

Many seq  $(0, 0, 0, 1, 3, 7, 15, 31, \dots)$

$$\sum_{i=0}^{\infty} a_i x^i = 0 + 0x + 0x^2 + 1x^3 + 3x^4 + 7x^5 + 15x^6 + 31x^7 + \dots$$

~~At~~

$$A(x) = x^2 (0 + 1x + 3x^2 + 7x^3 + 15x^4 + 31x^5 + \dots)$$

$$A(x) = x^2 ((2^0 - 1)x^0 + (2^1 - 1)x^1 + (2^2 - 1)x^2 + (2^3 - 1)x^3 + (2^4 - 1)x^4 + (2^5 - 1)x^5 + \dots)$$

$$A(x) = x^2 \sum_{i=0}^{\infty} (2^i - 1)x^i$$

$$A(x) = x^2 \left( \sum_{i=0}^{\infty} (2x)^i - \sum_{i=0}^{\infty} x^i \right)$$

$$A(x) = x^2 \left( \frac{1}{1-2x} - \frac{1}{1-x} \right)$$