

## 2dª Práctica Movimiento de Projectiles

### **Planteamiento del problema.**

Se pide hallar la gráfica de la trayectoria, se verá que es parabólica, de un objeto con las siguientes características:

- posición inicial (0,0)
- diámetro de 6cm
- masa de  $m = 100\text{g}$
- velocidad inicial de  $25\text{m/s}$
- ángulo de  $37^\circ$ , con respecto a la horizontal

Se tratarán estas condiciones con influencia del aire y sin él.

**Para analizar el movimiento con resistencia del aire se empleará lo siguiente:**

- Fuerza de resistencia del aire  $= 0.5 \cdot C \cdot A \cdot \rho \cdot v^2$   
donde  $C$  es el coeficiente de resistencia,  $A$  es la sección transversal,  $\rho$  es la densidad del aire y  $v$  es la magnitud de la velocidad.
- Dado que el aire interviene en las componentes  $x$  e  $y$ , y que tanto velocidad como aceleración son vectores, habrá que hacer:

$$a_x = -a \cdot \cos\theta = -0.5 \cdot C \cdot A \cdot \rho \cdot v^2 / m \cdot \cos\theta$$

$$a_y = -a \cdot \sin\theta = -0.5 \cdot C \cdot A \cdot \rho \cdot v^2 / m \cdot \sin\theta$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v_x = v_{\text{inicial}} \cdot \cos\theta + a_x \cdot t$$

$$v_y = v_{\text{inicial}} \cdot \sin\theta + a_y \cdot t$$

$$x = v_{\text{inicial}} \cdot \cos\theta \cdot t + 0.5 \cdot (a_x) \cdot t^2$$

$$y = v_{\text{inicial}} \cdot \sin\theta \cdot t + 0.5 \cdot (a_y) \cdot t^2$$

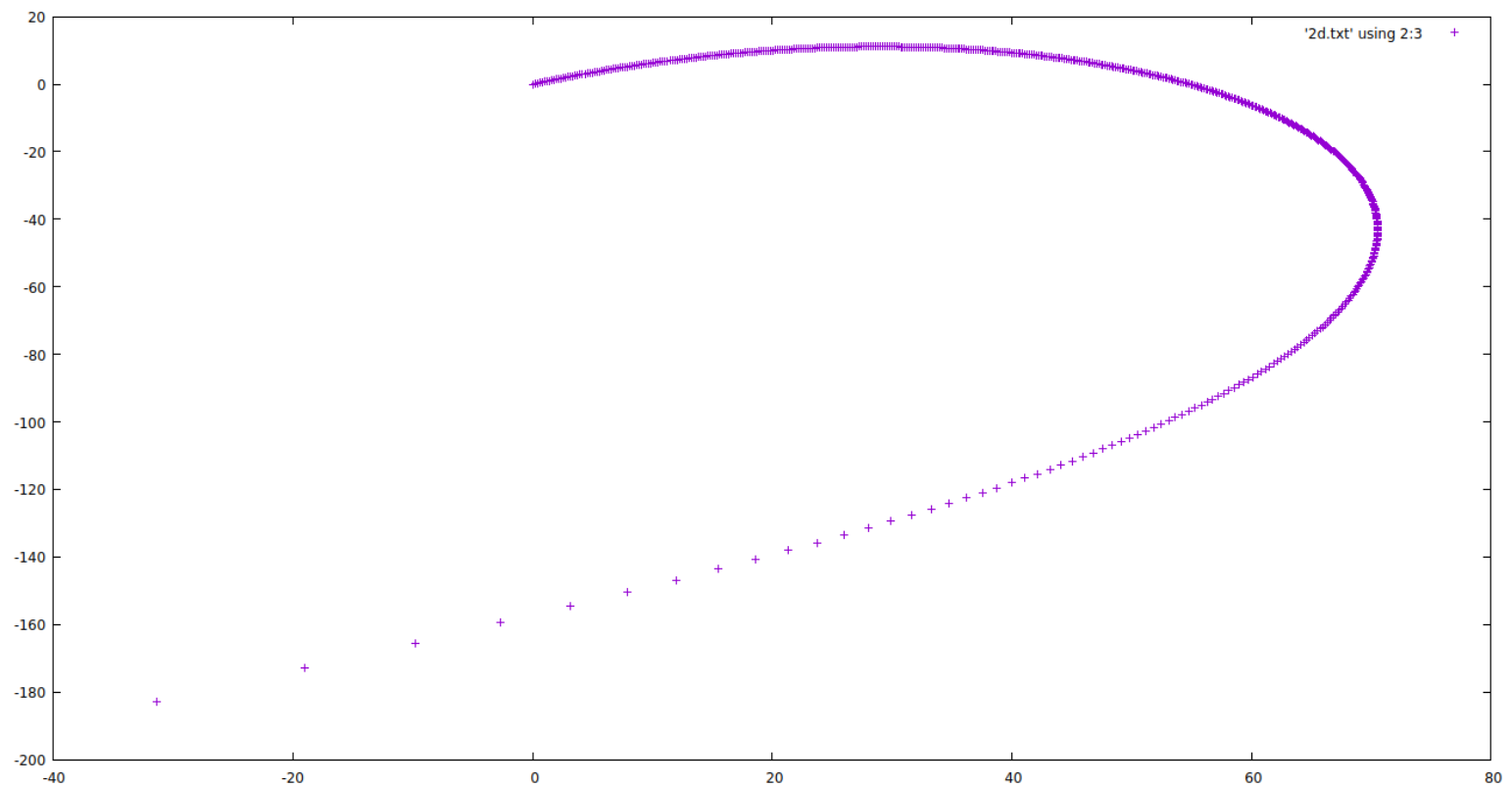
## Resultados

Los incrementos temporales fueron de 0.01 s. Se obtuvo la siguiente tabla:

| tiempo      | x              | y              | $V_x$          | $V_y$             |
|-------------|----------------|----------------|----------------|-------------------|
| 0           | 0              | 0              | 19.9662        | 15.045            |
| 0.01        | 0.199619       | 0.149927       | 19.9575        | 14.9405           |
| 0.02        | 0.399152       | 0.29881        | 19.949         | 14.836            |
| 0.03        | 0.598601       | 0.44665        | 19.9405        | 14.7317           |
| 0.04        | 0.797967       | 0.593447       | 19.9322        | 14.6274           |
| 0.05        | 0.997252       | 0.739203       | 19.9239        | 14.5231           |
| 0.06        | 1.19646        | 0.883919       | 19.9157        | 14.419            |
| 0.07        | 1.39558        | 1.0276         | 19.9076        | 14.3149           |
| 0.08        | 1.59463        | 1.17023        | 19.8997        | 14.2109           |
| 0.09        | 1.79361        | 1.31184        | 19.8917        | 14.1069           |
| 0.1         | 1.9925         | 1.4524         | 19.8839        | 14.003            |
| 0.11        | 2.19133        | 1.59193        | 19.8762        | 13.8992           |
| 0.12        | 2.39008        | 1.73043        | 19.8685        | 13.7954           |
| 0.13        | 2.58877        | 1.86789        | 19.861         | 13.6917           |
| 0.14        | 2.78738        | 2.00432        | 19.8535        | 13.5881           |
| 0.15        | 2.98592        | 2.13971        | 19.8461        | 13.4845           |
| ...         | ...            | ...            | ...            | ...               |
| 1.32        | 25.9037        | 10.9813        | 19.2819        | 1.59336           |
| 1.33        | 26.0971        | 10.9972        | 19.2776        | 1.49214           |
| 1.34        | 26.2905        | 11.0121        | 19.2733        | 1.39091           |
| 1.35        | 26.4837        | 11.0259        | 19.269         | 1.28966           |
| 1.36        | 26.677         | 11.0387        | 19.2647        | 1.18838           |
| 1.37        | 26.8701        | 11.0505        | 19.2603        | 1.08709           |
| 1.38        | 27.0632        | 11.0612        | 19.2559        | 0.985786          |
| 1.39        | 27.2563        | 11.071         | 19.2515        | 0.884457          |
| 1.4         | 27.4492        | 11.0797        | 19.247         | 0.783107          |
| 1.41        | 27.6422        | 11.0873        | 19.2426        | 0.681736          |
| 1.42        | 27.835         | 11.094         | 19.2381        | 0.580344          |
| 1.43        | 28.0278        | 11.0996        | 19.2335        | 0.478929          |
| 1.44        | 28.2205        | 11.1042        | 19.229         | 0.377491          |
| 1.45        | 28.4131        | 11.1077        | 19.2244        | 0.27603           |
| 1.46        | 28.6057        | 11.1103        | 19.2197        | 0.174545          |
| <b>1.47</b> | <b>28.7982</b> | <b>11.1118</b> | <b>19.2151</b> | <b>0.0730347</b>  |
| <b>1.48</b> | <b>28.9907</b> | <b>11.1122</b> | <b>19.2104</b> | <b>-0.0285005</b> |
| 1.49        | 29.183         | 11.1116        | 19.2057        | -0.130061         |
| 1.5         | 29.3753        | 11.11          | 19.2009        | -0.231649         |
| 1.51        | 29.5675        | 11.1074        | 19.1961        | -0.333263         |
| 1.52        | 29.7597        | 11.1037        | 19.1913        | -0.434905         |
| 1.53        | 29.9517        | 11.0989        | 19.1864        | -0.536575         |
| 1.54        | 30.1437        | 11.0932        | 19.1815        | -0.638274         |
| 1.55        | 30.3356        | 11.0864        | 19.1766        | -0.740002         |
| 1.56        | 30.5274        | 11.0785        | 19.1716        | -0.84176          |

|      |          |          |          |           |
|------|----------|----------|----------|-----------|
| 1.57 | 30.7192  | 11.0696  | 19.1665  | -0.943549 |
| 1.58 | 30.9108  | 11.0597  | 19.1615  | -1.04537  |
| ...  | ...      | ...      | ....     | ...       |
| 5.54 | 34.7903  | -124.173 | -7.40649 | -59.873   |
| 5.55 | 33.2852  | -125.851 | -7.97152 | -60.3967  |
| 5.56 | 31.6728  | -127.61  | -8.57308 | -60.948   |
| 5.57 | 29.9381  | -129.463 | -9.21642 | -61.5308  |
| 5.58 | 28.0623  | -131.423 | -9.90799 | -62.1499  |
| 5.59 | 26.0225  | -133.507 | -10.6558 | -62.8114  |
| 5.6  | 23.7896  | -135.738 | -11.4699 | -63.5228  |
| 5.61 | 21.3265  | -138.143 | -12.3631 | -64.2939  |
| 5.62 | 18.5849  | -140.759 | -13.3523 | -65.1373  |
| 5.63 | 15.5009  | -143.635 | -14.4596 | -66.0697  |
| 5.64 | 11.9874  | -146.834 | -15.7153 | -67.1139  |
| 5.65 | 7.9236   | -150.45  | -17.1614 | -68.3015  |
| 5.66 | 3.13562  | -154.612 | -18.8582 | -69.6781  |
| 5.67 | -2.63485 | -159.515 | -20.8956 | -71.3113  |
| 5.68 | -9.78854 | -165.462 | -23.4128 | -73.3061  |
| 5.69 | -18.9792 | -172.944 | -26.6372 | -75.8338  |
| 5.7  | -31.3416 | -182.818 | -30.9632 | -79.1915  |

Se aprecia que la altura máxima se dará entre 1.47 s y 1.48 s pues se registra una velocidad en el eje y de cero en ese intervalo, luego va incrementando en sentido negativo. Esta altura máxima está entre 11.1118 m y 11.1122 m.



### ***Para analizar el movimiento sin resistencia del aire:***

La única aceleración que interviene es la de la gravedad.

$$a_y = -g$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v_x = v_{\text{inicial}} \cdot \cos\theta$$

$$v_y = v_{\text{inicial}} \cdot \sin\theta + a_y \cdot t$$

$$x = v_{\text{inicial}} \cdot \cos\theta \cdot t$$

$$y = v_{\text{inicial}} \cdot \sin\theta \cdot t + 0.5 \cdot (a_y) \cdot t^2$$

### ***Resultados***

Los incrementos temporales fueron de 0.01 s. Se obtuvo la siguiente tabla:

| tiempo | x        | y       | v <sub>x</sub> | v <sub>y</sub> |
|--------|----------|---------|----------------|----------------|
| 0      | 0        | 0       | 19.9662        | 15.045         |
| 0.01   | 0.199662 | 0.14996 | 19.9662        | 14.947         |
| 0.02   | 0.399323 | 0.29894 | 19.9662        | 14.849         |
| 0.03   | 0.598985 | 0.44694 | 19.9662        | 14.751         |
| 0.04   | 0.798647 | 0.59396 | 19.9662        | 14.653         |
| 0.05   | 0.998309 | 0.74    | 19.9662        | 14.555         |
| 0.06   | 1.19797  | 0.88506 | 19.9662        | 14.457         |
| 0.07   | 1.39763  | 1.02914 | 19.9662        | 14.359         |
| 0.08   | 1.59729  | 1.17224 | 19.9662        | 14.261         |
| 0.09   | 1.79696  | 1.31436 | 19.9662        | 14.163         |
| 0.1    | 1.99662  | 1.4555  | 19.9662        | 14.065         |
| 0.11   | 2.19628  | 1.59566 | 19.9662        | 13.967         |
| ...    | ...      | ...     | ...            | ...            |
| 1.37   | 27.3537  | 11.4148 | 19.9662        | 1.619          |
| 1.38   | 27.5533  | 11.4305 | 19.9662        | 1.521          |
| 1.39   | 27.753   | 11.4453 | 19.9662        | 1.423          |
| 1.4    | 27.9526  | 11.459  | 19.9662        | 1.325          |
| 1.41   | 28.1523  | 11.4718 | 19.9662        | 1.227          |
| 1.42   | 28.352   | 11.4835 | 19.9662        | 1.129          |
| 1.43   | 28.5516  | 11.4943 | 19.9662        | 1.031          |
| 1.44   | 28.7513  | 11.5042 | 19.9662        | 0.932995       |
| 1.45   | 28.951   | 11.513  | 19.9662        | 0.834995       |
| 1.46   | 29.1506  | 11.5209 | 19.9662        | 0.736995       |
| 1.47   | 29.3503  | 11.5277 | 19.9662        | 0.638995       |
| 1.48   | 29.5499  | 11.5336 | 19.9662        | 0.540995       |
| 1.49   | 29.7496  | 11.5386 | 19.9662        | 0.442995       |
| 1.5    | 29.9493  | 11.5425 | 19.9662        | 0.344995       |
| 1.51   | 30.1489  | 11.5455 | 19.9662        | 0.246995       |

|             |                |                |                |                   |
|-------------|----------------|----------------|----------------|-------------------|
| 1.52        | 30.3486        | 11.5474        | 19.9662        | 0.148995          |
| <b>1.53</b> | <b>30.5482</b> | <b>11.5484</b> | <b>19.9662</b> | <b>0.0509953</b>  |
| <b>1.54</b> | <b>30.7479</b> | <b>11.5485</b> | <b>19.9662</b> | <b>-0.0470047</b> |
| 1.55        | 30.9476        | 11.5475        | 19.9662        | -0.145005         |
| 1.56        | 31.1472        | 11.5456        | 19.9662        | -0.243005         |
| 1.57        | 31.3469        | 11.5426        | 19.9662        | -0.341005         |
| 1.58        | 31.5466        | 11.5387        | 19.9662        | -0.439005         |
| 1.59        | 31.7462        | 11.5339        | 19.9662        | -0.537005         |
| 1.6         | 31.9459        | 11.528         | 19.9662        | -0.635005         |
| 1.61        | 32.1455        | 11.5212        | 19.9662        | -0.733005         |
| 1.62        | 32.3452        | 11.5133        | 19.9662        | -0.831005         |
| ...         | ...            | ...            | ...            | ...               |
| 5.64        | 112.609        | -71.0133       | 19.9662        | -40.227           |
| 5.65        | 112.809        | -71.416        | 19.9662        | -40.325           |
| 5.66        | 113.009        | -71.8198       | 19.9662        | -40.423           |
| 5.67        | 113.208        | -72.2245       | 19.9662        | -40.521           |
| 5.68        | 113.408        | -72.6302       | 19.9662        | -40.619           |
| 5.69        | 113.608        | -73.0369       | 19.9662        | -40.717           |
| 5.7         | 113.807        | -73.4445       | 19.9662        | -40.815           |

Se aprecia que la altura máxima se dará entre 1.53 s y 1.54 s pues se registra una velocidad en el eje y de cero en ese intervalo, luego va incrementando en sentido negativo. Esta altura máxima está entre 11.5484 m y 11.5485 m.

