Problem 1. Rewriting the problem in standard form:

$$\begin{aligned}
\min \quad & -e^{-w^T x} \\
\text{s.t} \quad & w^T Aw - w^T Ay - w^T x \leq -a \\
& y^T w - w^T x = b
\end{aligned}$$

Problem 5. Let m be the number of milk bottles and k be the number of knobs. Then

$$\begin{aligned}
\min \quad & -(0.07m + 0.05k) \\
\text{s.t} \quad & 4m + 3k \leq 240,000 \\
& 2m + 1k \leq 6000
\end{aligned}$$

Problem 6. Solving for the critical points:

$$\begin{aligned}
f(x,y) &= 2x^2 y + 4xy^2 + xy \\
f_x &= 4xy + 4y^2 + y \\
f_y &= 2x^2 + 8xy + x \\
f_{xx} &= 4y \\
f_{yy} &= 8x \\
f_{xy} = f_{yx} &= 4x + 8y + 1
\end{aligned}$$

Set $f_y = 0$, then

$$2x^2 + 8xy + x = 0 \Rightarrow x(2x + 8y + 1) = 0 \Rightarrow x = 0, y = \frac{-2x-1}{8}$$

Set $f_x = 0$, then

$$\begin{gathered}
4xy + 4y^2 + y = 0 \\
x = 0 \Rightarrow y(4y+1) = 0 \Rightarrow y = 0, -\frac{1}{4} \\
y = \frac{-2x-1}{8} \Rightarrow x = -\frac{1}{6}, -\frac{1}{2} \Rightarrow y = -\frac{1}{12}, 0
\end{gathered}$$

Therefore the critical points are $\{(0,0), (0, -\frac{1}{4}), (-\frac{1}{6}, -\frac{1}{12}), (-\frac{1}{2}, 0)\}$. To determine whether the critical points are local min, max, or saddle points, refer to the determinant of the second derivative matrix. We have that:

$$\begin{aligned}
D(0,0) &= -1 \\
D(0, -\frac{1}{4}) &= -1 \\
D(-\frac{1}{6}, -\frac{1}{12}) &= \frac{1}{3} \\
D(-\frac{1}{2}, 0) &= -1
\end{aligned}$$

Therefore $(-\frac{1}{6}, -\frac{1}{12})$ is a local maximum and the rest are saddle points.

Problem 11. Let $f(x) = ax^2 + bx + c$ where $a > 0$ and $b, c \in \mathbb{R}$. Then

$$x_1 = x_0 - \frac{f'(x_0)}{f''(x_0)} = x_0 - \frac{2ax_0 + b}{2a} = -\frac{b}{2a}$$

The first derivative of f is $2ax + b = 0$. Then the critical point is $-\frac{b}{2a}$. Checking the second derivative $2a > 0$ so the point is a local minimizer and Newton's method converges within one iteration.

Problem 14. Refer to relevant jupyter notebook.