Problem 1. We differentiate equation (5) in the perturbation notes with respect to u and suppress the function arguments for the sake of clarity. This gives us:

$$F_{xxx}x_u^3 + 3F_{xx}x_u x_{uu} + F_{xxu}(x_u^2 + x_u) + F_x x_{uuu} + 2F_{xu}x_{uu} + F_{uuu} + 3F_{uux}x_u = 0$$

Problem 2-5. Refer to the appropriate python notebook.