

Automating Abstract Interpretation of Abstract Machines

Thesis Defense of
J. Ian Johnson

2015, March 30



People make their own languages.

People make their own languages.
They write interpreters.

People make their own languages.

They write interpreters.

Higher-order. Dynamically typed.

People make their own languages.

They write interpreters.

Higher-order. Dynamically typed.

Some languages become popular.

People make their own languages.

They write interpreters.

Higher-order. Dynamically typed.

Some languages become popular.

Lots of code that *does stuff*.

People make their own languages.

They So many new languages!
High So many new interpreters!

~ Hal Abelson, foreword to EOPLv3

ped.

Some languages become popular.

Lots of code that *does stuff*.

People make their own languages.

They So many ways to go wrong!
High ~ me, right now ped.

Some languages become popular.

Lots of code that *does stuff*.

Two questions a dev asks:

Two questions a dev asks:

Does it do what it's supposed to do?

Two questions a dev asks:

Does it do what it's supposed to do?

Does it *not* do what it's *not* supposed to do?

Two questions a dev asks:

Does it do what it's supposed to do?

Test. Test. Test.

Does it *not* do what it's *not* supposed to do?

Two questions a dev asks:

Does it do what it's supposed to do?

Test. Test. Test.

Does it *not* do what it's *not* supposed to do?

Test?

Two questions a dev asks:

Does it do what it's supposed to do?

Test. Test. Test.

Does it *not* do what it's *not* supposed to do?

Test. But what else?

Two questions a dev asks:

Does it do what it's supposed to do?

Test. Test. Test.

Does it *not* do what it's *not* supposed to do?

Test. But what else?

Static types

Two questions a dev asks:

Does it do what it's supposed to do?

Test. Test. Test.

Does it *not* do what it's *not* supposed to do?

Test. But what else?

Static ~~types~~

Two questions a dev asks:

Does it do what it's supposed to do?

Test. Test. Test.

Does it *not* do what it's *not* supposed to do?

Test. But what else?

Static analysis

Static analysis is wonderful!

Understand code

Static analysis is wonderful!

Understand code

Value flow

Control flow

Type inference

Static analysis is wonderful!

Understand code

Harden code

Value flow

Control flow

Type inference

Static analysis is wonderful!

Understand code

Value flow

Control flow

Type inference

Harden code

Crash-freedom

Contract verification

Information flow

Data race detection

Static analysis is wonderful!

Understand code

- Value flow
- Control flow
- Type inference

Harden code

- Crash-freedom
- Contract verification
- Information flow
- Data race detection

Improve code

Static analysis is wonderful!

Understand code

- Value flow
- Control flow
- Type inference

Harden code

- Crash-freedom
- Contract verification
- Information flow
- Data race detection

Improve code

- no runtime enforcement
- optimization

Static analysis is terrible!

It's too hard to make right

Static analysis is terrible!

It's too hard to make right

It's too hard to make fast

Static analysis is terrible!

It's too hard to make right

It's too hard to make fast

It's too hard to make precise

Static analysis is terrible!

It's too hard to make right

It's too hard to make fast

It's too hard to make precise

It's too hard to understand

Static analysis is terrible!

It's too hard to make right

It's too hard to make fast

It's too hard to make precise

It's too hard to understand

It's too easy to waste time

Static analysis is terrible!

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too hard to make right

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too easy to waste time

Static analysis is terrible!

- › hard to make right
- › hard to make fast
- › hard to make precise
- › hard to understand
- › easy to waste time

Static analysis is terrible!

hard to make right

hard to make fast

hard to make precise

hard to understand

easy to waste time

Static analysis is terrible!

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Static analysis is doable!

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Static analysis is **doable!**

make right: systematic/automatic

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Static analysis is **doable!**

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make fast: systematize folklore

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Static analysis is **doable!**

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make fast: systematize folklore

make precise: memoize = pushdown

understand:

waste time:

Static analysis is **doable!**

make right: systematic/automatic

make fast: systematize folklore

make precise: memoize = pushdown

understand: interpreters

waste time:

Static analysis is **doable**!

make right:systematic/automatic

make fast:systematize folklore

make precise:memoize = pushdown

understand:interpreters

waste time:already wasted

Static analysis is doable!

make right: systematic/automatic

make fast: systematize folklore

make pi I made a languageishdown

understand: interpreters

waste time: already wasted

Thesis:

Precise and performant analyses for higher-order languages
can be systematically and algorithmically constructed
from their semantics.

Thesis:

Precise and performant analyses for higher-order languages

can be systematically and algorithmically constructed
from their semantics.

I built and proved

I evaluated

Thesis:

Precise and performant analyses for higher-order languages

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from their semantics.

I built and proved

Thesis:

Precise and performant analyses for higher-order languages
can be systematically and algorithmically constructed
from their semantics.

What is semantics?

Meaning of programs

What *is* semantics?

Intensional Meaning of programs

What *is* semantics?

Intensional Meaning of programs

What *is* semantics?

Intensional Meaning of programs

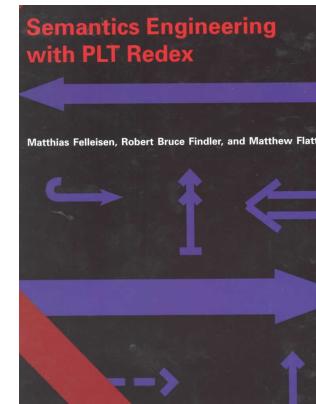
What *is* semantics?

Intensional Meaning of programs

What *is* semantics?

Intensional Meaning of programs

Abstract machines

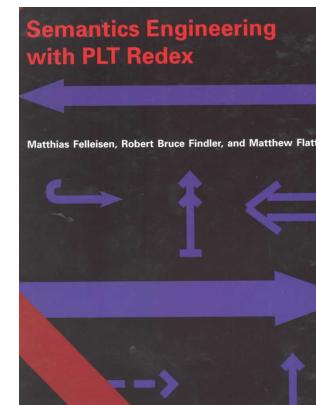


What *is* semantics?

Intensional meaning of programs

Abstract machines

Reasonably efficient



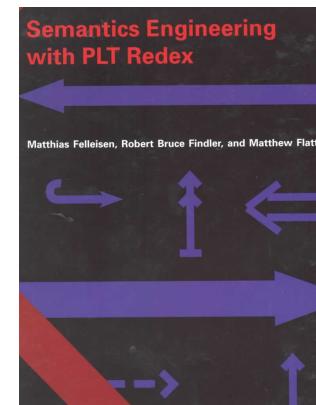
What *is* semantics?

Intensional meaning of programs

Abstract machines

Reasonably efficient

Approachable



What *is* semantics?

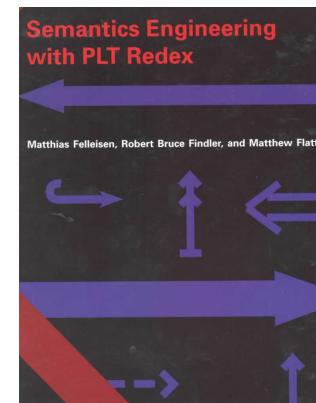
Intensional meaning of programs

Abstract machines

Reasonably efficient

Approachable

Easily abstracted



Abstract
abstract machines?

a

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Abstracting Abstract Machines

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Abstract

We describe a derivational approach to abstract interpretation that yields novel and transparently sound static analyses when applied to well-established abstract machines. To demonstrate the technique and support our claim, we transform the CEK machine of Felleisen and Friedman, a lazy variant of Krivine's machine, and the stack-inspecting CM machine of Clements and Felleisen into abstract interpretations of themselves. The resulting analyses bound temporal ordering of program events; predict return-flow and stack-inspection behavior; and approximate the flow and evaluation of by-need parameters. For all of these machines, we find that a series of well-known concrete machine refactorings, plus a technique we call store-allocated continuations, leads to machines that abstract into static analyses simply by bounding their stores. We demonstrate that the techniques scale up uniformly to allow static analysis of realistic language features, including tail calls, conditionals, side effects, exceptions, first-class continuations, and even garbage collection.

Categories and Subject Descriptors F.3.2 [Logics and Meanings of Programs]: Semantics of Programming Languages—Program analysis, Operational semantics; F.4.1 [Mathematical Logic and Formal Languages]: Mathematical Logic—Lambda calculus and related systems

General Terms Languages, Theory

Keywords abstract machines, abstract interpretation

1. Introduction

Abstract machines such as the CEK machine and Krivine's machine are first-order state transition systems that represent the core of a real language implementation. Store-based programming analysis, on the other hand, is concerned with safety and maintaining intensional properties of such a machine as it runs a program. It seems natural then to want to systematically derive analyses from machines to approximate the core of realistic run-time systems.

Our goal is to develop a technique that enables direct abstract interpretations of abstract machines by methods for transforming a given machine description into another that computes its finite approximation.

* Supported by the National Science Foundation under grant 0937060 to the Computing Research Association for the CIFellow Project.

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ICFP '10, September 27–29, 2010, Baltimore, Maryland, USA.
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We demonstrate that the technique of refactoring a machine with **store-allocated continuations** allows a direct structural abstraction¹ by bounding the machine's store. Thus, we are able to convert semantic techniques used to model language features into static analysis techniques for reasoning about the behavior of those very same features. By abstracting well-known machines, our technique delivers static analyzers that can reason about by-need evaluation, higher-order functions, tail calls, side effects, stack structure, exceptions and first-class continuations.

The basic idea behind store-allocated continuations is not new. SML/NJ has allocated continuations in the heap for well over a decade [28]. At first glance, modeling the program stack in an abstract machine with store-allocated continuations would not seem to provide any real benefit. Indeed, for the purpose of defining the meaning of a program, there is no benefit, because the meaning of the program does not depend on the stack-implementation strategy. Yet, a closer inspection finds that store-allocating continuations eliminate recursion from the definition of the state-space of the machine. With no recursive structure in the state-space, an abstract machine becomes eligible for conversion into an abstract interpreter through a simple structural abstraction.

To demonstrate the applicability of the approach, we derive abstract interpreters of:

- a call-by-value λ -calculus with state and control based on the CESK machine of Felleisen and Friedman [13],
- a call-by-need λ -calculus based on a tail-recursive, lazy variant of Krivine's machine derived by Ager, Danvy and Midgaard [1], and
- a call-by-value λ -calculus with stack inspection based on the CM machine of Clements and Felleisen [3];

and use abstract garbage collection to improve precision [25].

Overview

In Section 2, we begin with the CEK machine and attempt a structural abstract interpretation, but find ourselves blocked by two recursive structures in the machine: environments and continuations. We make three refactorings to:

1. store-allocate bindings,
2. store-allocate continuations, and
3. time-stamp machine states;

resulting in the CESK, CESK*, and time-stamped CESK* machines, respectively. The time-stamps encode the history (context) of the machine's execution and facilitate context-sensitive abstractions. We then demonstrate that the time-stamped machine abstracts directly into a parameterized, sound and computable static analysis.

¹ A structural abstraction distributes component-, point-, and member-wise.

a

Abstracting Abstract Machines

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Matthew Might
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Abstract

We describe a derivational approach to abstract interpretation that yields novel and transparently sound static analyses when applied to well-established abstract machines. To demonstrate the technique and support our claim, we transform the Cek machine of Felleisen and Friedman, a lazy variant of Krivine's machine, and the stack-inspecting CM machine of Clements and Felleisen into abstract interpretations of themselves. The resulting analyses bound temporal ordering of program events; predict return-flow and stack-inspection behavior; and approximate the flow and evaluation of by-need parameters. For all of these machines, we find that a series of well-known techniques we call static abstraction that abstract into static analysis of real conditional, side effects, even garbage collection.

Categories and Subject Terms: Sema analysis, Operational Formal Languages]; related systems

General Terms: Languages

Keywords: abstract

1. Introduction

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Our goal is to develop a technique that enables direct abstract interpretations of abstract machines by methods for transforming a given machine description into another that computes its finite approximation.

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Overview

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¹ A structural abstraction distributes component-, point-, and member-wise.

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SEMANTICS



→ ANALYSIS

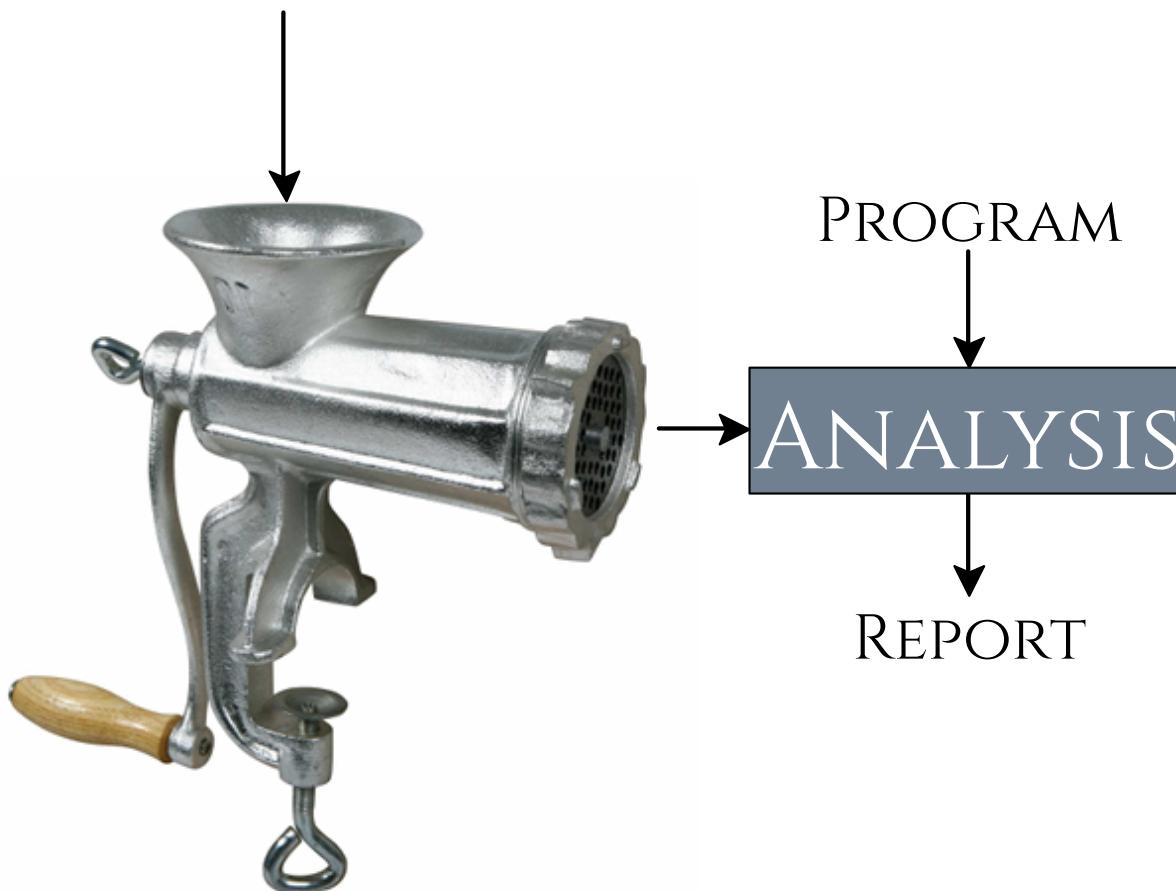
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→ ANALYSIS

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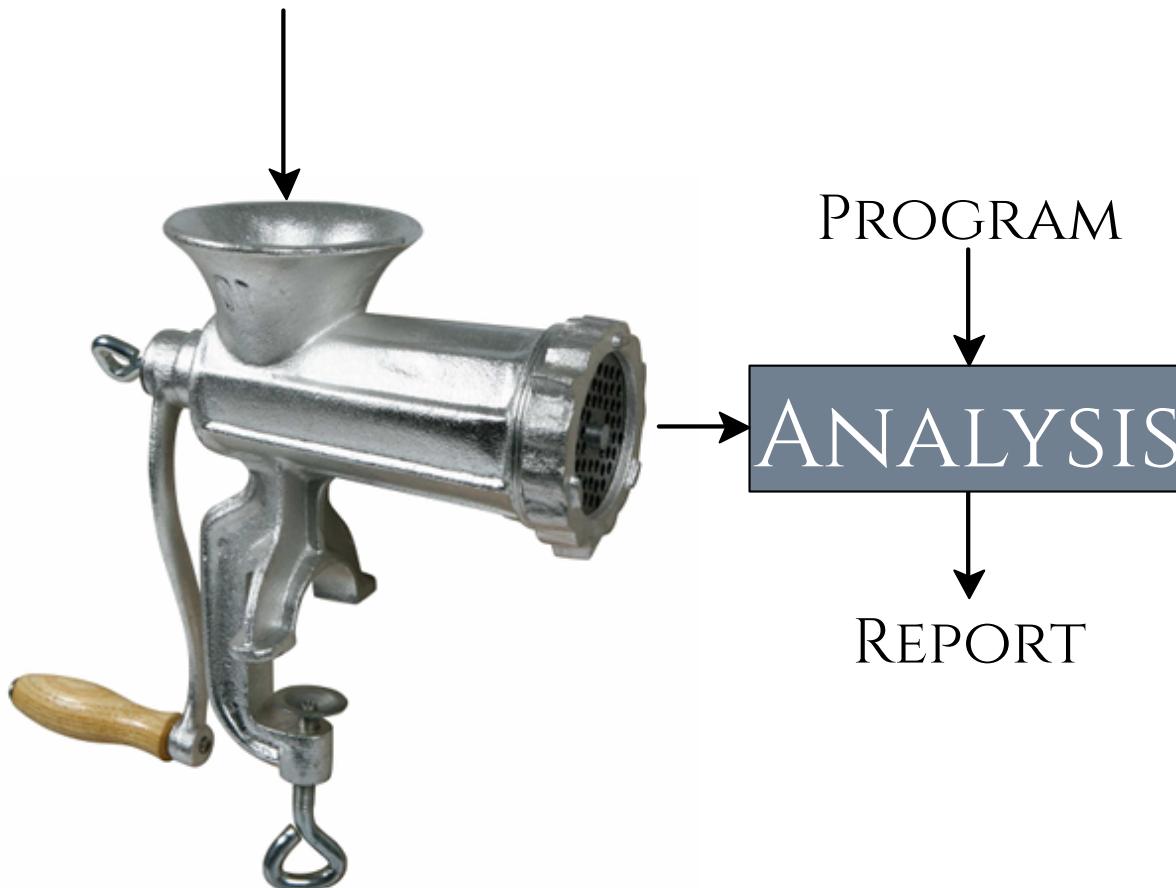


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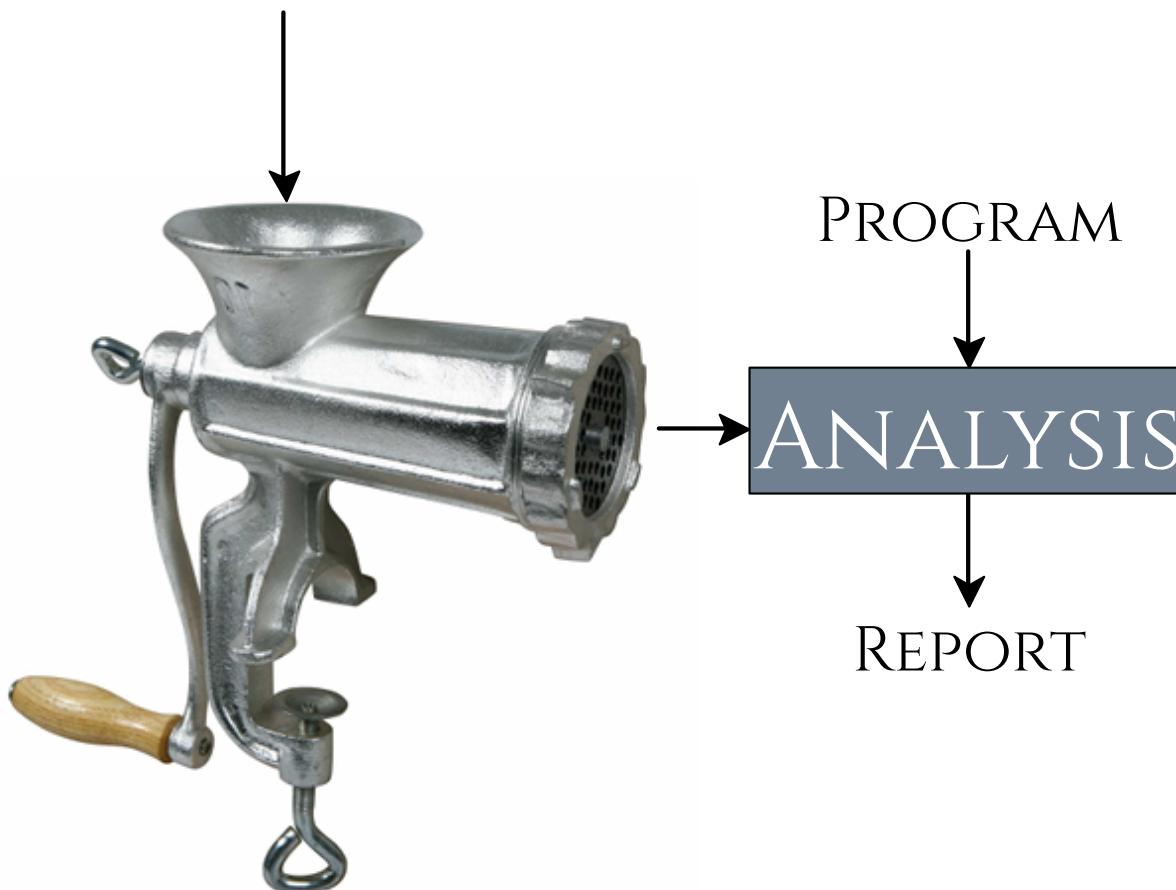


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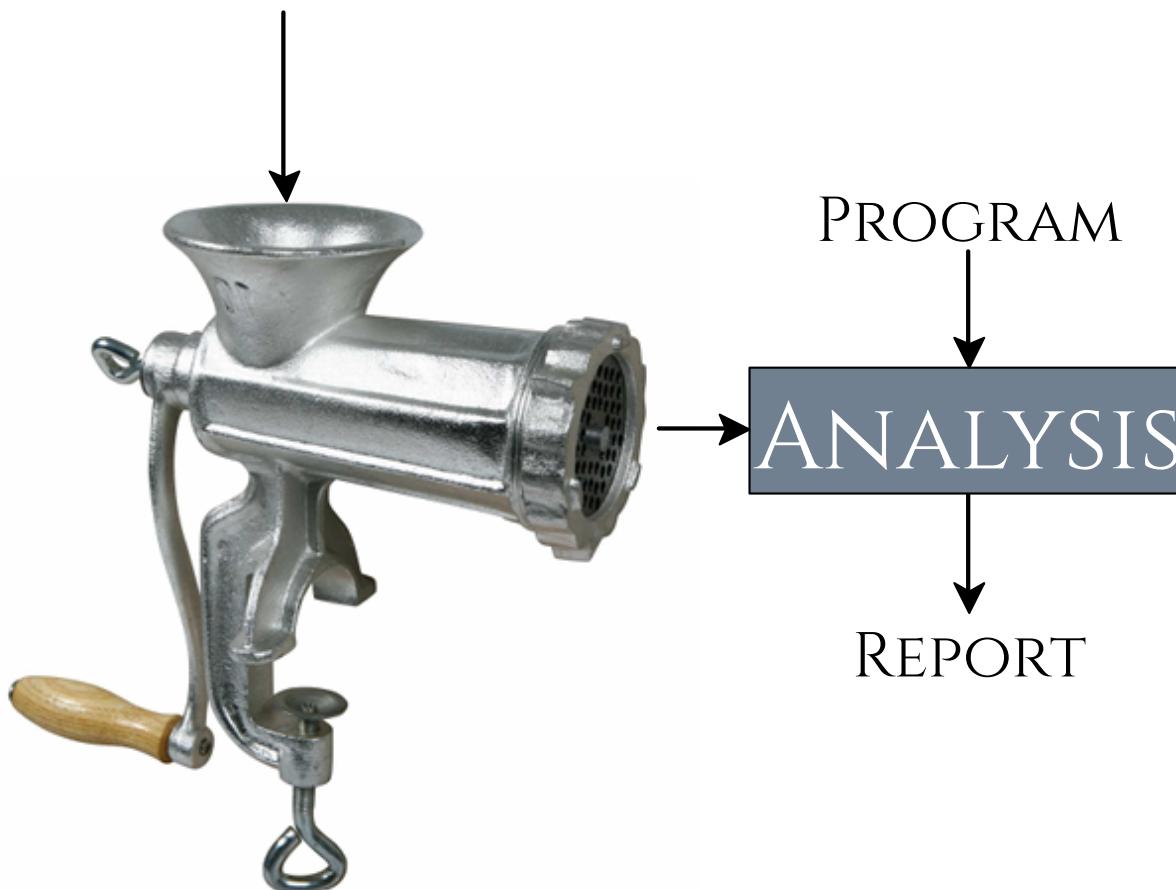


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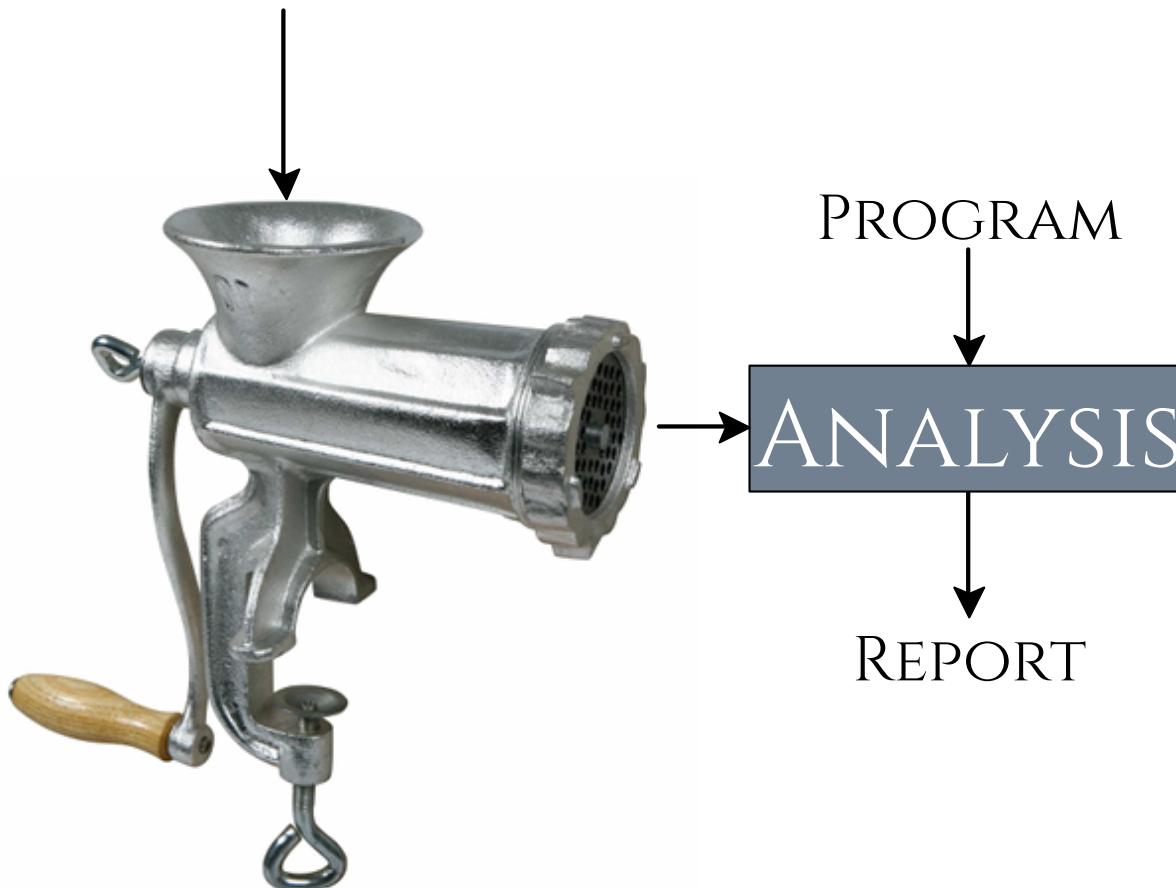


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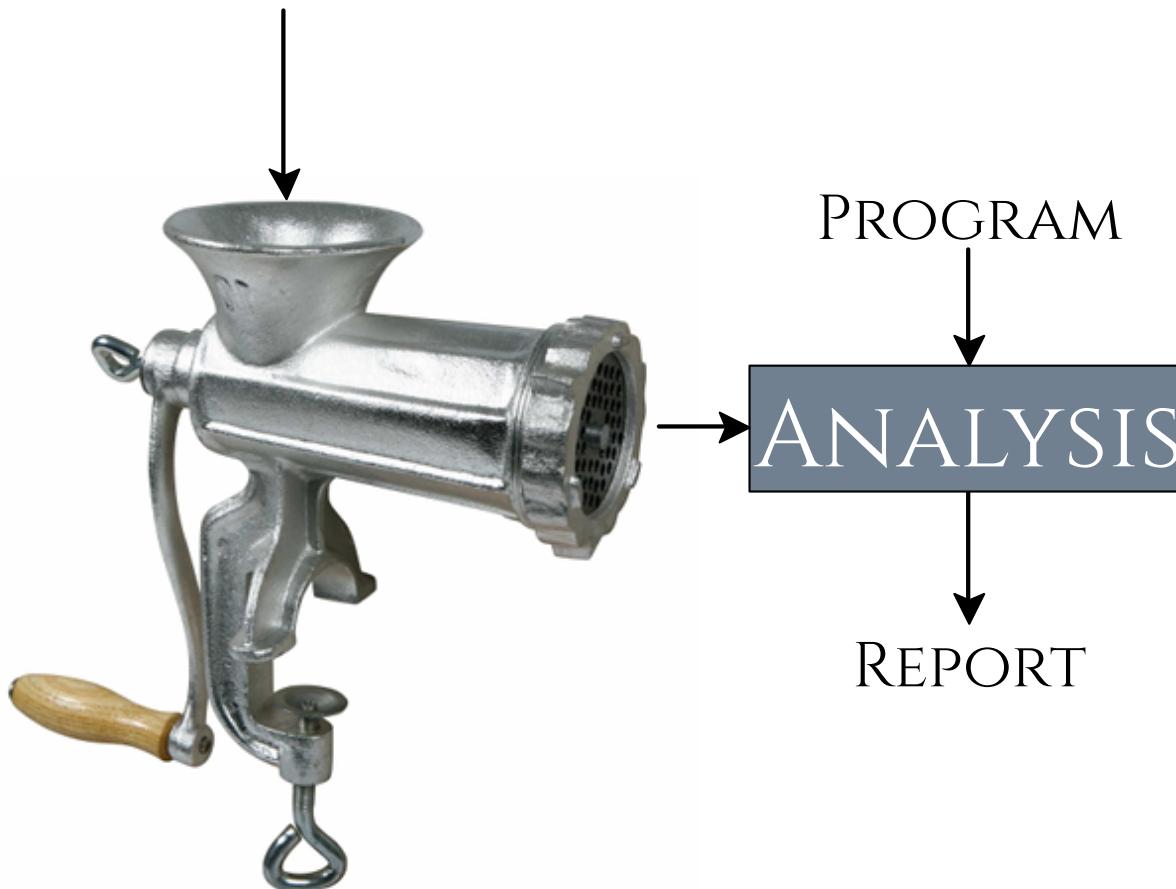


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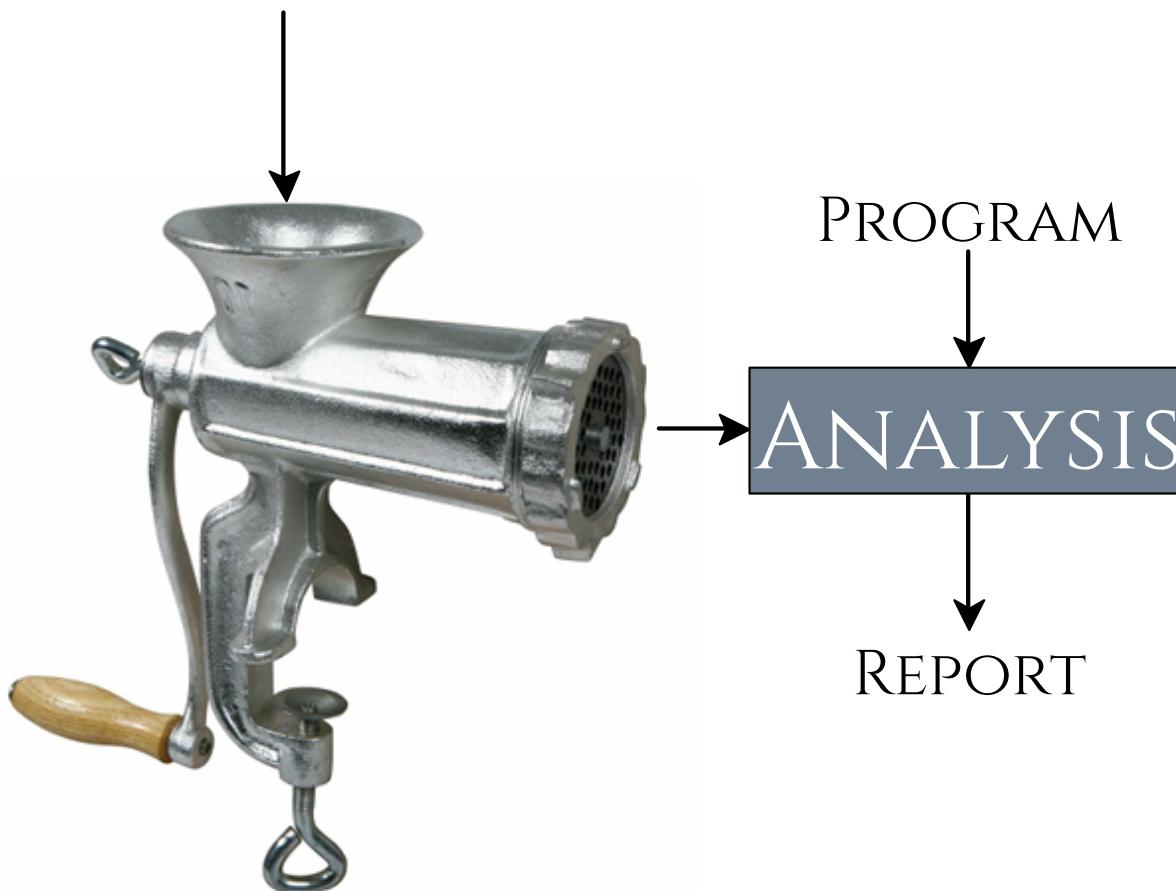


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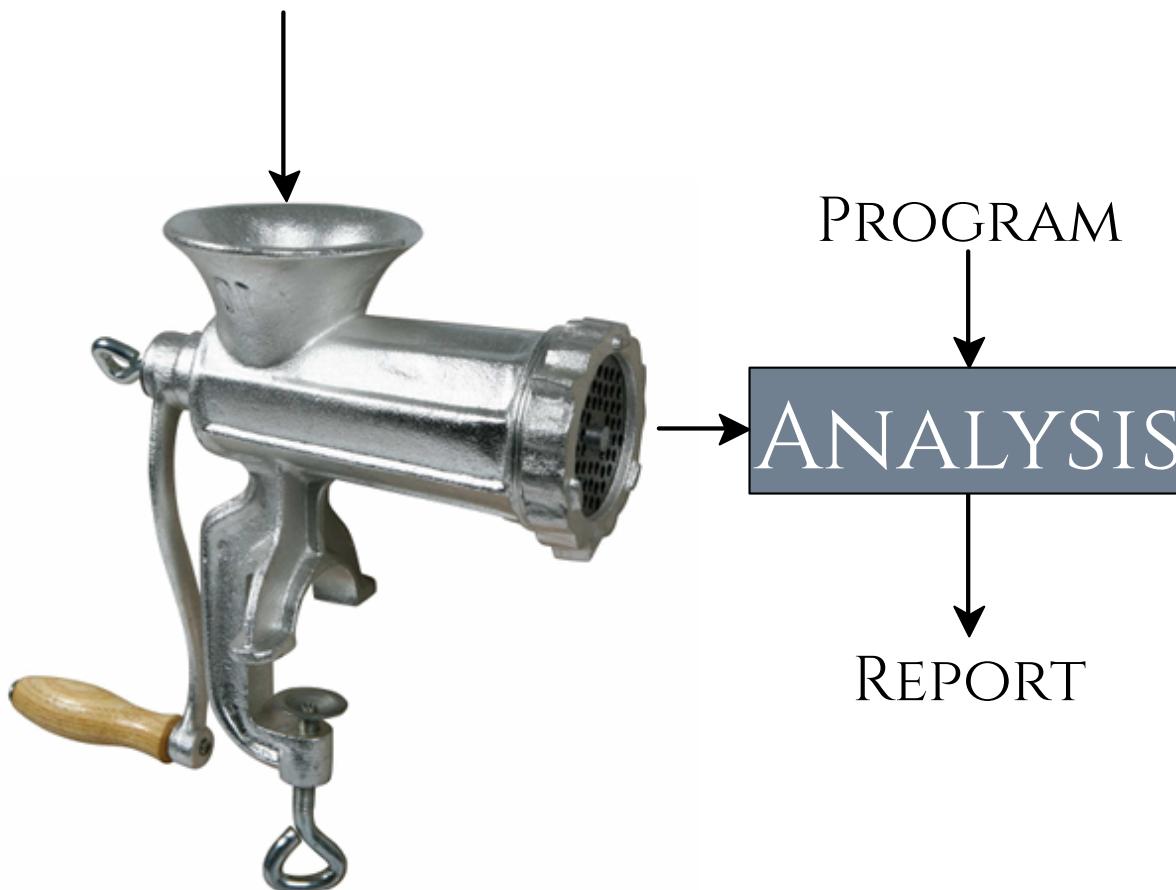
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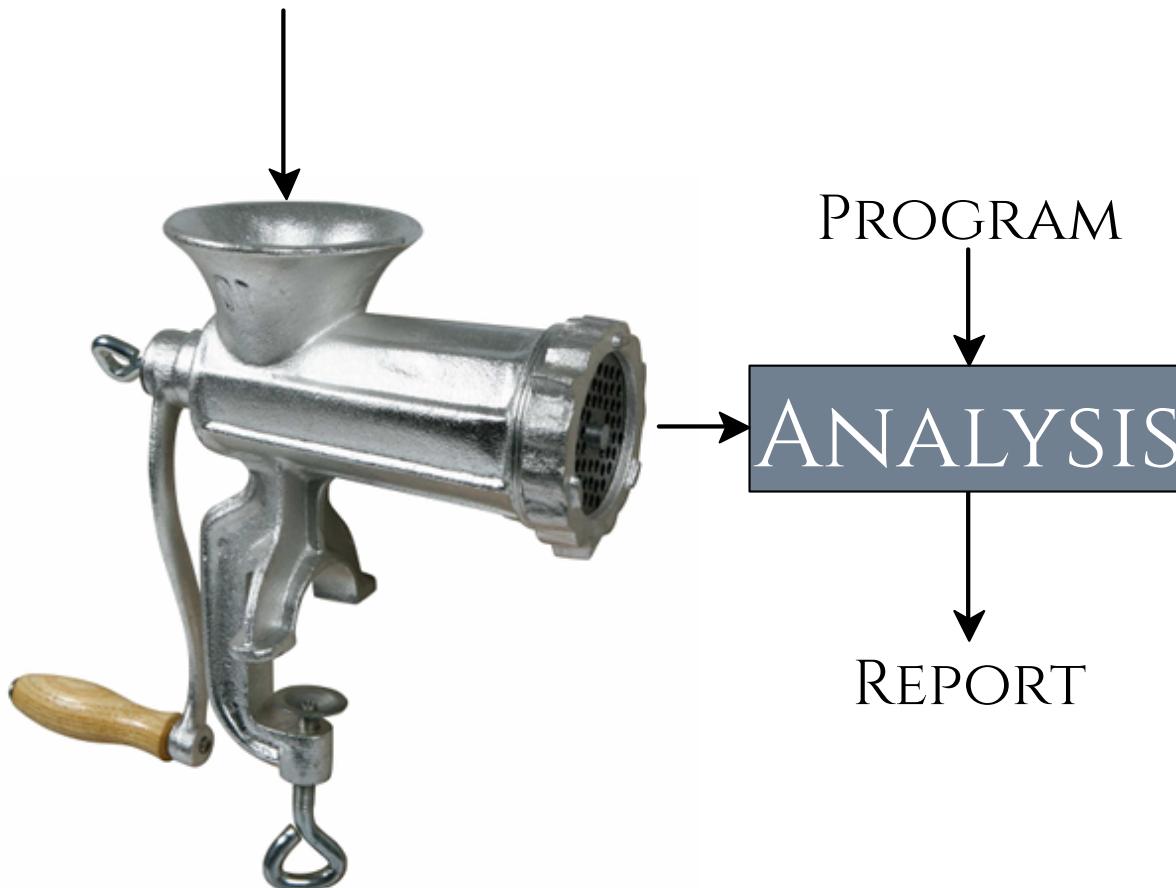


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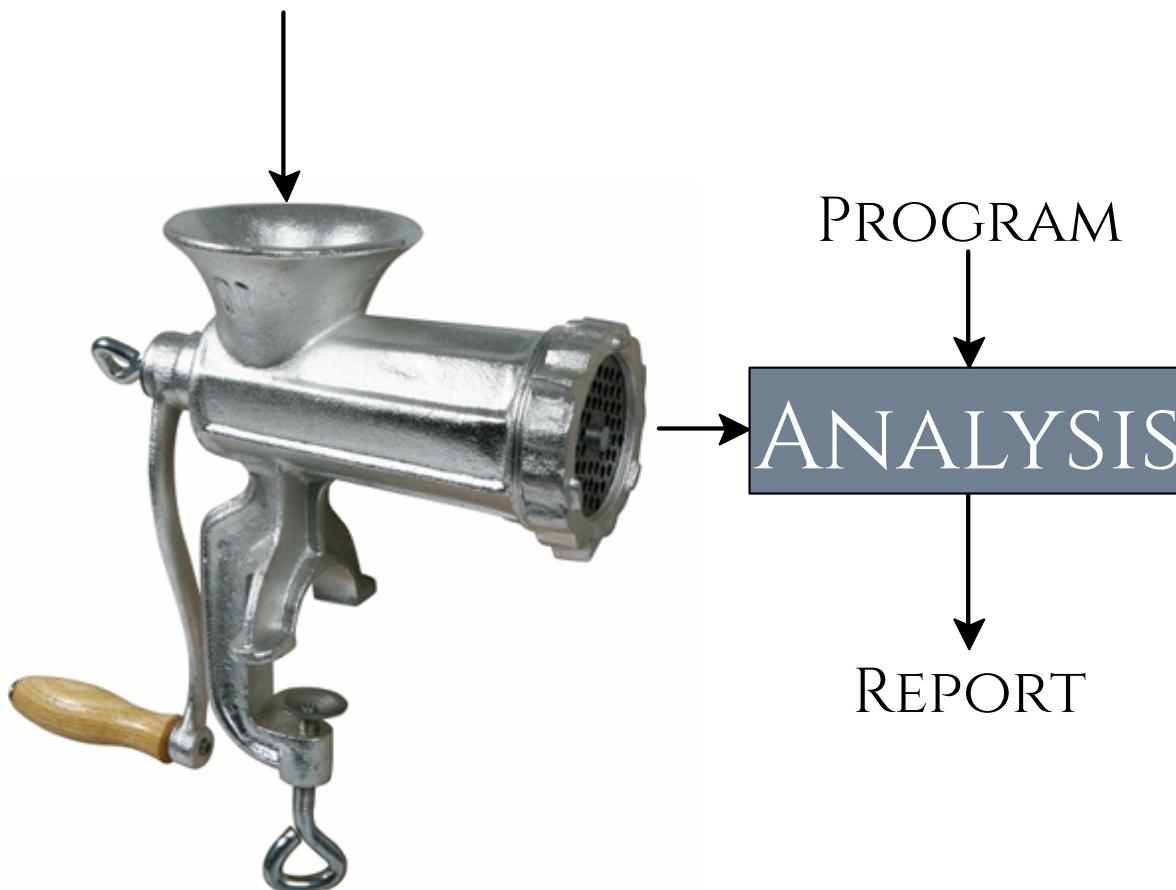


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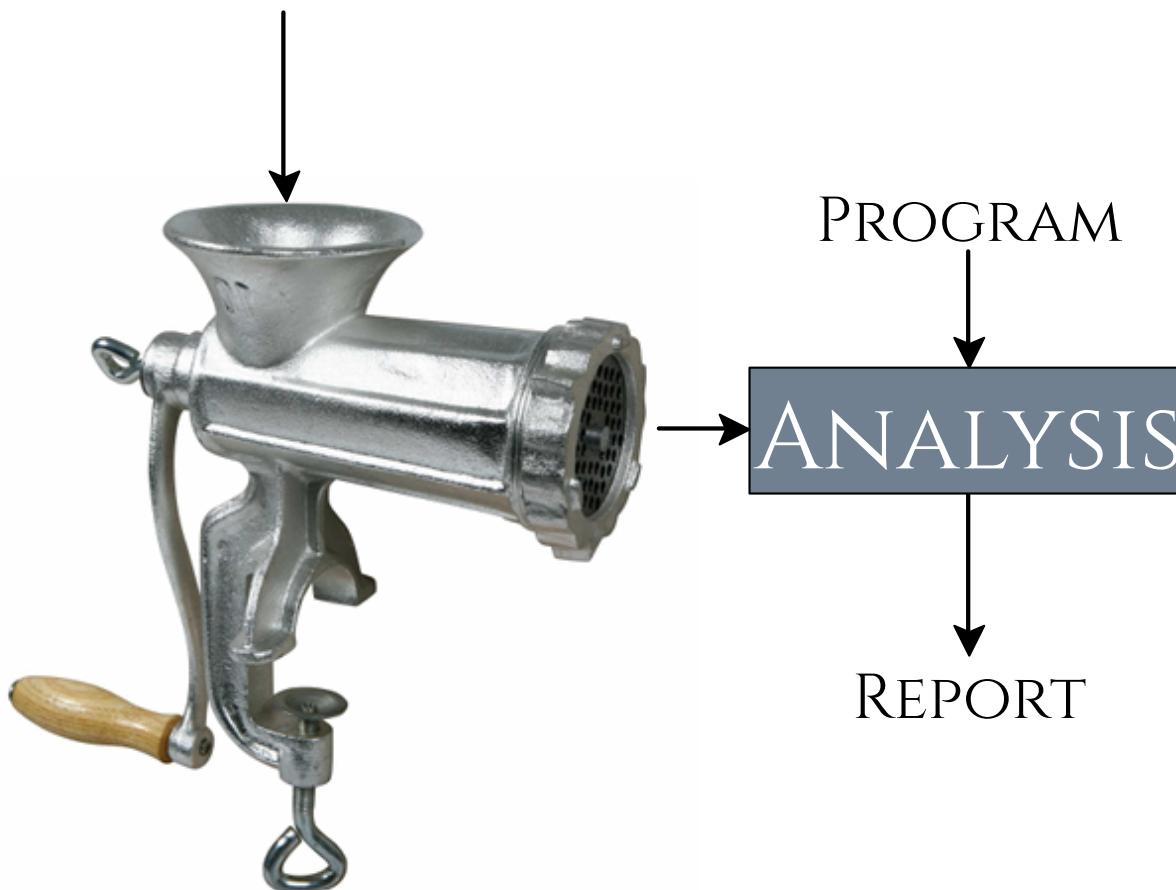


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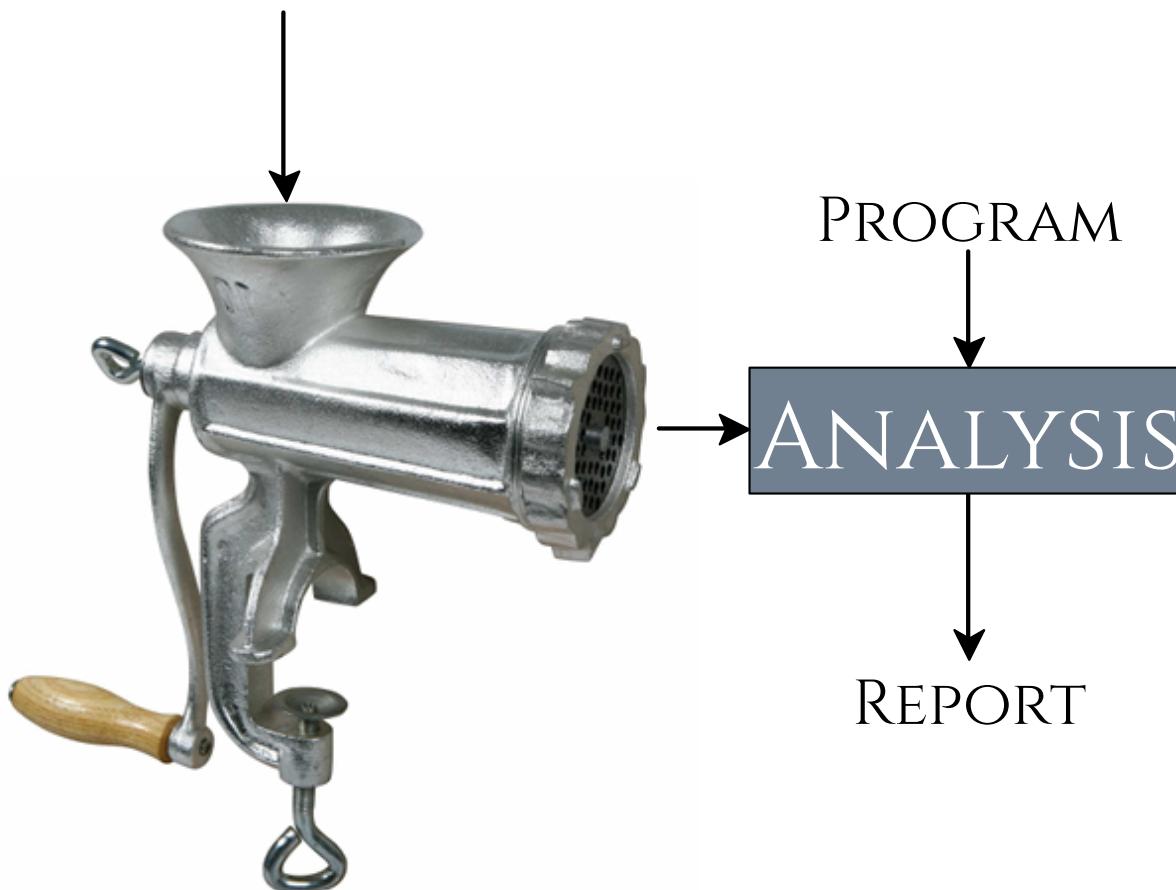


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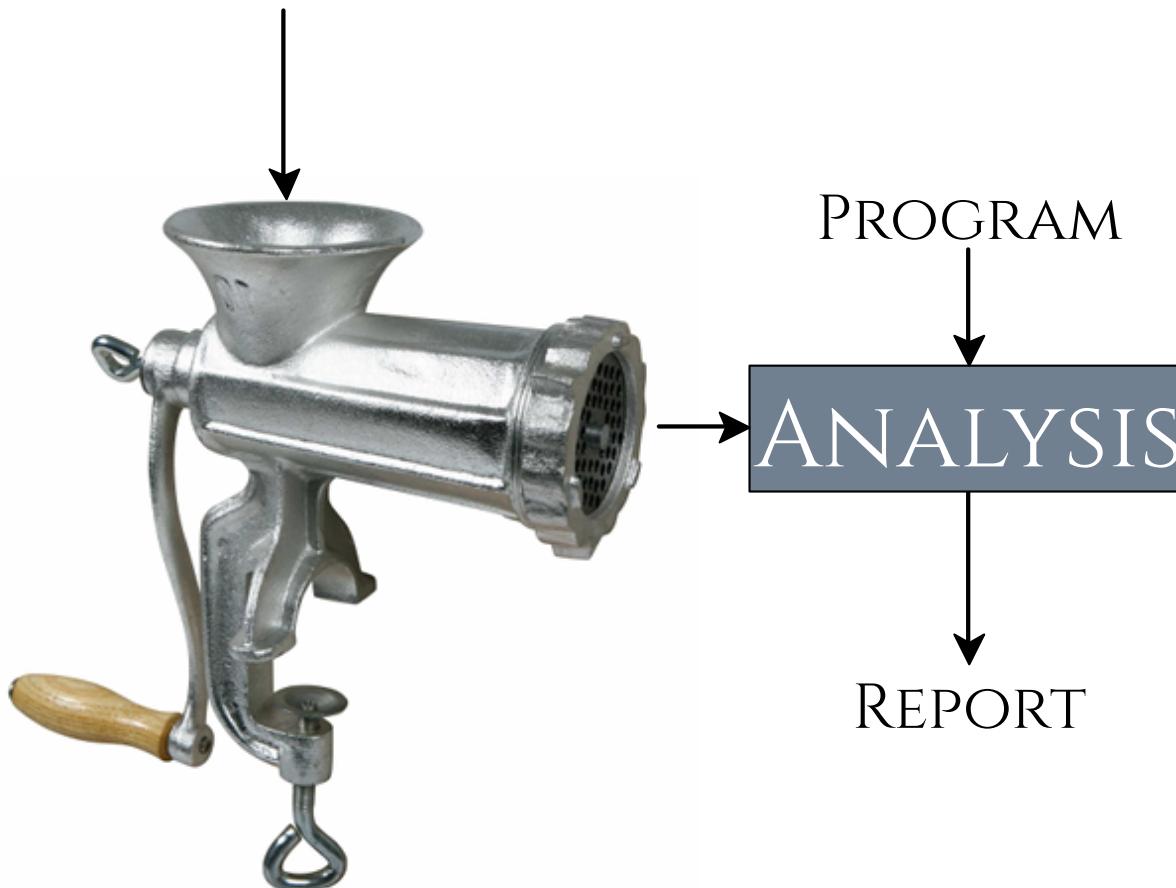


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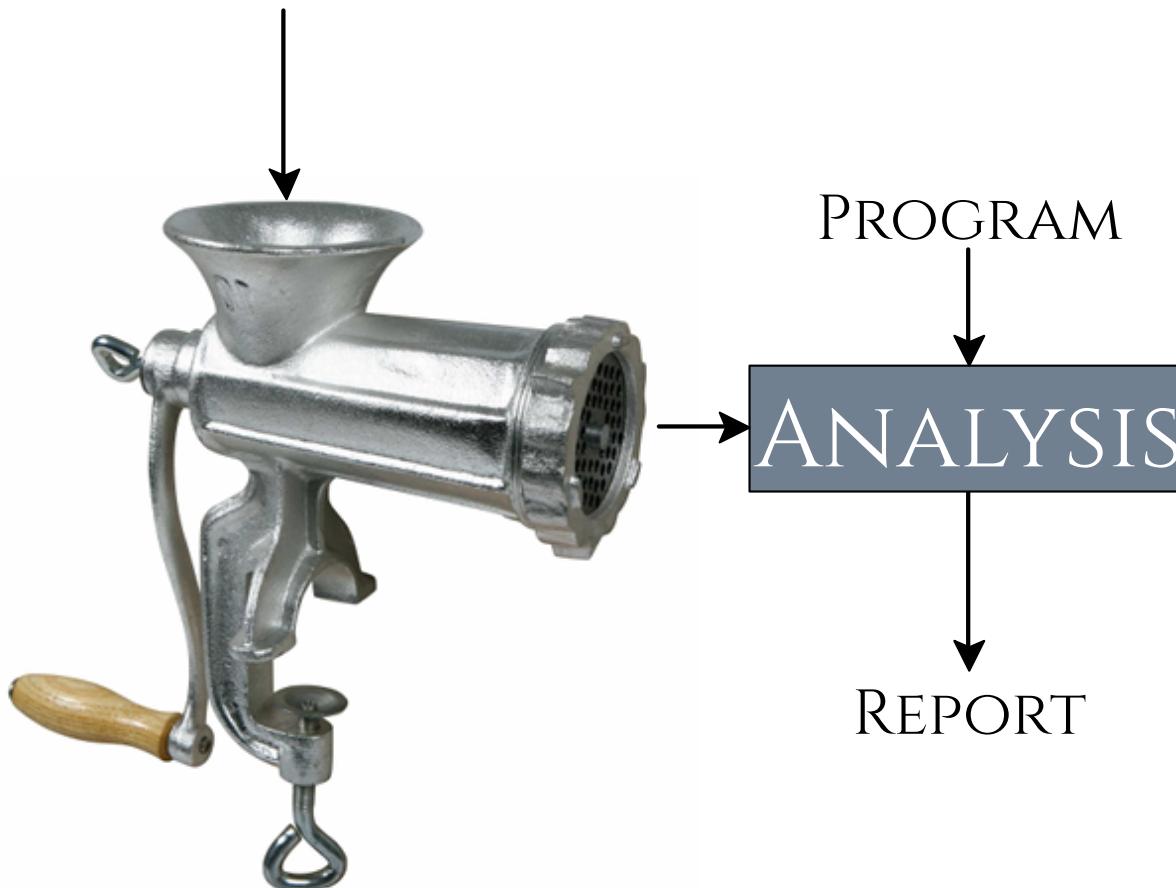


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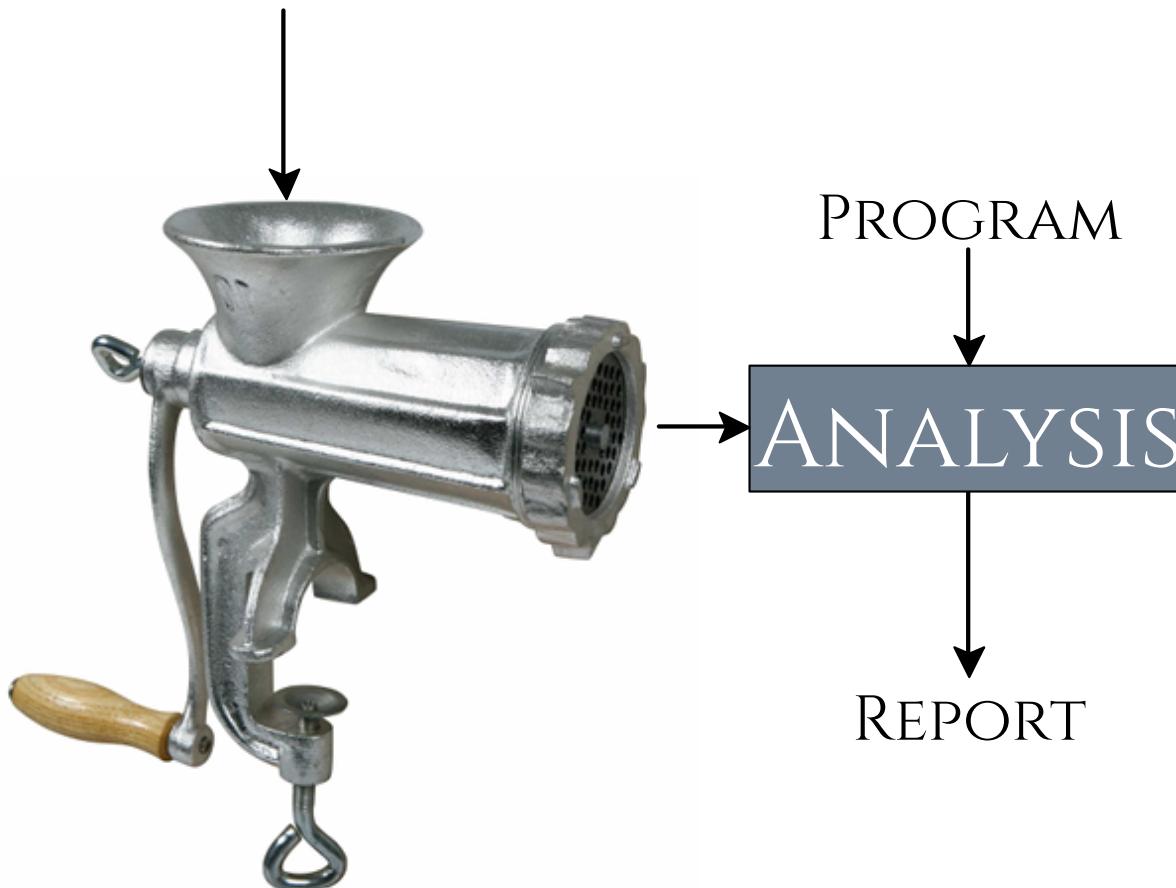


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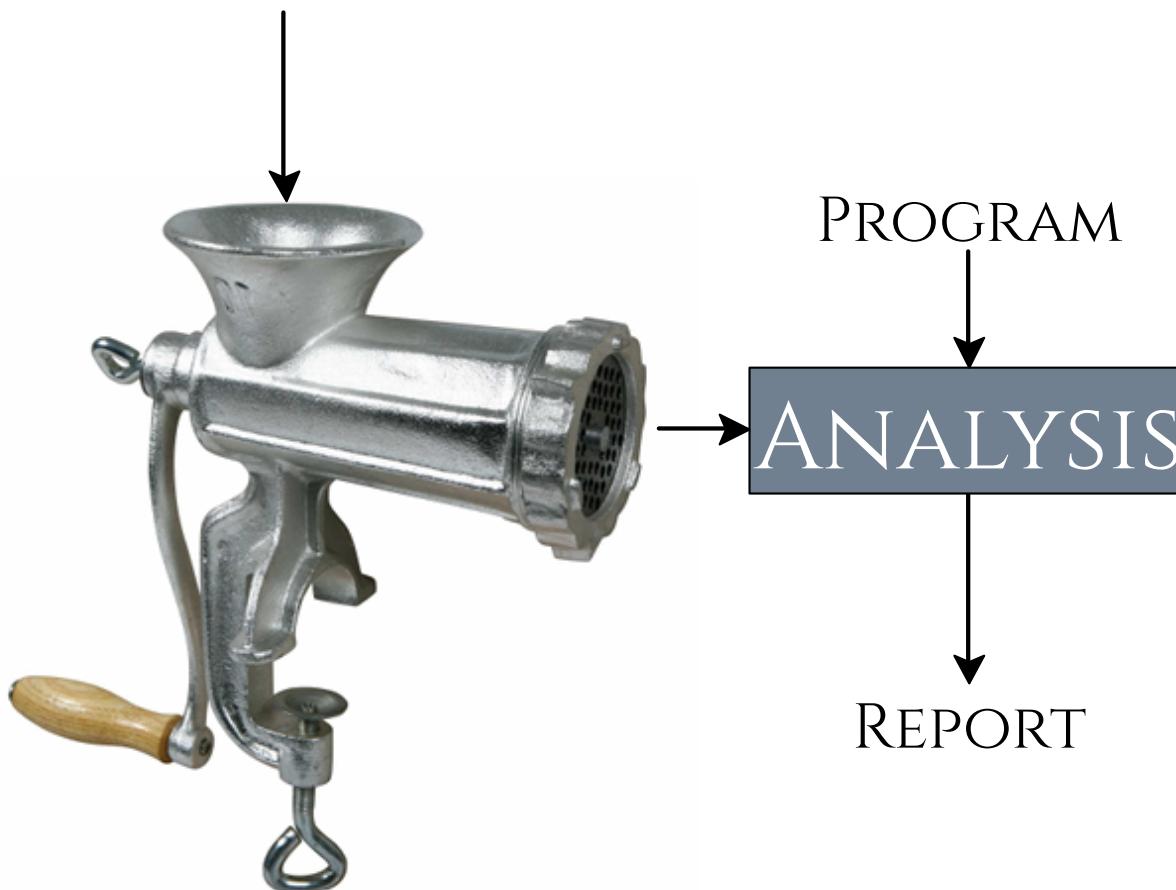


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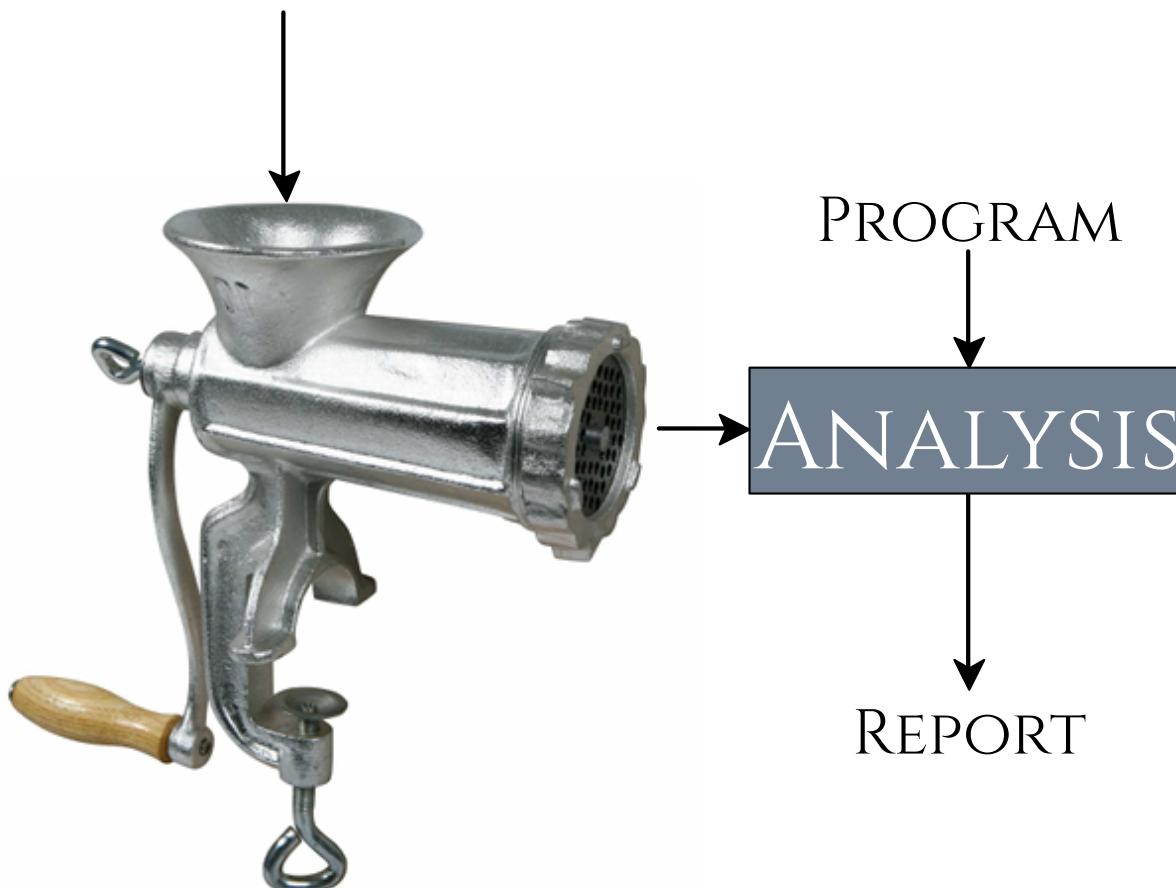


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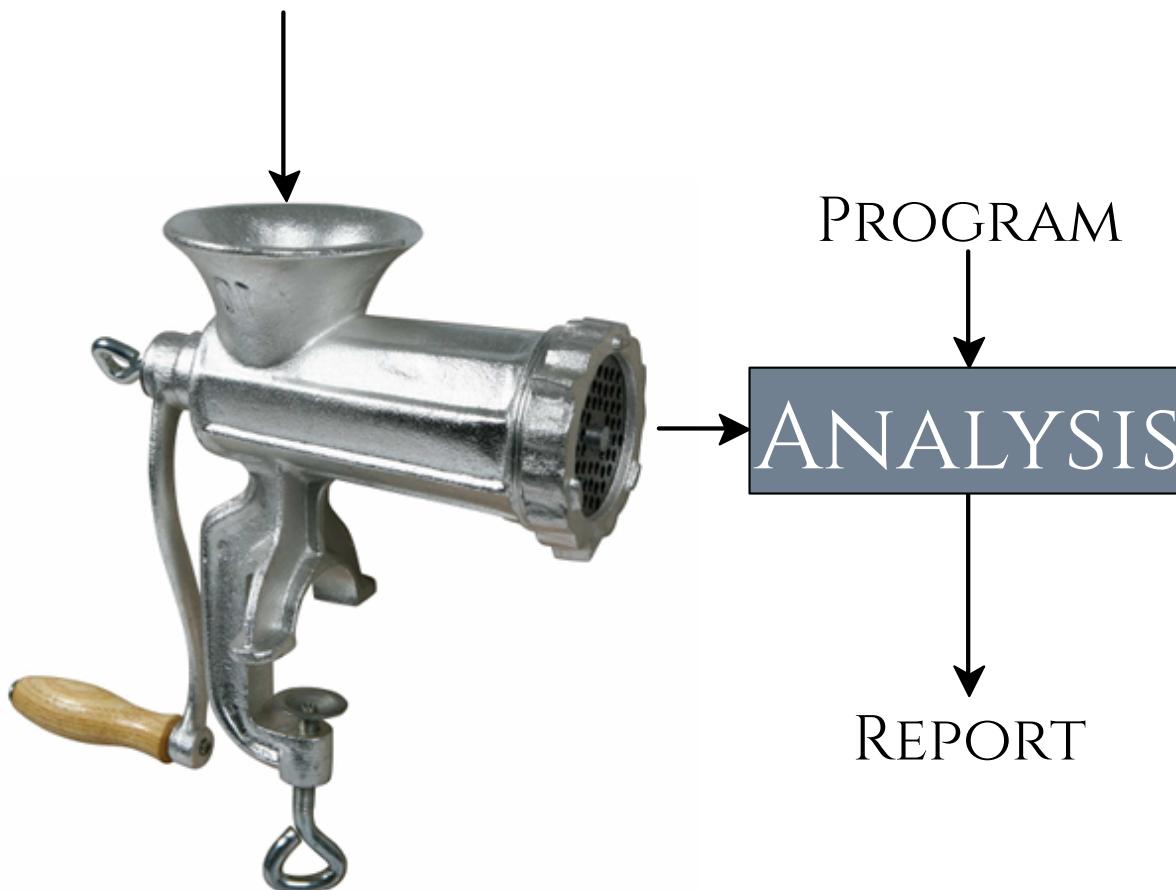


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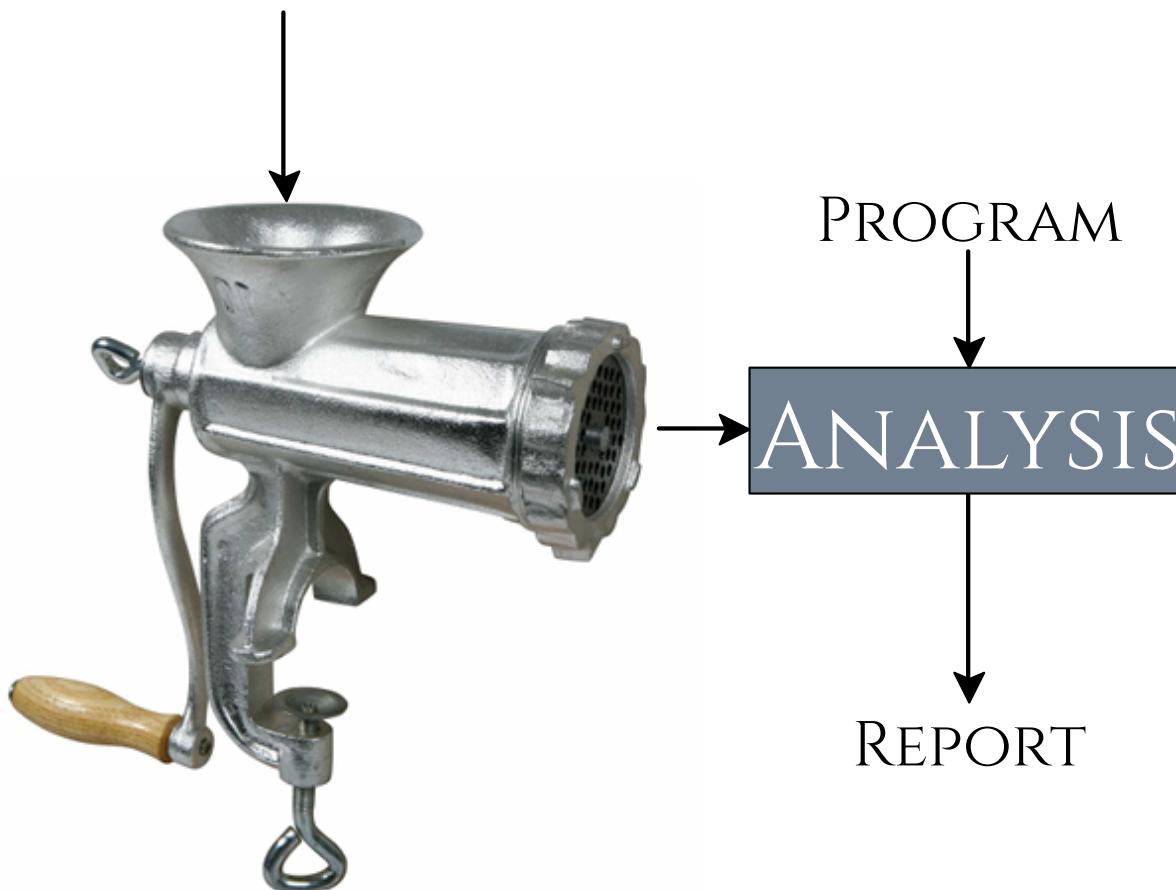


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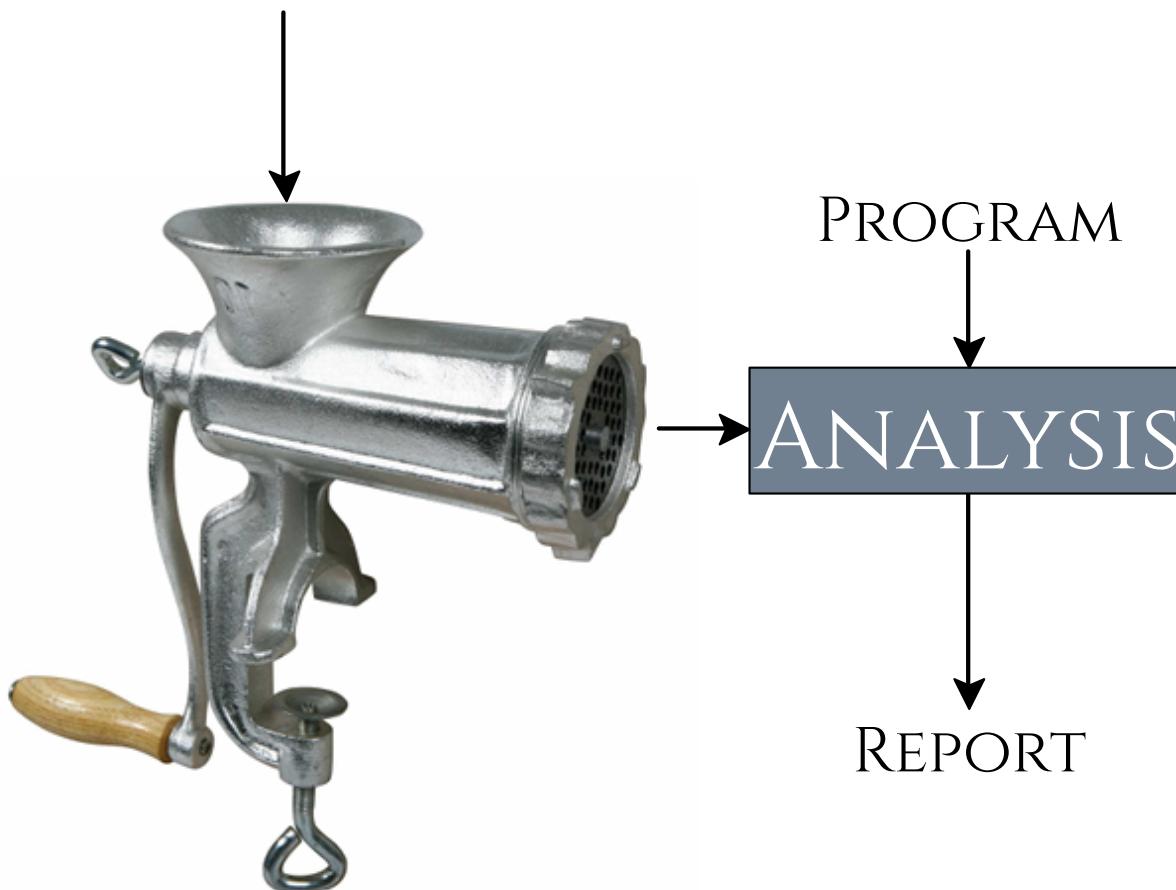


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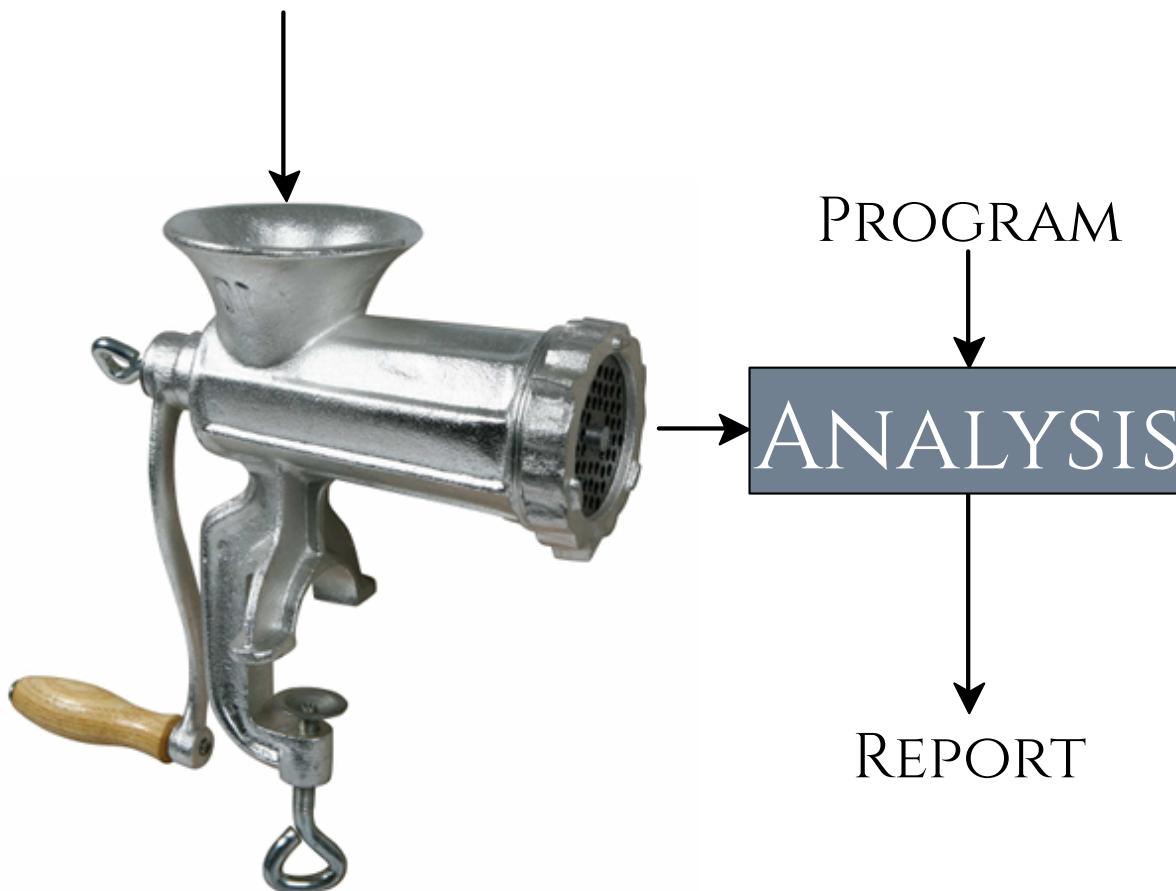


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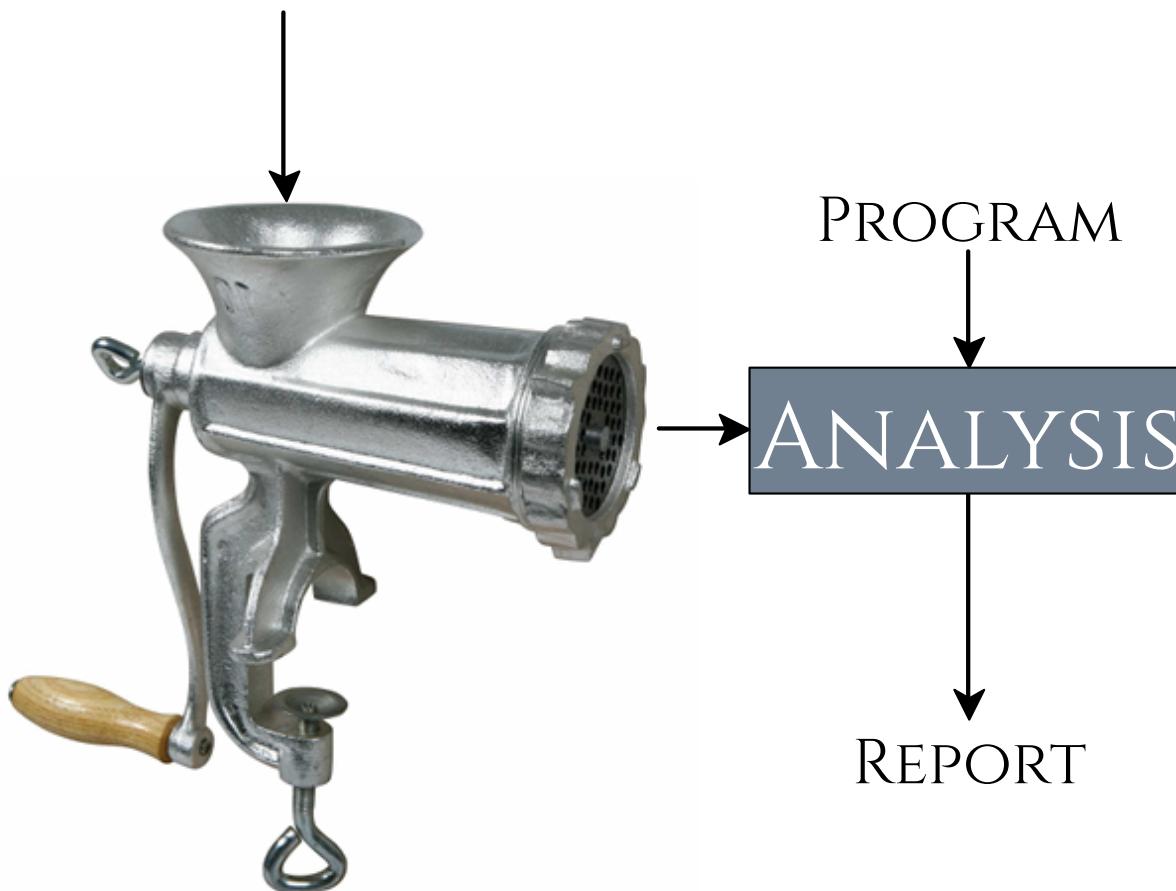


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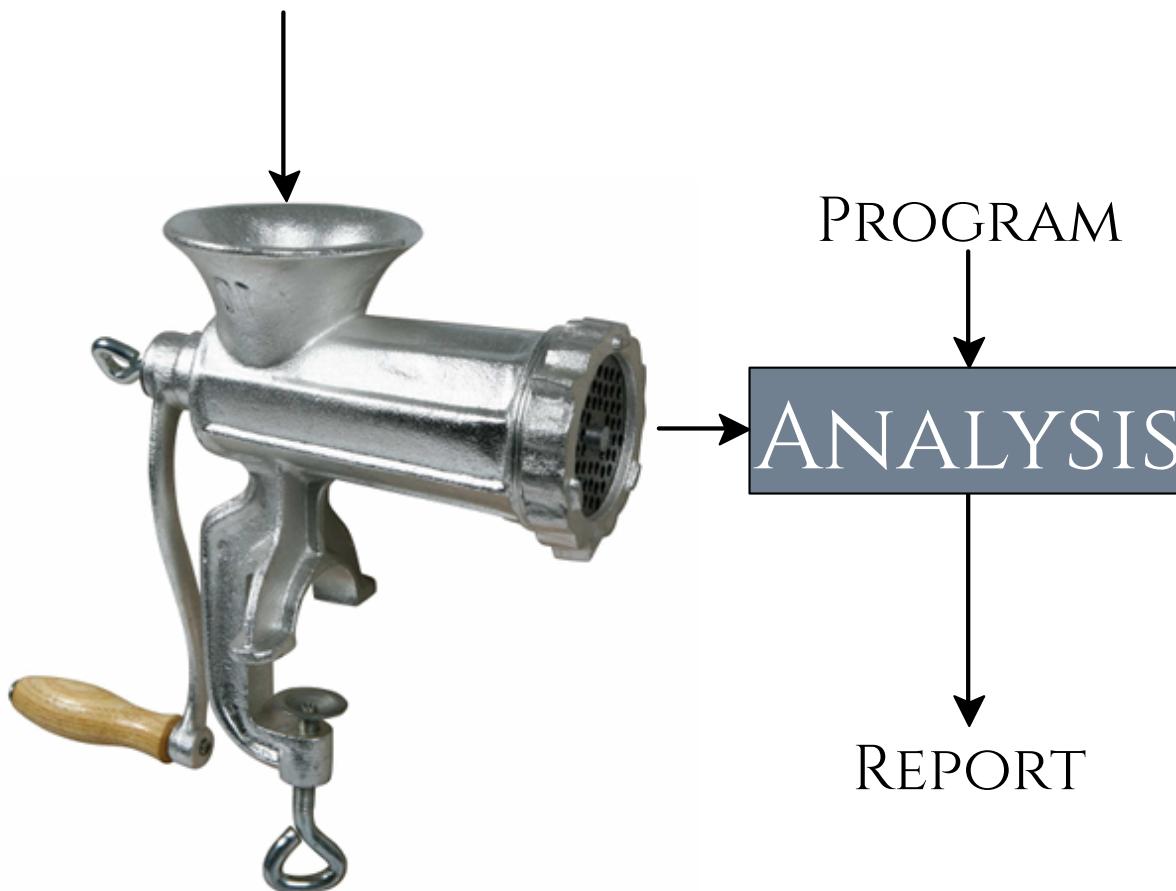


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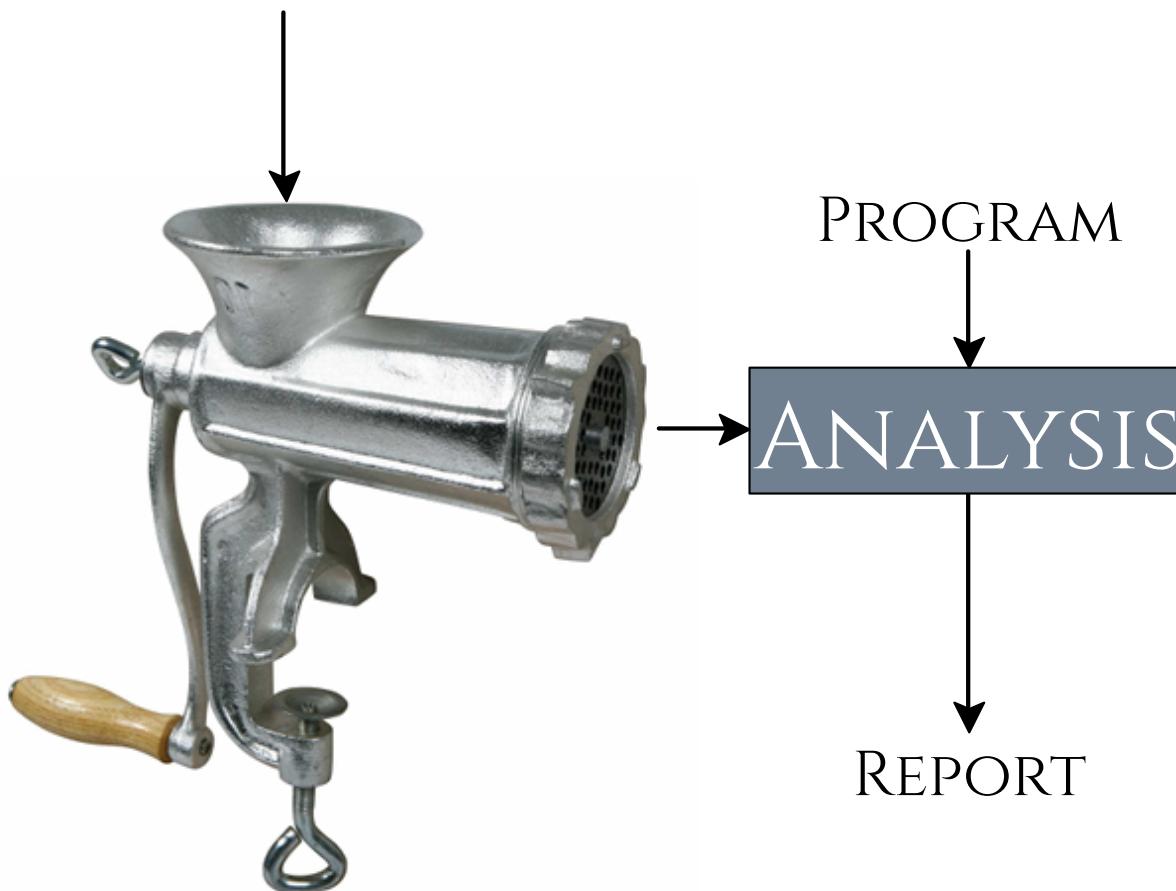


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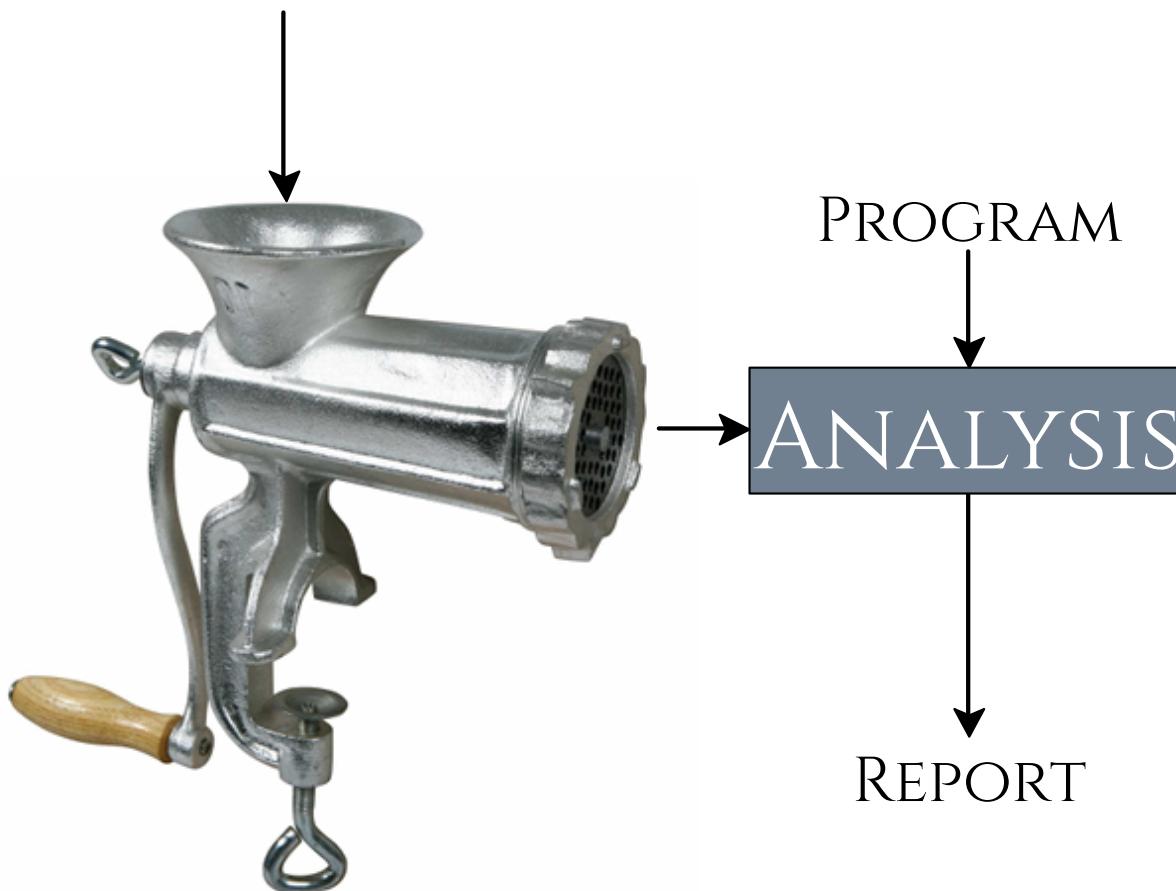


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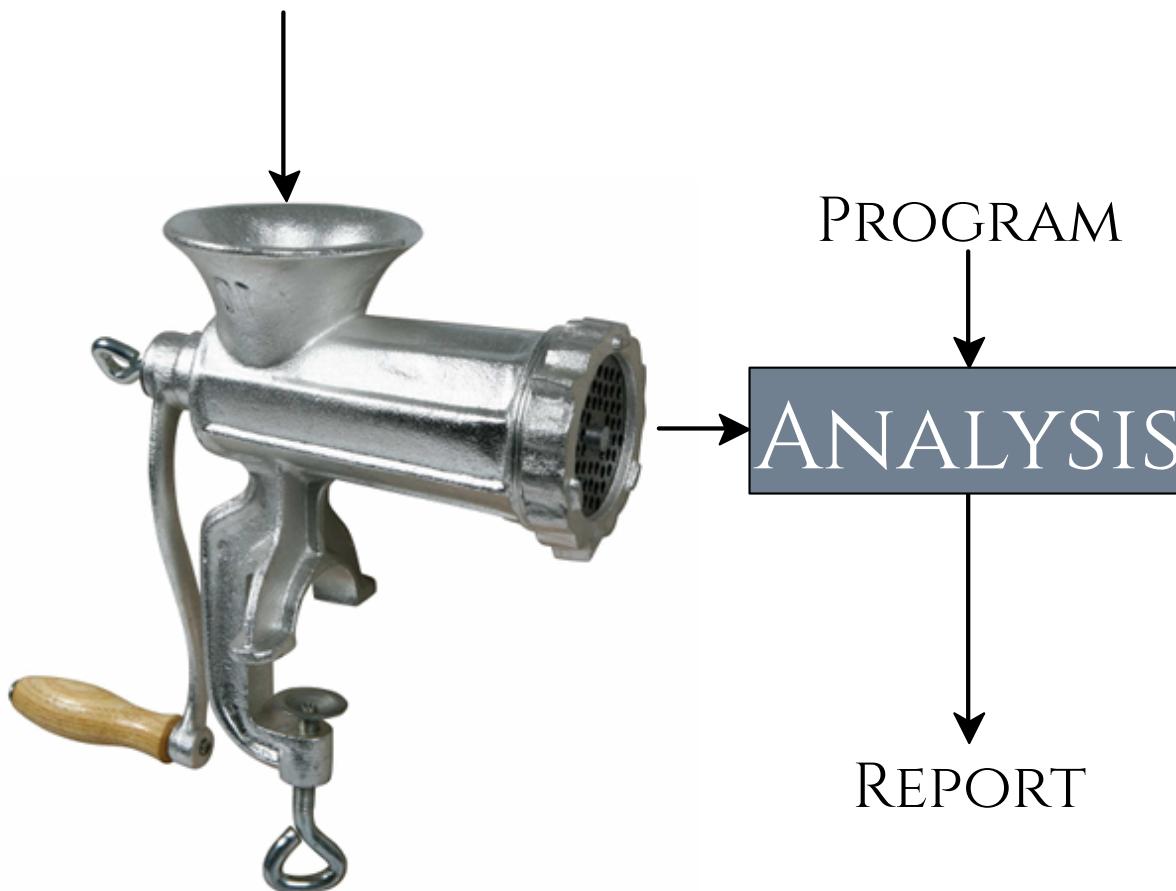


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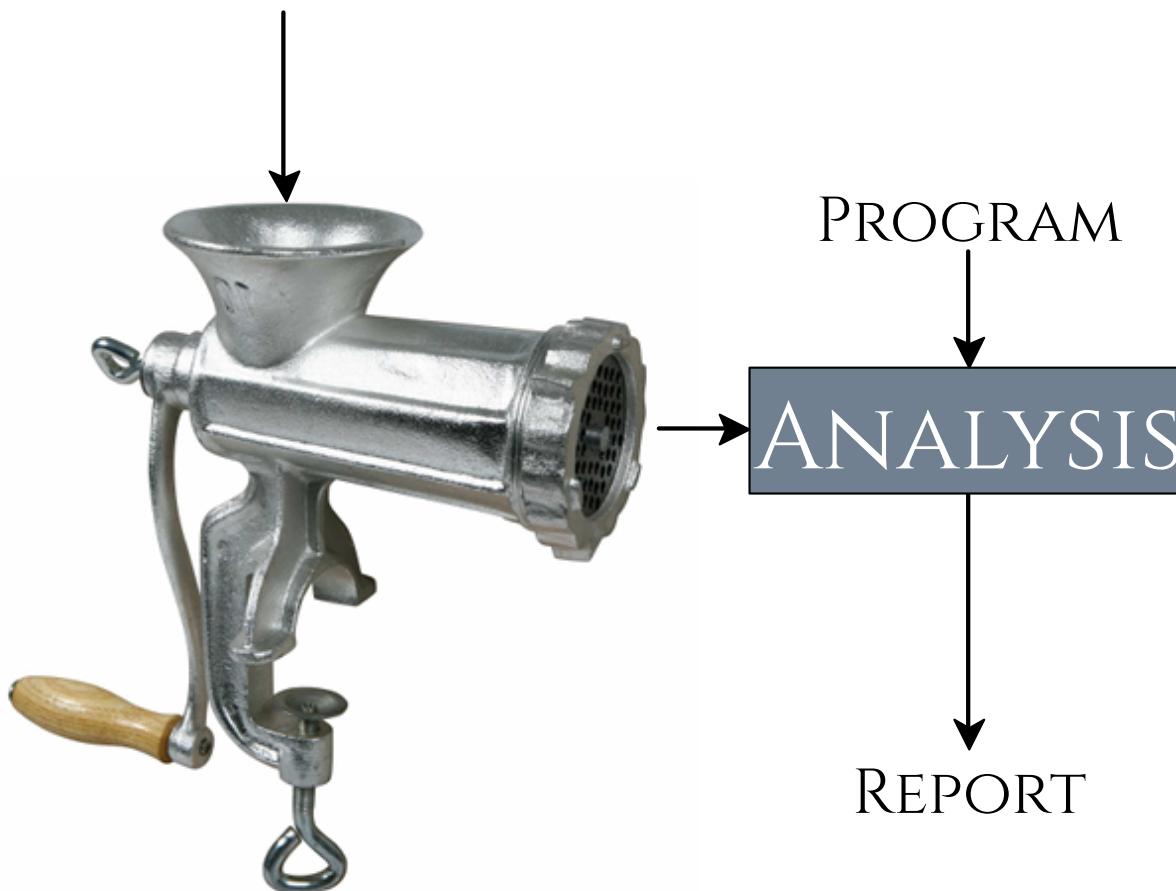


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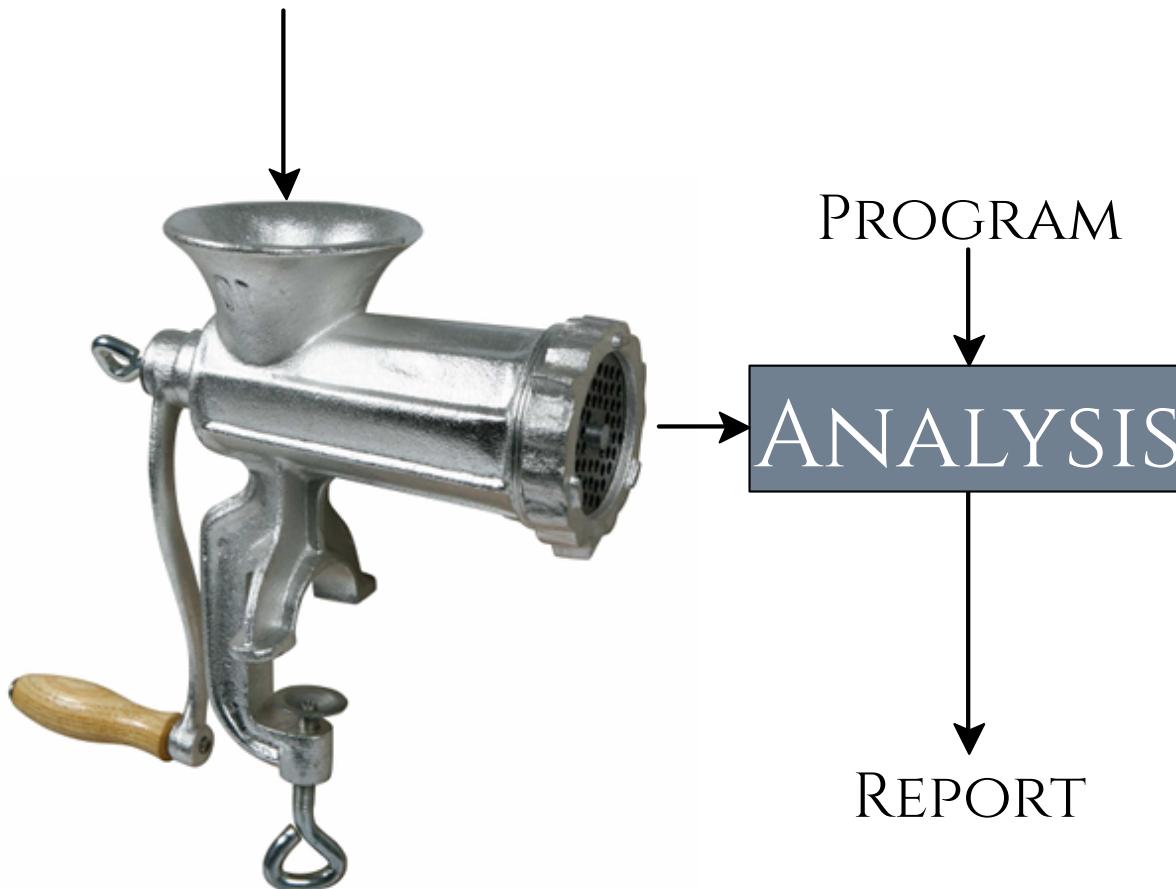


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REPORT

SEMANTICS

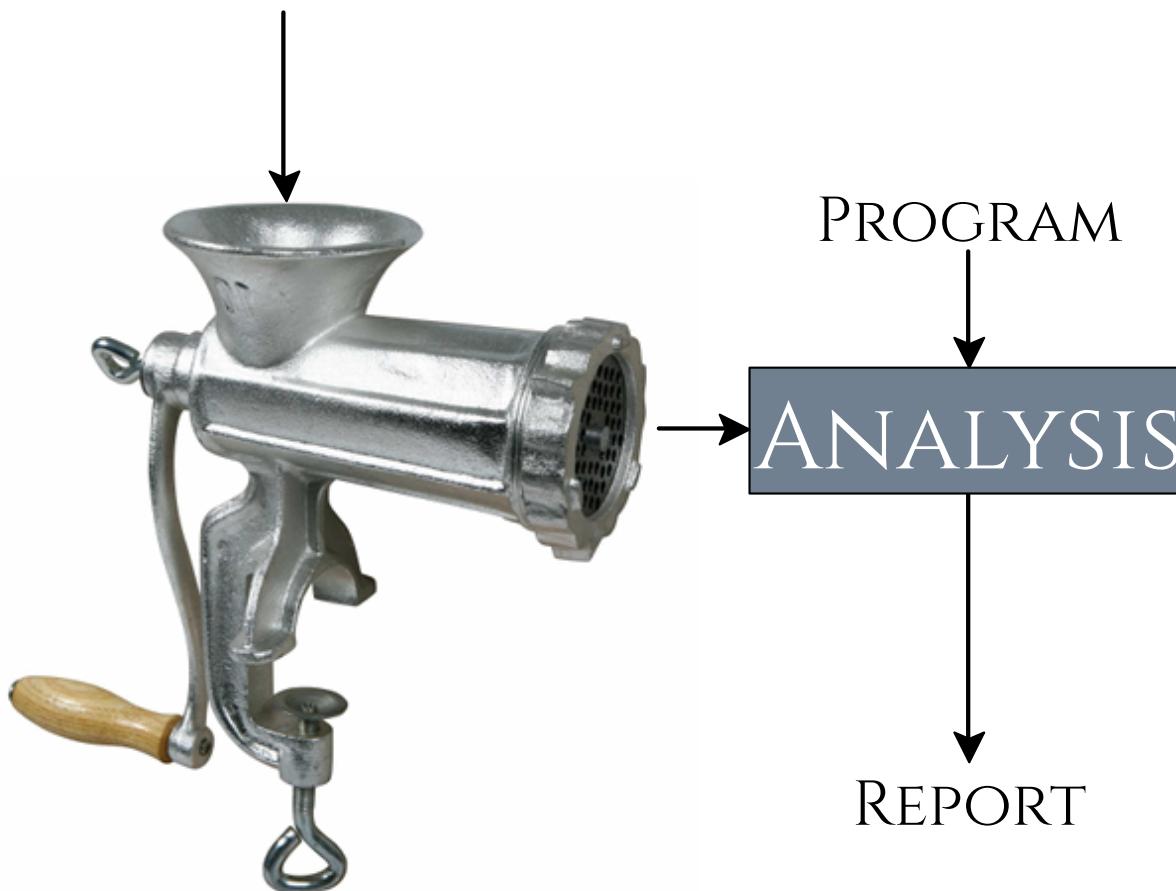


PROGRAM

ANALYSIS

REPORT

SEMANTICS

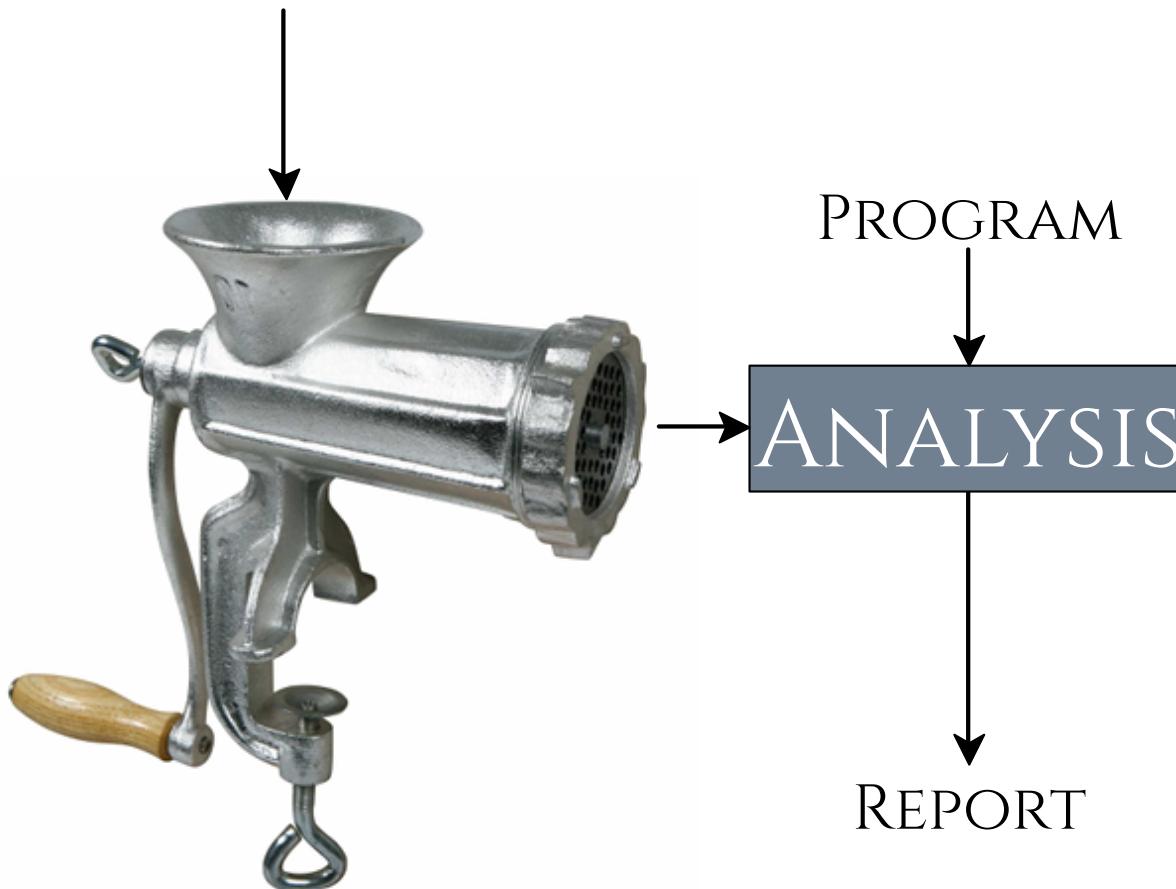


PROGRAM

ANALYSIS

REPORT

SEMANTICS

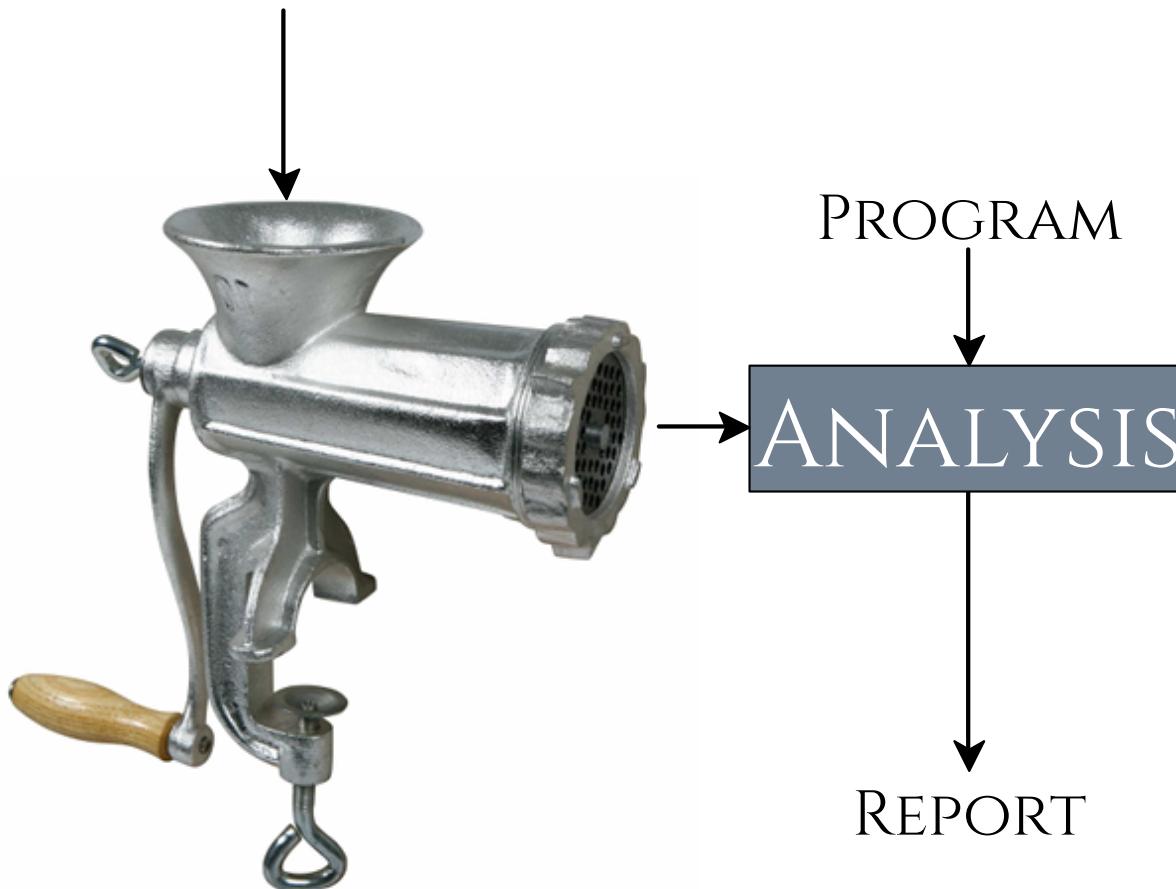


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ANALYSIS

REPORT

SEMANTICS

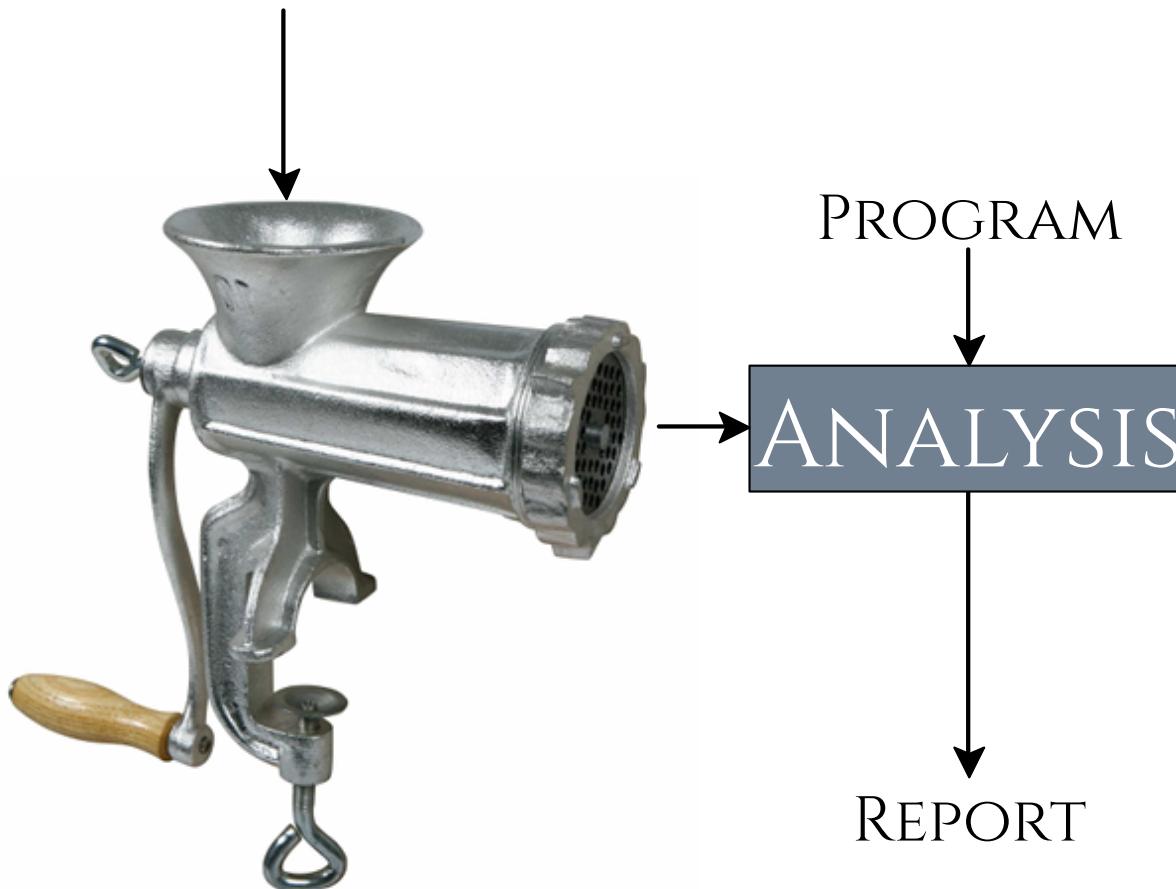


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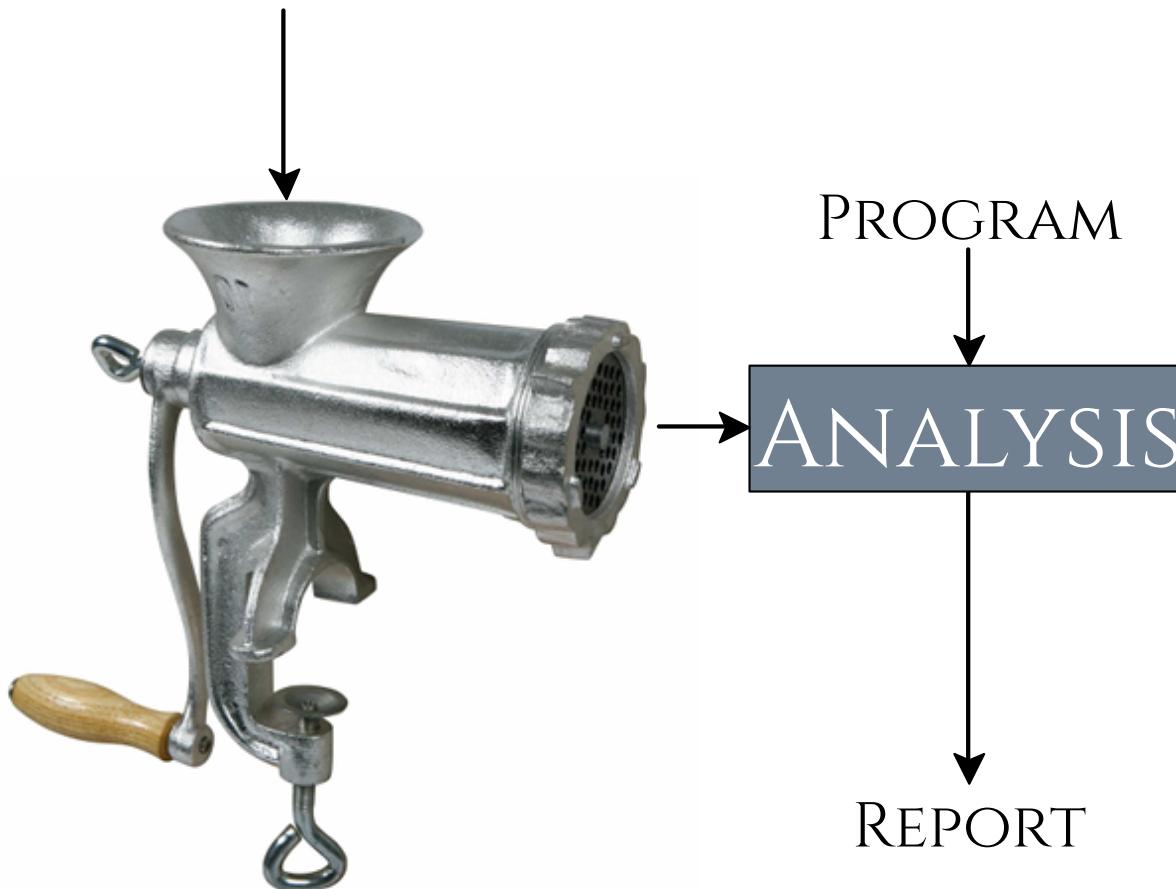


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ANALYSIS

REPORT

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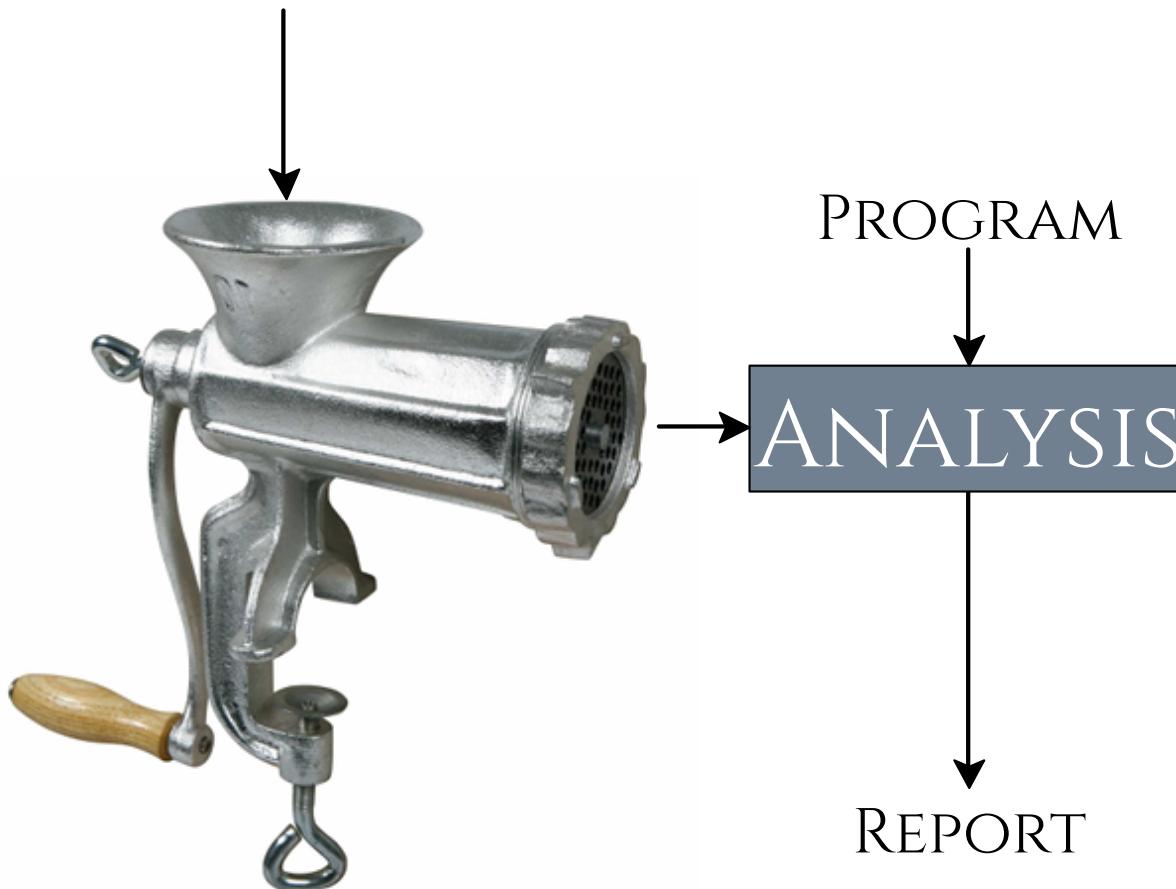


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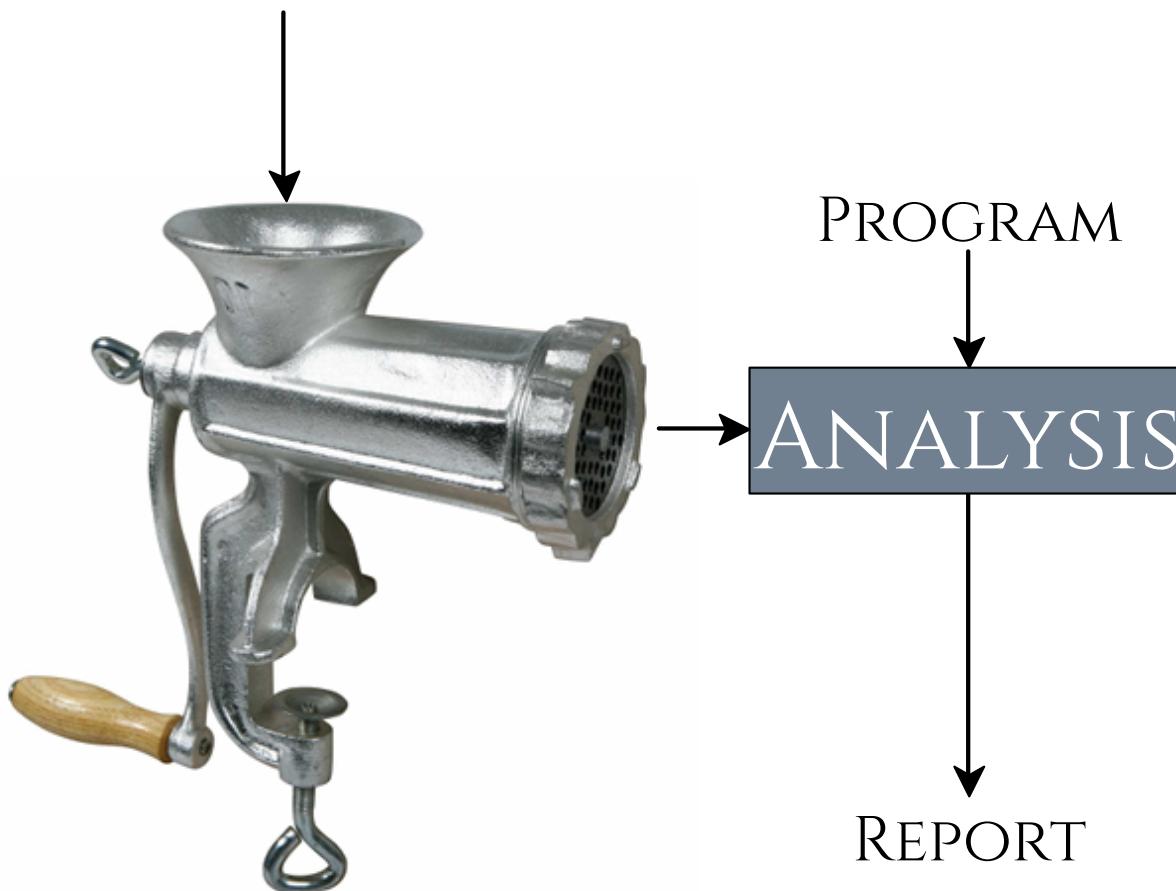


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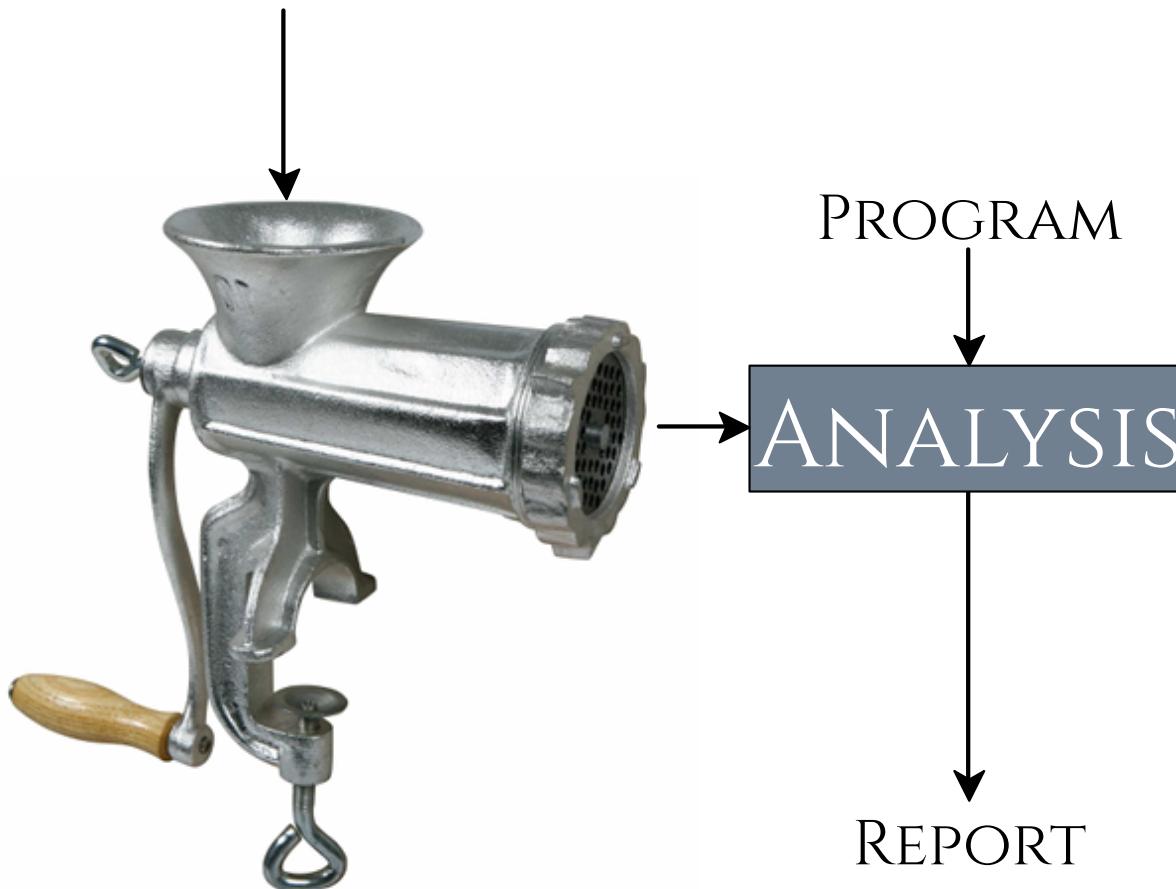


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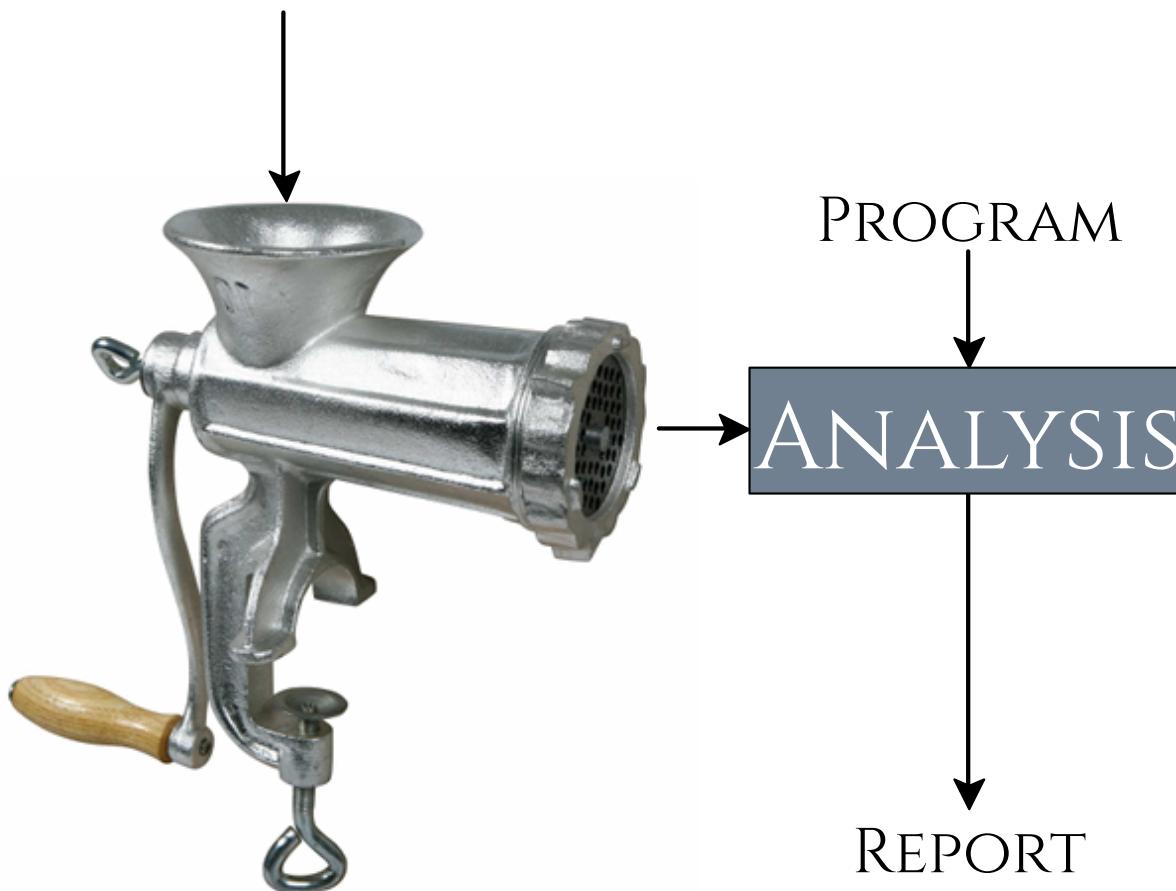


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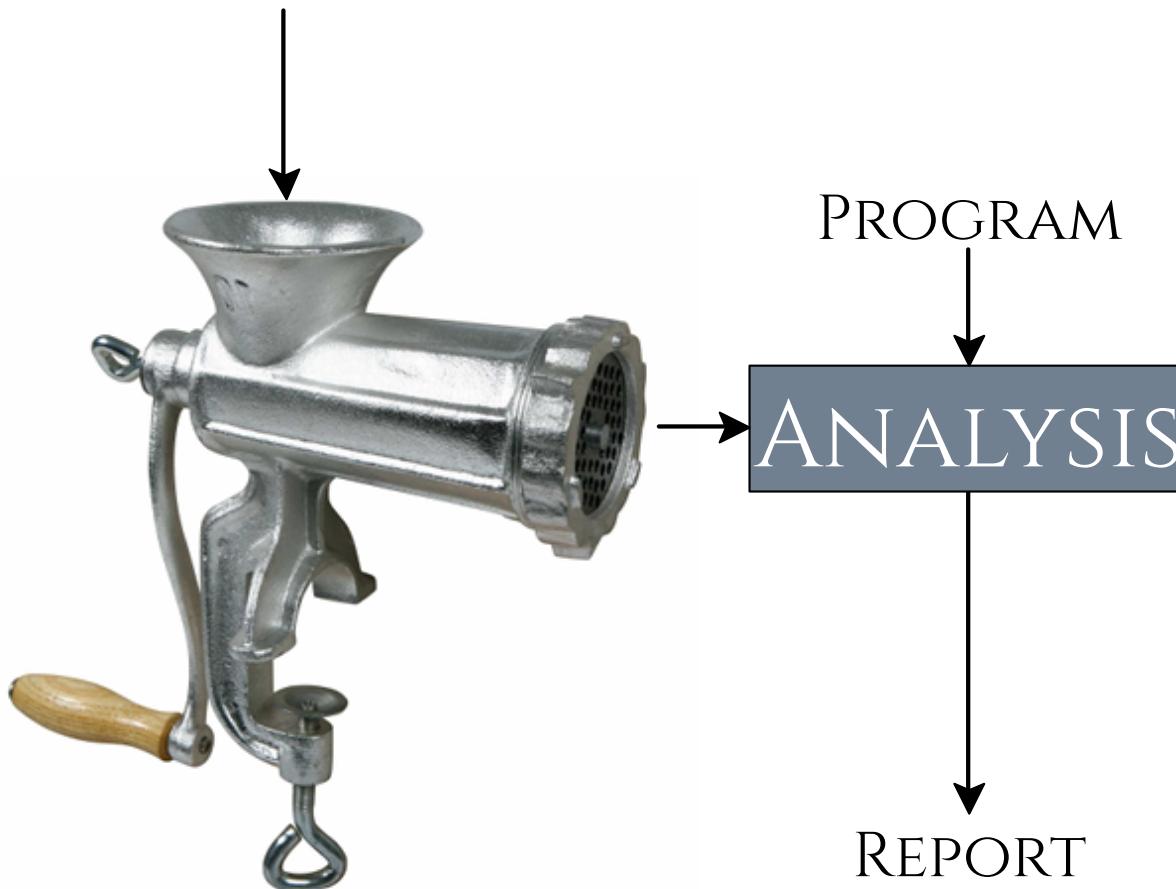


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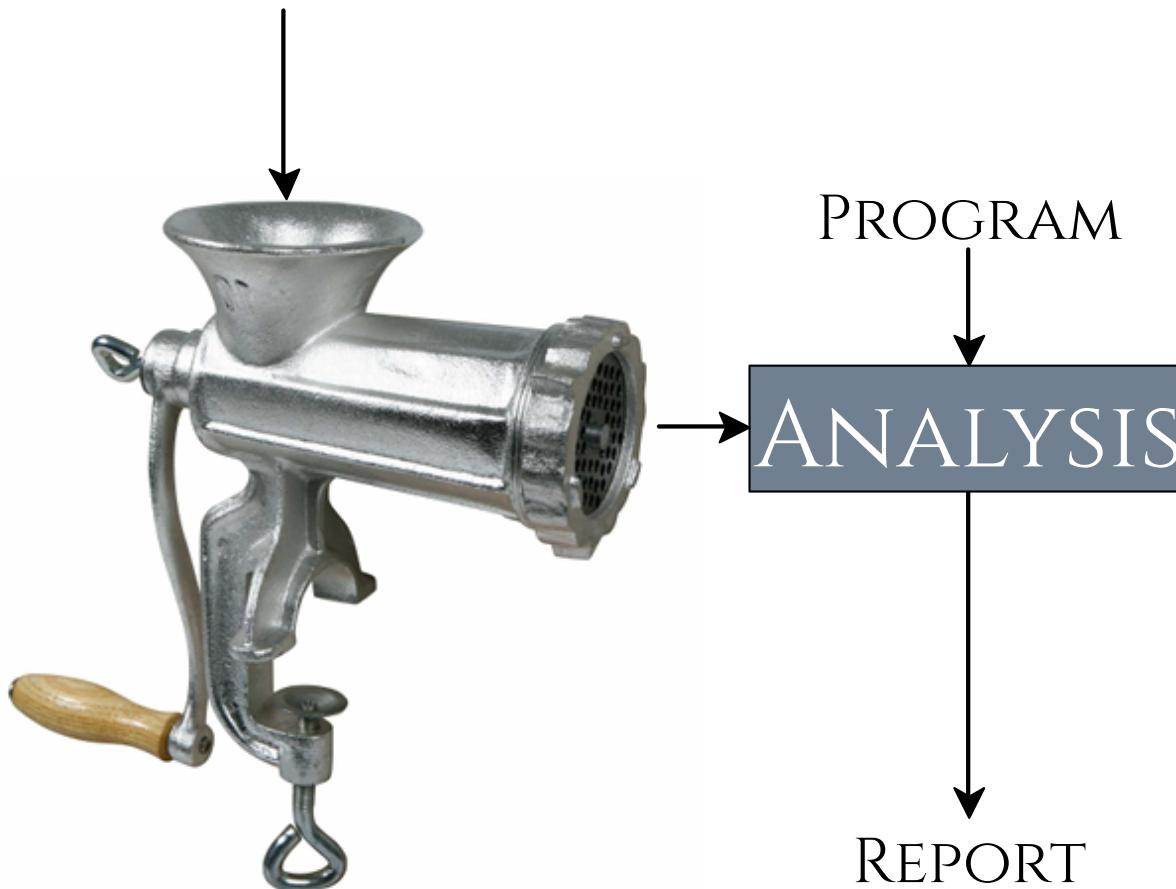


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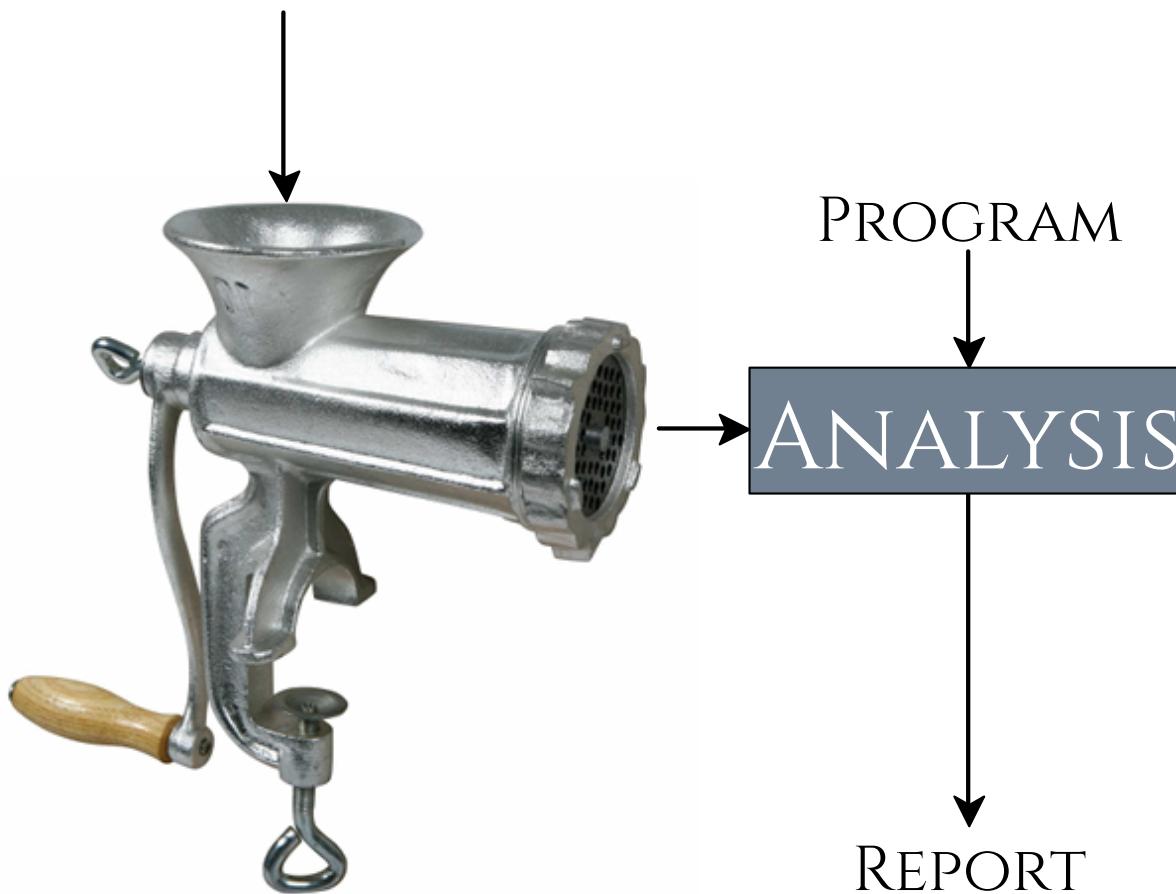


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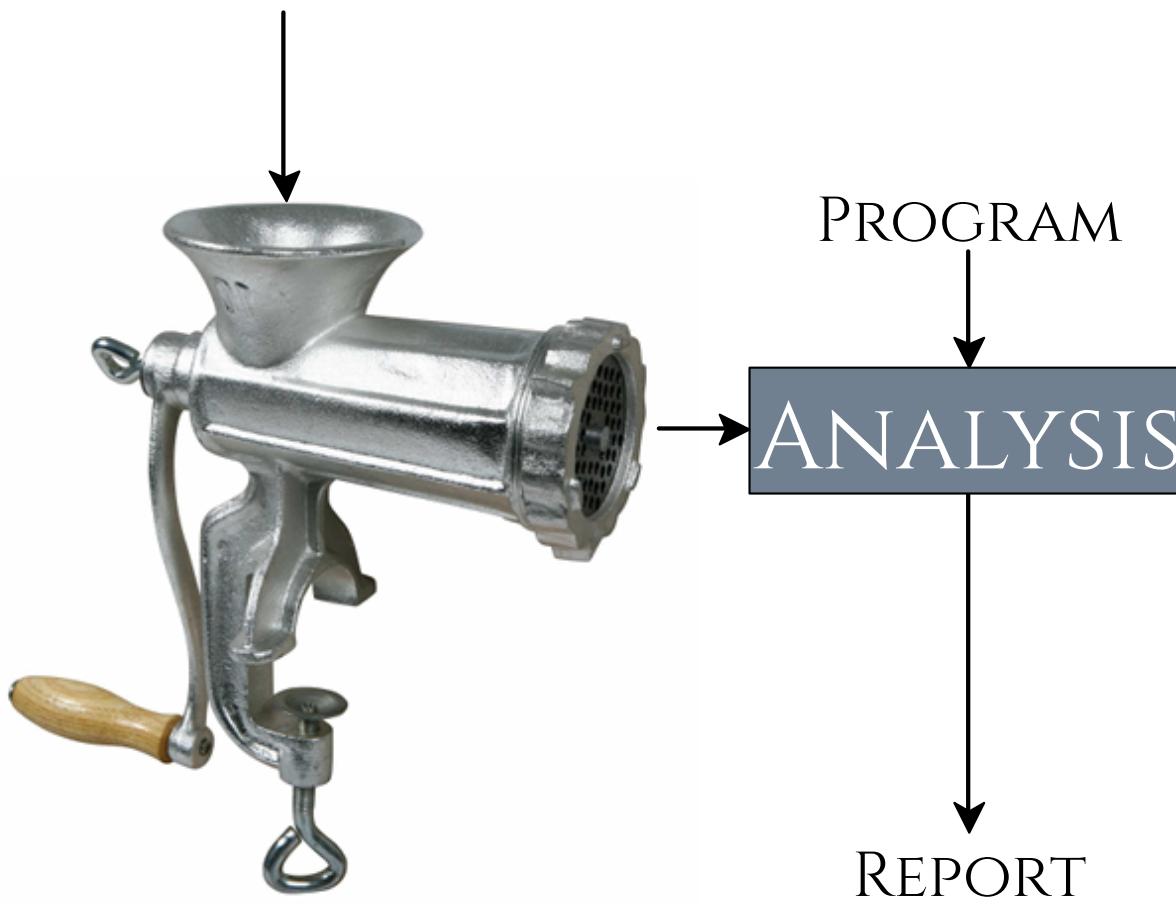


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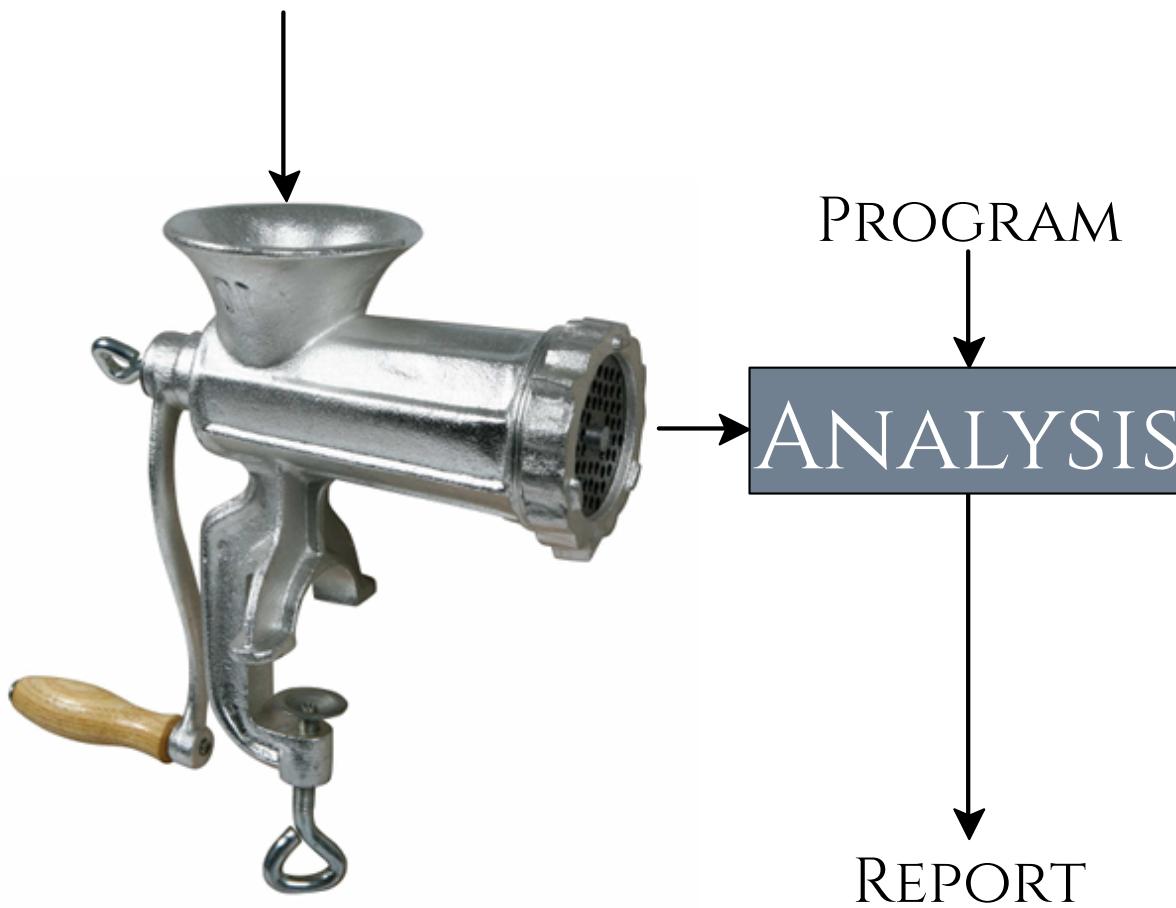


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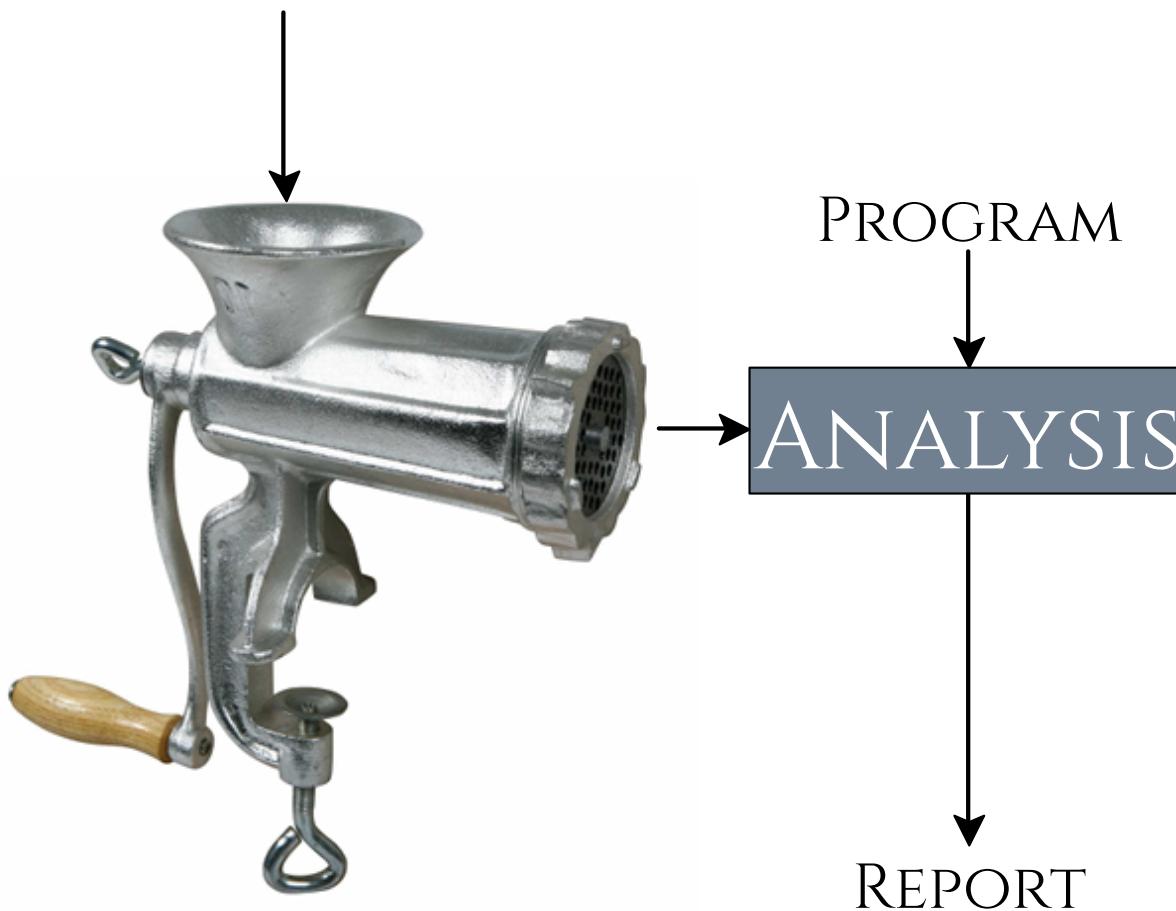


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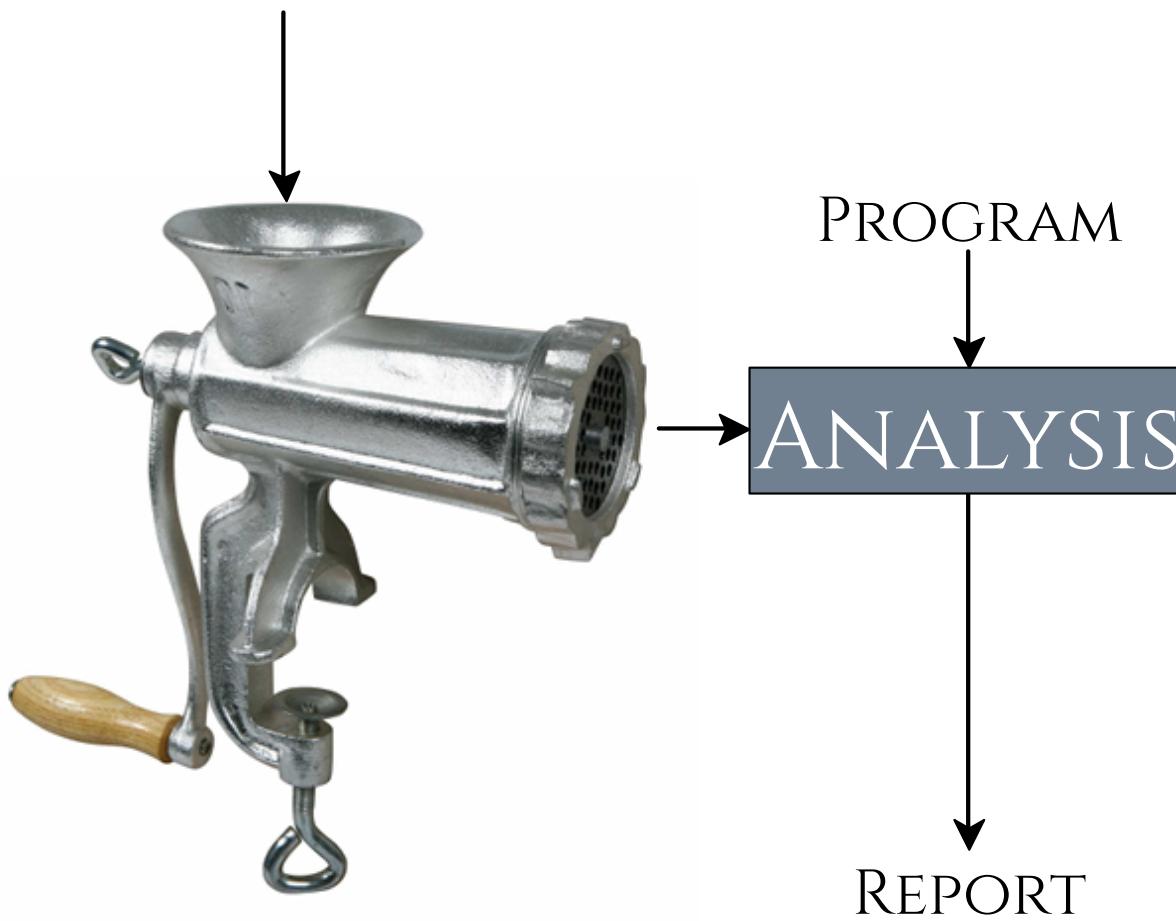


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ANALYSIS

REPORT

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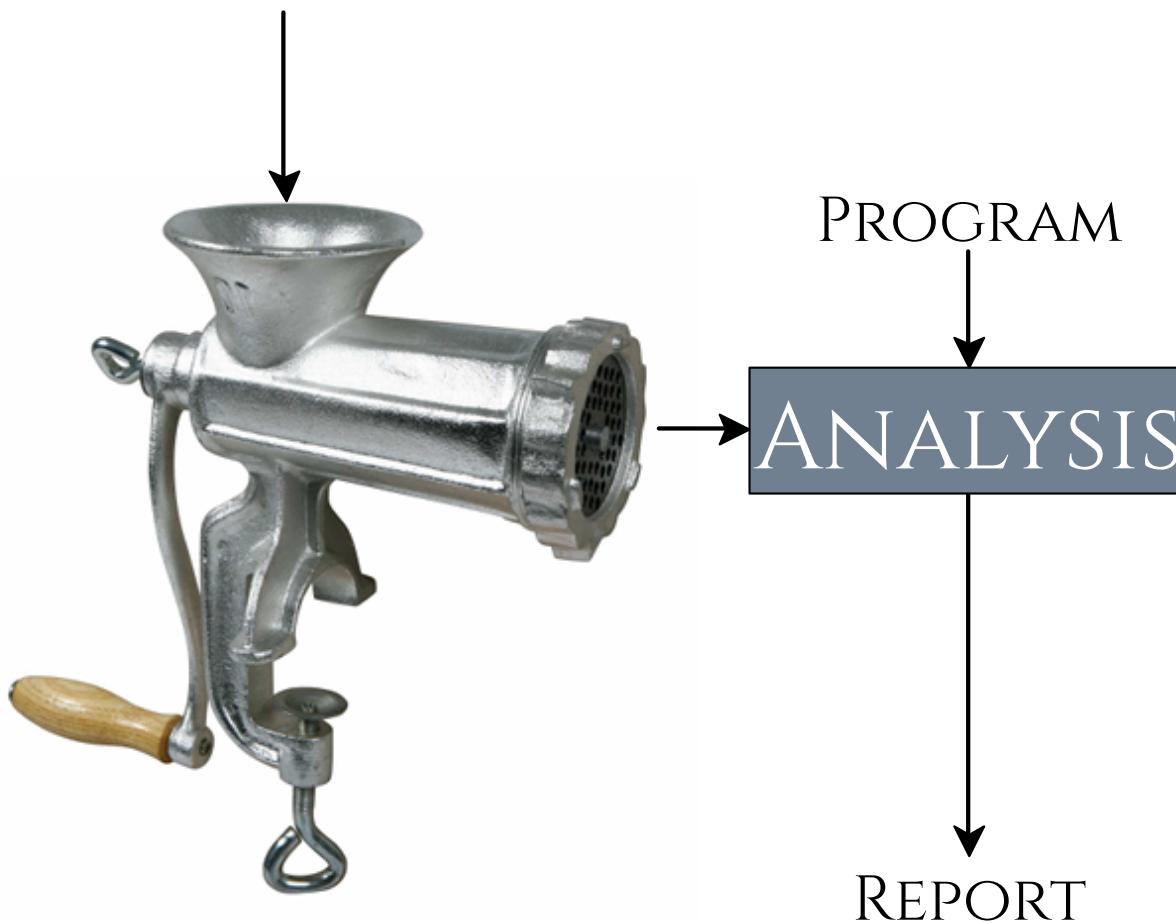


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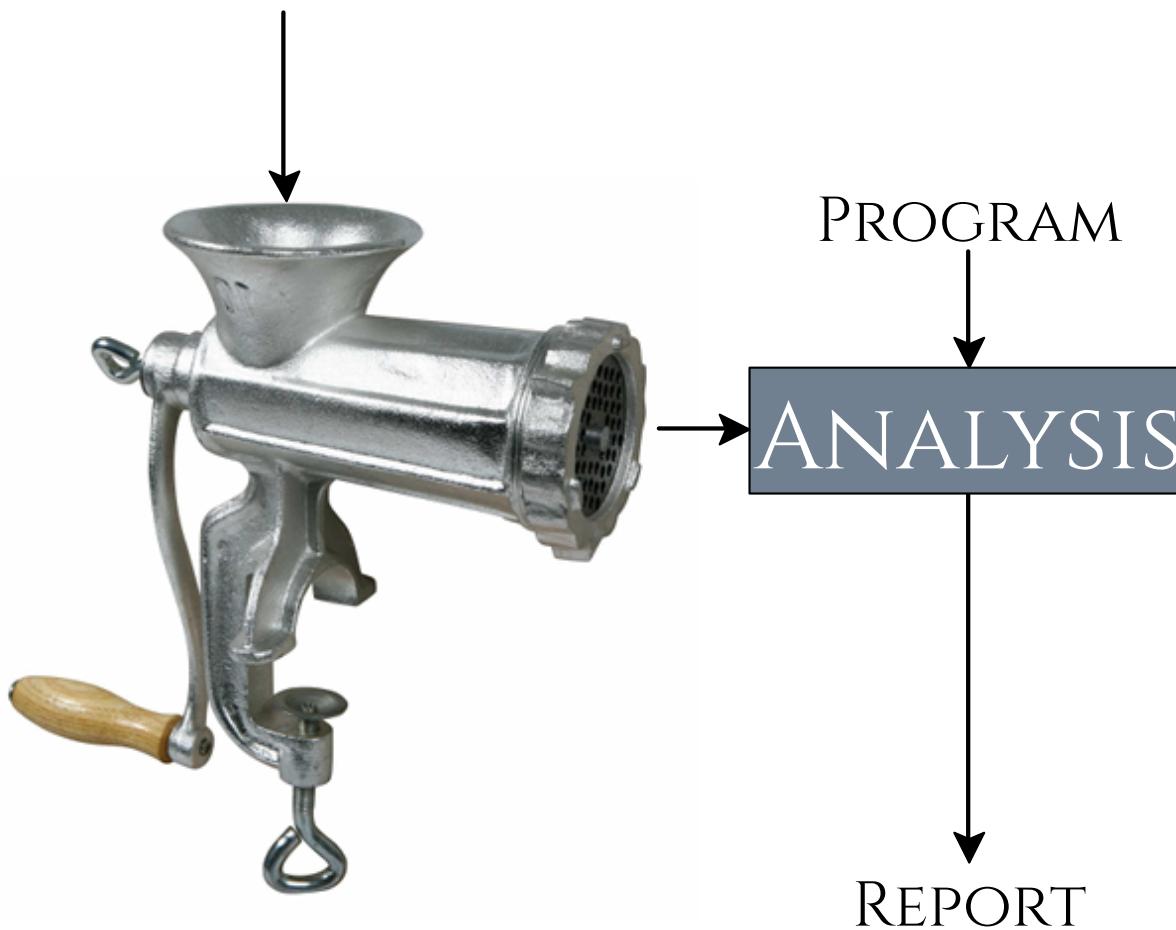


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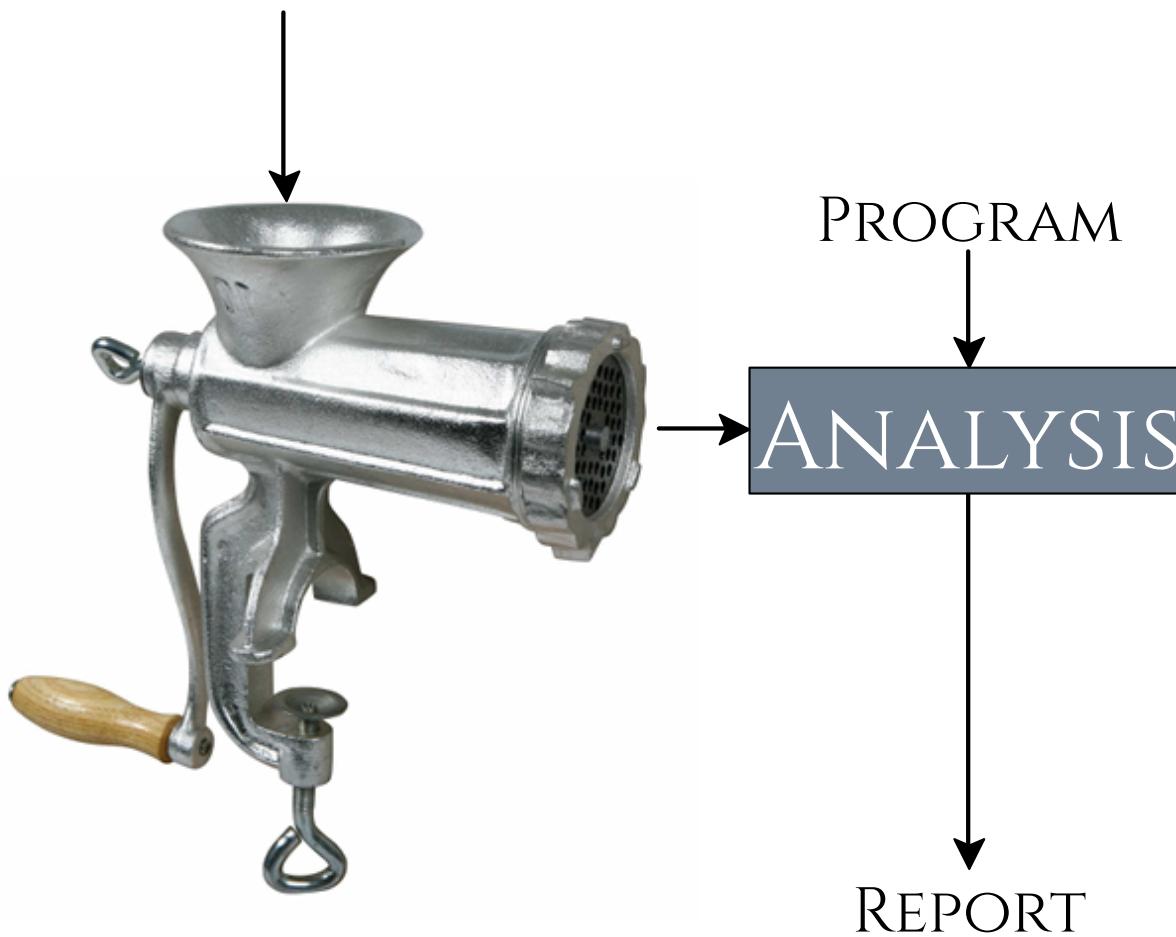


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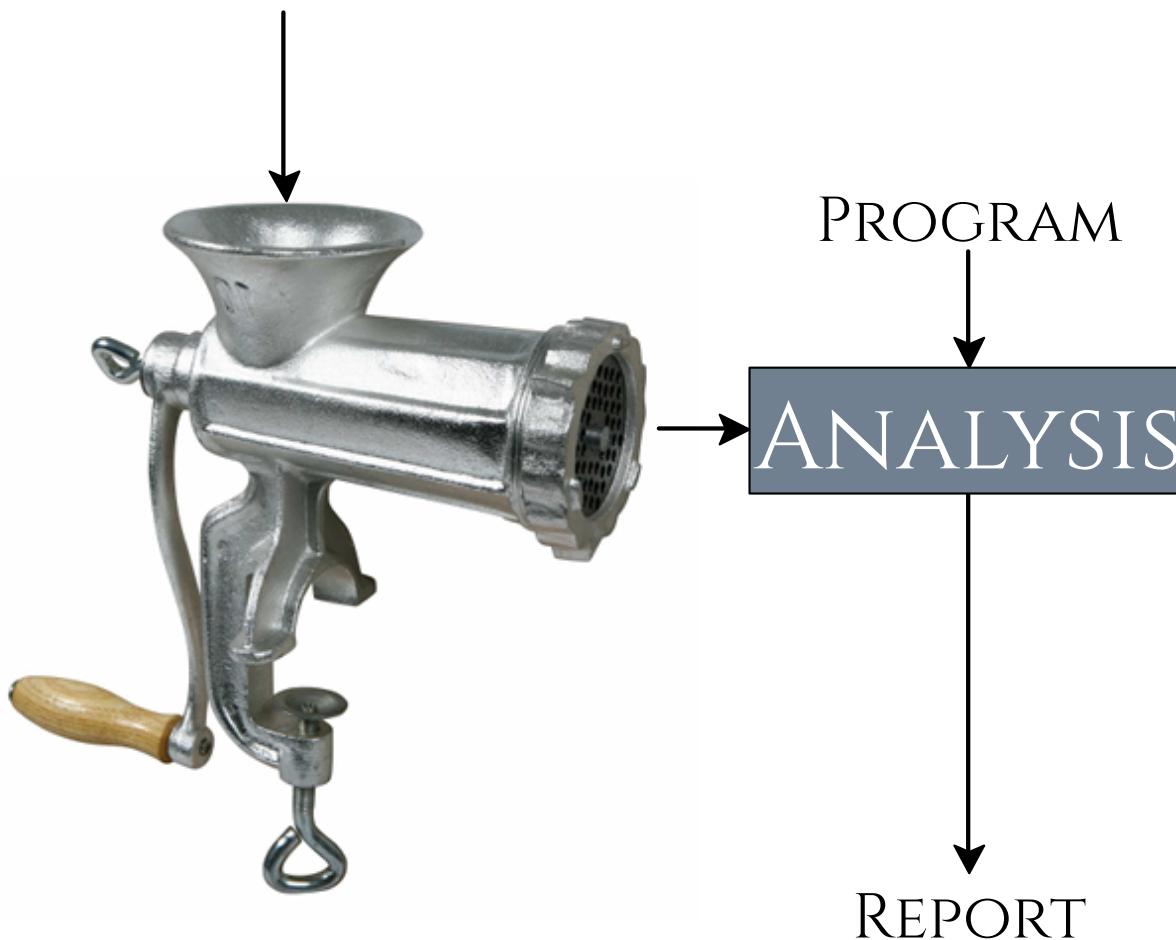


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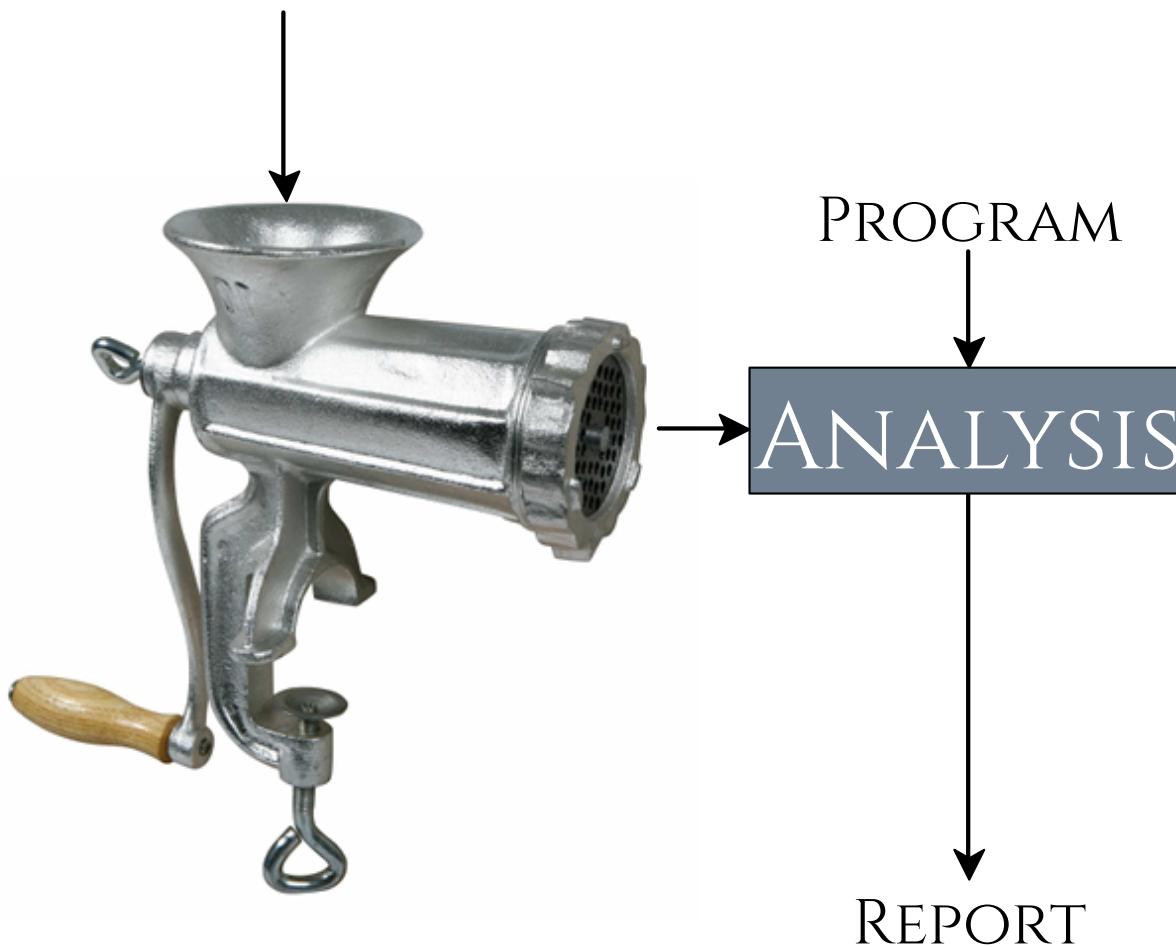


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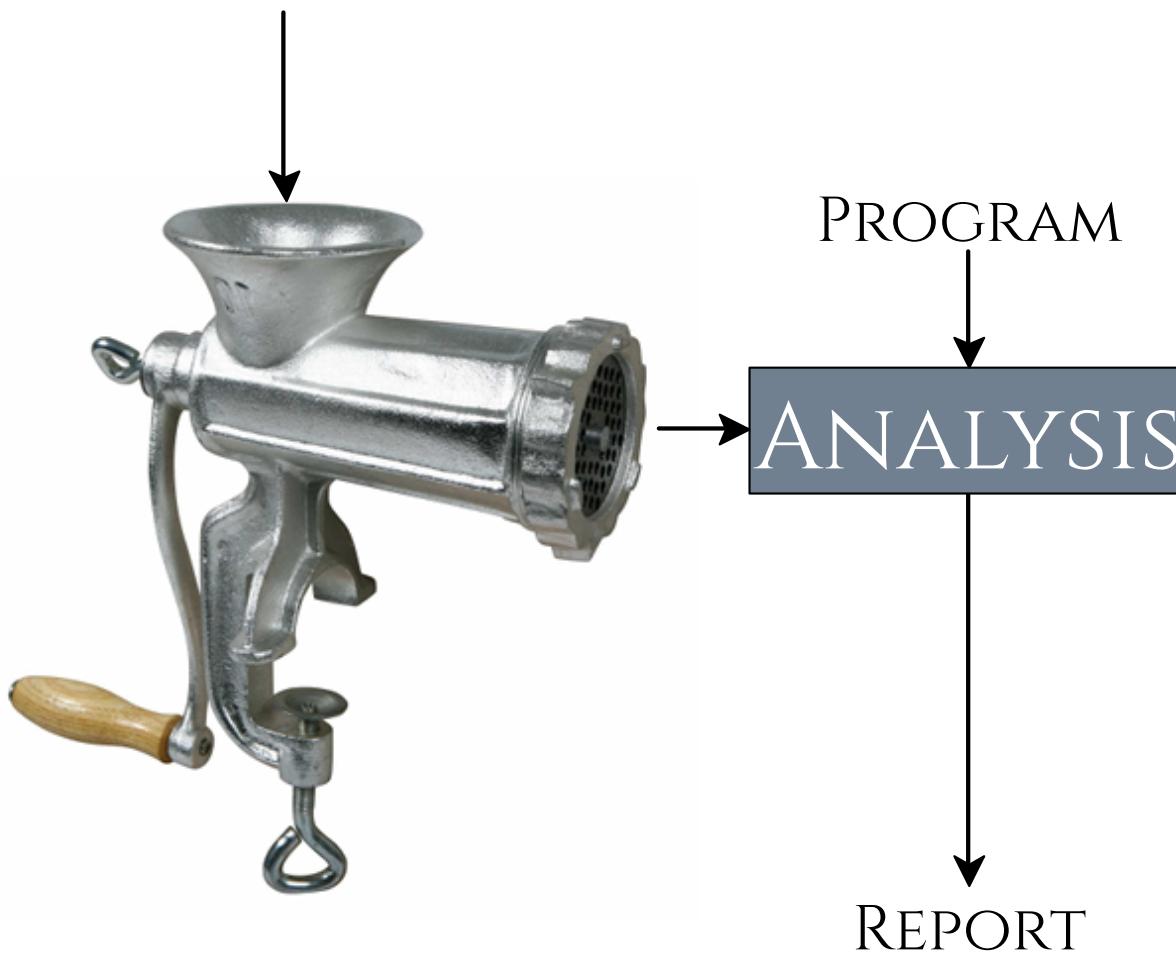


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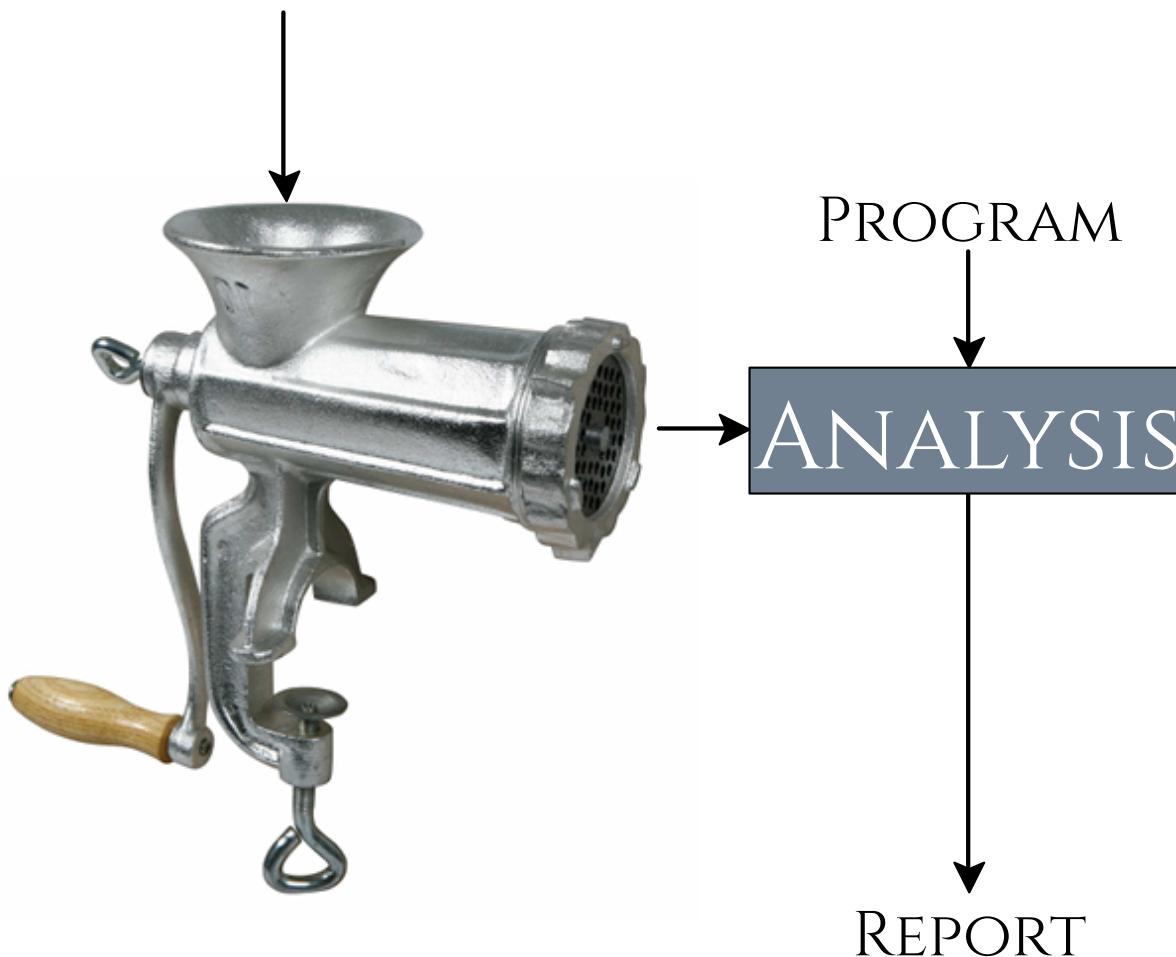


PROGRAM

ANALYSIS

REPORT

SEMANTICS

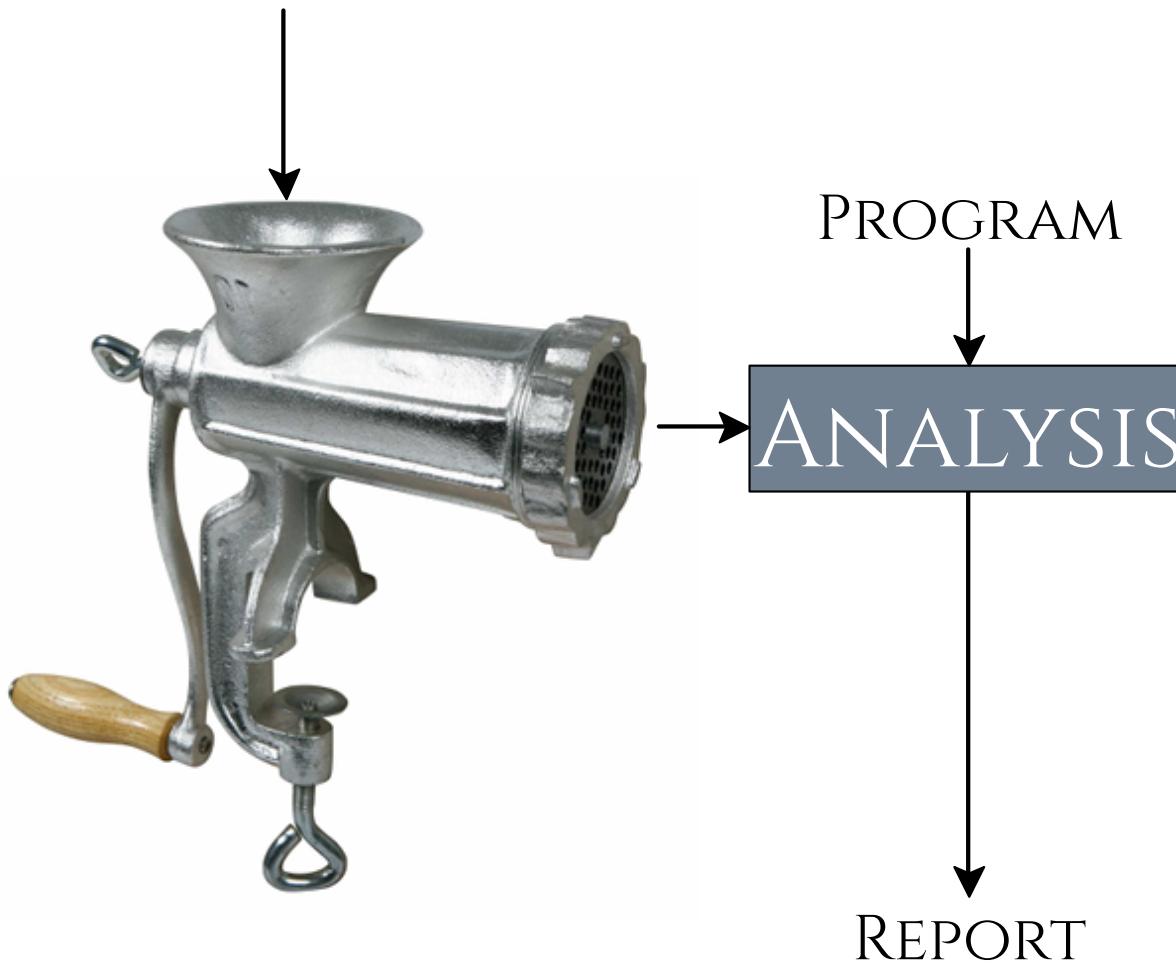


PROGRAM

ANALYSIS

REPORT

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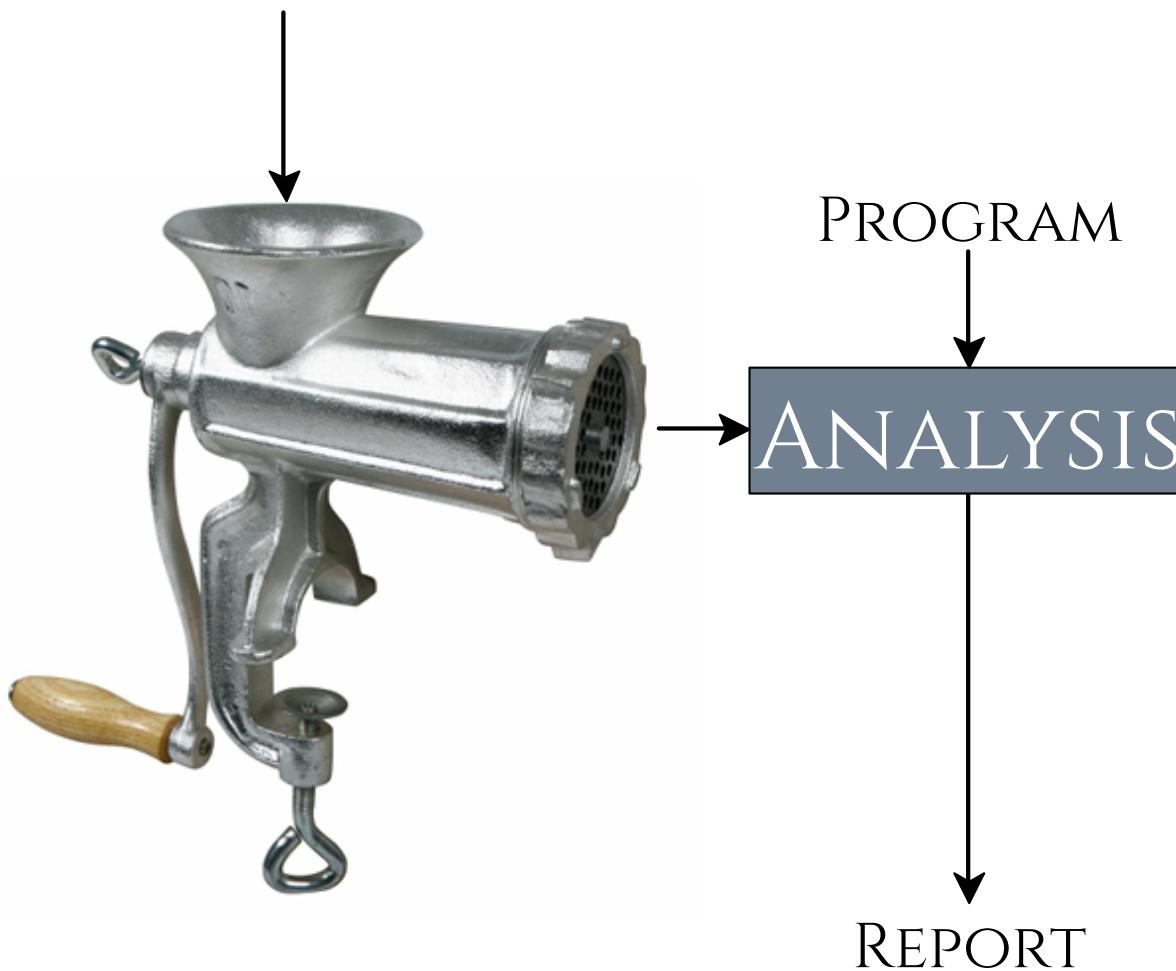


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ANALYSIS

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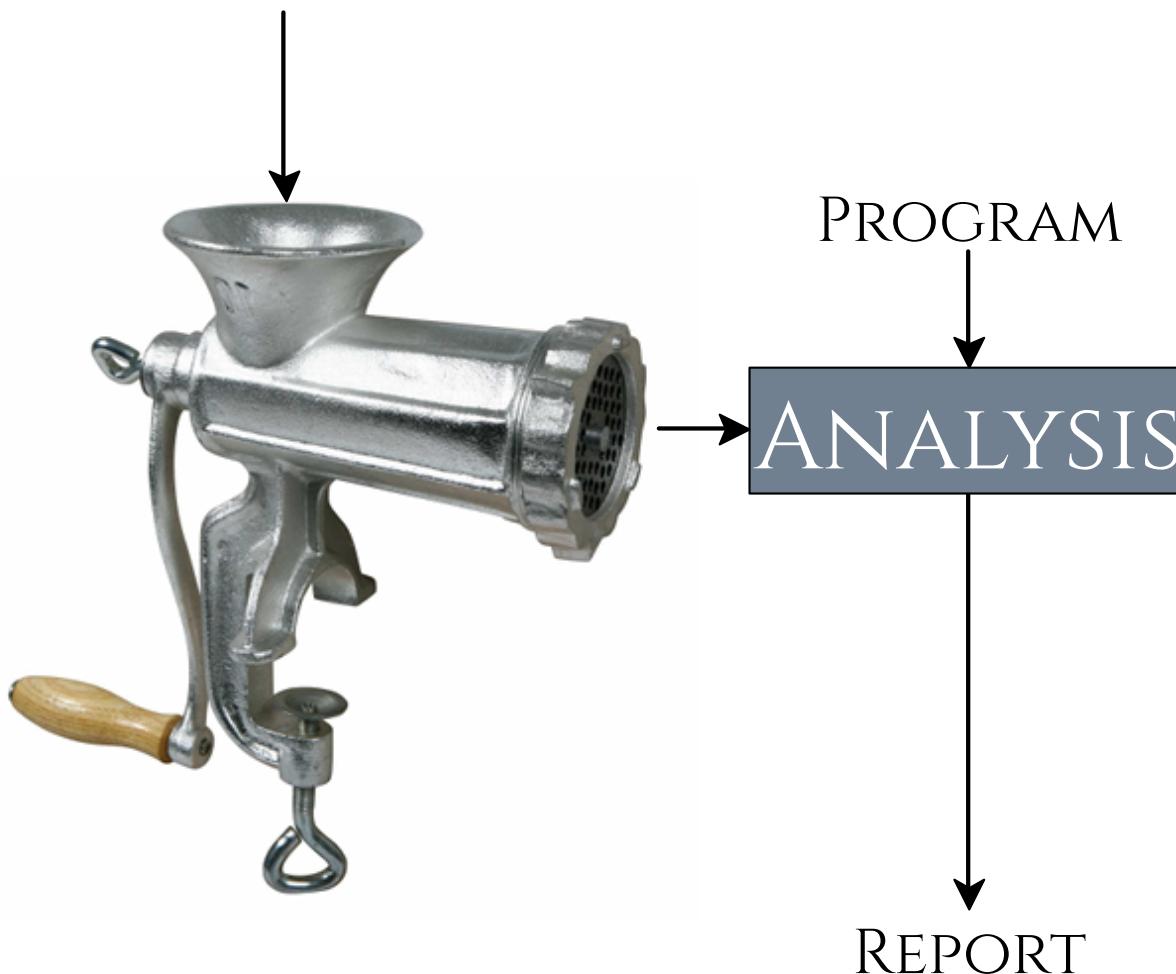


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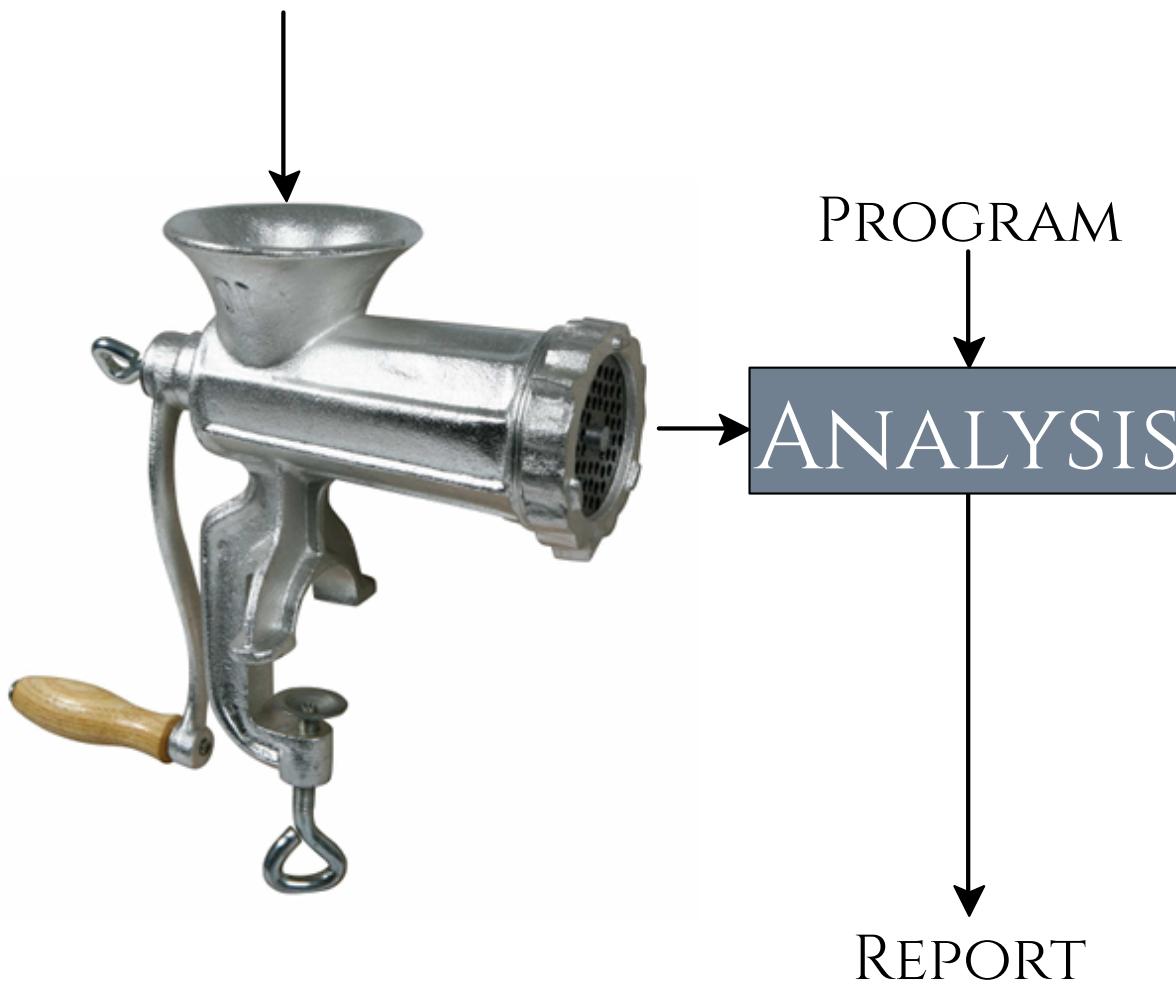


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ANALYSIS

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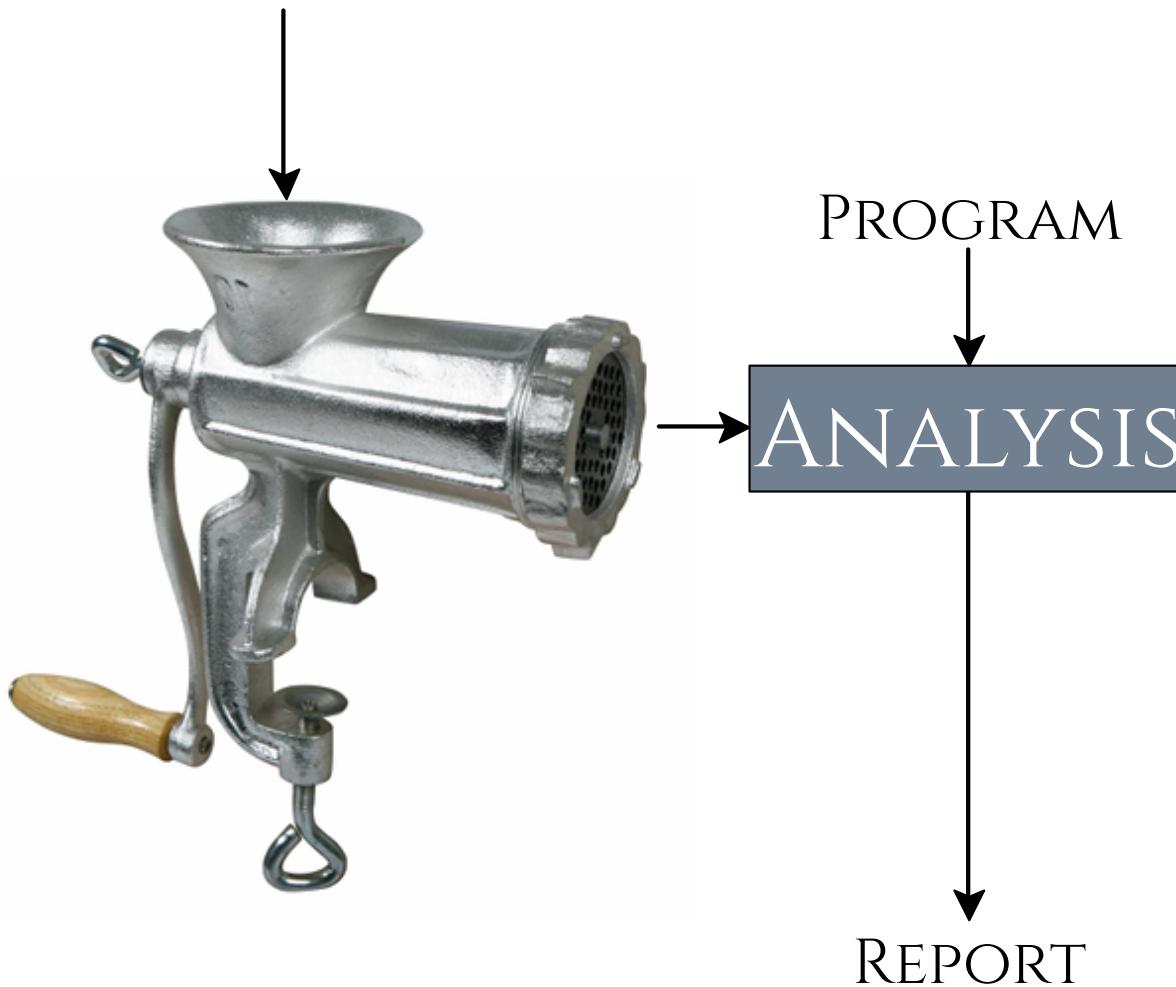


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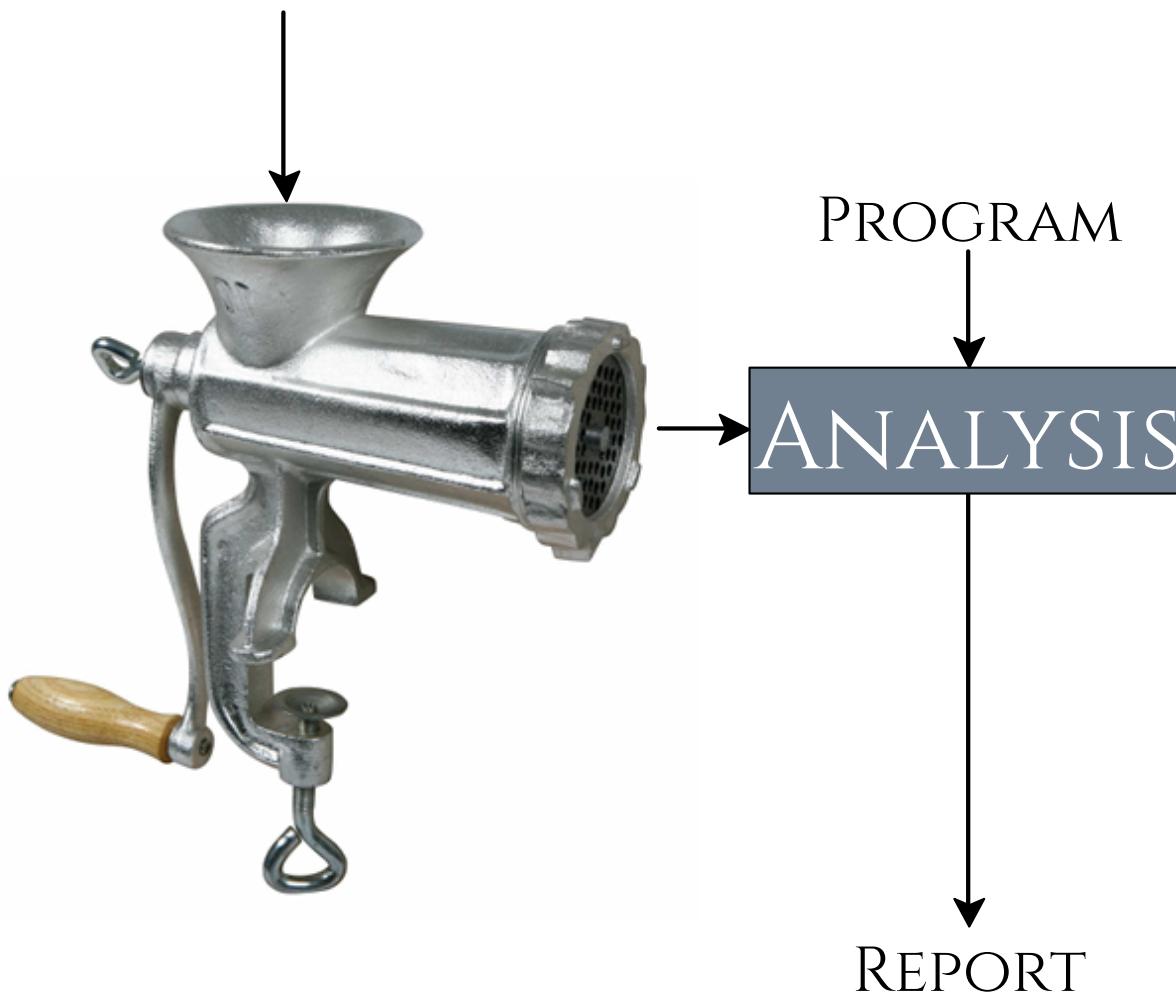
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REPORT

SEMANTICS



SEMANTICS

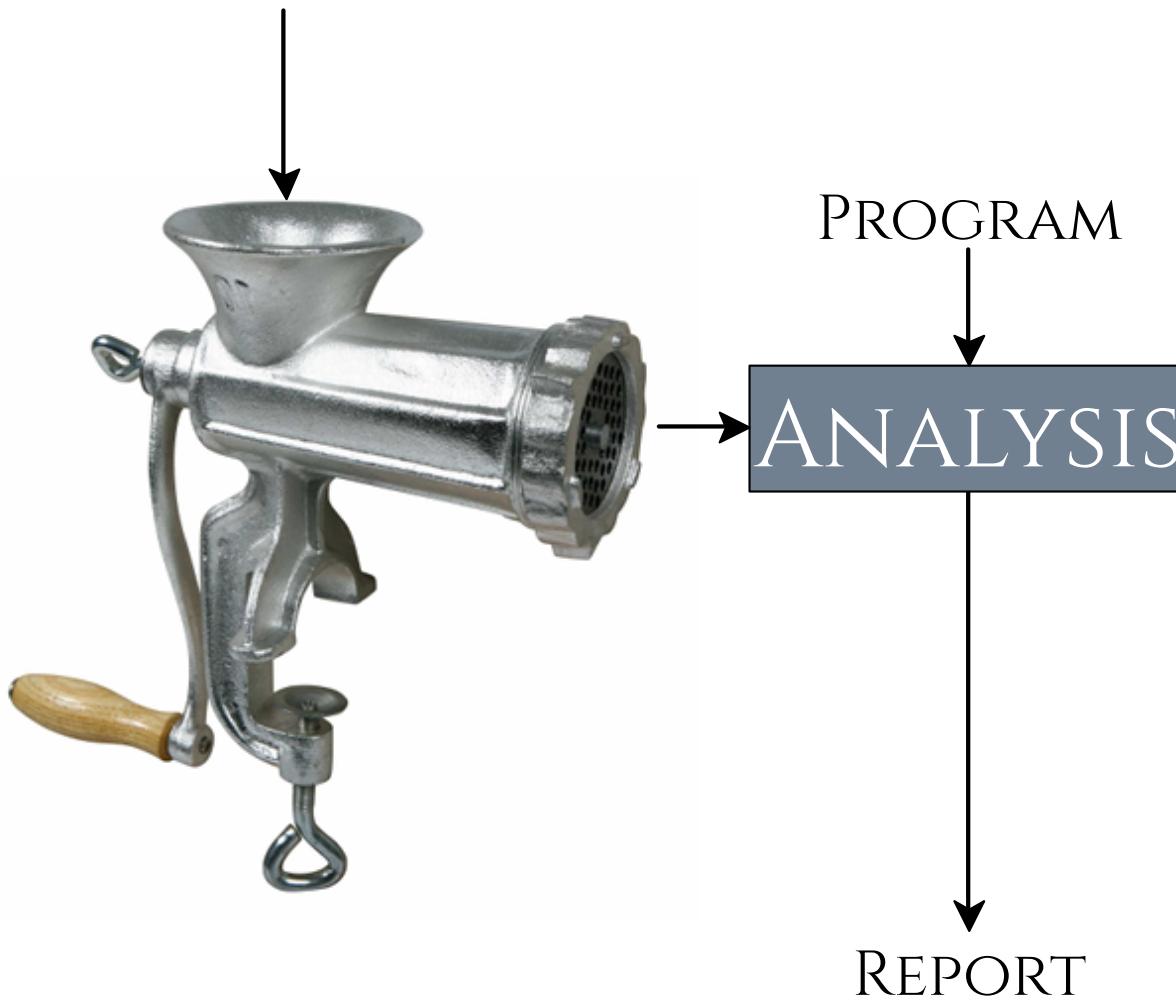


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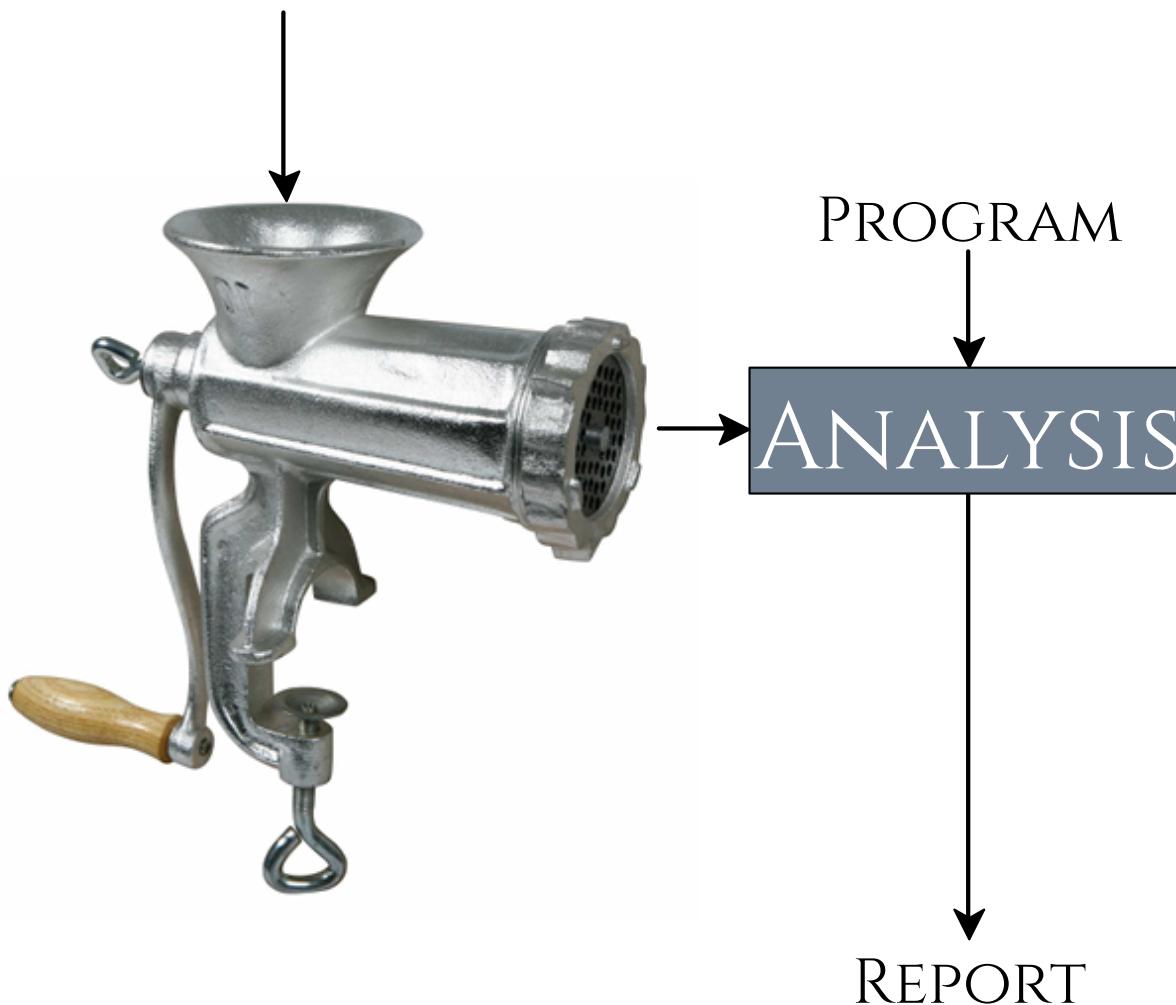
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SEMANTICS



SEMANTICS

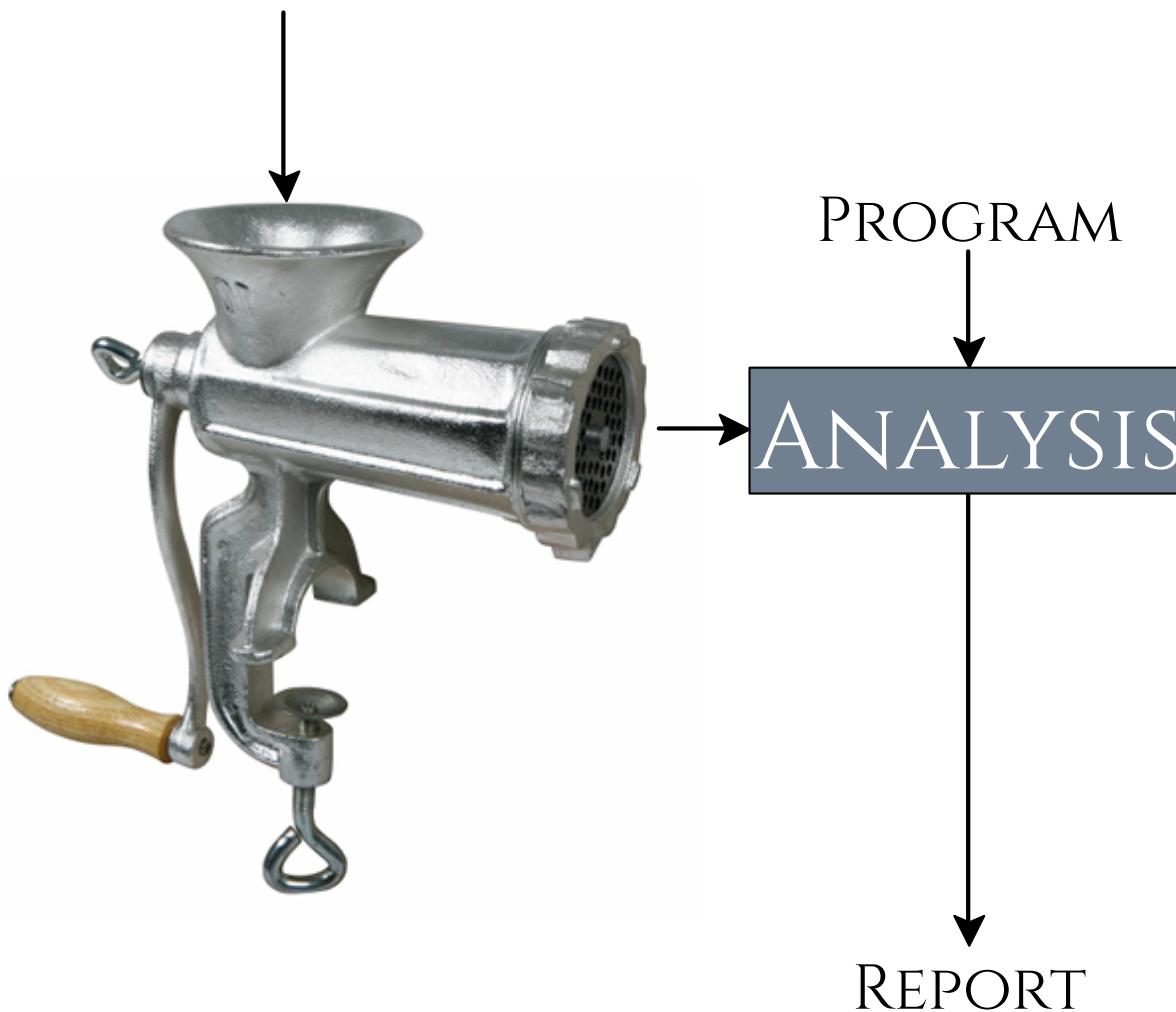


PROGRAM

ANALYSIS

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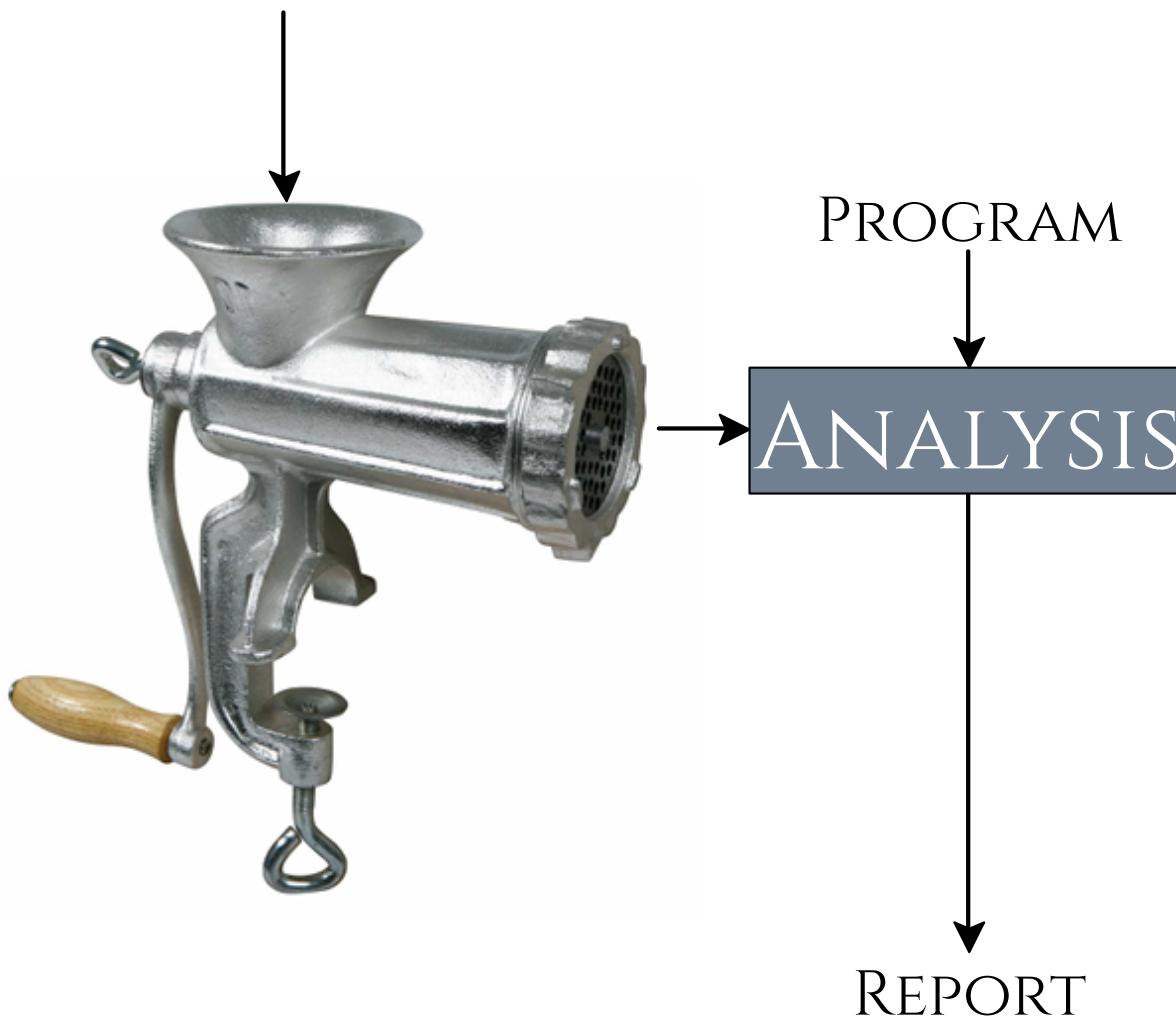


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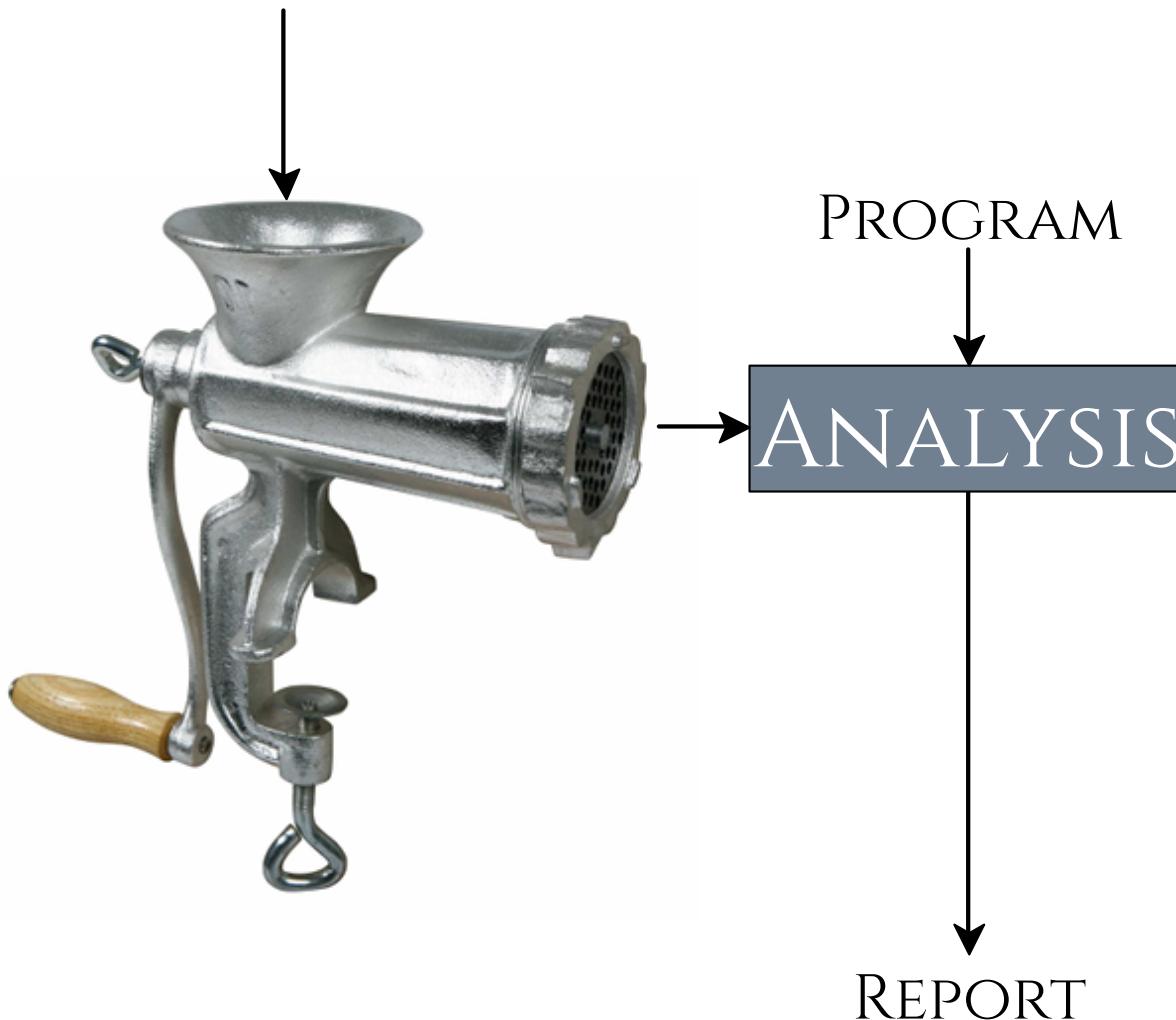


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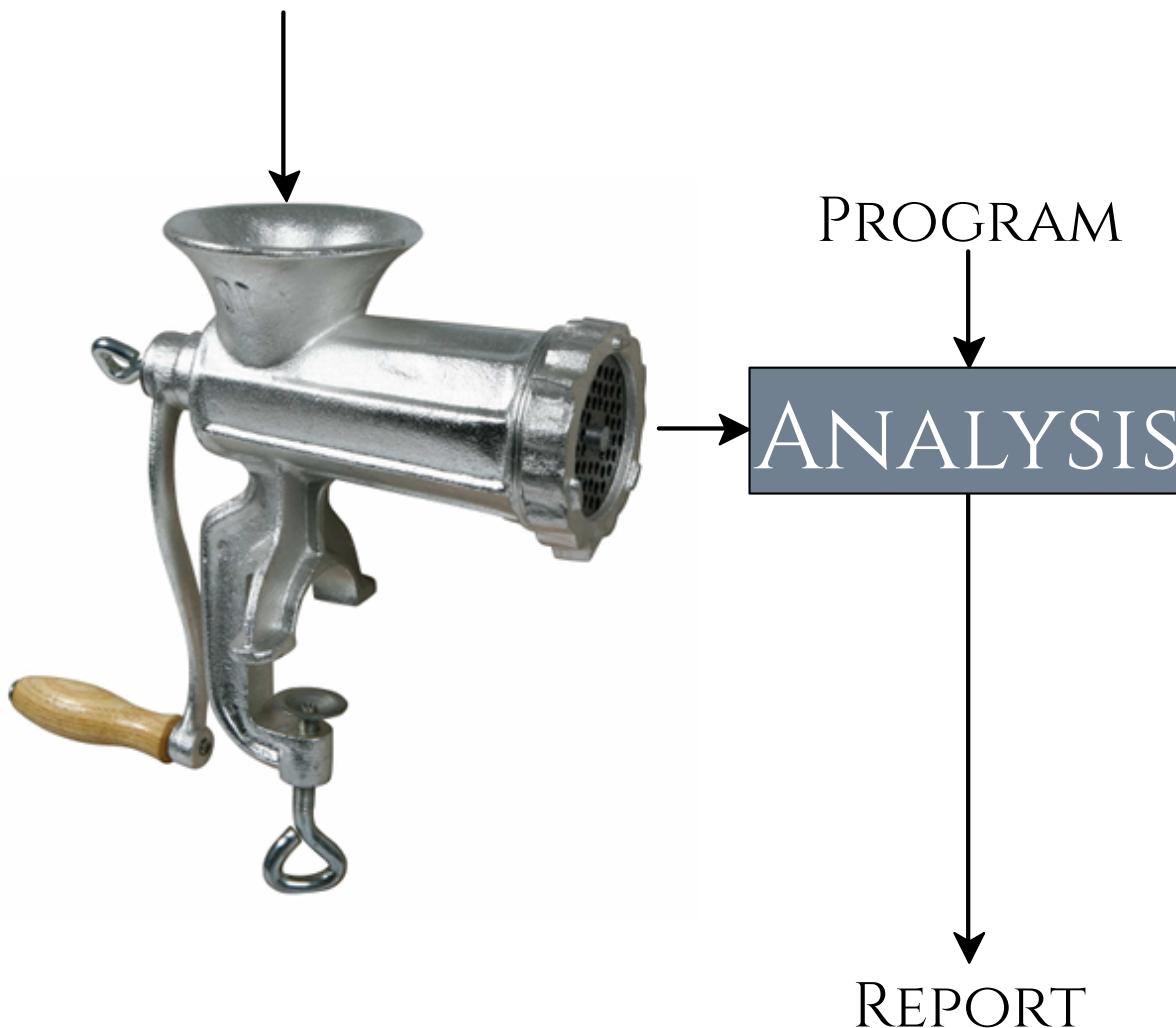
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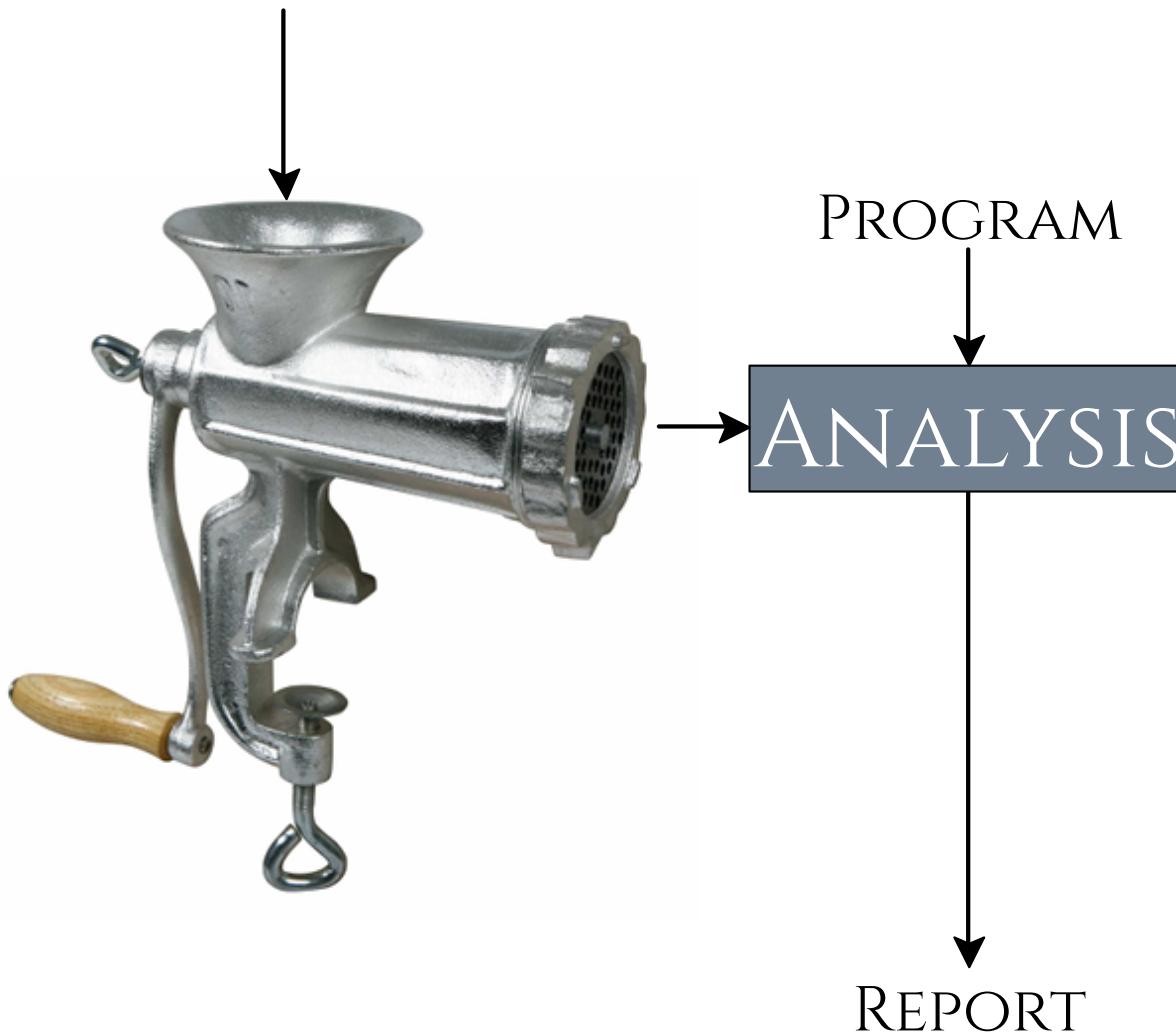


SEMANTICS



REPORT

SEMANTICS

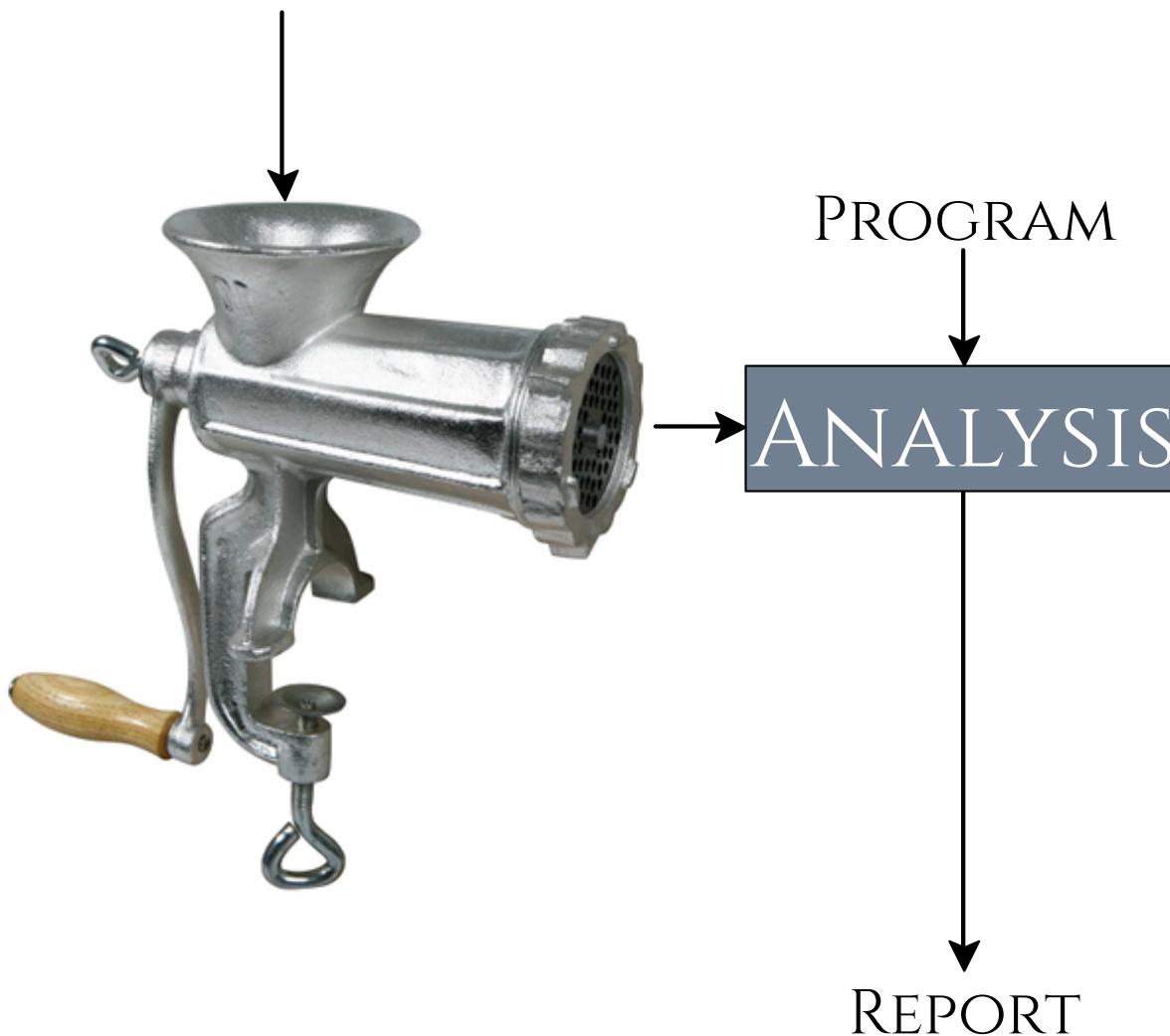


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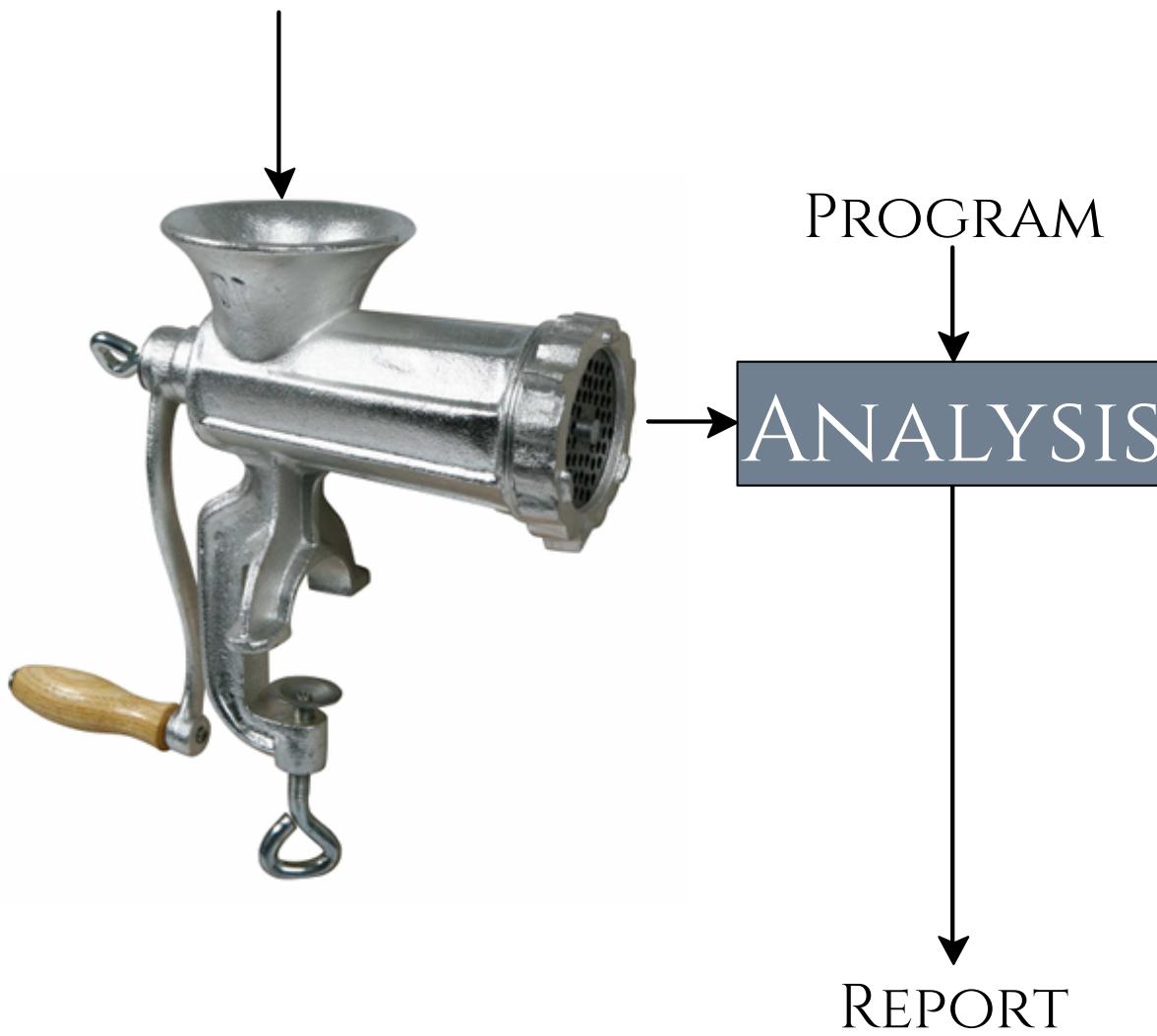
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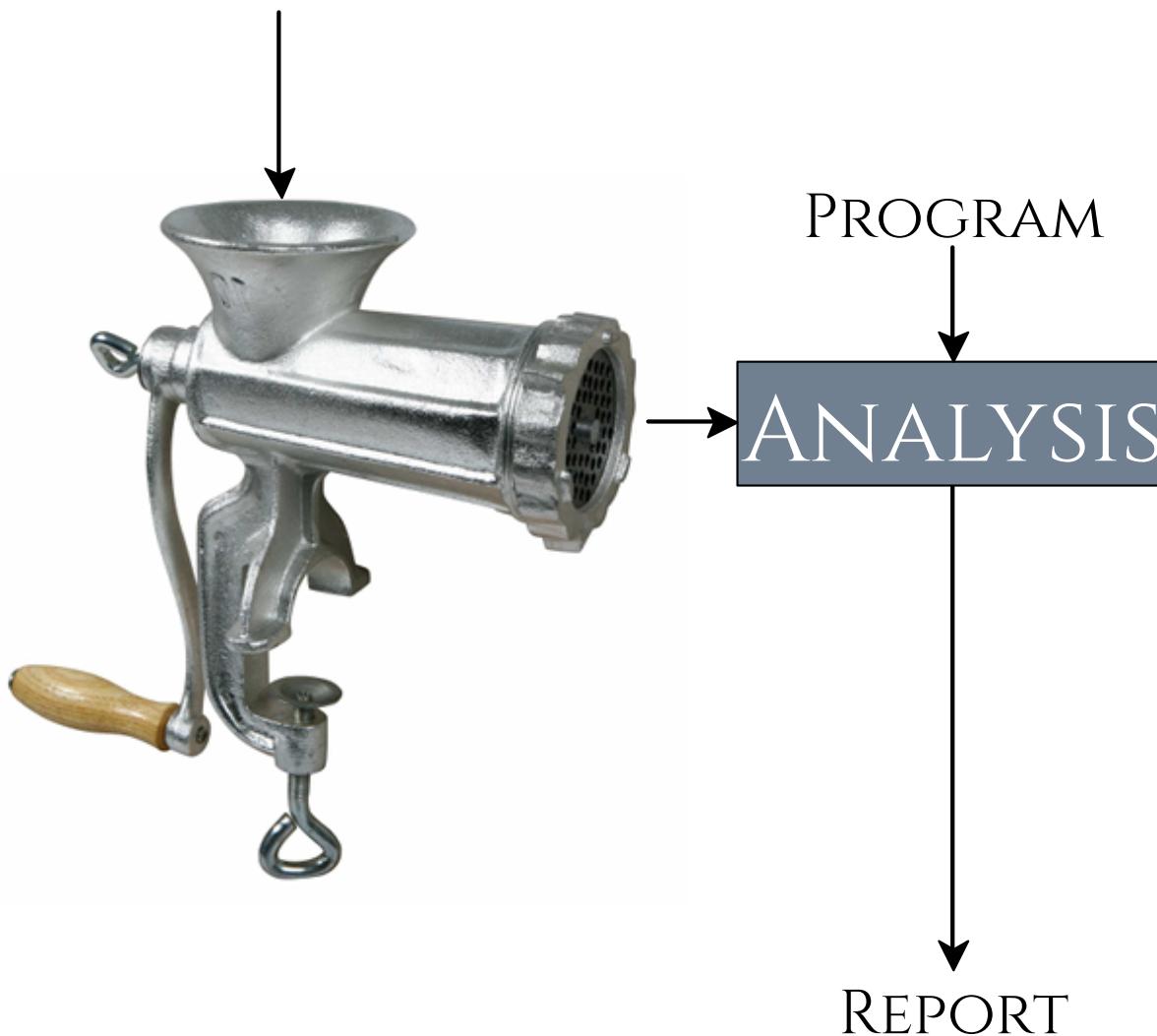


PROGRAM

ANALYSIS

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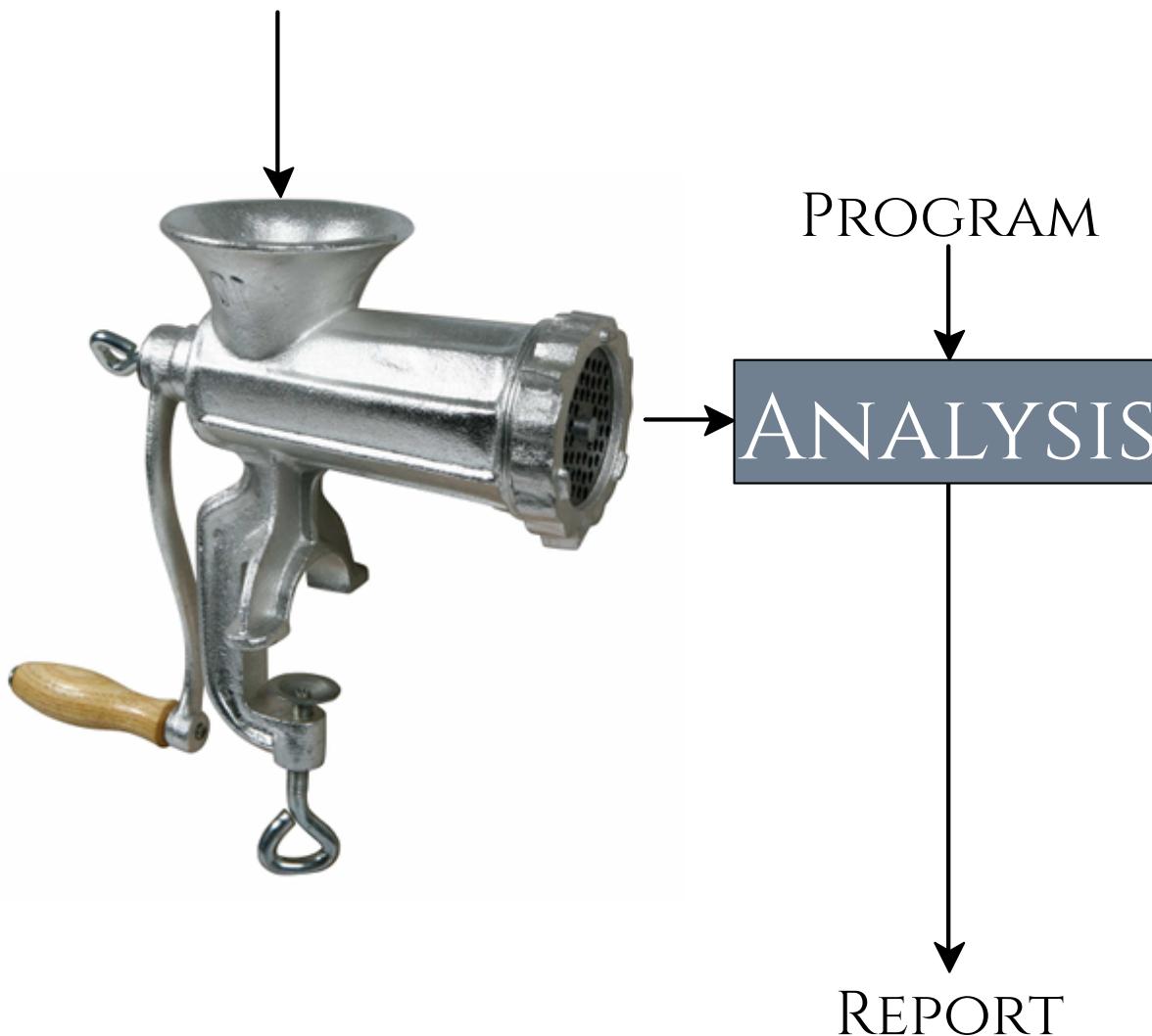


PROGRAM

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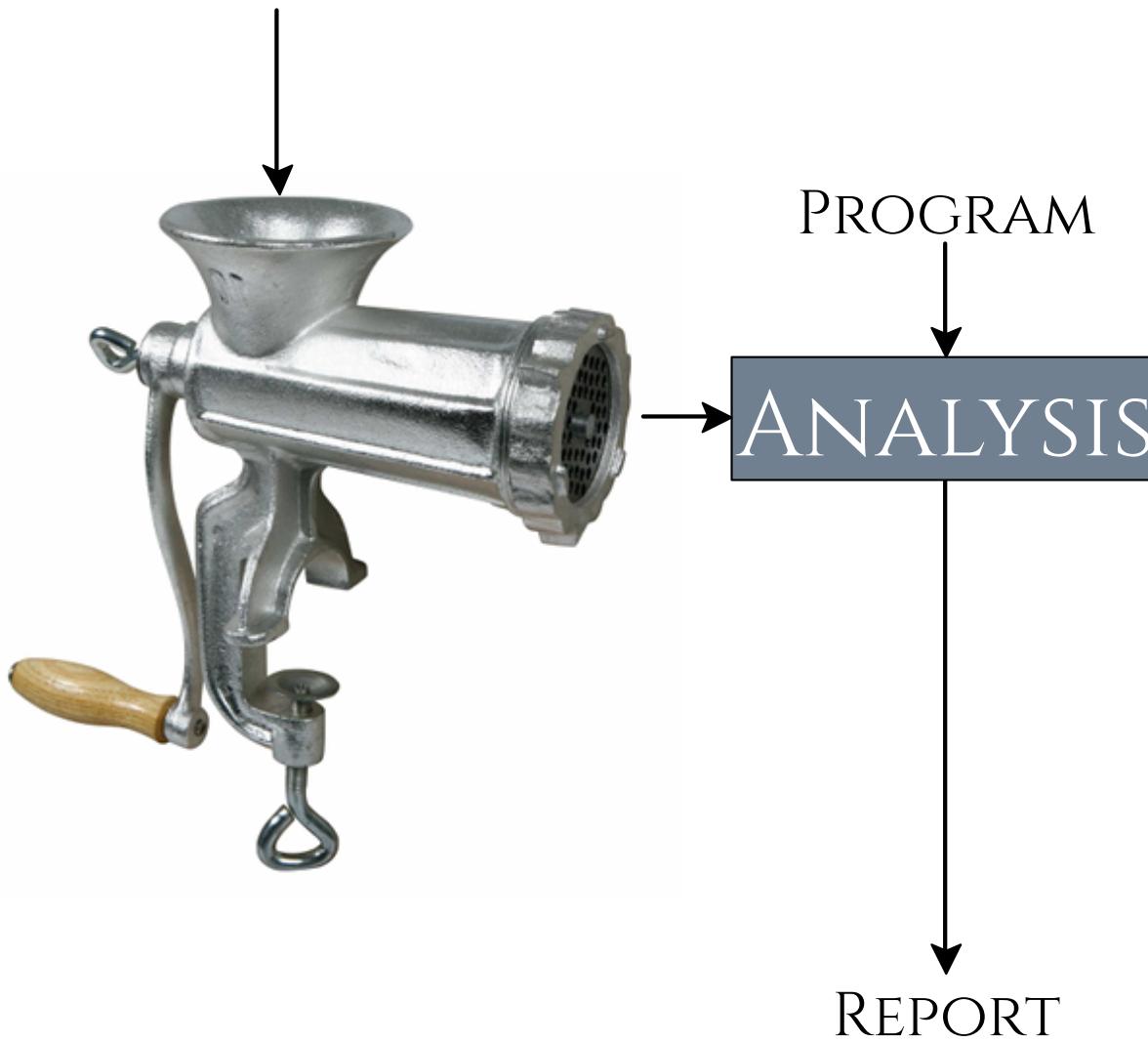
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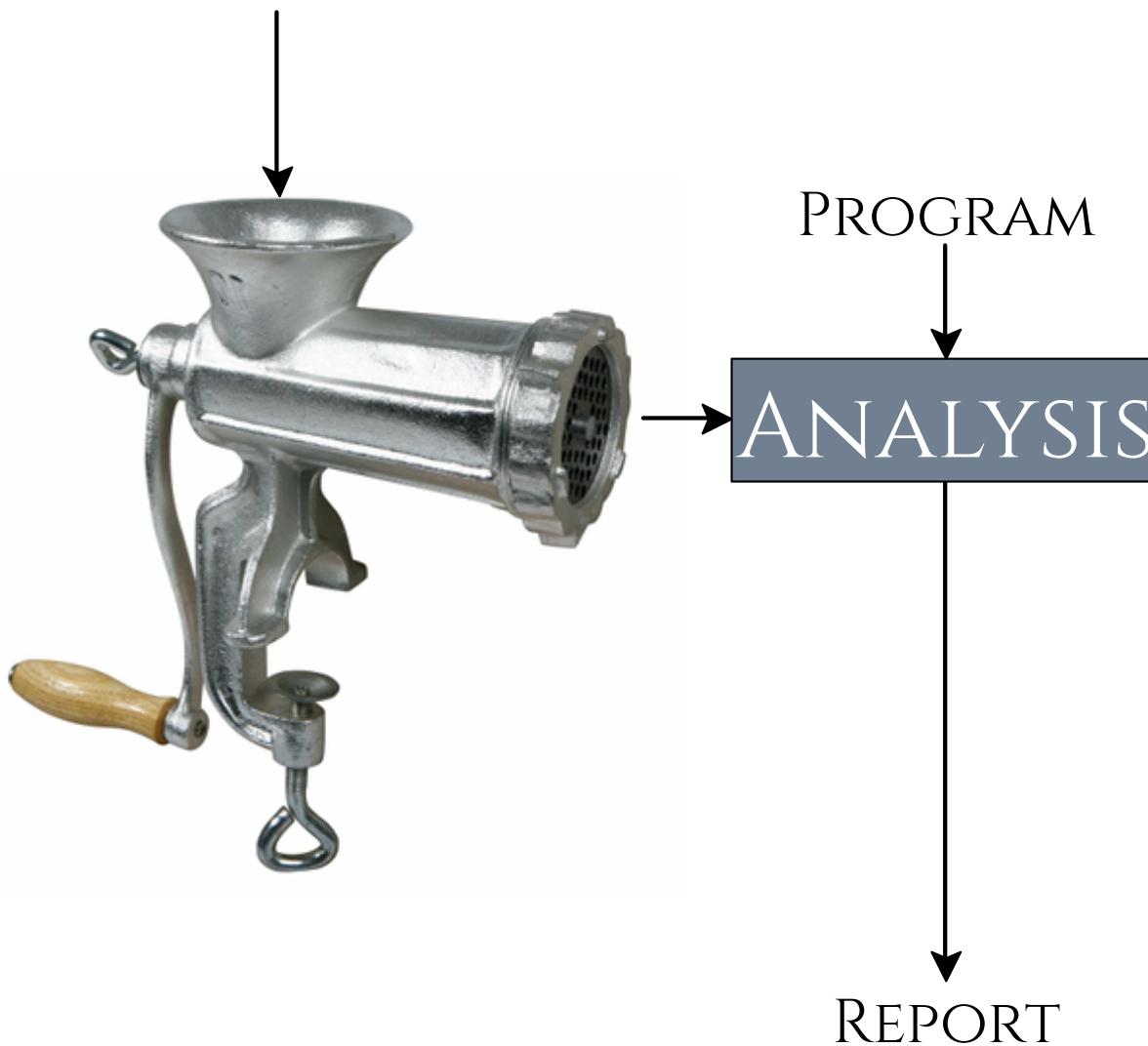
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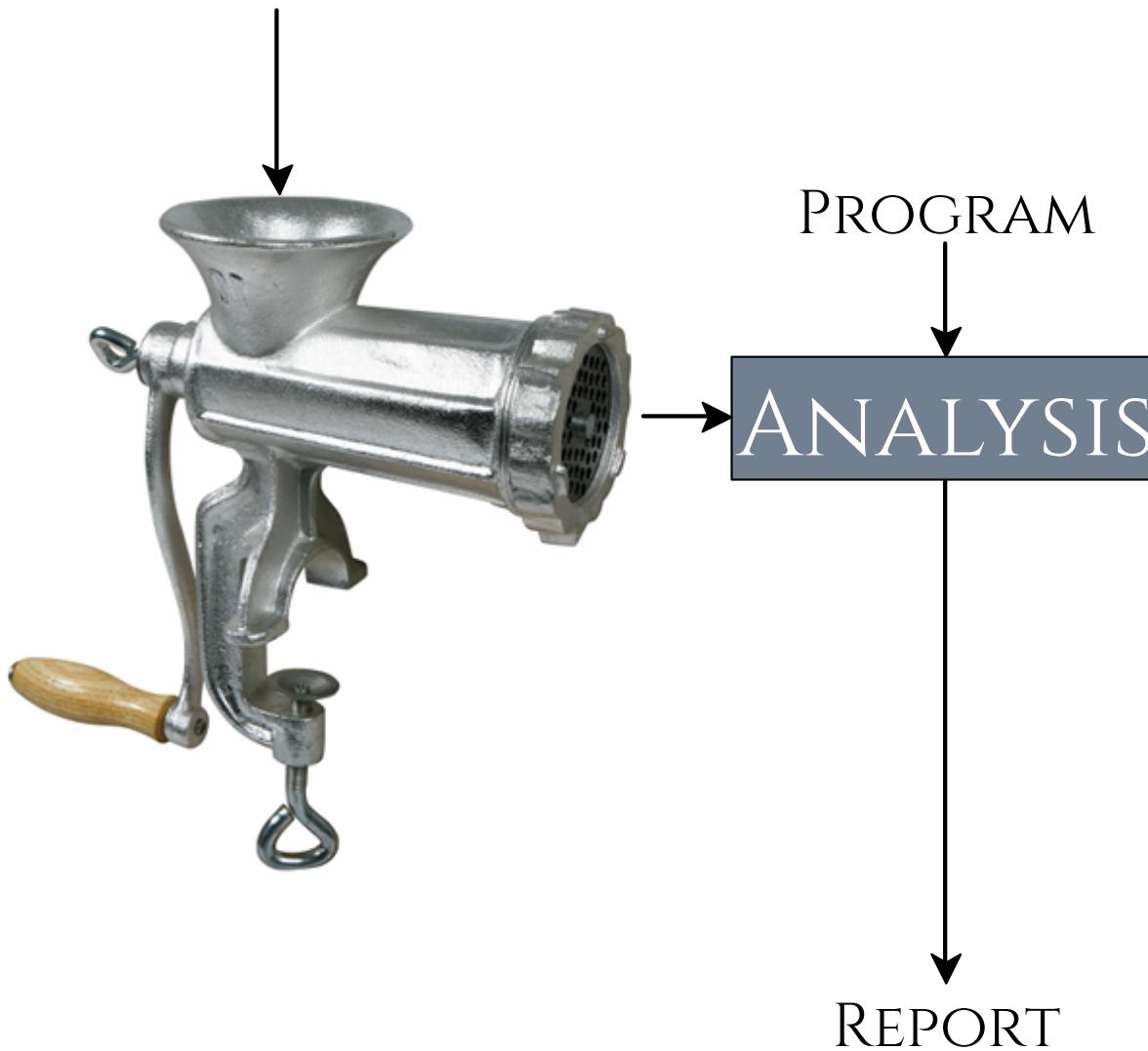


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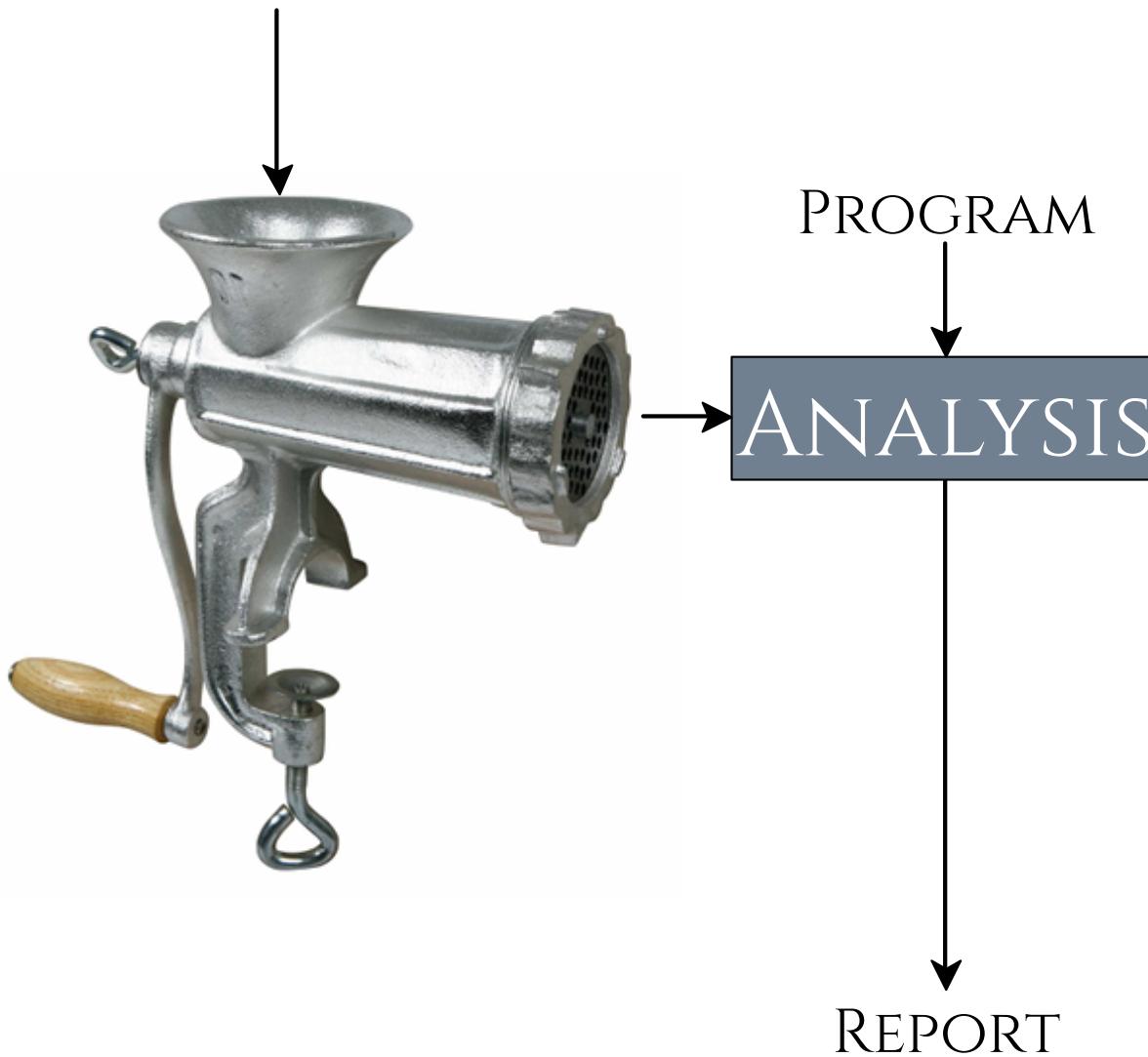
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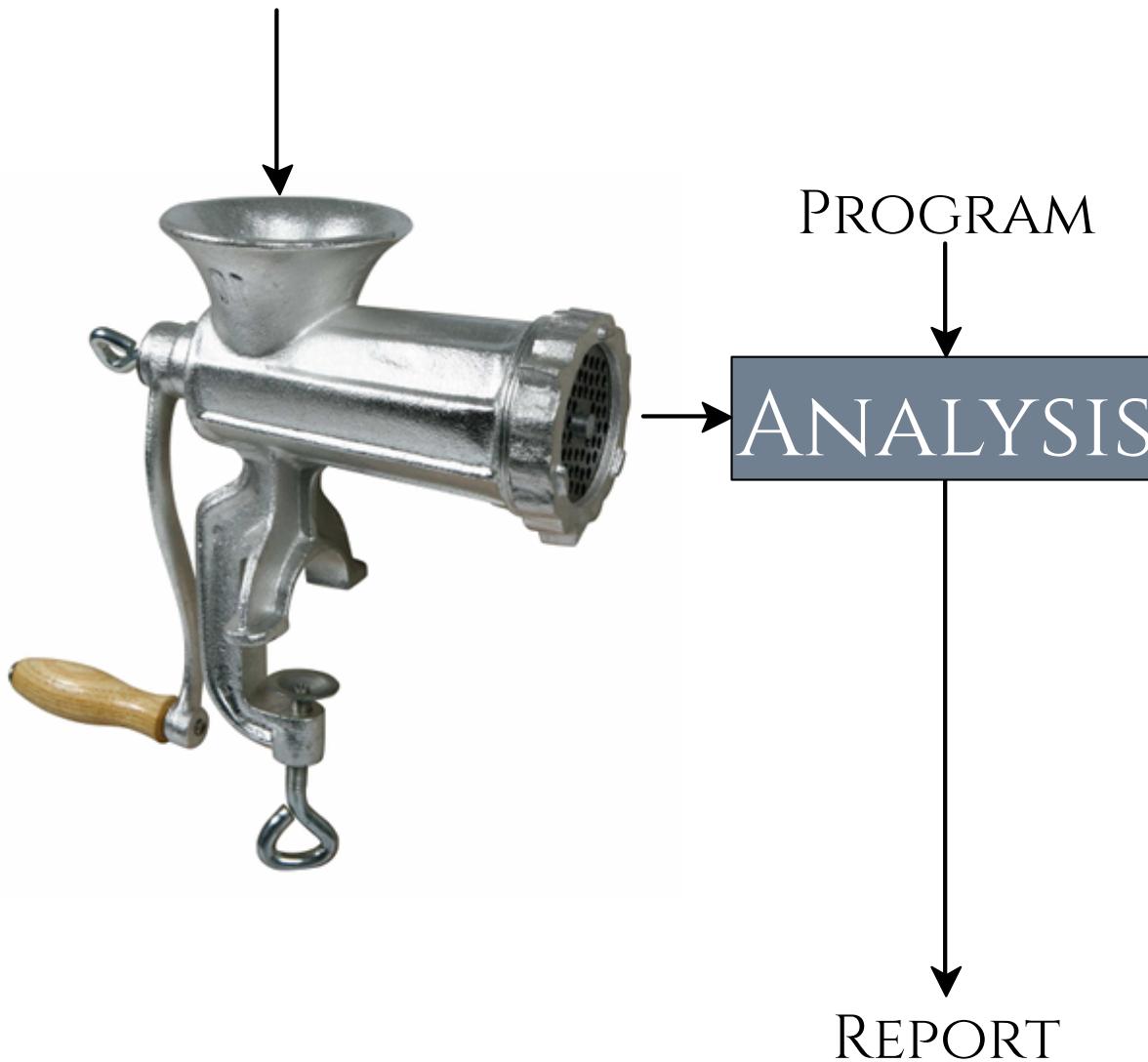
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SEMANTICS

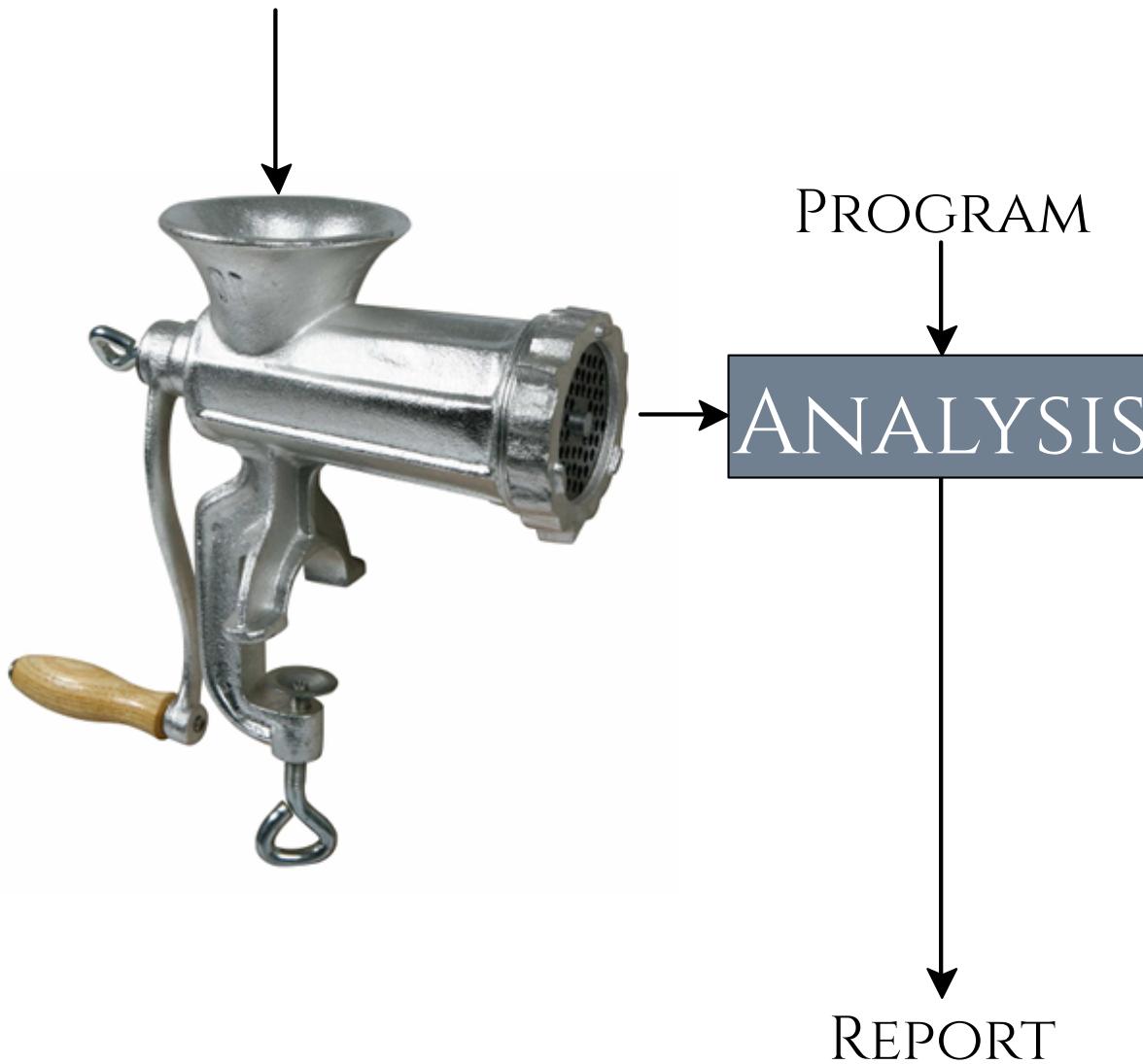


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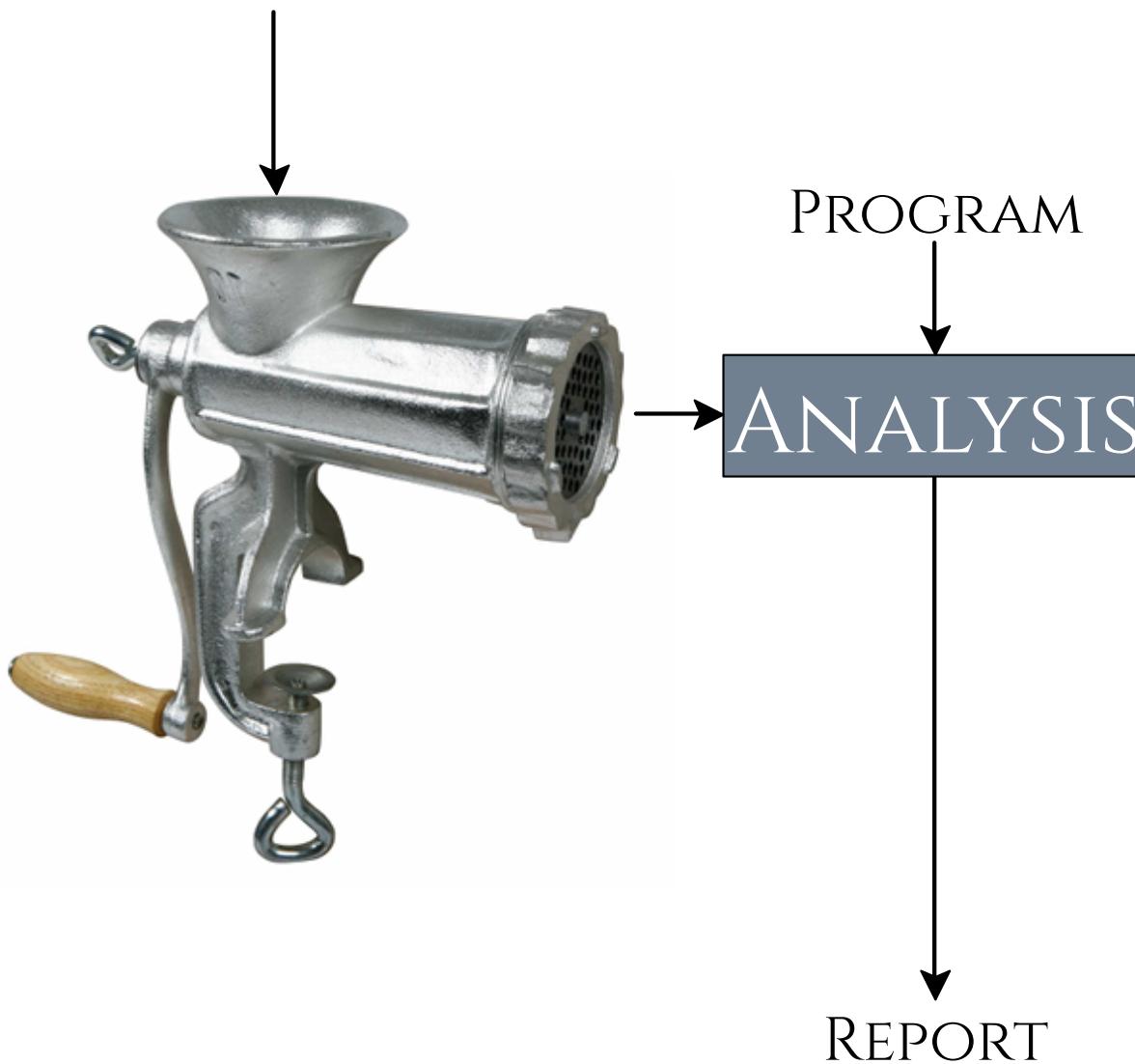


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SEMANTICS



SEMANTICS

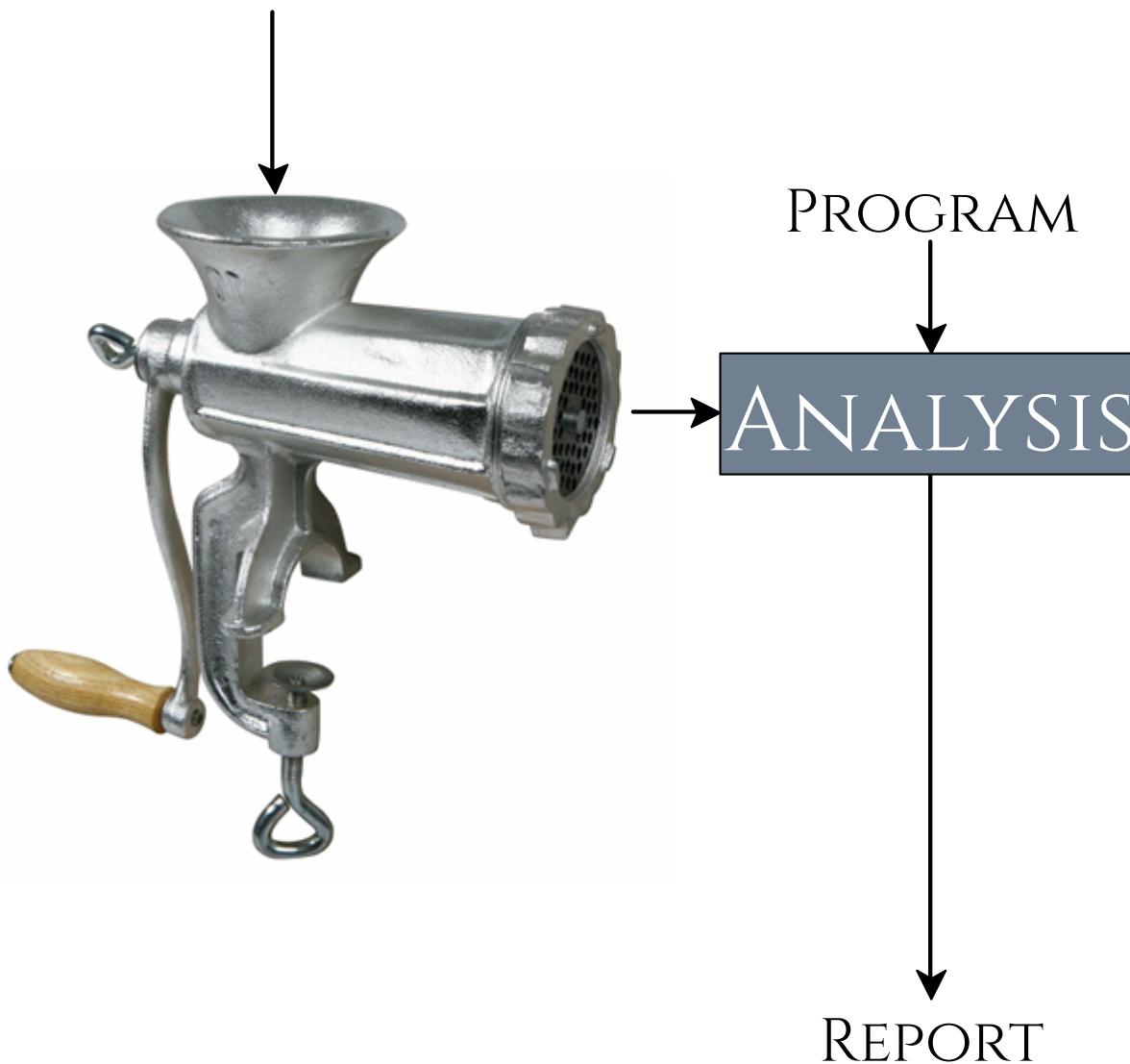


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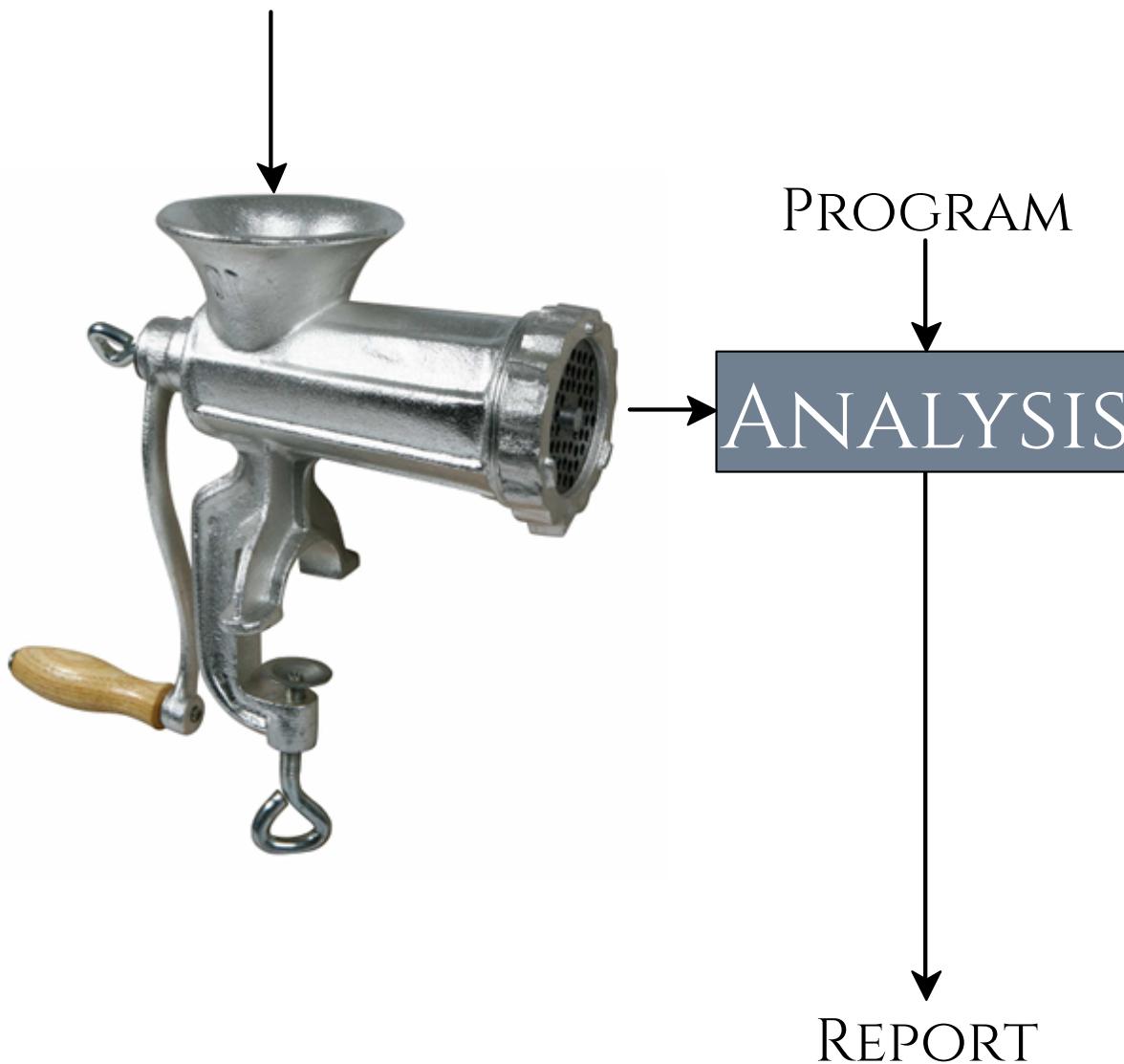


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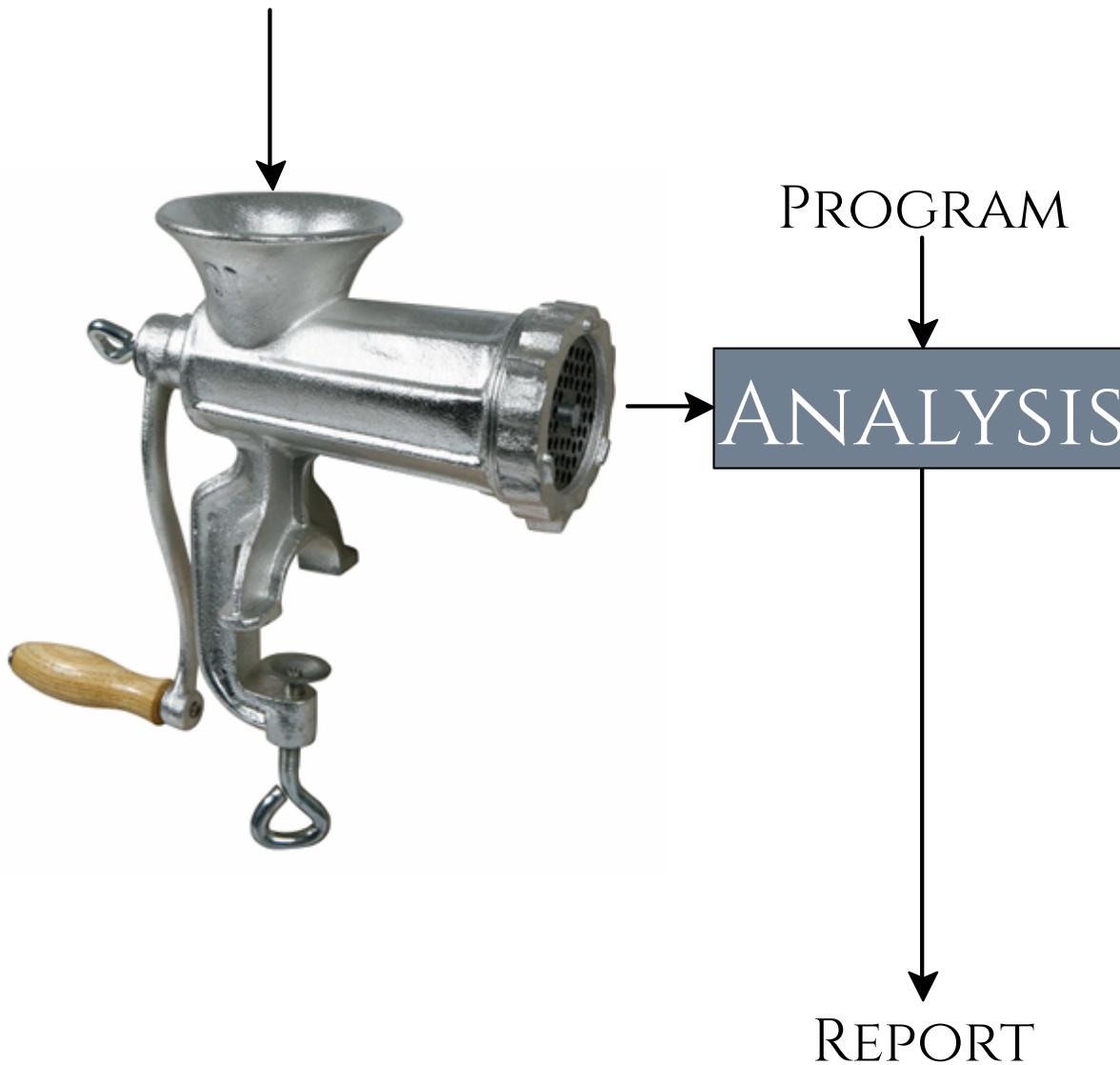


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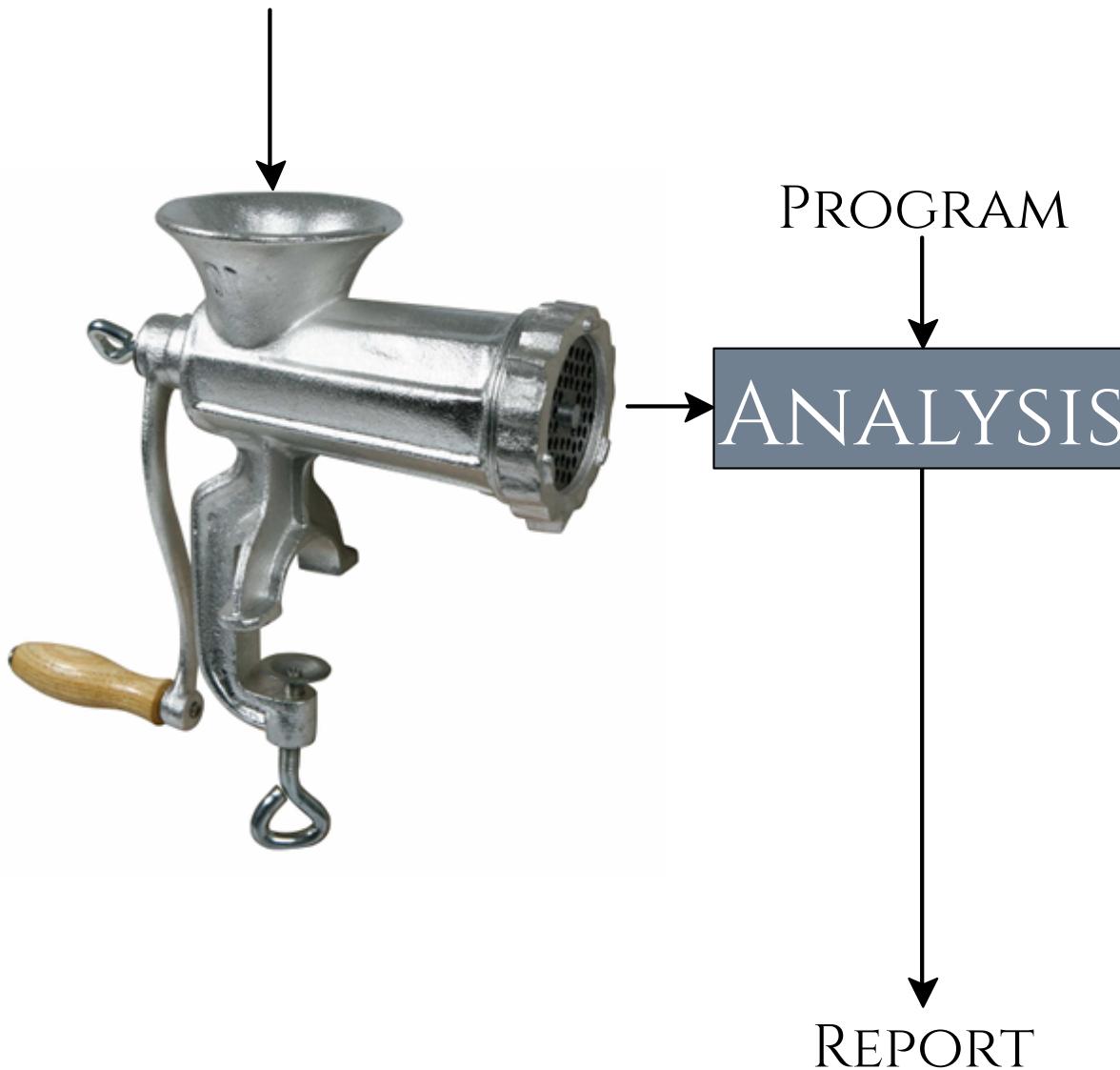


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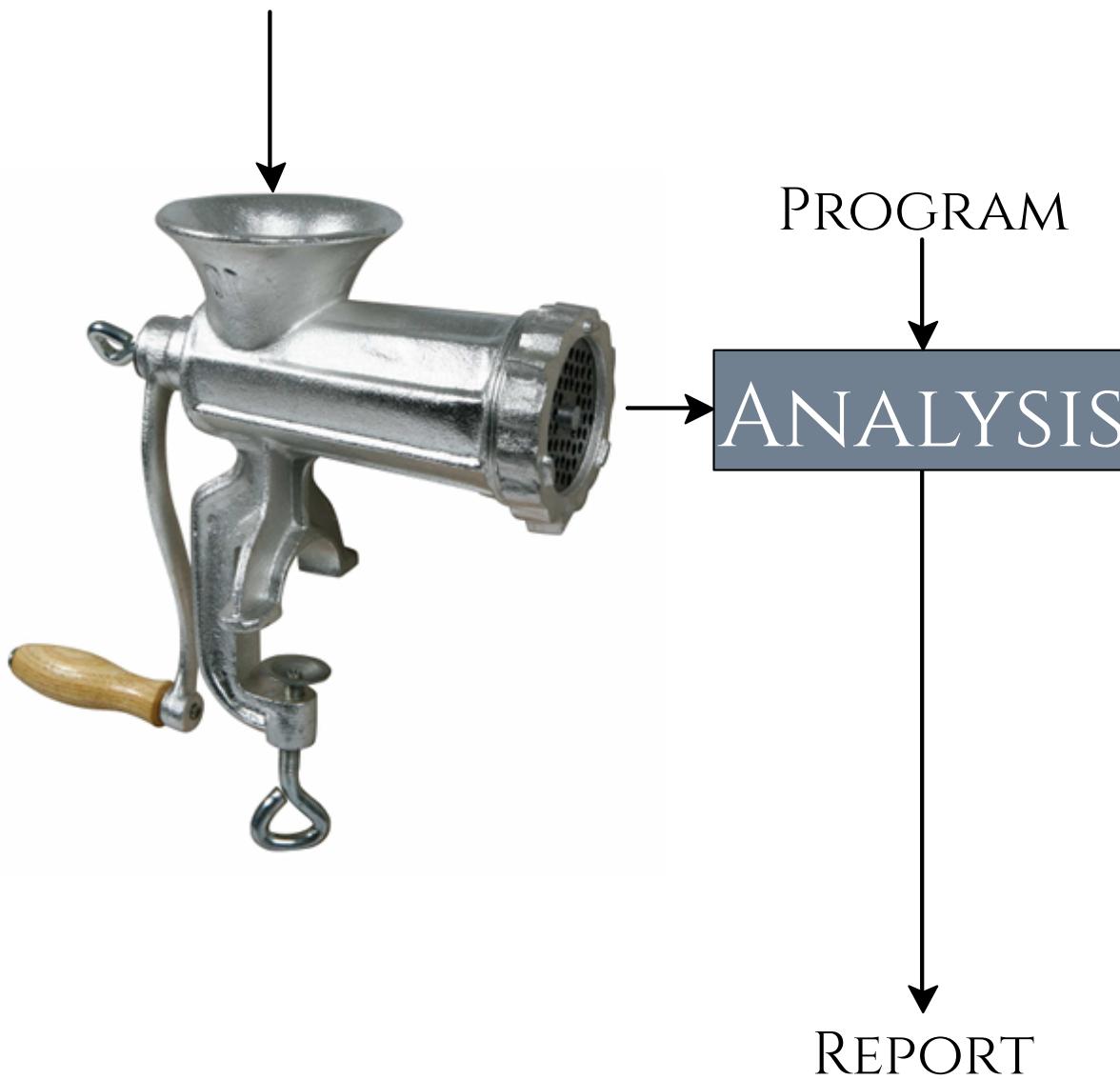


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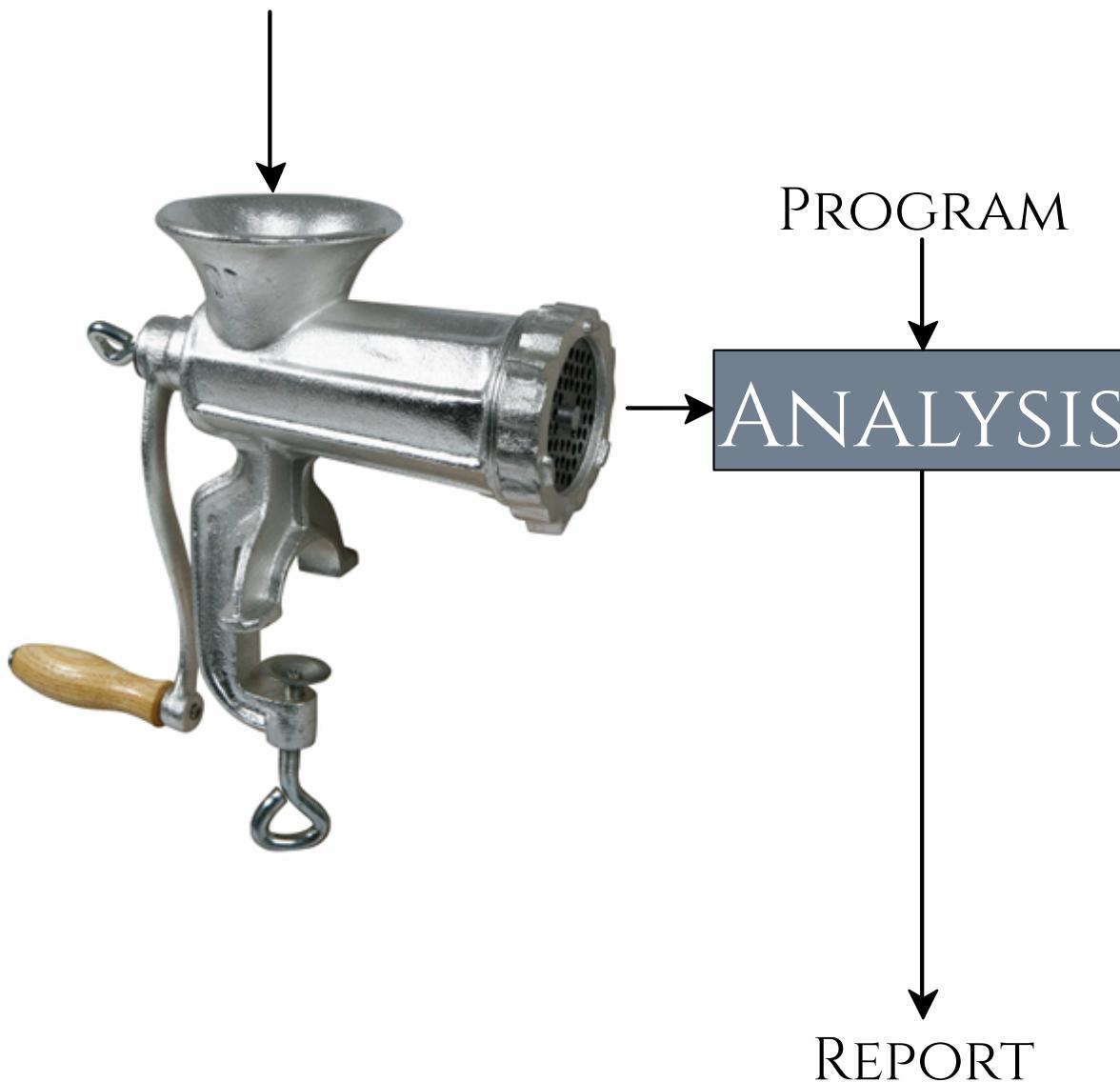
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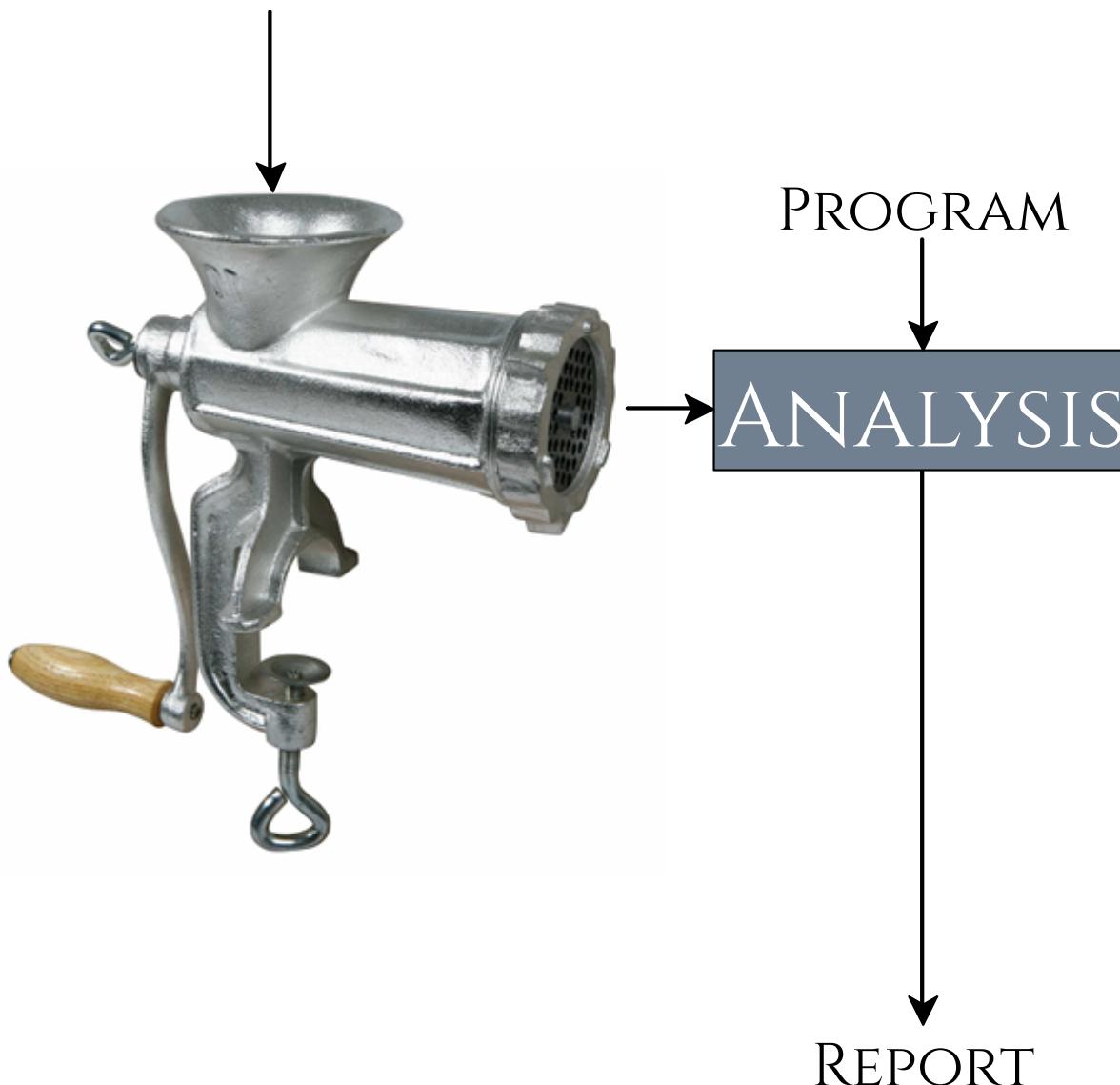
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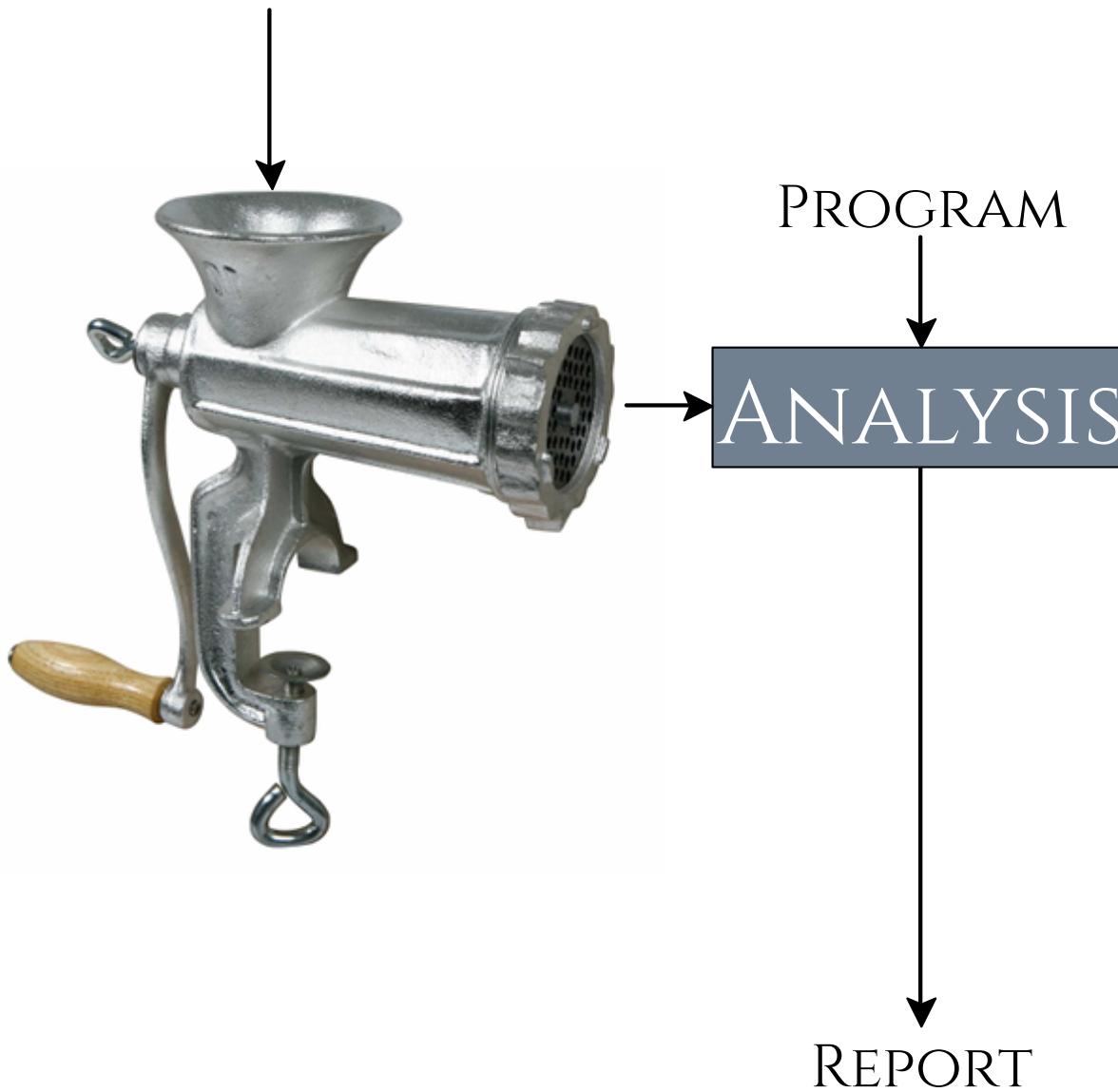
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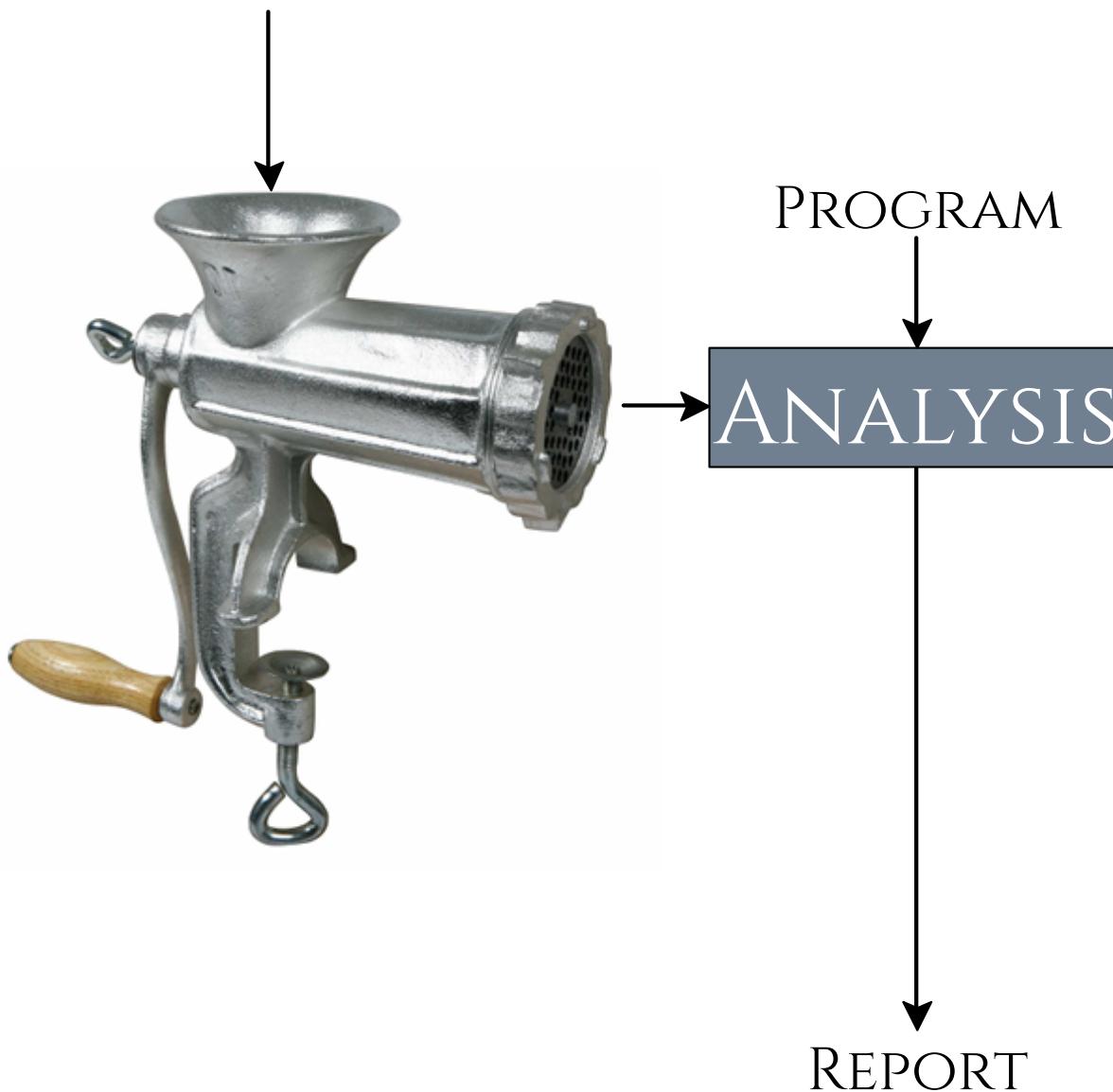


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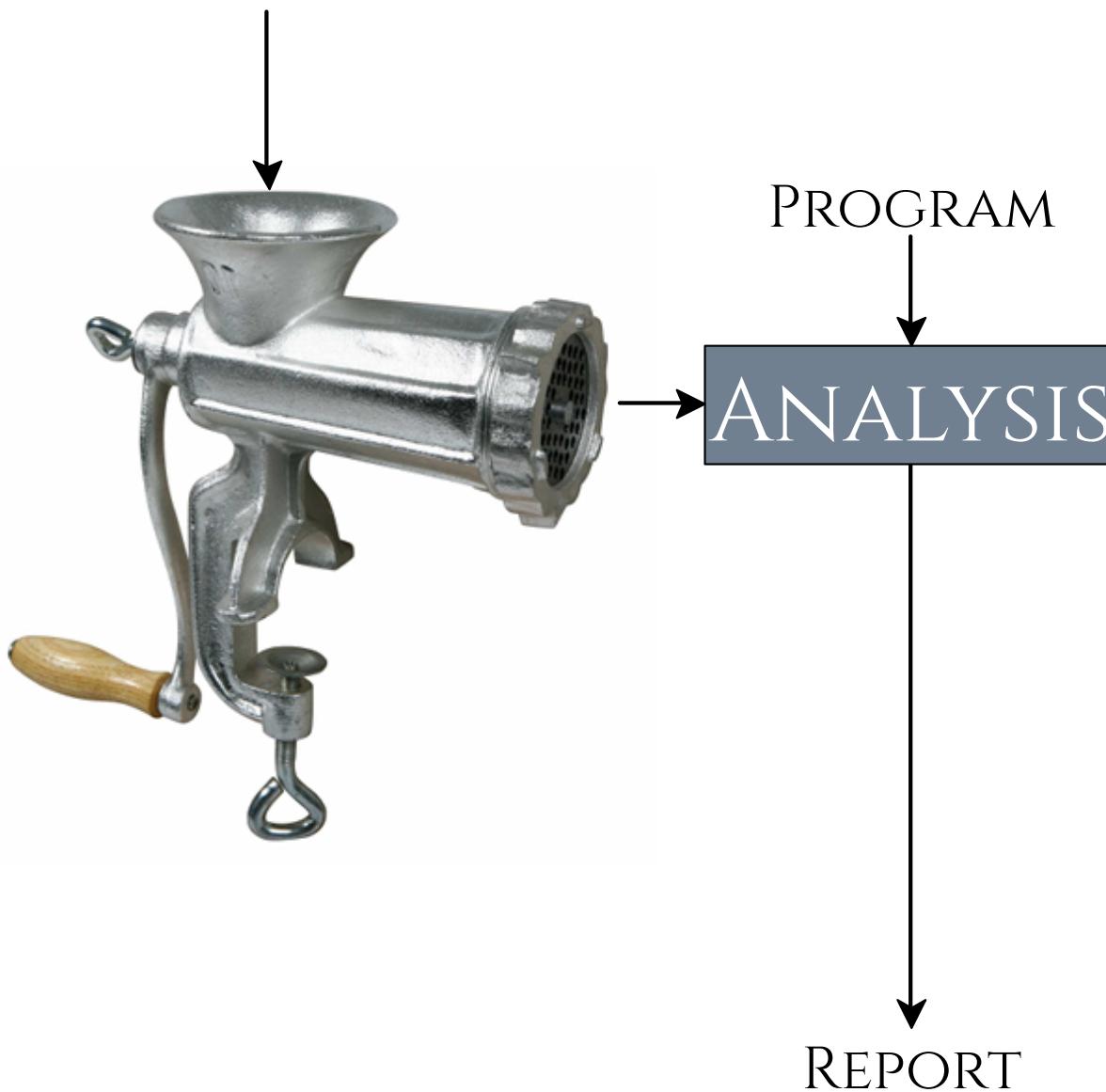


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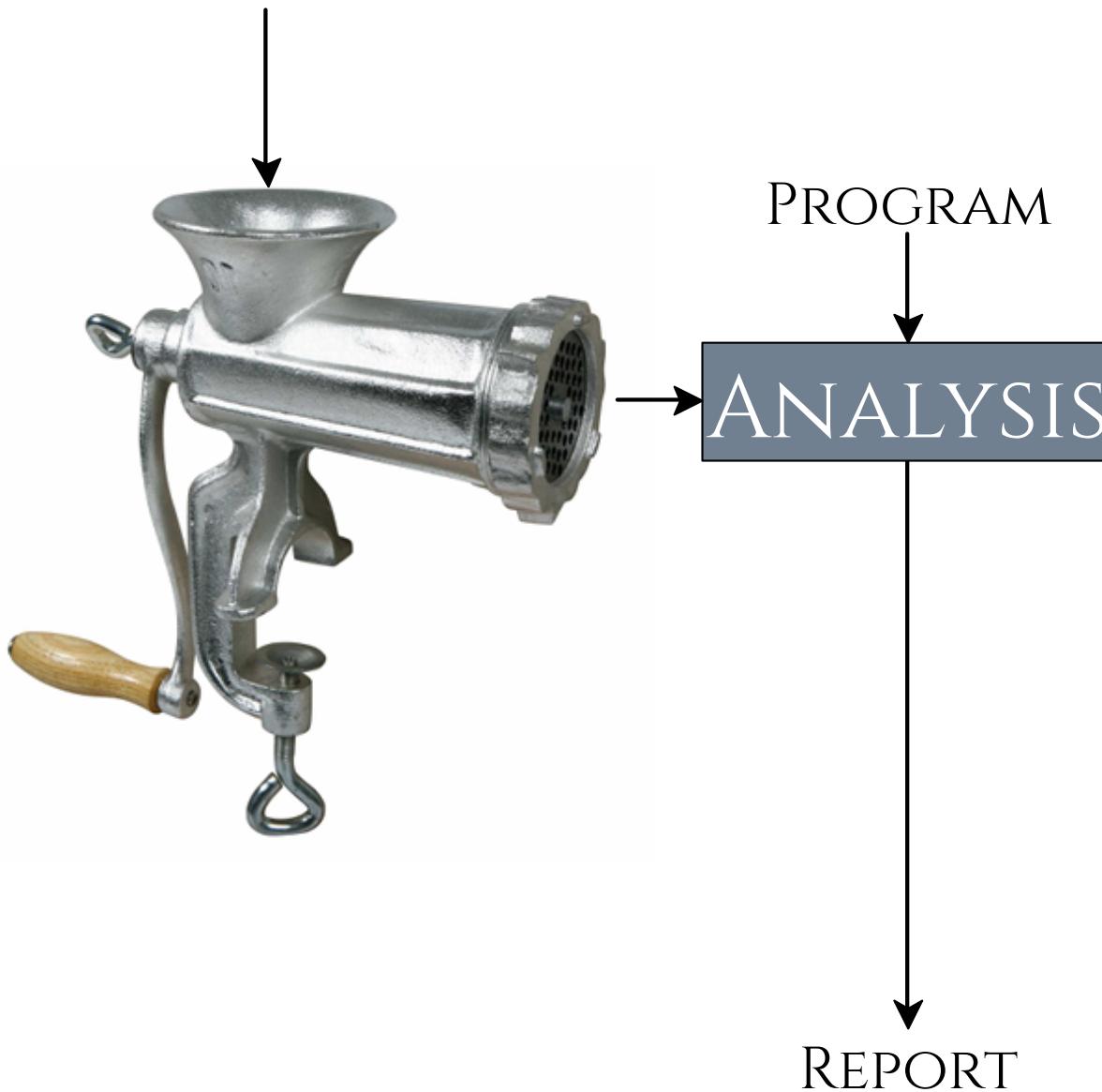
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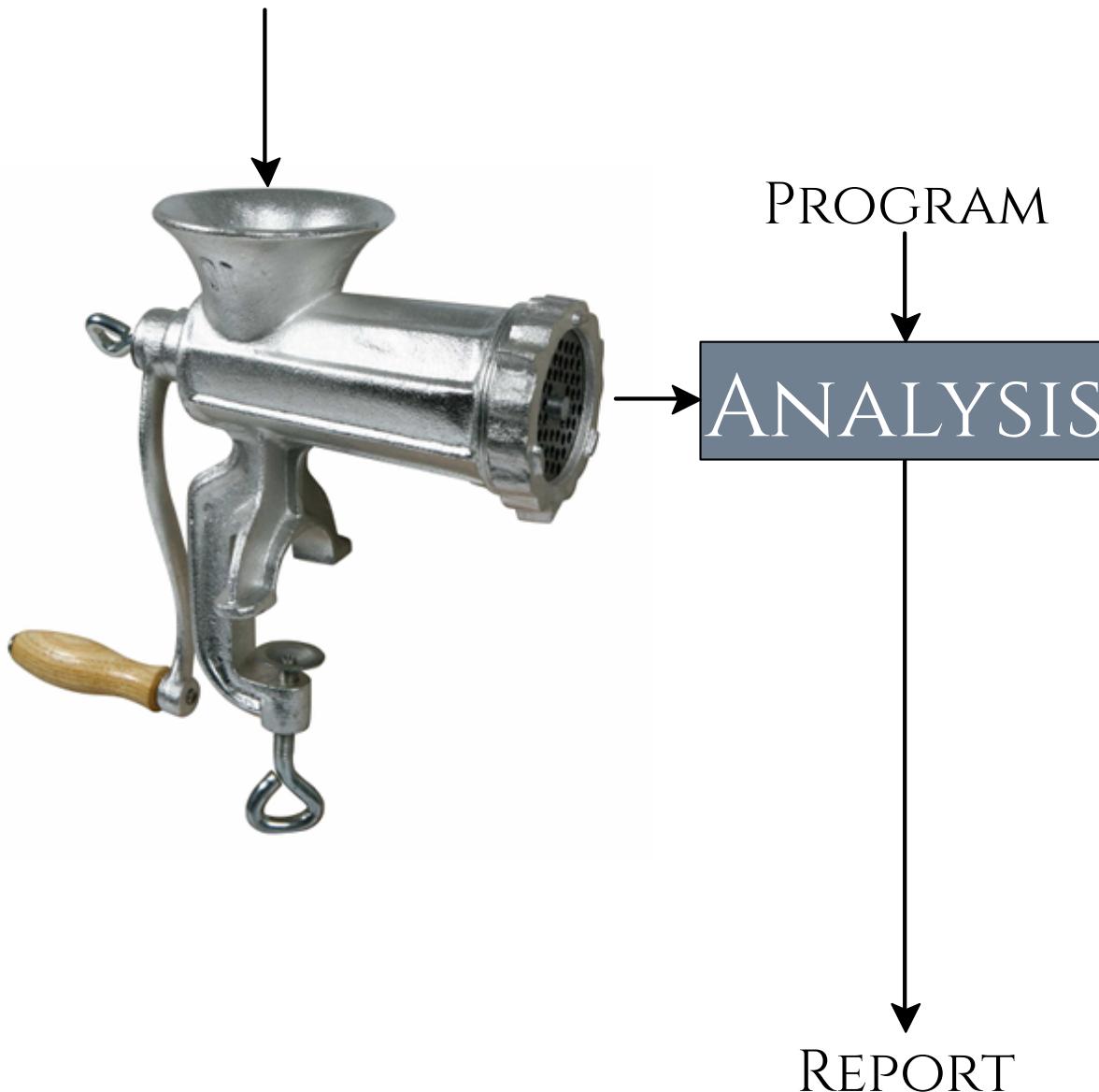
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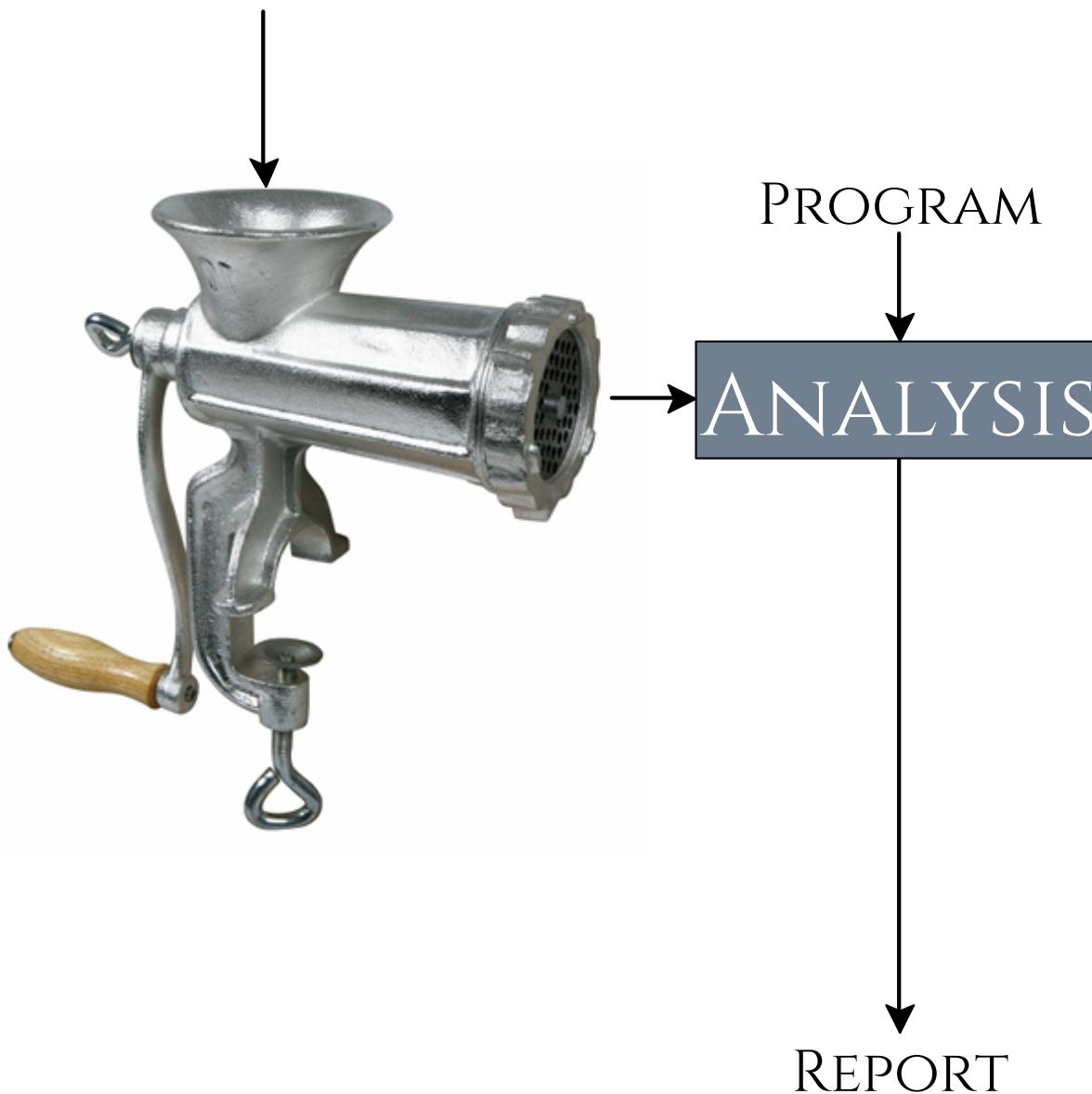
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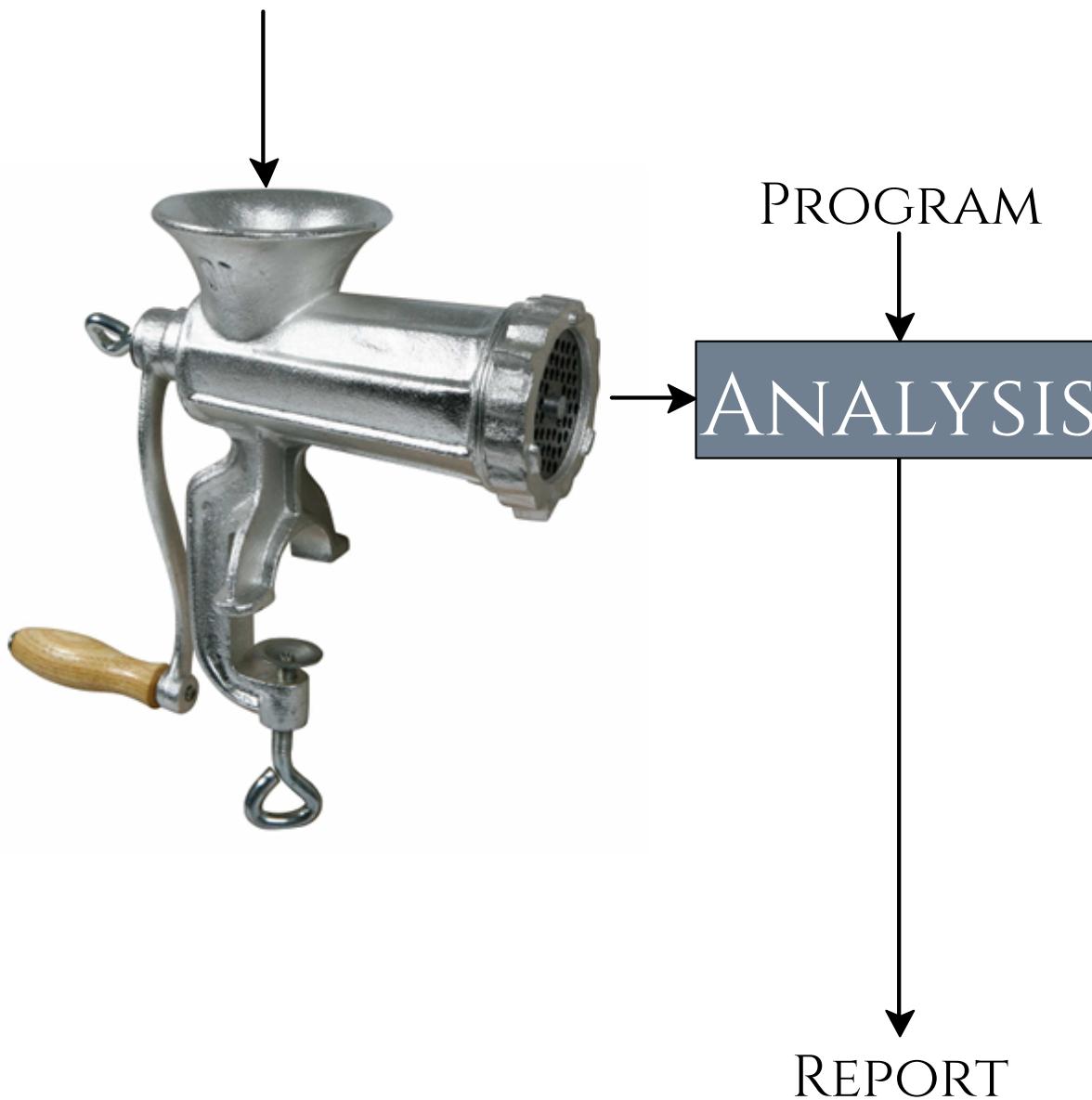


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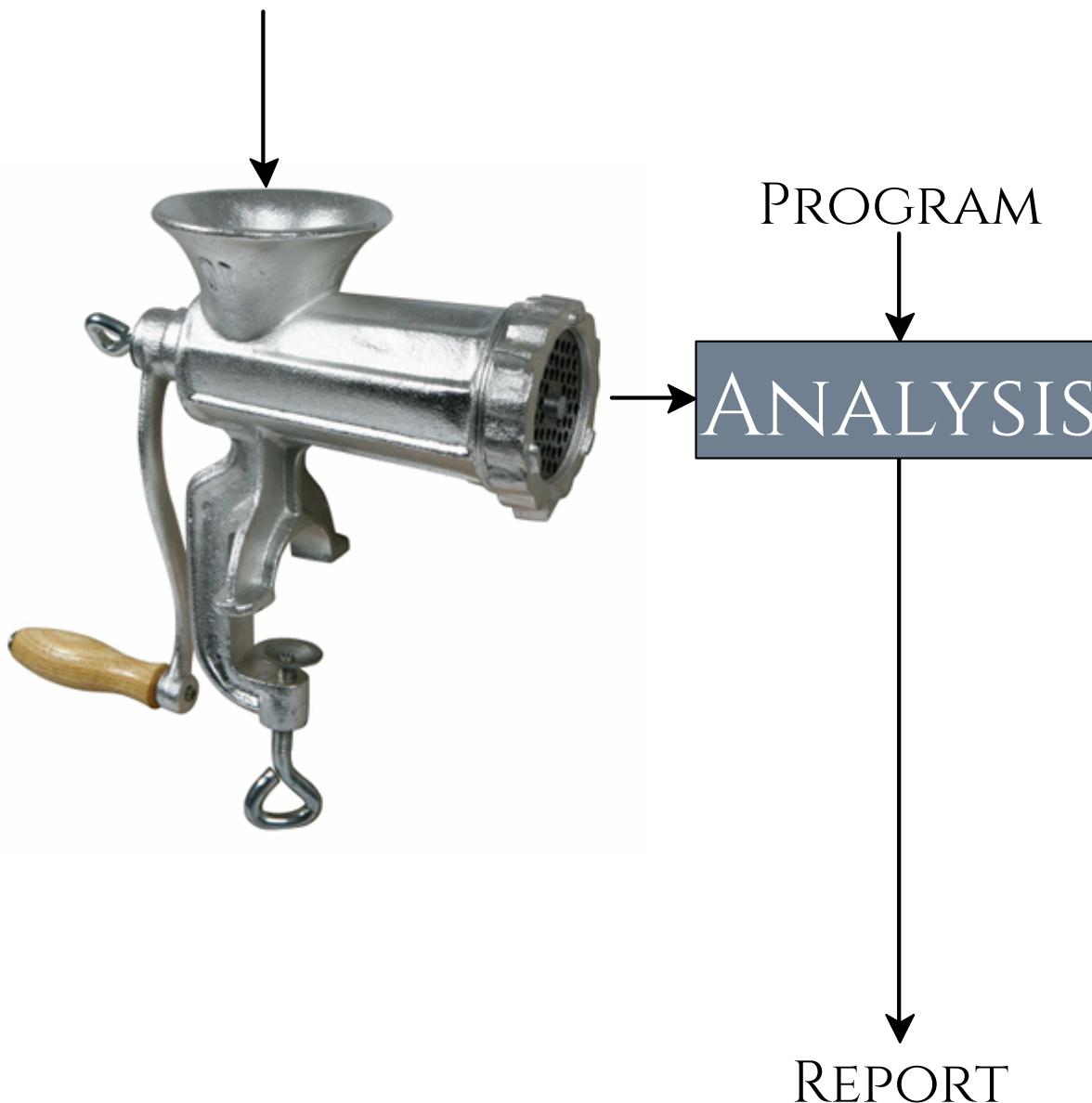
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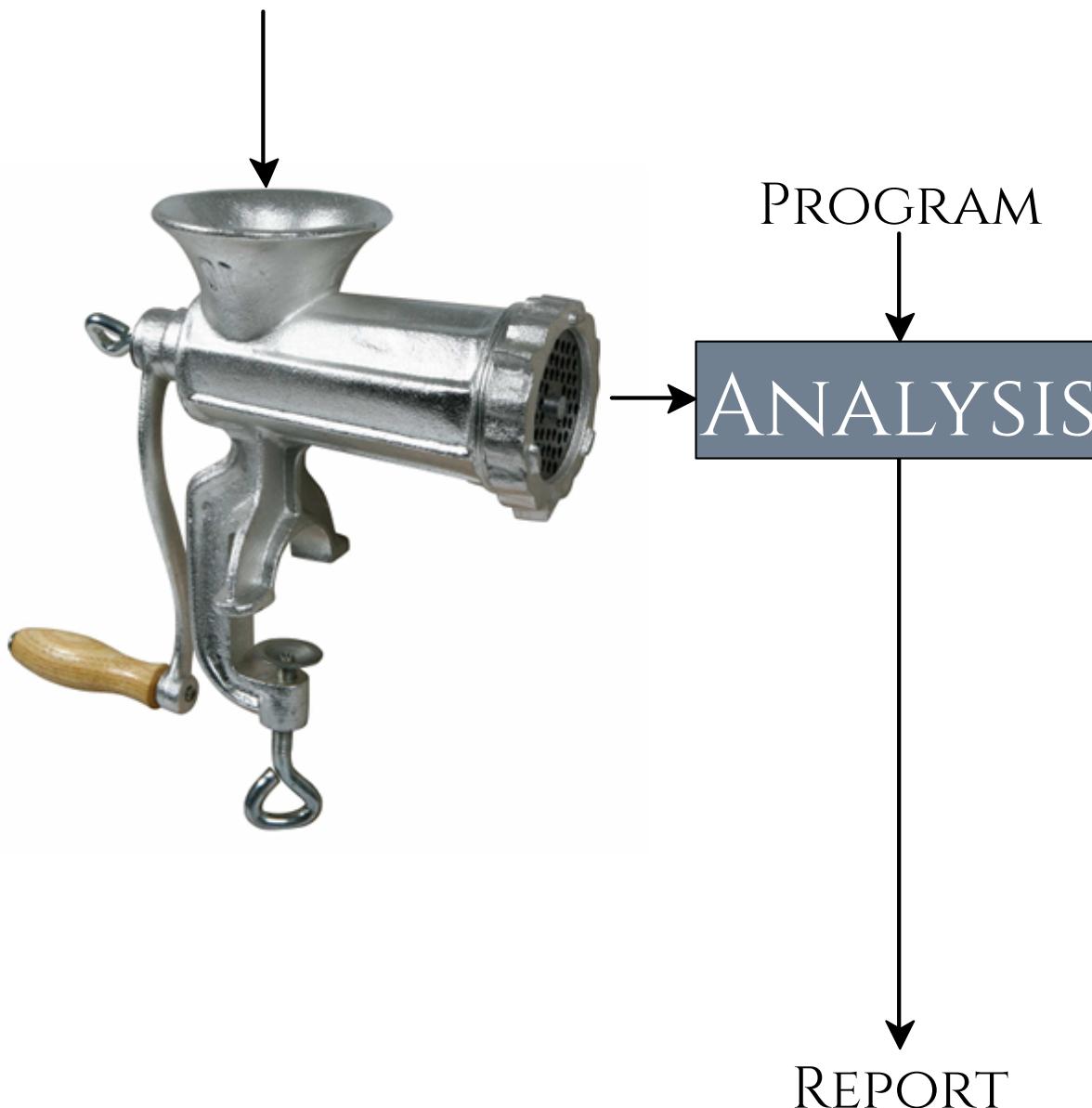


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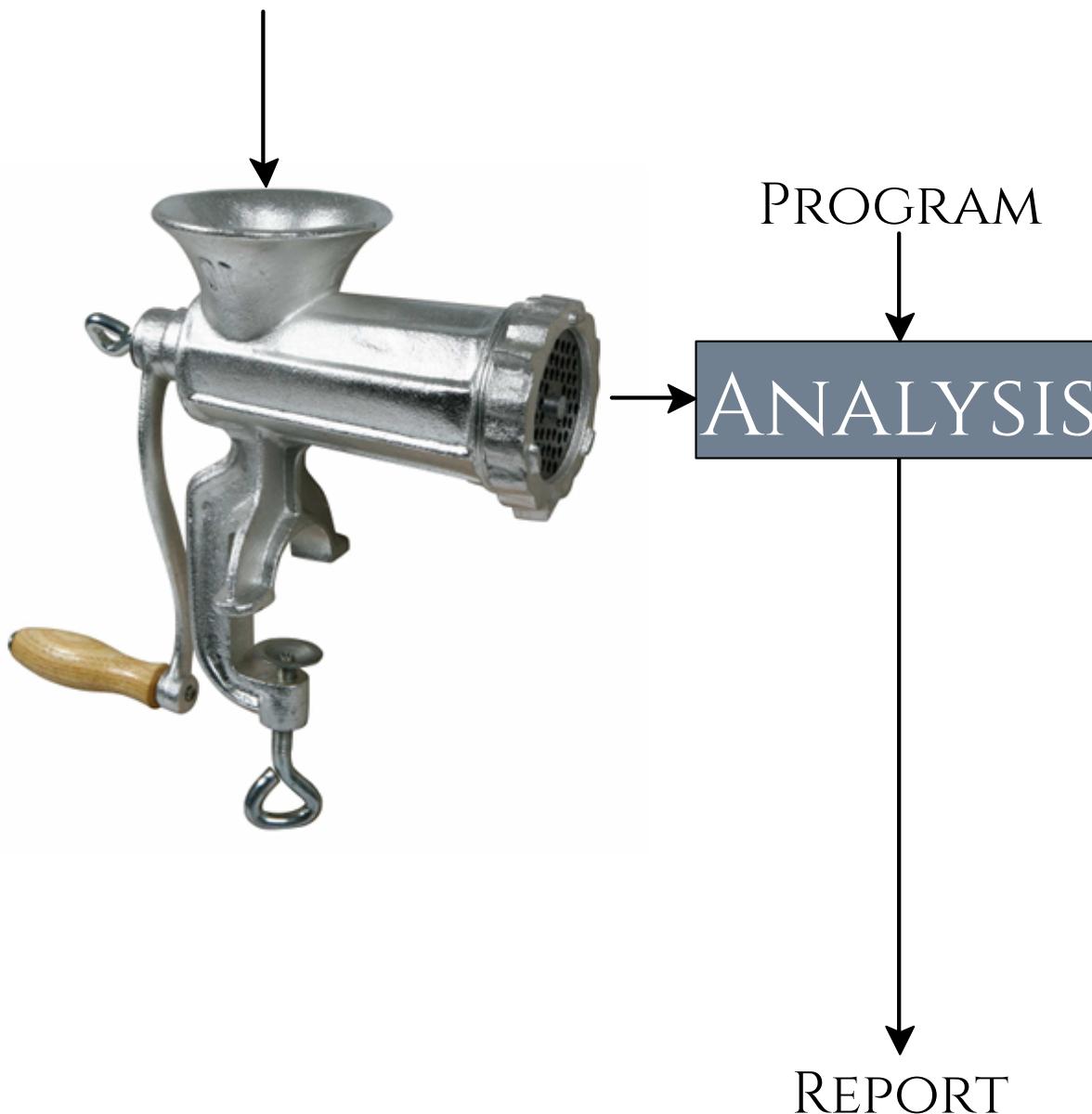


SEMANTICS



REPORT

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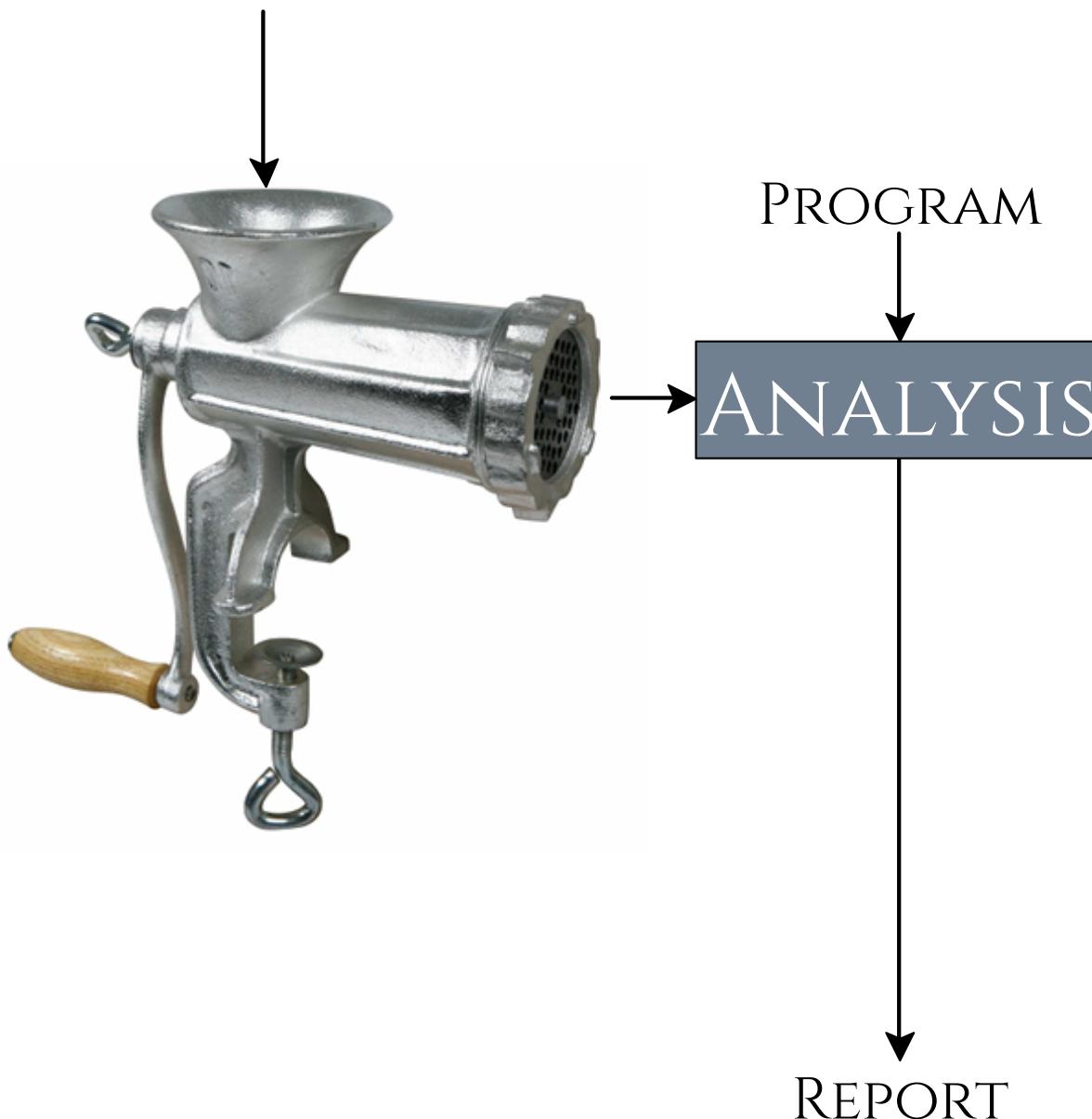


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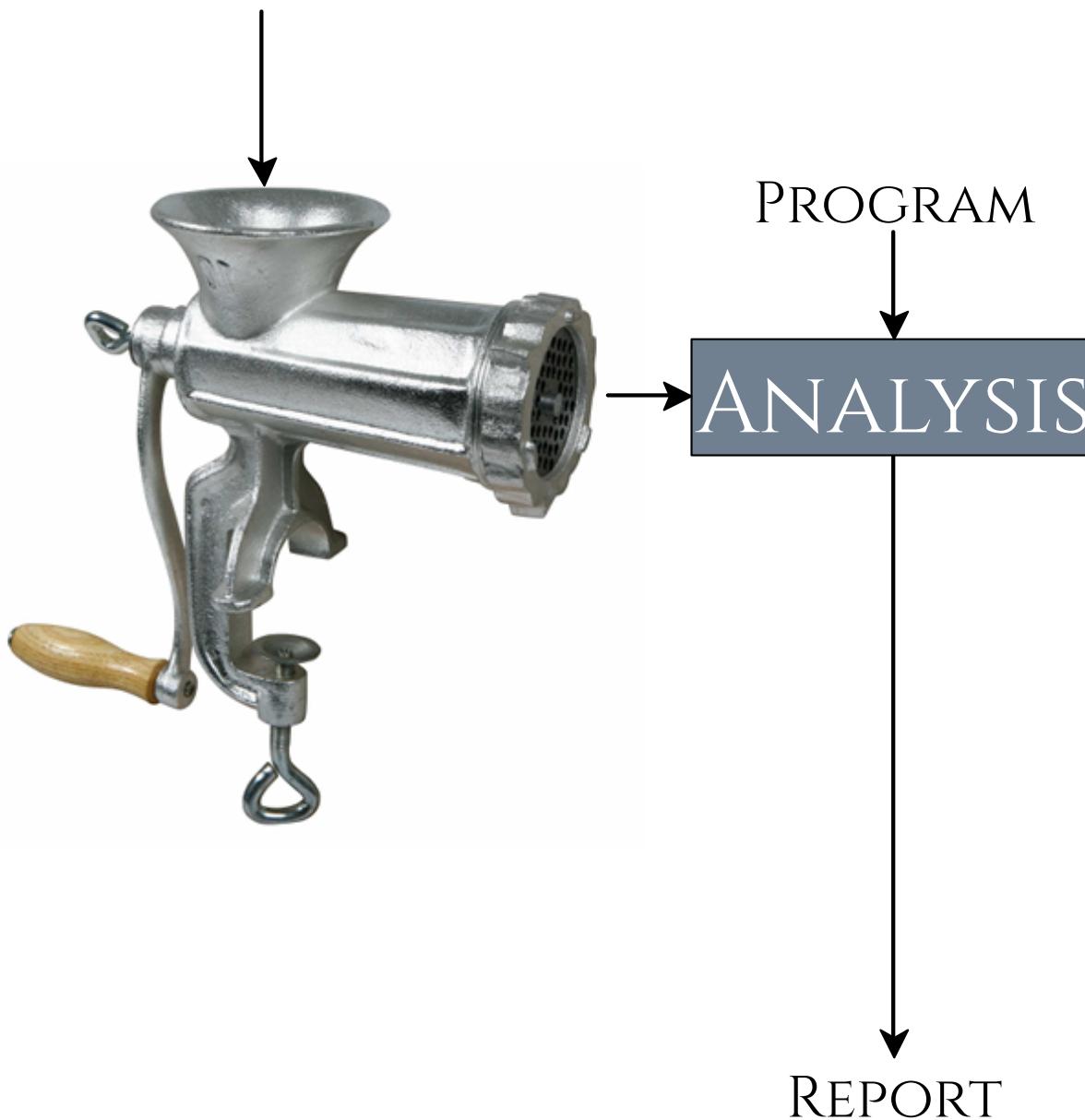
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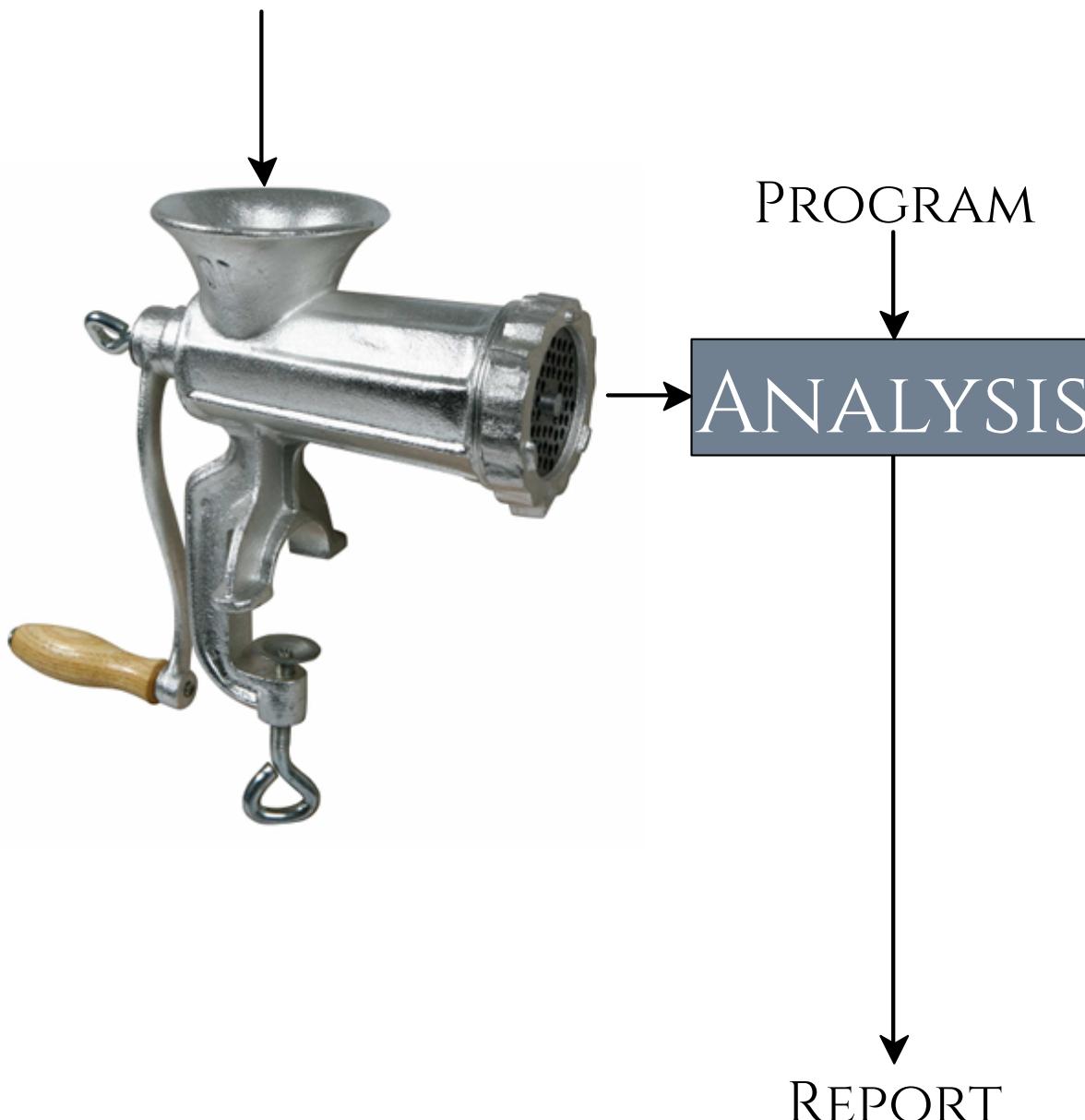
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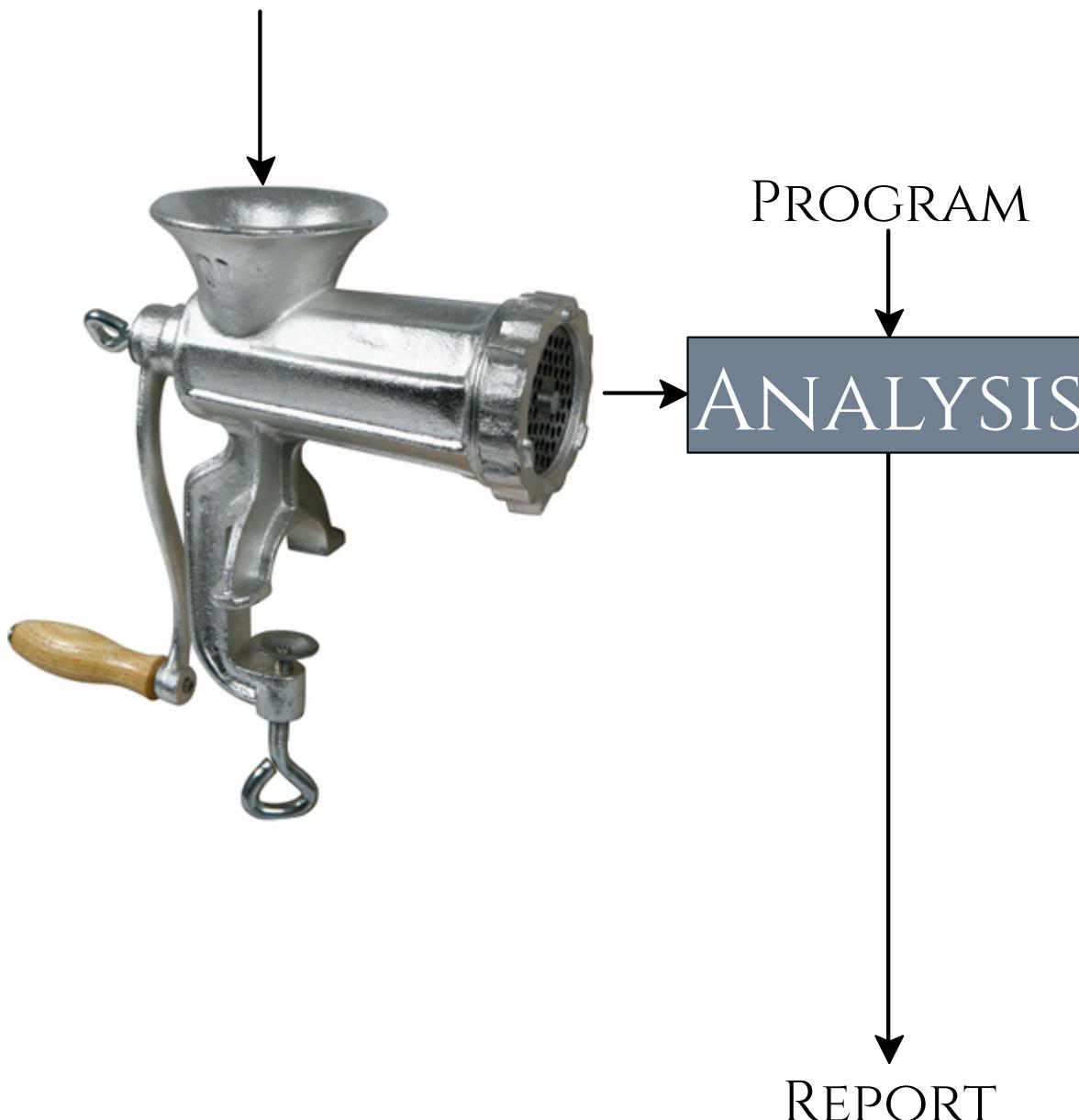
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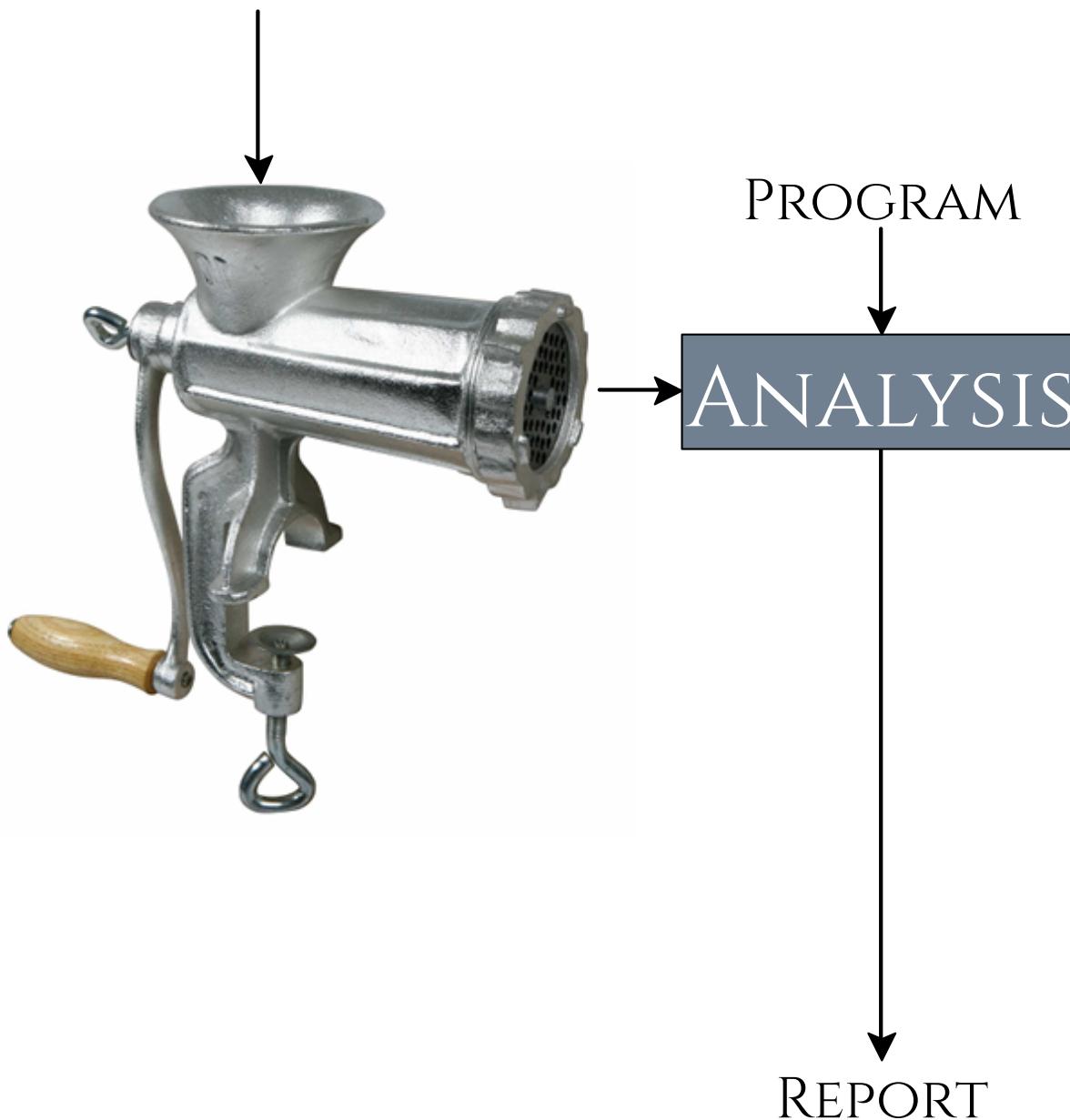


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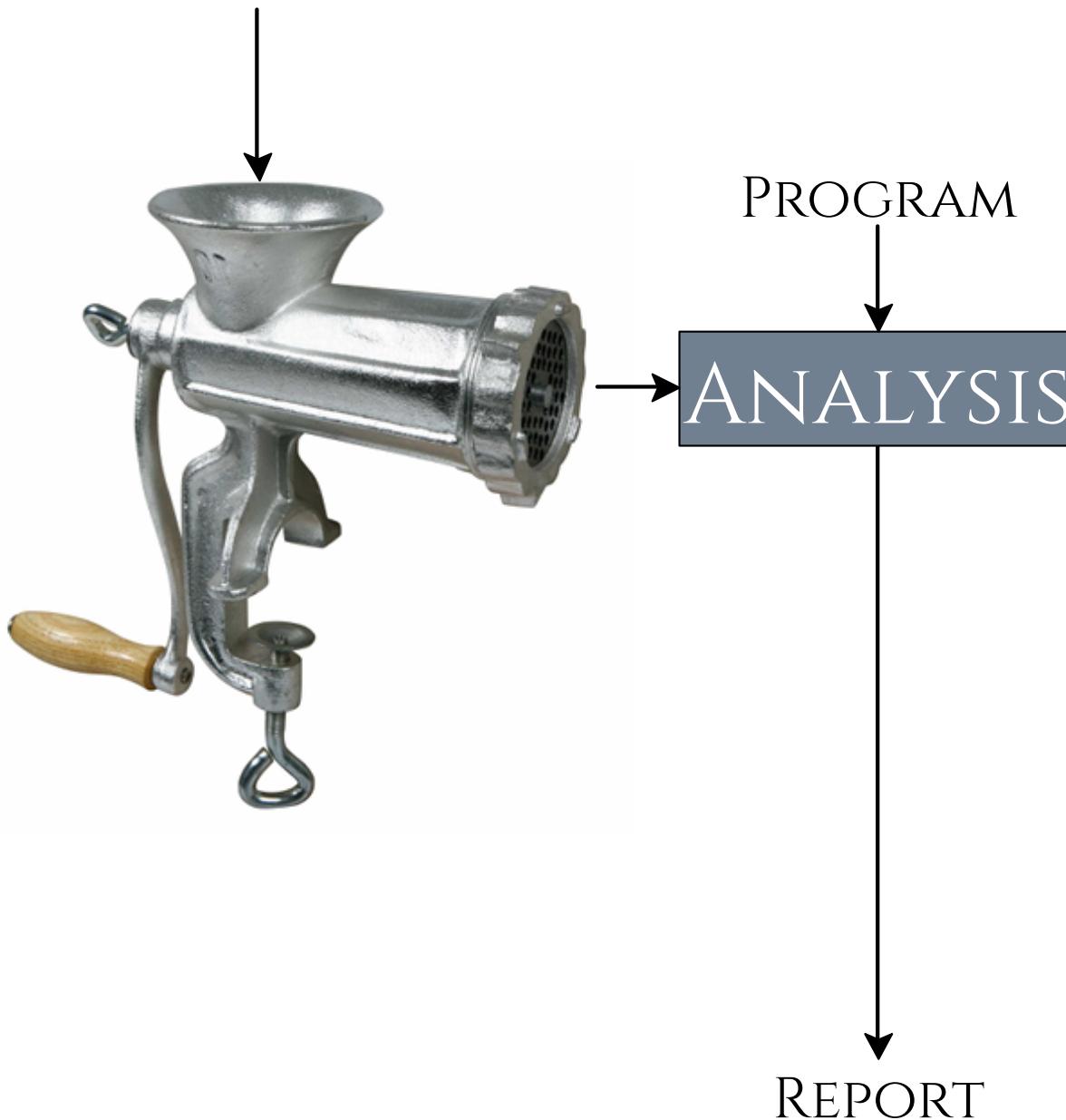


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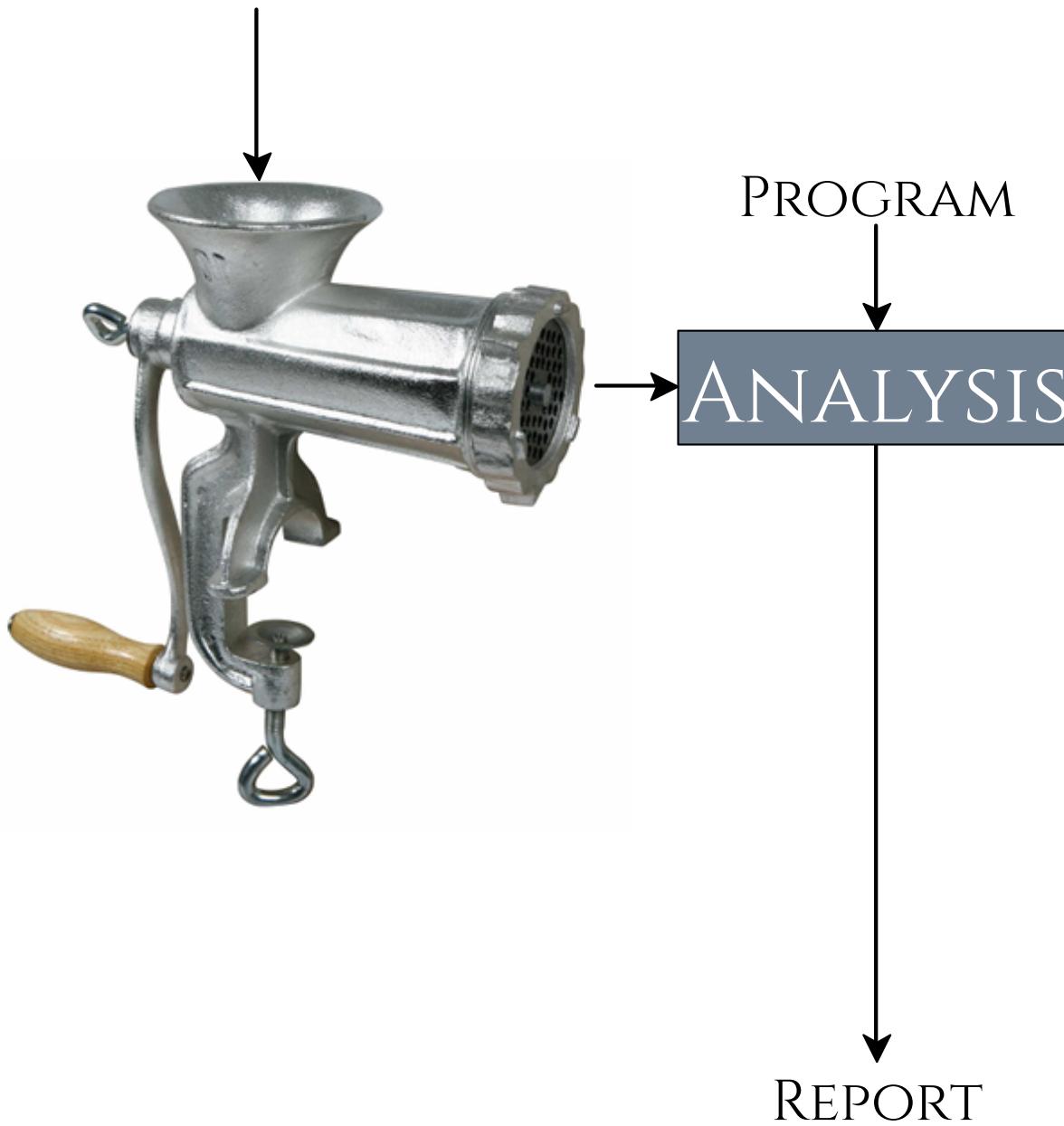
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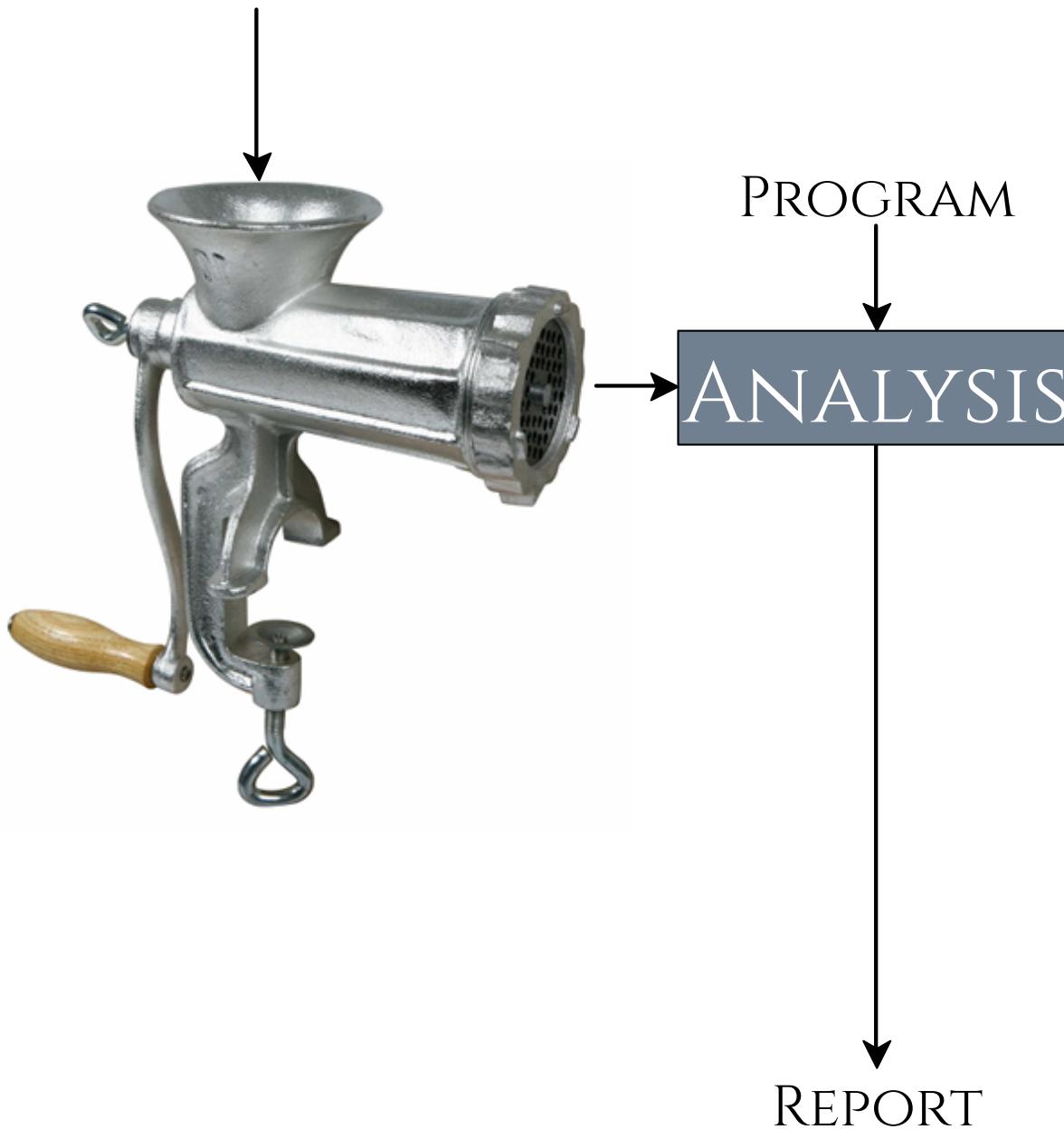
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SEMANTICS



REPORT

SEMANTICS

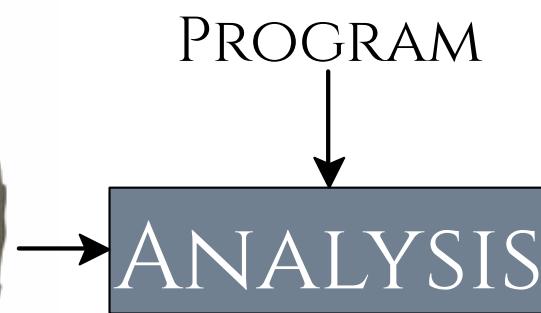


REPORT

SEMANTICS



PROGRAM

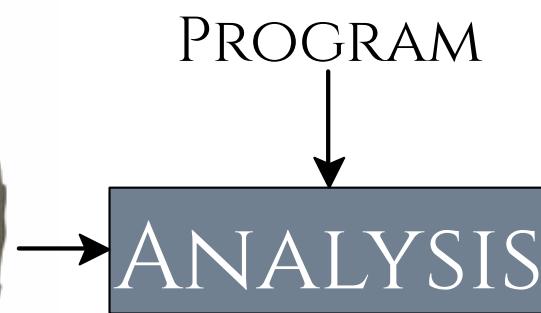


MAYBE

SEMANTICS



PROGRAM

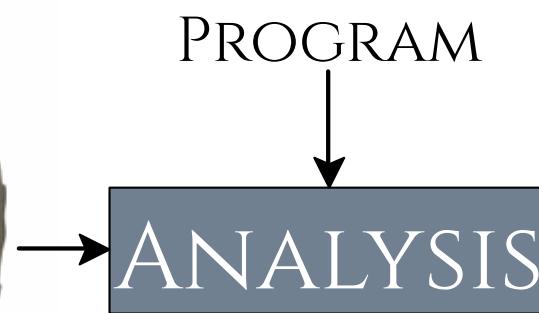


MAYBE

SEMANTICS



PROGRAM

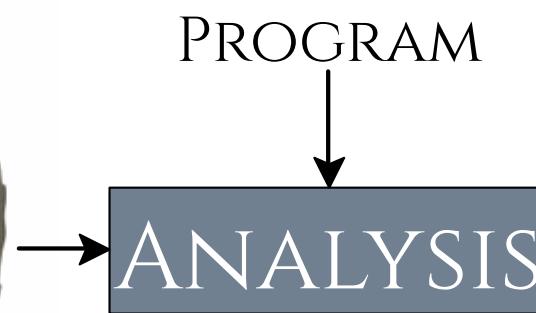


MAYBE

SEMANTICS



PROGRAM

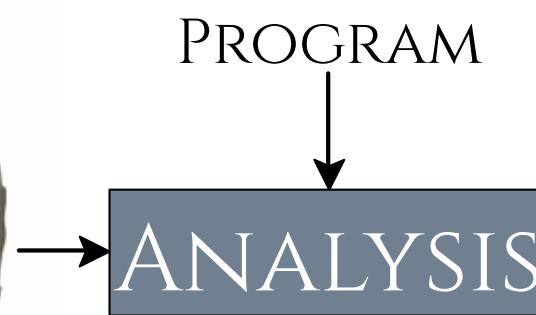


MAYBE

SEMANTICS



PROGRAM

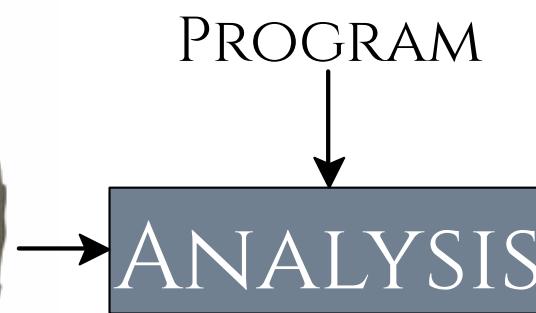


MAYBE

SEMANTICS



PROGRAM



MAYBE

SEMANTICS



PROGRAM

ANALYSIS

YES

MAYBE

SEMANTICS



PROGRAM

ANALYSIS

YES

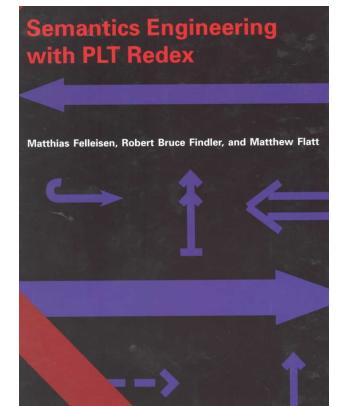
MAYBE

START WITH CESK MACHINE

CONTROL



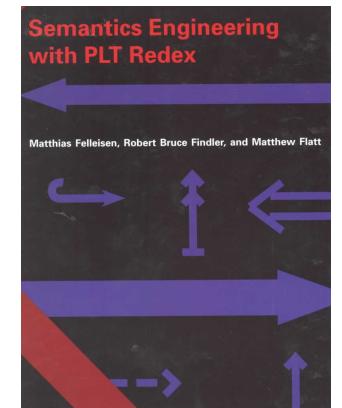
$\langle e, \rho, \sigma, K \rangle$



START WITH CESK MACHINE

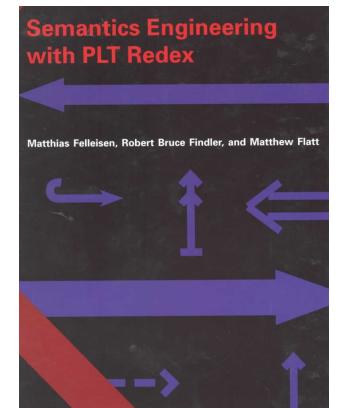
$\langle e, p, \sigma, K \rangle$

ENVIRONMENT



START WITH CESK MACHINE

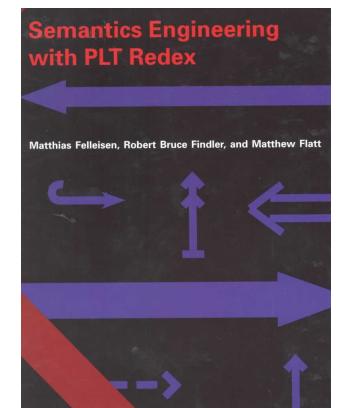
STORE
↓
 $\langle e, \rho, \sigma, K \rangle$



START WITH CESK MACHINE

$\langle e, \rho, \sigma, K \rangle$

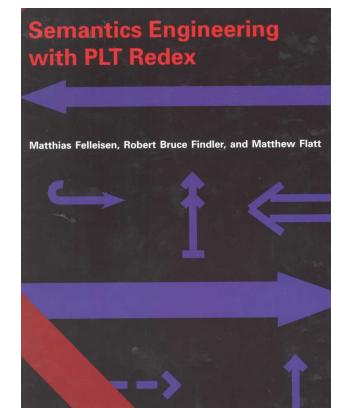
KONTINUATION



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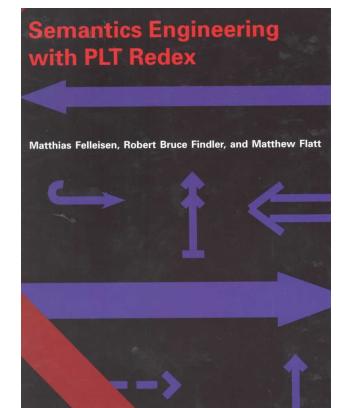
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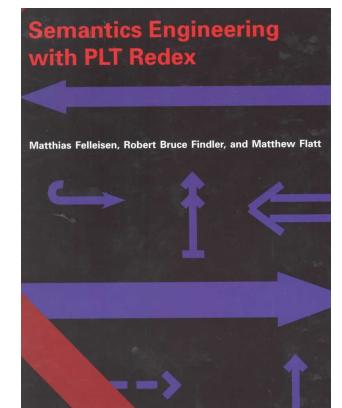


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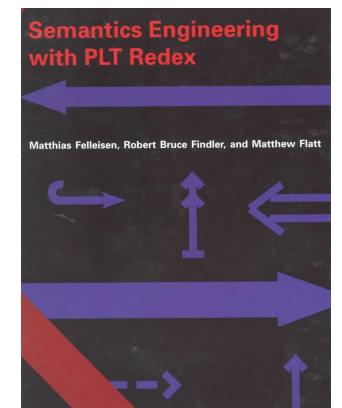


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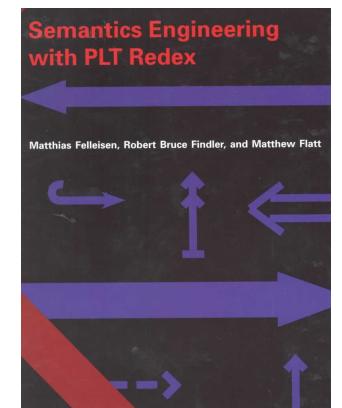


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$\langle e, \rho, \sigma, K \rangle$



CONTINUATION

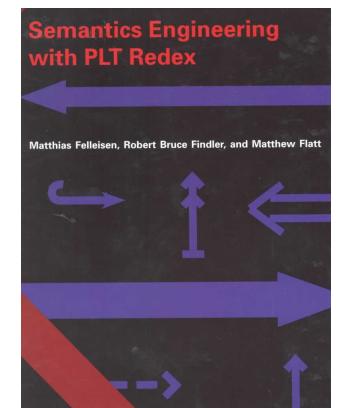


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CONTINUATION

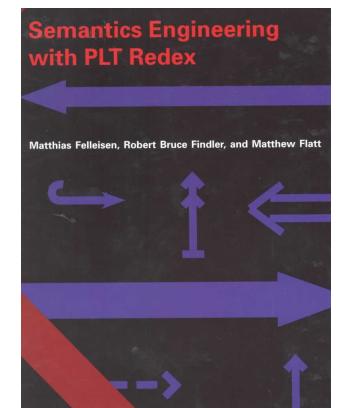


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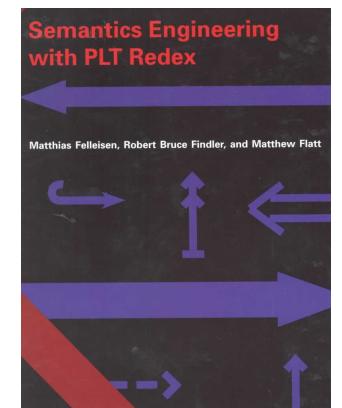


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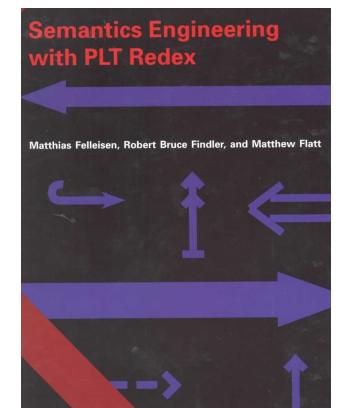


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K
CONTINUATION

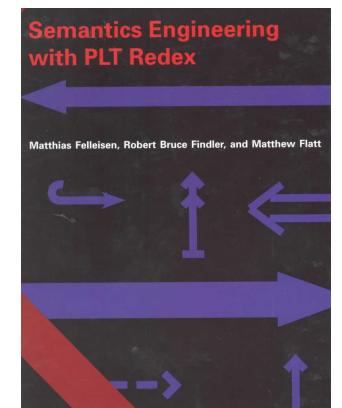


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K
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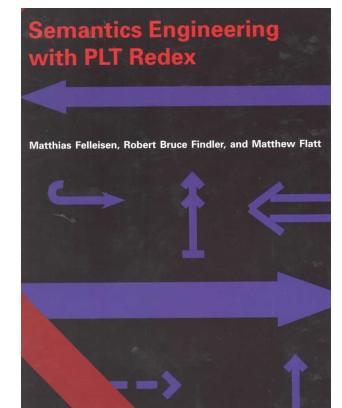


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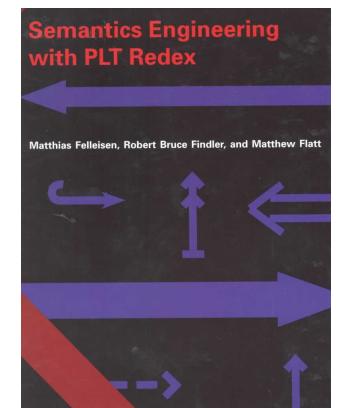


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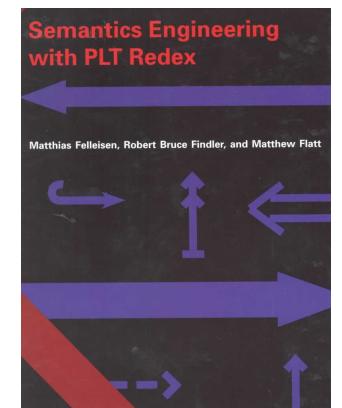


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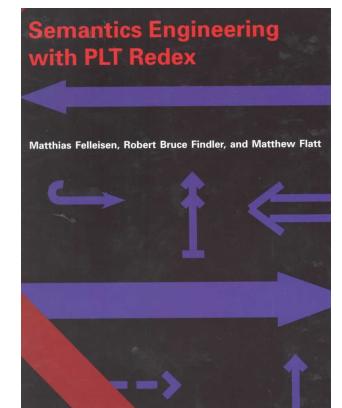


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CONTINUATION

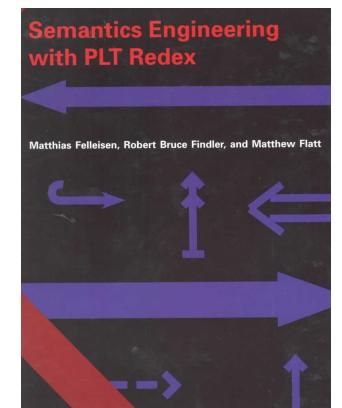


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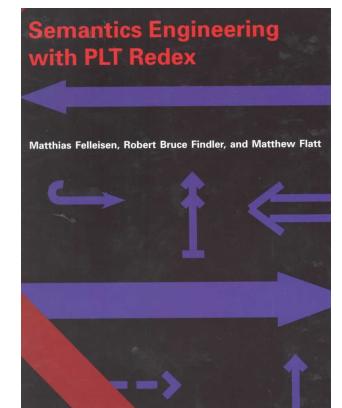


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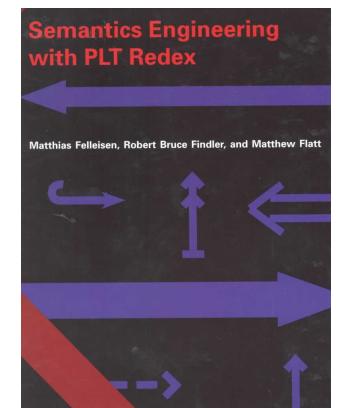


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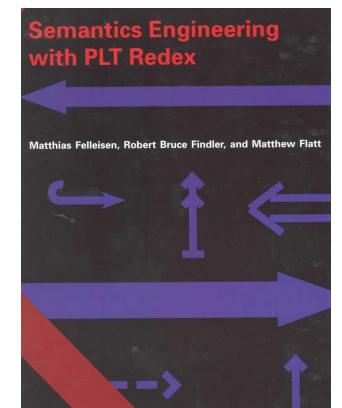


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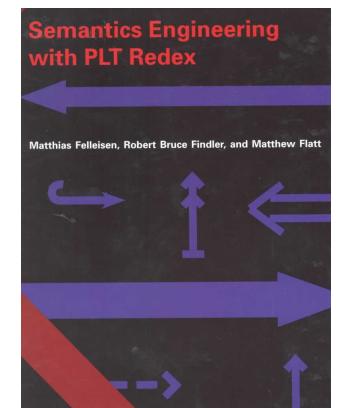


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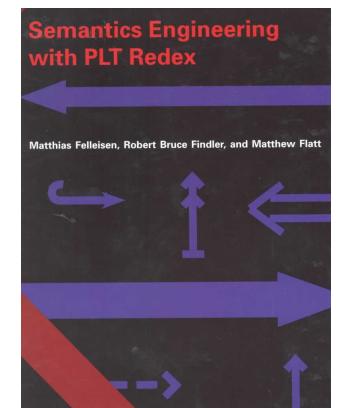


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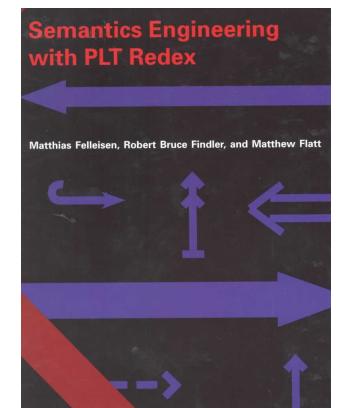


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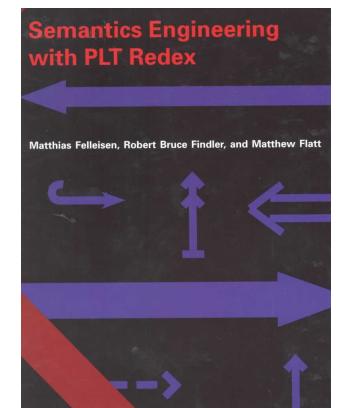
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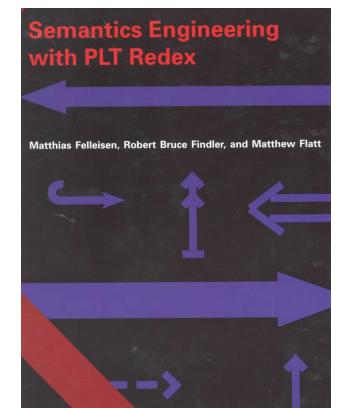
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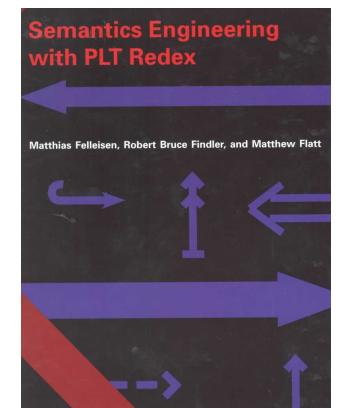


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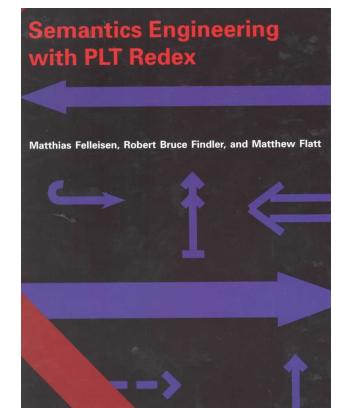


START WITH CESK MACHINE

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KONTINUATION



Let's evaluate $\Omega = ((\lambda x (x x)) (\lambda x (x x)))$

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Finitely many addresses \Rightarrow

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Problem: implicit allocation!

$\mapsto \langle x$

$a]) \rangle$

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$\mapsto \langle x$

The stack can grow unbounded $a] \rangle \rangle$

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What does it mean to finitize allocation?

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$[a \mapsto v_0]$ $[a \mapsto v_1]$

$[a \mapsto \{v_0, v_1\}]$

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Lookup a means *either* v_0 *or* v_1

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Lookup a means *either* v_0 *or* v_1 (or both)

Question: when do we lookup?

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AAM: whenever a rule says.

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$$\langle x \rho \sigma \kappa \rangle \mapsto \langle v \rho' \sigma \kappa \rangle \text{ where } \langle v, \rho' \rangle \in \sigma(\rho(x))$$

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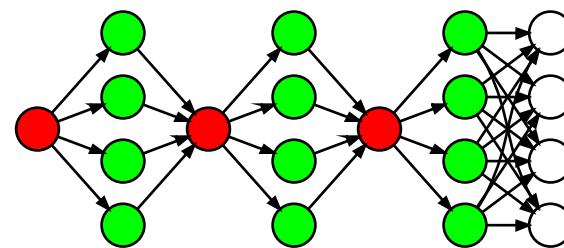
(**f** **x** **y**)

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(f x y)

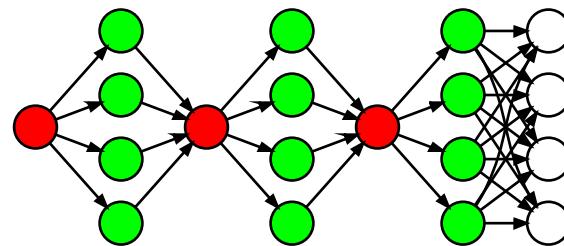


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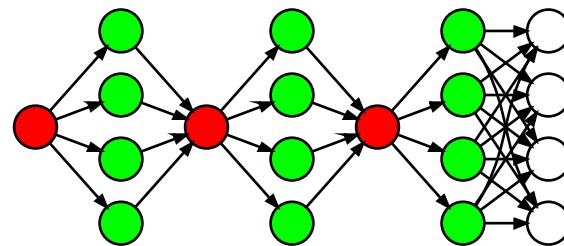
Me: whenever a looked up value is *inspected*.

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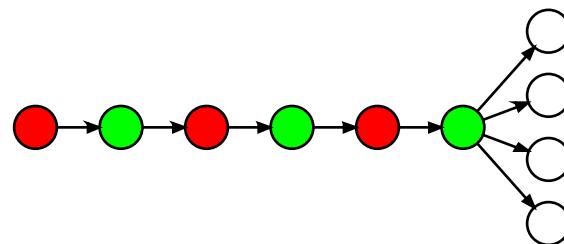
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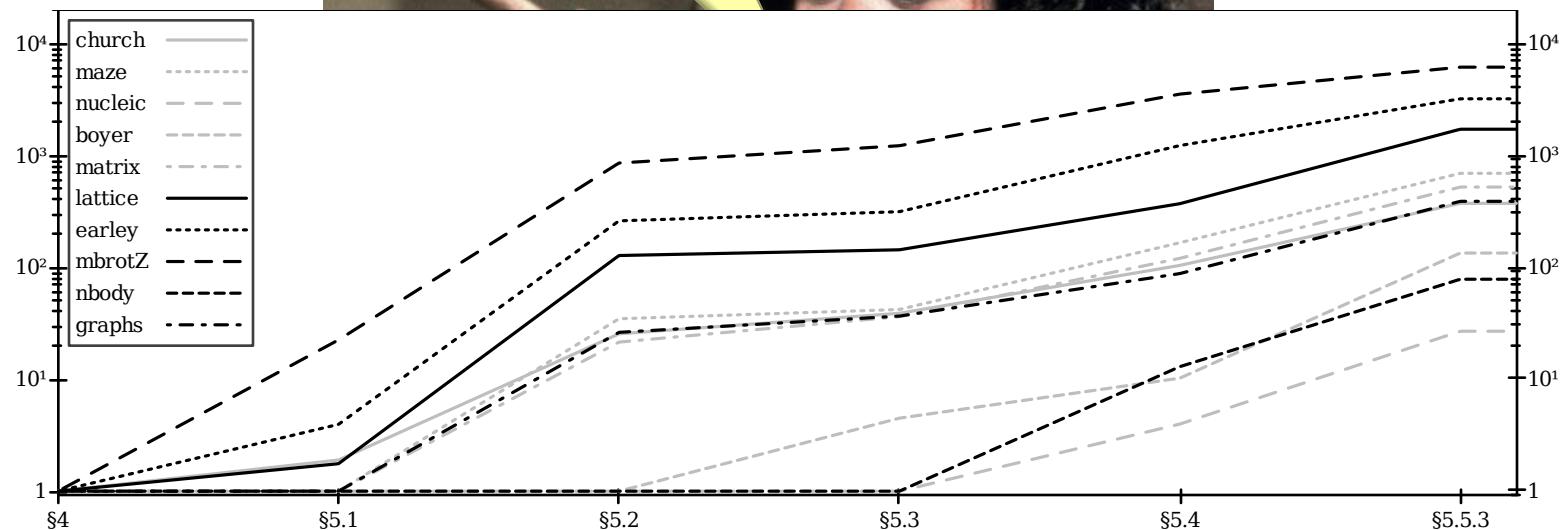
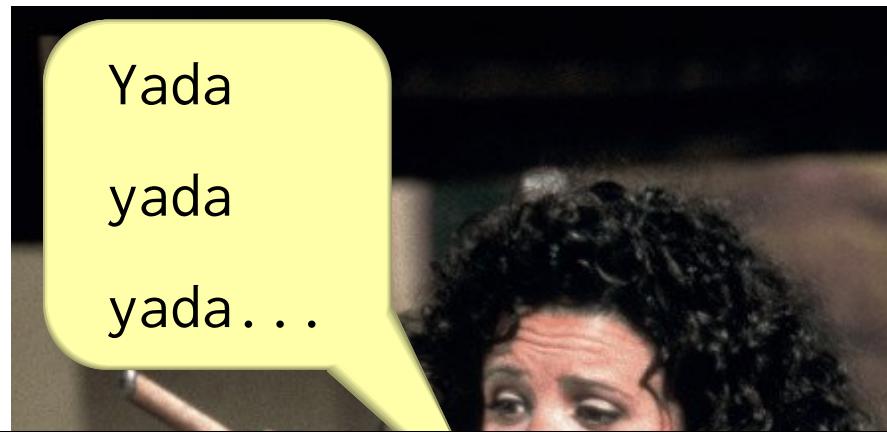


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Yada
yada
yada...



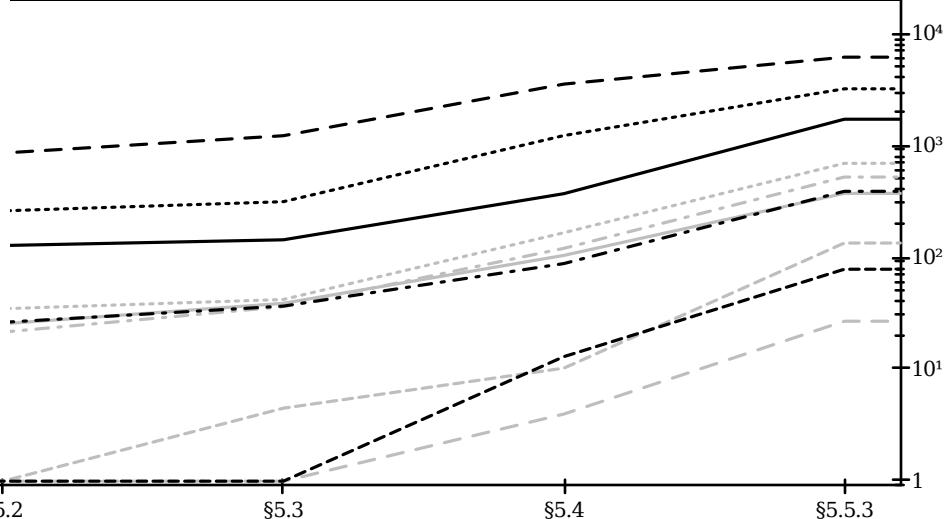


$traces(e) = \{\mathbf{ev}^{t_0}(e, \emptyset, \emptyset, \mathbf{halt}) \mapsto \varsigma\}$ where

$\varsigma \mapsto \varsigma'$ defined to be the following
let $t' = tick(\varsigma)$

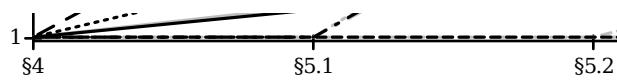
$\mathbf{ev}(\mathbf{var}(x), \rho, \sigma, \kappa) \mapsto \mathbf{co}(\kappa, v, \sigma)$ if $v \in \sigma(\rho(x))$
 $\mathbf{ev}(\mathbf{lit}(l), \rho, \sigma, \kappa) \mapsto \mathbf{co}(\kappa, l, \sigma)$
 $\mathbf{ev}^t(\mathbf{lam}(x, e), \rho, \sigma, \kappa) \mapsto \mathbf{co}(\kappa, \mathbf{clos}(x, e, \rho), \sigma)$
 $\mathbf{ev}^t(\mathbf{app}^\ell(e_0, e_1), \rho, \sigma, \kappa) \mapsto \mathbf{ev}^{t'}(e_0, \rho, \sigma', \mathbf{arg}_\ell^t(e_1, \rho, a_\kappa))$
 where $a_\kappa = \mathbf{alloc}_\ell^{cont}(e_1, \rho, \kappa)$
 $\sigma' = \sigma \sqcup [a_\kappa \mapsto \{\kappa\}]$
 $\mathbf{ev}^t(\mathbf{if}^\ell(e_0, e_1, e_2), \rho, \sigma, \kappa) \mapsto \mathbf{ev}^{t'}(e_0, \rho, \sigma', \mathbf{ifk}^t(e_1, e_2, \rho, a_\kappa))$
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 $\mathbf{co}(\mathbf{arg}_\ell^t(e, \rho, a_\kappa), v, \sigma) \mapsto \mathbf{ev}^t(e, \rho, \sigma, \mathbf{fun}_\ell^t(v, a_\kappa))$
 $\mathbf{co}(\mathbf{fun}_\ell^t(u, a_\kappa), v, \sigma) \mapsto \mathbf{ap}_\ell^t(u, v, \kappa, \sigma)$ if $\kappa \in \sigma(a_\kappa)$
 $\mathbf{co}(\mathbf{ifk}^t(e_0, e_1, \rho, a_\kappa), \mathbf{tt}, \sigma) \mapsto \mathbf{ev}^{t'}(e_0, \rho, \sigma, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
 $\mathbf{co}(\mathbf{ifk}^t(e_0, e_1, \rho, a_\kappa), \mathbf{ff}, \sigma) \mapsto \mathbf{ev}^{t'}(e_1, \rho, \sigma, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
 $\mathbf{ap}_\ell^t(\mathbf{clos}(x, e, \rho), v, \sigma, \kappa) \mapsto \mathbf{ev}^{t'}(e, \rho', \sigma', \kappa)$
 where $a = \mathbf{alloc}(\varsigma)$
 $\rho' = \rho[x \mapsto a]$
 $\sigma' = \sigma \sqcup [a \mapsto \{v\}]$
 $\mathbf{ap}_\ell^t(o, v, \sigma, \kappa) \mapsto \mathbf{co}(\kappa, v', \sigma)$ if $v' \in \Delta(o, v)$

Figure 5. Abstract abstract machine for ISWIM



$traces(e) = \{\text{ev}^{t_0}(e, \emptyset, \emptyset, \text{halt}) \mapsto \varsigma\}$ where
 $\varsigma \mapsto \varsigma'$ defined to be the following
 let $t' = \text{tick}(\varsigma)$
 $\text{ev}(\text{var}(x), \rho, \sigma, \kappa) \mapsto \text{co}(\kappa, v, \sigma)$ if $v \in \sigma(\rho(x))$
 $\text{ev}(\text{lit}(l), \rho, \sigma, \kappa) \mapsto \text{co}(\kappa, l, \sigma)$
 $\text{ev}^t(\text{lam}(x, e), \rho, \sigma, \kappa) \mapsto \text{co}(\kappa, \text{clos}(x, e, \rho), \sigma)$
 $\text{ev}^t(\text{app}^\ell(e_0, e_1), \rho, \sigma, \kappa) \mapsto \text{ev}^{t'}(e_0, \rho, \sigma', \text{arg}_\ell^t(e_1, \rho, a_\kappa))$
 where $a_\kappa = \text{alloccont}_\ell^t(\sigma, \kappa)$
 $\sigma' = \sigma \sqcup [a_\kappa \mapsto \{\kappa\}]$
 $\text{ev}^t(\text{if}^\ell(e_0, e_1, e_2), \rho, \sigma, \kappa) \mapsto \text{ev}^{t'}(e_0, \rho, \sigma', \text{ifk}^t(e_1, e_2, \rho, a_\kappa))$
 where $a_\kappa = \text{alloccont}_\ell^t(\sigma, \kappa)$
 $\sigma' = \sigma \sqcup [a_\kappa \mapsto \{\kappa\}]$
 $\text{co}(\text{arg}_\ell^t(e, \rho, a_\kappa), v, \sigma) \mapsto \text{ev}^t(e, \rho, \sigma, \text{fun}_\ell^t(v, a_\kappa))$
 $\text{co}(\text{fun}_\ell^t(u, a_\kappa), v, \sigma) \mapsto \text{ap}_\ell^t(u, v, \kappa, \sigma)$ if $\kappa \in \sigma(a_\kappa)$
 $\text{co}(\text{ifk}^t(e_0, e_1, \rho, a_\kappa), \text{tt}, \sigma) \mapsto \text{ev}^{t'}(e_0, \rho, \sigma, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
 $\text{co}(\text{ifk}^t(e_0, e_1, \rho, a_\kappa), \text{ff}, \sigma) \mapsto \text{ev}^{t'}(e_1, \rho, \sigma, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
 $\text{ap}_\ell^t(\text{clos}(x, e, \rho), v, \sigma, \kappa) \mapsto \text{ev}^{t'}(e, \rho', \sigma', \kappa)$
 where $a = \text{alloc}(\varsigma)$
 $\rho' = \rho[x \mapsto a]$
 $\sigma' = \sigma \sqcup [a \mapsto \{v\}]$
 $\text{ap}_\ell^t(o, v, \sigma, \kappa) \mapsto \text{co}(\kappa, v', \sigma)$ if $v' \in \Delta(o, v)$

Figure 5. Abstract abstract machine for ISWIM



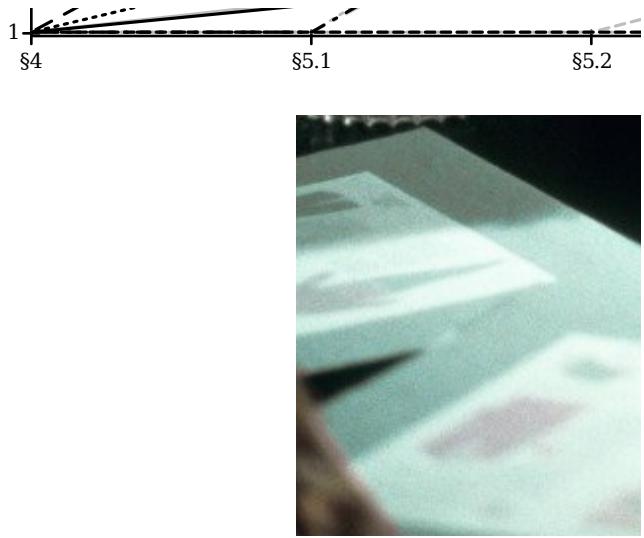
$\llbracket \cdot \rrbracket : Expr \rightarrow Store$
 $\rightarrow Env \times Store \Delta \times Kont \times Time$
 $\rightarrow State$
 $t' = \text{tick}(\ell, \rho, \sigma, t)$
 $\llbracket \text{var}(x) \rrbracket_\sigma = \lambda^t(\rho, \xi, \kappa). \text{co}(\kappa, \text{addr}(\rho(x))), \xi$
 $\llbracket \text{lit}(l) \rrbracket_\sigma = \lambda^t(\rho, \xi, \kappa). \text{co}(\kappa, l), \xi$
 $\llbracket \text{lam}(x, e) \rrbracket_\sigma = \lambda^t(\rho, \xi, \kappa). \text{co}(\kappa, \text{clos}(x, \llbracket e \rrbracket, \rho)), \xi$
 $\llbracket \text{app}^\ell(e_0, e_1) \rrbracket_\sigma = \lambda^t(\rho, \xi, \kappa). \llbracket e_0 \rrbracket^{t'}(\rho, \xi', \text{arg}_\ell^t(\llbracket e_1 \rrbracket, \rho, a_\kappa))$
 where $a_\kappa = \text{alloccont}_\ell^t(\sigma, \kappa)$
 $\xi' = (a_\kappa, \{\kappa\}): \xi$
 $\llbracket \text{if}^\ell(e_0, e_1, e_2) \rrbracket_\sigma = \lambda^t(\rho, \xi, \kappa). \llbracket e_0 \rrbracket^{t'}(\rho, \xi', \text{ifk}^t(\llbracket e_1 \rrbracket, \llbracket e_2 \rrbracket, \rho, a_\kappa))$
 where $a_\kappa = \text{alloccont}_\ell^t(\sigma, \kappa)$
 $\xi' = (a_\kappa, \{\kappa\}): \xi$

Figure 6. Abstract compilation



$traces(e) = \{\text{ev}^{t_0}(e, \emptyset, \emptyset, \text{halt}) \mapsto \varsigma\}$ where
 $\varsigma \mapsto \varsigma'$ defined to be the following
 let $t' = \text{tick}(\varsigma)$
 $\text{ev}(\text{var}(x), \rho, \sigma, \kappa) \mapsto \text{co}(\kappa, v, \sigma)$ if $v \in \sigma(\rho(x))$
 $\text{ev}(\text{lit}(l), \rho, \sigma, \kappa) \mapsto \text{co}(\kappa, l, \sigma)$
 $\text{ev}^t(\text{lam}(x, e), \rho, \sigma, \kappa) \mapsto \text{co}(\kappa, \text{clos}(x, e, \rho), \sigma)$
 $\text{ev}^t(\text{app}^\ell(e_0, e_1), \rho, \sigma, \kappa) \mapsto \text{ev}^{t'}(e_0, \rho, \sigma', \text{arg}_\ell^t(e_1, \rho, a_\kappa))$
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 $\text{co}(\text{arg}_\ell^t(e, \rho, a_\kappa), v, \sigma) \mapsto \text{ev}^t(e, \rho, \sigma, \text{fun}_\ell^t(v, a_\kappa))$
 $\text{co}(\text{fun}_\ell^t(u, a_\kappa), v, \sigma) \mapsto \text{ap}_\ell^t(u, v, \kappa, \sigma)$ if $\kappa \in \sigma(a_\kappa)$
 $\text{co}(\text{ifk}^t(e_0, e_1, \rho, a_\kappa), \text{tt}, \sigma) \mapsto \text{ev}^{t'}(e_0, \rho, \sigma, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
 $\text{co}(\text{ifk}^t(e_0, e_1, \rho, a_\kappa), \text{ff}, \sigma) \mapsto \text{ev}^{t'}(e_1, \rho, \sigma, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
 $\text{ap}_\ell^t(\text{clos}(x, e, \rho), v, \sigma, \kappa) \mapsto \text{ev}^{t'}(e, \rho', \sigma', \kappa)$
 where $a = \text{alloc}(\varsigma)$
 $\rho' = \rho[x \mapsto a]$
 $\sigma' = \sigma \sqcup [a \mapsto \{v\}]$
 $\text{ap}_\ell^t(o, v, \sigma, \kappa) \mapsto \text{co}(\kappa, v', \sigma)$ if $v' \in \Delta(o, v)$

Figure 5. Abstract abstract machine for ISWIM



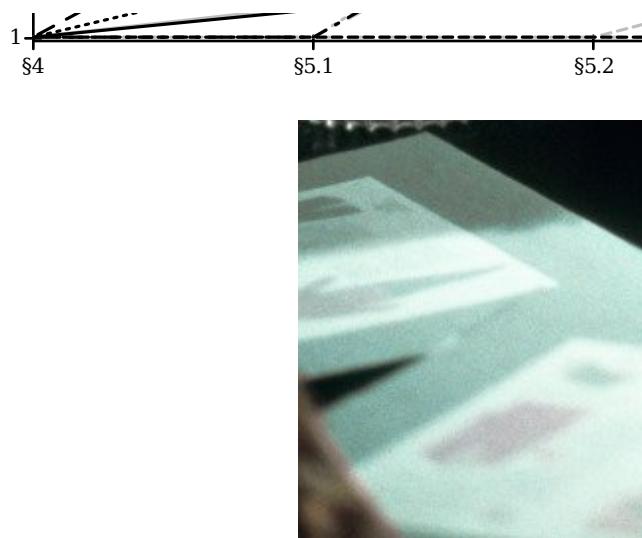
$\llbracket \cdot \rrbracket : Expr \rightarrow Store$
 $\rightarrow Env \times Store \Delta \times Kont \times Time$
 $\rightarrow State$
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 $\llbracket \text{var}(x) \rrbracket_\sigma = \lambda^t(\rho, \xi, \kappa). \text{co}(\kappa, \text{addr}(\rho(x))), \xi$
 $\llbracket \text{lit}(l) \rrbracket_\sigma = \lambda^t(\rho, \xi, \kappa). \text{co}(\kappa, l), \xi$
 $\llbracket \text{lam}(x, e) \rrbracket_\sigma = \lambda^t(\rho, \xi, \kappa). \text{co}(\kappa, \text{clos}(x, \llbracket e \rrbracket, \rho)), \xi$
 $\llbracket \text{app}^\ell(e_0, e_1) \rrbracket_\sigma = \lambda^t(\rho, \xi, \kappa). \llbracket e_0 \rrbracket^{t'}(\rho, \xi', \text{arg}_\ell^t(\llbracket e_1 \rrbracket, \rho, a_\kappa))$
 where $a_\kappa = \text{alloc}_\ell^t(\sigma, \kappa)$
 $\xi' = (a_\kappa, \{\kappa\}): \xi$
 $\llbracket \text{if}^\ell(e_0, e_1, e_2) \rrbracket_\sigma = \lambda^t(\rho, \xi, \kappa). \llbracket e_0 \rrbracket^{t'}(\rho, \xi', \text{ifk}^t(\llbracket e_1 \rrbracket, \llbracket e_2 \rrbracket, \rho, a_\kappa))$
 where $a_\kappa = \text{alloc}_\ell^t(\sigma, \kappa)$
 $\xi' = (a_\kappa, \{\kappa\}): \xi$

Figure 6. Abstract compilation

$traces(e) = \{ \text{inject}(\llbracket e \rrbracket_{\perp}^{t_0}(\perp, \epsilon, \text{halt})) \mapsto \varsigma \}$ where
 $\text{inject}(c, \xi) = \text{wn}(c, \text{replay}(\xi, \perp))$
 $\text{wn}(c, \sigma) \mapsto \text{wn}(c', \sigma') \iff c \llbracket \mapsto \rrbracket_\sigma c', \xi$
 ξ is such that $\text{replay}(\xi, \sigma) = \sigma'$
 $\text{co}(\text{arg}_\ell^t(k, \rho, a_\kappa), v) \llbracket \mapsto \rrbracket_\sigma k^t(\sigma)(\rho, \xi, \text{fun}_\ell^t(a_f, a_\kappa))$
 where $a_f = \text{alloc}(\varsigma)$
 $\xi = (a_f, \text{force}(\sigma, v)): \epsilon$
 $\text{co}(\text{fun}_\ell^t(a_f, a_\kappa), v) \llbracket \mapsto \rrbracket_\sigma \text{ap}_\ell^t(u, a, \kappa), (a, \text{force}(\sigma, v)): \epsilon$
 if $u \in \sigma(a_f), \kappa \in \sigma(a_\kappa)$
 $\text{co}(\text{ifk}^t(k_0, k_1, \rho, a_\kappa), \text{tt}) \llbracket \mapsto \rrbracket_\sigma k_0^t(\sigma)(\rho, \epsilon, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
 $\text{co}(\text{ifk}^t(k_0, k_1, \rho, a_\kappa), \text{ff}) \llbracket \mapsto \rrbracket_\sigma k_1^t(\sigma)(\rho, \epsilon, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
 $\text{ap}_\ell^t(\text{clos}(x, k, \rho), a, \kappa) \llbracket \mapsto \rrbracket_\sigma k^t(\sigma)(\rho', \xi, \kappa)$
 where $\rho' = \rho[x \mapsto a]$
 $\xi = (a, \sigma(a)): \epsilon$

$traces(e) = \{\text{ev}^{t_0}(e, \emptyset, \emptyset, \text{halt}) \mapsto \varsigma\}$ where
 $\varsigma \mapsto \varsigma'$ defined to be the following
let $t' = \text{tick}(\varsigma)$
 $\text{ev}(\text{var}(x), \rho, \sigma, \kappa) \mapsto \text{co}(\kappa, v, \sigma)$ if $v \in \sigma(\rho(x))$
 $\text{ev}(\text{lit}(l), \rho, \sigma, \kappa) \mapsto \text{co}(\kappa, l, \sigma)$
 $\text{ev}^t(\text{lam}(x, e), \rho, \sigma, \kappa) \mapsto \text{co}(\kappa, \text{clos}(x, e, \rho), \sigma)$
 $\text{ev}^t(\text{app}^\ell(e_0, e_1), \rho, \sigma, \kappa) \mapsto \text{ev}^{t'}(e_0, \rho, \sigma', \text{arg}_\ell^t(e_1, \rho, a_\kappa))$
where $a_\kappa = \text{alloc}_\ell^t(\sigma, \kappa)$
 $\sigma' = \sigma \sqcup [a_\kappa \mapsto \{\kappa\}]$
 $\text{ev}^t(\text{if}^\ell(e_0, e_1, e_2), \rho, \sigma, \kappa) \mapsto \text{ev}^{t'}(e_0, \rho, \sigma', \text{ifk}^t(e_1, e_2, \rho, a_\kappa))$
where $a_\kappa = \text{alloc}_\ell^t(\sigma, \kappa)$
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 $\text{co}(\text{arg}_\ell^t(e, \rho, a_\kappa), v, \sigma) \mapsto \text{ev}^t(e, \rho, \sigma, \text{fun}_\ell^t(v, a_\kappa))$
 $\text{co}(\text{fun}_\ell^t(u, a_\kappa), v, \sigma) \mapsto \text{ap}_\ell^t(u, v, \kappa, \sigma)$ if $\kappa \in \sigma(a_\kappa)$
 $\text{co}(\text{ifk}^t(e_0, e_1, \rho, a_\kappa), \text{tt}, \sigma) \mapsto \text{ev}^{t'}(e_0, \rho, \sigma, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
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Figure 5. Abstract abstract machine for ISWIM

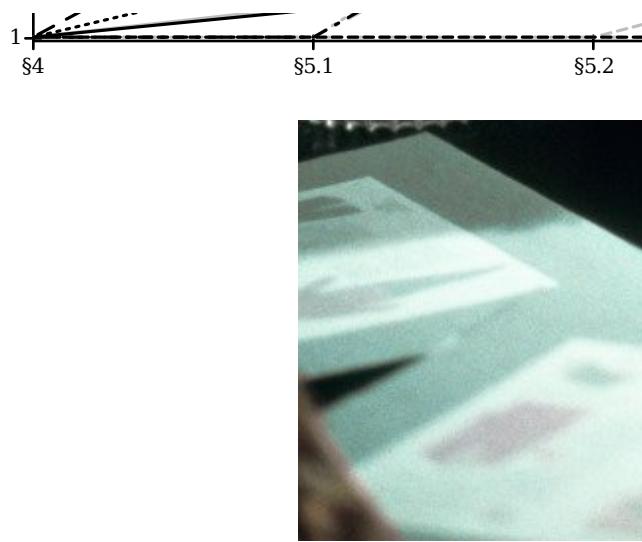


$\llbracket \cdot \rrbracket : Ex$
 $lookup(V, t) = \begin{cases} vs & \text{if } V = (t', vs):V', t' \leq t \\ vs' & \text{if } V = (t', vs):(t'', vs'):V', t' < t'' \end{cases}$
 $t' = ti$
 $\llbracket \text{var}(x) \rrbracket_\sigma = \lambda^t$
 $\llbracket \text{lit}(l) \rrbracket_\sigma = \lambda^t$
 $\llbracket \text{lam}(x, e) \rrbracket_\sigma = \lambda^t$
 $\llbracket \text{app}^\ell(e_0, e_1) \rrbracket_\sigma = \lambda^t$
 $\llbracket \text{if}^\ell(e_0, e_1, e_2) \rrbracket_\sigma = \lambda^t$
 $\llbracket \text{co}(\text{arg}_\ell^t(e, \rho, a_\kappa), v, \sigma) \rrbracket_\sigma = \text{ev}^t(e, \rho, \sigma, \text{fun}_\ell^t(v, a_\kappa))$
 $\llbracket \text{co}(\text{fun}_\ell^t(u, a_\kappa), v, \sigma) \rrbracket_\sigma = \text{ap}_\ell^t(u, v, \kappa, \sigma)$ if $\kappa \in \sigma(a_\kappa)$
 $\llbracket \text{co}(\text{ifk}^t(e_0, e_1, \rho, a_\kappa), \text{tt}, \sigma) \rrbracket_\sigma = \text{ev}^{t'}(e_0, \rho, \sigma, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
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 $\llbracket \text{ap}_\ell^t(\text{clos}(x, e, \rho), v, \sigma, \kappa) \rrbracket_\sigma = \text{ev}^{t'}(e, \rho', \sigma', \kappa)$
where $a = \text{alloc}(\varsigma)$
 $\rho' = \rho[x \mapsto a]$
 $\sigma' = \sigma \sqcup [a \mapsto \{v\}]$
 $\llbracket \text{co}(\text{fun}_\ell^t(a_f, a_\kappa), v) \rrbracket_\sigma = \text{ap}_\ell^t(u, a, \kappa), (a, \text{force}(\sigma, v)):\epsilon$
if $u \in \sigma(a_f), \kappa \in \sigma(a_\kappa)$
 $\llbracket \text{co}(\text{ifk}^t(k_0, k_1, \rho, a_\kappa), \text{tt}) \rrbracket_\sigma = \text{k}_0^t(\sigma)(\rho, \epsilon, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
 $\llbracket \text{co}(\text{ifk}^t(k_0, k_1, \rho, a_\kappa), \text{ff}) \rrbracket_\sigma = \text{k}_1^t(\sigma)(\rho, \epsilon, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
 $\llbracket \text{ap}_\ell^t(\text{clos}(x, k, \rho), a, \kappa) \rrbracket_\sigma = \text{k}^{t'}(\sigma)(\rho', \xi, \kappa)$
where $\rho' = \rho[x \mapsto a]$
 $\xi = (a, \sigma(a)):\epsilon$

Figure 5

$traces(e) = \{\text{ev}^{t_0}(e, \emptyset, \emptyset, \text{halt}) \mapsto \varsigma\}$ where
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 $\text{ev}^t(\text{lam}(x, e), \rho, \sigma, \kappa) \mapsto \text{co}(\kappa, \text{clos}(x, e, \rho), \sigma)$
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 $\text{co}(\text{fun}_\ell^t(u, a_\kappa), v, \sigma) \mapsto \text{ap}_\ell^t(u, v, \kappa, \sigma)$ if $\kappa \in \sigma(a_\kappa)$
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Figure 5. Abstract abstract machine for ISWIM



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 $\llbracket \text{co}(\text{fun}_\ell^t(u, a_\kappa), v, \sigma) \rrbracket_\sigma = \text{ap}_\ell^t(u, v, \kappa, \sigma)$ if $\kappa \in \sigma(a_\kappa)$
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 $\llbracket \text{ap}_\ell^t(\text{clos}(x, e, \rho), v, \sigma, \kappa) \rrbracket_\sigma = \text{ev}^{t'}(e, \rho', \sigma', \kappa)$
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 $\sigma' = \sigma \sqcup [a \mapsto \{v\}]$
 $\llbracket \text{ap}_\ell^t(o, v, \sigma, \kappa) \rrbracket_\sigma = \text{co}(\kappa, v', \sigma)$ if $v' \in \Delta(o, v)$

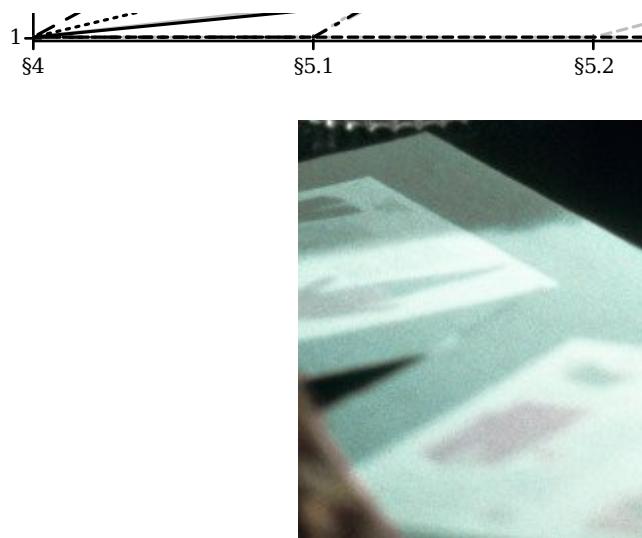
Figure 6. Abstract abstract machine for ISWIM

$traces(e)$
 $\text{inject}(c, \xi)$
 $wn(c, \sigma) \mapsto wn(c', \sigma')$
 ξ is such that
 $\text{co}(\text{arg}_\ell^t(k, \rho, c), \sigma) \mapsto \text{wn}(c', \sigma')$
wh
 $\text{co}(\text{fun}_\ell^t(a_f, a_\kappa), v) \llbracket \mapsto \rrbracket_\sigma \text{ap}_\ell^t(u, a, \kappa), (a, \text{force}(\sigma, v)):\epsilon$
if $u \in \sigma(a_f), \kappa \in \sigma(a_\kappa)$
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 $\text{ap}_\ell^t(\text{clos}(x, k, \rho), a, \kappa) \llbracket \mapsto \rrbracket_\sigma k^t(\sigma)(\rho', \xi, \kappa)$
where $\rho' = \rho[x \mapsto a]$
 $\xi = (a, \sigma(a)):\epsilon$

$\text{System} = (\widehat{\text{State}} \rightarrow \mathbb{N}^*) \times \omega(\widehat{\text{State}}) \times \text{State} \times$
 $replay([(a_i, vs_i), \dots], \sigma) = (\sigma', \delta?(vs_i, \sigma(a_i)) \vee \dots)$
where $\sigma' = \sigma \sqcup [a_i \mapsto vs_i] \sqcup \dots$
 $\delta?(vs, vs') = vs' \stackrel{?}{=} vs \sqcup vs'$
 $\sigma' = \text{mu}.lookup(\sigma(a_j), i_j)$
 $(\sigma', \Delta?) = replay(\text{appendall}(\{\xi \mid (_, \xi) \in I\}), \sigma)$
 $t' = \begin{cases} t+1 & \text{if } \Delta? \\ t & \text{otherwise} \end{cases}$
 $F' = \{c \mid (c, _) \in I, \Delta? \vee \hat{S}(c) \neq t:_\}$
 $\hat{S}' = \lambda c. \begin{cases} t':\hat{S}(c) & \text{if } c \in F' \\ \hat{S}(c) & \text{otherwise} \end{cases}$
 $s = (u_f, j, \sigma, \text{co}(v, w), \epsilon)$

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 $\text{ev}^t(\text{lam}(x, e), \rho, \sigma, \kappa) \mapsto \text{co}(\kappa, \text{clos}(x, e, \rho), \sigma)$
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 $\text{co}(\text{arg}_\ell^t(e, \rho, a_\kappa), v, \sigma) \mapsto \text{ev}^t(e, \rho, \sigma, \text{fun}_\ell^t(v, a_\kappa))$
 $\text{co}(\text{fun}_\ell^t(u, a_\kappa), v, \sigma) \mapsto \text{ap}_\ell^t(u, v, \kappa, \sigma)$ if $\kappa \in \sigma(a_\kappa)$
 $\text{co}(\text{ifk}^t(e_0, e_1, \rho, a_\kappa), \text{tt}, \sigma) \mapsto \text{ev}^{t'}(e_0, \rho, \sigma, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
 $\text{co}(\text{ifk}^t(e_0, e_1, \rho, a_\kappa), \text{ff}, \sigma) \mapsto \text{ev}^{t'}(e_0, \rho, \sigma, \kappa)$
 $\text{ap}_\ell^t(\text{clos}(x, e, \rho), v, \sigma, \kappa) \mapsto \text{ev}^{t'}(e, \rho, \sigma, \kappa)$
 where
 $\text{ap}_\ell^t(o, v, \sigma, \kappa) \mapsto \text{co}(\kappa, v', \sigma)$ if $v' \in \Delta(o, v)$

Figure 5. Abstract abstract machine for ISWIM



$\llbracket \cdot \rrbracket : Ex$
 $lookup(V, t) = \begin{cases} vs & \text{if } V = (t', vs):V', t' \leq t \\ vs' & \text{if } V = (t', vs):(t'', vs'):V', t' < t'' \end{cases}$
 $t' = ti$
 $\llbracket \text{var}(x) \rrbracket_\sigma = \lambda^i$
 $\llbracket \text{lit}(l) \rrbracket_\sigma = \lambda^i$
 $\llbracket \text{lam}(x, e) \rrbracket_\sigma = \lambda^i$
 $\llbracket \text{app}^\ell(e_0, e_1) \rrbracket_\sigma = \lambda^i$
 $\llbracket \text{if}^\ell(e_0, e_1, e_2) \rrbracket_\sigma = \lambda^i$
 $V \sqcup_t vs = V, \text{tt}$ if $vs_t \neq vs^*$
 $\text{where } vs_t = lookup(\sigma(a), t)$
 $vs^* = vs \sqcup vs_t$
 $V \sqcup_t vs = V, \text{ff}$ otherwise

$\widehat{\text{State}} \rightarrow \mathbb{N}^*$ $\vee \text{val}(\widehat{\text{State}}) \vee \text{State} \vee$
 $\cdot, \sigma) = (\sigma', \delta?(vs_i, \sigma(a_i)) \vee \dots)$
 $\text{where } \sigma' = \sigma \sqcup [a_i \mapsto vs_i] \sqcup \dots$
 $\text{ap}_\ell^t(\cdot, vs') = vs' \stackrel{?}{=} vs \sqcup vs'$
 $\sigma' = \text{append}(\sigma(u), v)$

$traces(e)$
 $\text{inject}(c, \xi)$
 $wn(c, \sigma) \mapsto wn(c', \sigma')$
 ξ is such that
 $\text{co}(\text{arg}_\ell^t(k, \rho, c)) \mapsto wn(c', \sigma')$
 wh
 $F' = \{c \mid (c, _) \in I, \Delta? \vee \hat{S}(c) \neq t:_\}$
 $\hat{S}' = \lambda c. \begin{cases} t':\hat{S}(c) & \text{if } c \in F' \\ \hat{S}(c) & \text{otherwise} \end{cases}$
 $s = (u_f, force(v, u), v, j, c)$
 $\text{co}(\text{fun}_\ell^t(a_f, a_\kappa), v) \llbracket \mapsto \rrbracket_\sigma \text{ap}_\ell^t(u, a, \kappa), (a, force(\sigma, v)):v$
 if $u \in \sigma(a_f), \kappa \in \sigma(a_\kappa)$
 $\text{co}(\text{ifk}^t(k_0, k_1, \rho, a_\kappa), \text{tt}) \llbracket \mapsto \rrbracket_\sigma k_0^t(\sigma)(\rho, \epsilon, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
 $\text{co}(\text{ifk}^t(k_0, k_1, \rho, a_\kappa), \text{ff}) \llbracket \mapsto \rrbracket_\sigma k_1^t(\sigma)(\rho, \epsilon, \kappa)$ if $\kappa \in \sigma(a_\kappa)$
 $\text{ap}_\ell^t(\text{clos}(x, k, \rho), a, \kappa) \llbracket \mapsto \rrbracket_\sigma k^t(\sigma)(\rho', \xi, \kappa)$
 where $\rho' = \rho[x \mapsto a]$
 $\xi = (a, \sigma(a)):\epsilon$

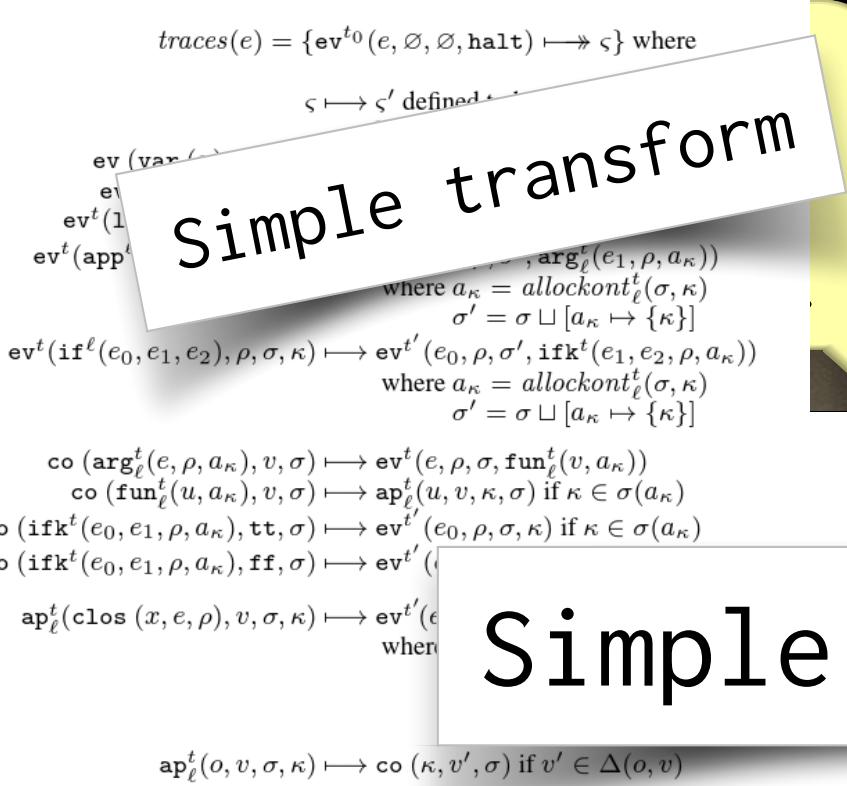
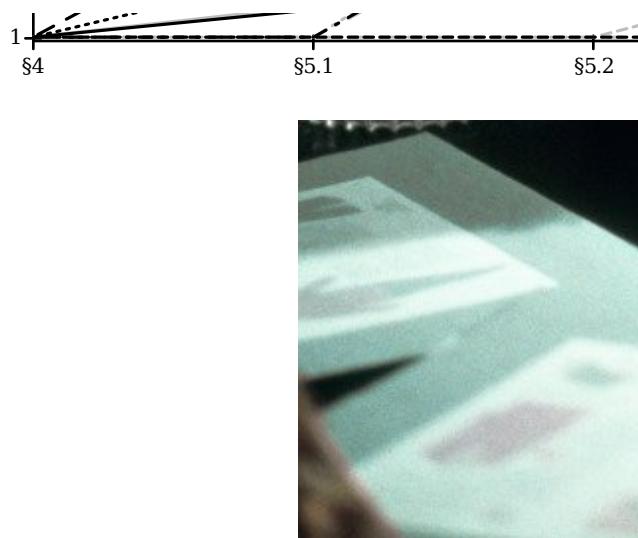


Figure 5. Abstract abstract machine for ISWIM



$\llbracket . \rrbracket : Ex$

$lookup(V, t) = \begin{cases} vs & \text{if } V = (t', vs):V', t' \leq t \\ vs' & \text{if } V = (t', vs):(t'', vs'):V', t' < t'' \end{cases}$

$t' = ti$

$\llbracket \text{var } (x) \rrbracket_\sigma = \lambda^t$

$\llbracket \text{lit } (l) \rrbracket_\sigma = \lambda^t$

$\llbracket \text{lam } (x, e) \rrbracket_\sigma = \lambda^t$

$\llbracket \text{app }^\ell(e_0, e_1) \rrbracket_\sigma = \lambda^t$
where

$\llbracket \text{if } \ell(e_0, e_1, e_2) \rrbracket_\sigma = \lambda^t$
where

$\sigma \sqcup_t [a \mapsto vs] = \sigma[a \mapsto V], \Delta?$
where $(V, \Delta?) = \sigma(a) \sqcup_t vs$

$\epsilon \sqcup_t vs = (t, vs), \text{tt}$

$(t', vs):V \sqcup_t vs' = (t', vs \sqcup vs'):V, \delta?(vs, vs') \text{ if } t' < t''$
 $V \sqcup_t vs = (t + 1, vs^*):V, \text{tt} \text{ if } vs_t \neq vs^*$
where $vs_t = lookup(\sigma(a), t)$

$vs^* = vs \sqcup vs_t$

$V \sqcup_t vs = V, \text{ff} \text{ otherwise}$

$\widehat{\text{State}} \rightarrow \mathbb{N}^* \setminus \{0\} \cup \{\widehat{\text{State}}\} \cup \text{State} \cup \dots$

$\llbracket . \rrbracket, \sigma) = (\sigma', \delta?(vs_i, \sigma(a_i)) \vee \dots)$
where $\sigma' = \sigma \sqcup [a_i \mapsto vs_i] \sqcup \dots$

$\text{ap}_\ell^t(o, v, \sigma, \kappa) = vs' \stackrel{?}{=} vs \sqcup vs'$
 $\sigma' = \text{appendup}(\sigma(a), v)$

$(\sigma', \Delta?) = \text{replay}(\text{appendall}(\{\xi \mid (_, \xi) \in I\}), \sigma)$

$t' = \begin{cases} t + 1 & \text{if } \Delta? \\ t & \text{otherwise} \end{cases}$

$F' = \{c \mid (c, _) \in I, \Delta? \vee \hat{S}(c) \neq t:_\}$

$\hat{S}' = \lambda c. \begin{cases} t':\hat{S}(c) & \text{if } c \in F' \\ \hat{S}(c) & \text{otherwise} \end{cases}$

$s = (u_f, force(v, u), _, \epsilon)$

$\text{co } (\text{fun}_\ell^t(a_f, a_\kappa), v) \llbracket \mapsto \rrbracket_\sigma \text{ap}_\ell^t(u, a, \kappa), (a, force(\sigma, v)):v$
if $u \in \sigma(a_f), \kappa \in \sigma(a_\kappa)$

$\text{co } (\text{ifk}^t(k_0, k_1, \rho, a_\kappa), \text{tt}) \llbracket \mapsto \rrbracket_\sigma k_0^t(\sigma)(\rho, \epsilon, \kappa) \text{ if } \kappa \in \sigma(a_\kappa)$
 $\text{co } (\text{ifk}^t(k_0, k_1, \rho, a_\kappa), \text{ff}) \llbracket \mapsto \rrbracket_\sigma k_1^t(\sigma)(\rho, \epsilon, \kappa) \text{ if } \kappa \in \sigma(a_\kappa)$

$\text{ap}_\ell^t(\text{clos } (x, k, \rho), a, \kappa) \llbracket \mapsto \rrbracket_\sigma k^t(\sigma)(\rho', \xi, \kappa)$
where $\rho' = \rho[x \mapsto a]$

$\xi = (a, \sigma(a)):\epsilon$

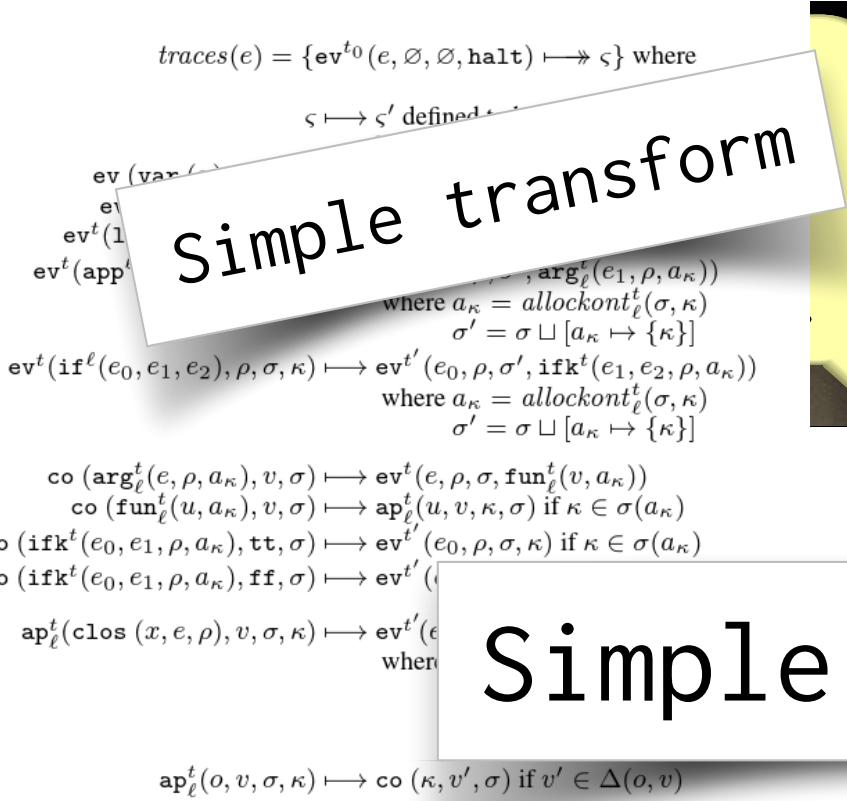


Figure 5. Abstract abstract machine for ISWIM



$\llbracket . \rrbracket : Ex$

$lookup(V, t) = \begin{cases} vs & \text{if } V = (t', vs):V', t' \leq t \\ vs' & \text{if } V = (t', vs):(t'', vs'):V', t' < t'' \end{cases}$

$\sigma \sqcup \epsilon \sqcup \Delta? \quad V \sqcup \Delta?$

Simple transform

$t' = ti$

$\llbracket \text{var } (x) \rrbracket_\sigma = \lambda^t$

$\llbracket \text{lit } (l) \rrbracket_\sigma = \lambda^t$

$\llbracket \text{lam } (x, e) \rrbracket_\sigma = \lambda^t$

$\llbracket \text{app }^\ell(e_0, e_1) \rrbracket_\sigma = \lambda^t$
where

$\llbracket \text{if } \ell(e_0, e_1, e_2) \rrbracket_\sigma = \lambda^t$
where

$(t', vs):V \sqcup_t vs' = (t', vs \sqcup vs') : V$
 $V \sqcup_t vs = (t + 1, vs^*):V, \text{tt} \text{ if } vs_t \neq vs^*$
where $vs_t = lookup(\sigma(a), t)$

$vs^* = vs \sqcup vs_t$

$V \sqcup_t vs = V, \text{ff} \text{ otherwise}$

$\widehat{\text{State}} \rightarrow \mathbb{N}^* \setminus \{0\} \times \omega(\widehat{\text{State}}) \times \text{State} \times \dots$

$\llbracket . \rrbracket, \sigma) = (\sigma', \delta?(vs_i, \sigma(a_i)) \vee \dots)$
where $\sigma' = \sigma \sqcup [a_i \mapsto vs_i] \sqcup \dots$

$\text{ap}_\ell^t(o, v, \sigma, \kappa) = vs' \stackrel{?}{=} vs \sqcup vs'$
 $\sigma' = \text{ap}_\ell^t(o, v, \kappa, \sigma)$

$(\sigma', \Delta?) = replay(\text{appendall}(\{\xi \mid (_, \xi) \in I\}), \sigma)$

$t' = \begin{cases} t + 1 & \text{if } \Delta? \\ t & \text{otherwise} \end{cases}$

$F' = \{c \mid (c, _) \in I, \Delta? \vee \hat{S}(c) \neq t:_\}$

$\hat{S}' = \lambda c. \begin{cases} t':\hat{S}(c) & \text{if } c \in F' \\ \hat{S}(c) & \text{otherwise} \end{cases}$

$s = (u_f, force(v, v_f), \dots)$

$\text{co } (\text{fun}_\ell^t(a_f, a_\kappa), v) \llbracket \mapsto \rrbracket_\sigma \text{ap}_\ell^t(u, a, \kappa), (a, force(\sigma, v)):v$
if $u \in \sigma(a_f), \kappa \in \sigma(a_\kappa)$

$\text{co } (\text{ifk}^t(k_0, k_1, \rho, a_\kappa), \text{tt}) \llbracket \mapsto \rrbracket_\sigma k_0^t(\sigma)(\rho, \epsilon, \kappa) \text{ if } \kappa \in \sigma(a_\kappa)$

$\text{co } (\text{ifk}^t(k_0, k_1, \rho, a_\kappa), \text{ff}) \llbracket \mapsto \rrbracket_\sigma k_1^t(\sigma)(\rho, \epsilon, \kappa) \text{ if } \kappa \in \sigma(a_\kappa)$

$\text{ap}_\ell^t(\text{clos } (x, k, \rho), a, \kappa) \llbracket \mapsto \rrbracket_\sigma k^t(\sigma)(\rho', \xi, \kappa)$
where $\rho' = \rho[x \mapsto a]$

$\xi = (a, \sigma(a)):\epsilon$

$traces(e) = \{ \text{ev}^{t_0}(e, \emptyset, \emptyset, \text{halt}) \mapsto \varsigma \}$ where
 $\varsigma \mapsto \varsigma' \text{ defined by}$

- $\text{ev}(\text{var } v) \mapsto v$
- $\text{ev}^t(1) \mapsto 1$
- $\text{ev}^t(\text{app}^t) \mapsto \text{app}^t$
- $\text{ev}^t(\text{if}^\ell(e_0, e_1, e_2), \rho, \sigma, \kappa) \mapsto \text{ev}^{t'}(e_0, \rho, \sigma')$ where $a_\kappa = \sigma' = e_1$
- $\text{co}(\text{arg}_\ell^t(e, \rho, a_\kappa), v, \sigma) \mapsto \text{ev}^t(e, \rho, \sigma, f)$
- $\text{co}(\text{fun}_\ell^t(u, a_\kappa), v, \sigma) \mapsto \text{ap}_\ell^t(u, v, \kappa, \sigma)$
- $\text{co}(\text{ifk}^t(e_0, e_1, \rho, a_\kappa), \text{tt}, \sigma) \mapsto \text{ev}^{t'}(e_0, \rho, \sigma)$
- $\text{co}(\text{ifk}^t(e_0, e_1, \rho, a_\kappa), \text{ff}, \sigma) \mapsto \text{ev}^{t'}(e_1, \rho, \sigma)$
- $\text{ap}_\ell^t(\text{clos}(x, e, \rho), v, \sigma, \kappa) \mapsto \text{ev}^{t'}(e, \rho, \sigma')$ where $\sigma' = \sigma \cup [x \mapsto v]$
- $\text{ap}_\ell^t(o, v, \sigma, \kappa) \mapsto \text{co}(\kappa, v', \sigma) \text{ if } v' \in \Delta(o, v)$

$\llbracket \cdot \rrbracket : Ex$

$lookup(V, t) = \begin{cases} vs & \text{if } V = (t', vs) : V', t' \leq t \\ vs' & \text{if } V = (t', vs) : (t'', vs') : V', t' : t'' \end{cases}$

$t' = t_i \quad \sigma \sqcup \quad \text{if } \Delta?$

$\llbracket \cdot \rrbracket : Ex$

$\llbracket t \rrbracket = \begin{cases} \sqcup_t vs' = (t', vs \sqcup vs') : V, \text{tt} & \text{if } t' \leq t \\ \sqcup_t vs = (t + 1, vs^*) : V, \text{ff} & \text{if } vs_t \neq vs^* \text{ where } vs_t = lookup(\sigma(a), t) \\ vs^* = vs \sqcup vs_t & \\ \sqcup_t vs = V, \text{ff} & \text{otherwise} \end{cases}$

$\widehat{\text{State}} \rightarrow \mathbb{N}^*$ $\vee \widehat{\text{Label}}$ $\vee \widehat{\text{Store}}$ $\vee \widehat{\cdot}, \sigma) = (\sigma', \delta?(vs_i, \sigma(a_i)) \vee \dots)$
 $\text{where } \sigma' = \sigma \sqcup [a_i \mapsto vs_i] \sqcup \dots$
 $\widehat{(o, vs')} = vs' \stackrel{?}{=} vs \sqcup vs'$
 $\widehat{o} = \text{closure}(o, vs)$

Figure 5. Abstract abstract machine for ISWIM

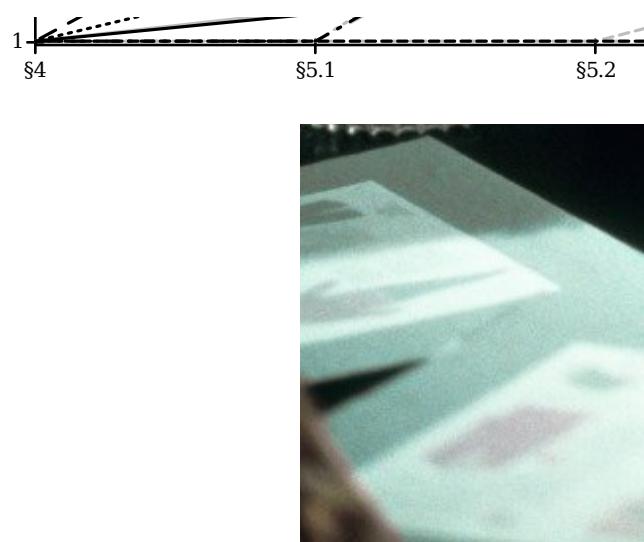




Figure 5. Abstracting away from state

1
§4

Human compiler





*“What we hope ever to do with ease,
we must first learn to do with diligence.”*

~SAMUEL JOHNSON

I made a language

I made a language

Pattern \mapsto Expression [Side-conditions]

I made a language

Pattern \mapsto Expression [Side-conditions]

Equality is built-in

I made a language

Pattern \mapsto Expression [Side-conditions]

Equality is built-in

First-order metafunctions

I made a language

Pattern \mapsto Expression [Side-conditions]

Equality is built-in

First-order metafunctions

All allocation is mediated

I made a language

Pattern \mapsto Expression [Side-conditions]

Equality is built-in

First-order metafunctions

All allocation is mediated

Any allocation strategy is sound

I made a language

Pattern \mapsto Expression [Side-conditions]

Equality is built-in

First-order metafunctions

All allocation is mediated

Any allocation strategy is sound

Fresh allocation ensures completeness

My language in a slide

My language in a slide



My language in a slide



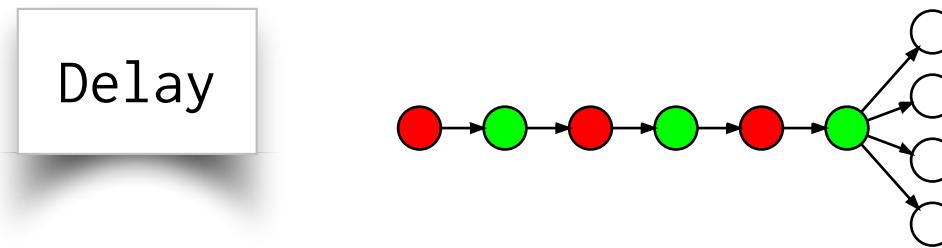
My language in a slide



My language in a slide

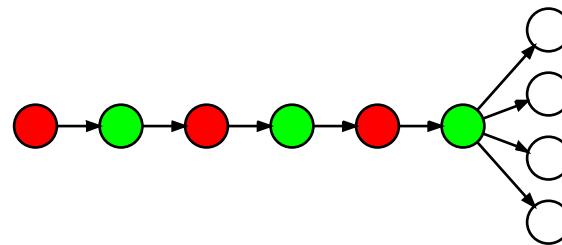


My language in a slide

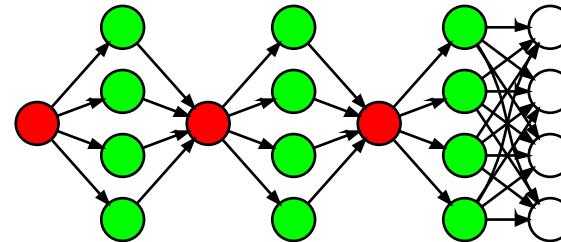


My language in a slide

Delay

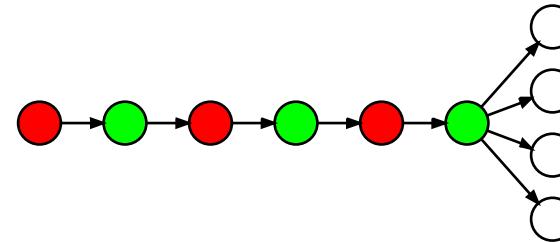


Resolve

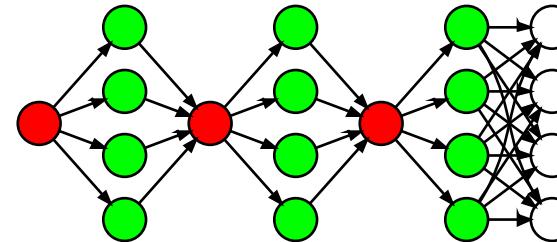


My language in a slide

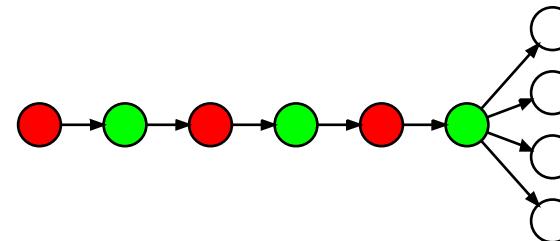
Delay



Resolve

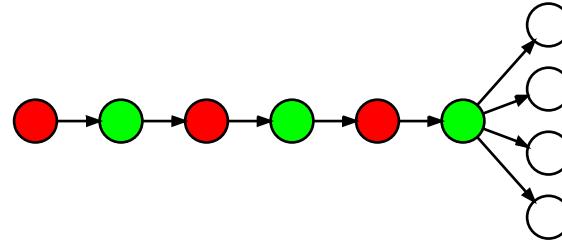


Deref

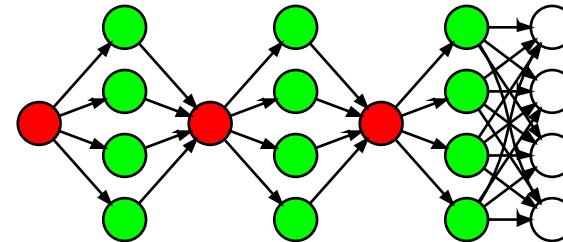


My language in a slide

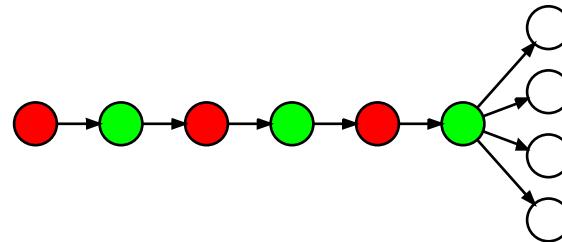
Delay



Resolve



Deref



Different behavior for fresh addresses

Definition: An abstract machine is

A finite set of rules

An address allocation function

A compound term allocation function

A collection of metafunctions

A function

An initial value

Todo:

Language design

Todo:

Related work

Todo:

Conclusion/future work

Todo:

What was proposed, what did