Exam 2 Review

CS2800

2009-10-01

1 Vocabulary and Definitions

Propositional logic deals with only two values, T and F, denoting the constant expressions true and false. There are *connectives* that are functions of n Booleans that return T or F.

The connectives we introduced in class are

<table>
<tr><th>English</th><th>Connective</th><th>A</th><th>B</th><th>C</th><th>Result</th></tr>
<tr><td rowspan="2">“not A”</td><td rowspan="2">$\neg A$</td><td>T</td><td rowspan="2">*</td><td rowspan="2">*</td><td>F</td></tr>
<tr><td>F</td><td>T</td></tr>
<tr><td rowspan="4">“A and B”</td><td rowspan="4">$A \wedge B$</td><td>T</td><td>T</td><td rowspan="4">*</td><td>T</td></tr>
<tr><td>T</td><td>F</td><td>F</td></tr>
<tr><td>F</td><td>T</td><td>F</td></tr>
<tr><td>F</td><td>F</td><td>F</td></tr>
<tr><td rowspan="4">“A or B” or “A unless B”</td><td rowspan="4">$A \vee B$</td><td>T</td><td>T</td><td rowspan="4">*</td><td>T</td></tr>
<tr><td>T</td><td>F</td><td>T</td></tr>
<tr><td>F</td><td>T</td><td>T</td></tr>
<tr><td>F</td><td>F</td><td>F</td></tr>
<tr><td rowspan="4">“exactly one of A or B”
or “only A or only B”</td><td rowspan="4">$A \oplus B$</td><td>T</td><td>T</td><td rowspan="4">*</td><td>F</td></tr>
<tr><td>T</td><td>F</td><td>T</td></tr>
<tr><td>F</td><td>T</td><td>T</td></tr>
<tr><td>F</td><td>F</td><td>F</td></tr>
<tr><td rowspan="4">“B if A” or “if A, then B” or
“A implies B” or “A, therefore B”</td><td rowspan="4">$A \Rightarrow B$</td><td>T</td><td>T</td><td rowspan="4">*</td><td>T</td></tr>
<tr><td>T</td><td>F</td><td>F</td></tr>
<tr><td>F</td><td>T</td><td>T</td></tr>
<tr><td>F</td><td>F</td><td>T</td></tr>
<tr><td rowspan="4">“A equals B”</td><td rowspan="4">$A \equiv B$</td><td>T</td><td>T</td><td rowspan="4">*</td><td>T</td></tr>
<tr><td>T</td><td>F</td><td>F</td></tr>
<tr><td>F</td><td>T</td><td>F</td></tr>
<tr><td>F</td><td>F</td><td>T</td></tr>
<tr><td rowspan="4">“B only if A”</td><td rowspan="4">$B \Rightarrow A$</td><td>T</td><td>T</td><td rowspan="4">*</td><td>T</td></tr>
<tr><td>T</td><td>F</td><td>T</td></tr>
<tr><td>F</td><td>T</td><td>F</td></tr>
<tr><td>F</td><td>F</td><td>T</td></tr>
<tr><td rowspan="8">“if A then B, else C”</td><td rowspan="8">$ite(A, B, C)$</td><td>T</td><td>T</td><td>T</td><td>T</td></tr>
<tr><td>T</td><td>T</td><td>F</td><td>T</td></tr>
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See the lecture notes for more terms. (`http://www.ccs.neu.edu/course/cs2800/Boolean-Logic-Notes.pdf`)

2 Propositional logic

Beyond evaluation of propositional formulas, you will need to be able to simplify formulas and construct formulas describing the logic of English statements. The following are practice problems with the answers on the final page.

2.1 Simplification

We say a formula is simplified when it is described with the fewest possible connectives (in this case we allow $\equiv$, *ite* and $\oplus$). One strategy to simplify a formula is to construct a truth table for the formula and infer a simpler formula that gives the same truth table. Another is to perform *validity-preserving transformations* to the formula, as can be found on page 12 of the lecture notes, section 6 "Useful Equalities". Since these transformations are validity-preserving, we can interchange the left and right sides of the equation. Of course there are (infinitely) many more transformations like this, since all valid equalities are essentially "transformations" in propositional logic.

Simplify the following formulas with any method you choose, as long as your logical steps are justified (commutativity and associativity rules need not be named).

1. $(A \equiv (B \vee C)) \wedge B \wedge ((C \vee \neg A) \Rightarrow B)$
2. $(P \Rightarrow (Q \Rightarrow R)) \equiv \neg P \vee \neg Q \vee R$
3. $(\neg A \vee (B \wedge ite(B, (C \vee \neg(C \Rightarrow A), A)))) \Rightarrow (\neg A \vee (\neg B \vee C))$
4. $\neg((\neg(P \vee \neg Q) \vee R) \vee S)$
5. $(A \vee \neg B \vee C \vee D) \wedge (\neg B \vee \neg A) \wedge (C \vee D \vee B)$

2.2 Logical English

English in general is very ambiguous, but if we choose our words carefully, we can construct unambiguous statements. The words we use are in the table on the first page, and their truth values correspond. An important observation in propositional logic is that an atom's truth value is independent of any other's - that is, if I decide to set $A = T$ in $A \equiv (B \vee C)$, then I am still free to choose values for B and C. Thus, it should not make sense to create separate representations of, "She is asleep" and "She is not asleep", since if one is true, the other must be false (otherwise $T \wedge F$ is a theorem, which we know is not true by definition of and).

In statements such as, "Shankar will publish only if Pete will referree. Shankar published. Therefore Pete referreed." the two first sentences should be seen as two things we know. In order for our "therefore" to make sense, we should utilize all that we know in order to make such a judgement. Thus the meaning of this is "((Shankar will publish $\Rightarrow$ Pete will referree) $\wedge$ Shankar will publish) $\Rightarrow$ Pete will referree". If you construct the truth table for this, you will see that this statement is valid. If such a statement is not valid, we would say the person making such a claim is full of something.

On the test and the homework you are expected to write the logic of the statement as it appears in English. An equivalent formula that does not translate 1-1 to the English will be penalized.

Translate the following statements into propositional formulas, stating which phrase corresponds to which symbol, and determine the statement's validity.

1. "A blue bird sings if and only if its songs don't suck. A blue bird's song sucks only if it just ate a bug. The sky is blue. Therefore, if a blue bird has just eaten a bug, it will not sing."
2. "Rock stars are cool if and only if they do cool things. Drinking Pepsi is cool if it is not endorsed by Britney Spears. Britney Spears is not doing commercials anymore. Therefore if rock stars drink Pepsi, they are cool."

3. “Susanne gets her own show on HBO only if she babysits the network director’s kid. Susanne does not babysit annoying kids. The network director’s kid is annoying. Does Susanne get her own show?”

4. “Violent people play violent video games. Jack plays violent video games. Is Jack violent?”

3 Solutions

3.1 Simplification

1. $A \wedge B$
   Notice that B is alone as a condition for the entire formula's truth $(B \wedge (...))$. Thus is B is false, then no matter what, the formula is false. If B is true, then the implication on the right will be true regardless of C and A, since an implication is only falsifiable if the consequent is false. This leads us to the last condition. Notice that we know $B = T$, so the value of C again doesn't matter, and that for $A \equiv (T \vee C)$ to be true, then A must be true. Thus the formula is true if and only if A is true and B is true.

2. T
   This is easily seen to be a tautology, thus it is true regardless of P, Q and R.

3. T
   First we should look at the $B \wedge ite(B, ...)$, since we know that the and is all F if $B = F$. When considering the ite, we should assume B is true. Thus we look at $(C \vee \neg(C \Rightarrow A))$. But notice that the truth of this statement depends entirely on C because if C is false, then since $F \Rightarrow$ *anything* is true, the negated implication is false. Therefore $(B \wedge ite(B, (C \vee \neg(C \Rightarrow A)), A)) \equiv (B \wedge C)$. Now considering that the overall implication will be false only when $\neg A \vee \neg B \vee C$ is false, let us assume that assignment and see if $\neg A \vee (B \wedge C)$ is true. $A = T$, $B = T$, $C = F$ is what we have, so $\neg A$ is false and $B \wedge C$ is false. Therefore this statement is not falsifiable and is thus T.

4. $\neg((Q \Rightarrow P) \Rightarrow (R \vee S))$

5. $(C \vee D) \wedge (A \oplus B)$

3.2 English Logic

1. b = a blue birds sings. w = a bird's song sucks. u = a bird ate a bug. s = the sky is blue.
   $((b \equiv \neg w) \wedge (w \implies u) \wedge s) \Rightarrow (u \Rightarrow \neg b)$.
   This statement is satisfiable but not valid, and is thus bad reasoning. A satisfying assignment is $u = F$ $b = w = s = T$. A falsifying assignment is $b = T$, $u = T$, $w = F$, $s = T$.

2. c = rock stars are cool. d = rock stars do cool things. p = rock stars drink Pepsi. e = Britney Spears endorses Pepsi.
   $((c \equiv d) \wedge ((p \Rightarrow d) \oplus e) \wedge \neg e) \Rightarrow (p \Rightarrow c)$.
   This statement is valid.

3. s = Susanne gets her own show on HBO. b = Susanne babysits the network director's kid. a = The network director's kid is annoying.
   $((s \equiv b) \wedge (a \Rightarrow \neg b) \wedge a) \Rightarrow s$
   This statement is satsfiable but not valid (i.e. Susanne does not get her own show). A satisfying assignment is $s = T$ $b = T$ $a = T$ (holds vacuously). A falsifying assignment is $s = F$ $b = F$ $a = T$.

4. v = (is a) violent person. g = (Jack) plays violent video games.
   $((v \Rightarrow g) \wedge g) \Rightarrow v$.
   This statement is satisfiable but not valid. A satisfying assignment is $v = T$ $g = T$. A falsifying assignment is $v = F$ $g = T$.