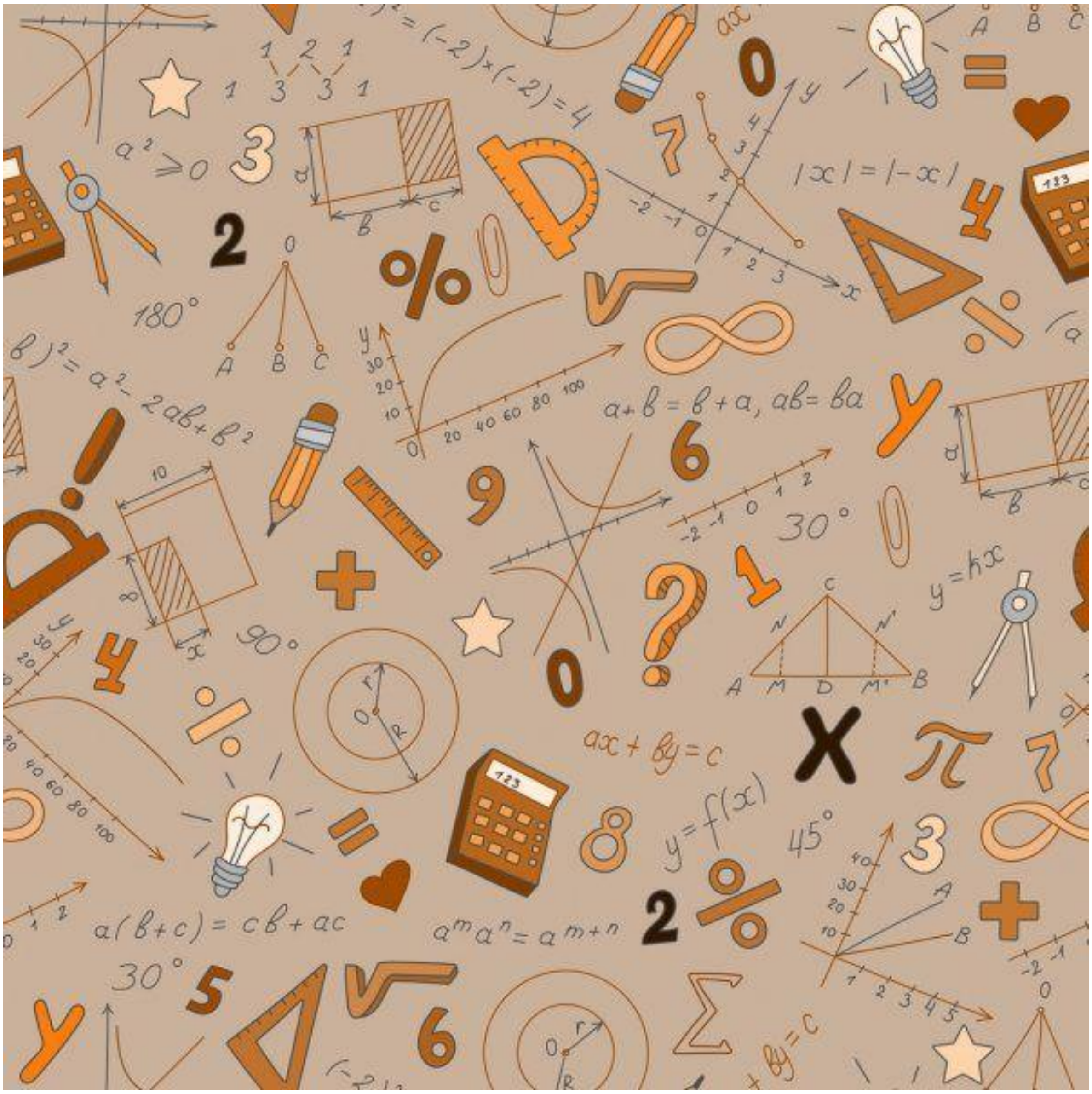
21st July 2022

UNIVERSITY OF MAURITIUS

DISCRETE STRUCTURES – SIS 1042Y

ASSIGNMENT 1



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Question 1

Problem description

Since my childhood I was always fascinated and fond of having many different footwear. As a result, I have grown up with a large collection of footwear in my closet and this makes my day to day decisions of what to wear a real hassle.

I am often indecisive to choose a footwear, be it to go to the University or for any other occasions. As a consequence, this delays my daily routine.

So I decided to classify all of my footwear as follows:

U= {Garment}

F= {Footwear}

T= {Sandals, Sneakers, Heels}

C= {Light colour footwear}, C'= {Dark colour footwear}

O= {Casual footwear}, O'= {Classic footwear}

- U is the universal set of all garments that are in my closet.
- F is the set containing all my footwear.
  $F \subset U$, F has some elements of U
- T is the set of the types of footwear I have; T* is sandal, T*** is sneakers and T*** is heels
- C is the set of the colour of my light coloured footwear and C* is the set of the colour of my dark coloured footwear.
- O is the set containing casual footwear and O* is the set containing classic

Relation between sets

$C \cap O$: The footwear is light coloured and casual.

$C \cap O^*$: The footwear is light coloured and classic.

$C^* \cap O$: The footwear is dark coloured and casual.

$C^* \cap O^*$: The footwear is dark coloured and classic.

$T^* \cap C \cap O$: The footwear is a sandal, light coloured and casual.

$T^* \cap C \cap O'$: The footwear is a sandal, light coloured and classic.

$T^* \cap C' \cap O$: The footwear is a sandal, dark coloured and casual.

$T^* \cap C' \cap O'$: The footwear is a sandal, dark coloured and classic.

$T^{**} \cap C \cap O$: The footwear is a sneakers, light coloured and casual.

$T^{**} \cap C \cap O'$: The footwear is a sneakers, light coloured and classic.

$T^{**} \cap C' \cap O$: The footwear is a sneakers, dark coloured and casual.

$T^{**} \cap C' \cap O'$: The footwear is a sneakers, dark coloured and classic.

$T^{***} \cap C \cap O$: The footwear is a heel, light coloured and casual.

$T^{***} \cap C \cap O'$: The footwear is a heel, light coloured and classic.

$T^{***} \cap C' \cap O$: The footwear is a heel, dark coloured and casual.

$T^{***} \cap C' \cap O'$: The footwear is a heel, dark coloured and classic.

SET PROPERTIES:

De Morgan's law:

1. $(C \cup O)^c = C^c \cap O^c$
2. $(C \cap O)^c = C^c \cup O^c$

Let $P = (C \cup O)^c$ and $Q = C^c \cap O^c$

Let x be an arbitrary element of P

$x \in P \Rightarrow x \in (C \cup O)^c$

$\Rightarrow x \in (C \cup O)^c$

$\Rightarrow x \notin (C \cup O)$

$\Rightarrow x \notin C$ and $x \notin O$

$\Rightarrow x \in C^c$ and $x \in O^c$

$\Rightarrow x \in C^c \cap O^c$

$\Rightarrow x \in Q$

$\therefore P \subset Q$ ---- (1)

Let y be an arbitrary element of Q

$y \in Q \Rightarrow y \in Cc \cap O^c$

$\Rightarrow y \in C^c \cap O^c$

$\Rightarrow y \in C^c$ and $y \in O^c$

$\Rightarrow y \notin C$ and $y \notin O$

$\Rightarrow y \notin (C \cup O)$

$\Rightarrow y \in (C \cup O)^c$

$\Rightarrow y \in P$

$\therefore Q \subset P$ ---- (2)

Combining (1) & (2), we get

P=Q, that is $(C \cup O)^c = C^c \cap O^c$

Commutative Law:

1. $C \cup O = O \cup C$
2. $C \cap O = O \cap C$

Let $x \in C \cup O$

If $x \in C \cup O$, then $x \in C$ or $x \in O$

($x \in C$ or $x \in O$ also means $x \in O$ or $x \in C$)

$x \in O$ or $x \in C$

$x \in O \cup C$

$\therefore x \in C \cup O \Leftrightarrow x \in O \cup C$ ---- (1)

Taking the first law in reverse, $O \cup C = C \cup O$

Let $x \in O \cup C$

If $x \in O \cup C$, then $x \in O$ or $x \in C$

($x \in O$ or $x \in C$ also means $x \in C$ or $x \in O$)

$x \in C$ or $x \in O$

$x \in C \cup O$

$\therefore x \in O \cup C \Leftrightarrow x \in C \cup O$ ---- (2)

Combining (1) & (2), we get

$C \cup O = O \cup C$

Associative Law:

1. $(T^* \cap C) \cap O = T^* \cap (C \cap O)$
2. $(T^* \cup C) \cup O = T^* \cup (C \cup O)$

Taking LHS of the first law, $(T^* \cap C) \cap O$

Let $x \in (T^* \cap C) \cap O$

If $x \in (T^* \cap C) \cap O$, then $x \in (T^* \cap C)$ and $x \in O$

$x \in (T^* \cap C)$ and $x \in O$

$x \in (T^* \cap C)$ further means that $x \in T^*$ and $x \in C$

Now having,

$x \in T^*$, $x \in C$ and $x \in O$

$x \in T^*$ and $x \in (C \cap O)$

$x \in T^* \cap (C \cap O)$

$(T^* \cap C) \cap O \Leftrightarrow T^* \cap (C \cap O)$

$\therefore (T^* \cap C) \cap O \subset T^* \cap (C \cap O)$ ---- (1)

Taking the first law in reverse (RHS), $T^* \cap (C \cap O) = (T^* \cap C) \cap O$

Let $x \in T^* \cap (C \cap O)$

If $x \in T^* (C \cap O)$, then $x \in T^*$ and $x \in (C \cap O)$

$x \in T^*$ and $x \in (C \cap O)$

$x \in (C \cap O)$ further means that $x \in C$ and $x \in O$

Now having,

$x \in T^*$, $x \in C$ and $x \in O$

$x \in (T^* \cap C)$ and $x \in O$

$x \in (T^* \cap C) \cap\ x \in O$

$T^* \cap (C \cap O) \Leftrightarrow (T^* \cap C) \cap O$

$\therefore T^* \cap (C \cap O) \subset (T^* \cap C) \cap O$ ---- (2)

Combining (1) & (2), we get $(T^* \cap C) \cap O = T^* \cap (C \cap O)$

Double Complement Law:

$(C^c)^c = \{x \mid x \notin C^c\}$

$(C^c)^c = \{x \mid \neg (x \in C^c)\}$

$(C^c)^c = \{x \mid \neg (x \notin C)\}$

$(C^c)^c = \{x \mid \neg (\neg(x \in C))\}$

$(C^c)^c = \{x \mid x \in C\}$

$\therefore (C^c)^c = C$

Distributive law:

1. $T^* \cup (C \cap O) = (T^* \cup C) \cap (T^* \cup O)$
2. $T^* \cap (C \cup O) = (T^* \cap C) \cup (T^* \cap O)$

Taking LHS of first law, $T^* \cup (C \cap O)$

Let $x \in T^* \cup (C \cap O)$

$x \in T^*$ or $x \in (C \cap O)$

$x \in T^*$ or $\{x \in C$ and $x \in O\}$

$\{x \in T^*$ or $x \in C\}$ and $\{x \in T^*$ or $x \in O\}$

$x \in (T^* \cup C)$ and $x \in (T^* \cup O)$

$x \in (T^* \cup C) \cap x \in (T^* \cup O)$

$x \in (T^* \cup C) \cap (T^* \cup O)$

$x \in T^* \cup (C \cap O) \Rightarrow x \in (T^* \cup C) \cap (T^* \cup O)$

$\therefore T^* \cup (C \cap O) \subset (T^* \cup C) \cap (T^* \cup O)$ ---- (1)

Taking RHS of first law, $(T^* \cup C) \cap (T^* \cup O)$

$x \in (T^* \cup C) \cap (T^* \cup O)$

$x \in (T^* \cup C)$ and $x \in (T^* \cup O)$

$x \in T^*$ or $x \in C$ and $x \in T^*$ or $x \in O$

$x \in T^*$ or ($x \in C$ and $x \in O$)

$x \in T^*$ or ($x \in C \cap O$)

$x \in T^* \cup (x \in C \cap O)$

$x \in T^* \cup (C \cap O)$

$x \in (T^* \cup C) \cap (T^* \cup O) \Rightarrow x \in T^* \cup (C \cap O)$

$\therefore (T^* \cup C) \cap (T^* \cup O) \subset T^* \cup (C \cap O)$ ---- (2)

Combining (1) & (2), we get $T^* \cup (C \cap O) = (T^* \cup C) \cap (T^* \cup O)$

Absorption law:

1. $C \cup (C \cap O) = C$
2. $C \cap (C \cup O) = C$

Let $x \in C \cup (C \cap O)$

$\Rightarrow x \in C$ or $x \in (C \cap O)$

$\Rightarrow x \in C$ or ($x \in C$ and $x \in O$)

Specifically, $x \in C$

Thus, $C \cup (C \cap O) \subset C$

Showing that $C \subset C \cup (C \cap O)$

Using the definition of union, $x \in C \cup (C \cap O)$ since $x \in C$

Thus, $C \subset C \cup (C \cap O)$

Therefore, by the above results,

$C \cup (C \cap O) = C$

Truth table

Let P(x) be the statement “x is a sandal”

Let Q(x) be the statement “x is a light colored sandal”

Let R(x) be the statement “x is a casual sandal”

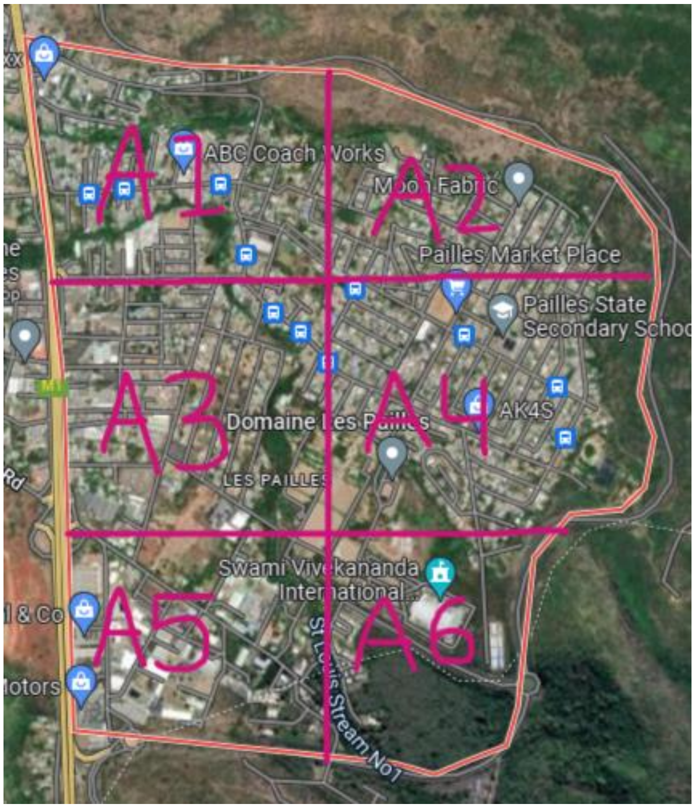
Statement	**Proposition**
I wear a sandal.	$\exists x(P(x))$
I wear a light colored sandal.	$\exists x(P(x) \wedge Q(x))$
I wear a light colored sandal or a casual sandal.	$\exists x(Q(x) \vee R(x))$
I do not wear a light colored sandal or a casual sandal.	$\forall x \neg(P(x) \vee Q(x) \vee R(x))$
I wear a sandal which is a light colored and casual.	$\forall x (P(x) \wedge Q(x) \wedge R(x))$

Proposition	**True when**	**False when**
$\exists x(P(x))$	There is an x for which P(x) is true.	P(x) is false for every x.
$\exists x(P(x) \wedge Q(x))$	There is an x for which P(x) and Q(x) is true.	P(x) and Q(x) is false for every x.
$\exists x(Q(x) \vee R(x))$	Q(x) or R(x) is true for every x.	There is an x for which Q(x) or R(x) is false.
$\forall x \neg(P(x) \wedge Q(x) \wedge R(x))$	P(x) or Q(x) or R(X) is true for every x.	There is an x for which P(x) or Q(x) or R(x) is false.
$\forall x (P(x) \wedge Q(x) \wedge R(x))$	P(x), Q(x) and R(x) are true for every x.	There is an x for which P(x), Q(x) and R(x) is false.

Question 2

Village: Pailles

Total number of residents: 10,622



(Source of information: https://worldpopulationreview.com/countries/cities/mauritius)

Number of residents per area:

Area	Number of residents
A1	1800
A2	1800
A3	1800
A4	1800
A5	1800
A6	1622

6 areas = 1 hour each slot

30 seconds = 1 test

1 second = 1/30

1 hour (3600 seconds) = 1/30 * 3600

=120 tests

1 officer = 120 tests

15 officers = 120*5

=1800 tests

1800 people are to be tested in each area.

Let break = B

Let working slot = S

Case 1: Assume that only one break can be taken between 2 working time slots, no break at the start and at the end

S B S B S B S B S B S

Therefore,

Number of ways to arrange the working slots= 6!

=720

Number of ways to choose the breaks=5C2

=10

Total number of ways = 720*10

=7200

Case 2: 2 breaks can be taken one after the other between 2 working time slots

↑↑ ↑↑ ↑↑ ↑↑ ↑↑

S BB S BB S BB S BB S BB S

Therefore,

Number of ways to arrange the working slots= 6!

=720

Number of ways to choose the breaks=5

Total number of ways = 720*5

=3600

Total number of unique schedules = 7200+3600

= 10800

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