodes.Add method), 190
get_shape() (bayespy.nodes.Categorical method), 133
                                                                                  has_plotter() (bayespy.nodes.Bernoulli method), 124
                      (bayespy.nodes.CategoricalMarkovChain
get_shape()
                                                                                  has_plotter() (bayespy.nodes.Beta method), 148
              method), 158
                                                                                  has_plotter() (bayespy.nodes.Binomial method), 129
get_shape() (bayespy.nodes.Dirichlet method), 153
                                                                                  has_plotter() (bayespy.nodes.Categorical method), 133
get_shape() (bayespy.nodes.Exponential method), 103
                                                                                  has_plotter()
                                                                                                        (bayespy.nodes.CategoricalMarkovChain
get_shape() (bayespy.nodes.Gamma method), 94
                                                                                                method), 158
get_shape() (bayespy.nodes.Gate method), 193
                                                                                  has_plotter() (bayespy.nodes.Dirichlet method), 153
get_shape() (bayespy.nodes.Gaussian method), 83
                                                                                  has_plotter() (bayespy.nodes.Exponential method), 104
get_shape() (bayespy.nodes.GaussianARD method), 89
                                                                                  has_plotter() (bayespy.nodes.Gamma method), 94
get_shape()
                          (bayespy.nodes.GaussianGammaARD
                                                                                  has_plotter() (bayespy.nodes.Gate method), 193
              method), 114
                                                                                  has_plotter() (bayespy.nodes.Gaussian method), 83
                            (bayespy.nodes.GaussianGammaISO
get_shape()
                                                                                  has_plotter() (bayespy.nodes.GaussianARD method), 89
              method), 109
                                                                                  has_plotter()
                                                                                                             (bayespy.nodes.GaussianGammaARD
get_shape()
                         (bayespy.nodes.GaussianMarkovChain
                                                                                                method), 114
              method), 164
                                                                                  has_plotter()
                                                                                                              (bayespy.nodes.GaussianGammaISO
get_shape() (bayespy.nodes.GaussianWishart method),
                                                                                                method), 109
                                                                                  has_plotter()
                                                                                                            (bayespy.nodes.GaussianMarkovChain
get_shape() (bayespy.nodes.Mixture method), 181
                                                                                                method), 164
get_shape() (bayespy.nodes.Multinomial method), 138
                                                                                  has_plotter() (bayespy.nodes.GaussianWishart method),
get_shape() (bayespy.nodes.Poisson method), 143
                                                                                                119
get_shape() (bayespy.nodes.SumMultiply method), 187
                                                                                  has_plotter() (bayespy.nodes.Mixture method), 182
get_shape() (bayespy.nodes.SwitchingGaussianMarkovChainhas_plotter() (bayespy.nodes.Multinomial method), 138
              method), 170
                                                                                  has_plotter() (bayespy.nodes.Poisson method), 143
get_shape() (bayespy.nodes.VaryingGaussianMarkovChain
                                                                                  has_plotter() (bayespy.nodes.SumMultiply method), 187
              method), 176
                                                                                  has_plotter() (bayespy.nodes.SwitchingGaussianMarkovChain
get_shape() (bayespy.nodes.Wishart method), 99
                                                                                                method), 170
```

has_plotter() (bayespy.nodes.VaryingGaussianMarkovChain method), 176	n initialize_from_parameters() (bayespy.nodes.Multinomial method), 138
has_plotter() (bayespy.nodes.Wishart method), 99 hinton() (in module bayespy.plot), 206	initialize_from_parameters() (bayespy.nodes.Poisson method), 143
HintonPlotter (class in bayespy.plot), 209	initialize_from_parameters() (bayespy.nodes.SwitchingGaussianMarkovChain method), 170
id() (bayespy.utils.misc.TestCase method), 325	initialize_from_parameters()
identity() (in module bayespy.utils.misc), 307	(bayespy.nodes.VaryingGaussianMarkovChain
ignore_bound_checks (bayespy.inference.VB attribute),	method), 176
198	initialize_from_parameters() (bayespy.nodes.Wishart method), 99
initialize_from_mean_and_covariance() (bayespy.nodes.GaussianARD method),	initialize_from_prior() (bayespy.inference.vmp.nodes.expfamily.Exponential
(bayespy.nodes.GaussianARD method),	method), 218
initialize_from_parameters()	initialize_from_prior() (bayespy.nodes.Bernoulli method),
(bayespy.inference.vmp.nodes.expfamily.Expone	entialFamily <sup>124</sup>
method), 218	initialize_from_prior() (bayespy.nodes.Beta method), 149 initialize_from_prior() (bayespy.nodes.Binomial method),
initialize_from_parameters() (bayespy.nodes.Bernoulli method), 124	129
initialize_from_parameters() (bayespy.nodes.Beta	initialize_from_prior() (bayespy.nodes.Categorical
method), 149	method), 133
initialize_from_parameters() (bayespy.nodes.Binomial method), 129	initialize_from_prior() (bayespy.nodes.CategoricalMarkovChain method), 159
initialize_from_parameters() (bayespy.nodes.Categorical method), 133	initialize_from_prior() (bayespy.nodes.Dirichlet method), 153
initialize_from_parameters()	initialize_from_prior() (bayespy.nodes.Exponential
(bayespy.nodes.CategoricalMarkovChain	method), 104 initialize_from_prior() (bayespy.nodes.Gamma method),
method), 159	94
initialize_from_parameters() (bayespy.nodes.Dirichlet method), 153	initialize_from_prior() (bayespy.nodes.Gaussian method),  84
initialize_from_parameters() (bayespy.nodes.Exponential method), 104	initialize_from_prior() (bayespy.nodes.GaussianARD
initialize_from_parameters() (bayespy.nodes.Gamma	method), 89
method), 94	initialize_from_prior() (bayespy.nodes.GaussianGammaARD
initialize_from_parameters() (bayespy.nodes.Gaussian	method), 114
method), 83	initialize_from_prior() (bayespy.nodes.GaussianGammaISO method), 109
initialize_from_parameters() (bayespy.nodes.GaussianARD method),	initialize_from_prior() (bayespy.nodes.GaussianMarkovChain
89	method), 164
initialize_from_parameters()	initialize_from_prior() (bayespy.nodes.GaussianWishart
(bayespy.nodes.GaussianGammaARD	method), 119
method), 114	initialize_from_prior() (bayespy.nodes.Mixture method), 182
<pre>initialize_from_parameters()           (bayespy.nodes.GaussianGammaISO method),</pre>	initialize_from_prior() (bayespy.nodes.Multinomial
(bayespy.nodes.GaussianGainmaisO method),	method), 138
initialize_from_parameters() (bayespy.nodes.GaussianMarkovChain	initialize_from_prior() (bayespy.nodes.Poisson method), 143
method), 164	$initialize\_from\_prior() \ (bayespy.nodes. Switching Gaussian Markov Chain$
initialize_from_parameters()	method), 170
(bayespy.nodes.GaussianWishart method),	initialize_from_prior() (bayespy.nodes.VaryingGaussianMarkovChain
119	method), 176 initialize_from_prior() (bayespy.nodes.Wishart method),
initialize_from_parameters() (bayespy.nodes.Mixture method), 182	99
method), 102	initialize_from_random() (bayespy.inference.vmp.nodes.expfamily.Exponen

method), 218	initialize_from_value() (bayespy.nodes.Exponential
initialize_from_random() (bayespy.nodes.Bernoulli method), 124	method), 104 initialize_from_value() (bayespy.nodes.Gamma method),
initialize_from_random() (bayespy.nodes.Beta method), 149	94 initialize_from_value() (bayespy.nodes.Gaussian method),
initialize_from_random() (bayespy.nodes.Binomial	84
method), 129 initialize_from_random() (bayespy.nodes.Categorical	initialize_from_value() (bayespy.nodes.GaussianARD method), 89
method), 134	initialize_from_value() (bayespy.nodes.GaussianGammaARD
initialize_from_random() (bayespy.nodes.CategoricalMarko	
method), 159 initialize_from_random() (bayespy.nodes.Dirichlet	initialize_from_value() (bayespy.nodes.GaussianGammaISO method), 109
method), 153	initialize_from_value() (bayespy.nodes.GaussianMarkovChain
initialize_from_random() (bayespy.nodes.Exponential	method), 164
method), 104 initialize_from_random() (bayespy.nodes.Gamma	initialize_from_value() (bayespy.nodes.GaussianWishart method), 119
method), 94	initialize_from_value() (bayespy.nodes.Mixture method),
initialize_from_random() (bayespy.nodes.Gaussian	182
method), 84	initialize_from_value() (bayespy.nodes.Multinomial
initialize_from_random() (bayespy.nodes.GaussianARD method), 89	method), 139 initialize_from_value() (bayespy.nodes.Poisson method),
initialize_from_random() (bayespy.nodes.GaussianGamma.	
method), 114	$initialize\_from\_value() \ (bayes py.nodes. Switching Gaussian Markov Chain$
initialize_from_random() (bayespy.nodes.GaussianGamma	
method), 109 initialize_from_random() (bayespy.nodes.GaussianMarkov	initialize_from_value() (bayespy.nodes.VaryingGaussianMarkovChain Chain method), 176
method), 164	initialize_from_value() (bayespy.nodes.Wishart method),
$initialize\_from\_random() \ (bayes py.nodes. Gaussian Wishart$	99
method), 119	inner() (in module bayespy.utils.linalg), 296
initialize_from_random() (bayespy.nodes.Mixture method), 182	integrated_logpdf_from_parents() (bayespy.nodes.Mixture method), 182
	intervals() (in module bayespy.utils.random), 300
method), 139	inv() (in module bayespy.utils.linalg), 296
initialize_from_random() (bayespy.nodes.Poisson	
mathad) 144	invgamma() (in module bayespy.utils.misc), 307
method), 144 initialize from random() (bayespy.nodes.SwitchingGaussia	invpsi() (in module bayespy.utils.misc), 307
method), 144 initialize_from_random() (bayespy.nodes.SwitchingGaussia method), 170	invpsi() (in module bayespy.utils.misc), 307
initialize_from_random() (bayespy.nodes.SwitchingGaussianmethod), 170 initialize_from_random() (bayespy.nodes.VaryingGaussianmethod)	invpsi() (in module bayespy.utils.misc), 307  nriMawkshaChaimd() (in module bayespy.utils.random), 301  is_callable() (in module bayespy.utils.misc), 307  MarkawaChiain() (in module bayespy.utils.misc), 307
initialize_from_random() (bayespy.nodes.SwitchingGaussian method), 170 initialize_from_random() (bayespy.nodes.VaryingGaussian method), 176	invpsi() (in module bayespy.utils.misc), 307 aniMawkshaChaimd() (in module bayespy.utils.random), 301 is_callable() (in module bayespy.utils.misc), 307 MisrkawaChriai() (in module bayespy.utils.misc), 307 is_shape_subset() (in module bayespy.utils.misc), 307
initialize_from_random() (bayespy.nodes.SwitchingGaussian method), 170 initialize_from_random() (bayespy.nodes.VaryingGaussian method), 176 initialize_from_random() (bayespy.nodes.Wishart	invpsi() (in module bayespy.utils.misc), 307  ariMavishaChaimd() (in module bayespy.utils.random), 301  is_callable() (in module bayespy.utils.misc), 307  MarkanaChain) (in module bayespy.utils.misc), 307  is_shape_subset() (in module bayespy.utils.misc), 307  is_string() (in module bayespy.utils.misc), 307
initialize_from_random() (bayespy.nodes.SwitchingGaussian method), 170 initialize_from_random() (bayespy.nodes.VaryingGaussian method), 176 initialize_from_random() (bayespy.nodes.Wishart method), 99 initialize_from_value() (bayespy.inference.vmp.nodes.expf	invpsi() (in module bayespy.utils.misc), 307  ariMawkshaChaimd() (in module bayespy.utils.random), 301  is_callable() (in module bayespy.utils.misc), 307  MarkovaChiti() (in module bayespy.utils.misc), 307  is_shape_subset() (in module bayespy.utils.misc), 307  is_string() (in module bayespy.utils.misc), 307  isinteger() (in module bayespy.utils.misc), 308
initialize_from_random() (bayespy.nodes.SwitchingGaussian method), 170 initialize_from_random() (bayespy.nodes.VaryingGaussian method), 176 initialize_from_random() (bayespy.nodes.Wishart method), 99 initialize_from_value() (bayespy.inference.vmp.nodes.expfamethod), 218	invpsi() (in module bayespy.utils.misc), 307 miMawkshaChnamd() (in module bayespy.utils.random), 301 is_callable() (in module bayespy.utils.misc), 307 MarkawaChnid) (in module bayespy.utils.misc), 307 is_shape_subset() (in module bayespy.utils.misc), 307 is_string() (in module bayespy.utils.misc), 307 isinteger() (in module bayespy.utils.misc), 308 amily.ExponentialFamily
initialize_from_random() (bayespy.nodes.SwitchingGaussian method), 170 initialize_from_random() (bayespy.nodes.VaryingGaussian method), 176 initialize_from_random() (bayespy.nodes.Wishart method), 99 initialize_from_value() (bayespy.inference.vmp.nodes.expformethod), 218 initialize_from_value() (bayespy.nodes.Bernoulli	invpsi() (in module bayespy.utils.misc), 307  ariMawkshaChaimd() (in module bayespy.utils.random), 301  is_callable() (in module bayespy.utils.misc), 307  MarkovaChiti() (in module bayespy.utils.misc), 307  is_shape_subset() (in module bayespy.utils.misc), 307  is_string() (in module bayespy.utils.misc), 307  isinteger() (in module bayespy.utils.misc), 308
initialize_from_random() (bayespy.nodes.SwitchingGaussian method), 170 initialize_from_random() (bayespy.nodes.VaryingGaussian method), 176 initialize_from_random() (bayespy.nodes.Wishart method), 99 initialize_from_value() (bayespy.inference.vmp.nodes.expfamethod), 218	invpsi() (in module bayespy.utils.misc), 307 miMawkshaChnamd() (in module bayespy.utils.random), 301 is_callable() (in module bayespy.utils.misc), 307 MarkawaChnid) (in module bayespy.utils.misc), 307 is_shape_subset() (in module bayespy.utils.misc), 307 is_string() (in module bayespy.utils.misc), 307 isinteger() (in module bayespy.utils.misc), 308 amily.ExponentialFamily
initialize_from_random() (bayespy.nodes.SwitchingGaussian method), 170 initialize_from_random() (bayespy.nodes.VaryingGaussian method), 176 initialize_from_random() (bayespy.nodes.Wishart method), 99 initialize_from_value() (bayespy.inference.vmp.nodes.expfi method), 218 initialize_from_value() (bayespy.nodes.Bernoulli method), 124 initialize_from_value() (bayespy.nodes.Beta method), 149 initialize_from_value() (bayespy.nodes.Binomial	invpsi() (in module bayespy.utils.misc), 307 miMawkshaChnamd() (in module bayespy.utils.random), 301 is_callable() (in module bayespy.utils.misc), 307 MarkawaChnid) (in module bayespy.utils.misc), 307 is_shape_subset() (in module bayespy.utils.misc), 307 is_string() (in module bayespy.utils.misc), 307 isinteger() (in module bayespy.utils.misc), 308 amily.ExponentialFamily
initialize_from_random() (bayespy.nodes.SwitchingGaussian method), 170 initialize_from_random() (bayespy.nodes.VaryingGaussian method), 176 initialize_from_random() (bayespy.nodes.Wishart method), 99 initialize_from_value() (bayespy.inference.vmp.nodes.expf method), 218 initialize_from_value() (bayespy.nodes.Bernoulli method), 124 initialize_from_value() (bayespy.nodes.Beta method), 149 initialize_from_value() (bayespy.nodes.Binomial method), 129	invpsi() (in module bayespy.utils.misc), 307  miMawkshaChnamd() (in module bayespy.utils.random), 301 is_callable() (in module bayespy.utils.misc), 307  Markowchnid) (in module bayespy.utils.misc), 307 is_shape_subset() (in module bayespy.utils.misc), 307 is_string() (in module bayespy.utils.misc), 307 isinteger() (in module bayespy.utils.misc), 308 amily.ExponentialFamily  kalman_filter() (in module bayespy.utils.misc), 308  L load() (bayespy.inference.VB method), 196 load() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily
initialize_from_random() (bayespy.nodes.SwitchingGaussian method), 170 initialize_from_random() (bayespy.nodes.VaryingGaussian method), 176 initialize_from_random() (bayespy.nodes.Wishart method), 99 initialize_from_value() (bayespy.inference.vmp.nodes.expfi method), 218 initialize_from_value() (bayespy.nodes.Bernoulli method), 124 initialize_from_value() (bayespy.nodes.Beta method), 149 initialize_from_value() (bayespy.nodes.Binomial	invpsi() (in module bayespy.utils.misc), 307  miMawkshaChnamd() (in module bayespy.utils.random), 301  is_callable() (in module bayespy.utils.misc), 307  MarkovoChrid() (in module bayespy.utils.misc), 307  is_shape_subset() (in module bayespy.utils.misc), 307  is_string() (in module bayespy.utils.misc), 307  isinteger() (in module bayespy.utils.misc), 308  amily.ExponentialFamily  kalman_filter() (in module bayespy.utils.misc), 308  L  load() (bayespy.inference.VB method), 196  load() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily method), 219
initialize_from_random() (bayespy.nodes.SwitchingGaussian method), 170 initialize_from_random() (bayespy.nodes.VaryingGaussian method), 176 initialize_from_random() (bayespy.nodes.Wishart method), 99 initialize_from_value() (bayespy.inference.vmp.nodes.expf method), 218 initialize_from_value() (bayespy.nodes.Bernoulli method), 124 initialize_from_value() (bayespy.nodes.Beta method), 149 initialize_from_value() (bayespy.nodes.Binomial method), 129 initialize_from_value() (bayespy.nodes.Categorical method), 134 initialize_from_value() (bayespy.nodes.CategoricalMarkovalue() (bayespy.nodes.Categorica	invpsi() (in module bayespy.utils.misc), 307  miMawksha@haimd() (in module bayespy.utils.random), 301 is_callable() (in module bayespy.utils.misc), 307  Markaw@hiif) (in module bayespy.utils.misc), 307 is_shape_subset() (in module bayespy.utils.misc), 307 is_string() (in module bayespy.utils.misc), 307 isinteger() (in module bayespy.utils.misc), 308 amily.ExponentialFamily  kalman_filter() (in module bayespy.utils.misc), 308  L load() (bayespy.inference.VB method), 196 load() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily method), 219 load() (bayespy.inference.vmp.nodes.stochastic.Stochastic
initialize_from_random() (bayespy.nodes.SwitchingGaussian method), 170 initialize_from_random() (bayespy.nodes.VaryingGaussian method), 176 initialize_from_random() (bayespy.nodes.Wishart method), 99 initialize_from_value() (bayespy.inference.vmp.nodes.expformethod), 218 initialize_from_value() (bayespy.nodes.Bernoulli method), 124 initialize_from_value() (bayespy.nodes.Beta method), 149 initialize_from_value() (bayespy.nodes.Binomial method), 129 initialize_from_value() (bayespy.nodes.Categorical method), 134 initialize_from_value() (bayespy.nodes.CategoricalMarkovalue() (bayespy.nodes.Categoric	invpsi() (in module bayespy.utils.misc), 307  miMawksha@haimd() (in module bayespy.utils.random), 301  is_callable() (in module bayespy.utils.misc), 307  Markawa@hii() (in module bayespy.utils.misc), 307  is_shape_subset() (in module bayespy.utils.misc), 307  is_string() (in module bayespy.utils.misc), 307  isinteger() (in module bayespy.utils.misc), 308  amily.ExponentialFamily  kalman_filter() (in module bayespy.utils.misc), 308  L  load() (bayespy.inference.VB method), 196  load() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily method), 219  load() (bayespy.inference.vmp.nodes.stochastic.Stochastic Chain method), 215  load() (bayespy.nodes.Bernoulli method), 124
initialize_from_random() (bayespy.nodes.SwitchingGaussian method), 170 initialize_from_random() (bayespy.nodes.VaryingGaussian method), 176 initialize_from_random() (bayespy.nodes.Wishart method), 99 initialize_from_value() (bayespy.inference.vmp.nodes.expf method), 218 initialize_from_value() (bayespy.nodes.Bernoulli method), 124 initialize_from_value() (bayespy.nodes.Beta method), 149 initialize_from_value() (bayespy.nodes.Binomial method), 129 initialize_from_value() (bayespy.nodes.Categorical method), 134 initialize_from_value() (bayespy.nodes.CategoricalMarkovalue() (bayespy.nodes.Categorica	invpsi() (in module bayespy.utils.misc), 307  aniMavkshaChaimd() (in module bayespy.utils.random), 301 is_callable() (in module bayespy.utils.misc), 307  MarkovvChrid) (in module bayespy.utils.misc), 307 is_shape_subset() (in module bayespy.utils.misc), 307 is_string() (in module bayespy.utils.misc), 307 isinteger() (in module bayespy.utils.misc), 308  amily.ExponentialFamily  kalman_filter() (in module bayespy.utils.misc), 308  L load() (bayespy.inference.VB method), 196 load() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily method), 219 load() (bayespy.inference.vmp.nodes.stochastic.Stochastic Chain method), 215

load() (bayespy.nodes.Categorical method), 134 load() (bayespy.nodes.CategoricalMarkovChain method), 159 load() (bayespy.nodes.Dirichlet method), 154 load() (bayespy.nodes.Exponential method), 104	logpdf() (bayespy.nodes.Mixture method), 182 logpdf() (bayespy.nodes.Multinomial method), 139 logpdf() (bayespy.nodes.Poisson method), 144 logpdf() (bayespy.nodes.SwitchingGaussianMarkovChain method), 171
load() (bayespy.nodes.Gamma method), 94	logpdf() (bayespy.nodes.VaryingGaussianMarkovChain
load() (bayespy.nodes.Gaussian method), 84	method), 177
load() (bayespy.nodes.GaussianARD method), 89	logpdf() (bayespy.nodes.Wishart method), 99
load() (bayespy.nodes.GaussianGammaARD method),	logsumexp() (in module bayespy.utils.misc), 308
114	longMessage (bayespy.utils.misc.TestCase attribute), 326
load() (bayespy.nodes.GaussianGammaISO method), 109	lower_bound_contribution()
load() (bayespy.nodes.GaussianMarkovChain method),	(bayespy.inference.vmp.nodes.constant.Constant
164	method), 224
load() (bayespy.nodes.GaussianWishart method), 119	lower_bound_contribution()
load() (bayespy.nodes.Mixture method), 182	(bayespy.inference.vmp.nodes.deterministic.Deterministic
load() (bayespy.nodes.Multinomial method), 139	method), 222
load() (bayespy.nodes.Poisson method), 144	lower_bound_contribution()
load() (bayespy.nodes.SwitchingGaussianMarkovChain method), 170	(bayespy.inference.vmp.nodes.expfamily.ExponentialFamily method), 219
load() (bayespy.nodes.VaryingGaussianMarkovChain	lower_bound_contribution()
method), 177	(bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDTo
load() (bayespy.nodes.Wishart method), 99	method), 230
logdet() (bayespy.utils.misc.CholeskyDense method),	lower_bound_contribution()
314	(bayes py. in ference. vmp. nodes. gaussian. Gaussian Gamma ISO ToGother and the state of the
logdet() (bayespy.utils.misc.CholeskySparse method),	method), 228
315	lower_bound_contribution()
logdet_chol() (in module bayespy.utils.linalg), 296	(bayespy.inference.vmp.nodes.gaussian.GaussianToGaussianGam
logdet_chol() (in module bayespy.utils.misc), 308	method), 226
logdet_cov() (in module bayespy.utils.linalg), 296	lower_bound_contribution()
logdet_tri() (in module bayespy.utils.linalg), 296	(bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGamma
loglikelihood_lowerbound() (bayespy.inference.VB	method), 234
method), 196	lower_bound_contribution()
logodds_to_probability() (in module	(bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGamma
bayespy.utils.random), 301	method), 232
logpdf() (bayespy.inference.vmp.nodes.expfamily.Exponen method), 219	(bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWishart
logpdf() (bayespy.nodes.Bernoulli method), 124	method), 236
logpdf() (bayespy.nodes.Beta method), 149	lower_bound_contribution() (bayespy.nodes.Add
logpdf() (bayespy.nodes.Binomial method), 129	method), 191
logpdf() (bayespy.nodes.Categorical method), 134	lower_bound_contribution() (bayespy.nodes.Bernoulli
logpdf() (bayespy.nodes.CategoricalMarkovChain	method), 124
method), 159	lower_bound_contribution() (bayespy.nodes.Beta
logpdf() (bayespy.nodes.Dirichlet method), 154	method), 149
logpdf() (bayespy.nodes.Exponential method), 104	lower_bound_contribution() (bayespy.nodes.Binomial
logpdf() (bayespy.nodes.Gamma method), 95	method), 129
logpdf() (bayespy.nodes.Gaussian method), 84	lower_bound_contribution() (bayespy.nodes.Categorical
logpdf() (bayespy.nodes.GaussianARD method), 89	method), 134
logpdf() (bayespy.nodes.GaussianGammaARD method),	lower_bound_contribution()
115	(bayespy.nodes.CategoricalMarkovChain
$logpdf() \ \ (bayespy.nodes. Gaussian Gamma ISO \ \ method),$	method), 159
109	lower_bound_contribution() (bayespy.nodes.Dirichlet
logpdf() (bayespy.nodes.GaussianMarkovChain method),	method), 154
165	lower_bound_contribution() (bayespy.nodes.Exponential
logpdf() (bayespy.nodes.GaussianWishart method), 119	method), 104

lower_bound_contribution() (bay method), 95	espy.nodes.Gamma	lowerbound() (bayespy.nodes.GaussianGammaISO method), 110
lower_bound_contribution() (lower_bound_contribution() (lower_bound_contribution())	bayespy.nodes.Gate	lowerbound() (bayespy.nodes.GaussianMarkovChain method), 165
	spy.nodes.Gaussian	lowerbound() (bayespy.nodes.GaussianWishart method),
lower_bound_contribution()		lowerbound() (bayespy.nodes.Mixture method), 183
(bayespy.nodes.GaussianAR	RD method),	lowerbound() (bayespy.nodes.Multinomial method), 139
89	,,	lowerbound() (bayespy.nodes.Poisson method), 144
lower_bound_contribution() (bayespy.nodes.GaussianGa	mmaARD	lowerbound() (bayespy.nodes.SwitchingGaussianMarkovChain method), 171
method), 115		lowerbound() (bayespy.nodes.VaryingGaussianMarkovChain
lower_bound_contribution()		method), 177
(bayespy.nodes.GaussianGa 110	mmaISO method),	lowerbound() (bayespy.nodes.Wishart method), 99
lower_bound_contribution()		M
(bayespy.nodes.GaussianMa	ırkovChain	m_digamma() (in module bayespy.utils.misc), 308
method), 165		m_dot() (in module bayespy.utils.linalg), 296
lower_bound_contribution()		m_dot() (in module bayespy.utils.misc), 308
(bayespy.nodes.GaussianWi	shart method),	m_outer() (in module bayespy.utils.misc), 309
119		m_solve_triangular() (in module bayespy.utils.misc), 309
	espy.nodes.Mixture	make_equal_length() (in module bayespy.utils.misc), 309
method), 182		make_equal_ndim() (in module bayespy.utils.misc), 309
lower_bound_contribution() (bayespy.	.nodes.Multinomial	mask() (in module bayespy.utils.random), 301
method), 139	1 5	maxDiff (bayespy.utils.misc.TestCase attribute), 326
	respy.nodes.Poisson	mean() (in module bayespy.utils.misc), 309
method), 144	nadaa CumMultimlu	minimize() (in module bayespy.utils.optimize), 302
lower_bound_contribution() (bayespy.i method), 187	nodes.Sumividitiply	Mixture (class in bayespy.nodes), 179
lower_bound_contribution()		mmdot() (in module bayespy.utils.linalg), 297
(bayespy.nodes.SwitchingG	aussianMarkovChair	Moments (class in bayespy.inference.vmp.nodes.node),
method), 171	aussiamviarko v Chan	
lower_bound_contribution()		move_plates() (bayespy.inference.vmp.nodes.constant.Constant method), 224
(bayespy.nodes.VaryingGau	ssianMarkovChain	move_plates() (bayespy.inference.vmp.nodes.deterministic.Deterministic
method), 177		method), 222
	espy.nodes.Wishart	move_plates() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily
method), 99		mathod) 210
lowerbound() (bayespy.inference.vmp.	.nodes.expfamily.Exp	Poneve all attention, 219 Poneve all attention, 219
method), 219		method), 230
method), 215		ochastic_plates() (bayespy.inference.vmp.nodes.gaussian.GaussianGammaISO method), 228
lowerbound() (bayespy.nodes.Bernoul		move_plates() (bayespy.inference.vmp.nodes.gaussian.GaussianToGaussian
lowerbound() (bayespy.nodes.Beta me		method), 226
lowerbound() (bayespy.nodes.Binomia		$move\_plates() \ (bayes py.inference.vmp.nodes.gaussian. Wrap To Gaussian $
lowerbound() (bayespy.nodes.Categor		method), 234
lowerbound() (bayespy.nodes.Categoriumethod), 159		move_plates() (bayespy.inference.vmp.nodes.gaussian.WrapToGaussianGarmethod), 232
lowerbound() (bayespy.nodes.Dirichle lowerbound() (bayespy.nodes.Exponer		move_plates() (bayespy.inference.vmp.nodes.gaussian.WrapToGaussianWismethod), 237
lowerbound() (bayespy.nodes.Gamma		move_plates() (bayespy.inference.vmp.nodes.node.Node
lowerbound() (bayespy.nodes.Gaussia		method), 212
lowerbound() (bayespy.nodes.Gaussia		move_plates() (bayespy.inference.vmp.nodes.stochastic.Stochastic
	ussianGammaARD	method), 215
method), 115		move_plates() (bayespy.nodes.Add method), 191

move_plates() (bayespy.nodes.Bernoulli method), 125	nodes() (bayespy.inference.vmp.transformations.RotateSwitchingMarkovCl method), 203
move_plates() (bayespy.nodes.Beta method), 149 move_plates() (bayespy.nodes.Binomial method), 130	nodes() (bayespy.inference.vmp.transformations.RotateVaryingMarkovChai
move_plates() (bayespy.nodes.Categorical method), 134	method), 204
move_plates() (bayespy.nodes.CategoricalMarkovChain	method), 20+
method), 159	0
move_plates() (bayespy.nodes.Dirichlet method), 154	
move_plates() (bayespy.nodes.Exponential method), 104	observe() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily method), 219
move_plates() (bayespy.nodes.Gamma method), 95	observe() (bayespy.inference.vmp.nodes.stochastic.Stochastic
move_plates() (bayespy.nodes.Gate method), 193	method), 215
move_plates() (bayespy.nodes.Gaussian method), 84	observe() (bayespy.nodes.Bernoulli method), 125
move_plates() (bayespy.nodes.GaussianARD method), 90	observe() (bayespy.nodes.Beta method), 149
move_plates() (bayespy.nodes.GaussianGammaARD	observe() (bayespy.nodes.Binomial method), 130
method), 115	observe() (bayespy.nodes.Categorical method), 134
move_plates() (bayespy.nodes.GaussianGammaISO	observe() (bayespy.nodes.CategoricalMarkovChain
method), 110	method), 160
move_plates() (bayespy.nodes.GaussianMarkovChain	observe() (bayespy.nodes.Dirichlet method), 154
method), 165	observe() (bayespy.nodes.Exponential method), 105
move_plates() (bayespy.nodes.GaussianWishart method),	observe() (bayespy.nodes.Gamma method), 95
120	observe() (bayespy.nodes.Gaussian method), 84
move_plates() (bayespy.nodes.Mixture method), 183	observe() (bayespy.nodes.GaussianARD method), 90
move_plates() (bayespy.nodes.Multinomial method), 139	observe() (bayespy.nodes.GaussianGammaARD
move_plates() (bayespy.nodes.Poisson method), 144	method), 115
move_plates() (bayespy.nodes.SumMultiply method), 187	observe() (bayespy.nodes.GaussianGammaISO method),
move_plates() (bayespy.nodes.SwitchingGaussianMarkovC	Chain 110
method), 171	observe() (bayespy.nodes.GaussianMarkovChain
$move\_plates() \ (bayespy.nodes. Varying Gaussian Markov Charlow Char$	
method), 177	observe() (bayespy.nodes.GaussianWishart method), 120
move_plates() (bayespy.nodes.Wishart method), 100	observe() (bayespy.nodes.Mixture method), 183
moveaxis() (in module bayespy.utils.misc), 309	observe() (bayespy.nodes.Multinomial method), 139
multidigamma() (in module bayespy.utils.misc), 309	observe() (bayespy.nodes.Poisson method), 144
Multinomial (class in bayespy.nodes), 136	observe() (bayespy.nodes.SwitchingGaussianMarkovChain
MultinomialDistribution (class in	method), 171
bayespy.inference.vmp.nodes.multinomial),	$observe () \ \ (bayes py.nodes. Varying Gaussian Markov Chain$
291	method), 177
MultinomialMoments (class in	observe() (bayespy.nodes.Wishart method), 100
bayespy.inference.vmp.nodes.multinomial),	optimize() (bayespy.inference.VB method), 196
251	orth() (in module bayespy.utils.random), 301
multiply_shapes() (in module bayespy.utils.misc), 309	outer() (in module bayespy.utils.linalg), 297
mvdot() (in module bayespy.utils.linalg), 297	n
N	P
	parse_command_line_arguments() (in module
nans() (in module bayespy.utils.misc), 309	bayespy.utils.misc), 310
nested_iterator() (in module bayespy.utils.misc), 309	pattern_search() (bayespy.inference.VB method), 196
Node (class in bayespy.inference.vmp.nodes.node), 211	pdf() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily
$nodes () \ (bayes py. inference. vmp. transformations. Rotate Gaussian and the following properties of the properties $	
method), 199	pdf() (bayespy.nodes.Bernoulli method), 125
$nodes () \ (bayes py. inference. vmp. transformations. Rotate Gaussian and the state of the st$	
method), 201	pdf() (bayespy.nodes.Binomial method), 130
nodes() (bayespy.inference.vmp.transformations.RotateGau	
method), 202	pdf() (bayespy.nodes.CategoricalMarkovChain method),
nodes() (bayespy.inference.vmp.transformations.RotateMu	
method), 205	pdf() (bayespy.nodes.Dirichlet method), 154
	pdf() (bayespy.nodes.Exponential method), 105

plates (bayespy.nodes.GaussianGammaISO attribute),

plates (bayespy.nodes.GaussianMarkovChain attribute),

pdf() (bayespy.nodes.GaussianMarkovChain method),	plates (bayespy.nodes.GaussianWishart attribute), 121 plates (bayespy.nodes.Mixture attribute), 184			
pdf() (bayespy.nodes.GaussianWishart method), 120	plates (bayespy.nodes.Multinomial attribute), 141			
pdf() (bayespy.nodes.Mixture method), 183	plates (bayespy.nodes.Poisson attribute), 146			
pdf() (bayespy.nodes.Multinomial method), 139	plates (bayespy.nodes.SumMultiply attribute), 188			
pdf() (bayespy.nodes.Poisson method), 144	plates (bayespy.nodes.SwitchingGaussianMarkovChain			
pdf() (bayespy.nodes.SwitchingGaussianMarkovChain	attribute), 172			
method), 171	plates (bayespy.nodes.VaryingGaussianMarkovChain at-			
pdf() (bayespy.nodes.VaryingGaussianMarkovChain	tribute), 179			
method), 177	plates (bayespy.nodes.Wishart attribute), 101			
pdf() (bayespy.nodes.Wishart method), 100	plates_from_parent() (bayespy.inference.vmp.nodes.bernoulli.BernoulliDistr			
pdf() (in module bayespy.plot), 206	method), 285			
PDFPlotter (class in bayespy.plot), 208	plates_from_parent() (bayespy.inference.vmp.nodes.beta.BetaDistribution			
plates (bayespy.inference.vmp.nodes.constant.Constant	method), 281			
attribute), 225	plates_from_parent() (bayespy.inference.vmp.nodes.binomial.BinomialDistr			
plates (bayespy.inference.vmp.nodes.deterministic.Determ				
attribute), 223	plates_from_parent() (bayespy.inference.vmp.nodes.categorical.Categorical			
plates (bayespy.inference.vmp.nodes.expfamily.Exponentia				
attribute), 220	plates_from_parent() (bayespy.inference.vmp.nodes.categorical_markov_cha			
plates (bayespy.inference.vmp.nodes.gaussian.GaussianGa				
attribute), 231	plates_from_parent() (bayespy.inference.vmp.nodes.dirichlet.DirichletDistri			
plates (bayespy.inference.vmp.nodes.gaussian.GaussianGa				
attribute), 229	plates_from_parent() (bayespy.inference.vmp.nodes.expfamily.ExponentialF			
plates (bayespy.inference.vmp.nodes.gaussian.GaussianTo				
attribute), 227	plates_from_parent() (bayespy.inference.vmp.nodes.gamma.GammaDistribu			
plates (bayespy.inference.vmp.nodes.gaussian.WrapToGau				
attribute), 235	plates_from_parent() (bayespy.inference.vmp.nodes.gaussian.GaussianARD			
plates (bayespy.inference.vmp.nodes.gaussian.WrapToGau				
attribute), 233	plates_from_parent() (bayespy.inference.vmp.nodes.gaussian.GaussianDistr			
plates (bayespy.inference.vmp.nodes.gaussian.WrapToGau				
attribute), 237	plates_from_parent() (bayespy.inference.vmp.nodes.gaussian.GaussianGami			
plates (bayespy.inference.vmp.nodes.node.Node at-	method), 264			
tribute), 213	plates_from_parent() (bayespy.inference.vmp.nodes.gaussian.GaussianGami			
plates (bayespy.inference.vmp.nodes.stochastic.Stochastic	method), 262			
attribute), 216	plates_from_parent() (bayespy.inference.vmp.nodes.gaussian.GaussianWish			
plates (bayespy.nodes.Add attribute), 191	method), 266			
plates (bayespy.nodes.Bernoulli attribute), 126	plates_from_parent() (bayespy.inference.vmp.nodes.gaussian_markov_chain.			
plates (bayespy.nodes.Beta attribute), 151	method), 269			
plates (bayespy.nodes.Binomial attribute), 131	plates_from_parent() (bayespy.inference.vmp.nodes.gaussian_markov_chain.			
plates (bayespy.nodes.Categorical attribute), 131	method), 272			
	plates_from_parent() (bayespy.inference.vmp.nodes.gaussian_markov_chain.			
plates (bayespy.nodes.CategoricalMarkovChain attribute), 161	method), 275			
	plates_from_parent() (bayespy.inference.vmp.nodes.multinomial.Multinomi			
plates (bayespy nodes Exponential attribute), 156	method), 292			
plates (bayespy.nodes.Exponential attribute), 106	plates_from_parent() (bayespy.inference.vmp.nodes.poisson.PoissonDistribu			
plates (bayespy.nodes.Gamma attribute), 96				
plates (bayespy.nodes.Gate attribute), 194	method), 294			
plates (bayespy.nodes.Gaussian attribute), 86	plates_from_parent() (bayespy.inference.vmp.nodes.stochastic.Distribution			
plates (bayespy.nodes.GaussianARD attribute), 91	method), 254			
plates (bayespy.nodes.GaussianGammaARD attribute),	plates_from_parent() (bayespy.inference.vmp.nodes.wishart.WishartDistribu			

116

pdf() (bayespy.nodes.Gamma method), 95

pdf() (bayespy.nodes.Gaussian method), 85

pdf() (bayespy.nodes.GaussianARD method), 90

pdf() (bayespy.nodes.GaussianGammaARD method), 115

pdf() (bayespy.nodes.GaussianGammaISO method), 110

```
method), 279
                                                                                                                                  attribute), 173
plates_multiplier (bayespy.inference.vmp.nodes.constant.Copstates_multiplier (bayespy.nodes.VaryingGaussianMarkovChain
                  attribute), 225
                                                                                                                                 attribute), 179
plates_multiplier (bayespy.inference.vmp.nodes.deterministipliates_multiplier (bayespy.nodes.Wishart attribute), 101
                  attribute), 223
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.bernoulli.BernoulliDistribu
plates_multiplier (bayespy.inference.vmp.nodes.expfamily.ExponentialFlathide), 285
                  attribute), 220
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.beta.BetaDistribution
plates_multiplier (bayespy.inference.vmp.nodes.gaussian.GaussianGammth\( R)\( D\) T\( GaussianWishart \)
                  attribute), 231
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.binomial.BinomialDistribu
plates_multiplier (bayespy.inference.vmp.nodes.gaussian.GaussianGammatl60)TaGaussianGammaARD
                  attribute), 229
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.categorical.CategoricalDis
plates_multiplier (bayespy.inference.vmp.nodes.gaussian.GaussianToGautsout)GathomaISO
                  attribute), 227
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.categorical_markov_chain.
plates_multiplier (bayespy.inference.vmp.nodes.gaussian.WrapToGaussiathGd)maaARD
                  attribute), 235
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.dirichlet.DirichletDistribut
plates_multiplier (bayespy.inference.vmp.nodes.gaussian.WrapToGaussiathGd)marasSO
                  attribute), 233
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.expfamily.ExponentialFam
plates_multiplier (bayespy.inference.vmp.nodes.gaussian.WrapToGaussiathWd)habto
                  attribute), 237
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.gamma.GammaDistributio
                                                                                                                                  method), 277
plates_multiplier (bayespy.inference.vmp.nodes.node.Node
                  attribute), 213
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.gaussian.GaussianARDDis
plates_multiplier (bayespy.inference.vmp.nodes.stochastic.Stochastic method), 261
                  attribute), 216
                                                                                                               plates\_to\_parent() \ (bayespy.inference.vmp.nodes.gaussian. Gaussian Distribution \ (bayespy.inference.vmp.nodes.gaussian. \ (bayespy.inference.vmp.nodes.gaus
plates_multiplier (bayespy.nodes.Add attribute), 191
                                                                                                                                  method), 258
plates_multiplier (bayespy.nodes.Bernoulli attribute), 126
                                                                                                               plates\_to\_parent() \ (bayespy.inference.vmp.nodes.gaussian. Gaussian Gamma. A substantial of the control of t
plates_multiplier (bayespy.nodes.Beta attribute), 151
                                                                                                                                  method), 264
plates_multiplier (bayespy.nodes.Binomial attribute), 131
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.gaussian.GaussianGamma
plates_multiplier (bayespy.nodes.Categorical attribute),
                                                                                                                                  method), 263
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.gaussian.GaussianWishart
plates_multiplier (bayespy.nodes.CategoricalMarkovChain
                                                                                                                                  method), 266
                  attribute), 161
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.gaussian_markov_chain.Ga
plates_multiplier (bayespy.nodes.Dirichlet attribute), 156
                                                                                                                                  method), 269
plates_multiplier (bayespy.nodes.Exponential attribute),
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.gaussian_markov_chain.Sw
                                                                                                                                  method), 272
plates_multiplier (bayespy.nodes.Gamma attribute), 96
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.gaussian_markov_chain.Va
plates_multiplier (bayespy.nodes.Gate attribute), 194
                                                                                                                                 method), 275
plates_multiplier (bayespy.nodes.Gaussian attribute), 86
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.multinomial.MultinomialD
plates_multiplier
                                     (bayespy.nodes.GaussianARD
                                                                                                                                  method), 292
                  tribute), 91
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.poisson.PoissonDistribution
plates_multiplier (bayespy.nodes.GaussianGammaARD
                                                                                                                                  method), 294
                  attribute), 116
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.stochastic.Distribution
plates_multiplier (bayespy.nodes.GaussianGammaISO at-
                                                                                                                                  method), 254
                  tribute), 112
                                                                                                               plates_to_parent() (bayespy.inference.vmp.nodes.wishart.WishartDistributio
plates_multiplier (bayespy.nodes.GaussianMarkovChain
                                                                                                                                  method), 279
                  attribute), 167
                                                                                                               plot() (bayespy.inference.VB method), 197
                                                                                                                               (bayespy.inference.vmp.nodes.constant.Constant
plates_multiplier (bayespy.nodes.GaussianWishart at-
                                                                                                               plot()
                  tribute), 121
                                                                                                                                  method), 224
plates_multiplier (bayespy.nodes.Mixture attribute), 184
                                                                                                               plot() (bayespy.inference.vmp.nodes.deterministic.Deterministic
plates_multiplier (bayespy.nodes.Multinomial attribute),
                                                                                                                                  method), 222
                                                                                                               plot() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily
plates_multiplier (bayespy.nodes.Poisson attribute), 146
                                                                                                                                  method), 219
plates_multiplier (bayespy.nodes.SumMultiply attribute),
                                                                                                              plot() (bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDToGau
                                                                                                                                  method), 231
```

plates\_multiplier (bayespy.nodes.SwitchingGaussianMarkovfolbath) (bayespy.inference.vmp.nodes.gaussian.GaussianGammaISOToGaussianGauss

method), 229	R
plot() (bayespy.inference.vmp.nodes.gaussian.GaussianToC	Gaussian CammaLSO.inference.vmp.nodes.bernoulli.BernoulliDistribution
memod), 227	method) 285
method), 233	ssianGamm(bayespy.inference.vmp.nodes.beta.BetaDistribution
memod), 255	ssianGamm(USQ), inference.vmp.nodes.binomial.BinomialDistribution
plot() (bayespy.inference.vmp.nodes.gaussian.WrapToGausmethod), 237	ssian Wish (bayespy.inference.vmp.nodes.categorical.CategoricalDistribution method), 289
plot() (bayespy.inference.vmp.nodes.node.Node method), 213	random() (bayespy.inference.vmp.nodes.categorical_markov_chain.Categorical_method), 291
plot() (bayespy.inference.vmp.nodes.stochastic.Stochastic method), 215	random() (bayespy.inference.vmp.nodes.dirichlet.DirichletDistribution method), 283
plot() (bayespy.nodes.Add method), 191 plot() (bayespy.nodes.Bernoulli method), 125	random() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily method), 219
plot() (bayespy.nodes.Beta method), 150 plot() (bayespy.nodes.Binomial method), 130	random() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamilyDistr method), 256
plot() (bayespy.nodes.Categorical method), 135 plot() (bayespy.nodes.CategoricalMarkovChain method),	random() (bayespy.inference.vmp.nodes.gamma.GammaDistribution method), 277
plot() (bayespy.nodes.Dirichlet method), 154	random() (bayespy.inference.vmp.nodes.gaussian.GaussianARDDistribution method), 261
plot() (bayespy.nodes.Exponential method), 105 plot() (bayespy.nodes.Gamma method), 95	random() (bayespy.inference.vmp.nodes.gaussian.GaussianDistribution method), 259
plot() (bayespy.nodes.Gate method), 193 plot() (bayespy.nodes.Gaussian method), 85	random() (bayespy.inference.vmp.nodes.gaussian.GaussianGammaARDDismethod), 264
plot() (bayespy.nodes.GaussianARD method), 90 plot() (bayespy.nodes.GaussianGammaARD method),	random() (bayespy.inference.vmp.nodes.gaussian.GaussianGammaISODistimethod), 263
plot() (bayespy.nodes.GaussianGammaISO method), 110	random() (bayespy.inference.vmp.nodes.gaussian.GaussianWishartDistribut method), 266
plot() (bayespy.nodes.GaussianMarkovChain method),	random() (bayespy.inference.vmp.nodes.gaussian_markov_chain.GaussianMmethod), 269
plot() (bayespy.nodes.GaussianWishart method), 120 plot() (bayespy.nodes.Mixture method), 183	random() (bayespy.inference.vmp.nodes.gaussian_markov_chain.Switching(method), 272
plot() (bayespy.nodes.Multinomial method), 140 plot() (bayespy.nodes.Poisson method), 145	random() (bayespy.inference.vmp.nodes.gaussian_markov_chain.VaryingGamethod), 275
plot() (bayespy.nodes.SumMultiply method), 187 plot() (bayespy.nodes.SwitchingGaussianMarkovChain	random() (bayespy.inference.vmp.nodes.multinomial.MultinomialDistribution method), 293
method), 171 plot() (bayespy.nodes.VaryingGaussianMarkovChain	random() (bayespy.inference.vmp.nodes.poisson.PoissonDistribution method), 294
method), 177 plot() (bayespy.nodes.Wishart method), 100	random() (bayespy.inference.vmp.nodes.stochastic.Distribution method), 254
plot() (in module bayespy.plot), 206 plot_iteration_by_nodes() (bayespy.inference.VB	random() (bayespy.inference.vmp.nodes.stochastic.Stochastic method), 215
method), 197 plotmatrix() (bayespy.nodes.GaussianGammaISO	random() (bayespy.inference.vmp.nodes.wishart.WishartDistribution method), 279
method), 110	random() (bayespy.nodes.Bernoulli method), 125
Plotter (class in bayespy.plot), 207	random() (bayespy.nodes.Beta method), 150
Poisson (class in bayespy.nodes), 141	random() (bayespy.nodes.Binomial method), 130
PoissonDistribution (class in	random() (bayespy.nodes.Categorical method), 135
bayespy.inference.vmp.nodes.poisson), 293 PoissonMoments (class in	random() (bayespy.nodes.CategoricalMarkovChain
PoissonMoments (class in bayespy.inference.vmp.nodes.poisson), 252	method), 160
caj copjimerence. impinoacorpoisson, 202	random() (bayespy.nodes.Dirichlet method), 155 random() (bayespy.nodes.Exponential method), 105
	random() (bayespy.nodes.Exponential method), 103

	v.nodes.Gaussian method), 85 v.nodes.GaussianARD method), 90		bayespy.inference.vmp.	transformations),	
random() method) random() (bayasan			RotateSwitchingMarkovChain bayespy.inference.vmp. 202	(class transformations),	in
110	y.nodes.GaussianGammaISO method		RotateVaryingMarkovChain	(class	in
	(bayespy.nodes.GaussianMarkovChai		bayespy.inference.vmp.	•	III
	v.nodes.GaussianWishart method), 120	)	RotationOptimizer	(class	in
	v.nodes.Mixture method), 183		bayespy.inference.vmp.	transformations),	
	v.nodes.Multinomial method), 140		198		
random() (bayespy	v.nodes.Poisson method), 145		rts_smoother() (in module bayesp	y.utils.misc), 311	
random() (bayespy method)	nodes.SwitchingGaussianMarkovCha, 171			e method), 325	
method)			S safe_indices() (in module bayespy	y.utils.misc), 312	
	nodes.Wishart method), 100		save() (bayespy.inference.VB me		
	e() (in module bayespy.utils.misc), 31	4	save() (bayespy.inference.vmp.no		entialFamily
	in module bayespy.utils.misc), 311		method), 220		·
	bayespy.utils.misc), 311		save() (bayespy.inference.vmp.no	des.stochastic.Stocha	stic
	nference.vmp.transformations.RotateC		111041104), =10		
method)		~	save() (bayespy.nodes.Bernoulli r	method), 125	
rotate() (bayespy.ii	nference.vmp.transformations.RotateC				
method)		20116	save() (bayespy.nodes.Binomial r	nethod), 130	
method)	nference.vmp.transformations.RotateC	Jaus	save() (bayespy:Hodes.Categorica	l method), 135	
	nference.vmp.transformations.RotateN	Mult	- 100		od),
	nference.vmp.transformations.RotateS	Swit		al method), 105	
		Jary	save() (bayespy.nodes.Gamma mo	ethod), 95	
method)	nference.vmp.transformations.RotateV				
	nference.vmp.transformations.Rotation	nOr	save() (bayespy.nodes.GaussianA	RD method), 90 Commo APD - motho	v4)
method)	, 199 nodes.Gaussian method), 85		115		
	odes.Gaussian Method), 90		save() (bayespy.nodes.GaussianG		
	nodes.GaussianMarkovChain method	).	save() (bayespy.nodes.Gaussian) 166	viarkovCnain metno	00),
165			save() (bayespy.nodes.GaussianW	lichart method) 120	
rotate() (bayespy.i	nodes.SwitchingGaussianMarkovChai		save() (bayespy.nodes.Mixture m		
method)	•		save() (bayespy.nodes.Multinomi	* *	
rotate() (bayesp	y.nodes.VaryingGaussianMarkovChai		save() (bayespy.nodes.Poisson me		
method)	, 178		save() (bayespy.nodes.Switchin		ain
0 (	yespy.nodes.Gaussian method), 85		method), 172	C	
	espy.nodes.GaussianARD method), 9	0	save() (bayespy.nodes.Varyin	gGaussianMarkovCh	ain
RotateGaussian	`	n	method), 178	-	
	inference.vmp.transformations),		save() (bayespy.nodes.Wishart me	ethod), 100	
199			set_annealing() (bayespy.inferenc	e.VB method), 197	
RotateGaussianAF	`		set_autosave() (bayespy.inference		
	inference.vmp.transformations),		set_callback() (bayespy.inference		
200 Poteta Goussian Me	owkouChoin (alaca :		set_parameters() (bayespy.inferen		
RotateGaussianMa	arkovChain (class i inference.vmp.transformations),	in	set_parameters() (bayespy.inferen	ce.vmp.nodes.expfan	nily.ExponentialFami
201	imerence.vinp.transformations),		method), 220	Darmoulli mathad\ 10	05
RotateMultiple	(class i		set_parameters() (bayespy.nodes.l set_parameters() (bayespy.nodes.l		دي
1	`		ser_parameters() (oayespy.nodes.i	50m momou <i>j</i> , 150	

```
set_parameters() (bayespy.nodes.Binomial method), 130
                                                                                                           set_plotter() (bayespy.nodes.Binomial method), 130
set_parameters() (bayespy.nodes.Categorical method),
                                                                                                           set_plotter() (bayespy.nodes.Categorical method), 135
                                                                                                            set_plotter()
                                                                                                                                         (bayespy.nodes.CategoricalMarkovChain
set_parameters() (bayespy.nodes.CategoricalMarkovChain
                                                                                                                              method), 160
                                                                                                           set_plotter() (bayespy.nodes.Dirichlet method), 155
                 method), 160
set_parameters() (bayespy.nodes.Dirichlet method), 155
                                                                                                            set_plotter() (bayespy.nodes.Exponential method), 105
set_parameters() (bayespy.nodes.Exponential method),
                                                                                                           set_plotter() (bayespy.nodes.Gamma method), 96
                                                                                                            set_plotter() (bayespy.nodes.Gate method), 193
set_parameters() (bayespy.nodes.Gamma method), 96
                                                                                                           set_plotter() (bayespy.nodes.Gaussian method), 86
set_parameters() (bayespy.nodes.Gaussian method), 85
                                                                                                            set_plotter() (bayespy.nodes.GaussianARD method), 91
set_parameters() (bayespy.nodes.GaussianARD method),
                                                                                                           set_plotter()
                                                                                                                                              (bayespy.nodes.GaussianGammaARD
                                                                                                                              method), 116
set_parameters()
                                 (bayespy.nodes.GaussianGammaARD
                                                                                                                                                (bayespy.nodes.GaussianGammaISO
                                                                                                           set_plotter()
                 method), 116
                                                                                                                              method), 111
set_parameters()
                                    (bayespy.nodes.GaussianGammaISO
                                                                                                           set_plotter()
                                                                                                                                             (bayespy.nodes.GaussianMarkovChain
                  method), 111
                                                                                                                              method), 166
set_parameters() (bayespy.nodes.GaussianMarkovChain
                                                                                                           set_plotter() (bayespy.nodes.GaussianWishart method),
                                                                                                                              120
                 method), 166
set_parameters()
                                            (bayespy.nodes.GaussianWishart
                                                                                                           set_plotter() (bayespy.nodes.Mixture method), 183
                                                                                                            set_plotter() (bayespy.nodes.Multinomial method), 140
                  method), 120
                                                                                                            set_plotter() (bayespy.nodes.Poisson method), 145
set_parameters() (bayespy.nodes.Mixture method), 183
set_parameters() (bayespy.nodes.Multinomial method),
                                                                                                           set_plotter() (bayespy.nodes.SumMultiply method), 187
                  140
                                                                                                            set_plotter() (bayespy.nodes.SwitchingGaussianMarkovChain
set_parameters() (bayespy.nodes.Poisson method), 145
                                                                                                                              method), 172
set_parameters() (bayespy.nodes.SwitchingGaussianMarkov&taphotter() (bayespy.nodes.VaryingGaussianMarkovChain
                 method), 172
                                                                                                                              method), 178
set_parameters() (bayespy.nodes.VaryingGaussianMarkovChantplotter() (bayespy.nodes.Wishart method), 100
                 method), 178
                                                                                                           set_value() (bayespy.inference.vmp.nodes.constant.Constant
                                                                                                                              method), 225
set_parameters() (bayespy.nodes.Wishart method), 100
set_plotter() (bayespy.inference.vmp.nodes.constant.Constantetup() (bayespy.inference.vmp.transformations.RotateGaussian
                  method), 225
                                                                                                                              method), 200
set_plotter() (bayespy.inference.vmp.nodes.deterministic.Detertmin) (bayespy.inference.vmp.transformations.RotateGaussianARD
                 method), 222
                                                                                                                              method), 201
method), 220
                                                                                                                              method), 202
set_plotter() (bayespy.inference.vmp.nodes.gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gauss
                                                                                                                              method), 205
                 method), 231
set_plotter() (bayespy.inference.vmp.nodes.gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gaussian.Gauss
                  method), 229
                                                                                                                              method), 203
set_plotter() (bayespy.inference.vmp.nodes.gaussian.GaussianeTunGa(thaiyanGyuinfearEstQe.vmp.transformations.RotateVaryingMarkovChai
                 method), 227
                                                                                                                              method), 204
set_plotter() (bayespy.inference.vmp.nodes.gaussian.WrapTooEduips()attragrayatRDmisc.TestCase method), 325
                  method), 235
                                                                                                           setUpClass() (bayespy.utils.misc.TestCase method), 325
set_plotter() (bayespy.inference.vmp.nodes.gaussian.WrapToxGartDiaxoCiption().StDayespy.utils.misc.TestCase method),
                 method), 233
set_plotter() (bayespy.inference.vmp.nodes.gaussian.WrapToxGaux(s)iduxWispartnference.vmp.nodes.expfamily.ExponentialFamily
                  method), 237
                                                                                                                              method), 220
set_plotter()
                            (bayespy, inference, vmp.nodes, node, Node show() (bayespy, inference, vmp.nodes, stochastic, Stochastic
                                                                                                                              method), 215
                  method), 213
set_plotter() (bayespy.inference.vmp.nodes.stochastic.Stochasticw() (bayespy.nodes.Bernoulli method), 125
                 method), 215
                                                                                                            show() (bayespy.nodes.Beta method), 150
set_plotter() (bayespy.nodes.Add method), 191
                                                                                                           show() (bayespy.nodes.Binomial method), 130
set_plotter() (bayespy.nodes.Bernoulli method), 125
                                                                                                           show() (bayespy.nodes.Categorical method), 135
set_plotter() (bayespy.nodes.Beta method), 150
                                                                                                           show()
                                                                                                                                         (bayespy.nodes.CategoricalMarkovChain
```

method), 160	trace_solve_gradient() (bayespy.utils.misc.CholeskyDense		
show() (bayespy.nodes.Dirichlet method), 155	method), 314		
show() (bayespy.nodes.Exponential method), 105	trace_solve_gradient() (bayespy.utils.misc.CholeskySparse		
show() (bayespy.nodes.Gamma method), 96	method), 315		
show() (bayespy.nodes.Gaussian method), 86	tracedot() (in module bayespy.utils.linalg), 297		
show() (bayespy.nodes.GaussianARD method), 91	transpose() (in module bayespy.utils.linalg), 297		
show() (bayespy.nodes.GaussianGammaARD method),	trues() (in module bayespy.utils.misc), 313		
116			
show() (bayespy.nodes.GaussianGammaISO method),	U		
111	unique() (in module bayespy.utils.misc), 313		
show() (bayespy.nodes.GaussianMarkovChain method),	unobserve() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily		
166	method), 220		
show() (bayespy.nodes.GaussianWishart method), 120	unobserve() (bayespy.inference.vmp.nodes.stochastic.Stochastic		
show() (bayespy.nodes.Mixture method), 184	method), 216		
show() (bayespy.nodes.Multinomial method), 140	unobserve() (bayespy.nodes.Bernoulli method), 126		
show() (bayespy.nodes.Poisson method), 145	unobserve() (bayespy.nodes.Beta method), 150		
show() (bayespy.nodes.SwitchingGaussianMarkovChain	unobserve() (bayespy.nodes.Binomial method), 131		
method), 172	unobserve() (bayespy.nodes.Categorical method), 135		
show() (bayespy.nodes.VaryingGaussianMarkovChain	unobserve() (bayespy.nodes.CategoricalMarkovChain		
method), 178	method), 160		
show() (bayespy.nodes.Wishart method), 100	unobserve() (bayespy.nodes.Dirichlet method), 155		
skipTest() (bayespy.utils.misc.TestCase method), 326	unobserve() (bayespy.nodes.Exponential method), 105		
solve() (bayespy.utils.misc.CholeskyDense method), 314	unobserve() (bayespy.nodes.Gamma method), 96		
solve() (bayespy.utils.misc.CholeskySparse method), 315	unobserve() (bayespy.nodes.Gaussian method), 86		
solve_triangular() (in module bayespy.utils.linalg), 297	unobserve() (bayespy.nodes.GaussianARD method), 91		
sphere() (in module bayespy.utils.random), 301	unobserve() (bayespy.nodes.GaussianGammaARD		
squeeze() (in module bayespy.utils.misc), 312	method), 116		
squeeze_to_dim() (in module bayespy.utils.misc), 312	unobserve() (bayespy.nodes.GaussianGammaISO		
Stochastic (class in bayespy.inference.vmp.nodes.stochastic	c), method), 111		
213	unobserve() (bayespy.nodes.GaussianMarkovChain		
subTest() (bayespy.utils.misc.TestCase method), 326	method), 166		
sum_multiply() (in module bayespy.utils.misc), 312	unobserve() (bayespy.nodes.GaussianWishart method),		
<pre>sum_multiply_to_plates() (in module bayespy.utils.misc),</pre>	121		
312	unobserve() (bayespy.nodes.Mixture method), 184		
sum_product() (in module bayespy.utils.misc), 313	unobserve() (bayespy.nodes.Multinomial method), 140		
<pre>sum_to_dim() (in module bayespy.utils.misc), 313</pre>	unobserve() (bayespy.nodes.Poisson method), 145		
<pre>sum_to_shape() (in module bayespy.utils.misc), 313</pre>	unobserve() (bayespy.nodes.SwitchingGaussianMarkovChain		
SumMultiply (class in bayespy.nodes), 185	method), 172		
svd() (in module bayespy.utils.random), 301	unobserve() (bayespy.nodes.VaryingGaussianMarkovChain		
SwitchingGaussianMarkovChain (class in	method), 178		
bayespy.nodes), 167	unobserve() (bayespy.nodes.Wishart method), 101		
SwitchingGaussianMarkovChainDistribution (class in	update() (bayespy.inference.VB method), 197		
bayespy.inference.vmp.nodes.gaussian_markov_c	Chapitate() (bayespy.inference.vmp.nodes.expfamily.ExponentialFamily		
269	method), 220		
symm() (in module bayespy.utils.misc), 313	update() (bayespy.inference.vmp.nodes.stochastic.Stochastic		
<b>-</b>	method), 216		
Т	update() (bayespy.nodes.Bernoulli method), 126		
T() (in module bayespy.utils.misc), 304	update() (bayespy.nodes.Beta method), 150		
t_logpdf() (in module bayespy.utils.random), 301	update() (bayespy.nodes.Binomial method), 131		
tearDown() (bayespy.utils.misc.TestCase method), 326	update() (bayespy.nodes.Categorical method), 135		
tearDownClass() (bayespy.utils.misc.TestCase method),	update() (bayespy.nodes.CategoricalMarkovChain		
326	method), 161		
tempfile() (in module bayespy.utils.misc), 313			
tempine() (in module oujespj.umse); 313	update() (bayespy.nodes.Dirichlet method), 155		

```
update() (bayespy.nodes.Gamma method), 96
update() (bayespy.nodes.Gaussian method), 86
update() (bayespy.nodes.GaussianARD method), 91
update() (bayespy.nodes.GaussianGammaARD method),
update() (bayespy.nodes.GaussianGammaISO method),
update() (bayespy.nodes.GaussianMarkovChain method),
update() (bayespy.nodes.GaussianWishart method), 121
update() (bayespy.nodes.Mixture method), 184
update() (bayespy.nodes.Multinomial method), 140
update() (bayespy.nodes.Poisson method), 145
update() (bayespy.nodes.SwitchingGaussianMarkovChain
         method), 172
update() (bayespy.nodes.VaryingGaussianMarkovChain
         method), 178
update() (bayespy.nodes.Wishart method), 101
use_logging() (bayespy.inference.VB method), 197
VaryingGaussianMarkovChain (class in bayespy.nodes),
         173
VaryingGaussianMarkovChainDistribution
         bayespy.inference.vmp.nodes.gaussian_markov_chain),
         273
VB (class in bayespy.inference), 194
W
Wishart (class in bayespy.nodes), 97
wishart() (in module bayespy.utils.random), 301
wishart_rand() (in module bayespy.utils.random), 302
WishartDistribution
                                (class
                                                   in
         bayespy.inference.vmp.nodes.wishart), 278
WishartMoments
                               (class
                                                   in
         bayespy.inference.vmp.nodes.wishart), 244
WrapToGaussianGammaARD
                                     (class
                                                   in
         bayespy.inference.vmp.nodes.gaussian), 233
WrapToGaussianGammaISO
                                    (class
                                                   in
         bayespy.inference.vmp.nodes.gaussian), 231
WrapToGaussianWishart
                                  (class
                                                   in
         bayespy.inference.vmp.nodes.gaussian), 235
write_to_hdf5() (in module bayespy.utils.misc), 313
Ζ
zipper_merge() (in module bayespy.utils.misc), 313
```