CS2800 Homework 8 Extra Problems

Definitions

Below are definitions that will be used in the problem statements. `fact`, `app`, etc. are omitted here since they are given in the homework.

```
;; choose : nat x nat -> nat
;; Binomial choose function
(defun choose (n k)
  (/ (fact n) (* (fact k) (fact (- n k)))))

;; posp : any -> Boolean
;; T iff given a positive integer
(defun posp (n)
  (not (zp n)))
```

Problems

Prove the following theorems. If you use induction, clearly indicate what functions were used to generate the induction schemes.

1.
```
(implies (and (posp n)
              (posp k)
              (< k n))
         (equal (+ (choose (- n 1) (- k 1)) (choose (- n 1) k))
                (choose n k)))
```

The above is not a theorem without the hypotheses. Why not?

2.
```
(implies (and (natp n)
              (natp k)
              (<= k n))
         (natp (choose n k)))
```

3.
```
(booleanp (in a X))
```

4.
```
(implies (and (not (= a b))
              (in a (rem-el b X)))
         (in a X))
```

5.
```
(implies (and (not (= a b))
              (in a X))
         (in a (rem-el b X)))
```

6.
```
(implies (not (= a b))
         (= (in a (rem-el b X))
            (in a X)))
```

7.
```
(implies (in a X)
         (not (in a (diff Y X))))
```

8.
```
(implies (consp X)
         (not (=< X (diff Y X))))
```