Deeksha - CET-12

Subject	Topic	Test – KAR	Date
Mathematics	Board Exam 2023	CET – 12	20th May 2023

Max. Marks: 60 **Duration: 1 Hour 20 min**

1. **This paper consists of 60 questions.**

   - *Multiple Choice Questions with one correct answer. A correct answer carries 1 Mark. No Negative marks.*

2. **The OMR sheet for 200 questions is to be used**

3. **Use of calculators and log tables is prohibited**

4. **Darken the appropriate bubble using a pen in the OMR sheet provided to you. Once entered, the answer cannot be changed. Any corrections or modifications will automatically draw a penalty of 1 mark**

5. **No clarification will be entertained during the examination. Doubts in the paper can be reported to the coordinator after the exam**

6. **If the details in the OMR Sheet are not filled, If the OMR sheet is mutilated, torn, white Ink used, the circles filled and scratched, then the OMR sheet will not be graded**

All the best!!

Useful Data

At. Wt.:

$N = 14;\ O = 16;\ H = 1;\ S = 32;\ Cl = 35.5;\ Mn = 55;\ Na = 23;\ C = 12;\ Ag = 108;\ K = 39;\ Fe = 56;\ Pb = 207$

Physical Constants:

$h = 6.626\times10^{-34}\,\mathrm{Js}$, $\mathrm{N_a} = 6.022\times10^{23}\,\mathrm{mol^{-1}}$, $\mathrm{c} = 2.998\times10^{8}\,\mathrm{m\,s^{-1}}$, $\mathrm{m_e} = 9.1\times10^{-31}\,\mathrm{kg}$, $R = 8.314\ \mathrm{J\,mol^{-1}\ K^{-1}}$

Multiple Choice Questions with one correct answer. A correct answer carries 1 mark. No negative mark. **60 x 1 = 60**

1. The value of $\cot^{-1}\left[\dfrac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$ where $x \in \left(0, \dfrac{\pi}{4}\right)$ is

   (A) $\pi - \dfrac{x}{3}$ (B) $\dfrac{x}{2}$ (C) $\pi - \dfrac{x}{2}$ (D) $\dfrac{x}{2} - \pi$

Sol: $\cot^{-1}\left[\dfrac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right], x \in \left(1, \dfrac{\pi}{4}\right)$

$$\sqrt{1+\sin x} = \cos\frac{x}{2} + \sin\frac{x}{2}$$

$$\sqrt{1-\sin x} = \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2} = \cos\frac{x}{2} - \sin\frac{x}{2}$$

$$\text{G.E.} = \cot^{-}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]$$

$$= \cot^{-1}\left[\cot\frac{x}{2}\right] = \frac{x}{2}$$

Ans: (B)

2. If $x\begin{bmatrix}3\\2\end{bmatrix} + y\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}15\\5\end{bmatrix}$ then the value of x and y are

   (A) $x=-4, y=-3$ (B) $x=4, y=3$ (C) $x=-4, y=3$ (D) $x=4, y=-3$

Sol:

$3x + y = 15$

$2x - y = 5$

$5x = 20$

$x = 4$

$3(4) + y = 15$

$y = 15 - 12$

$y = 3$

Ans: (B)

3. If A and B are two matrices such that $AB = B$ and $BA = A$ then $A^2 + B^2 =$

   (A) AB (B) A + B (C) 2BA (D) 2AB

Sol: $A^2 + B^2 = A \cdot A + B \cdot B = A(BA) + B(AB) = (AB)A + (BA)B$

$= BA + AB = A + B$

Ans: (B)

4. If $A=\begin{bmatrix} 2-\text{k} & 2 \\ 1 & 3-\text{k} \end{bmatrix}$ is singular matrix, then the value of $5\text{k}-\text{k}^2$ is equal to

(A) -4 (B) 4 (C) 6 (D) -6

Sol: $(2-k)(3-k)-2=0$

$6-5k+k^2-2=0$

$4=5k-k^2$

Ans: (B)

5. The area of a triangle with vertices $(-3,0),(3,0)$ and $(0,k)$ is 9 sq. units, the value of k is

(A) 6 (B) 9 (C) 3 (D) -9

Sol: $9=\dfrac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix}$

$\pm 18=-3(-k)+1(3k)$

$\pm 18=6k \Rightarrow k=\pm 3$

Ans: (C)

6. If $\Delta=\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ and $\Delta_1=\begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a & b & c \end{vmatrix}$ then

(A) $\Delta_1 \neq \Delta$ (B) $\Delta_1=\Delta$ (C) $\Delta_1=-\Delta$ (D) $\Delta_1=3\Delta$

Sol: $D=\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

$R_2 \to R_1-R_2$ and $R_3 \to R_3-R_1$

$$\Delta=\begin{vmatrix} 1 & a & a^2 \\ 0 & a-b & (a+b)(a-b) \\ 0 & c-a & (c+a)(c-a) \end{vmatrix}$$

$\Delta=(a-b)(c-b)(c-a)$

$$\Delta_1=\begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a & b & c \end{vmatrix}$$

$C_2 \to C_2-C_1, C_3 \to C_3-C_1$

$$\Delta_1=\begin{vmatrix} 1 & 0 & 0 \\ bc & c(a-b) & b(a-c) \\ a & b-a & c-a \end{vmatrix}$$

$\Delta_1=(a-b)(c-b)(c-a)$

Ans: (B)

7. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ where $a, x \in (0,1)$ then the value of x is

(A) $\frac{2a}{1+a^2}$ (B) 0 (C) $\frac{2a}{1-a^2}$ (D) $\frac{a}{2}$

Sol: $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right),\ a,\ x \in (0,1)$

$\Rightarrow\ 2\tan^{-1}a + 2\tan^{-1}a = 2\tan^{-1}x$

$\Rightarrow\ 2\tan^{-1}a = \tan^{-1}x$

$\Rightarrow\ \tan^{-1}\left(\frac{2a}{1-a^2}\right)=\tan^{-1}x \qquad \therefore\ x=\frac{2a}{1-a^2}$

Ans: (C)

8. If $u=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ then $\frac{du}{dv}$ is

(A) $\frac{1-x^2}{1+x^2}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

Sol: $u=\sin^{-1}\left(\frac{2x}{1+x^2}\right) \qquad v=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$u=2\tan^{-1}x \qquad v=2\tan^{-1}x$

$\frac{du}{dx}=\frac{2}{1+x^2} \qquad \frac{dv}{dx}=\frac{2}{1+x^2}$

$\therefore\ \frac{du}{dv}=1$

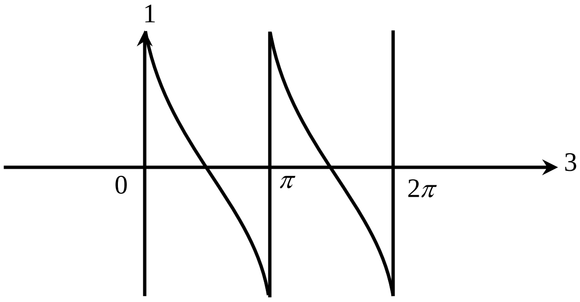
Ans: (C)

9. The function $f(x)=\cot x$ is discontinuous on every point of the set

(A) $\left\{x=(2n+1)\frac{\pi}{2}; n \in Z\right\}$ (B) $\{x=n\pi; n \in Z\}$

(C) $\left\{x=\frac{n\pi}{2}; n \in Z\right\}$ (D) $\{x=2n\pi; n \in Z\}$

Sol:



Graph of $\cot x$

$\cot x=\frac{\cos x}{\sin x}$ is undefined when $\sin x=0$

$\therefore$ $\cot x$ is undefined at $x = n\pi, n \in z$

Ans: (B)

10. If the function is $f(x) = \frac{1}{x+2}$, then the point of discontinuity of the composite function $y = f(f(x))$ is

(A) $\frac{2}{5}$ (B) $\frac{-5}{2}$ (C) $\frac{1}{2}$ (D) $\frac{5}{2}$

Sol: $f(x) = \frac{1}{x+2}$ is undefined at $x = -2$

$$f(f(x)) = \frac{1}{f(x)+2} = \frac{1}{\frac{1}{x+2}} = \frac{x+2}{2x+5} \text{ is undefined at } x = -\frac{5}{2}$$

Ans: (B)

11. If $y = a\sin x + b\cos x$, then $y^2 + \left(\frac{dy}{dx}\right)^2$ is a

(A) function of x and y (B) function of x

(C) constant (D) function of y

Sol: $y = a\sin x + b\cos x$

$$y^2 + \left(\frac{dy}{dx}\right)^2 = (a\sin x + b\cos x)^2 + (a\cos x - b\sin x)^2$$

$$= a^2\sin^2 x + b^2\cos^2 x + 2ab\sin x\cos x + a^2\cos^2 x + b^2\sin^2 x - 2ab\sin x\cos x$$

$= a^2 + b^2$ is constant

Ans: (C)

12. If $f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \ldots\ldots + x^n$ then $f''(1) =$

(A) $n(n-1)2^n$ (B) $(n-1)2^{n-1}$ (C) 2^{n-1} (D) $n(n-1)2^{n-2}$

Sol: $f(x) = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + x^n$

$= (1+x)^n$

$f'(x) = n(1+x)^{n-1}$

$f''(x) = n(n-1)(1+x)^{n-2}$

$f''(1) = n(n-1)\cdot 2^{n-2}$

Ans: (D)

13. If $A = \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}$ and $AB = I$ then $B =$

(A) $\cos^2\alpha/2 \cdot I$ (B) $\cos^2\alpha/2 \cdot A^T$ (C) $\sin^2\alpha/2 \cdot A$ (D) $\cos^2\alpha/2 \cdot A$

Sol: $B = A^{-1}$

$$A = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

$$|A| = 1 + \tan^2\frac{\alpha}{2}$$

$$= \sec^2\frac{\alpha}{2}$$

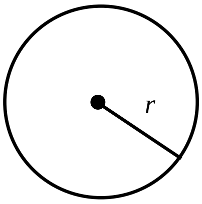
$$A^{-1} = \frac{1}{\sec^2\frac{\alpha}{2}}\begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} = \cos^2\frac{\alpha}{2}A^T$$

Ans: (B)

14. A circular plate of radius 5cm is heated. Due to expansion, its radius increases at the rate of $0.05\text{cm}/\sec$. The rate at which its area is increasing when the radius is 5.2cm is

(A) $5.05\pi\text{cm}^2/\sec$ (B) $5.2\pi\text{cm}^2/\sec$ (C) $0.52\pi\text{cm}^2/\sec$ (D) $27.4\pi\text{cm}^2/\sec$

Sol: $\frac{dr}{dt} = 0.05\ cm/\sec$



$A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

$$\left(\frac{dA}{dt}\right) = 2\pi(5.2)(0.05) = 0.52\pi$$

Ans: (C)

15. The distance ' s ' in meters travelled by a particle in ' t ' seconds is given by $s = \frac{2t^3}{3} - 18t + \frac{5}{3}$. The acceleration when the particle comes to rest is

(A) $12\text{m}^2/\sec$. (B) $3\text{m}^2/\sec$. (C) $18\text{m}^2/\sec$. (D) $10\text{m}^2/\sec$.

Sol: $s = \frac{2t^3}{3} - 18t + \frac{5}{3}$

$$v = \frac{ds}{dt} = 2t^2 - 18$$

$$a = \frac{dv}{dt} = 4t \qquad v = 0 \Rightarrow t = 3$$

$\therefore\ a = 12$

Ans: (A)

16. A particle moves along the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$. When the rate of change of abscissa is 4 times that of its ordinate, then the quadrant in which the particle lies is

(A) III or IV (B) I or III (C) II or III (D) II or IV

Sol: $\frac{x^2}{16}+\frac{y^2}{4}=1$

$\frac{2x}{16}\frac{dx}{dt}+\frac{2y}{4}\frac{dy}{dt}=0$

$\frac{x}{8}\frac{dx}{dt}+\frac{y}{2}\frac{dy}{dt}=0$

$\Rightarrow \frac{x}{8}\left(4\frac{dy}{dt}\right)+\frac{y}{2}\frac{dy}{dt}=0$

$\Rightarrow \frac{dy}{dt}\left(\frac{x}{2}+\frac{y}{2}\right)=0$

$\Rightarrow x=-y$

$\Rightarrow (x, y)$ lie in II or IV

Ans: (D)

17. An enemy fighter jet is flying along the curve given by $y=x^2+2$. A soldier is placed at $(3,2)$ wants to shoot down the jet when it is nearest to him. Then the nearest distance is

(A) 2 units (B) $\sqrt{3}$ units (C) $\sqrt{5}$ units (D) $\sqrt{6}$ units

Sol: $y=x^2+2$ $\therefore\ p\left(t, t^2+2\right)$

$A(3, 2)$

Slope of AP $=\frac{t^2}{t-3}$

$y=x^2+2 \Rightarrow y'=2x$

Slope of normal = Slope of AP

$\frac{-1}{2t}=\frac{t^2}{t-3}$

$2t^3=-t+3$

$2t^3=-t+3$

$2t^3+t-3=0$

$t=0$ $\therefore\ p=(1, 3)$

$\therefore$ AP $=\sqrt{4+1}=\sqrt{5}$

Ans: (C)

18. $\int_2^8 \frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}}+5^{\sqrt{10-x}}}\,dx=$

(A) 4 (B) 5 (C) 3 (D) 6

Sol: $I=\int_2^8 \frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}}+5^{\sqrt{10-x}}}\,dx$ … (1)

$$I=\int_2^8 \frac{5^{\sqrt{x}}}{5^{\sqrt{10-x}}+5^{\sqrt{x}}}\,dx \qquad \ldots (2)$$

$$2I=\int_2^8 1dx = x\Big|_2^8 = 8-2=6$$

$I=3$

Ans: (C)

19. $\int\sqrt{\text{cosec}x-\text{sin}x}\,dx=$

(A) $2\sqrt{\text{sin}x}+C$ (B) $\sqrt{\text{sin}x}+C$ (C) $\frac{2}{\sqrt{\text{sin}x}}+C$. (D) $\frac{\sqrt{\text{sin}x}}{2}+C$

Sol: $\int\sqrt{\cos ecx-\sin x}\;dx=\int\sqrt{\frac{1}{\sin x}-\sin x}\;dx$

$$=\int\sqrt{\frac{1-\sin^2 x}{\sin x}}\;dx$$

$$\int\frac{\cos x}{\sqrt{\sin x}}\,dx \qquad \sin x=t,\ \cos x\ dx=dt$$

$$=\int\frac{1}{\sqrt{t}}\,dt$$

$$=2\sqrt{\sin x}+C$$

Ans: (A)

20. If $\text{f}(x)$ and $\text{g}(x)$ are two functions with $\text{g}(x)=x-\frac{1}{x}$ and fog $(x)=x^3-\frac{1}{x^3}$ then $\text{f}'(x)=$

(A) $x^2-\frac{1}{x^2}$ (B) $3x^2+3$ (C) $1-\frac{1}{x^2}$ (D) $3x^2+\frac{3}{x^4}$

Sol: $g(x)=x-\frac{1}{x}(fog)(x)=x^3-\frac{1}{x^3}$

$$f(g(x))=x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)^3+3\left(x-\frac{1}{x}\right)$$

$$\Rightarrow f(g(x))=(g(x))^3+3g(x)$$

$$\Rightarrow f(x)=x^3+3x$$

$$\Rightarrow f'(x)=3x^2+3$$

Ans: (B)

21. $\int\frac{1}{1+3\sin^2 x+8\cos^2 x}dx=$

(A) $\frac{1}{6}\tan^{-1}\left(\frac{2\text{tan}x}{3}\right)+C$ (B) $\frac{1}{6}\tan^{-1}(2\text{tan}x)+C$ (C) $6\tan^{-1}\left(\frac{2\text{tan}x}{3}\right)+C$ (D) $\tan^{-1}\left(\frac{2\text{tan}x}{3}\right)+C$

Sol: $\int\frac{1}{1+3\sin^2 x+8\cos^2 x}dx$

$$= \int \frac{\sec^2 x}{\sec^2 x + 3\tan^2 x + 8} dx$$

$$= \int \frac{\sec^2 x \, dx}{9 + 4\tan^2 x}$$

$$= \int \frac{dt}{3^2 + (2t)^2} \qquad \tan x = t,\ \sec^2 x \, dx = dt$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{2t}{3}\right)\cdot\frac{1}{2}$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)$$

Ans: (A)

22. $\int_{-2}^{0}\left(x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\right)dx =$

(A) 4 (B) 0 (C) 1 (D) 3

Sol: $\int_{-2}^{0}\left((x+1)^3 + 2 + (x+1)\cos(x+1)\right)dx$

$$\int_{-1}^{1}\left(x^3 + 2 + x\cos x\right) dx$$

$$2x]_{-1}^{1} = 2(1+1) = 4$$

Ans: (A)

23. $\int_0^{\pi} \frac{x\tan x}{\sec x \cdot \operatorname{cosec} x} dx =$

(A) $\pi/2$ (B) $\pi/4$ (C) $\pi^2/2$ (D) $\pi^2/4$

Sol: $\int_0^{\pi} \frac{x\tan x}{\sec x \cdot \operatorname{cosec} x} dx$

$$= \frac{\pi}{2}\int_0^{\pi} \frac{\tan x}{\sec x \operatorname{cosec} x} dx$$

$$= \frac{\pi}{2}\int_0^{\pi} \sin^2 x \, dx$$

$$= \frac{\pi}{2}\int_0^{\pi} \frac{1-\cos 2x}{2}$$

$$= \frac{\pi}{2}\left[\frac{x}{2} - \frac{1}{4}\sin 2x\right]_0^{\pi}$$

$$= \frac{\pi}{2}\left[\left(\frac{\pi}{2} - 0\right) - (0-0)\right] = \frac{\pi^2}{4}$$

Ans: (D)

24. $\int \sqrt{5-2x+x^2}\,dx =$

(A) $\frac{x-1}{2}\sqrt{5+2x+x^2}+2\log\left|(x-1)+\sqrt{5+2x+x^2}\right|+C$

(B) $\frac{x-1}{2}\sqrt{5-2x+x^2}+2\log\left|(x+1)+\sqrt{x^2+2x+5}\right|+C$

(C) $\frac{x-1}{2}\sqrt{5-2x+x^2}+2\log\left|(x-1)+\sqrt{5-2x+x^2}\right|+C$

(D) $\frac{x}{2}\sqrt{5-2x+x^2}+4\log\left|(x+1)+\sqrt{x^2-2x+5}\right|+\mathrm{C}$

Sol: $\int\sqrt{5-5x+x^2}\,dx=\int\sqrt{(x-1)^2+2^2}\,dx$

$=\frac{x-1}{2}\sqrt{5-2x+x^2}+2\log\left|x-1+\sqrt{5-2x-x^2}\right|+c$

Ans: (C)

25. The area of the region bounded by the line $y=x+1$, and the lines $x=3$ and $x=5$ is

(A) $\frac{11}{2}$ sq. units (B) 10 sq. units (C) 7 sq. units (D) $\frac{7}{2}$ sq. units

Sol: $A=\int_3^5 x+1dx=\left.\frac{x^2}{2}+1\right|_3^5=\frac{25}{2}-\frac{9}{2}+5-3$

$=\frac{16}{2}+2=8+2=10$

Ans: (B)

26. If a curve passes through the point $(1,1)$ and at any point (x,y) on the curve, the product of the slope of its tangent and x co-ordinate of the point is equal to the y co-ordinate of the point, then the curve also passes through the point

(A) $(-1,2)$ (B) $(2,2)$ (C) $(\sqrt{3},0)$ (D) $(3,0)$

Sol: $x\frac{dy}{dx}=y$

$\Rightarrow \frac{1}{y}dy=\frac{1}{x}dx$

$\Rightarrow \log y=\log x+C$

$(1,1)$ lie on it

$\Rightarrow \log 1=\log 1+C \Rightarrow C=0$

$\therefore \log y=\log x$

$\Rightarrow y=x$

Ans: (B)

27. The degree of the differential equation $1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2=\sqrt[3]{\frac{d^2y}{dx^2}+1}$ is

(A) 1 (B) 6 (C) 2 (D) 3

Sol: $1+(y')^2+(y'')^2=(y''+1)^{1/3}$

Cubing on B – S

$\left(1+(y')^2+(y'')^2\right)^3=y''+1$

On verification degree = 6

$\therefore$ (B) is the correct answer

Ans: (B)

28. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ then

(A) $\vec{a}$ and $\vec{b}$ are coincident. (B) $\vec{a}$ and $\vec{b}$ are perpendicular.

(C) Inclined to each other at 60°. (D) $\vec{a}$ and $\vec{b}$ are parallel.

Sol: $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}| \Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=|a|^2+|b|^2-2\vec{a}\cdot b$

$4\vec{a}\cdot\vec{b}=0$

$\vec{a}\cdot\vec{b}=0$

$\vec{a}\perp\vec{b}$

Ans: (B)

29. The component of $\hat{i}$ in the direction of the vector $\hat{i}+\hat{j}+2\hat{k}$ is

(A) $6\sqrt{6}$ (B) $\sqrt{6}$ (C) $\frac{\sqrt{6}}{6}$ (D) 6

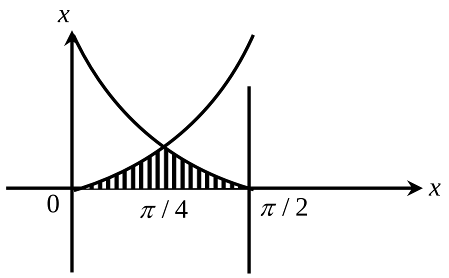
Sol: $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=\frac{1}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}=\frac{\sqrt{6}}{6}$

Ans: (C)

30. In the interval $(0,\pi/2)$, area lying between the curves $y=\tan x$ and $y=\cot x$ and the X-axis is

(A) $4\log 2$ sq. units (B) $3\log 2$ sq. units (C) $\log 2$ sq. units (D) $2\log 2$ sq. units

Sol:



$$A=2\int_0^{\frac{\pi}{4}}\tan x\,dx=2\log\sec x\Big|_0^{\pi/4}=2\log\sqrt{2}=\log 2$$

Ans: (C)

31. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ and $\left(\vec{a}\times\vec{b}\right)+\left(\vec{b}\times\vec{c}\right)+\left(\vec{c}\times\vec{a}\right)=\lambda\left(\vec{b}\times\vec{c}\right)$ then the value of λ is equal to

(A) 4 (B) 2 (C) 6 (D) 3

Sol: $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ [$\times\vec{b}$ both sides]

$\vec{a}\times\vec{b}+3\vec{c}\times\vec{b}=\vec{0}$ -

$\vec{a}\times\vec{b}=3\vec{b}\times\vec{c}$

$\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ $\left(\vec{c}\times\right)$ on both sides)

$\vec{c}\times\vec{a}+2\vec{c}\times\vec{b}=\vec{0}$

$\vec{c}\times\vec{a}=2\vec{b}\times\vec{c}$

Ans: (C)

32. If a line makes an angle of $\frac{\pi}{3}$ with each X and Y axis then the acute angle made by Z -axis is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

Sol: $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

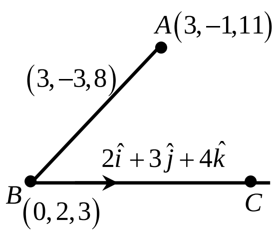
$\frac{1}{2}+\frac{1}{2}+\cos^2\gamma=1\Rightarrow\gamma=90°$

Ans: (A)

33. The length of perpendicular drawn from the point $(3,-1,11)$ to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is

(A) $\sqrt{33}$ (B) $\sqrt{66}$ (C) $\sqrt{53}$ (D) $\sqrt{29}$

Sol:



$\overrightarrow{BA}=3\hat{i}-3\hat{j}+8\hat{k}$

$\overrightarrow{BC}=2\hat{i}+3\hat{j}+4\hat{k}$

$$\overrightarrow{BA}\times\overrightarrow{BC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&-3&8\\2&3&4\end{vmatrix}=-36\hat{i}+4\hat{j}+15\hat{k}$$

$\overrightarrow{BC}=2\hat{i}+3\hat{j}+4k$

$\perp$r distance $=\frac{\sqrt{1296+16+225}}{\sqrt{29}}=\sqrt{53}$

Ans: (C)

34. The equation of the plane through the points $(2,1,0),(3,2,-2)$ and $(3,1,7)$ is

(A) $6x-3y+2z-7=0$ (B) $3x-2y+6z-27=0$ (C) $7x-9y-z-5=0$ (D) $2x-3y+4z-27=0$

Sol: $\begin{vmatrix} x-2 & y-1 & z \\ 1 & 1 & -2 \\ 1 & 0 & 7 \end{vmatrix}=0$

$\Rightarrow 7x-9y-z-5=0$

Ans: (C)

35. The point of intersection of the line $x+1=\frac{y+3}{3}=\frac{-z+2}{2}$ with the plane $3x+4y+5z=10$ is

(A) $(2,6,-4)$ (B) $(-2,6,-4)$ (C) $(2,6,4)$ (D) $(2,-6,-4)$

Sol: Any point on the line $=(\lambda-1,3\lambda-3,-2\lambda+2)$

This lies on plane

$\therefore \lambda=3$

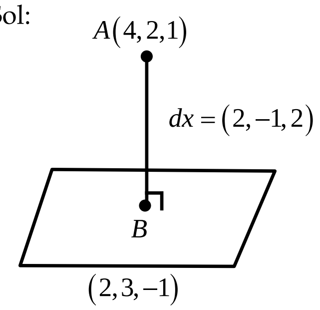
$\therefore$ Point $(2,6,-4)$

Ans: (A)

36. If $(2,3,-1)$ is the foot of the perpendicular from $(4,2,1)$ to a plane, then the equation of the plane is

(A) $2x-y+2z=0$ (B) $2x-y+2z+1=0$ (C) $2x+y+2z-5=0$ (D) $2x+y+2z-1=0$

Sol:



Equation of the plane is $2x-y+2z+k=0$ …. (1)

$(2,3,-1)$ lies on (1)

$\therefore k=1$

Ans: (B)

37. $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=144$ and $|\vec{a}|=4$ then $|\vec{b}|$ is equal to

(A) 8 (B) 12 (C) 4 (D) 3

Sol: $\left(|\vec{a}||\vec{b}|\right)^2=144$

$\Rightarrow |\vec{a}||\vec{b}|=12$

$\Rightarrow |\vec{b}|=\frac{12}{4}=3$

Ans: (D)

38. If A and B are events such that $P(A)=\frac{1}{4}, P(A/B)=\frac{1}{2}$ and $P(B/A)=\frac{2}{3}$ then $P(B)$ is

(A) $\frac{2}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Sol: $P(A).P\left(\frac{B}{A}\right)=P(B).P\left(\frac{A}{B}\right)$

$=\frac{1}{4}\times\frac{2}{3}=P(B).\frac{1}{2}$

$P(B)=\frac{1}{3}$

Ans: (D)

39. A bag contains $2n+1$ coins. It is known that n of these coins have head on both sides whereas the other $n+1$ coins are fair. One coin is selected at random and tossed. If the probability that toss results in heads is $\frac{31}{42}$, then the value of n is

(A) 8 (B) 5 (C) 10 (D) 6

Sol: n coins – Two headers (TH)

$n+1$ coins – Fair

$P(x)=\frac{n}{2n+1}.1+\frac{n+1}{2n+1}.\frac{1}{2}=\frac{31}{42} \qquad \Rightarrow n=10$

Ans: (C)

40. Let $A=\{x,y,z,u\}$ and $B=\{a,b\}$. A function $f:A\to B$ is selected randomly. The probability that the function is an onto function is

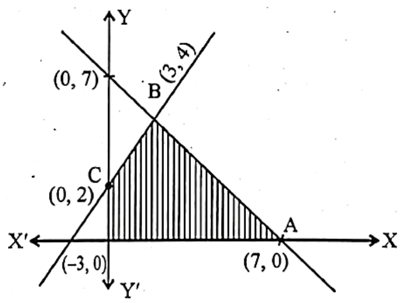
(A) $\frac{5}{8}$ (B) $\frac{7}{8}$ (C) $\frac{1}{35}$ (D) $\frac{1}{8}$

Sol: $n(s)=2^4=16$

$n(E)=2^4-2=14 \qquad P(E)=\frac{14}{16}=\frac{7}{8}$

Ans: (B)

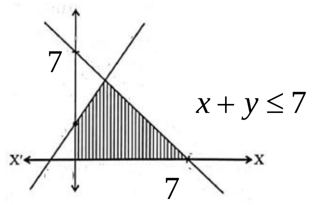
41. The shaded region in the figure given is the solution of which of the inequations?



(A) $x+y\geq 7, 2x-3y+6\geq 0, x\geq 0, y\geq 0$ (B) $x+y\leq 7, 2x-3y+6\geq 0, x\geq 0, y\geq 0$

(C) $x+y\leq 7, 2x-3y+6\leq 0, x\geq 0, y\geq 0$ (D) $x+y\geq 7, 2x-3y+6\leq 0, x\geq 0, y\geq 0$

Sol:



Ans: (B)

42. If $f(x)=ax+b$, where a and b are integers, $f(-1)=-5$ and $f(3)=3$ then a and b are respectively

(A) $0,2$ (B) $-3,-1$ (C) $2,3$ (D) $2,-3$

Sol: $f(-1)=-5$ $f(3)=3$

$-a+b=-5$ ... (1) $3a+b=3$ ... (2)

(1) – (2) $-4a=-8$ $\Rightarrow a=2$

$-2+b=-5$

$b=-3$

$(a,b)=(2,-3)$

Ans: (D)

43. The value of $e^{\log_{10}\tan 1^\circ+\log_{10}\tan 2^\circ+\log_{10}\tan 3^\circ+...+\log_{10}\tan 89^\circ}$ is

(A) $\frac{1}{e}$ (B) 0 (C) 1 (D) 3

Sol: $e^{\log_{10}\tan 1^\circ\cdot\cot 1^\circ+...+\log_{10}\tan 45^\circ}=e^{\log_{10}1}=e^0=1$

Ans: (C)

44. The value of $\begin{vmatrix}\sin^2 14^\circ & \sin^2 66^\circ & \tan 135^\circ\\ \sin^2 66^\circ & \tan 135^\circ & \sin^2 14^\circ\\ \tan 135^\circ & \sin^2 14^\circ & \sin^2 66^\circ\end{vmatrix}$ is

(A) 1 (B) -1 (C) 2 (D) 0

Sol: WRONG QUESTION

Ans: ()

45. The modulus of the complex number $\frac{(1+i)^2(1+3i)}{(2-6i)(2-2i)}$ is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{4}{\sqrt{2}}$ (C) $\frac{\sqrt{2}}{4}$ (D) $\frac{2}{\sqrt{2}}$

Sol: $\left|\frac{(1+i)^2(1+3i)}{(2-6i)(2-2i)}\right|=\frac{|(1+i)|^2|1+3i|}{|2-6i||2-2i|}$

$$=\frac{\left(\sqrt{1+1}\right)^2\sqrt{1+9}}{\sqrt{4+36}\sqrt{4+4}}=\frac{2\times\sqrt{10}}{\sqrt{40}2\sqrt{2}}=\frac{1}{2\sqrt{2}}=\frac{\sqrt{2}}{4}$$

Ans: (C)

46. Given that a, b and x are real numbers and $a < b, x < 0$ then

(A) $\frac{a}{x} < \frac{b}{x}$ (B) $\frac{a}{x} > \frac{b}{x}$ (C) $\frac{a}{x} \le \frac{b}{x}$ (D) $\frac{a}{x} \ge \frac{b}{x}$

Sol: $a < b$

$4 > 5$

$x < 0$

$x = -2$

$$\left.\begin{array}{l}\frac{4}{-2} > \frac{5}{-2}\\ -2 > -2.5\end{array}\right\}\frac{a}{x} > \frac{b}{x}$$

Ans: (B)

47. Ten chairs are numbered as 1 to 10. Three women and two men wish to occupy one chair each. First the women choose the chairs marked 1 to 6, then the men choose the chairs from the remaining. The number of possible ways is

(A) ${}^6C_3 \times {}^4P_2$ (B) ${}^6C_3 \times {}^4C_2$ (C) ${}^6P_3 \times {}^4C_2$ (D) ${}^6P_3 \times {}^4P_2$

Sol: Women select 3 chairs in ${}^6C_3$ ways. Men select 2 chairs in ${}^4C_2$ ways

Total $= {}^6C_3 \times {}^4C_2$

Ans: (B)

48. Which of the following is an empty set?

(A) $\{x : x^2 - 9 = 0, x \in \mathrm{R}\}$ (B) $\{x : x^2 - 1 = 0, x \in \mathrm{R}\}$ (C) $\{x : x^2 = x + 2, x \in \mathrm{R}\}$ (D) $\{x : x^2 + 1 = 0, x \in \mathrm{R}\}$

Sol: $x^2 + 1 = 0$, $x \in R$ is an empty set

Ans: (D)

49. n^{th} term of the series $1 + \frac{3}{7} + \frac{5}{7^2} + \frac{1}{7^2} + \ldots$ is

(A) $\frac{2n-1}{7^n}$ (B) $\frac{2n-1}{7^{n-1}}$ (C) $\frac{2n+1}{7^{n-1}}$ (D) $\frac{2n+1}{7^n}$

Sol: $P(n): 1 + \frac{3}{7} + \frac{5}{7^2} + \frac{1}{7^2} + \ldots$

$$P(1): \frac{2\cdot 1 - 1}{7^{1-1}} = 1$$

$$P(2): \frac{2\times 2 - 1}{7^{2-1}} = \frac{3}{7}$$

Ans: (B)

50. If $p\left(\frac{1}{q}+\frac{1}{r}\right), q\left(\frac{1}{r}+\frac{1}{p}\right), r\left(\frac{1}{p}+\frac{1}{q}\right)$ are in A.P., then p, q, r

(A) are in A.P. (B) are not in A.P. (C) are not in G.P. (D) are in G.P.

Sol:

$p\left(\frac{1}{q}+\frac{1}{r}+\frac{1}{p}-\frac{1}{p}\right), q\left(\frac{1}{r}+\frac{1}{p}+\frac{1}{q}-\frac{1}{q}\right)$

$r\left(\frac{1}{p}+\frac{1}{q}+\frac{1}{r}-\frac{1}{r}\right)$

$pk-1,\ qk-1,\ rk-1$

p, q, r are A.P

k is $\left(\frac{1}{p}+\frac{1}{q}+\frac{1}{r}\right)$ are in A.P

Ans: (A)

51. A line passes through $(2,2)$ and is perpendicular to the line $3x+y=3$. Its y-intercept is

(A) 1 (B) $\frac{1}{3}$ (C) $\frac{4}{3}$ (D) $\frac{2}{3}$

Sol: $3x+y=3$

$y=-3x+3$

$m_{1^2}=3$

$m_{2^2}=\frac{1}{3}$

$y-2=\frac{1}{3}(x-2)$

$x=0$

$y-2=-\frac{2}{3}$

$y=2-\frac{2}{3}=\frac{4}{3}$

Ans: (C)

52. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is

(A) $2x^2-3y^2=7$ (B) $x^2-y^2=32$ (C) $y^2-x^2=32$ (D) $\frac{x^2}{4}-\frac{y^2}{9}=1$

Sol: $2ae=16$

$2a\sqrt{2}=16$

$a=\frac{16}{2\sqrt{2}}$

$a=4\sqrt{2}$

$x^2-y^2=32$

Ans: (B)

53. If $\lim_{x\to 0}\dfrac{\sin(2+x)-\sin(2-x)}{x} = \text{A}\cos\text{B}$, then the values of A and B respectively are

(A) 2,1 (B) 2,2 (C) 1,1 (D) 1,2

Sol: $\lim\limits_{x\to 0} 2\cos[2]\dfrac{\sin x}{x}$

$\lim\limits_{x\to 0} 2\cos 2\cdot\dfrac{\sin x}{x}$

$2\cos 2$

$A=2,\ B=2$

Ans: (B)

54. If n is even and the middle term in the expansion of $\left(x^2+\dfrac{1}{x}\right)^{\text{n}}$ is $924x^6$, then n is equal to

(A) 12 (B) 10 (C) 8 (D) 14

Sol: Middle term $T_{\frac{n}{2}+1} = {}^nC_{\frac{n}{2}}\left(x^2\right)^{\frac{n}{2}}\left(\dfrac{1}{x}\right)^{\frac{n}{2}} = 924x^6$

${}^nC_{\frac{n}{2}}x^{\frac{n}{2}} = 924x^6$

$\dfrac{n}{2}=6 \Rightarrow n=12$

Ans: (A)

55. The mean of 100 observations is 50 and their standard deviation is 5 . Then the sum of squares of all observations is

(A) 250000 (B) 50000 (C) 255000 (D) 252500

Sol: $\dfrac{\sum x_i}{100}=50;\ \sigma=5$

$\dfrac{\sum x_i^2}{100}-\left(\dfrac{\sum x_i}{100}\right)^2=25$

$\dfrac{\sum x_i^2}{100}=2500+25$

$\Rightarrow \sum x_i^2 = 252500$

Ans: (D)

56. $f:R\to R$ and $g:[0,\infty)\to R$ are defined by $f(x)=x^2$ and $g(x)=\sqrt{x}$. Which one of the following is not true ?

(A) $(f\,o\,g)(2)=2$ (B) $(g\,o\,f)(4)=4$ (C) $(g\,o\,f)(-2)=2$ (D) $(f\,o\,g)(-4)=4$

Sol: $f(g(2))=(g(2))^2=\left(\sqrt{2}\right)^2=2\to \text{True}$

$g(f(4))=\sqrt{f(4)}=\sqrt{4^2}=4\to \text{True}$

$g(f(-2))=\sqrt{f(-2)}=\sqrt{(-2)^2}=2\to$ True

$f(g(-4))=(g(-4))^2=(\sqrt{-4})^2\neq 4\to$ False

Ans: (D)

57. Let $f:R\to R$ be defined by $f(x)=3x^2-5$ and $g:R\to R$ by $g(x)=\dfrac{x}{x^2+1}$ then gof is

(A) $\dfrac{3x^2}{x^4+2x^2-4}$ (B) $\dfrac{3x^2-5}{9x^4-30x^2+26}$ (C) $\dfrac{3x^2}{9x^4+30x^2-2}$ (D) $\dfrac{3x^2-5}{9x^4-6x^2+26}$

Sol: $g(f(x))=\dfrac{3x^2-5}{(3x^2-5)^2+1}=\dfrac{3x^2-5}{9x^4-30x^2+26}$

Ans: (B)

58. Let the relation R be defined in N by aRb if $3a+2b=27$ then R is

(A) $\{(1,12),(3,9),(5,6),(7,3),(9,0)\}$ (B) $\{(1,12),(3,9),(5,6),(7,3)\}$

(C) $\{(2,1),(9,3),(6,5),(3,7)\}$ (D) $\left\{\left(0,\dfrac{27}{2}\right),(1,12),(3,9),(5,6),(7,3)\right\}$

Sol: $a\in N,\ b\in N$

From option B all satisfies condition

Ans: (B)

59. Let $\mathrm{f}(x)=\sin 2x+\cos 2x$ and $\mathrm{g}(x)=x^2-1$, then $g(f(x))$ is invertible in the domain

(A) $x\in\left[\dfrac{-\pi}{2},\dfrac{\pi}{2}\right]$ (B) $x\in\left[\dfrac{-\pi}{4},\dfrac{\pi}{4}\right]$ (C) $x\in\left[0,\dfrac{\pi}{4}\right]$ (D) $x\in\left[\dfrac{-\pi}{8},\dfrac{\pi}{8}\right]$

Sol: $g(f(x))$

$g\left[\left(\dfrac{1}{\sqrt{2}}\sin 2x+\dfrac{1}{\sqrt{2}}\cos 2x\right)\sqrt{2}\right]$

$g\left[\sqrt{2}\sin\left(2x+\dfrac{\pi}{4}\right)\right]$

$g(f(x))=2\sin^2\left(2x+\dfrac{\pi}{4}\right)-1$

$=-\cos\left[4x+\dfrac{\pi}{2}\right]$

$g(f(x))$ is invertible

$0\le 4x+\dfrac{\pi}{2}\le\pi$

$-\dfrac{\pi}{8}\le x\le\dfrac{\pi}{8}$

Ans: (D)

60. The contrapositive of the statement, "if two lines do not intersect in the same plane, then they are parallel" is

(A) If two lines are not parallel then they do not intersect in the same plane.

(B) If two lines are not parallel then they intersect in the same plane.

(C) If two lines are parallel then they do not intersect in the same plane.

(D) If two lines are parallel then they intersect in the same plane.

Sol: p : Two lines do not intersect in same plane

Q : Two lines are parallel

Contrapositive: $\sim q \rightarrow \sim p$

If two lines not parallel they intersect in same plane

Ans: (B)