

2.2

$$a) \begin{cases} a_0 = a_1 = 1 \\ a_{n+1} = \sqrt{a_n^2 + a_{n-1}^2} \end{cases}$$

$$a_{n+1}^2 = a_n^2 + a_{n-1}^2$$

$$b_n = a_n^2$$

$$b_{n+1} = b_n + b_{n-1}$$

Anihilator dla takiego ciągu jest znany i wynosi $E^2 - E - 1$
tak więc

$$b_n = \alpha \left(\frac{1-\sqrt{5}}{2} \right)^n + \beta \left(\frac{1+\sqrt{5}}{2} \right)^n$$

$$b_1 = 1^2 \quad b_0 = 1^2$$

$$1 = \alpha + \beta$$

$$\beta = 1 - \alpha$$

$$1 = \alpha \frac{1-\sqrt{5}}{2} + \beta \frac{1+\sqrt{5}}{2}$$

$$1 = \alpha \frac{1-\sqrt{5}}{2} + \frac{1+\sqrt{5}}{2} - \alpha \frac{1+\sqrt{5}}{2} \quad / \cdot 2$$

$$2 = \cancel{\alpha} - \alpha\sqrt{5} + 1 + \sqrt{5} - \cancel{\alpha} - \sqrt{5}\alpha$$

$$1 - \sqrt{5} = -2\sqrt{5}\alpha$$

$$\frac{\sqrt{5}-1}{2\sqrt{5}} = \alpha$$

$$\frac{5-\sqrt{5}}{10} = \alpha$$

$$a_n = \sqrt{\frac{5-\sqrt{5}}{10} \left(\frac{1-\sqrt{5}}{2} \right)^n + \frac{5+\sqrt{5}}{10} \left(\frac{1+\sqrt{5}}{2} \right)^n}$$

$$\beta = 1 - \frac{5-\sqrt{5}}{10} = \frac{5+\sqrt{5}}{10}$$