Deeksha - K CET

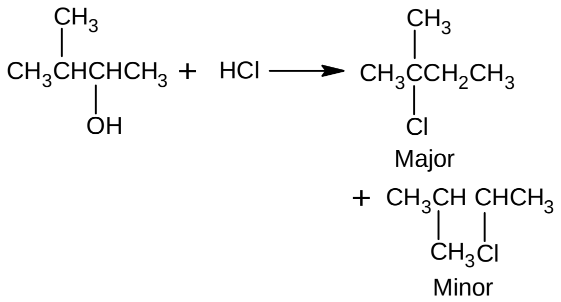
Subject	Topic		Date
C + M + P	Complete Syllabus	CET – 12 – CT	

C+M+P Key Answers:

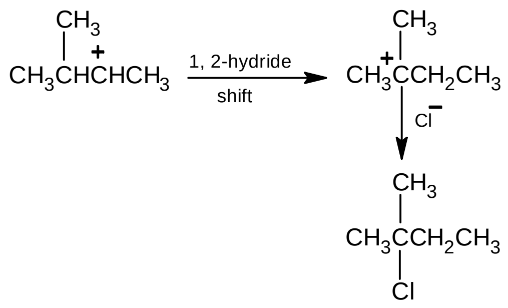
1. c	2. c	3. a	4. d	5. c	6. c	7. b	8. b	9. a	10. b
11. d	12. a	13. c	14. d	15. c	16. c	17. b	18. b	19. a	20. b
21. a	22. d	23. c	24. c	25. b	26. c	27. b	28. b	29. c	30. a
31. c	32. d	33. b	34. a	35. b	36. a	37. d	38. a	39. d	40. d
41. c	42. d	43. b	44. c	45. b	46. d	47. c	48. c	49. b	50. c
51. b	52. b	53. a	54. b	55. a	56. b	57. d	58. c	59. c	60. b
61. d	62. a	63. b	64. b	65. c	66. c	67. a	68. c	69. b	70. b
71. b	72. c	73. c	74. c	75. c	76. a	77. a	78. a	79. b	80. b
81. c	82. a	83. c	84. d	85. b	86. b	87. b	88. c	89. a	90. d
91. c	92. a	93. a	94. b	95. c	96. c	97. d	98. a	99. a	100.d
101.d	102.c	103.b	104.b	105.c	106.c	107.a	108.a	109.d	110.a
111.d	112.d	113.c	114.d	115.a	116.a	117.a	118.c	119.a	120.d
121.c	122.d	123.d	124.b	125.d	126.d	127.c	128.c	129.b	130.b
131.b	132.a	133.b	134.c	135.a	136.b	137.a	138.a	139.b	140.b
141.d	142.d	143.c	144.c	145.d	146.b	147.d	148.a	149.b	150.a
151.b	152.b	153.d	154.c	155.b	156.a	157.b	158.c	159.d	160.a
161.b	162.b	163.c	164.a	165.b	166.a	167.d	168.b	169.d	170.d
171.d	172.b	173.b	174.c	175.a	176.d	177.a	178.a	179.d	180.c

Chemistry Solutions:

1. Pentaacetate of glucose does not react with NH_2OH indicating absence of free aldehyde group.
2. Sol:



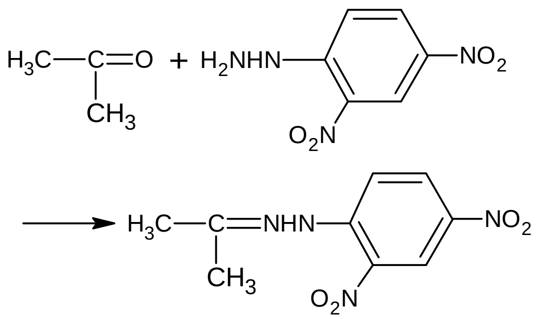
The intermediate in the reaction is secondary carbocation that can change to more stable tertiary carbocation by 1,2-hydride shift.



3. Hence, Cr^{3+} is oxidised to $Cr_2O_7^{2-}$
4. $C_2H_5OCH_3 + HI \rightarrow C_2H_5OH + CH_3I$

   Halogen goes with the smaller group on cleavage of ether.
5. Highest O.N. of Mn in K_2MnO_4 is $2 + x - 8 = 0$ or $x = +6$ while in all other compounds O.N. of Mn is lower than $\overset{+4}{Mn}O_2$, $\overset{+8/3}{Mn_3}O_4$, $\overset{+2}{Mn}SO_4$.

6. Sol:



7. $2Pb(NO_3)_2 \xrightarrow{673\text{K}} 4NO_2 + 2PbO + O_2$

8. $K_1 = \dfrac{[H_2O]}{[H_2]}, \; K_2 = \dfrac{[CO_2]}{[CO]}$

$$K = \frac{[CO_2][H_2]}{[CO][H_2O]} = \frac{K_2}{K_1} = \frac{500}{65} = 7.69$$

9. Positive $E°$ value of copper accounts for its inability to liberate H_2 from acids.

10. $CuSO_4 + H_2S \xrightarrow{\text{dil. } HCl} \underset{\text{Black ppt.}}{CuS \downarrow} + H_2SO_4$

$$CuS + 2HNO_{3(\text{dil.})} \xrightarrow{\Delta} \underset{\text{Blue solution}}{Cu(NO_3)_2} + H_2S$$

$$Cu(NO_3)_2 + 4NH_3 \longrightarrow \underset{\text{Deep blue solution}}{\left[Cu(NH_3)_4\right]^{2+}} + 2NO_3^-$$

11. Carbon $\left(1s^2\, 2s^2\, 2p^2\right)$ is isoelectronic with $N^+\left(1s^2\, 2s^2\, p^2\right)$.

12. $CH_3CH_2Cl \xrightarrow{NaCN} \underset{(X)}{CH_3CH_2CN} \xrightarrow{Ni/H_2}$

$$\underset{(Y)}{CH_3CH_2CH_2NH_2} \xrightarrow{(CH_3CO)_2O} \underset{(Z)}{CH_3CH_2CH_2NHCOCH_3}$$

13. Me_3SiCl is not a monomer for a high molecular mass silicone polymer.

14. The compounds $\left[Co(SO_4)(NH_3)_5\right]Br$ and $\left[Co(SO_4)(NH_3)_5\right]Cl$ do not have same chemical formula.

15. Bond order of O_2^- is 1.5

Bond order $= \dfrac{8-5}{2} = 1.5$

16. Number of X atoms $= \dfrac{1}{8} \times 8 = 1$

Number of Y atoms $= 1$

Number of Z atoms $= 12 \times \dfrac{1}{4} = 3$

Formula of the compound $= X\,Y\,Z_3$

17. $\kappa = C \times \dfrac{l}{A}$

$$\frac{l}{A} = \kappa \times \frac{1}{C} = \kappa \times R = 0.0212 \times 55 = 1.166 \text{ cm}^{-1}$$

18. Step growth polymers are condensation polymers. They are formed by the loss of simple molecules like water, alcohol, etc., and lead to the formation of high molecular mass condensation polymers.

19. Aim: $\dfrac{1}{2}N_2 + \dfrac{3}{2}Cl_2 \longrightarrow NCl_3, \quad \Delta H = ?$

Operate $\dfrac{1}{2}$Eqn. (ii) + Eqn. (i) $- \dfrac{3}{2}$Eqn, (iii)

We get $\Delta H = \dfrac{1}{2}(-\Delta H_2) + (-\Delta H_1) - \dfrac{3}{2}\Delta H_3$

$$= -\Delta H_1 - \frac{\Delta H_2}{2} - \frac{3}{2}\Delta H_3$$

20. $6XeF_4 + 12H_2O \longrightarrow 4Xe + 2XeO_3 + \underset{\text{(i)}}{24HF} + \underset{\text{(ii)}}{3O_2}$

$$XeF_6 + 3H_2O \longrightarrow \underset{\text{(iii)}}{XeO_3} + 6HF$$

21. Salvarsan was the first effective treatment discovered for syphilis.

22. Moles of $CO_2 = \dfrac{2.2}{44} = 0.05$

Moles of carbon $= 0.05$ mole atoms of nitrogen

$$= \frac{6.02 \times 10^{21}}{6.02 \times 10^{23}} = 0.01$$

Mole atoms of sulphur $= 0.03$

Molar ratio

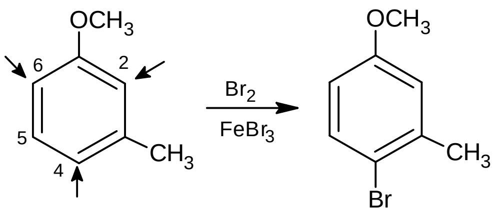
0.05 : 0.01 : 0.03

5 : 1 : 3

23. IUPAC name of $\left[Pt(NH_3)_2\,Cl(NO_2)\right]$ is Diamminechloridonitrito-N-platinum(II).

24. Actinoid series has elements from atomic number 90 to 103. Thulium (Tm) has atomic number 69.

25. Both OCH_3 and CH_3 are $o, p-$directing groups. The possible positions of attack which are facilitated both by OCH_3 and CH_3 groups as shown by arrows below:



26. Ionic bond cannot be formed between two similar atoms.

27. At CMC, the particles of an electrolyte aggregate and form associated colloids known as micelles.

28. Structure (a), (c) and (d) have the same. M.F. $(C_6H_{12}O)$ as that of $CH_3COCH_2CH_2CH_2CH_3$ and hence all are isomers. However, the M,F, of structure (b) is $C_6H_{10}O$ and hence it is not isomer.

29. Most naturally occurring amino acids have L-configuration.

30. 12

31. $Na_2SO_4 \rightleftharpoons 2Na^+ + SO_4^{2-}$

1	0	0
$1-\alpha$	2α	α

Van't Hoff factor $(i) = \frac{1-\alpha+2\alpha+\alpha}{1} = 1+2\alpha$

32. If the volume is kept constant and an inert gas such as argon is added which does not take part in the reaction, the equilibrium remains undisturbed.

33. $p\pi - p\pi$ bonding in nitrogen is strong hence it can form triple bond with another N. Single $N-N$ bond is weaker than $P-P$ bond due to high interionic repulsion of non-bonding electrons. Hence, $N \equiv N$ is stable and P_2 is not.

34. If some other metal is used as the anode other than graphite then the oxygen liberated at anode will convert some Al to Al_2O_3.

35. $W = Z \times I \times t$

$$= 4\times10^{-4} \times 12 \times \frac{75}{100} \times 3 \times 3600 = 38.8\,g$$

36. For endothermic reaction, $\Delta H = +ve$. For reaction to be spontaneous, ΔS must be positive and also $T\Delta S$ must be greater than ΔH in magnitude. The reaction is then said to be entropy driven.

37. $\log\, k = \log\, A - \frac{E_a}{2.303\,RT}$

$$\log\, k = \log\left(1.00\times10^{11}\right) - \frac{35\times10^3}{2.303\times2\times542}$$

$$\log\, k = 11.0 - 14.01$$

Taking antilog, $k = 10^{-3}\,s^{-1}$

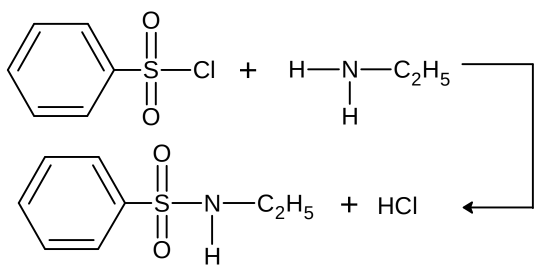
38. $\Delta p = m \times \Delta\upsilon$

$$\therefore\ 1.8\times10^{-18}\,g\ cm\ s^{-1} = 9\times10^{-28}\,g \times \Delta\upsilon$$

or $\Delta\upsilon = 2\times10^{9}\ cm\ s^{-1}$

39. $$\underset{}{CH_3OH} \xrightarrow{PI_3} \underset{(X)}{CH_3I} \xrightarrow{KCN} \underset{(Y)}{CH_3CN} \xrightarrow{\text{Hydrolysis}} \underset{(Z)}{CH_3COOH}$$

40. Sol:



N-Ethylbenzenesulphonamide
(soluble in alkali)

41. The gas formed in the upper layers of atmosphere by action of UV radiations is ozone.

42. At the anode, the following oxidation reactions are possible:

(d) $Cl^-_{(aq)} \rightarrow \frac{1}{2}Cl_{2(g)} + e^{-1}$; $E^\circ_{cell} = 1.36\,V$

(b) $2H_2O_{(l)} \rightarrow O_{2(g)} + 4H^+_{(aq)} + 4e^{-1}$; $E^\circ_{\text{cell}} = 1.23\,\text{V}$

The reaction at anode with lower value of E° is preferred and therefore, water should get oxidised in preference to $Cl^-_{(aq)}$. However, on account of over potential of oxygen reaction (d) is preferred.

43. Hydrolysis of an ester is pseudo first order reaction.

44. Number of moles of urea $= \frac{10}{60} = \frac{1}{6}$

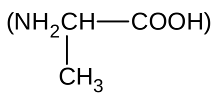
Weight of solute, $X = 6\,\text{g}$

Number of moles of $X = \frac{6}{M}$

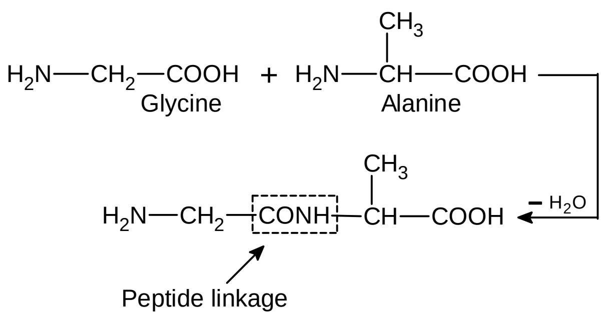
For isotonic solutions, $n_1 = n_2$

or $\frac{1}{6} = \frac{6}{M}$ or $M = 36\,\text{g mol}^{-1}$.

45. The peptide linkage formed between glycine $(H_2N - CH_2 - COOH)$ and alanine



to give glycylalanine can be shown as



Glycylalanine (Gly-Ala)

46. $T_c = \frac{8a}{27Rb}$ $\qquad T_c \propto a$

In the given problem, $'b'$ value remains almost constant.

Hence, the order is $D > B > A > C$

47. Acetone + Ethyl alcohol solution shows positive deviation while acetone + chloroform shows negative deviation.

Other examples:

Positive deviation – Acetone + Ethyl alcohol

Acetone + Benzene, Water + Ethyl alcohol

Negative deviation - Nitric acid + Water

Benzene + Chloroform.

48. Williamson's synthesis - $C_2H_5ONa + C_2H_5Br$

ROR' -Unsymmetrical ether

$p-$nitrophenol - Intermolecular hydrogen bonding

Acetylation - $C_6H_5OH + CH_3COCl$ in presence of pyridine

49. (I) is basic $(pH > 7)$, (II) is acidic $(pH < 7)$, (III) is neutral $(pH \approx 7)$, (IV) is strongly basic $(pH >> 7)$, i.e., (IV) > (I) > (III) > (II).

50. $10\,\text{g}$ of the salt will contain $NaCl = 9.5\,\text{g}$

1 mole, i.e., $58.5\,\text{g}$ of $NaCl$ contains 6.02×10^{23} molecules

$\therefore$ $9.5\,\text{g}$ of $NaCl$ will contains $= \frac{6.02 \times 10^{23}}{58.5} \times 9.5 \simeq 10^{23}$

51. $CH_3CH = CHCHO \xrightarrow{H_2/Pt} CH_3CH = CHCH_2OH$

52. Hydrogen is produced during the reaction

$Mg + H_2O \xrightarrow{\text{Heat}} Mg(OH)_2 + H_2$

All others are used for manufacture of H_2O_2

$H_2S_2O_8 + 2H_2O \longrightarrow H_2O_2 + 2H_2SO_4$

53. Sol:

$$C_2H_5NH_2 \xrightarrow{HNO_2} \underset{(X)}{C_2H_5OH} \xrightarrow{PCl_5} \underset{(Y)}{C_2H_5Cl}$$

$$\xrightarrow{KCN} \underset{(Z)}{\underset{\text{(Propanenitrile)}}{C_2H_5CN}}$$

54. Exhaled air contains moisture and CO_2 . Moisture reacts with superoxide to generate O_2 and KOH . KOH thus produced reacts with CO_2 to form $KHCO_3$.

$4KO_2 + 2H_2O \longrightarrow 4KOH + 3O_2$

$KOH + CO_2 \longrightarrow KHCO_3$

55. Sol:

$$\left[MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O\right] \times 2$$

$$10H_2O + 5SO_2 \longrightarrow 5SO_4^{2-} + 2OH^+ + 10e^-$$

$$2MnO_4^- + 5SO_2 + 2H_2O \longrightarrow 5SO_4^{2-} + 2Mn^{2+} + 4H^+$$

56. At equilibrium, $\Delta G = 0$

$\Delta G = \Delta H - T\Delta S$

$\therefore \ \Delta H = T\Delta S$

57. Water soluble chemical pollutants are sulphuric acid, inorganic metal compounds, etc.

58. Unit of rate constant for the reaction is

$$k = \frac{\text{rate}}{[H_2][NO]^2} = \frac{\text{mol L}^{-1}\ \text{s}^{-1}}{\left(\text{mol L}^{-1}\right)\left(\text{mol L}^{-1}\right)^2} = \text{mol}^{-2}\text{L}^2\text{s}^{-1}$$

59. Limestone is added as flux and S, Si and P change to their oxides and pass into the slag

60. The correct order of acidity is

$$ICH_2COOH < BrCH_2COOH < ClCH_2COOH < FCH_2COOH$$

Mathematics Solutions:

61. By data A is of order $m \times n$.

$A \cdot B'$ is defined means, B' must be of order $n \times k$.

Again, $B'A$ is defined means, B' must be of order $l \times m$. Thus, $l = n$ and $k = m$.

$\therefore$ order of $B = m \times n$

62. Consider, $(AB' - BA') = (AB')' - (BA')'$

$= BA' - AB'$

$= -(AB' - BA')$

$\therefore \ AB' - BA'$ is a skew symmetric matrix.

63. $\begin{pmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{pmatrix} = \begin{pmatrix} 0 & y-2 \\ 8 & 4 \end{pmatrix}$

$\Rightarrow \ x = -\frac{7}{3}, \qquad y = 7, \qquad x = -\frac{2}{3}$

But, $x = -\frac{7}{3}, x = -\frac{2}{3}$ is not consistent

$\therefore$ not possible to find.

64. Consider $C_1 \to C_1 + C_2 + C_3$

$$\Delta = \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+c & a \\ x+a+b+c & a & x+b \end{vmatrix}$$

$$= (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+c & a \\ 1 & a & x+b \end{vmatrix}$$

$\Rightarrow \ x+a+b+c$ is a factor of Δ

65. $\Delta = \begin{vmatrix} 1 & 2 & 6 \\ 2 & 6 & 24 \\ 6 & 24 & 120 \end{vmatrix}$

Consider $R_2 \to R_1 - 2R_1, R_3 \to R_3 - 6R_1$

$$\Delta = \begin{vmatrix} 1 & 2 & 6 \\ 0 & 2 & 12 \\ 0 & 12 & 84 \end{vmatrix} = 1(168 - 144) = 24 = 4!$$

66. We know that $|adj\ A| = |A|^{n-1}$

By data $|adj\ A| = |B| = 81$ and $n = 3$

$\therefore \ |A|^{3-1} = 81 \Rightarrow |A|^2 = 81 \Rightarrow |A| = \pm 9.$

67. We have, $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$

$f(t) = \cos t\left(-t^2\right) + t(0) + 1(t\sin t)$

$f(t) = -t^2\cos t + t\sin t$

$$\lim_{t\to 0}\frac{f(t)}{t^2} = \lim_{t\to 0}(-\cos t) + \lim_{t\to 0}\frac{\sin t}{t} = -1 + 1 = 0$$

68. The number of injection $= nP_m$

Where $n(A) = m$ and $n(B) = n$ and $n \geq m$.

$\therefore$ number of injection $= 4P_3 = \frac{4!}{(4-3)!} = 24$

(If $n < m$, then the number of bijection =0)

69. We have, $f(x) = 3x + 4$.

Let, $f(x_1) = f(x_2) \Rightarrow 3x_1 + 4 = 3x_2 + 4$

$\Rightarrow \ 3x_1 = 3x_2 \Rightarrow x_1 = x_2$

Thus f is injective.

Let $y = 3x + 4 \Rightarrow 3x = y - 4$

$\Rightarrow \ x = \frac{y-4}{3}$

If $y = 5$, then $x = \frac{1}{3} \notin N$.

Hence $5 \in N$ has no preimage in N.

Thus f is not surjective.

70. Clearly T is an equivalence relation on R.

We have, $S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$

Now,

$(x,x)\in S \Rightarrow x = x+1 \Rightarrow 1=0$ which is not true.

$\therefore$ S is not reflexive. Thus S is not an equivalence relation.

71. The function $f: Z \to Z$ defined by $f(x)=x+2$ is a bijection

72. We have, $\tan\left(\frac{1}{2}\tan^{-1}\frac{12}{5}\right)$

Let $\tan^{-1}\frac{12}{5}=\theta \Rightarrow \tan\theta=\frac{12}{5}$

$\therefore$ Also we have

$$\tan\theta=\frac{2\tan\theta/2}{1-\tan^2\theta/2} \Rightarrow \frac{12}{5}=\frac{2\tan\theta/2}{1-\tan^2\theta/2}$$

$$\Rightarrow 6\tan^2\frac{\theta}{2}+5\tan\frac{\theta}{2}-6=0$$

$$\Rightarrow (2\tan\theta/2+3)(3\tan\theta/2-2)=0$$

$$\Rightarrow \tan\theta=\frac{2}{3} \quad \text{(rejecting -ve value)}$$

Or $\tan^{-1}\frac{12}{5}=\theta \Rightarrow \tan\theta=\frac{12}{5}$

$$\Rightarrow \cos\theta=\frac{5}{13}$$

$$\text{G.E}=\tan\frac{\theta}{2}=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\frac{1-\frac{5}{13}}{1+\frac{5}{13}}=\sqrt{\frac{8}{18}}=\frac{2}{3}$$

73. $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{5x}{1-6x^2}\right)=\frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{1-6x^2}=1 \Rightarrow 6x^2+5x-1=0$$

$$\Rightarrow (6x-1)(x+1)=0$$

$$\Rightarrow x=\frac{1}{6}\text{(rejecting -ve value)}$$

74. G.E. $=\tan^{-1}\left[\dfrac{\frac{yz}{xr}+\frac{zx}{yr}+\frac{xy}{zr}-\frac{xyz}{r^3}}{1-\left(\frac{yz\cdot zx}{xyr^2}+\frac{zx\cdot xy}{yzr^2}+\frac{xy\cdot yz}{zxr^2}\right)}\right]$

$$=\tan^{-1}\left[\frac{\cdots\cdots\cdots}{1-\frac{1}{r^2}\left(z^2+y^2+x^2\right)}\right]$$

$$=\tan^{-1}\infty=\frac{\pi}{2} \qquad \left(\because\ x^2+y^2+z=r^2\right)$$

75. G.E. $\Rightarrow \tan\left[\frac{1}{2}2\tan^{-1}x+\frac{1}{2}2\tan^{-1}x\right]$

$$=\tan\left(2\tan^{-1}x\right)=\frac{2x}{1-x^2}$$

76. Clearly, $\tan\left(\cos^{-1}x\right)=\tan\left[\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right]$

$$=\frac{\sqrt{1-x^2}}{x}$$

77. $f(x)$ is continuous at $x=0$.

$$\Rightarrow f(0)=\lim_{x\to 0} f(x)$$

$$\Rightarrow k=\lim_{x\to 0}(\cos x)^{1/x}$$

$$\Rightarrow K=e^{\lim_{x\to 0}\frac{1}{x}(\cos x-1)}=e^{\lim_{x\to 0}-\frac{(1-\cos x)}{x}}=e^0$$

$$\left(\because \lim_{x\to 0}\frac{1-\cos x}{x}=0\right)$$

$$\Rightarrow k=1$$

78. By data $f(x)$ is continuous at $x=2$.

$$\Rightarrow \lim_{x\to 2}f(x)=f(2)\Rightarrow \lim_{x\to 2}f(x)=2\left(\because\ f(2)=2\right)$$

Now, $\lim_{x\to 2}f(x)=\lim_{x\to 2}(3x-4)=2$

$$\lim_{x\to 2^+}f(x)=\lim_{x\to 2}(2x+\lambda)=4+\lambda$$

$$4+\lambda=2\Rightarrow\lambda=-2$$

79. $x=a(\cos\theta+\theta\sin\theta);\ y=a(\sin\theta-\theta\cos\theta)$

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{a(\cos\theta+\theta\sin\theta-\cos\theta)}{a(-\sin\theta+\theta\cos\theta+\sin\theta)}$$

$$=\frac{a\theta\sin\theta}{a\theta\cos\theta}=\tan\theta$$

80. $y=\tan^{-1}\left(\frac{x+a}{1-ax}\right)\Rightarrow y=\tan^{-1}x+\tan^{-1}a$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{1+x^2}$$

81. $y = \log\log(\log x)$

$$\frac{dy}{dx} = \frac{1}{\log(\log x)} \cdot \frac{1}{\log x} \cdot \frac{1}{x}$$

82. We have, $\sin(x+y) + \cos(x+y) - \log(x+y) = 0$

This is an expression in $x + y$

$$\therefore \frac{d}{dx}(x+y) = 0 \Rightarrow 1 + \frac{d^2y}{dx^2} = 0$$

OR Differentiate w.r.t. x,

$$\left[\cos(x+y) - \sin(x+y) - \frac{1}{x+y}\right]\frac{d}{dx}(x+y) = 0$$

$$\Rightarrow \frac{d}{dx}(x+y) = 0 \Rightarrow 1 + \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = 1 \Rightarrow \frac{d^2y}{dx^2} = 0$$

83. $g(x) = [f(x)]^2 + [f'(x)]^2$

$$g'(x) = 2f(x)\cdot f'(x) + 2f'(x)\cdot f''(x)$$

$$= 2f'(x)[f(x) + f''(x)]$$

$$= 2f'(x)\cdot 0 \qquad \text{(by data)}$$

$\therefore g'(x) = 0 \Rightarrow g(x)$ is a constant function.

Now, $g(3) = 8$ $\qquad \therefore g(8) = 8$

84. We have, $y = 2x^2 + 3\sin x$

Slope of the normal $= -\dfrac{1}{(dy/dx)}$

$$\frac{dy}{dx} = 4x + 3\cos x, \left(\frac{dy}{dx}\right)_{x=0} = 3$$

Slope of the normal $= -\dfrac{1}{3}$

85. We have, $x^3 - 3xy^2 + 2 = 0$

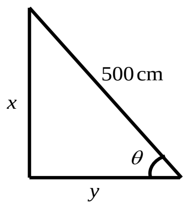
$$\Rightarrow \frac{dy}{dx} = \left(\frac{x^2 - y^2}{2xy}\right) = m_1$$

$$3x^2y - y^3 = 2 \Rightarrow \frac{dy}{dx} = -\left(\frac{2xy}{x^2 - y^2}\right) = m_2$$

Clearly, $m_1 \cdot m_2 = -1$.

Thus curves cut at right angle.

86. From the figure we have, $\sin\theta = \dfrac{x}{500}$



$$\Rightarrow \cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{500}\frac{dx}{dt}$$

$$\Rightarrow \cos\theta \cdot \frac{d\theta}{dt} = \frac{10}{500} = \frac{1}{50}$$

$$\left(\frac{d\theta}{dt}\right)_{y=200} = \frac{1}{50}\frac{1}{(\cos\theta)_{y=200}}$$

$$= \frac{1}{50}\frac{500}{200}$$

$$\therefore \left(\frac{d\theta}{dt}\right)_{y=200} = \frac{1}{20} \text{ radian/sec}$$

87. Let, $y = \left(\dfrac{1}{x}\right)^{2x^2}$

$$\frac{dy}{dx} = \left(\frac{1}{x}\right)^{2x^2}\left[\frac{2x^2}{(1/x)}\cdot\left(-\frac{1}{x^2}\right) + \log\left(\frac{1}{x}\right)\cdot 4x\right]$$

$$\frac{dy}{dx} = 0 \quad \Rightarrow -2x - 4x\log x = 0$$

$$\Rightarrow -2x(1 + 2\log x) = 0$$

$$\Rightarrow x = 0 \qquad \text{or } x = e^{(-1/2)}$$

Max. value $= \left(\sqrt{e}\right)^{2/e} = e^{1/e} = \sqrt[e]{e}$

88. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2\int e^{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}dx = 2e^{\sqrt{x}}$

$$\int e^{f(x)}\cdot f'(x)dx = e^{f(x)}$$

89. Let, $I = \displaystyle\int e^x\left(\frac{1-x}{1+x}\right)^2 dx$

$$= \int e^x\left[\frac{1 + x^2 - 2x}{(1+x^2)^2}\right]dx$$

$$= \int e^x\left[\frac{1}{1+x^2} + \frac{-2x}{(1+x^2)^2}\right]dx$$

$$= e^x \cdot \frac{1}{1+x^2} + c$$

$$\left(\because \frac{dy}{dx}\left(\frac{1}{1+x^2}\right) = \frac{-2x}{\left(1+x^2\right)^2} \right)$$

90. We have, $I = \int \frac{x^9}{\left(4x^2+1\right)^6} dx$

$$= \int \frac{x^9 dx}{x^{12}\left(4+\frac{1}{x^2}\right)^6}$$

$$= -\frac{1}{2}\int \frac{\left(-\frac{2}{x^3}\right)dx}{\left(4+\frac{1}{x^2}\right)^6}$$

$$= -\frac{1}{2}\frac{\left(4+\frac{1}{x^2}\right)^{-5}}{(-5)} \left(\because \frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{-2}{x^3} \right)$$

$$= \frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5} + c$$

91. Now, $\frac{1}{(x+2)\left(x^2+1\right)} = \frac{1}{5(x+2)} + \frac{Ax+B}{x^2+1}$

$$\Rightarrow 1 = \frac{1}{5}\left(x^2+1\right) + (Ax+B)(x+2)$$

Coeff. of $x^2 : A + \frac{1}{5} = 0 \quad \Rightarrow A = -\frac{1}{5}$

Constants : $2B + \frac{1}{5} = 1 \Rightarrow B = \frac{2}{5}$

$$\therefore\ a = \frac{1}{2}\left(-\frac{1}{5}\right) = -\frac{1}{10}; b = \frac{2}{5}$$

92. $\int_{-1}^{1} xe^x dx = xe^x - 1\cdot e^x\Big]_{-1}^{1}$

(applying generalised integration by parts)

$$= e^x (x-1)\Big]_{-1}^{1}$$

$$= \left[e(1-1)\right] - \left[e^{-1}(-1-1) = \frac{2}{e}\right.$$

93. $\int_0^1 \frac{\left(\sin^{-1}x\right)^2}{\sqrt{1-x^2}} dx = \int_0^1 \left(\sin^{-1}x\right)^2 \cdot \frac{1}{\sqrt{1-x^2}}$

It is of the form $\int \left[f(x) \right]^n \cdot f'(x) dx$

$$= \frac{\left(\sin^{-1}x\right)^3}{3}\Bigg]_0^1 = \frac{\pi^3}{24}$$

94. $\int_0^{\pi/2} \frac{\sin^3 x}{\cos^3 x + \sin^3 x} dx = \frac{\pi}{4}$

Standard result.

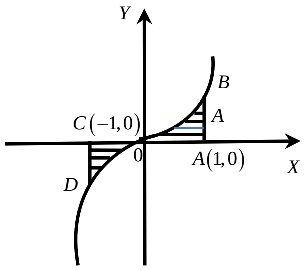
95. $|1-x| = 1-x$ if $-1 < x \le 1$

$$\therefore\ I = \int_{-1}^{1} (1-x) dx = x - \frac{x^2}{2}\Bigg]_{-1}^{1}$$

$$= \left(1-\frac{1}{2}\right) - \left(-1-\frac{1}{2}\right)$$

$$= 1 - \frac{1}{2} + 1 + \frac{1}{2} = 2$$

96. We have, $y = x|x|, x = -1, x = 1$



Clearly required area is

2 Area OBA $= 2\int_0^1 x^2 dx$

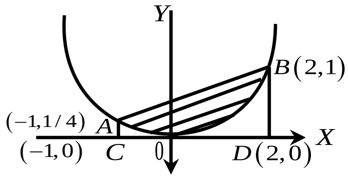
$$= 2\cdot\frac{x^3}{3}\Bigg]_0^1 = \frac{2}{3}$$

97. Now, $x^2 = 4y$ and $x = 4y - 2$ intersect $A\left(-1, \frac{1}{4}\right)$

And $B(2,1)$.

Required area = Area AOBA

= Area CDBAC – Area CAOBDA



$$= \int_{-1}^{2} f(x) dx - \int_{-1}^{2} g(x) dx$$

Where, $f(x)=\dfrac{x+2}{4}$,

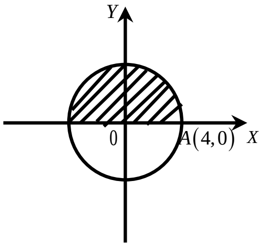
$g(x)=\dfrac{x^2}{4}$

$=\dfrac{1}{4}\int_{-1}^{2}\left(x+2-x^2\right)dx$

$\dfrac{1}{4}\left[\dfrac{x^2}{2}+2x-\dfrac{x^3}{3}\right]_{-1}^{1}=\dfrac{9}{8}$sq.units

98. Now, $y=\sqrt{16-x^2}$

$\Rightarrow\ x^2+y^2=16.$



Required area

$=2\int_{0}^{4}\sqrt{16-x^2}\,dx$

$=2\left[\dfrac{x^2}{2}\sqrt{16-x^2}+\dfrac{16}{2}\sin^{-1}\dfrac{x}{4}\right]_0^4$

$=2\left(8\times\dfrac{\pi}{2}\right)=8\pi$ sq.units

99. Projection of $\vec{a}$ along $\vec{b}$ is $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Projection vector $=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\cdot\hat{b}$

$=\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}\qquad\left(\because\ \hat{b}=\dfrac{1}{|\vec{b}|}\vec{b}\right)$

100.Now, $\vec{a}\cdot\vec{b}=12$

$\Rightarrow|\vec{a}|\cdot|\vec{b}|\cos\theta=12$

$\Rightarrow\ \cos\theta=\dfrac{12}{10\times2}\left(\because\ |\vec{a}|=10,|\vec{b}|=2\right)$

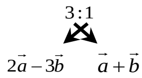
$\Rightarrow\ \cos\theta=\dfrac{3}{5}\Rightarrow\sin\theta=\dfrac{4}{5}$

Now, $|\vec{a}\times\vec{b}|=|\vec{a}|\cdot|\vec{b}|\sin\theta$

$=10\times2\times\dfrac{4}{5}=16$

101.Required vector is $\dfrac{3\left(\vec{a}+\vec{b}\right)+1\left(2\vec{a}-3\vec{b}\right)}{3+1}$

$\dfrac{3\left(\vec{a}+\vec{b}\right)+1\left(2\vec{a}-3\vec{b}\right)}{3+1}=\dfrac{5\vec{a}}{4}$



102.Let, $\vec{a}=i-2j+2k$.

Required area $=9\cdot a$

$=9\cdot\dfrac{i-2j+2k}{\sqrt{1+4+4}}=3(i-2j+2k)$

103.Now, $\vec{a}\times\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot\sin\theta$

$|\vec{a}\times\vec{b}|=1\Rightarrow3\cdot\dfrac{\sqrt{2}}{3}\cdot\sin\theta=1$

$\Rightarrow\sin\theta=\dfrac{1}{\sqrt{2}}\Rightarrow\ \theta=\dfrac{\pi}{4}$

104.Power of highest order derivative is 2.

$\therefore$ degree =2.

105.We have, $\dfrac{dy}{dx}+y=-2xe^{-x}$

I.F = $e^{\int 1dx}=e^x$

Thus the solution is

$ye^x=\int-2xe^{-x}\cdot e^x dx+c$

$\Rightarrow\ ye^x=-x^2+c\Rightarrow\ ye^x+x^2=c$

106.We have, $\dfrac{dy}{dx}-\dfrac{x}{1-x^2}y=\dfrac{1}{1-x^2}$.

I.F $=e^{-\int\frac{x}{1-x^2}dx}=e^{\frac{1}{2}\int\frac{-2x}{1-x^2}dx}$

$=e^{\frac{1}{2}\log\left(1-x^2\right)}=\sqrt{1-x^2}$

107.We have, $e^x\cos y\,dx-e^x\sin y\,dy=0$

Clearly, $\dfrac{d}{dx}\left(e^x\cos y\right)=e^x\cos y-e^x\sin y\dfrac{dy}{dx}$

$\therefore\ e^x\cos y=k$ is the general solution.

108.The lines

$\dfrac{x-1}{-3}=\dfrac{y-2}{2k}=\dfrac{z-3}{2}$ and $\dfrac{x-1}{3k}=\dfrac{y-5}{1}=\dfrac{z-6}{-5}$

Are at right angles if

$-9k+2k-10=0 \Rightarrow -7k=10 \Rightarrow k=\frac{-10}{7}$

109.The given planes are

$2x+3y+4z=4$ and $4x+6y+8z=12$

$\Rightarrow 2x+3y+4z-4=0$ and $2x+3y+4z-6=0$

Clearly these two planes are parallel.

$\therefore$ distance between them $=\left|\frac{-6-(-4)}{\sqrt{4+9+16}}\right|=\frac{2}{\sqrt{29}}$

110.Clearly foot of the perpendicular for (2, 5, 7) on the x – axis is (2, 0, 0).

111.Clearly $k=+\frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$, because the line make equal angles with axis.

112.We have, $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$

And $2x-2y+z=5$

$\sin\theta=\left|\frac{(3\times 2)+4(-2)+5(1)}{\sqrt{9+16+25}\sqrt{4+4+1}}\right|$

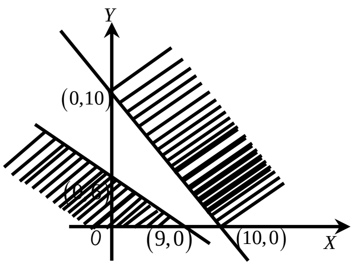
$=\frac{3}{\sqrt{50}\sqrt{9}}=\frac{1}{5\sqrt{2}}=\frac{\sqrt{2}}{10}$

113.Now, d.c's of x-axis are 1, 0, 0

We have the plane $2x-3y+6z-11=0$.

$\alpha=\sin\left(\sin^{-1}\alpha\right)=\left|\frac{2-0+0}{\sqrt{1+0+0}\sqrt{4+9+36}}\right|=\frac{2}{7}$

114.The required region is given in the following figure.



The shaded region represents the inequations.

Clearly, the inequation represents a null set.

115.Clearly, the region is represented by the in equations

$x, y\geq 0, x+y\geq 5, x\geq 4, y\leq 2$

116.Clearly,

$E=\{(1,1),(1,2),(2,1),(1,4),(2,3),(3,2),(4,1),$

$(1,6),(2,5),(3,4),(4,3),(5,2),(6,1),(5,6),(6,5)\}$

Thus $n(E)=15$ and $n(S)=6\times 6=36$

$\therefore$ required probability $=\frac{n(E)}{n(S)}=\frac{15}{36}=\frac{5}{12}$

117.We have, $\sum p(x_i)=1 \Rightarrow k+3k+3k+k=1$

$\Rightarrow 8k=1 \Rightarrow k=\frac{1}{8}$

Variance $=\sum p_i\cdot x_1^2-\left(\sum p_i x_i\right)^2$

$=\left(0^2\cdot k^2+l^2\cdot 3k+2^2\cdot 3k+3^2\cdot k\right)$

$-\left(0\cdot k+1\cdot 3k+2\cdot 3k+3\cdot k\right)^2$

$=3k+12k+9k-\left(3k+6k+3k\right)^2$

$=24k-(12k)^2=3-\frac{9}{4}=\frac{3}{4}$

$k=\frac{1}{8}$ and variance $=\frac{3}{4}$

118.We have, $P(A/B)=\frac{P(A\cap B)}{P(B)}$

Since $P(B)=0$, $P(A/B)$ is not defined.

119.Required probability

$=P\left[(G\cap G\cap B)\cup(G\cap B\cap G)\cup(B\cap G\cap G)\right]$

$=P(G\cap G\cap B)+P(G\cap B\cap G)+P(B\cap G\cap G)$

$=\left(\frac{3}{8}\times\frac{2}{7}\times\frac{2}{6}\right)+\left(\frac{3}{8}\times\frac{2}{7}\times\frac{2}{6}\right)+\left(\frac{2}{8}\times\frac{3}{7}\times\frac{3}{6}\right)$

$=3\left(\frac{1}{28}\right)=\frac{3}{28}$

120.Required probability $=P(D\cap D)=\frac{3}{8}\times\frac{2}{7}=\frac{3}{28}$

Physics Solutions:

121.By homogeneity of dimensions of LHS and RHS,

Distance (LHS) $=[\gamma]+\left[T^2\right]$

$\therefore[\gamma]=\left[LT^{-2}\right]$

122.Time of flight $=\dfrac{2u\sin\theta}{g}$

$=\dfrac{2\times 9.8\times\sin 30°}{9.8}=2\times\dfrac{1}{2}=1\,\text{s}$

123. $x=a_0+a_1t+a_2t^2$

$\therefore$ velocity, $v=\dfrac{dx}{dt}=0+a_1+2a_2t$

Acceleration, $a=\dfrac{dv}{dt}=0+2a_2=2a_2$

124.Total time of motion on circular track

$=2\times 60+20=140\,\text{s}$. Time period of revolution $=40\text{s}$.

Therefore displacement in time $(=3\times 40=120\,\text{s})=$

zero as athlete will be reaching at the starting point.

Thus, displacement in 140 seconds $=$ displacement in

20 seconds $=2R$

125.Since nitrogen is a diatomic gas

$C_P=\dfrac{7}{2}R$

Heat supplied $Q=nC_P\Delta T$ where n is the number of moles, C_P is the molar specific heat at constant pressure, ΔT is the rise in temperature.

$\therefore 1163.4=(1)\dfrac{7}{2}\times 8.31\Delta T$

$\Delta T=\dfrac{1163.4\times 2}{7\times 8.31}=40\,\text{K}$

126.Potential energy stored in the spring when it is extended by x is

$U_1=\dfrac{1}{2}kx^2$

Potential energy stored in the spring when it is further extended by y is

$U_2=\dfrac{1}{2}k(x+y)^2$

$\therefore$ work done $=U_2-U_1=\dfrac{1}{2}k(x+y)^2-\dfrac{1}{2}kx^2$

$=\dfrac{1}{2}ky(2x+y)$

127.Loss of kinetic energy in a perfectly inelastic collision

$=\Delta E$

$\Delta E=\dfrac{1}{2}\dfrac{m_1m_2}{(m_1+m_2)}(u_1-u_2)^2$ or

$\Delta E=\dfrac{1}{2}\dfrac{M\times M}{(M+M)}(u_1-u_2)^2=\dfrac{M}{4}(u_1-u_2)^2$

128.Time period of a simple pendulum is $T=2\pi\sqrt{\dfrac{l}{g}}$

Time period of a simple pendulum is independent on the material of the bob. Hence when a metal bob is replaced by a wooden bob its time period remains the same.

129.Given: radius vector $\vec{r}=2\hat{i}+\hat{j}+\hat{k}$

Linear momentum $\vec{p}=2\hat{i}-3\hat{j}+\hat{k}$

Angular momentum $\vec{L}=\vec{r}\times\vec{p}$

$$\therefore\vec{L}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & 1\\ 2 & -3 & 1\end{vmatrix}$$

$=\hat{i}(1-(-3))-\hat{j}(2-2)+\hat{k}(-6-2)=4\hat{i}-8\hat{k}$

130.The position of centre of mass of a system does not depend upon the forces on the particle.

131.Frequency of the tuning fork, $\upsilon=260\,\text{Hz}$

The fundamental frequency of a closed organ pipe,

$\upsilon=\dfrac{v}{4l}$

Where v is the velocity of sound in air

Therefore, length of air column $(l)=\dfrac{v}{4(\upsilon)}=\dfrac{330}{4\times 260}$

$=0.317\,\text{m}=31.7\,\text{cm}$

132.Here, $\mu=10^{-4}\,\text{kg/m}$,

$\dfrac{2\pi}{\lambda}=1,\dfrac{2\pi}{T}=30$

$v=\dfrac{\lambda}{T}=30\,\text{m}\,\text{s}^{-1}$

As $v=\sqrt{\dfrac{T}{\mu}}$

$\therefore T = v^2 \times \mu = (30)^2 \times 10^{-4} = 0.09\,\text{N}$

133. $g_h = \dfrac{gR^2}{(R+h)^2}$. Given $g_h = \dfrac{g}{100}$. Therefore, we have

$$\frac{gR^2}{(R+h)^2} = \frac{g}{100} \text{ or } R+h = 10R \text{ or } h = 9R$$

134. Applying principle of conservation of linear momentum,

$$p = \sqrt{p_1^2 + p_2^2}$$

$$(m_1+m_2)v = \sqrt{(m_1v_1)^2 + (m_2v_2)^2}$$

$$\Rightarrow (30+20)v = \sqrt{(30\times1)^2 + (20\times2)^2} = 50$$

$$\therefore v = \frac{50}{50} = 1\,\text{m s}^{-1}$$

135. Here,

Mass of stone, $m = 2\,\text{kg}$

Length of a string, $r = 0.5\,\text{m}$

Breaking tension, $T = 900\,\text{N}$

As $T = mr\omega^2$ or $\omega^2 = \dfrac{T}{mr} = \dfrac{900}{2\times0.5} = 900$

$\omega = 30\,\text{rad s}^{-1}$

136. $J = \sigma E$ or $\sigma = J/E$

137. Here, $l_1 = 240\,\text{cm}, l_2 = 120\,\text{cm}, R = 2\Omega, r = ?$

$$r = \left(\frac{l_1 - l_2}{l_2}\right)R = \left(\frac{240-120}{120}\right)\times 2 = 2\Omega$$

138. The tolerance level of a resistor with colour code red, blue, orange, gold is $\pm5\%$.

139. Cells in series, current through the external resistance,

$$I = \frac{n\varepsilon}{R+nr}$$

Cells in parallel, $I = \dfrac{\varepsilon}{R+\dfrac{r}{n}} = \dfrac{n\varepsilon}{nR+r}$

According to problem, $\dfrac{n\varepsilon}{R+nr} = \dfrac{n\varepsilon}{nR+r}$

Or, $R+nr = nR+r \Rightarrow R(1-n) = r(1-n)$

$\Rightarrow R = r$

140. Let R be the resistance to be connected in parallel to 12Ω, then effective resistance of AD arm $= \dfrac{12\times R}{12+R}$.

Potential difference between B and D is zero if bridge is balanced, i.e.,

$$\frac{\text{Resistance of } AB \text{ arm}}{\text{Resistance of } BC \text{ arm}} = \frac{\text{Resistance of } AD \text{ arm}}{\text{Resistance of } DC \text{ arm}}$$

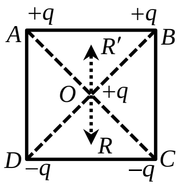
$$\frac{6}{12} = \frac{12R/(12+R)}{6}$$

On solving, we get $R = 4\Omega$

141. $\phi = \dfrac{q}{\varepsilon_0}$

$q = 0 \qquad \therefore \phi = 0$

142. Since $ABCD$ is a square, the centre O is equi-spaced from all the corners of the square. Let the charge at O be $+q$.



Therefore force on the charge at O due to charges at A and C will be along $\overrightarrow{OC}$ both as one force is repulsive and the other is attractive. So the resultant of these two forces will be along $\overrightarrow{OC}$. Similarly, the resultant force due to charges at B and D will be along $\overrightarrow{OD}$. Hence the resultant of all the forces at O will be along $\overrightarrow{OR}$. In case the charge at O be $-q$, then the resultant will be along $\overrightarrow{OR}$. Since both $\overrightarrow{OR}$ and $\overrightarrow{OR'}$ are perpendicular to the side AB, so the resultant force on any charge placed at O due to the given configuration of charges will be perpendicular to the side AB.

143. The potential at the centre of the sphere

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{2R}$$

We require a point where $V = \dfrac{V_C}{2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Q}{4R}$

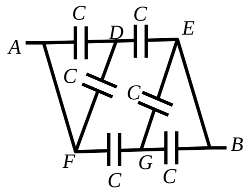
This point cannot be inside the sphere where

$$V \geq \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{4R}$$

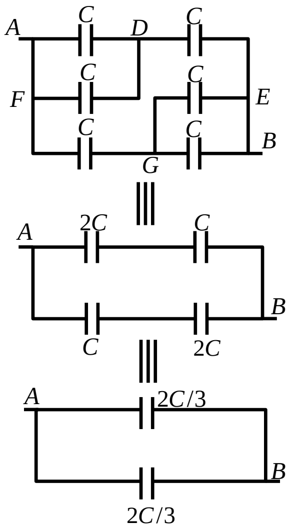
$$\therefore r = \frac{4}{3}R$$

Distance from the surface $= r - R = \frac{4}{3}R - R = \frac{R}{3}$

144.Sol:



The equivalent circuit diagrams are as shown in the figure below.

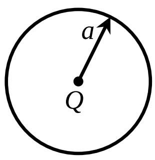


The equivalent capacitance between A and B is

$$C_{AB} = \frac{2C}{3} + \frac{2C}{3} = \frac{4C}{3}$$

145.The electrical potential at any point on circle of radius a due to charge Q at its centre is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}$$



It is an equipotential surface. Hence, work done is carrying a charge q round the circle is zero.

146.For an un-electrified soap bubble force due to excess pressure from inside is balanced by the force due to surface tension. When it is given an electric charge, there is an outward normal force ($\sigma^2/2\varepsilon_0$ per unit area, where σ is the surface charge density) that expands the bubble.

147.De Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mqV}}$

For the same value of V or $\lambda \propto \frac{1}{\sqrt{mq}}$

$$\therefore \lambda_e : \lambda_p : \lambda_\alpha = \frac{1}{\sqrt{m_e q_e}} : \frac{1}{\sqrt{m_p q_p}} : \frac{1}{\sqrt{m_\alpha q_\alpha}}$$

$$= \frac{1}{\sqrt{\left(\frac{m_p}{1840}\right)\times e}} : \frac{1}{\sqrt{m_p \times e}} : \frac{1}{\sqrt{4m_p \times 2e}}$$

$$\therefore \lambda_e > \lambda_p > \lambda_\alpha$$

148.Energy of photon, $E = h\upsilon = \frac{hc}{\lambda_{ph}}$

where λ_{ph} is the wavelength of a photon

$$\lambda_{ph} = \frac{hc}{E}$$

Wavelength of the electron, $\lambda_e = \frac{h}{\sqrt{2mE}}$

$$\therefore \frac{\lambda_{ph}}{\lambda_e} = \frac{hc}{E} \times \frac{\sqrt{2mE}}{h} = c\sqrt{\frac{2m}{E}}$$

149. $B_{\text{axis}} = \frac{\mu_0}{4\pi} \times \frac{2\pi IR^2}{\left(R^2 + x^2\right)^{3/2}}$

At centre, $B_{\text{centre}} = \frac{\mu_0}{4\pi} \times \frac{2\pi I}{R}$

In the given problem,

$$\frac{\mu_0}{4\pi} \times \frac{2\pi IR^2}{\left(R^2 + x^2\right)^{3/2}} = \frac{1}{8}\left[\frac{\mu_0}{4\pi} \times \frac{2\pi I}{R}\right]$$

Or $\left(R^2 + x^2\right)^{3/2} = 8R^3$

Solving, we get $x = \sqrt{3}R$

150. Work done in changing the orientation of a dipole of moment M in a magnetic field B from position θ_1 to θ_2 is given by

$W = MB(\cos\theta_1 - \cos\theta_2)$

Here, $\theta_1 = 0°$ and $\theta_2 = 180°$

So, $W = 2MB = 2\times 2.5\times 0.2 = 1\text{J}$

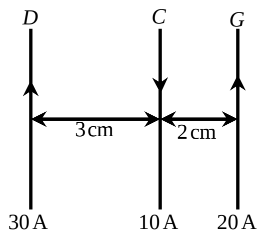
151. For ferromagnetic substances, χ_m is large and positive.

152. As $\mu_r = 1 + \chi_m$

$\therefore \chi_m = \mu_r - 1 = 5500 - 1 = 5499$

153. $S = \frac{I_g G}{I - I_g} = \frac{0.01\times 25}{10 - 0.01} = \frac{25}{999}\Omega$

154. The magnetic field due to wire D at wire C is



$B_D = \left(\frac{\mu_0}{4\pi}\right)\frac{2I}{r} = \frac{10^{-7}\times 2\times 30}{0.03} = 2\times 10^{-4}\text{T}$ which is directed into the page. The magnetic field due to wire G at C is

$B = B_G - B_D = 2\times 10^{-4} - 2\times 10^{-4}$ = zero.

The force on 25 cm of wire C is $F = Bl\sin\theta$ = zero.

155. Current due to orbital motion of electron $I = e\upsilon$.

Magnetic dipole moment,

$M = AI = (\pi r^2)(e\upsilon) = \pi r^2 \upsilon e$

156. Total torque on the coil, $\tau = BIAN\sin\theta$

Where θ is the angle which the normal to the plane of the coil makes with the direction of magnetic field.

Here, $\theta = 0°$

$\therefore \tau = 0$

157. The decay constant λ is the reciprocal of the mean life τ.

$\Rightarrow \lambda = \frac{1}{\tau}$

$\lambda_\alpha = \frac{1}{1620}$ per year and $\lambda_\beta = \frac{1}{405}$ per year

$\therefore$ Total decay constant, $\lambda = \lambda_\alpha + \lambda_\beta$ or

$\lambda = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$ per year

According to radioactive decay, $N = N_0 e^{-\lambda t}$

When $(3/4)^{\text{th}}$ part of the sample has disintegrated,

$N = \frac{N_0}{4}$ or $e^{\lambda t} = 4$

Taking logarithm on both sides, we get

$\lambda t = \ln 4$ or $t = \frac{1}{\lambda}\ln 2^2 = \frac{2}{\lambda}\ln 2$

$= 2\times 324\times 0.693 = 449$ year

158. Gamma rays are packets of energy. They carry no charge and no mass. Therefore, in gamma ray emission, there is no change in proton number and neutron number.

159. $_7X^{15} + \underset{\text{alpha particle}}{_2He^4} \to \underset{\text{Proton}}{_1H^1} + {_zY^A}$

According to the law of conservation of charge number, we get

$7 + 2 = 1 + Z$

$Z = 8$

According to the law of conservation of mass number, we get

$15 + 4 = 1 + A$

$A = 18$

Hence, the resulting atom has mass number 18 and atomic number 8.

160. Nuclear radius, $R = R_0(A)^{1/3}$ where A is the mass number of a nucleus.

Given: $R = 3.6\,\text{fm}$

$\therefore 3.6\,\text{fm} = (1.2\,\text{fm})(A^{1/3})$ $[\because R_0 = 1.2\,\text{fm}]$

or $A = (3)^3 = 27$

161. $\frac{1}{\lambda}=R\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$

For shortest wavelength in Balmer series,

$n_1=2, n_2=\infty$

$\frac{1}{\lambda}=R\left[\frac{1}{2^2}-\frac{1}{\infty^2}\right]$ or $\lambda=\frac{4}{R}$

For shortest wavelength in Brackett series,

$\frac{1}{\lambda'}=R\left[\frac{1}{4^2}-\frac{1}{\infty^2}\right]$

$\lambda'=\frac{16}{R}=4\times\frac{4}{R}=4\lambda$

162. $E_H=-13.6\,\text{eV}$

For $n=2$ of Li^{++}

$$E_n\left(Li^{++}\right)=\frac{E_H Z^2}{n^2}=\frac{-13.6\times(3)^2}{(2)^2}=-30.6\,\text{eV}$$

163. $|\varepsilon|=L\frac{dI}{dt}=\frac{0.5(10-0)}{2}=2.5\,\text{Volt}$

164. $\tan\phi=\left(\frac{X_L}{R}\right)$

$X_L=\omega L=(2\pi\upsilon L)=(2\pi)(50)(0.01)=\pi\,\Omega$

Also, $R=1\Omega$

$\therefore\phi=\tan^{-1}(\pi)$

165. Here, $V_R=100V, R=1k\Omega=10^{-3}\Omega,$

$C=2\,\mu\text{F}=2\times10^{-6}F, \omega=200\,\text{rad s}^{-1}$

Current, $I=\frac{V}{Z}$ where $V=\sqrt{(V_R)^2+(V_L-V_C)^2}$,

$Z=\sqrt{R^2+(X_L-X_C)^2}$

At resonance, $X_L=X_C$ $\therefore Z=R$

$V_L=V_C$ $\therefore V=V_R$

$\therefore I=\frac{V_R}{R}=\frac{100\text{V}}{10^3\Omega}=10^{-1}\text{A}$

At resonance, $V_L=V_C=IX_C=\frac{1}{\omega C}$

$=\frac{10^{-1}}{200\times2\times10^{-6}}=\frac{10^3}{4}=250\,\text{V}$

166. The three coils are in parallel.

$\therefore\frac{1}{L_p}=\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$ or

$L_p=1\text{H}$

167. The core of a transformer is laminated to reduce eddy current.

168. At absolute zero temperature, a semiconductor acts as an insulator.

169. Both $n-$type and $p-$type germanium are electrically neutral.

170. The Boolean expression for NAND gate is $Y=\overline{A\cdot B}$

Therefore the truth table of NAND gate is as given below

Inputs		Output
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

171. The radio waves can be sent from one place to another through ground wave propagation, sky wave propagation and space wave propagation.

172. As magnifying power, $M=\frac{f_0}{f_e}$, therefore, M can be increased by increasing focal length of objective lens.

173. For a path difference $(\mu-1)t$, the shift is

$x=(\mu-1)t\frac{D}{d}$

174. $\mu=\frac{\text{Real depth}}{\text{Apparent depth}}$

$\Rightarrow$ Apparent depth $=\frac{\text{Real depth}}{\mu}$

For 1st liquid, apparent depth $=\frac{d}{\mu_1}$

For 2nd liquid, apparent depth $=\frac{d}{\mu_2}$

$\therefore$ Total apparent depth $=\frac{d}{\mu_1}+\frac{d}{\mu_2}=d\left[\frac{1}{\mu_1}+\frac{1}{\mu_2}\right]$

175. $\mu_1 L_1=\mu_2 L_2$

$1.9d=\mu_2\times1.5d$

$\therefore \mu_2 = \frac{1.9d}{1.5d} = 1.27$

176. $i = e = \frac{3}{4}A$ where i is the angle of incidence, e is the angle of emergence and A is the angle of prism

As $A + \delta = i + e$

$\therefore \delta = i + e - A$

$= \frac{3}{4}A + \frac{3}{4}A - A = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ$

177.Here, distance between object and screen $D = 90\,\text{cm}$.

Distance between two locations of convex lens

$d = 20\,\text{cm},\ f = ?$

As $f = \frac{D^2 - d^2}{4D}$

$\therefore f = \frac{90^2 - 20^2}{4 \times 90} = \frac{(90+20)(90-20)}{360}$

$= \frac{110 \times 70}{360} = \frac{770}{36}\text{cm}$

178. $\frac{\Delta V}{V} = \frac{\Delta(\pi r^2 l)}{\pi r^2 l} = \frac{r^2 \Delta l + 2rl\Delta r}{r^2 l}$

$\frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{2\Delta r}{r} \Rightarrow 0 = \frac{\Delta l}{l} + \frac{2\Delta r}{r}$

$\Rightarrow \frac{\Delta l}{l} = -\frac{2\Delta r}{r}$

179.Here, $R = \frac{2.8}{2} = 1.4\,\text{mm} = 0.14\,\text{cm}$

$\frac{4}{3}\pi R^3 = 125 \times \frac{4}{3}\pi r^3$ or $r = \frac{R}{5} = \frac{0.14}{5} = 0.028\,\text{cm}$

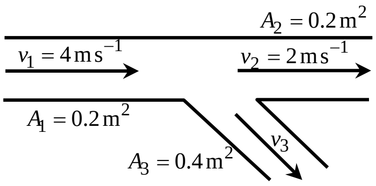
Change in energy = surface tension $\times$ increase in area

$= 75 \times \left[125 \times 4\pi r^2 - 4\pi R^2\right]$

$= 75 \times 4\pi \times \left[125 \times (0.028)^2 - (0.14)^2\right]$

$= 74\,\text{erg}$

180.Sol:



According to steady flow,

$A_1 v_1 = A_2 v_2 + A_3 v_3$ or $A_3 v_3 = A_1 v_1 - A_2 v_2$

or $v_3 = \frac{1}{A_3}[A_1 v_1 - A_2 v_2]$

$= \frac{1}{0.4}[0.2 \times 4 - 0.2 \times 2] = 1\,\text{m s}^{-1}$