MOCK TEST PAPER -1 (KEY)

Physics

1) 2	2) 2	3) 3	4) 2	5) 3	6) 3	7) 4	8) 1	9) 3	10) 1
11) 2	12) 4	13) 1	14) 4	15) 1	16) 1	17) 3	18) 1	19) 2	20) 3
21) 1	22) 4	23) 2	24) 3	25) 4	26) 3	27) 2	28) 3	29) 2	30) 2
31) 2	32) 1	33) 1	34) 3	35) 1	36) 2	37) 4	38) 2	39) 4	40) 4
41) 1	42) 4	43) 1	44) 1	45) 3	46) 4	47) 2	48) 4	49) 1	50) 4
51) 4	52) 2	53) 3	54) 4	55) 2	56) 3	57) 4	58) 3	59) 4	60) 1

Chemistry

61) 1	62) 3	63) 3	64) 1	65) 4	66) 2	67) 4	68) 2	69) 2	70) 1
71) 1	72) 3	73) 3	74) 2	75) 3	76) 4	77) 1	78) 1	79) 1	80) 1
81) 4	82) 2	83) 4	84) 1	85) 2	86) 3	87) 1	88) 4	89) 3	90) 1
91) 1	92) 2	93) 2	94) 1	95) 4	96) 3	97) 1	98) 4	99) 1	100) 3
101) 1	102) 3	103) 4	104) 2	105) 2	106) 1	107) 1	108) 3	109) 1	110) 4
111) 2	112) 4	113) 1	114) 4	115) 2	116) 2	117) 3	118) 3	119) 4	120) 3

Mathematics

121) 1	122) 1	123) 1	124) 3	125) 3	126) 1	127) 3	128) 2	129) 2	130) 3
131) 3	132) 2	133) 4	134) 2	135) 3	136) 3	137) 1	138) 3	139) 4	140) 3
141) 3	142) 4	143) 4	144) 3	145) 3	146) 4	147) 3	148) 4	149) 1	150) 2
151) 2	152) 2	153) 4	154) 3	155) 2	156) 1	157) 2	158) 3	159) 3	160) 4
161) 4	162) 4	163) 1	164) 3	165) 3	166) 2	167) 3	168) 1	169) 2	170) 1
171) 3	172) 2	173) 2	174) 3	175) 3	176) 3	177) 3	178) 4	179) 4	180) 4

HINTS AND SOLUTIONS

SUBJECT : PHYSICS

1. (2)

When the ball is moving upward, both its weight and the air resistance are retarding its motion but during its descent,only the air resistance opposes its motion as its weight accelerates it downwards.

Hence magnitude of deceleration during ascent is higher than the magnitude of acceleration during downward motion.

$\therefore$ Time taken for ascent is less than the time taken for descent

2. (2)

Power radiated $\propto A\left(T^4\right)$

$\therefore 4\pi r_1^2 \times \left(T_1\right)^4 = 4\pi r_2^2 \times \left(T_2\right)^4$

or $\dfrac{r_1}{r_2} = \left(\dfrac{T_2}{T_1}\right)^2$

3. (3)

When the angle of contact between a liquid drop and a solid surface is obtuse, the cohesive forces within the liquid dominate over the adhesive forces between the solid and the liquid and hence the liquid does not wet the solid surface.

4. (2)

The minimum possible length of a closed organ pipe for emitting sound of a given wavelength is quarter wavelength. Since audible frequency range is 20 Hz to 20000 Hz and speed of sound = 336 m/s (given), minimum audible wavelength is (336/20000) m = 16.8 mm.

$\therefore$ Minimum length of closed pipe that produces sound of 16.8 mm wavelength = (16.8/4) mm = 4.2 mm.

5. (3)

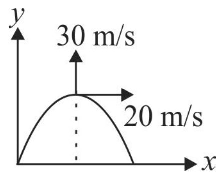
Resultant force

$= \sqrt{P^2 + P^2 + 2P \times P \times \cos 120^o}$

$= \sqrt{P^2 + P^2 + 2 \times P^2 \times \left(-\dfrac{1}{2}\right)} = P$

6. (3)

Let m kg be the mass of the projectile



Initial momentum of the projectile

$= \mathrm{m} \times 20\hat{\mathrm{i}}\ \mathrm{kgm/s}$

After the internal explosion the mass of each part will be m/2

Momentum of the first part

$= \dfrac{\mathrm{m}}{2} \times 30\,\hat{\mathrm{j}}\ \mathrm{kgm/s}$

Let $\vec{v}$ be the velocity of the second part.

Then from the principle of conservation of linear momentum, $\mathrm{m} \times 20\hat{\mathrm{i}} = \dfrac{\mathrm{m}}{2} \times 30\,\hat{\mathrm{j}} + \dfrac{m}{2}\vec{v}$

$\therefore \vec{v} = 40\,\hat{\mathrm{i}} - 30\,\hat{\mathrm{j}}$

$\therefore \quad \left|\vec{v}\right| = \sqrt{40^2 + \left(-30\right)^2} = 50\ \mathrm{m/s}$

7. (4) If a capacitor of capacitance C is charged with the help of a charging battery of constant voltage V, then energy supplied by the battery = QV = CV^2

Energy stored in the capacitor = $\dfrac{1}{2}QV = \dfrac{1}{2}CV^2$

Hence 50% of the energy drawn from the battery is stored in the capacitor

8. (1)

Capacitance of spherical drop of radius R is

$C = 4\pi\varepsilon_0 R$

When the bigger drop is divided into 8 equal drops of radius r, using conservation of volume:

$\frac{4}{3}\pi R^3 = 8\times\frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{2}$

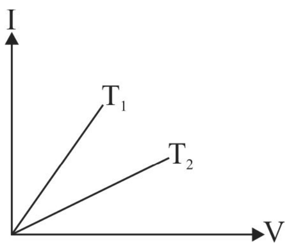
Thus capacitance of each small drop is

$C' = 4\pi\varepsilon_0 r = 4\pi\varepsilon_0 \frac{R}{2} = \frac{C}{2} = \frac{1}{2}\mu F$

9. (3)

When a dielectric slab is slipped between the plates of a capacitor connected to a battery, its capacitance increases and that causes both the charge on the capacitor plates and the energy stored in the capacitor to increase as charge flows from the battery to the capacitor.The potential difference and the net electric field between the capacitor plates does not change due to presence of the dielectric.

10. (1)



Slope of I-V curve gives reciprocal of resistance of the conductor at the given temperature.

$\therefore$ The resistance of the metallic conductor is less in case of temperature T_1 compared to T_2.

Also, we know that the resistance of a conductor increases with temperature.

Therefore, $T_1 < T_2$.

11. (2) Reduction of chemical energy in the battery

$= VIt = 6\times5\times6\times60\,J$

$= 10800\ J = 1.08\times10^4\ J$

12. (4)

For 30V, 90W bulb

$\text{Resistance } r = \frac{V^2}{P} = \frac{30\times30}{90} = 10\Omega;$

$\text{Current } I = \frac{P}{V} = \frac{90}{30} = 3A$

To work with 120V line, a resistance R is connected in series to keep the current same.

$\therefore\ I = V/(r+R)$

or, $3 = \frac{120}{10+R} \Rightarrow R = 30\Omega$

13. (1)

Efficiency of Carnot's engine is given by

$\eta = 1 - \frac{T_2}{T_1}$

Where T_1 is the source (higher) temperature and T_2 is the sink (lower) temperature in Kelvin

$\therefore \eta_1 = 1 - \frac{0+273}{200+273} = \frac{200}{473}$

$\eta_2 = 1 - \frac{-200+273}{0+273} = \frac{200}{273}$

$\therefore\ \frac{\eta_1}{\eta_2} = \frac{273}{473} \approx \frac{1}{1.73} = 1:1.73$

14. (4)

For given quantity of ideal gas, PV = constant and independent of the volume V.

15. (1)

The positive charge Q will execute an oscillatory motion along x axis with the origin as its mean position as the net electrostatic force on the positive charge by the two negative charges is always acting towards origin and parallel to x axis due to symmetry.

As the resultant force on Q is not proportional to its distance from origin, the motion is not SHM.

16. (1)

According to Newton's law of cooling rate of heat loss is proportional to the excess of mean temperature of the body over the surroundings where the mean is calculated between initial and final temperature.

As the amount of heat lost (Q) is same in three cases, rates of heat loss are:

$(Q/t_1) = k(72.5 - \theta)$,

$(Q/t_2) = k(67.5 - \theta)$ and

$(Q/t_3) = k(62.5 - \theta)$,

where $\theta =$ room temperature

and k = proportionality constant.

Hence $t_1 < t_2 < t_3$.

17. (3)

According to the Stefan's law , the total energy emitted per second by a unit area of a black body is proportional to the fourth power of its absolute temperature. Thus blue glass is at higher temperature than red glass therefore it shines brighter than red glass.

18. (1)

Initial number of nuclei = N_0

After time t_1 and t_2 the number of nuclei reduces to N_1 and N_2 respectively.

Half life $T_{1/2} = 20$ minutes.

As $\dfrac{N_1}{N_0} = e^{-\frac{\ln 2}{T_{1/2}} t_1}$ and $\dfrac{N_2}{N_0} = e^{-\frac{\ln 2}{T_{1/2}} t_2}$,

dividing the first equation with the second,

$$\Rightarrow \frac{N_1/N_0}{N_2/N_0} = e^{\frac{\ln 2}{T_{1/2}}(t_2 - t_1)}$$

$$\Rightarrow (t_2 - t_1) = \frac{T_{1/2}}{\ln 2} \cdot \ln\left(\frac{N_1/N_0}{N_2/N_0}\right)$$

Given $N_1/N_0 = 80\%$ and $N_2/N_0 = 20\%$

$$(t_2 - t_1) = 20 \times \frac{\ln(0.8/0.2)}{\ln 2} = 40\,\text{minutes}$$

19. (2)

Nuclear force binds the protons and neutrons in the nucleus of an atom

20. (3)

According to Bohr's postulates of atomic structure, the angular momentum of the electron in a stable orbit is integral multiple of $h/2\pi$

$\therefore$ Angular momentum, $J = mVR = nh/2\pi$

or $VR = nh/2\pi m$

where h is Planck's constant, m is the mass of electron

$\therefore$ $h/2\pi m$ is constant

$\therefore$ $VR \propto n$ (n is the quantum number)

21. (1)

$$\phi = 5t^3 - 100t + 300$$

$$e = -\frac{d\phi}{dt} = -\left[15t^2 - 100\right]$$

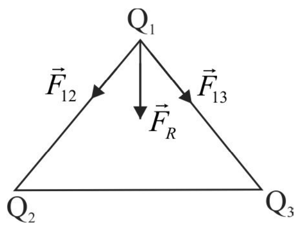
For $t = 2s, e = -(60 - 100) = 40\,V$

22. (4)

When the magnet M is falling through the ring R, according to the Lenz's law, the direction of the induced current in the ring will be such that it opposes the change in magnetic flux through the ring. Hence the resultant magnetic field will always oppose the motion of the falling magnet and the acceleration of M willbe less than g.

23. (2)

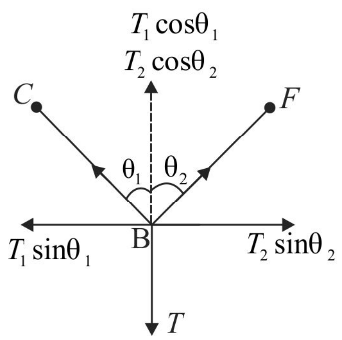
Each charge will be acted upon by two forces due to two other charges placed at the two other corners of the equilateral triangle.



Since the lines of action or the directions of those two forces are different irrespective of the magnitude and sign of the charges, the system can never be in equilibrium. One such case is shown in figure,

where $\overrightarrow{F_R} = \overrightarrow{F_{12}} + \overrightarrow{F_{13}}, \overrightarrow{F_R}$ is the resultant force on the charge Q_1

24. (3)



From figure, $T_1 \sin\theta_1 = T_2 \sin\theta_2$

As $\theta_1 = \theta_2 = 60^\circ, T_1 = T_2$

Also $T_1 \cos\theta_1 + T_2 \cos\theta_2 = T$

$\Rightarrow 2T_1 \cos\theta = 10\,\text{N}\left(\text{as } \theta_1 = \theta_2, \text{ and } T_1 = T_2\right)$

$\Rightarrow 2 \times T_1 \times \cos 60^o = 10N$

$\Rightarrow T_1 = 10N = T_2$

25. (4)

Specific resistance $\rho = 10^{-7}\,\Omega\text{m}$

Current I = 0.1 A

Cross-section area $A = 10^{-6}\,\text{m}^2$

$\Rightarrow$ Current density $j = I/A = 0.1/10^{-6} = 10^5\,\text{Am}^{-2}$

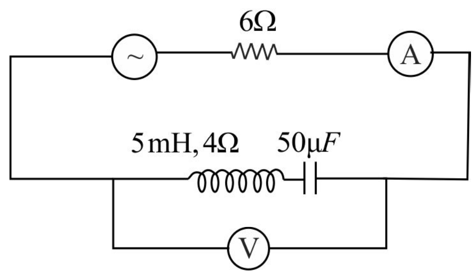
$\therefore$ Potential gradient

$= \text{Electric field } E = \rho j = 10^{-7} \times 10^5 = 10^{-2}\,\text{V/m}$

26. (3)

V = 20cos(2000t)

$\therefore$ Peak value of the a.c. voltage = 20V



$\therefore\ V_{\text{r.m.s}} = \frac{20}{\sqrt{2}} = 10\sqrt{2}\ \text{V}$

and angular frequency $\omega = 2000$ rad/s

Inductive reactance of the coil =

$X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\Omega$

Capacitive reactance

$X_C = \frac{1}{\omega C} = \frac{1}{2000 \times 50 \times 10^{-6}} = 10\Omega$

As capacitive and inductive reactances are equal, the circuit is in resonance and the load is effectively purely resistive.

$\therefore$ Impedance $Z = 6 + 4 = 10\,\Omega$

$\therefore$ Current, $I = \frac{V_{\text{r.m.s}}}{Z} = \frac{10\sqrt{2}}{10} = \sqrt{2}\text{A} \approx 1.4\,\text{A}$

$\therefore$ Voltage difference across the coil and the capacitance

$= I \times 4 = \sqrt{2} \times 4 \approx 5.6\ \text{V}$

27. (2)

For a given frequency, the shortest length of closed pipe that resonates to it is quarter wavelength.

$$\therefore\ l_{\min} = \frac{1}{4}\lambda = \frac{1}{4}\frac{v}{n} = \frac{v}{4n}$$

28. (3)

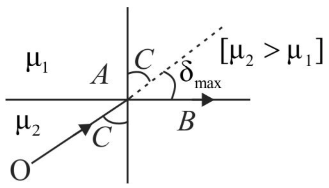
Corpuscular theory was postulated on the basis of rectilinear propagation of light.

29. (2)

Generally reverberation time is proportional to linear dimensions of the room. Hence, it will be proportional to the volume of the room

30. (2)

In the limiting case, $\overline{AB}$ will be the refracted ray which grazes along the surface AB corresponding to the incident ray OA



Here , angle of incidence = c (critical angle) and angle of refraction $= \pi/2$

Therefore, angle of deviation = $\delta_{\max} = \frac{\pi}{2} - c$

31. (2) Cannot cycle thermal efficiency

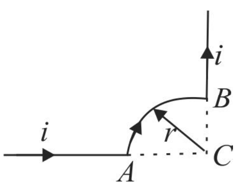
$$\eta = \frac{W}{Q} = 1 - \frac{T_c}{T_h}$$

where T_h and T_C are temperatures of source and sink, and Q and W are total heat absorbed and work output of the cycle respectively.

$\therefore$ The amount of heat converted to work = work output of the cycle = nQ

$$= \left[1 - \frac{(127+273)}{(227+273)}\right] \times 6 \times 10^4 = 1.2 \times 10^4 J$$

32. (1)



The magnetic field at C due to the straight portions of the conductor is zero because the position vectors $(\vec{r})$ and current elements $(id\vec{l})$ are parallel. So the field at C is only due to the circular portion. The direction will be perpendicular to the plane of the paper and directed into the paper according to right hand thumb rule.

33. (1) For adiabatic compression: $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

Given, $\frac{V_2}{V_1} = \frac{8}{27}, \gamma = \frac{5}{3}$ (ideal monoatomic gas)

and, $T_1 = 27 + 273 = 300K$

$$\Rightarrow T_2 = \left(\frac{27}{8}\right)^{5/3-1} \times T_1$$

$$= \left(\frac{3}{2}\right)^2 \times 300 = 675K$$

$\therefore$ Increase in temperature:

$$= 675 - 300 = 375^\circ C$$

34. (3)

With the introduction of a thin transparent medium, the fringe system gets a displacement of m fringes, then we have $(\mu - 1)t = m\lambda$

For the first case

$$(\mu - 1) \times t_1 = m_1 \lambda \qquad (1)$$

and for the second case

$$(\mu - 1) \times t_2 = m_2 \lambda \qquad (2)$$

Dividing (ii) and (i) , we get

$$\frac{t_2}{t_1} = \frac{m_2}{m_1} = \frac{20}{30}$$

$$\therefore t_2 = \frac{20}{30} \times t_1 = \frac{20}{30} \times 4.8\,\text{mm} = 3.2\,\text{mm}$$

35. (1)

At height h, $g' = g\left(1 - \frac{2h}{R_e}\right)$

$\frac{g' - g}{g} = -\frac{2h}{R_e} = -1\%$ (given)

At depth h, $g'' = g\left(1 - \frac{h}{R_e}\right)$

$\Rightarrow \frac{g'' - g}{g} = -\frac{h}{R_e} = -\frac{1}{2}\% = -0.5\%$

Hence weight in mine will decrease by 0.5%

36. (2)

As the ammeter and the voltmeter are connected in parallel and in series with the battery respectively, there will be a large current flowing through the ammeter and almost zero current flowing through the voltmeter. Hence the ammeter is likely to be damaged.

37. (4)

If v_O and v_E represents the velocities, μ_O and μ_E the respective indices of ordinary and extraordinary rays for a doubly refracting crystal, then

$\frac{\mu_O}{\mu_E} = \frac{v_E}{v_O}$

Now, for a quartz crystal (positive crystal)

$\mu_O \le \mu_E \therefore v_O \ge v_E$

and for a calcite crystal (negative crystal)

$\mu_O \ge \mu_E \therefore v_O \le v_E$

38. (2)

During the propagation of transverse waves, restoring force for oscillations of the particles of the medium is provided by its shear resistance which is represented by the shear modulus or the modulus of rigidity of the material.

Gaseous medium like air offer very little shear resistance and hence waves can travel through air only in form of longitudinal waves where the restoring force during oscillation is provided by the resistance of the gas against volume strain when change of pressure occurs i.e., its bulk modulus.

39. (4)

$\beta = \frac{D}{d}.\lambda$

$\therefore \frac{\beta}{\beta'} = \frac{\lambda}{\lambda'} = n \Rightarrow \beta' = \frac{\beta}{n}$

40. (4)

Two beams are equally energetic. Hence,

$n_1 h v_1 = n_2 h v_2 \Rightarrow \frac{n_1}{n_2} = \frac{v_2}{v_1}$

41. (1)

We have for radioactive decay

$\frac{N}{N_0} = e^{-\lambda t}$, where N_0 = initial number of active nuclei and N = number of active nuclei after the mean life

Here, t = mean life = $1/\lambda$ (λ is decay constant)

$\therefore \frac{N}{N_0} = e^{-\lambda \times \frac{1}{\lambda}} = \frac{1}{e}$

or, $1 - \frac{N}{N_0} = 1 - \frac{1}{e} = 1 - \frac{1}{2.718} = 0.632$

or, $\frac{N_0 - N}{N_0} = 0.632$

$\therefore$ Percentage of decay

$= \frac{N_0 - N}{N_0} \times 100 = 63.2 \simeq 63\%$

42. (4)

Speed of light in vacuum is a fundamental physical constant.

43. (1)

Time of flight t = $\frac{2u\sin\theta}{g}$

and Range R = $\frac{u^2\sin 2\theta}{g}$

We get same range for angles of projection θ and $(90° - \theta)$.

$$\therefore t_1 = \frac{2u\sin\theta}{g} \text{ and } t_2 = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\Rightarrow t_1 t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{2u^2\sin 2\theta}{g^2} = \frac{2}{g}\times R$$

$$\Rightarrow t_1 t_2 \propto R$$

44. (1)

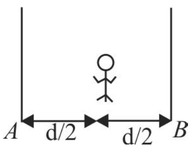
Kinetic energy = $\frac{p^2}{2m}$

Ratio of kinetic energy of two bodies of masses 1kg and 2kg having equal momentum is

$$\frac{K.E_1}{K.E_2} = \frac{p^2/2m_1}{p^2/2m_2} = \frac{m_2}{m_1} = \frac{2}{1}$$

45. (3)

Let the man hear first echo of claps after reflecting from cliff A. Thus the sound wave covers distance 2(d/2).



Therefore, $2(d/2) = vt$

$\Rightarrow d = 340m$

46. (4)

$$C = \frac{\varepsilon_0 A}{d} = 2pF......(i)$$

when the separation of the plates is doubled and the interspace between the plates is filled with wax, then capacitance becomes

$$C' = \frac{K\varepsilon_0 A}{2d} = 6pF.......(ii)$$

where K is dielectric constant of wax

From equations (i) and (ii)

$$\frac{K}{2}\times 2 = 6 \Rightarrow K = 6$$

47. (2)

The mass of atomic nucleus is slightly less than the sum of masses of the nucleons (protons and neutrons) present in the nucleus. This mass difference is called mass defect

Therefore, $m < (A-Z)m_n + Zm_p$

48. (4)

The width of central maximum decreases in Fresnel diffraction and increases in Fraunhofer diffraction when the slit width is decreased.

49. (1)

When resistance at 0°C is R_0, resistance R at t°C is given as, $R = R_0(1+\beta t)$, where β = temperature coefficient.

Given $R_1 = 1\Omega$ at $t_1 = (300-273) = 27^\circ C$,

$R_2 = 2\Omega(\text{at } t_2\,^\circ C)$ and $\beta = 0.00125/^\circ C$

$$\Rightarrow \frac{R_1}{R_2} = \frac{1+\beta t_1}{1+\beta t_2}$$

$$\therefore \frac{1}{2} = \frac{1+0.00125\times 27}{1+0.00125 t_2}$$

$$\Rightarrow t_2 = 854^\circ C \equiv 854 + 273 = 1127K$$

50. (4)

The value of resistance to be connected in series with galvanometer is

$$R = \frac{V}{I_g} - G = \frac{50}{10\times 10^{-3}} - 40$$

= 5000 - 40 = 4960

51. (4)

Here, the frequency of the reflected sound as heard by the driver is twice the actual frequency of the sound of the horn because apparent pitch and hence frequency (since pitch $\propto$ frequency) is one octave higher than the actual one.

Due to motion of the source, the apparent frequency of sound reaching the stationary cliff

$n_1 = n.\dfrac{v}{v-v_c}$ where v is the velocity of sound and v_c is the velocity of the car

Also, after reflection from the cliff, the relative direction of motion of the observer, i.e., the driver and direction of the sound propagation become opposite to each other, for which there will be further modification of frequency

The changed frequency is given by

$$n_2 = n_1.\frac{v+v_c}{v} = n.\frac{v}{v-v_c}\frac{v+v_c}{v} = n.\frac{v+v_c}{v-v_c}$$

Now, from the problem , $n_2 = 2n$

$$\therefore n.\frac{v+v_c}{v-v_c} = 2n \Rightarrow v_c = \frac{v}{3}$$

52. (2)

For a ideal transformer without any loss, output and input powers are equal.

53. (3)

Total power drawn = nP

54. (4)

The focal length of the plano convex lens is 0.4 because after the plane surface is silvered, the combination of lens + mirror will behave like a concave mirror of focal length = $\dfrac{f_{lens}}{2}$

As $f_{mirror} = 0.2m, f_{lens} = 0.4m$

From lens maker's formula

$$\frac{1}{f_{lens}} = (\mu-1)\left[\frac{1}{R_C} - \frac{1}{R_P}\right]$$

As, $\mu = 1.5, R_p = \infty,$

$$\Rightarrow \frac{1}{0.4} = 0.5\frac{1}{R_C} \Rightarrow R_C = 0.2m$$

55. (2)

When a source in Young's double slit experiment is white, the central fringe is white as all colours meet there in phase because phase difference is zero.

Fringe pattern for different colours (i.e., different wavelengths) appear at different locations as the locations for constructive interference is different for different wavelengths.

56. (3)

At the higher temperature electron crosses forbidden gap and reaches the conduction bond, leaving behind holes

57. (4)

58. (3)

$v = \sqrt{\dfrac{\gamma RT}{M}}$; γ is the same as both hydrogen and oxygen are diatomic

$$\therefore \frac{v_H}{v_O} = \sqrt{\frac{M_O}{M_H}} = \sqrt{\frac{16}{1}} \Rightarrow v_H : v_O = 4:1$$

59. (4)

Force on the particle

F = qE

Acceleration, $a = \dfrac{F}{m} = \dfrac{qE}{m}$

As the particle starts from rest, its velocity after time t :

$$v = a = \frac{qE}{m}t$$

$\therefore$ Kinetic energy:

$$\text{K.E.}=\frac{1}{2}m\times\left(\frac{qE}{m}t\right)^2=\frac{q^2E^2t^2}{2m}$$

60. (1)

Using $\text{emf}=-L\frac{dI}{dt}$ and

$\text{emf}=\frac{\text{work done}}{\text{charge}}=\frac{W}{Q}$ simultaneously,

$$[L]\frac{[I]}{[t]}=\frac{[W]}{[Q]}\Rightarrow[L]=\frac{[W][t]}{[I][Q]}$$

$$\Rightarrow[L]=\frac{(ML^2T^{-2})\times(T)}{(A)\times(AT)}=ML^2T^{-2}A^{-2}$$

SUBJECT : CHEMISTRY

61. (1)

62. (3) $P_1=P, V_1=4\,dm^3, T_1=T$

$P_2=2P, V_2=?, T_2=2T$

From gas equation,

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}\Rightarrow\frac{P\times4}{T}=\frac{2P\times V}{2T}$$

$\Rightarrow V=4\,dm^3$.

63. (3)

64. (1) pK_a is the negative logarithm of dissociation constant.

$pK_a=-\log K_a$

Strong acids have lower pK_a values.

65. (4)

Copper is extracted from copper pyrites by auto reduction. The reaction involved is

$$2Cu_2O+Cu_2S\xrightarrow[\text{reduction}]{\text{auto}}6Cu+SO_2$$

66. (2)

For the reaction to be spontaneous, ΔG should be negative. As in the given reaction solid phase changes to the liquid phase entropy increases for ΔG must be negative.

$\Delta H<T\Delta S$

67. (3)

Reactivity towards electrophilic substitution reaction increases as the electron density in the benzene ring increases. Activating groups i.e., -OH, $-CH_3$ increases electron density on the ring. -OH has a strong + R effect and $-CH_3$ has +I effect and hyperconjugative effect, +R effect being stronger than the +I effect and hyperconjugative effect, PhOH will undergo the reaction more readily than $PhCH_3$.-COOH group is deactivating group and decreases the electron density on the ring so less reactive than PhOH and $PhCH_3$.PhH has neither a deactivating nor an activating group, so it will be more reactive than the compounds with deactivating groups.

Phenol undergoes bromination easily even in absence of any halogen carrier.

68. (2) The methyl group in toluene is electron releasing group, (+I effect) and thus, it activates the benzene nucleus and gives electrophilic substitution reactions more readily than benzene. Sulphonation of toluene is approximately ten times more reactive than sulphonation of benzene. However, chloro and nitro group deactivates the benzene nucleus.

69. (2) Prolonged exposure of chloroform in human may cause damage to liver due to the formation of phosgene gas $(COCl_2)$.

The reaction involved is

$$CHCl_3+\frac{1}{2}O_2\xrightarrow{hv}\underset{\left(\substack{\text{Phosgene}\\\text{poisonous}}\right)}{COCl_2}+HCl$$

70. **(1)**

71. **(1)**

$EMF = E^o$ (cathode) $-E^o$ (anode)

$E^o\left(Cu^{+2}/Cu\right) - E^o\left(Al^{+3}/Al\right) = 2\,V$

$\Rightarrow E^o\left(Al^{+3}/Al\right) = -2 + 0.34 = -1.66\,V$

72. **(3)**

When the electrons of hydrogen atom return to L shell i.e., 2^{nd} energy level from shells of higher energy then the obtained series is called Balmer series.

73. **(3)** Hydrogen is smallest among the species.

74. **(2)** 1000 mL of *N* NaOH contains = 40 g of NaOH = 1 mole

100 mL of N NaOH will contain

= 0.4 gm NaOH $\equiv \frac{1}{10}$ mole

Now, $\underset{0.1\text{mole}}{OH^-} + \underset{0.1\text{mole}}{H^+} \rightarrow \underset{0.1\text{mole}}{H_2O}$

$\because$ When 1 mole H_2O is formed, the heat produced = 57.3 kJ

For 0.1 mole H_2O is formed, the heat produced = 5.73 kJ

75. **(3)** $\text{Rate}_1 = k[A]^2$

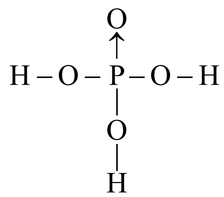
$\text{Rate}_2 = k[2A]^2 = 4k[A]^2$

$\Rightarrow \frac{\text{Rate}_2}{\text{Rate}_1} = 4 \Rightarrow \text{Rate}_2 = 4 \times \text{Rate}_1$

76. **(4)** Order of a reaction is the sum of the exponents to which the molar concentrations are raised in the rate determining step. Order is determined by the mechanism of reaction and concentration of reactants.

77. **(1)** The structure of orthophosphoric acid,

H_3PO_4 is



It is a tribasic acid i.e., 3-hydroxyl groups are present. Phosphorus atom is in sp^3 hybrid state.

78. **(1)** $CH_3I > CH_3Br > CH_3Cl$

79. **(1)** Molecular formula of $A = C_2Cl_3OH$ As (A) reduces Fehling's solution and on oxidation gives a monocarboxylic acid (B). It means (A) must be an aldehyde.

$\underset{(A)}{CCl_3CHO}$

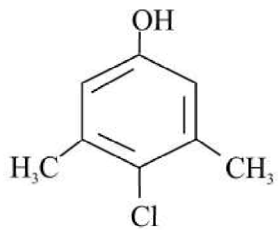
This is further confirmed by the reaction,

$C_2H_5OH + Cl_2 \xrightarrow{[O]} CH_3CHO$

$\xrightarrow{Cl_2} CCl_3CHO$

$A = \text{chloral}\left(CCl_3CHO\right)$

80. **(1)**



chloroxylenol
(4-chloro-3, 5-dimethylphenol)

81. **(4)** The outermost configuration for 6^{th} period of *p*-block is $[Xe]4f^{14}5d^{10}6s^2 6p^{1-6}$.

Last element of *p*-block element in 6^{th} period is Rn. Therefore, its electronic configuration is

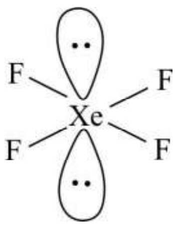
$^{86}Rn = [Xe]4f^{14}5d^{10}6s^2 6p^6$

82. **(2)** The hybridization of XeF_4 is

Hyb.=

$$\frac{[V+M+A-C]}{2}=\frac{[8+4+0-0]}{2}=6\left(sp^3d^2\right)$$

Therefore, the structure of XeF_4 is square planar



83. **(4)** Buna - S is a polymer of 1,3-butadiene and styrene.

84. **(1)** After dilution with equal volume of water

$\left[H^+\right]=0.5\times10^{-4}M$.

$\Rightarrow\left[OH^-\right]=\dfrac{10^{-14}}{0.5\times10^{-4}}=2\times10^{-10}\ mol.dm^{-3}$.

85. **(2)**

When radius ratio is between 0.732 to 1.000 the coordination number is 8.

86. **(3)** Due to absence of hydrogen bonding in ether, its boiling point is lowest among the given species.

87. **(1)** Noble gases have fully filled valence orbitals hence have high ionisation potential.

88. **(4)** C^{14} is a natural radioactive isotope.

89. **(3)**

44 g H_2 = 22 mol H_2

44 g CO_2 = 1 mol CO_2

$\because$ Pressure of 1 mol CO_2 = 1 atm.

$\therefore$ Pressure of 22 mol. H_2 = 22 atm.

90. **(1)** $\overset{1}{H_3C}-\underset{\underset{Cl}{|}}{\overset{2}{C}}=\overset{3}{CH}-\underset{\underset{CH_3}{|}}{\overset{4}{CH}}-\overset{5}{CH_3}$

2–chloro–4–methyl–2–pentene

91. **(1)** At STP,

22400 cm^3 of CH_4 gas = 16 g

112 cm^3 of CH_4 gas = 0.08 g

92. **(2)**

30 gram of Mg and 30 gram of O_2 are reacted.

$2Mg+O_2\rightarrow 2MgO$

2×24 gram of magnesium combines with 32 gram of oxygen.

$\therefore$ 30 gram of magnesium combines with $\dfrac{32\times30}{2\times24}$ gram of O_2 = 20 gram of O_2.

Residual mixture contains (30–20) gram = 10 gram of O_2 and 50 gram of magnesium oxide

93. **(2)**

94. **(1)**

The hybrid orbitals used for bonding in methane $\left(sp^3\right)$, ethene $\left(sp^2\right)$ and ethyne (sp) have 25, 33 and 50% s-character.

95. **(4)** HCHO - Methanal

96. **(3)** : $\left(CH_3COO\right)_2Ca+\left(HCOO\right)_2Ca$

$\rightarrow \underset{\text{acetaldehyde}}{2CH_3CHO}+2CaCO_3$

Ketones are produced when calcium salts other than formate are heated.

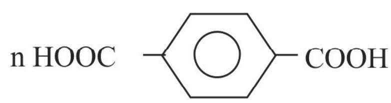
97. **(1)**

Given reaction is known as Carbylamine reaction.

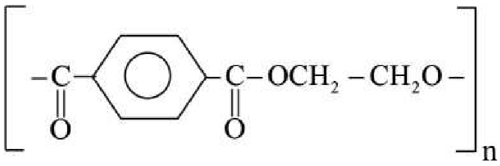
98 **(4)** The carbon compound is CH_3NH_2. As,

$CH_3NH_2\xrightarrow[NaNO_2]{HCl}CH_3OH+N_2+H_2O$

99. **(1)** Dacron is obtained by the condensation polymerisation of terepthalic acid and ethylene glycol.



$+n.HOCH_2-CH_2OH\xrightarrow[300°C]{-n\,H_2O}$



100. (3) Due to the presence of the unpaired electrons in valence shell, ions show paramagnetic behavior. The electronic configuration of the ions is

$Ti(Z=22):[Ar]4s^2 3d^2$

$Ti^{3+}:[Ar]4s^0 3d^1 \rightarrow$ One unpaired electron.

$Mn(Z=25):[Ar]4s^2 3d^5$

$Mn^{3+}:[Ar]4s^0 3d^4 \rightarrow$ Four unpaired electrons.

$Cu(Z=29):[Ar]4s^1 3d^{10}$

$Cu^{2+}:[Ar]4s^0 3d^9 \rightarrow$ Four unpaired electrons

101. (1)

From the reaction involved, we have

$$Cr_2O_7^{2-} + 14H^+ + 6I^- \rightarrow 3I_2 + 2Cr^{3+} + 7H_2O$$

1 mole of $K_2Cr_2O_7$ = 3 moles of I_2

6 moles of I_2 liberated by

$= 6 \times \frac{1}{3}$ mole of $K_2Cr_2O_7$

$= 2$ moles of $K_2Cr_2O_7$

102. (3)

According to Faraday's second law,

$$\frac{\text{Mass of Ag}}{\text{Mass of H}} = \frac{\text{Equivalent mass of silver}}{\text{Equivalent mass of hydrogen}}$$

or $\frac{1.08}{\text{Mass of H}} = \frac{108}{1}$ or Mass of, H = 0.01 g

$\because$ 2.00 gm of H_2 occupies 22400 mL at STP

$\therefore$ 0.01 gm of H_2 will occupy $= \frac{22400 \times 0.01}{2}$

$= 112\,cm^3$

103. (4)

pH = 3

$E = -0.0591\,pH = -0.0591 \times 3 = -0.1773\,V$

104. (2) For a 2nd order reaction, $t_{1/2} = \frac{1}{k \times a}$

Where k = rate constant, a = initial conc.

(For 'n' th order $t_{1/2} \propto a^{1-n}$)

105. (2) CO : Molecular weight = 28

N_2 : Molecular weight = 28

Hence CO is the answer.

106. (1)

107. (1)

Grignard reagent is not prepared in aqueous solution because the reagent produced reacts with the active hydrogen of water to yield alkane.

108. (3)

For liberation of 11200 cm^3 of H_2 at STP requires 96500 C.

For liberation 112 cm^3 of H_2 at STP requires $96500 \times 112/11200 = 965\,C$

109. (1) A smuggler could not carry gold by deposting iron on gold surface. We know that greater is the standard electrode potential of an electrode, the more is the tendency of its ion to get induced. Since the standard reduction potential of Au^{3+}/Au is greater than Fe^{2+}/Fe, iron cannot be deposited on gold by electroplating

110. (4) $N_1V_1 = N_2V_2$; 0.5 × 100 = 0.1 × V

V = 500 cm^3

111. (2) 10 volume solution of H_2O_2 is 3.035% solution i.e. 3.035 g of H_2O_2 is present in 100 ml of the solution..

112. (4) It does not cause acidity in the soil.

113. (1) During adsorption of gas on a solid, the movement of its molecules becomes restricted and hence it results in negative entropy, that is, $\Delta S = -ve$ or $\Delta S < 0$.

Process is spontaneous, therefore, ΔG becomes –ve or $\Delta G < 0$.

Adsorption is an exothermic process, therefore, $\Delta H = -ve$ or $\Delta H < 0$.

114. (4)

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$$

For $1dm^3$, $R = k[SO_2]^2[O_2]$

$$R = k\left[\frac{1}{1}\right]^2\left[\frac{1}{1}\right] = 1$$

For $2dm^3$, $R = k\left[\frac{1}{2}\right]^2\left[\frac{1}{2}\right] = \frac{1}{8}$

So, the ratio is 8:1

115. (2) The magnitude of adsorption depends on

(i) the nature of the adsorbent and its state of subdivisison

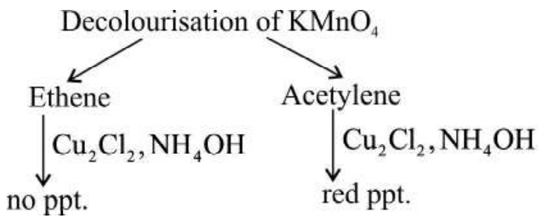
(ii) the nature of the adsorbate and its concentration

(iii) the temperature.

116. (2)

117. (3) High Blood pressure.

118. (3)



119. (4) The order of reactivity of alkyl halides is,

Iodide > Bromide > Chloride > Fluoride (For same alkyl group)

$3^o > 2^o > 1^o >$ methyl (For same halogen)

120. (3)

$$C_2H_5Cl + \underset{(alc.)}{AgCN} \rightarrow \underset{(X)}{C_2H_5NC} + AgCl$$

Functional isomer of $C_2H_5NC(X)$ is C_2H_5CN

SUBJECT : MATHS

121. (4)

$$I = \int_0^{\frac{\pi}{2}} \log(\tan x)dx \qquad (1)$$

$$I = \int_0^{\frac{\pi}{2}} \log\left[\tan\left(\frac{\pi}{2} - x\right)\right]dx$$

$$I = \int_0^{\frac{\pi}{2}} \log(\cot x)dx \qquad (2)$$

(1) + (2)

$$2I = \int_0^{\frac{\pi}{2}} [\log(\tan x) + \log(\cot x)]dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log(\tan x \cdot \cot x)dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log(1)dx$$

$2I = 0$

$I = 0$

122. (1)

$$A(\alpha_1) = \begin{bmatrix} \cos\alpha_1 & \sin\alpha_1 \\ -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix}$$

$$A(\alpha_2)=\begin{bmatrix}\cos\alpha_2 & \sin\alpha_2\\ -\sin\alpha_2 & \cos\alpha_2\end{bmatrix}$$

$$A(\alpha_1)\cdot A(\alpha_2)$$

$$=\begin{bmatrix}\cos\alpha_1\cdot\cos\alpha_2-\sin\alpha_1\sin\alpha_2 & \cos\alpha_1\cdot\sin\alpha_2+\sin\alpha_1\cos\alpha_2\\ -\sin\alpha_1\cdot\cos\alpha_2-\cos\alpha_1\sin\alpha_2 & -\sin\alpha_1\cdot\sin\alpha_2+\cos\alpha_1\cos\alpha_2\end{bmatrix}$$

$$=\begin{bmatrix}\cos(\alpha_1+\alpha_2) & \sin(\alpha_1+\alpha_2)\\ -\sin(\alpha_1+\alpha_2) & \cos(\alpha_1+\alpha_2)\end{bmatrix}$$

$$=A(\alpha_1+\alpha_1)$$

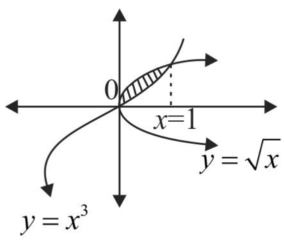
123. (1)

$$t_7=40\Rightarrow a+6d=40$$

$$Now\quad S_{13}=\frac{13}{2}[2a+12d]=13(a+6d)$$

$$=13\times 40=520$$

124. (3)



$$Area=\int_0^1\left(\sqrt{x}-x^3\right)dx$$

$$=\left(\frac{x^{3/2}}{3/2}-\frac{x^4}{4}\right)_0^1$$

$$=\frac{2}{3}-\frac{1}{4}$$

$$=\frac{5}{12}$$

125. (3)

Equation of conic is $x^2+10x-16y+25=0$ i.e,

$$(x+5)^2=16y$$

$\therefore$ conic is parabola with focus (-5, 4)

Now since focus is mid point of latus rectum.

$\therefore$ only points given in option (3) can be end points of the latus rectum.

126. (1) $\begin{vmatrix}1 & \omega & \omega^2\\ \omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\end{vmatrix}\left[C_1'=C_1+C_2+C_3\right]$

$$=\begin{vmatrix}1+\omega+\omega^2 & \omega & \omega^2\\ 1+\omega+\omega^2 & \omega^2 & 1\\ 1+\omega+\omega^2 & 1 & \omega\end{vmatrix}=\begin{vmatrix}0 & \omega & \omega^2\\ 0 & \omega^2 & 1\\ 0 & 1 & \omega\end{vmatrix}=0.$$

127. (3)

$$.\lim_{\theta\to\frac{\pi}{2}}\frac{\frac{\pi}{2}-\theta}{\cot\theta}$$

Put $\theta-\frac{\pi}{2}=t$

$$\lim_{t\to 0}\frac{-t}{\cot\left(\frac{\pi}{2}+t\right)}$$

$$=\lim_{t\to 0}\frac{-t}{-\tan t}$$

$$=\lim_{t\to 0}\frac{t}{\tan t}=1$$

128. (2)

Given that

$$2^x+2^y=2^{x+1}$$

$$\frac{dy}{dx}=\frac{2^x(1-2^y)}{2^y(2^x-1)}=\frac{2^x-2^x2^y}{2^x2^y-2^y}$$

$$=\frac{2^x-2^{x+y}}{2^{x+y}-2^y}$$

From Eq.(1), we get

$$\frac{dy}{dx}=\frac{2^x-2^x-2^y}{2^x+2^y-2^y}=\frac{-2^y}{2^x}=-2^y\cdot 2^{-x}$$

Therefore, $\frac{dy}{dx}=-2^{y-x}$

129. (2)

$$|\bar{a}|=1,|\bar{b}|=1,|\bar{c}|=1$$

$$\bar{a}\cdot\bar{b}=0,\bar{b}\cdot\bar{c}=0,\bar{a}\cdot\bar{c}=0$$

$$|\bar{a}+\bar{b}+\bar{c}|^2$$

$$=|\bar{a}|^2+|\bar{b}|^2+|\bar{c}|^2+2(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a})$$

$$=1+1+1+2(0)$$

$$\therefore |\bar{a}+\bar{b}+\bar{c}|^2=3$$

$$\therefore |\bar{a}+\bar{b}+\bar{c}|=\sqrt{3}$$

130. (3)

Equation of straight line with intercept a on both axis is $\frac{x}{a}+\frac{y}{a}=1\Rightarrow x+y=a$ it passes through (2, 4); $\therefore 6=a$

$\Rightarrow$ equation of line is $x+y-6=0$

131. (3)

$$\cos^{-1}\sqrt{p}+\cos^{-1}\sqrt{1-p}+\cos^{-1}\sqrt{1-q}=\frac{3\pi}{4}$$

or, $\cos^{-1}\sqrt{p}+\sin^{-1}\sqrt{p}+\cos^{-1}\sqrt{1-q}=\frac{3\pi}{4}$

or, $\frac{\pi}{2}+\cos^{-1}\sqrt{1-q}=\frac{3\pi}{4}$

or $\cos^{-1}\sqrt{1-q}=\frac{\pi}{4}$

$$\therefore \sqrt{1-q}=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$

$$1-q=\frac{1}{2}$$

$$q=\frac{1}{2}$$

132. (2)

$$\sin x\frac{1}{\sqrt{2}}-\cos x\frac{1}{\sqrt{2}}=1\Rightarrow\cos\left(x+\frac{\pi}{4}\right)=-1$$

$$x+\frac{\pi}{4}=2n\pi\pm\pi\Rightarrow x=2n\pi\pm\pi-\frac{\pi}{4}$$

$$\Rightarrow x=2n\pi\pm\frac{3\pi}{4}$$

133. (4)

Both the circles passes through the origin and so they must have the same tangent at (0, 0).

The general equation of tangents of the given circles are

$$xx_1+yy_1+g(x+x_1)+f(y+y_1)=0$$

$$xx_1+yy_1+g'(x+x_1)+f'(y+y_1)=0$$

Substituting $x_1=0$ and $y_1=0$, we get

$$gx+fy=0 \quad\Rightarrow\quad g'x+f'y=0$$

or $-\frac{f}{g}=-\frac{f'}{g'}$ or $g'f=f'g$

134. (2)

Given that, a die is thrown 10 times.

For a single throw of die, probability that odd number comes

$=\frac{1}{2}=p$ and probability that even number comes

$==\frac{1}{2}=q$.

So, probability that odd number will come up at least one time in n=10 throws will be

$$P(X \geq 1) = 1 - P(X = 0)$$

$$= 1 - \left[{}^{n}C_0 (q)^{n-0} (p)^0 \right]$$

$$= 1 - \left[{}^{10}C_0 \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0 \right]$$

$$= 1 - \frac{1}{2^{10}} = 1 - \frac{1}{1024}$$

$$= \frac{1023}{1024}$$

So, required probability is $\frac{1023}{1024}$

135. (3)

$f(x) = 2x + \cos x$

$\therefore f'(x) = 2 - \sin x > 0$

$\therefore f(x)$ is an increasing function.

136. (3)

General term in the expansion of $\left(x - \frac{3}{x^2}\right)^9$ is

$t_{r+1} = {}^{n}C_r\, x^{n-r} a^r$

$= {}^{9}C_r (x)^{9-r} \left(\frac{-3}{x^2}\right)^r = {}^{9}C_r (-3)^r \cdot x^{9-3r}$

For term independent of x, exponent of x should be zero.

$\therefore 9 - r - 2r = 0 \Rightarrow r = 3$

$t_4 = {}^{9}C_3 (-3)^3 = -2268$

137. (1)

Since $a * 3 = 3 * a = a$

$\therefore$ 3 is the identity element of given group.

$\therefore if\ 4^{-1} = b \Rightarrow \frac{9}{4}; \quad \therefore 5 * x = 4^{-1}$

$$\Rightarrow \frac{5x}{3} = \frac{9}{4} \Rightarrow x = \frac{27}{20}$$

138. (3)

Let $I = \int_0^{\pi} \frac{x\,dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

$$I = \int_0^{\pi} \frac{(\pi - x)\,dx}{a^2 \cos^2(\pi - x) + b^2 \sin^2(\pi - x)}$$

$$I = \int_0^{\pi} \frac{(\pi - x)\,dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$I = \int_0^{\pi} \frac{\pi\,dx}{a^2 \cos^2 x + b^2 \sin^2 x} - \int_0^{\pi} \frac{x\,dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$I = \pi \int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} - I$$

$$2I = \pi \int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$I = \frac{\pi}{2} \times 2 \int_0^{\frac{\pi}{2}} \frac{dx}{b^2 \cos^2 x \left(\frac{a^2}{b^2} + \tan^2 x\right)}$$

$$= \frac{\pi}{b^2} \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{\left(\frac{a}{b}\right)^2 + \tan^2 x} dx$$

$$= \frac{\pi}{b^2} \frac{1}{\left(\frac{a}{b}\right)} \tan^{-1} \left(\frac{\tan x}{\frac{a}{b}} \right)_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{ab} \left[\tan^{-1} \infty - \tan^{-1} 0 \right] = \frac{\pi}{ab} \left(\frac{\pi}{2} - 0 \right) = \frac{\pi^2}{2ab}$$

139. (4)

$$\int_0^{\infty} \frac{xdx}{(1+x)(1+x^2)}$$

Put $x=\tan\theta$

$$=\int_0^{\frac{\pi}{2}} \frac{\tan\theta\sec^2\theta d\theta}{(1+\tan\theta)\sec^2\theta} = \int_0^{\frac{\pi}{2}} \frac{\sin\theta/\cos\theta}{1+\frac{\sin\theta}{\cos\theta}} d\theta$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\cos\theta+\sin\theta} = I \text{, say} \qquad ....(1)$$

Also $$I=\int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-\theta\right)}{cos\left(\frac{\pi}{2}-\theta\right)+\sin\left(\frac{\pi}{2}-\theta\right)} d\theta$$

$$\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sin\theta+\cos\theta} d\theta \qquad ...(2)$$

Adding (1) and (2) we get

$$2I=\int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\sin\theta+\cos\theta} d\theta + \int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sin\theta+\cos\theta} d\theta$$

$$2I=\int_0^{\frac{\pi}{2}} \frac{\sin\theta+\cos\theta}{\sin\theta+\cos\theta} d\theta = \int_0^{\frac{\pi}{2}} (1)\, d\theta$$

$$2I=[\theta]_0^{\frac{\pi}{2}} = \frac{\pi}{2} \quad \therefore I=\frac{\pi}{4}$$

140. (3)

$x^2+y^2-3x-4y+2=0$ cuts x axis

Put $y=0$

Hence, $x^2-3x+2=0$

$\Rightarrow (x-2)(x-1)=0 \Rightarrow x=2,1$

$\therefore$ points (1,0), (2,0)

141. (3)

$Z_5=\{1,2,3,4\}$

Under operation mulitiplication module 5, 1 is the identity element.

$\left(3^{-1}\right)^{-1}=3.$

142. (4)

$$\int \frac{\sin 2x}{1+\cos^2 x} dx \quad let\ 1+\cos^2 x = t$$

$\therefore -\sin 2x dx = dt$

$$\therefore \int \frac{\sin 2x}{1+\cos^2 x} dx = -\int \frac{dt}{t} = -\log|t|+C$$

$$\therefore I = -\log\left|1+\cos^2 x\right| + C = C - \log(1+\cos^2 x)$$

143. (4)

Given that, A and B are square matrices of same order and B is skew symmetric matrix. So,

$B' = -B \quad or \quad B=-B'$

Now taking transpose of $\left(A'BA\right)$, we have

$$\left(A'BA\right)' = \left(A'\right)\left(B'\right)\left(A'\right)' = A'(-B)A = -A'BA$$

$$\Rightarrow \left(A'BA\right)' = -A'BA$$

So, it is skew symmetric.

144. (3)

Since $a_1, a_2, a_3, a_4, ...a_9$ in A.P

$\therefore a_2-a_1=d, a_3-a_2=d ... a_9-a_8=d$

Now

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$$

$C_3 \to C_3 - C_2$

$$\Delta = \begin{vmatrix} a_1 & a_2 & d \\ a_4 & a_5 & d \\ a_7 & a_8 & d \end{vmatrix}$$

$C_2 \to C_2 - C_1$

$$\Delta = \begin{vmatrix} a_1 & d & d \\ a_4 & d & d \\ a_7 & d & d \end{vmatrix} = 0$$

($\because$ Two columns identical)

$\Delta = 0$

Also $= 156$

$= \log_e(1) = 0$

145. (3)

A and B are square matrices and A^{-1}, B^{-1} exist

then : $\left(AB^{-1}\right) = B^{-1}A^{-1}$

146. (4) Conceptual

147. (3)

Given implication is $p \to q$

Its contrapositive is $\sim q \to \sim p$, converse is $\sim p \to \sim q$

148. (4)

$$\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{\left(x^2+2x+1\right)+1} = \int \frac{dx}{1+\left(x+1\right)^2}$$

$= \tan^{-1}\left(x+1\right) + C$

149. (1)

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1+\cos x \cdot \sin x}\, dx \qquad (1)$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2}-x\right) - \sin\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)\cdot \sin\left(\frac{\pi}{2}-x\right)}\, dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1+\sin x \cdot \cos x}\, dx \qquad (2)$$

(1) + (2)

$$2I = \int_0^{\frac{\pi}{2}} 0\, dx$$

$I = 0$

150. (2)

Given that, points of feasible region are (0, 3) and (1, 1) and (3, 0).

$z = px + qy; \quad p, q > 0$

Minimum occurs at (3, 0) and (1, 1).

At point (3, 0): $z = 3p + 0 \Rightarrow z = 3p$

At point (1,1): $z = p + q$

$\Rightarrow 3p = p + q \Rightarrow 2p = q \Rightarrow p = \dfrac{q}{2}$

This is required condition.

151. (2)

Projection of $\vec{a}$ on $\vec{b}$ = $\dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

$\therefore \vec{a}.\vec{b} = 2 + 6 - 6 = 2$

$|b| = \sqrt{14} \Rightarrow$ projection is $\dfrac{2}{\sqrt{14}}$

152. (2)

$$k = \lim_{x\to 0} \frac{\sin \pi x}{5x}$$

$$k = \lim_{x\to 0} \frac{\sin \pi x}{\pi x} \times \frac{\pi}{5}$$

$$k = (1)\frac{\pi}{5}$$

$$k = \frac{\pi}{5}$$

153. (4)

Here $\frac{dr}{dt}=2cm/\sec, \text{Now } A=4\pi r^2$

$$\frac{dA}{dt}=8\pi r\frac{dr}{dt}=8\pi r(2)=16\pi r$$

$$\therefore \frac{dA}{dt}\propto r$$

154. (3)

Clock strikes appropriate number of times at each hour.

It will strike in a day

$=2X(1+2+3+4+5+6+7+8+9+10+11+12)$

$=156$

155. (2)

Probability that solving a problem by three persons A, B and C independently is

$$P(A)=\frac{1}{2}, P(B)=\frac{1}{4} \text{ and } P(C)=\frac{1}{3}$$

Probability that problem will be solved by any two is

$$P=P(A)P(B)P(C^{'})+P(A)P(B^{'})P(C)$$

$$+P(A^{'})P(B)P(C)$$

where, $P(A)P(B)P(C^{'})$ is problem will be solved by A and B but not C,

$P(A)P(B^{'})P(C)$ problem will be solved by A and C but not B and $P(A^{'})P(B)P(C)$ problem will be solved by B and C but not A.

Now $P(A^{'})=1-P(A)=1-\frac{1}{2}=\frac{1}{2}$

$$P(B^{'})=1-P(B)=1-\frac{1}{4}=\frac{3}{4}$$

$$P(C^{'})=1-P(C)=1-\frac{1}{3}=\frac{2}{4}$$

Therefore,

$$P=\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{2}{3}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{3}$$

$$=\frac{2}{24}+\frac{3}{24}+\frac{1}{24}=\frac{6}{24}=\frac{1}{4}$$

156. (1)

$$f(0)=0;\ \lim_{x\to 0}f(x)=\lim_{x\to 0}x\sin(1/x)=0$$

$\therefore f(x)$ is continuous at $x=0$

$$f(0)=\lim_{k\to 0}\frac{f(h)-f(0)}{h}=\lim_{h\to 0}\frac{h\sin(1/h)-0}{h}$$

$=\lim_{h\to 0}\sin(1/h)$ which does not exist.

$\therefore f(x)$ is not differentiable at $x=0$.

157. (2)

$$\frac{1}{1+\cos\theta+i\sin\theta}\times\frac{(1+\cos\theta)-i\sin\theta}{(1+\cos\theta)-i\sin\theta}$$

$$=\frac{1+\cos\theta-i\sin\theta}{(1+\cos\theta)^2+\sin^2\theta}$$

$$=\frac{1+\cos\theta-i\sin\theta}{1+\cos^2\theta+2\cos\theta+\sin^2\theta}=\frac{1+\cos\theta-i\sin\theta}{1+1+2\cos\theta}$$

$$=\frac{1}{2}\left[\frac{1+\cos\theta-i\sin\theta}{1+\cos\theta}\right] \quad \therefore \text{real part is } \frac{1}{2}$$

158. (3)

$$\begin{vmatrix}1&\omega&\omega^2\\\omega&\omega^2&1\\\omega^2&1&\omega\end{vmatrix}=\begin{vmatrix}1+\omega+\omega^2&\omega&\omega^2\\1+\omega+\omega^2&\omega^2&1\\1+\omega+\omega^2&1&\omega\end{vmatrix}$$

$$(C_1\to C_1+C_2+C_3)$$

$$=\begin{vmatrix}0&\omega&\omega^2\\0&\omega^2&1\\0&1&\omega\end{vmatrix}=0$$

159. (3)

Given that, $\frac{\log_e x}{x}$ if x>0

Let, $y = \frac{\log_e x}{x}$

Differentiate with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\log_e x}{x}\right) = \frac{x\frac{d}{dx}(\log_e x) - \log_e x\frac{dx}{dx}}{x^2}$$

$$= \frac{x\left(\frac{1}{x}\right) - \log_e x}{x^2} = \frac{1 - \log_e x}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - \log_e x}{x^2}$$

To find maximum value put $\frac{dy}{dx} = 0$

$$\Rightarrow \frac{1 - \log_e x}{x^2}$$

$$\Rightarrow 1 - \log_e x = 0$$

$$\Rightarrow \log_e x = 1$$

$$\Rightarrow x = e$$

So maximum value of y from Eq (1) is

$$y = \frac{\log_e^{\ e}}{e} = \frac{1}{e}$$

160. (4)

$${}^nP_4 = 24\,{}^nC_5$$

$$\frac{n!}{(n-4)!} = 24 \cdot \frac{n!}{5!(n-5)!}$$

$$\frac{1}{n-4} = \frac{24}{5!}$$

$$\frac{1}{n-4} = \frac{24}{120}$$

$$n - 4 = \frac{120}{24}$$

$$n - 4 = 5$$

$$\therefore \quad n = 9$$

161. (4)

$$\frac{d}{dx}\left(e^{x^3}\right) = e^{x^3}.3x^2 \ ; \qquad \frac{d}{dx}(\log x) = \frac{1}{x}.$$

$$\therefore \frac{d\left(e^{x^3}\right)}{d(\log x)} = \frac{3x^2 e^{x^3}}{1/x} = 3x^3 e^{x^3}.$$

162. (4)

The radical axis of two circles is along their radius. So, the radical axis of the line centres of those circles area parallel.

163. (1)

$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ is the inverse of itself.

164. (3)

Slope of given line$= m = \frac{0-1}{2-0} = -\frac{1}{2}$

Point on the line (4, -1)

$\therefore$ equation of line, $y + 1 = -\frac{1}{2}(x - 4)$

$$2y + 2 = -x + 4; \Rightarrow 2y + x = 2$$

165. (3)

We have $y^2 = 4x$

$$x^2 = 4y$$

From (1) and (2) we get

$$\left(\frac{x^2}{4}\right)^2 = 4x \ \text{ i.e. } \ x^4 - 64x = 0$$

i.e. $x\left(x^3-4^3\right)=0 \quad \therefore x=0\, or\, x=4$

$$\therefore \text{Required area}=\left|\int_0^4 \sqrt{4x}dx-\int_0^4 \frac{x^2}{4}dx\right|$$

$$=2\left[\frac{x^{3/2}}{3/2}\right]_0^4-\frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$$

$$=\frac{4}{3}\left[4^{3/2}\right]-\frac{1}{4}\left[\frac{4^3}{3}\right]$$

$$=\frac{4}{3}(8)-\frac{16}{3}$$

$$=\frac{16}{3}$$

166. (2)

Given that,

$$f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Let $x=\tan\theta$ *then*

$$f(x)=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

We know that, $\sin 2\theta=\dfrac{2\tan\theta}{1+\tan^2\theta}$

So, $f(x)=\sin^{-1}(\sin 2\theta)$

$\Rightarrow f(x)=2\theta$

but $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$

Therefore, $f(x)=2\tan^{-1}x$

Differentiating it with respect to x, we get

$$f'(x)=\frac{d}{dx}(2\tan^{-1}x)=2\left(\frac{1}{1+x^2}\right)=\frac{2}{1+x^2}$$

At $x=\sqrt{3}$, we get

$$f'(\sqrt{3})=\frac{2}{1+(\sqrt{3})^2}=\frac{2}{1+3}=\frac{2}{4}=\frac{1}{2}$$

167. (3)

$$12\cot^2\theta-31\,\text{cosec}\,\theta+32=0$$

$$12\left(\text{cosec}^2\theta-1\right)-31\,\text{cosec}\,\theta+32=0$$

$$12\,\text{cosec}^2\theta-31\,\text{cosec}\,\theta+20=0$$

$$12\,\text{cosec}^2\theta-16\,\text{cosec}\,\theta-15\,\text{cosec}\,\theta+20=0$$

$$\left(4\,\text{cosec}\,\theta-5\right)\left(3\,\text{cosec}\,\theta-4\right)=0$$

$$\therefore\ \text{cosec}\,\theta=\frac{5}{4},\frac{4}{3}$$

168. (1)

a=3, b=5, c=4, $s=\dfrac{a+b+c}{2}=\dfrac{12}{2}=6$

$$\sin\frac{B}{2}=\sqrt{\frac{(s-c)(s-a)}{ca}}=\sqrt{\frac{2.3}{12}}=\sqrt{\frac{1}{2}}$$

$$\cos\frac{B}{2}=\sqrt{\frac{s(s-b)}{ca}}=\sqrt{\frac{6.1}{12}}=\sqrt{\frac{1}{2}}$$

$$\therefore \sin\frac{B}{2}+\cos\frac{B}{2}=\frac{2}{\sqrt{2}}=\sqrt{2}$$

169. (2)

Suppose if possible $\log_2 7$ is rational say p/q where p and q are integers prime to each other. Then $p/q=\log_2 7 \Rightarrow 7 \Rightarrow 2^{p/q} \Rightarrow 2^p=7^q$. This is false because LHS is even whereas the RHS is odd.

170. (1)

(1) Standard deviation is given by

$$\sigma=\sqrt{\frac{\sum\left(x_n-\bar{x}\right)^2}{n}}$$

where, x_n value of each element, $\bar{x}$ is mean and n is total elements.

Mean of the given data is

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
6	6-8=-2	$(-2)^2=4$
7	7-8=-1	$(-1)^2=1$
8	8-8=0	0
9	9-8=1	$(1)^1=1$
10	10-8=2	$(2)^2=4$

$$\bar{x} = \frac{6+7+8+9+10}{5} = 8$$

Therefore, standard deviation is

$$\sigma = \sqrt{\frac{4+1+0+1+4}{5}} = \sqrt{\frac{10}{5}} = \sqrt{2}$$

171. (3)

$$\frac{dx}{x} + \frac{dy}{y} = 0$$

Integrating, $\log x + \log y = \log C$

i.e. $\log(xy) = \log C; \quad \therefore xy = C$

172. (2)

$$\lim_{x\to\infty}\left(1 - \frac{4}{x-1}\right)^{3x-1}$$

$$= \lim_{x\to\infty}\left(1 + \frac{-4}{x-1}\right)^{-\left(\frac{x-1}{4}\right)\left(\frac{-4}{x-1}\right)(3x-1)}$$

$$= e^{\lim_{x\to\infty}\frac{-4(3x-1)}{x-1}}$$

$$= e^{\lim_{x\to\infty}\frac{-4\left[3-\frac{1}{x}\right]}{1-\frac{1}{x}}}$$

$$= e^{\frac{-4(3-0)}{1-0}} = e^{-12}$$

173. (2)

$$\frac{dy}{dx}\left(\log(\sin x)\right) = \frac{1}{\sin x}.\cos x = \cot x$$

174. (3)

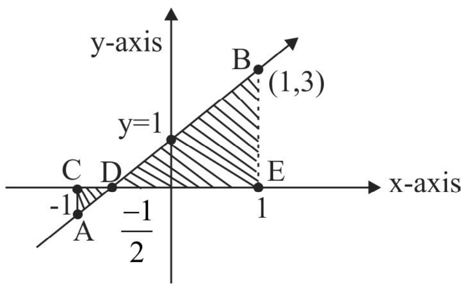
Given that, lines $y = 2x+1$, $x - axis$, and ordinates $x = -1$ and $x = 1$.

Consider equation of line

$$y = 2x + 1$$

Point of intersection with $y - axis$ is y=1

And point of intersection with $x - axis$ $x = \frac{-1}{2}$



For

$x = 1, y = 2x + 1 \Rightarrow y = 3$ So, point B is $(1,3)$

For

$x = -1, y = 2x + 1 \Rightarrow y = -1$

So, point A is $(-1,-1)$

Then required area

A=area of triangle ACD + area of triangle BDE

$$A = \frac{1}{2}\cdot\frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\frac{3}{2}\cdot 3$$

$$A = \frac{1}{4} + \frac{9}{4}$$

$$A = \frac{5}{2}$$

175. (3)

$$\omega = \frac{-1 + i\sqrt{3}}{2}$$

$$\left(3+\omega+3w^2\right)^4=\left[3\left(1+w^2\right)+w\right]^4$$

$$=\left[3(-w)+w\right]^4=(-2w)^4=16w^4=16w$$

176. (3)

$$k+\frac{1}{2}=\lim_{x\to 0}\frac{\sin 5x}{x^2+2x}$$

$$k+\frac{1}{2}=\lim_{x\to 0}\frac{\sin 5x}{x}\times\frac{1}{x+2}$$

$$k+\frac{1}{2}=\lim_{x\to 0}\frac{\sin 5x}{5x}\times\frac{5}{x+2}$$

$$k+\frac{1}{2}=1\times\frac{5}{0+2}$$

$$k+\frac{1}{2}=\frac{5}{2}$$

$$k=\frac{5}{2}-\frac{1}{2}$$

$$k=2$$

177. (3)

$$y=ae^{mx}+be^{-mx}$$

d.w.r.t. x

$$\therefore \frac{dy}{dx}=ame^{mx}-bme^{-mx}$$

d.w.r.t. x

$$\therefore \frac{d^2y}{dx^2}=am^2e^{mx}+bm^2e^{-mx}$$

$$=m^2\left(ae^{mx}+be^{-mx}\right)=m^2y \quad or, \frac{d^2y}{dx^2}-m^2y=0$$

178. (4)

Here we have to put the 3 vowels on 3 specified even places and the 4 consonants in the remaining places. The first operation can be done in 3! ways and the second in 4! ways.

$\therefore$ The required number of ways = $3!\times 4!$

$$=6\times 24=144$$

179. (4)

$$x=\sin t, y=\cos pt$$

$$\frac{dx}{dt}=\cos t;\frac{dy}{dt}=-p\sin pt;\frac{dy}{dx}=\frac{-p\sin pt}{\cos t}$$

$$\frac{d^2y}{dx^2}=\frac{-\cos t\, p^2\cos pt\left(\frac{dt}{dx}\right)-p\sin pt\sin t\left(\frac{dt}{dx}\right)}{\cos^2 t}$$

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-\frac{xdy}{dx}+p^2y=0$$

180. (4) $I=\int_0^{\frac{\pi}{8}}\cos^3 4\theta\, d\theta$

Put $4\theta=x$

$$I=\int_0^{\frac{\pi}{2}}\frac{\cos^3 x}{4}dx$$

$$I=\frac{1}{4}\int_0^{\frac{\pi}{2}}\frac{1}{4}\left[\cos 3x+3\cos x\right]dx$$

$$I=\frac{1}{16}\left[\frac{\sin 3x}{3}+3\sin x\right]_0^{\frac{\pi}{2}}$$

$$I=\frac{1}{16}\left[\frac{1}{3}sin\frac{3\pi}{2}+3\sin\frac{\pi}{2}-0-0\right]$$

$$I=\frac{1}{16}\left[-\frac{1}{3}+3\right]$$

$$I=\frac{1}{16}\times\frac{8}{3}$$

$$I=\frac{1}{6}$$