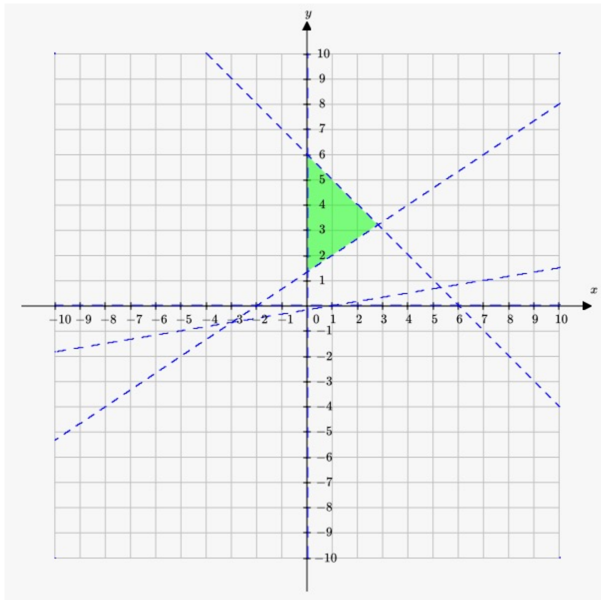
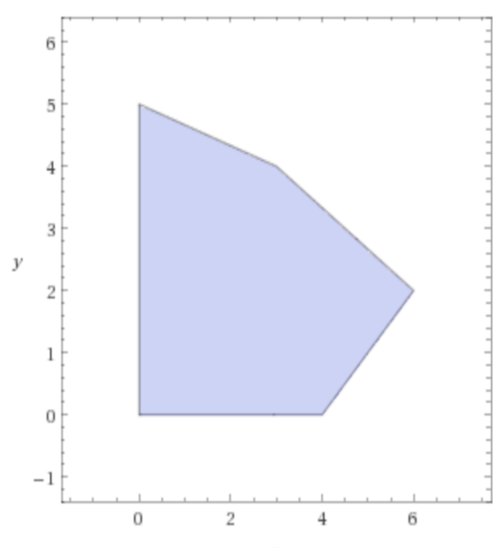
Problem 8.1. We graph the feasible set below. To find the optimizer, plot the objective function over the feasible set and find the maximum point. The optimizer is at $(14/5, 16/5)$ and the objective function has a value of $6/5$.

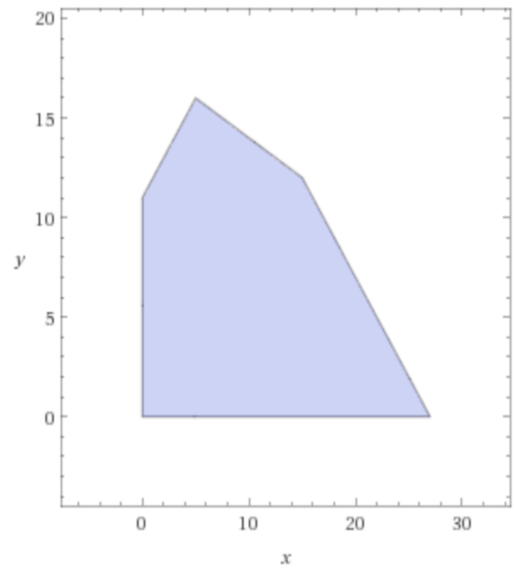


Problem 8.2.

(i) We graph the feasible set below. The optimizer is at $(6, 2)$ and the objective function has a value of 20 at the optimizer.



(ii) We graph the feasible set below. The optimizer is at $(15, 12)$ and the objective function has a value of 132 at the optimizer.



Problem 8.5.

(i) We use the simplex algorithm to solve the optimization (both by hand and using the Simplex Algorithm we coded for a computation problem set). The result after the first pivot is: $\{2 : 11, 3 : 10, 4 : 4, 0 : 0, 1 : 0\}$. After the second pivot the results are, $\{2 : 3, 1 : 2, 0 : 6, 3 : 0, 4 : 0\}$. We see that the results to the optimization are the same as in problem 8.2.

(ii) We use the simplex algorithm to solve the optimization(both by hand and using the Simplex Algorithm we coded for a computation problem set). The result after the first pivot is: $\{2 : 38, 0 : 27, 4 : 36, 3 : 0, 1 : 0\}$. After the second pivot the results are, $\{2 : 14, 0 : 15, 1 : 12, 3 : 0, 4 : 0\}$. We see that the results to the optimization are the same as in problem 8.2.

Problem 8.7.

(i) We use the simplex algorithm we coded for a computation problem set to solve for the optimizer. The optimizer is at $(3, 4)$ and the objective function has a value of 11 at the optimizer.

(ii) This optimization is unsolvable. The program returns an error.

(iii) We use the simplex algorithm we coded for a computation problem set to solve for the optimizer. The optimizer is at $(0, 2)$ and the objective function has a value of 2 at the optimizer.

Problem 8.13. Consider the linear problem

$$\begin{aligned} \max \quad & c^T x \\ \text{s.t} \quad & Ax \geq 0 \\ & x \geq 0 \end{aligned}$$

Let $x = 0$ not be an optimum point. Then by the second assumption there exists a solution $x > 0$ that satisfies the conditions. However, if x satisfies the conditions that $Ax \geq 0$ and $x > 0$ then the vector λx where $\lambda > 1$ also satisfies $A\lambda x \geq 0$ and $\lambda x > 0$. Therefore the problem is unbounded. Now let the problem be bounded. Then by the above proof there does not exist an $x > 0$ that satisfies the conditions. But $x = 0$ satisfies the conditions $Ax \geq 0$ and $x \geq 0$ and therefore is the optimal point.

Problem 8.17. We have that the primal problem can be written as

$$\begin{aligned} \max \quad & c^T x \\ \text{s.t} \quad & Ax \geq b \\ & x \geq 0 \end{aligned}$$

This implies that the dual problem can be written as

$$\begin{aligned} \min \quad & b^T y \\ \text{s.t} \quad & A^T y \geq c \\ & y \geq 0 \end{aligned}$$

Then the dual to the dual problem can be written as

$$\begin{aligned} \min \quad & -c^T x \\ \text{s.t} \quad & -Ax \leq -b \\ & x \geq 0 \end{aligned}$$

which when one removes the negatives is the primal problem.

Problem 8.18. We have that the linear problem is

$$\begin{aligned} \max \quad & x_1 + x_2 \\ \text{s.t} \quad & 2x_1 + x_2 \leq 30 \\ & x_1 + 3x_2 \leq 5 \\ & 2x_1 + 3x_2 \leq 4 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Then the dual problem is

$$\begin{aligned} \min \quad & 3y_1 + 5y_2 + 4y_3 \\ \text{s.t} \quad & 2y_1 + y_2 + 2y_3 \geq 1 \\ & y_1 + 3y_2 + 3y_3 \geq 1 \\ & y_1, y_2, y_3 \geq 0 \end{aligned}$$

We solve both using the Simplex Algorithm code and get optimizers of $(1.25, 0.5)$ and $(0.25, 0, 0.25)$ respectively and equal optimal values of 1.75 for both problems.