Problem Set #2 Part 1

OSM Lab, Professor Stachurski

Dan Ehrlich

Problem 1 First checking for the spectral radius condition, the largest eigenvalue of matrix A has a value of $0.965538166352 < 1$ so the spectral radius condition holds and the equation $x = Ax + b$ has a unique solution. Solving for x using both matrix algebra and successive approximations we get that:

$$x = \begin{bmatrix} -0.89552239 \\ 13.34328358 \\ 45.64179104 \end{bmatrix}$$

Problem 2 In a standard job search model, we have that $\bar{w}$ should satisfy

$$\bar{w} = c(1 - \beta) + \beta \sum_{k=1}^{K} \max \left\{ w_k, \bar{w} \right\} p_k$$

We need to show that $T(x) : \mathbb{R}_+ \to \mathbb{R}_+$ is a contraction mapping where $T(x) = c(1 - \beta) + \beta \sum_{k=1}^{K} \max \left\{ w_k, x \right\} p_k$. Consider

$$|T(x) - T(y)| = \left| c(1 - \beta) + \beta \sum_{k=1}^{K} \max \left\{ w_k, x \right\} p_k - c(1 - \beta) + \beta \sum_{k=1}^{K} \max \left\{ w_k, y \right\} p_k \right| =$$

$$= \beta \left| \sum_{k=1}^{K} \max \left\{ w_k, x \right\} p_k - \sum_{k=1}^{K} \max \left\{ w_k, y \right\} p_k \right| =$$

$$= \beta \left| \sum_{k=1}^{K} (\max \left\{ w_k, x \right\} - \max \left\{ w_k, y \right\}) p_k \right|$$

$$\leq \beta \sum_{k=1}^{K} \left| (\max \left\{ w_k, x \right\} - \max \left\{ w_k, y \right\}) \right| p_k$$

$$\leq \beta \sum_{k=1}^{K} |x - y| p_k$$

$$\leq \beta \sum_{k=1}^{K} \rho(x, y) p_k = \beta \rho(x, y)$$

And finally $\rho(T(x), T(y)) \leq \beta \rho(x, y)$ which proves that T is a contraction mapping and there exists a fixed point solution, or in other words, a $\bar{w}$ such that unemployed individuals offered a wage above $\bar{w}$ will choose to accept it, and will reject those below. They are indifferent if offered wage $\bar{w}$. Furthermore, $\bar{w}$ can be found by guessing an initial value and using successive approximations as in problem 1.

Problem 3 For any and all code refer to the file "DP_part1.py". As unemployment benefits increase, $\bar{w}$ increases as well. Increased benefits increase the value of staying unemployed, so individuals are willing to wait for a higher wage offer. We can solve for $\frac{\partial \bar{w}}{\partial c}$ explicitly:

$$\frac{\partial \bar{w}}{\partial c} = \frac{1}{1 + \frac{\beta}{1-\beta}[1 - G(\bar{w})]} > 0$$

where G is the wage offer distribution. A graph is show below.