

Part 5

IPDFA Using Value Contexts

Value Contexts: Key Ideas

Consider call chains σ_1 and σ_2 reaching S_p

- Data flow value invariant:
If the data flow reaching S_p along σ_1 and σ_2 are identical, then



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Value Contexts: Key Ideas

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- Data flow value invariant:

If the data flow reaching S_p along σ_1 and σ_2 are identical, then

- ▶ the data flow values reaching E_p for the two contexts will also be identical
- We can reduce the amount of effort by using
 - ▶ Data flow values at S_p as value contexts
 - ▶ Maintaining distinct data flow values in p for each value context

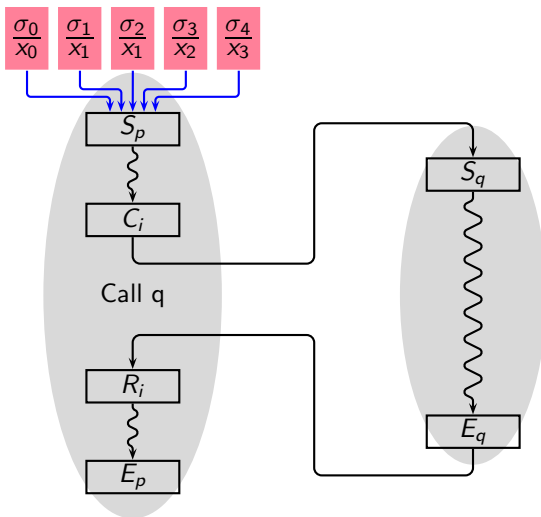


Interprocedural Data Flow Analysis Using Value Contexts

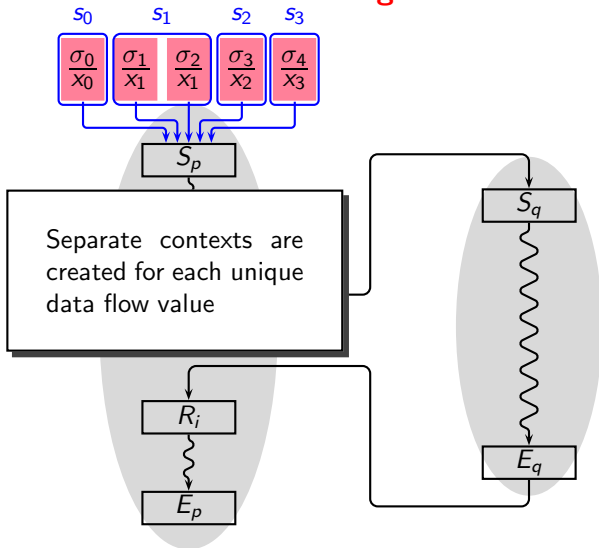
- A value context is defined by a particular input data flow value reaching a procedure
- It is used to enumerate the summary flow functions in terms of (input \mapsto output) pairs
- In order to compute these pairs, data flow analysis within a procedure is performed separately for each context (i.e. input data flow value)
- When a new call to a procedure is encountered, the pairs are consulted to decide if the procedure needs to be analysed again
 - ▶ If it was already analysed once for the input value, output can be directly processed
 - ▶ Otherwise, a new context is created and the procedure is analysed for this new context



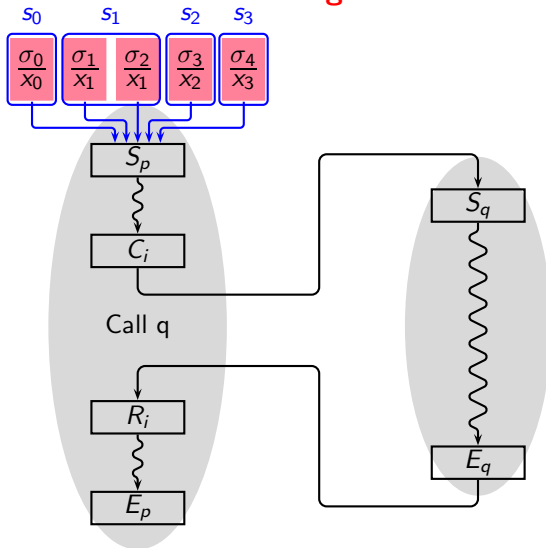
Understanding Value Contexts



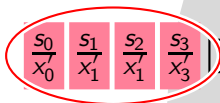
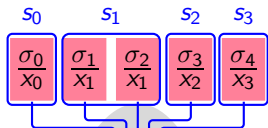
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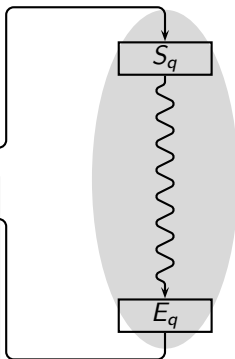
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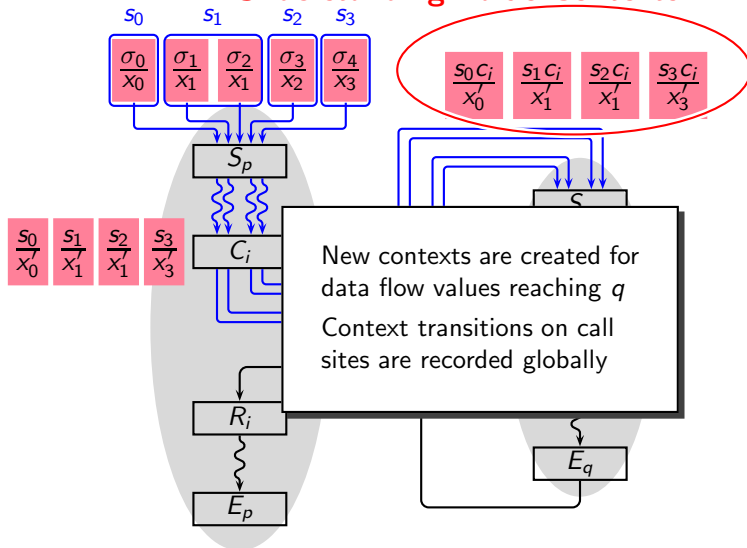
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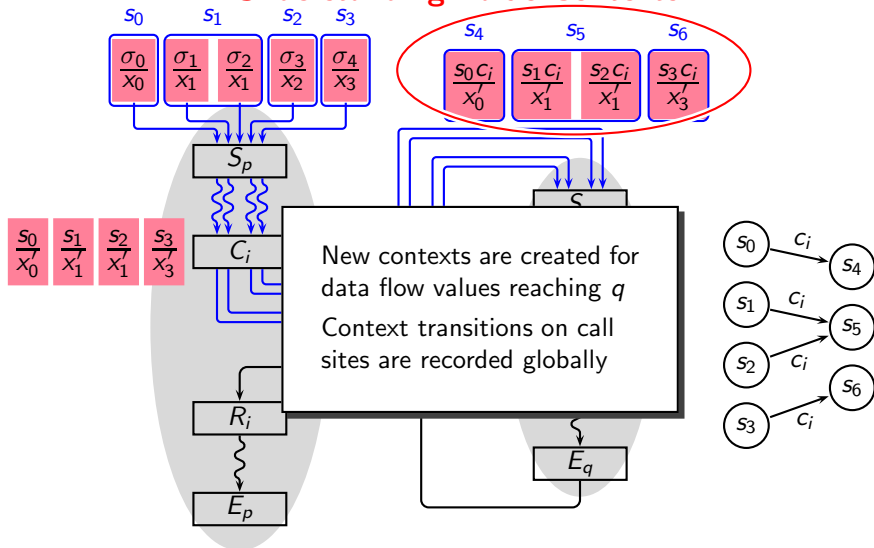
Distinct data flow values are maintained for each context (i.e. each procedure is analysed separately for each context)



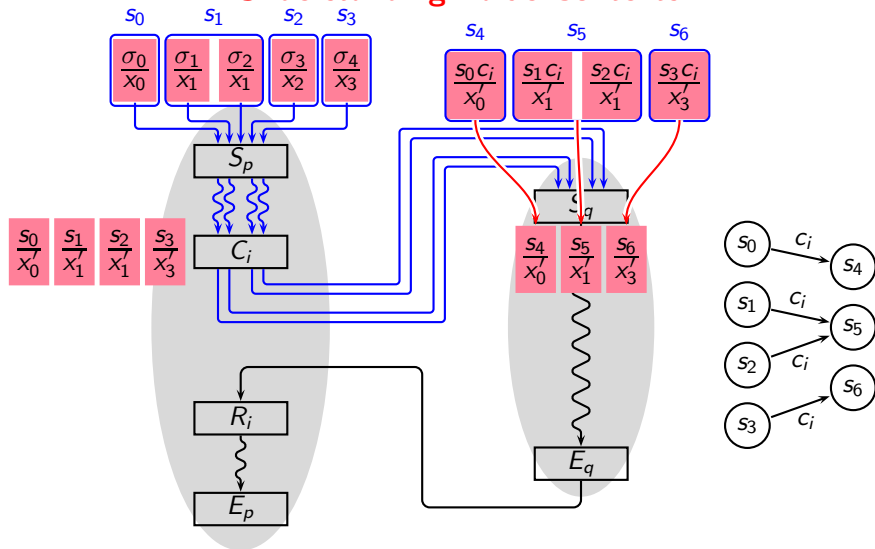
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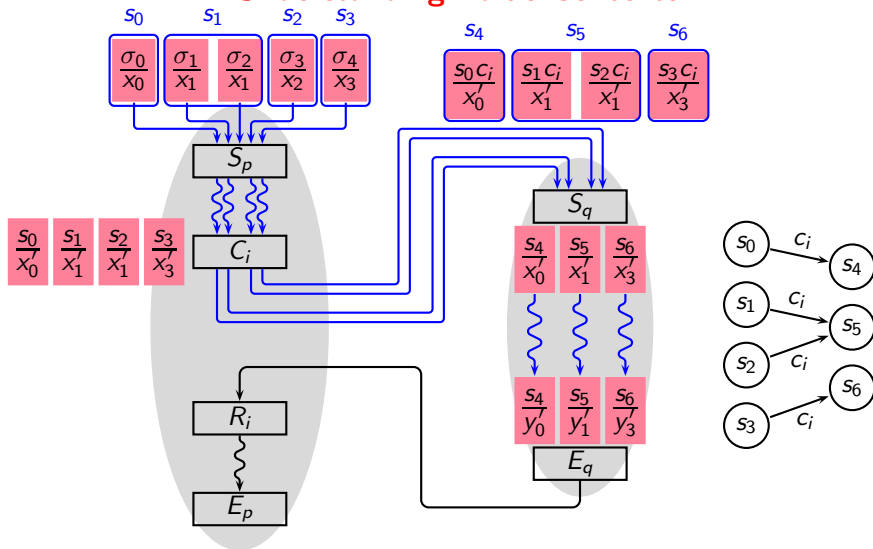
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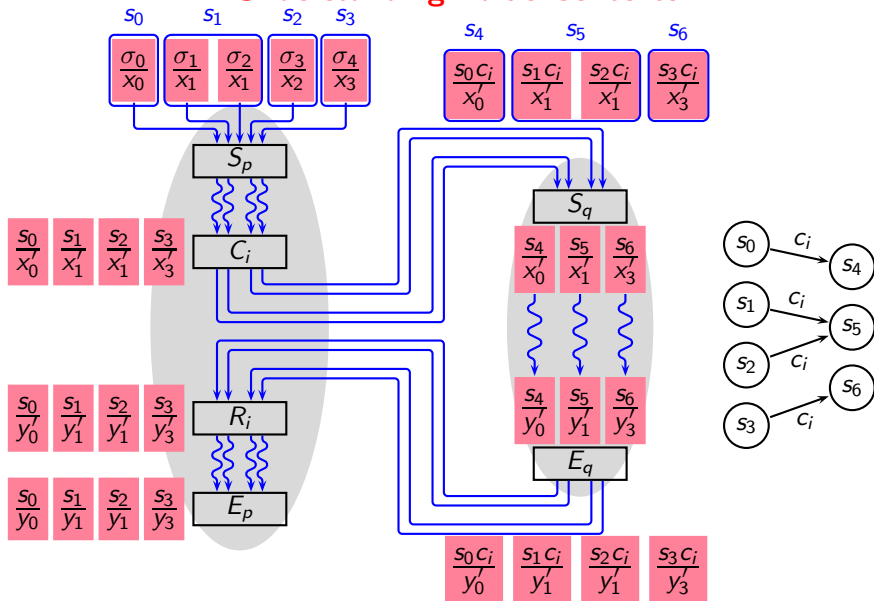


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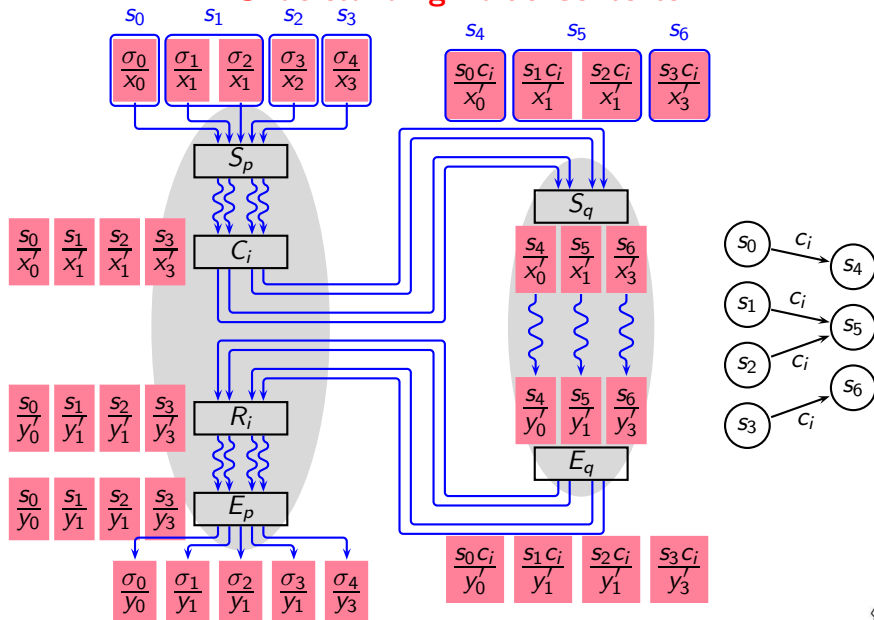




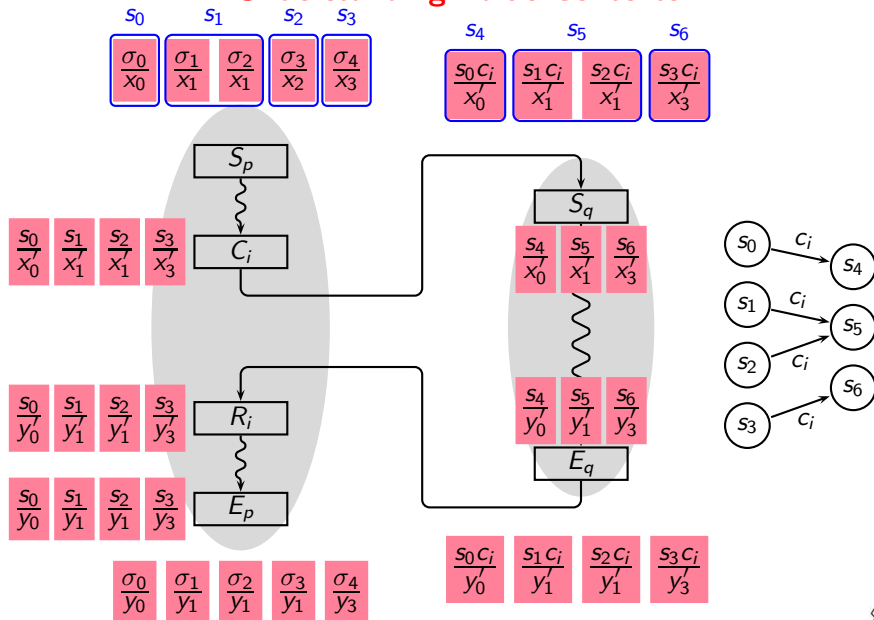
Understanding Value Contexts



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Understanding Value Contexts



Defining Value Contexts

- The set of value contexts is $VC = Procs \times L$

A value context $X = \langle proc, entryValue \rangle \in VC$
where $proc \in Procs$ and $entryValue \in L$



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- Supporting functions (CS is the set of call sites)
 - ▶ $exitValue : VC \mapsto L$
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- Supporting functions (CS is the set of call sites)

- ▶ $exitValue : VC \mapsto L$

eg. $exitValue(X) = v$

- ▶ $transitions : (VC \times CS) \mapsto VC$

eg. $X \xrightarrow{C_i} Y$



Interprocedural Data Flow Analysis Using Value Contexts

- The method works with a collection of control flow graphs

No need of supergraph

- ▶ No need to distinguish between C_i and R_i
 - ▶ No need of call $(C_i \rightarrow S_p)$ and return $(E_p \rightarrow E_i)$ edges
- Maintain a work list WL of entries $\langle context, node \rangle$
(in reverse post order of nodes within a procedure for forward flows)
- Notation:

$\langle p, v \rangle$	Context for procedure p with data flow value v
$X m$	Work list entry for context X for node m
$X.v$	Data flow value in context X is v
$Out_m[X]$	Data flow value of context X in Out_m
$X \xrightarrow{C_i} Y$	Transition from context X to context Y at call site C_i



Interprocedural Data Flow Analysis Using Value Contexts: An Overview

- Select $X|_n$ from WL . Compute ln_n .



Interprocedural Data Flow Analysis Using Value Contexts: An Overview

- Select $X|n$ from WL . Compute In_n .
 - ▶ If $n = C_i$ calling procedure p
 - ▶ If $n = E_p$
 - ▶ If n is some other node



Interprocedural Data Flow Analysis Using Value Contexts: An Overview

- Select $X|n$ from WL . Compute In_n .
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Propagate In_n to appropriate value context of the callee procedure p
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Compute Out_n



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Propagate In_n to appropriate value contexts of the callers of p
 - ▶ If n is some other node
Compute Out_n

Update WL

- Repeat until WL is empty



Interprocedural Data Flow Analysis Using Value Contexts (2)

Select $X|_n$ from WL . Compute In_n . Let $X.v$ be in In_n



Interprocedural Data Flow Analysis Using Value Contexts (2)

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Interprocedural Data Flow Analysis Using Value Contexts (2)

Select $X|n$ from WL . Compute In_n . Let $X.v$ be in In_n

- If $n = C_j$ calling procedure p
 - ▶ If some context $\langle p, v \rangle$ exists (say Y) /* p is the callee */
 - ▶ If it does not exist



Interprocedural Data Flow Analysis Using Value Contexts (2)

Select $X|n$ from WL . Compute In_n . Let $X.v$ be in In_n

- If $n = C_i$ calling procedure p
 - ▶ If some context $\langle p, v \rangle$ exists (say Y) /* p is the callee */
 - record the transition $X \xrightarrow{C_i} Y$
 - $Out_{C_i}[X] = Out_{C_i}[X] \sqcap exitValue(Y)$
 - if there is a change, add $X|m, \forall m \in succ(C_i)$ to WL
 - ▶ If it does not exist



Interprocedural Data Flow Analysis Using Value Contexts (2)

Select $X|n$ from WL . Compute In_n . Let $X.v$ be in In_n

- If $n = C_i$ calling procedure p
 - ▶ If some context $\langle p, v \rangle$ exists (say Y) /* p is the callee */

 - ▶ If it does not exist
 - create a new context $Y = \langle p, v \rangle$ /* p is the callee */
 - initialize $exitValue(Y) = \top$
 - record the transition $X \xrightarrow{C_i} Y$
 - initialize $Out_m[Y] = \top$ for all nodes m of procedure p
 - add entries $Y|m$ for all nodes m of procedure p to WL



Interprocedural Data Flow Analysis Using Value Contexts (2)

Select $X|_n$ from WL . Compute In_n . Let $X.v$ be in In_n

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Interprocedural Data Flow Analysis Using Value Contexts (3)

Select $X|_n$ from WL . Compute In_n . Let $X.v$ be in In_n



Interprocedural Data Flow Analysis Using Value Contexts (3)

Select $X|n$ from WL . Compute In_n . Let $X.v$ be in In_n

- If $n = E_p$
- For all other nodes



Interprocedural Data Flow Analysis Using Value Contexts (3)

Select $X|n$ from WL . Compute In_n . Let $X.v$ be in In_n

- If $n = E_p$

- ▶ Set $exitValue(X) = v$

/* E_p is an empty block */

- For all other nodes



Interprocedural Data Flow Analysis Using Value Contexts (3)

Select $X|n$ from WL . Compute In_n . Let $X.v$ be in In_n

- If $n = E_p$
 - ▶ Set $exitValue(X) = v$ /* E_p is an empty block */
 - ▶ Find out all transitions $Z \xrightarrow{C_j} X$
 - Set $Out_{C_j}[Z] = Out_{C_j}[Z] \sqcap v$
 - If there is a change, add $Z|m, \forall m \in succ(C_j)$ to WL
- For all other nodes



Interprocedural Data Flow Analysis Using Value Contexts (3)

Select $X|n$ from WL . Compute In_n . Let $X.v$ be in In_n

- If $n = E_p$
- For all other nodes
 - ▶ Set $Out_n[X] = f_n(v)$



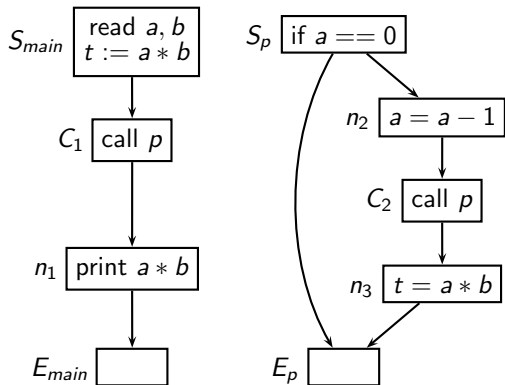
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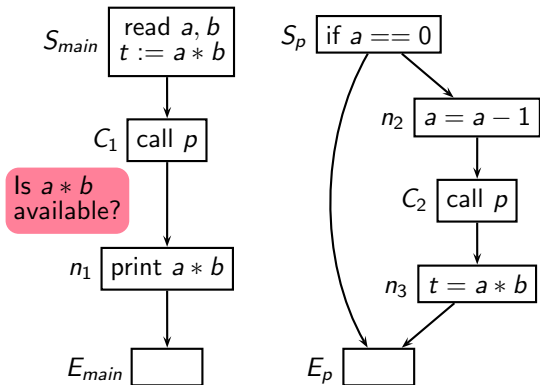
- If $n = E_p$
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 - Set $Out_{C_j}[Z] = Out_{C_j}[Z] \sqcap v$
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- For all other nodes
 - ▶ Set $Out_n[X] = f_n(v)$
 - ▶ If there is a change, add $X|m, \forall m \in succ(n)$ to WL



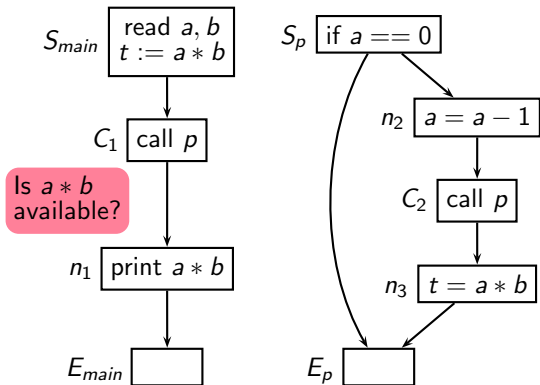
Available Expressions Analysis Using Value Contexts



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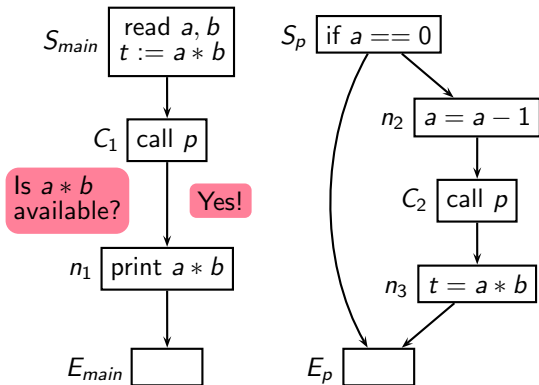
Available Expressions Analysis Using Value Contexts



```
int a, b, t;  
void p()  
{  
  if (a == 0)  
  {  
    a = a-1;  
    p();  
    t = a*b;  
  }  
}
```



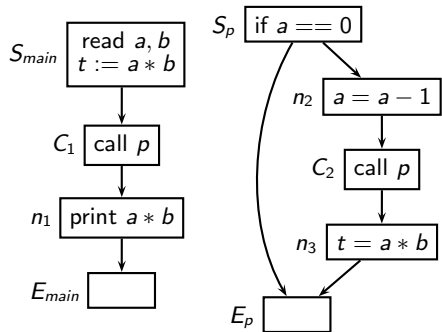
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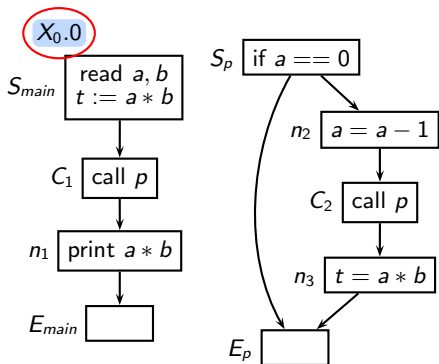
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Available Expressions Analysis Using Value Contexts



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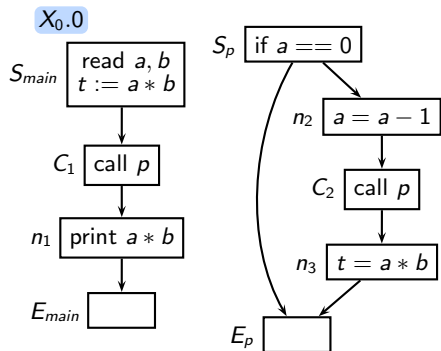
Create a new context X_0 with BI which is 0 for available expressions analysis



Available Expressions Analysis Using Value Contexts

$$WL = [X_0 | S_m, X_0 | C_1, X_0 | n_1, X_0 | E_m]$$

X_0



Context	<i>exitValue</i>
$X_0 = \langle \text{main}, 0 \rangle$	1

Create a new context X_0 with BI which is 0 for available expressions analysis

Initialize $exitValue(X_0)$ to $\top = 1$

Initialize the work list with all nodes in procedure main for X_0

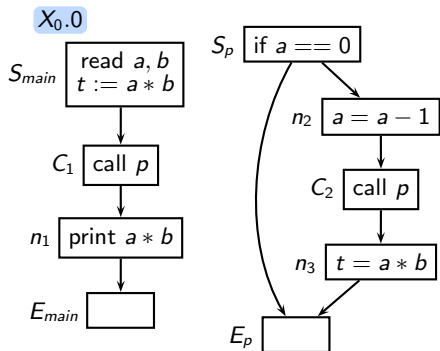
Initialize $Out_n[X_0]$ for all n in main to \top



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X_0



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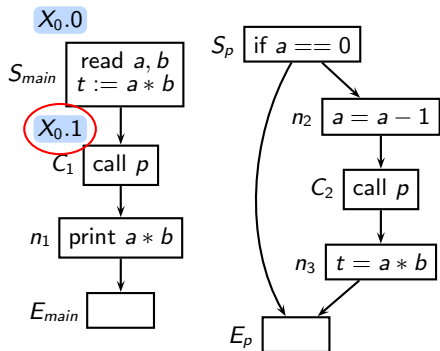
Compute the data flow values for S_m for context X_0



Available Expressions Analysis Using Value Contexts

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X_0



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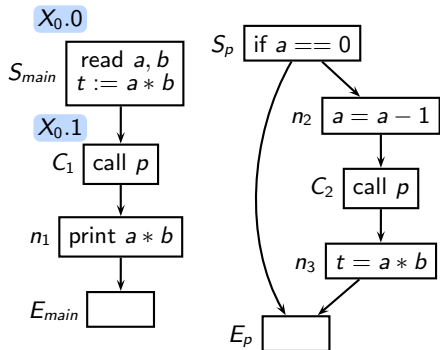
Compute the data flow values for S_m for context X_0
It does not change



Available Expressions Analysis Using Value Contexts

$$WL = [X_0 | C_1, X_0 | n_1, X_0 | E_m]$$

X_0

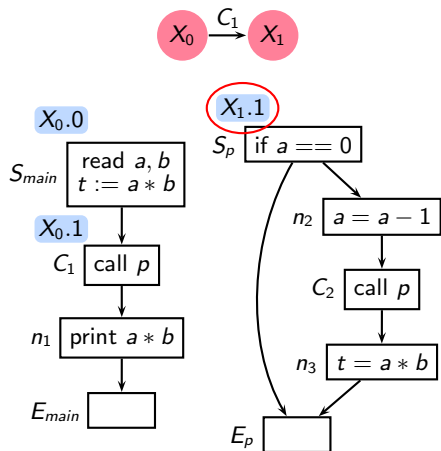


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Available Expressions Analysis Using Value Contexts

$$WL = [X_1 | S_p, X_1 | n_2, X_1 | C_2, X_1 | n_3, X_1 | E_p, X_0 | n_1, X_0 | E_m]$$



Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle p, 1 \rangle$	1

Create a new context X_1 with entry value 1

Record the transition to X_1

Initialize $\text{exitValue}(X_1)$ to $\top = 1$

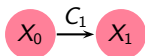
Add all nodes of procedure p to the work list for X_1

Initialize $\text{Out}_n[X_1]$ for all n in p to \top

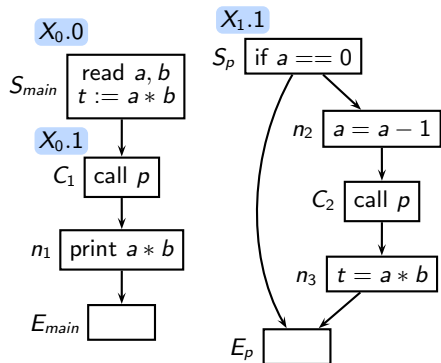


Available Expressions Analysis Using Value Contexts

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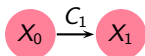


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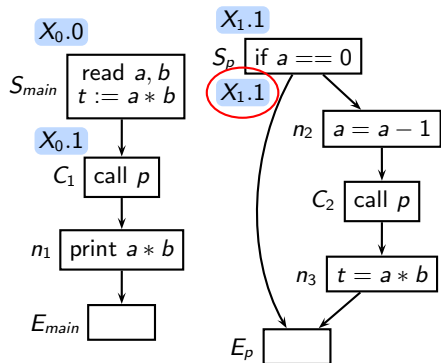


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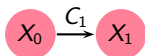


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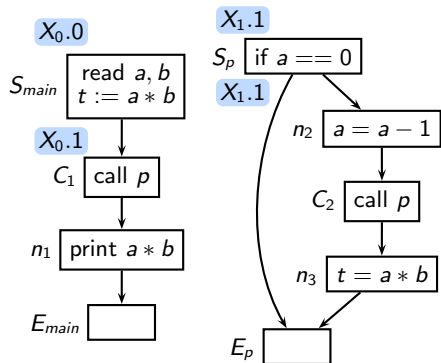


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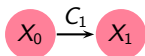


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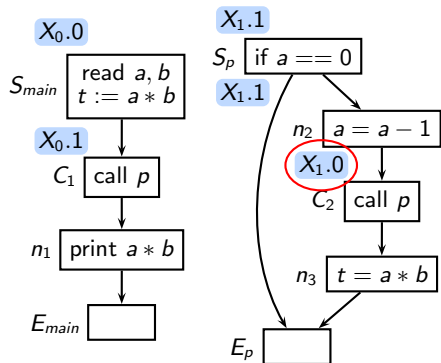


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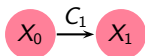


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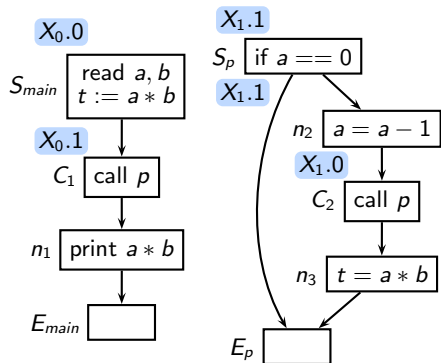


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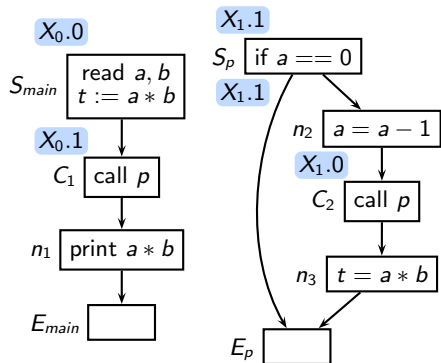
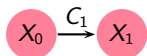


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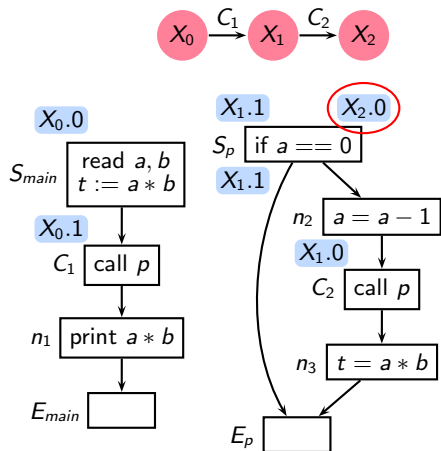


Context	exitValue
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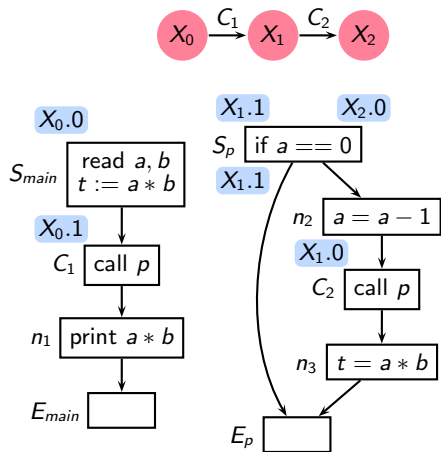
Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle p, 1 \rangle$	1
$X_2 = \langle p, 0 \rangle$	1

Since there is no context for p with value 0, create context X_2
 Record the transition to X_2
 Initialize $\text{exitValue}(X_2)$ to $\top = 1$
 Add all nodes of procedure p to the work list for X_2 Initialize $\text{Out}_n[X_2]$ for all n in p to \top



Available Expressions Analysis Using Value Contexts

$$WL = [X_2|S_p, X_2|n_2, X_2|C_2, X_2|n_3, X_2|E_p, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$

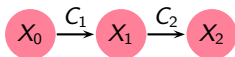


Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle p, 1 \rangle$	1
$X_2 = \langle p, 0 \rangle$	1

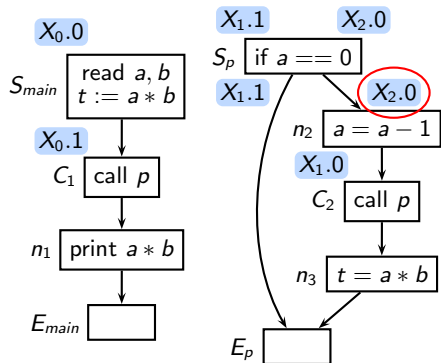


Available Expressions Analysis Using Value Contexts

$$WL = [X_2|S_p, X_2|n_2, X_2|C_2, X_2|n_3, X_2|E_p, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$

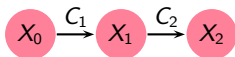


Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	1

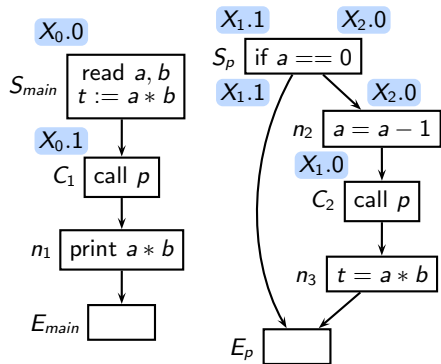


Available Expressions Analysis Using Value Contexts

$$WL = [X_2|n_2, X_2|C_2, X_2|n_3, X_2|E_p, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$

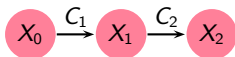


Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	1

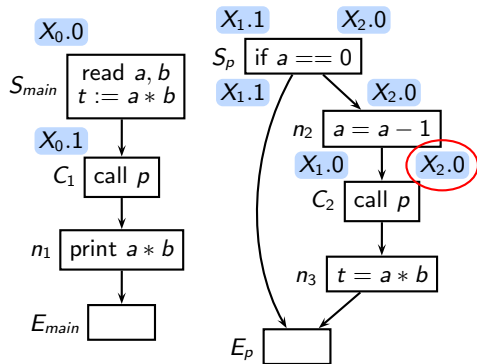


Available Expressions Analysis Using Value Contexts

$$WL = [X_2|n_2, X_2|C_2, X_2|n_3, X_2|E_p, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$

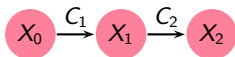


Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	1

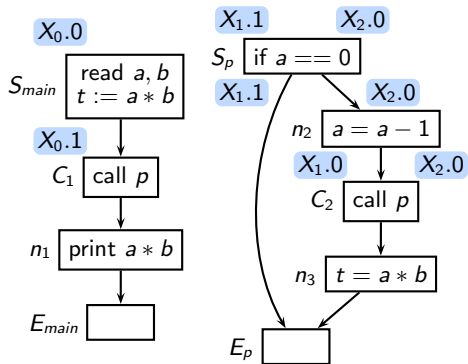


Available Expressions Analysis Using Value Contexts

$$WL = [X_2|C_2, X_2|n_3, X_2|E_p, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$

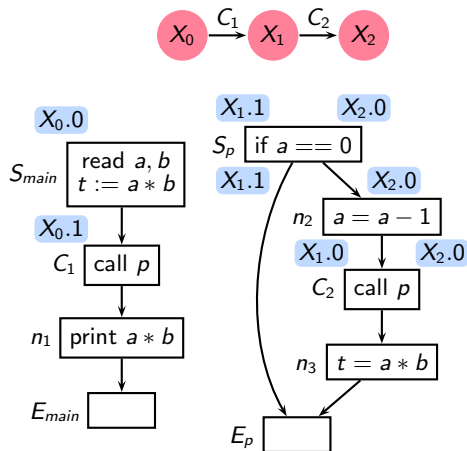


Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle p, 1 \rangle$	1
$X_2 = \langle p, 0 \rangle$	1



Available Expressions Analysis Using Value Contexts

$$WL = [X_2|C_2, X_2|n_3, X_2|E_p, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$



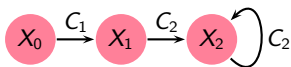
Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	1

p has context X_2 with value 0 so no need to create a new context

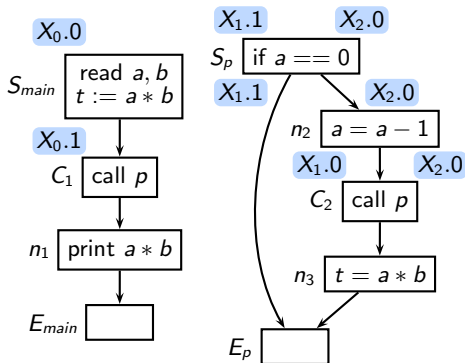


Available Expressions Analysis Using Value Contexts

$$WL = [X_2|C_2, X_2|n_3, X_2|E_p, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$



Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	1

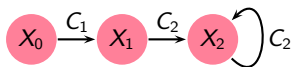


p has context X_2 with value 0 so no need to create a new context
Record the transition from context X_2 to itself

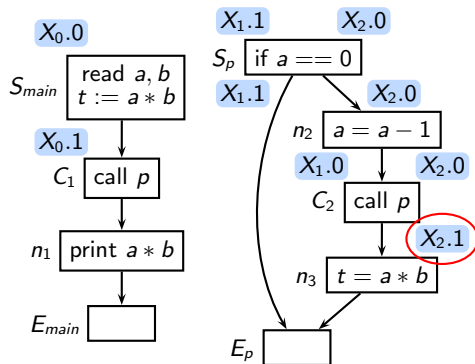


Available Expressions Analysis Using Value Contexts

$$WL = [X_2|C_2, X_2|n_3, X_2|E_p, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$



Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	1



p has context X_2 with value 0 so no need to create a new context

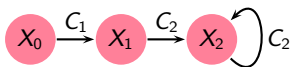
Record the transition from context X_2 to itself

Use the $\text{exitValue}(X_2)$ to compute $\text{Out}_{C_2}[X_2]$

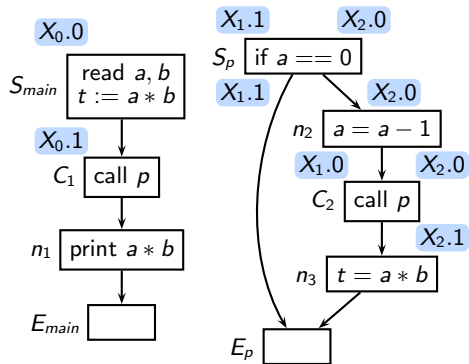


Available Expressions Analysis Using Value Contexts

$$WL = [X_2|n_3, X_2|E_p, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$

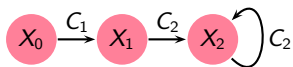


Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	1

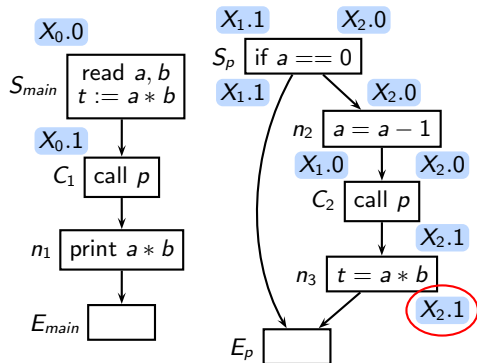


Available Expressions Analysis Using Value Contexts

$$WL = [X_2|n_3, X_2|E_p, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$

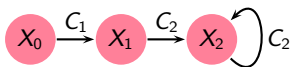


Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle p, 1 \rangle$	1
$X_2 = \langle p, 0 \rangle$	1

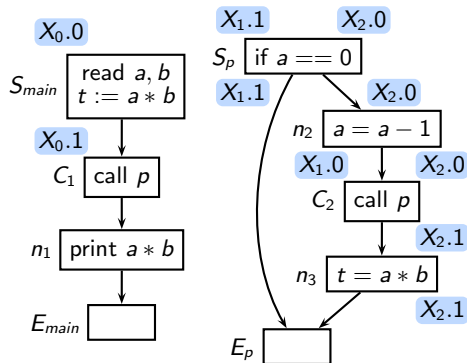


Available Expressions Analysis Using Value Contexts

$$WL = [X_2|E_p, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$

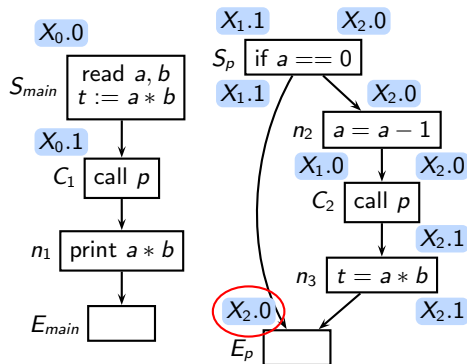
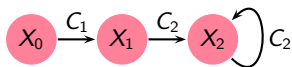


Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle p, 1 \rangle$	1
$X_2 = \langle p, 0 \rangle$	1



Available Expressions Analysis Using Value Contexts

$$WL = [X_2|E_p, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$



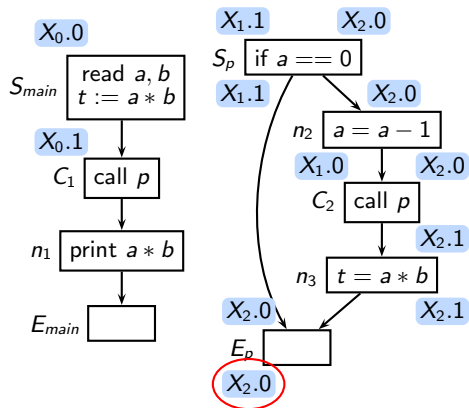
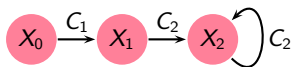
Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	1

At E_p the values from S_p and n_3 are merged for context X_2



Available Expressions Analysis Using Value Contexts

$$WL = [X_2|E_p, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$



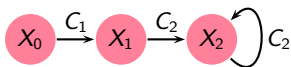
Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0

At E_p the values from S_p and n_3 are merged for context X_2
 $\text{exitValue}(X_2)$ is set to 0

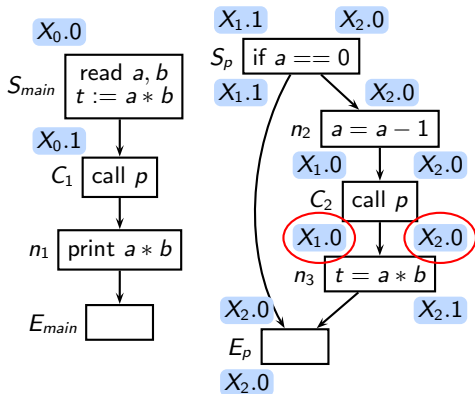


Available Expressions Analysis Using Value Contexts

$$WL = [X_2|E_p, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$



Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0



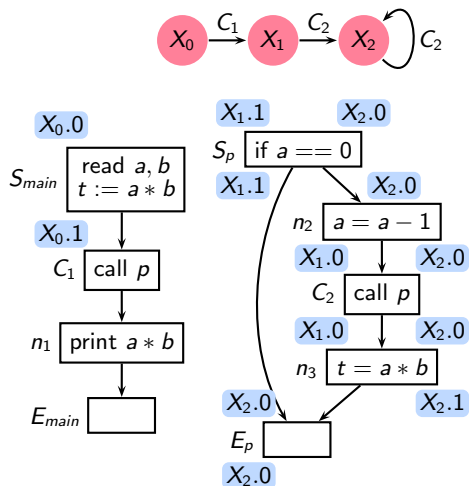
At E_p the values from S_p and n_3 are merged for context X_2
 $\text{exitValue}(X_2)$ is set to 0

Since X_2 has transitions $X_1 \xrightarrow{C_2} X_2$
 and $X_2 \xrightarrow{C_2} X_2$, $\text{Out}_{C_2}[X_1]$ and
 $\text{Out}_{C_2}[X_2]$ become 0



Available Expressions Analysis Using Value Contexts

$$WL = [X_2|n_3, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$



Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0

At E_p the values from S_p and n_3 are merged for context X_2
 $\text{exitValue}(X_2)$ is set to 0

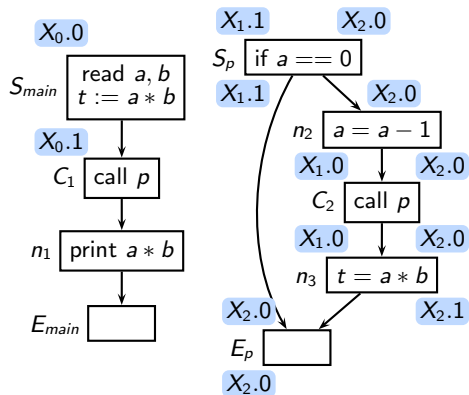
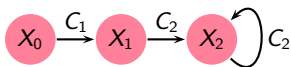
Since X_2 has transitions $X_1 \xrightarrow{C_2} X_2$
 and $X_2 \xrightarrow{C_2} X_2$, $\text{Out}_{C_2}[X_1]$ and
 $\text{Out}_{C_2}[X_2]$ become 0

Since $\text{Out}_{C_2}[X_2]$ changes, $X_2|n_3$
 is added to the work list



Available Expressions Analysis Using Value Contexts

$$WL = [X_2|n_3, X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$



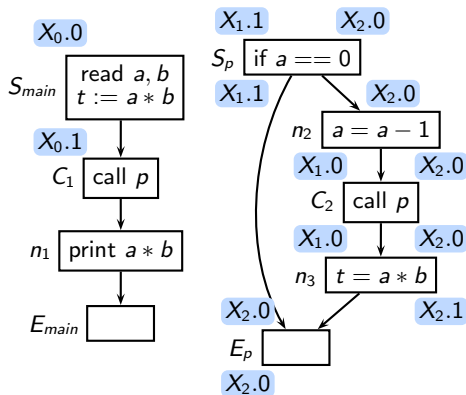
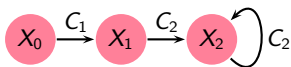
Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0

There is no change in $Out_{n_3}[X_2]$
(because it was initialized to \top)



Available Expressions Analysis Using Value Contexts

$$WL = [X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$



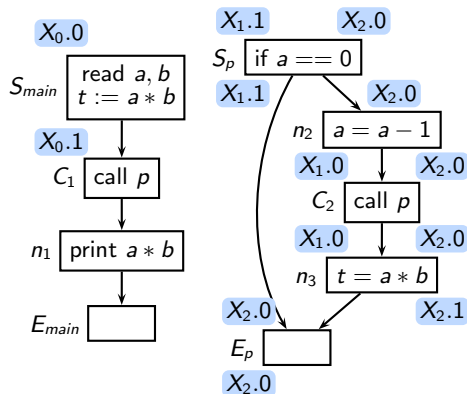
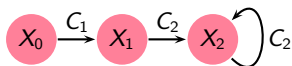
Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0

There is no change in $Out_{n_3}[X_2]$
(because it was initialized to \top)



Available Expressions Analysis Using Value Contexts

$$WL = [X_1|n_3, X_1|E_p, X_0|n_1, X_0|E_m]$$



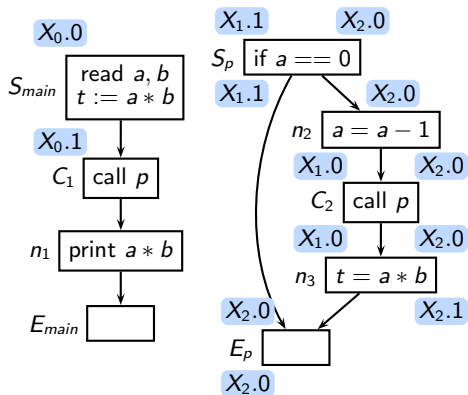
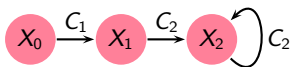
Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0

There is no change in $Out_{n_3}[X_1]$ either



Available Expressions Analysis Using Value Contexts

$$WL = [X_1|E_p, X_0|n_1, X_0|E_m]$$



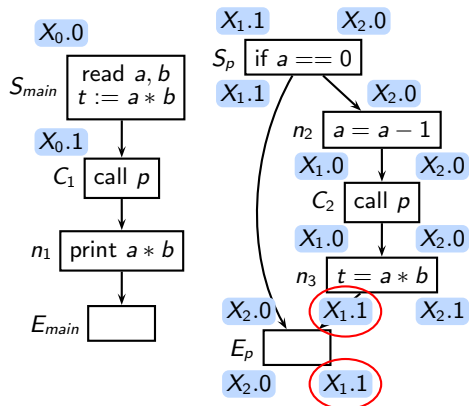
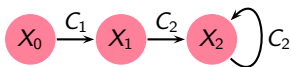
Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0

There is no change in $Out_{n_3}[X_1]$ either



Available Expressions Analysis Using Value Contexts

$$WL = [X_1|E_p, X_0|n_1, X_0|E_m]$$



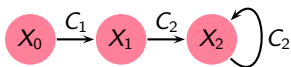
Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0

At E_p the values from S_p and n_3 are merged for context X_1

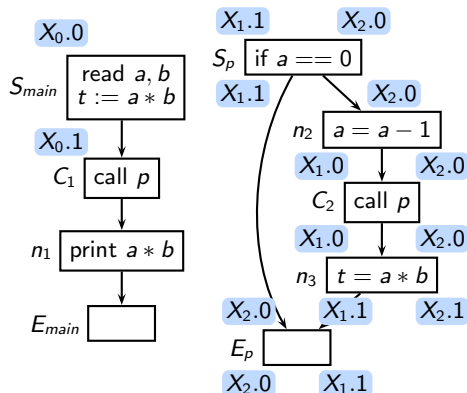


Available Expressions Analysis Using Value Contexts

$$WL = [X_1|E_p, X_0|n_1, X_0|E_m]$$



Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0

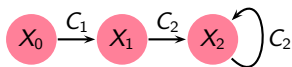


At E_p the values from S_p and n_3 are merged for context X_1
 $\text{exitValue}(X_1)$ remains 1

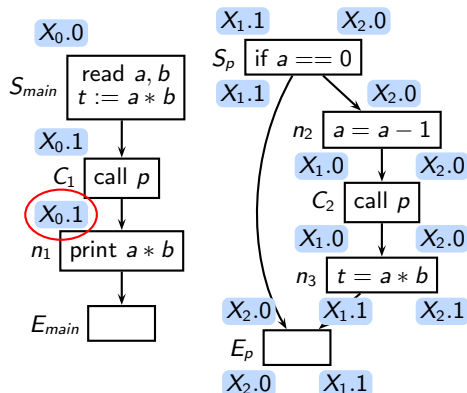


Available Expressions Analysis Using Value Contexts

$$WL = [X_1|E_p, X_0|n_1, X_0|E_m]$$



Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0



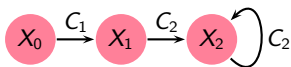
At E_p the values from S_p and n_3 are merged for context X_1
 $\text{exitValue}(X_1)$ remains 1

Since X_1 has transition $X_0 \xrightarrow{C_1} X_1$,
 $\text{Out}_{C_1}[X_0]$ becomes 1

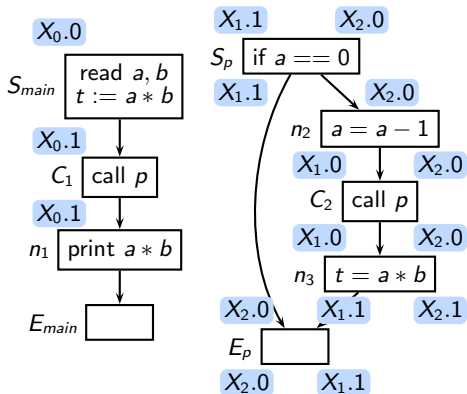


Available Expressions Analysis Using Value Contexts

$$WL = [X_0 | n_1, X_0 | E_m]$$

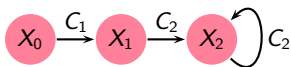


Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0

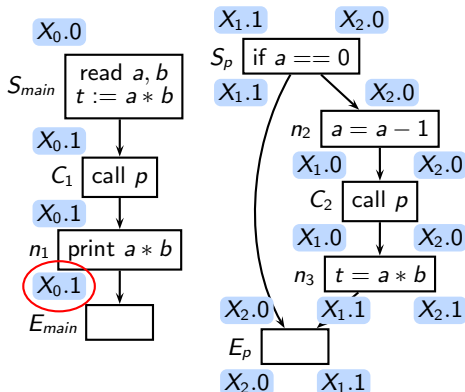


Available Expressions Analysis Using Value Contexts

$$WL = [X_0 | n_1, X_0 | E_m]$$

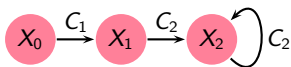


Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0

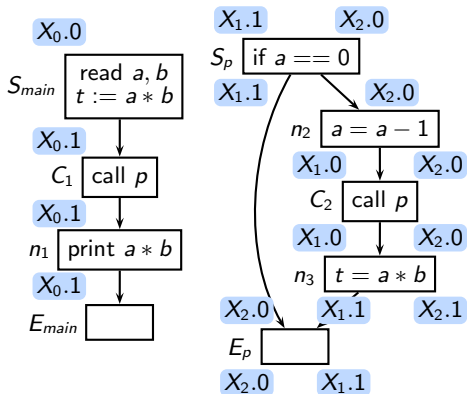


Available Expressions Analysis Using Value Contexts

$$WL = [X_0 | E_m]$$

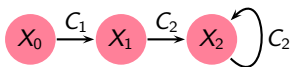


Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0

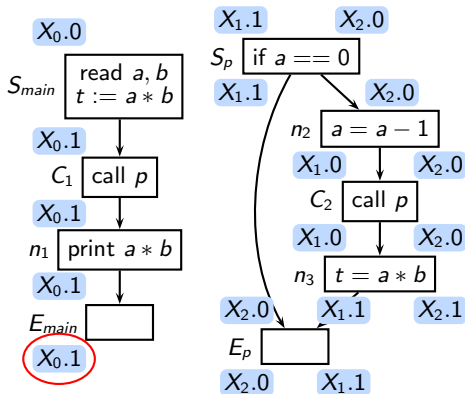


Available Expressions Analysis Using Value Contexts

$$WL = [X_0 | E_m]$$

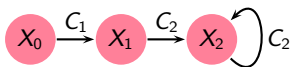


Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0

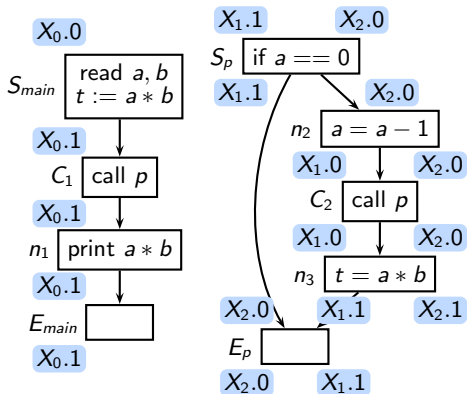


Available Expressions Analysis Using Value Contexts

$$WL = [X_0 | E_m]$$

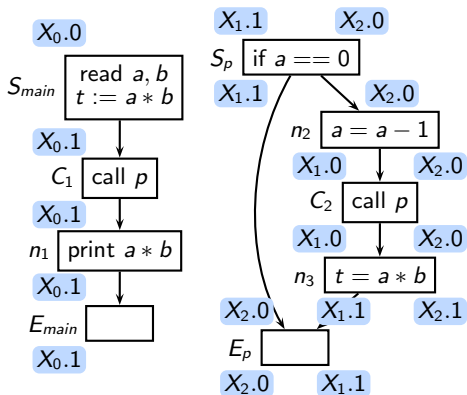
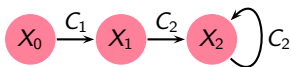


Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0



Available Expressions Analysis Using Value Contexts

$WL = []$



Context	exitValue
$X_0 = \langle \text{main}, 0 \rangle$	1
$X_1 = \langle \text{p}, 1 \rangle$	1
$X_2 = \langle \text{p}, 0 \rangle$	0

Work list is empty and the analysis is over



A Trace of Value Context Based Analysis (1)

S. No.	Work List	Sel. node	Data flow value	New context	New trans.	exit value	Addition to the work list
1				$X_0 = \langle m, 0 \rangle$		$X_{0.1}$	$X_0 S_m, X_0 C_1,$ $X_0 n_1, X_0 E_m$
2	$X_0 S_m, X_0 C_1, X_0 n_1,$ $X_0 E_m$	S_m	$Out_{S_m}[X_0] = 1$				
3	$X_0 C_1, X_0 n_1, X_0 E_m$	C_1		$X_1 = \langle p, 1 \rangle$	$X_0 \xrightarrow{C_1} X_1$	$X_{1.1}$	$X_1 S_p, X_1 n_2,$ $X_1 C_2, X_1 n_3,$ $X_1 E_p$
4	$X_1 S_p, X_1 n_2, X_1 C_2,$ $X_1 n_3, X_1 E_p, X_0 n_1,$ $X_0 E_m$	S_p	$Out_{S_p}[X_1] = 1$				
5	$X_1 n_2, X_1 C_2, X_1 n_3,$ $X_1 E_p, X_0 n_1, X_0 E_m$	n_2	$Out_{n_2}[X_1] = 0$				
6	$X_1 C_2, X_1 n_3, X_1 E_p,$ $X_0 n_1, X_0 E_m$	C_2		$X_2 = \langle p, 0 \rangle$	$X_1 \xrightarrow{C_2} X_2$	$X_{2.1}$	$X_2 S_p, X_2 n_2,$ $X_2 C_2, X_2 n_3,$ $X_2 E_p$
7	$X_2 S_p, X_2 n_2, X_2 C_2,$ $X_2 n_3, X_2 E_p, X_1 n_3,$ $X_1 E_p, X_0 n_1, X_0 E_m$	S_p	$Out_{S_p}[X_2] = 0$				



A Trace of Value Context Based Analysis (2)

S. No.	Work List	Sel. node	Data flow value	New context	New trans.	exit value	Addition to the work list
8	$X_2 n_2, X_2 C_2, X_2 n_3, X_2 E_p, X_1 n_3, X_1 E_p, X_0 n_1, X_0 E_m$	n_2	$Out_{n_2}[X_2] = 0$				
9	$X_2 C_2, X_2 n_3, X_2 E_p, X_1 n_3, X_1 E_p, X_0 n_1, X_0 E_m$	C_2	$Out_{C_2}[X_2] = 1$		$X_2 \xrightarrow{C_2} X_2$		
10	$X_2 n_3, X_2 E_p, X_1 n_3, X_1 E_p, X_0 n_1, X_0 E_m$	n_3	$Out_{n_3}[X_2] = 1$				
11	$X_2 E_p, X_1 n_3, X_1 E_p, X_0 n_1, X_0 E_m$	E_p	$Out_{E_p}[X_2] = 0$ $Out_{C_2}[X_2] = 0$ $Out_{C_2}[X_1] = 0$			$X_2.0$	$X_2 n_3$
12	$X_2 n_3, X_1 n_3, X_1 E_p, X_0 n_1, X_0 E_m$	n_3	No change				
13	$X_1 n_3, X_1 E_p, X_0 n_1, X_0 E_m$	n_3	$Out_{n_3}[X_1] = 1$				
14	$X_1 E_p, X_0 n_1, X_0 E_m$	E_p	$Out_{E_p}[X_1] = 1$ $Out_{C_1}[X_0] = 1$			$X_1.1$	
15	$X_0 n_1, X_0 E_m$	n_1	$Out_{n_1}[X_0] = 1$				
16	$X_0 E_m$	E_m	$Out_{E_m}[X_0] = 1$				



Merging ExitValue with Previous Out Value at the Call Site

Select $X|n$ from WL . Compute In_n . Let $X.v$ be in In_n

- If $n = C_i$ calling procedure p
 - ▶ If some context $\langle p, v \rangle$ exists (say Y) /* p is the callee */
 - record the transition $X \xrightarrow{C_i} Y$
 - $Out_{C_i}[X] = Out_{C_i}[X] \sqcap exitValue(Y)$
 - if there is a change, add $X|m, \forall m \in succ(C_i)$ to WL



Merging ExitValue with Previous Out Value at the Call Site

Select $X|n$ from WL . Compute In_n . Let $X.v$ be in In_n

- If $n = C_i$ calling procedure p
 - ▶ If some context $\langle p, v \rangle$ exists (say Y) /* p is the callee */
 - record the transition $X \xrightarrow{C_i} Y$
 - $Out_{C_i}[X] = \boxed{Out_{C_i}[X] \sqcap exitValue(Y)}$
 - if there is a change, add $X|m, \forall m \in succ(C_i)$ to WL



Merging ExitValue with Previous Out Value at the Call Site

Select $X|n$ from WL . Compute In_n . Let $X.v$ be in In_n

- If $n = C_i$ calling procedure p
 - ▶ If some context $\langle p, v \rangle$ exists (say Y) /* p is the callee */
 - record the transition $X \xrightarrow{C_i} Y$
 - $Out_{C_i}[X] = \boxed{Out_{C_i}[X] \sqcap exitValue(Y)}$
 - if there is a change, add $X|m, \forall m \in succ(C_i)$ to WL

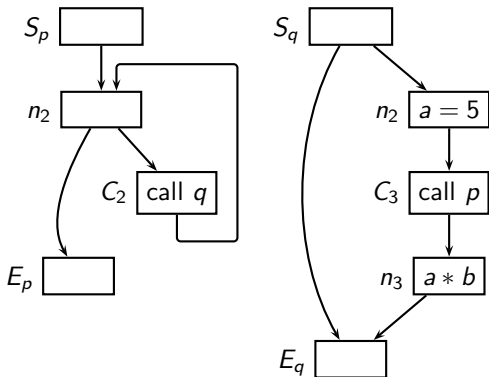
Analogy:

- ▶ At the intraprocedural level, we merge the values at the entry of a loop to compute the glb across all iterations of the loop
- ▶ At the interprocedural level, we want to compute the glb across repeated calls at the same call site (perhaps in a loop)



Partially Available Expressions Analysis Using Value Contexts

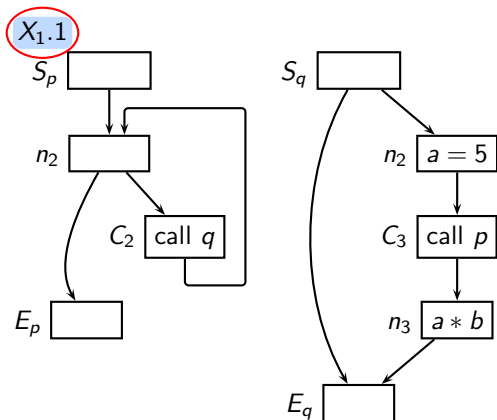
This example illustrates non-termination of analysis if the *exitValue* is not merged with the previous *Out* value



We assume that procedure main calls procedure p (and not q) and the expression $a * b$ is partially available on entry to p



Partially Available Expressions Analysis Using Value Contexts

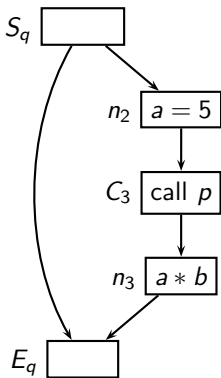
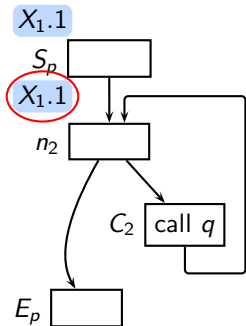


Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0

We create context X_1 for entry value 1 with *exitValue* as 0 (\top for partially available expressions analysis)



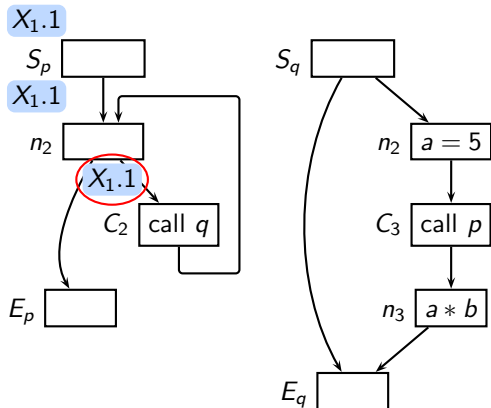
Partially Available Expressions Analysis Using Value Contexts



Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0



Partially Available Expressions Analysis Using Value Contexts

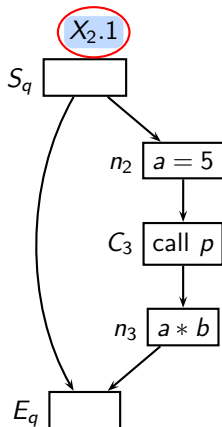
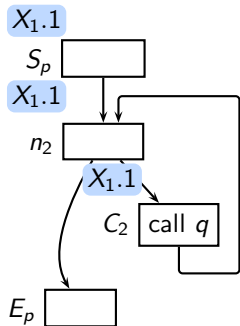
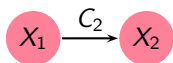


Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0

Value 1 reaches q
and a new context
must be created for it



Partially Available Expressions Analysis Using Value Contexts

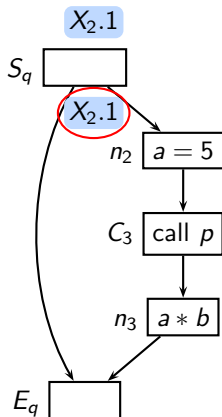
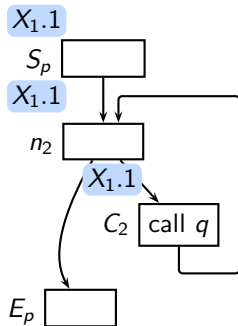
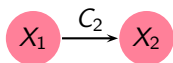


Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0

We create context X_2 for value 1 reaching q and record a transition from X_1 to X_2 on C_2



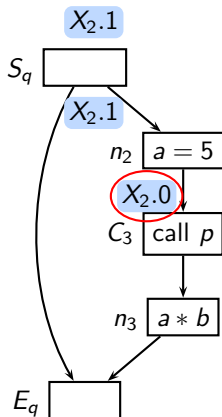
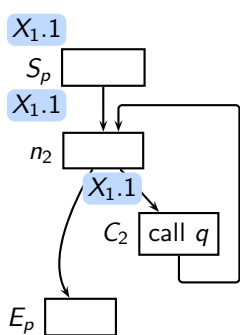
Partially Available Expressions Analysis Using Value Contexts



Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0



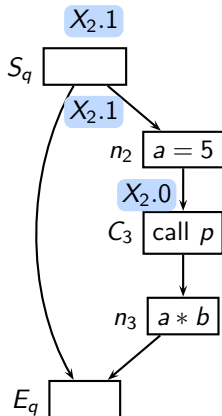
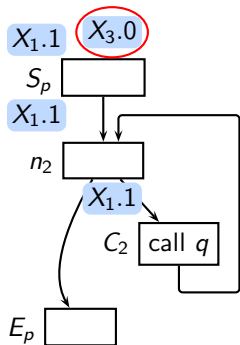
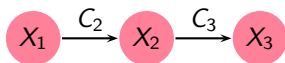
A diagram showing a directed edge from node x_1 to node x_2 with weight c_2 . The nodes are represented by pink circles, and the edge is a black arrow pointing from x_1 to x_2 with the label c_2 above it.



Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0

The expression is killed in node n_2 and data flow value 0 reaches the call site C_3 that calls p

Partially Available Expressions Analysis Using Value Contexts



Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0

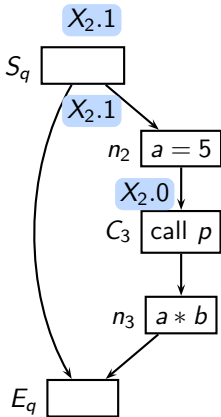
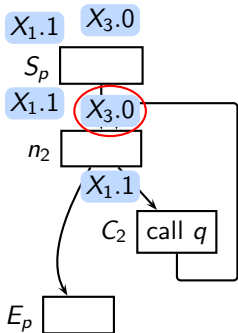
We create context X_3 for the new value (0) reaching p and record transition from X_2 to X_3 on C_3



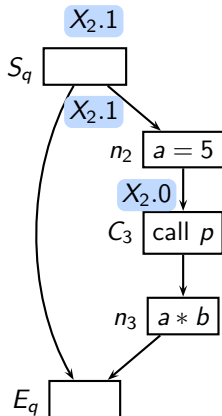
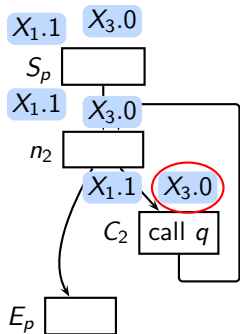
Partially Available Expressions Analysis Using Value Contexts



Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0



Partially Available Expressions Analysis Using Value Contexts

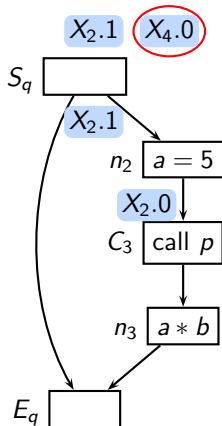
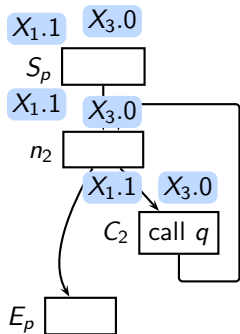
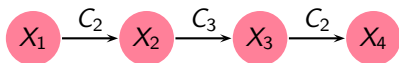


Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0

And now the value 0 reaches q at call site C_2



Partially Available Expressions Analysis Using Value Contexts

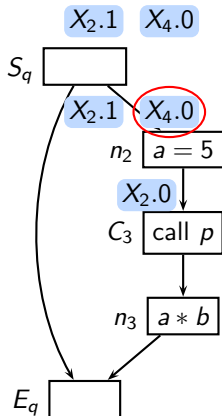
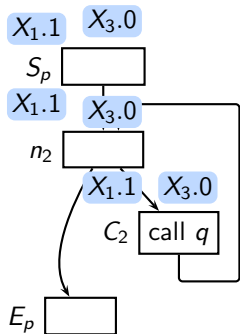
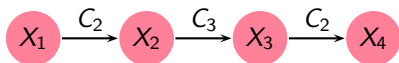


Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	0

We create context X_4 for the new value (0) reaching p and record transition from X_3 to X_4 on C_2



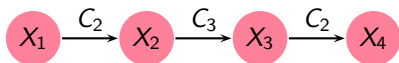
Partially Available Expressions Analysis Using Value Contexts



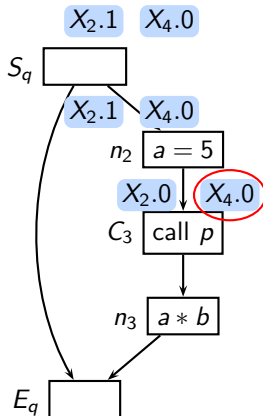
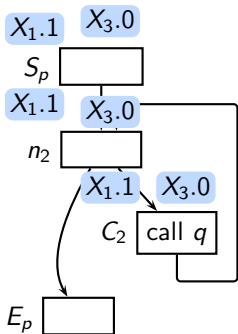
Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	0



Partially Available Expressions Analysis Using Value Contexts



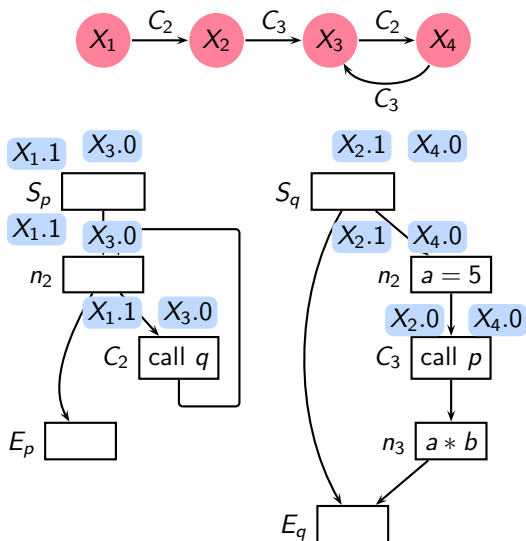
Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	0



And now the value 0 reaches p at call site C_3



Partially Available Expressions Analysis Using Value Contexts

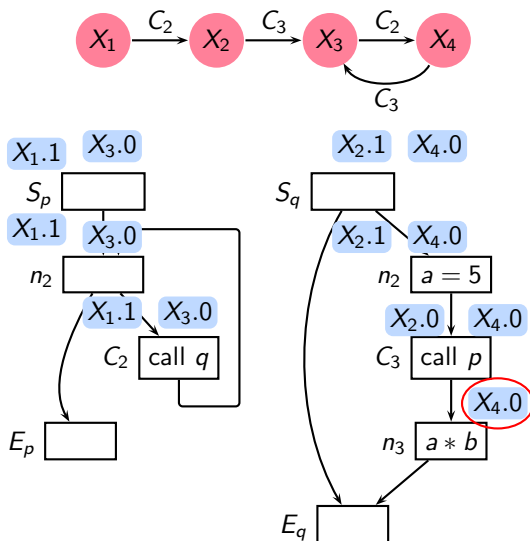


Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	0

We already have context X_3 with entry value 0 for p so no need to analyse p again



Partially Available Expressions Analysis Using Value Contexts



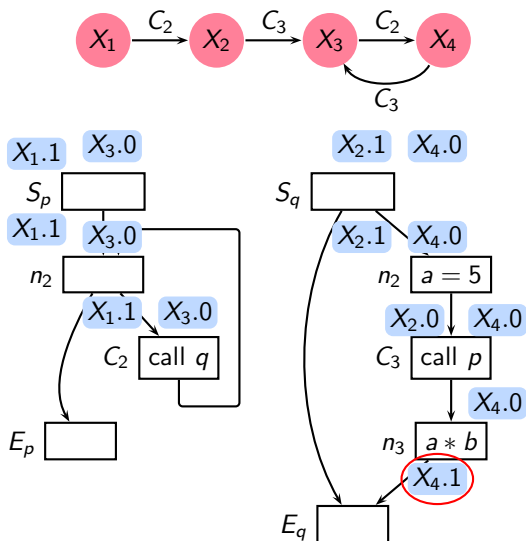
Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	0

We use the *exitValue* for X_3 to compute Out_{C_3} for the context X_4 (because of the transition $X_4 \xrightarrow{C_3} X_3$)

The analysis of p is not yet over for any context, and so we get the \top value



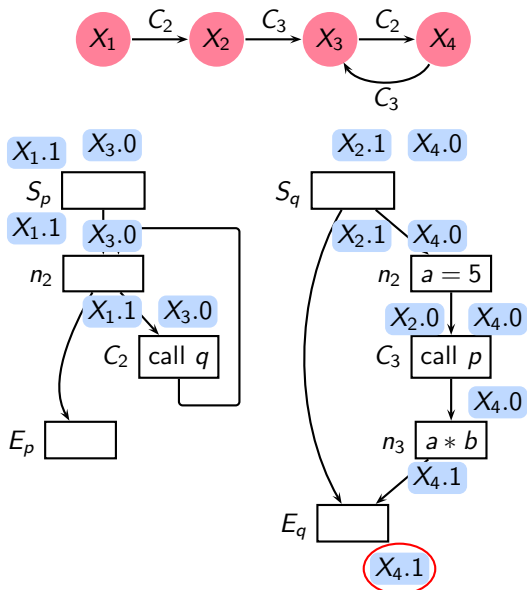
Partially Available Expressions Analysis Using Value Contexts



Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	0



Partially Available Expressions Analysis Using Value Contexts



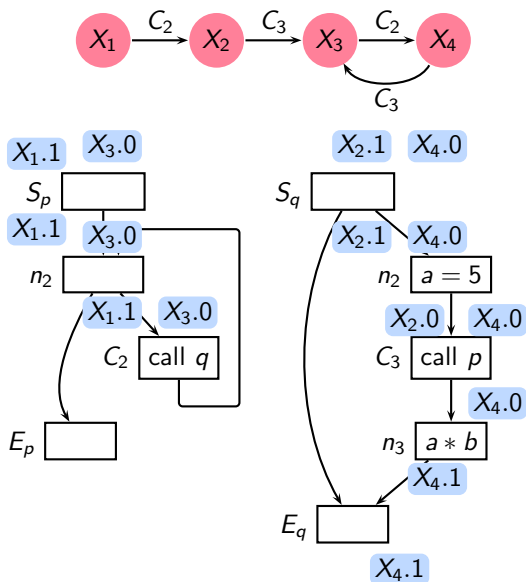
Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	0

The analysis of q for X_4 is now and the *exitValue* of X_4 becomes 1

This change in X_4 must be propagated to X_3 in the caller p (identified from the transition $X_3 \xrightarrow{C_2} X_4$)



Partially Available Expressions Analysis Using Value Contexts



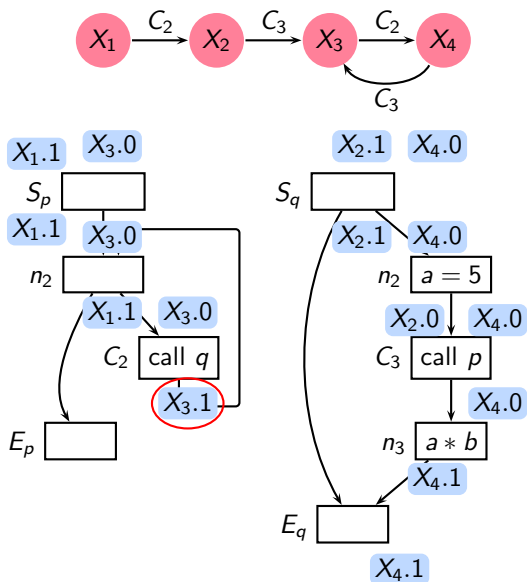
Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	1

The analysis of q for X_4 is now and the *exitValue* of X_4 becomes 1

This change in X_4 must be propagated to X_3 in the caller p (identified from the transition $X_3 \xrightarrow{C_2} X_4$)



Partially Available Expressions Analysis Using Value Contexts

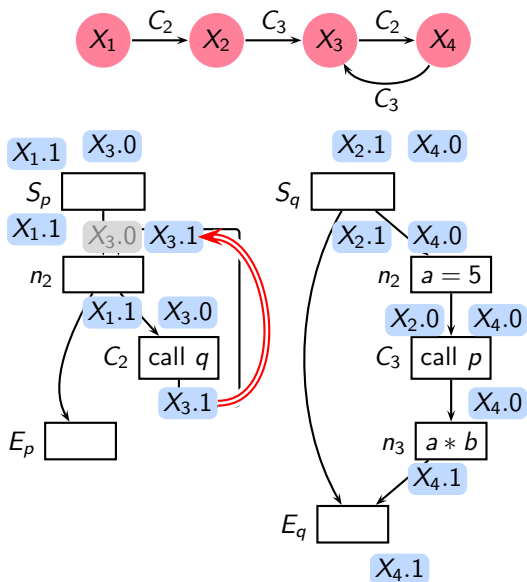


Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	1

Out_{C_2} becomes 1 for X_3 which changes the value at In_{n_2} for X_3 to 1



Partially Available Expressions Analysis Using Value Contexts

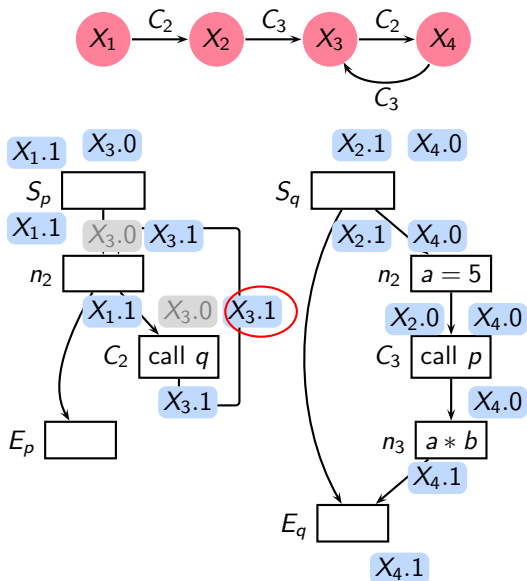


Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	1

Out_{C_2} becomes 1 for X_3 which changes the value at In_{n_2} for X_3 to 1



Partially Available Expressions Analysis Using Value Contexts

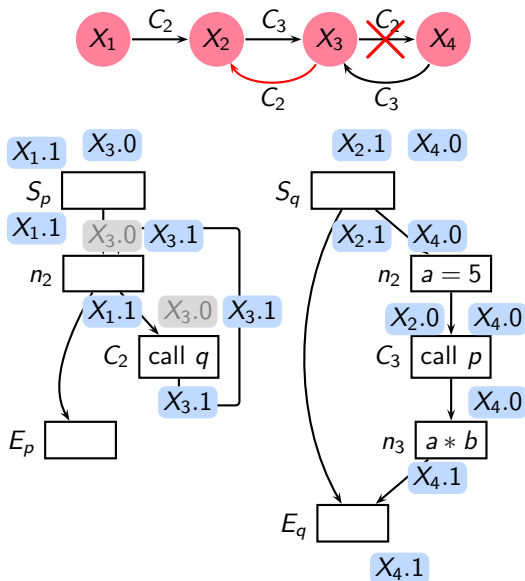


Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	1

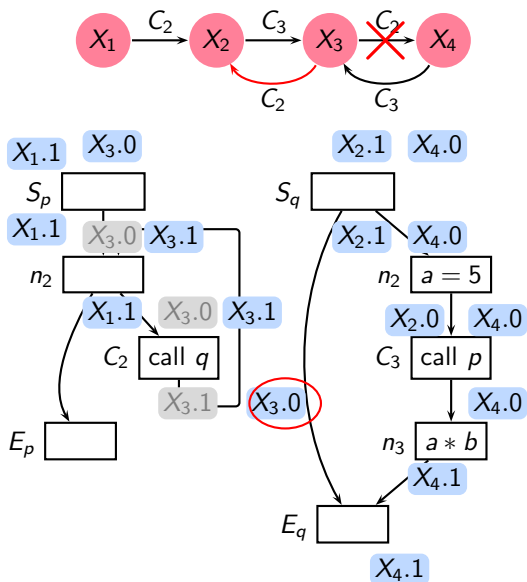
in_{C_2} becomes 1 for X_3
 Since we have a context for q with entry value 1 (X_2), we remove the transition $X_3 \xrightarrow{C_3} X_4$ and add the transition $X_3 \xrightarrow{C_2} X_2$



Partially Available Expressions Analysis Using Value Contexts



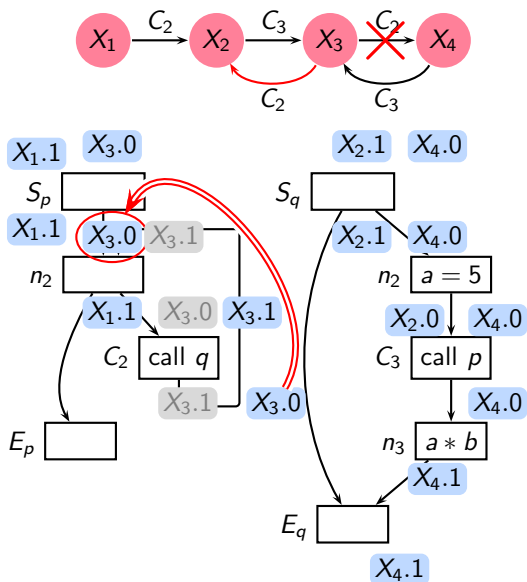
Partially Available Expressions Analysis Using Value Contexts



Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	1



Partially Available Expressions Analysis Using Value Contexts

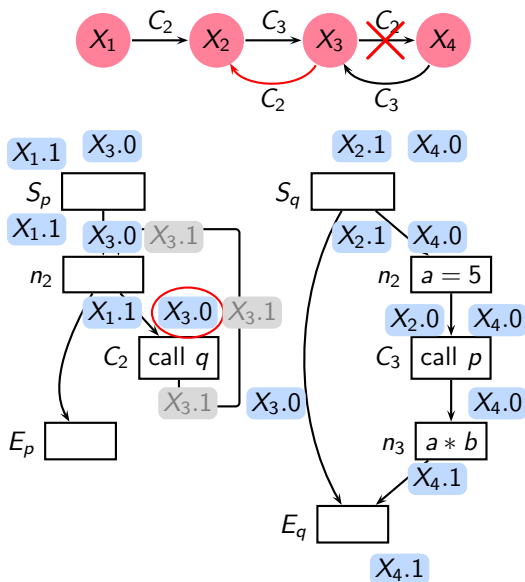


Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	1

The value of X_3 in In_{n_2} once again becomes 0



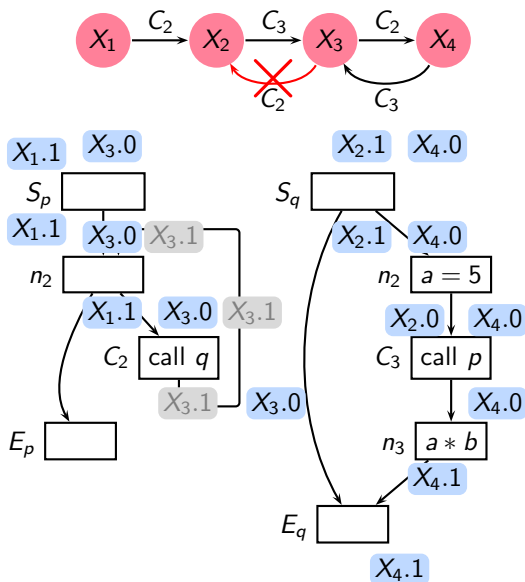
Partially Available Expressions Analysis Using Value Contexts



The value of X_3 in In_{C_2} once again becomes 0
 The transition from X_3 needs to be restored to $X_3 \xrightarrow{C_3} X_4$ removing the transition $X_3 \xrightarrow{C_2} X_2$



Partially Available Expressions Analysis Using Value Contexts

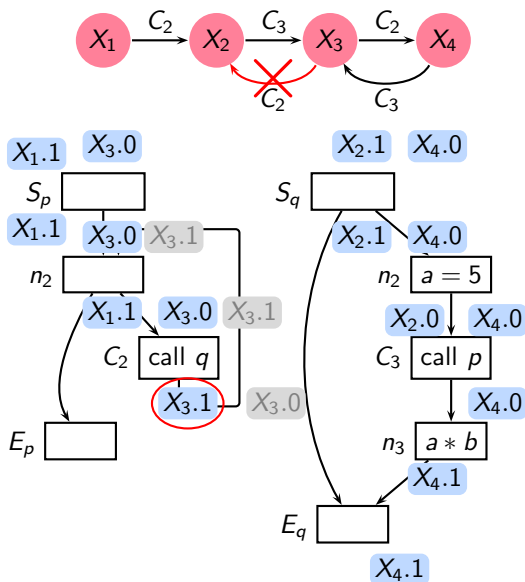


Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	1

The value of X_3 in In_{C_2} once again becomes 0
 The transition from X_3 needs to be restored to $X_3 \xrightarrow{C_2} X_4$ removing the transition $X_3 \xrightarrow{C_2} X_2$



Partially Available Expressions Analysis Using Value Contexts



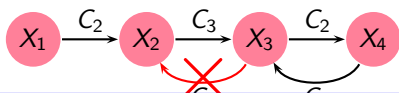
Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0
$X_2 = \langle q, 1 \rangle$	0
$X_3 = \langle p, 0 \rangle$	0
$X_4 = \langle q, 0 \rangle$	1

We use the *exitValue* of X_4 to compute Out_{C_2} for X_3 which once again becomes 0

Thus we are back to the same situation



Partially Available Expressions Analysis Using Value Contexts



Context	<i>exitValue</i>
$X_1 = \langle p, 1 \rangle$	0

- The process would not terminate so long as the processing of the nodes in the loop continues
- If the work list organization allows processing of E_p , then the *exitValue* of X_3 will also change to 1 which will lead to termination
- Our underlying flow functions are monotonic and a fixed point exists; non-termination is caused by the algorithm because its progress depends on the order of the nodes in the work list
- We avoid this problem by taking a meet at the exit of call nodes when the exit values of existing contexts are used at the call sites in the callers

$X_4.1$



Defining Value Context Method Using Data Flow Equations

- The overall data flow values Γ are sets of $X.v$ where X is a context and $v \in L$ is the underlying data flow value.
- We merge underlying data flow values only if the contexts are same

$$\Gamma_1 \uplus \Gamma_2 = \left\{ X.w \mid \begin{array}{l} X.u \in \Gamma_1 \wedge X.v \in \Gamma_2 \Rightarrow w = u \sqcap v, \\ X.u \in \Gamma_1 \wedge X.v \notin \Gamma_2 \Rightarrow w = u, \\ X.u \notin \Gamma_1 \wedge X.v \in \Gamma_2 \Rightarrow w = v \end{array} \right\}$$

Effectively, if a context does not exist in Γ , its value is \top in Γ

- Data flow variables for node n in procedure p are $In(p, n)$ and $Out(p, n)$
- The flow function for node n in procedure p is $f(p, n)$



Defining Value Context Method Using Data Flow Equations

We assume the following auxiliary functions

- Function *context* maintains the context information
context(*p*, *v*) returns the context of procedure *p* for entry value *v*
If no such context exists, the function creates a new context and returns it
- Function *exitValue*(*X*) returns the exit value of context *X*
If context *X* does not exist, the function returns $\top \in L$
- Function *gpred* extends the predecessor relation *pred* (which is local to a procedure) to a global level across procedures

$$gpred(p, n) = \begin{cases} \{(q, m) \mid \text{call site } m \text{ in } q \text{ calls } p\} & n \text{ is } S_p \\ \{(p, m) \mid m \in pred(n)\} & \text{otherwise} \end{cases}$$



Defining Value Context Method Using Data Flow Equations

We define data flow equations for a forward data flow analysis

$$In(p, n) = \begin{cases} \{X.v \mid X = context(p, v), Y.v \in In(q, m), \\ \quad (q, m) \in gpred(p, n)\} & n \text{ is } S_p \\ \biguplus_{(p, m) \in gpred(p, n)} Out(p, m) & \text{otherwise} \end{cases}$$

$$Out(p, n) = \begin{cases} Out(p, n) \biguplus \{X.v \mid X.v' \in In(p, m), \\ \quad Y = context(q, v'), \\ \quad v = exitValue(Y)\} & n \text{ calls } q \\ \{X.v \mid X.v' \in In(p, m), v = f(p, n)(v')\} & \text{otherwise} \end{cases}$$



Value Contexts and Interprocedurally Valid Paths

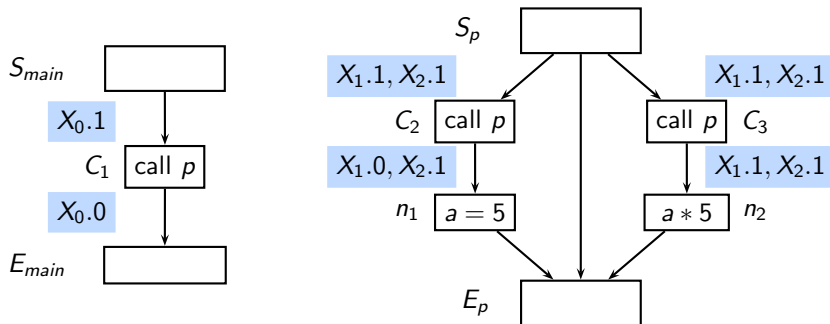
The role of value contexts in context sensitivity

- Value contexts preserve interprocedurally valid paths
- Value contexts consider only interprocedurally valid paths

We explain this with the help of an example by illustrating paths using a staircase diagram



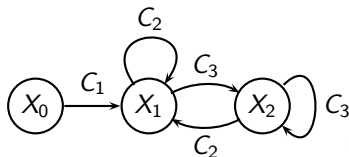
Value Contexts and Interprocedurally Valid Paths: Example



Context Transition Table

Context	<i>exitValue</i>
$X_0 : \langle \text{main}, 0 \rangle$	1
$X_1 : \langle p, 0 \rangle$	1
$X_2 : \langle p, 1 \rangle$	1

Context Transition Graph



Value Contexts and Interprocedurally Valid Paths: Example

We explain the data flow value at the entry of C_2 by dividing the paths into the following two categories:

- A. Paths in which the innermost recursion is along the call at C_2 .
- B. Paths in which the innermost recursion is along the call at C_3 .

We draw the staircase diagrams of the example paths in the two categories

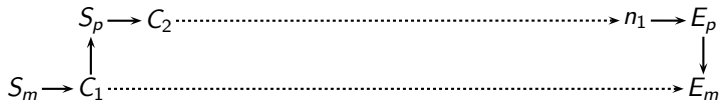


Innermost Recursion Along the Call at C_2

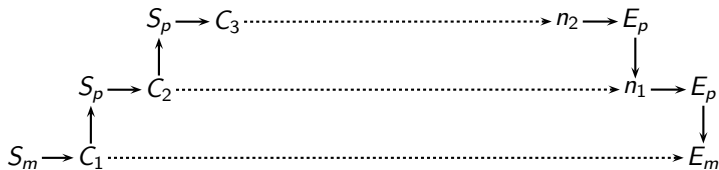
$S_m \rightarrow C_1 \cdots \cdots \cdots \rightarrow E_m$



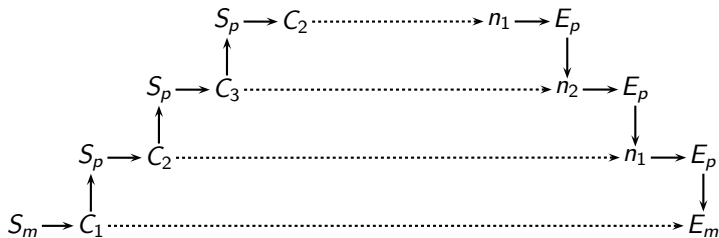
Innermost Recursion Along the Call at C_2



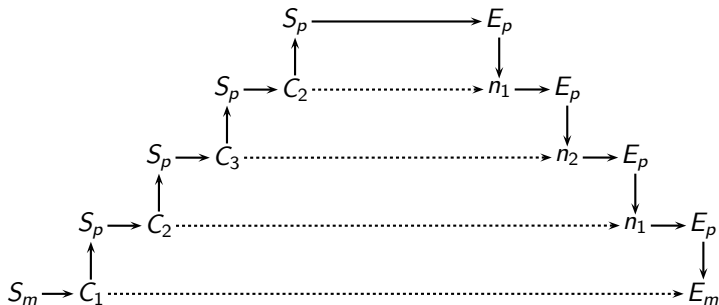
Innermost Recursion Along the Call at C_2

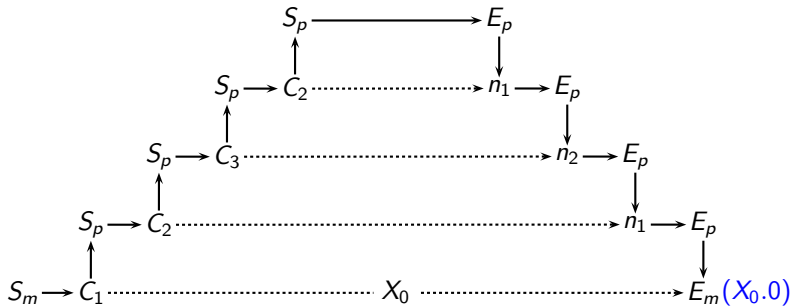


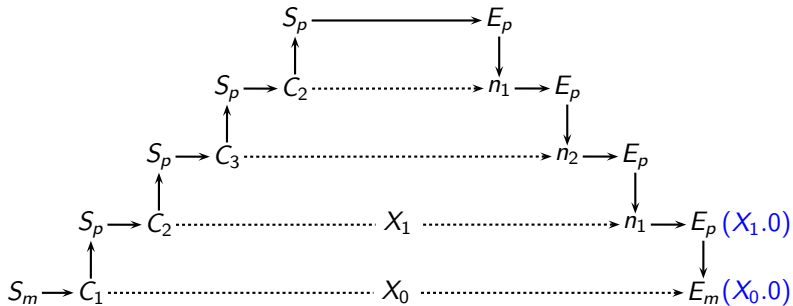
Innermost Recursion Along the Call at C_2



Innermost Recursion Along the Call at C_2

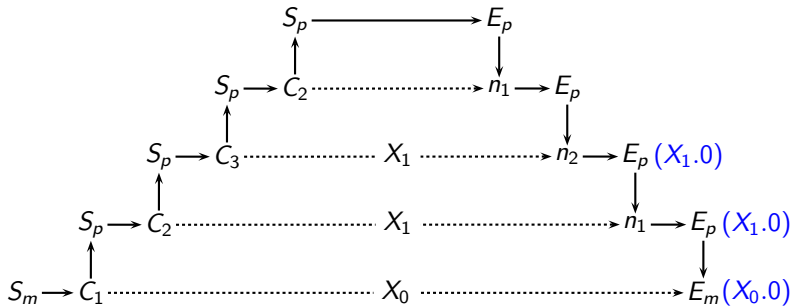


$$Blis\ 0$$


$$Blis\ 0$$


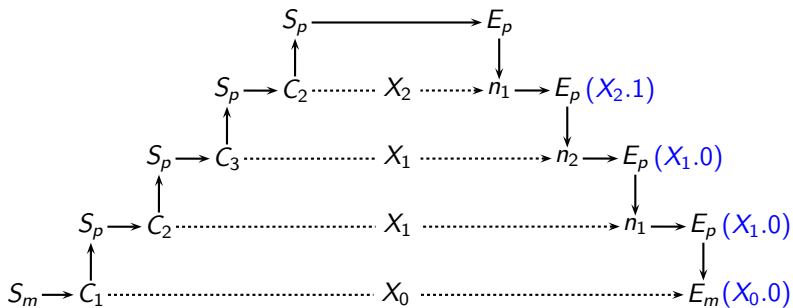
Innermost Recursion Along the Call at C_2

n_1 kills the liveness of a
New context is not required



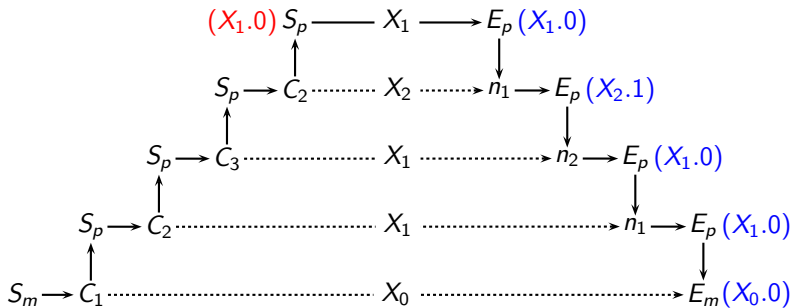
Innermost Recursion Along the Call at C_2

n_2 generates the liveness of a
New context is required



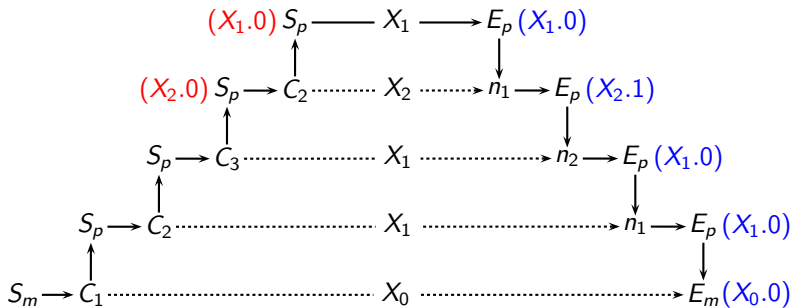
Innermost Recursion Along the Call at C_2

exitValue of X_1 is 0



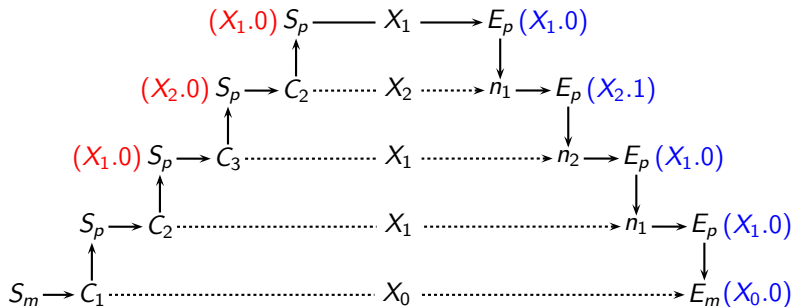
Innermost Recursion Along the Call at C_2

exitValue of X_2 is 0



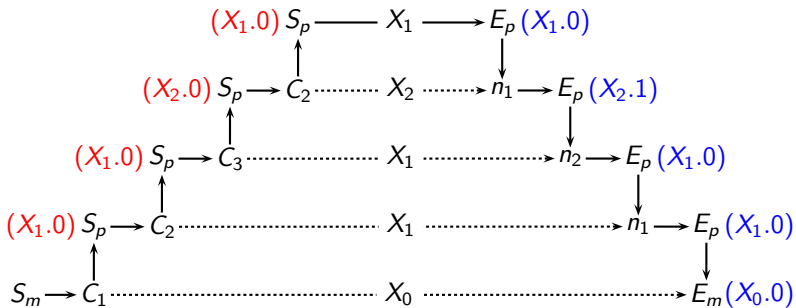
Innermost Recursion Along the Call at C_2

exitValue of
 X_1 remains 0



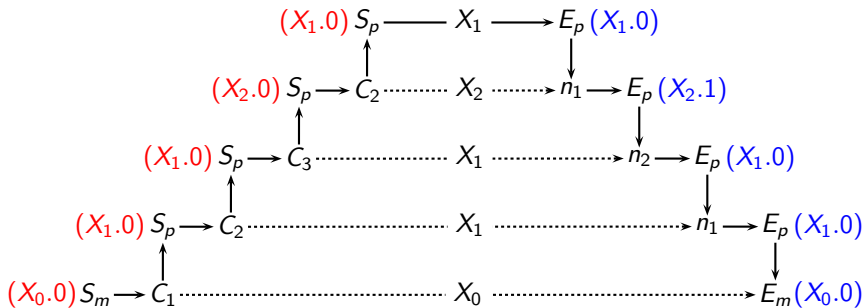
Innermost Recursion Along the Call at C_2

exitValue of
 X_1 remains 0



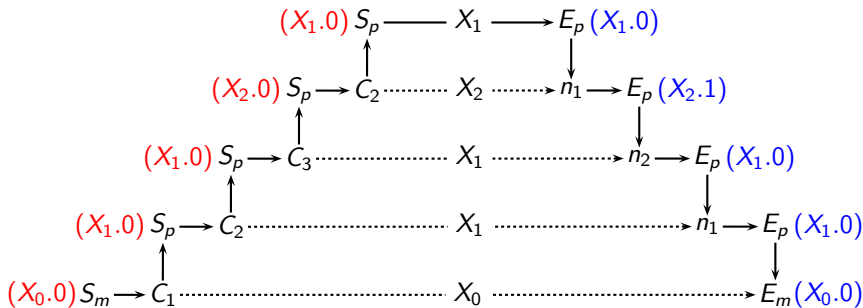
Innermost Recursion Along the Call at C_2

exitValue of X_0 is 0



Innermost Recursion Along the Call at C_2

For this example, the innermost call determines the *exitValue* of contexts

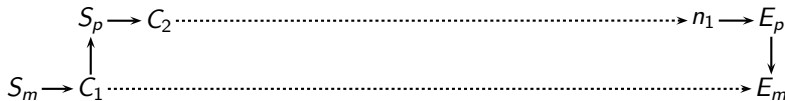


Innermost Recursion Along the Call at C_3

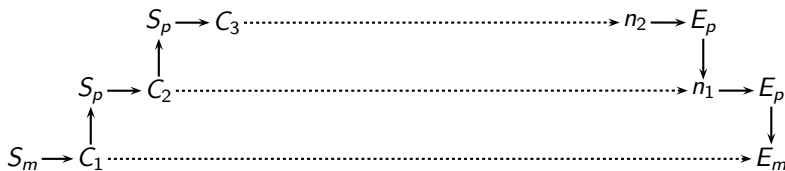
$S_m \rightarrow C_1 \cdots \cdots \cdots \rightarrow E_m$



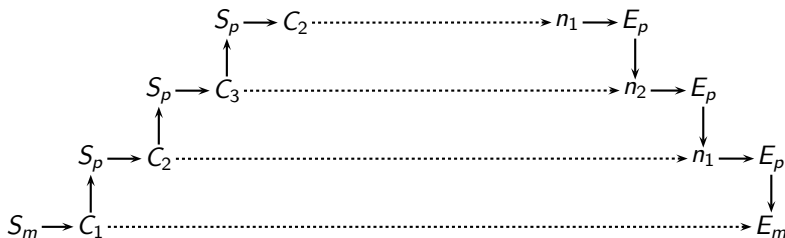
Innermost Recursion Along the Call at C_3



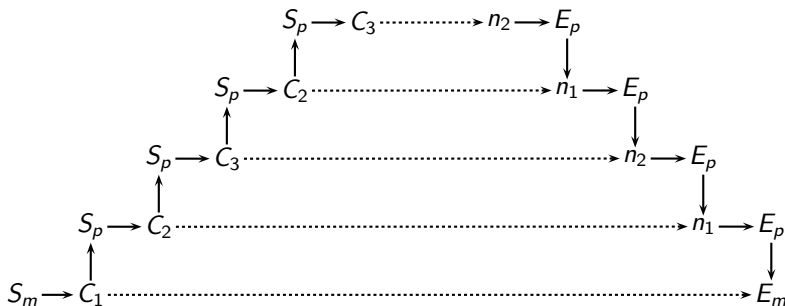
Innermost Recursion Along the Call at C_3



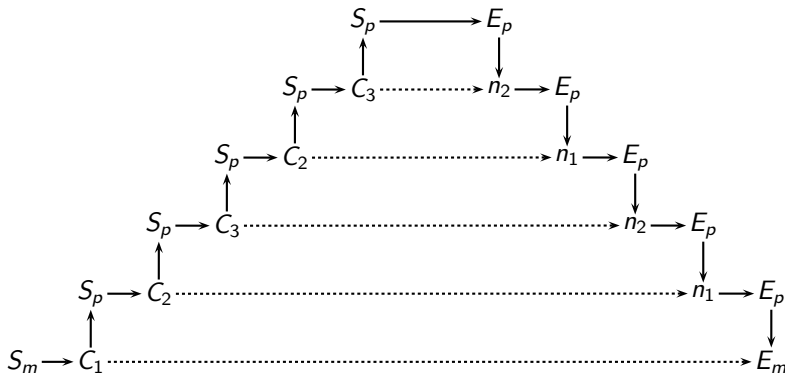
Innermost Recursion Along the Call at C_3



Innermost Recursion Along the Call at C_3

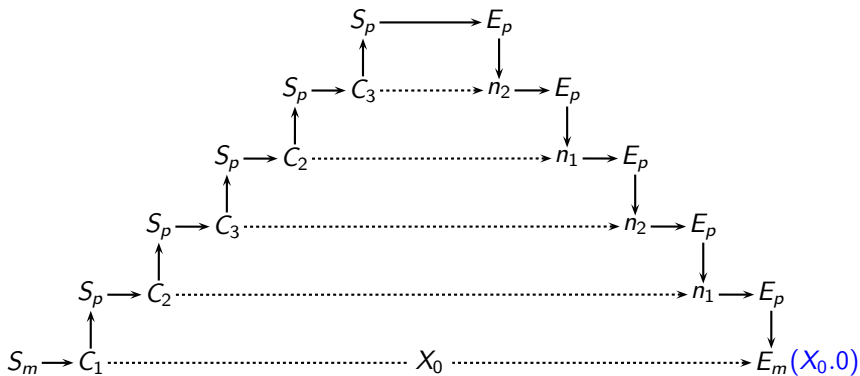


Innermost Recursion Along the Call at C_3



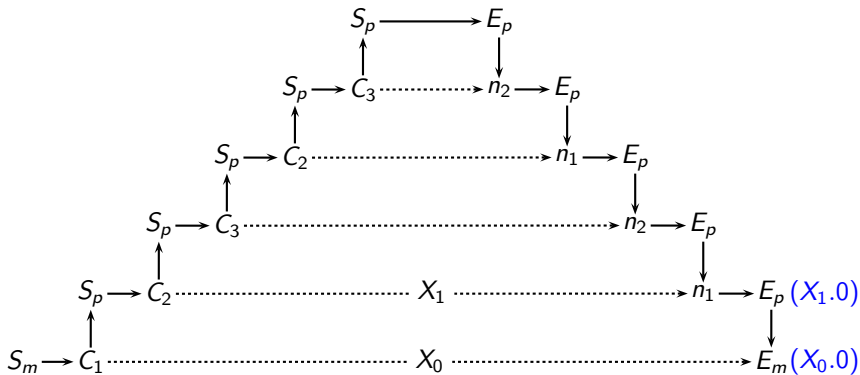
Innermost Recursion Along the Call at C_3

Blis 0

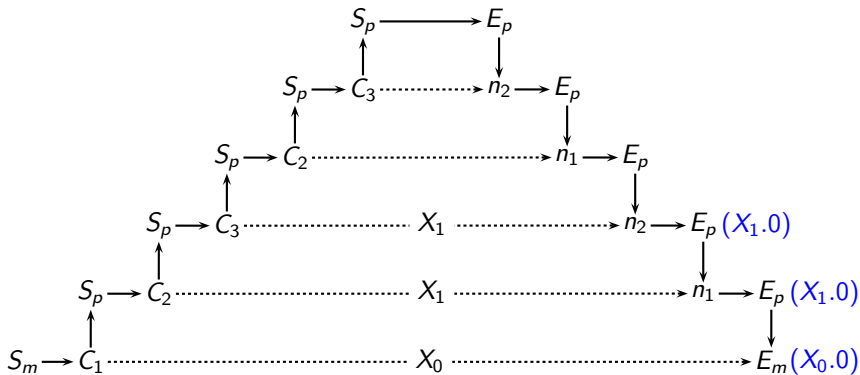


Innermost Recursion Along the Call at C_3

n_1 kills the liveness of a
New context is not required

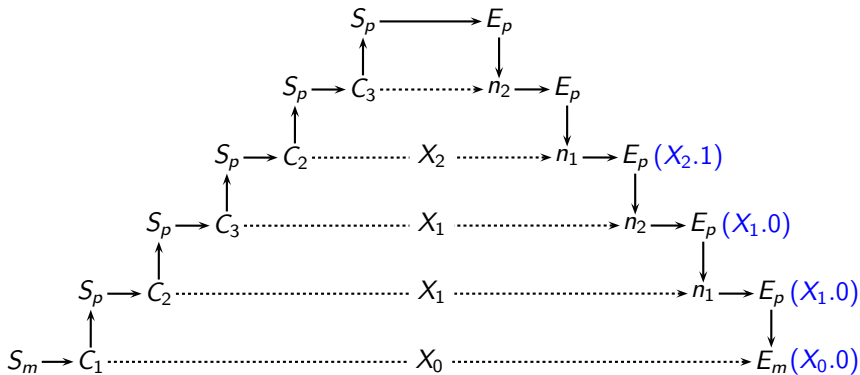


n_1 kills the liveness of a
New context is not required

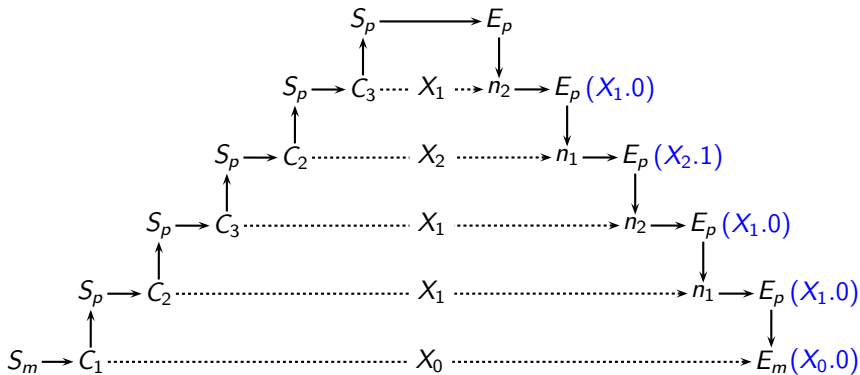


Innermost Recursion Along the Call at C_3

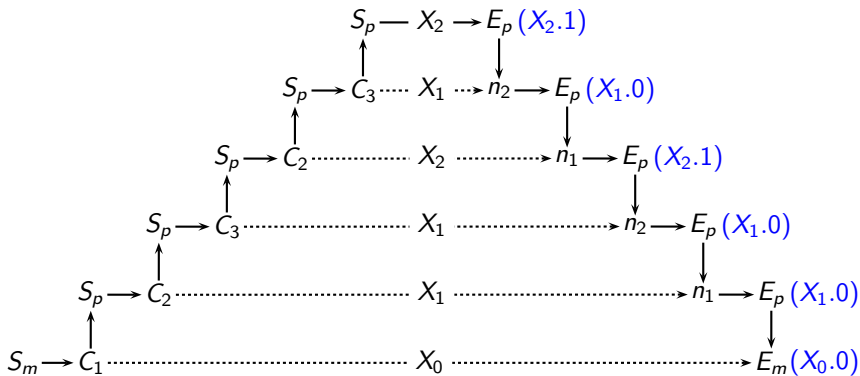
n_2 generates the liveness of a New context is required



n_1 kills the liveness of a
New context is not required

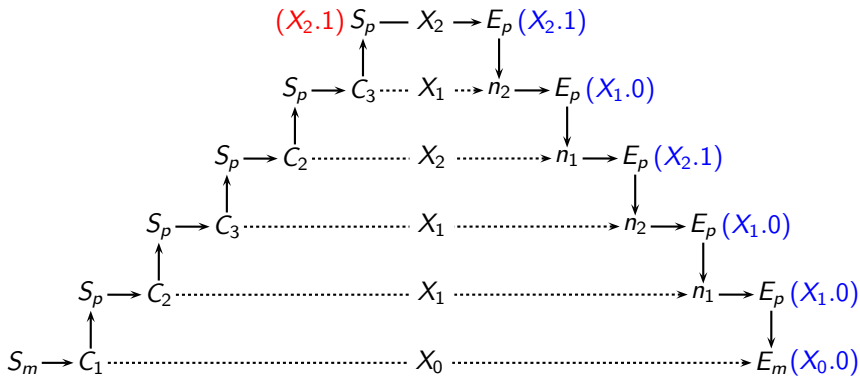


n_2 generates the liveness of a
New context is not required



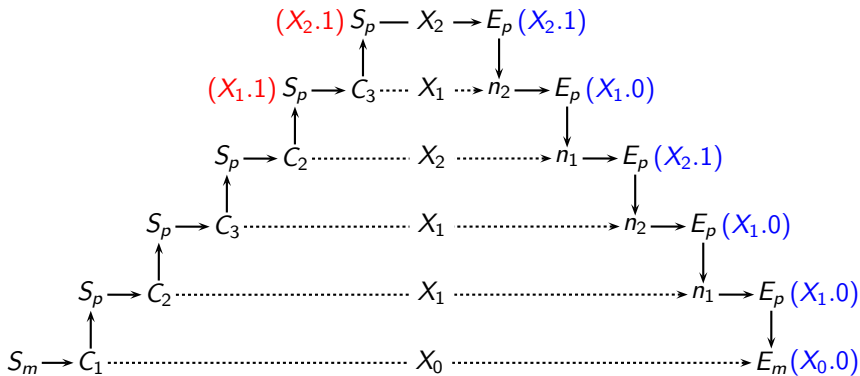
Innermost Recursion Along the Call at C_3

exitValue of X_2 is 1 (after merging with previous value 0)



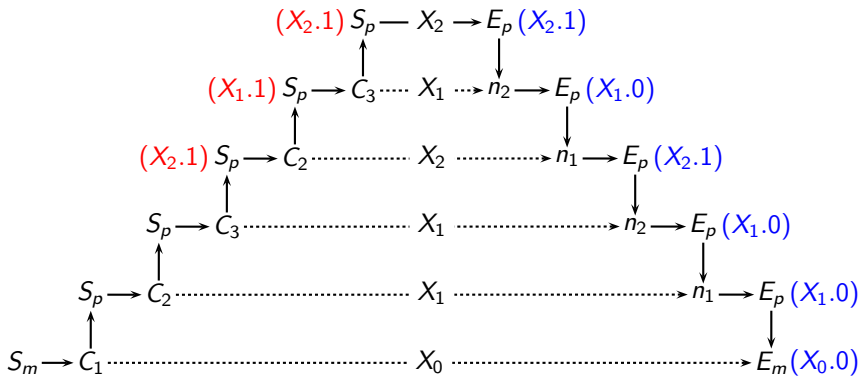
Innermost Recursion Along the Call at C_3

exitValue of X_1 is 1 (after merging with previous value 0)



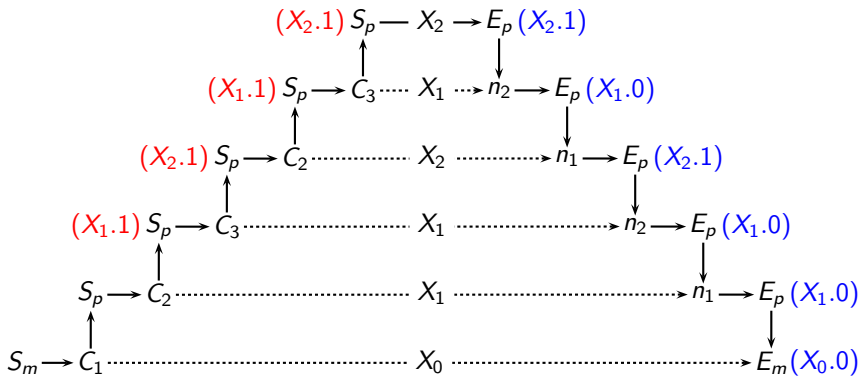
Innermost Recursion Along the Call at C_3

exitValue of
 X_2 remains 1



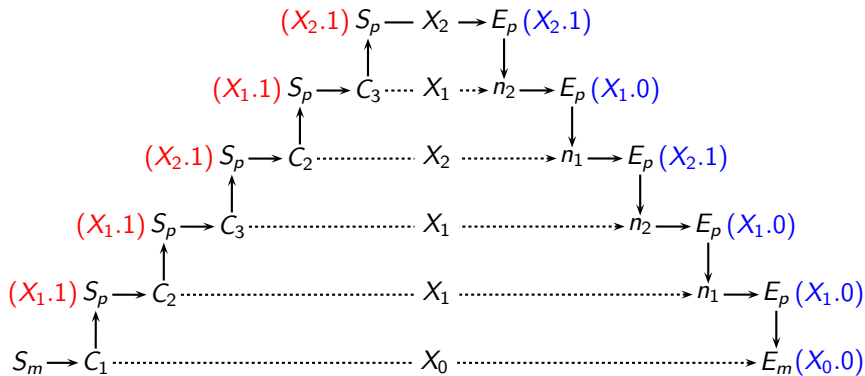
Innermost Recursion Along the Call at C_3

exitValue of
 X_1 remains 1



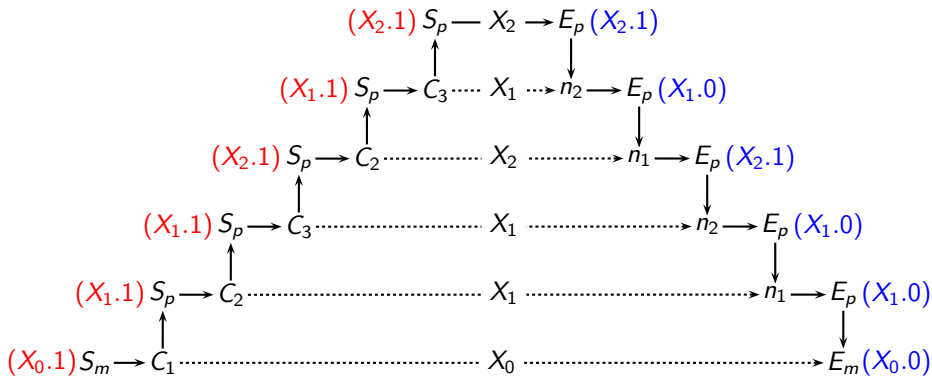
Innermost Recursion Along the Call at C_3

exitValue of
 X_1 remains 1



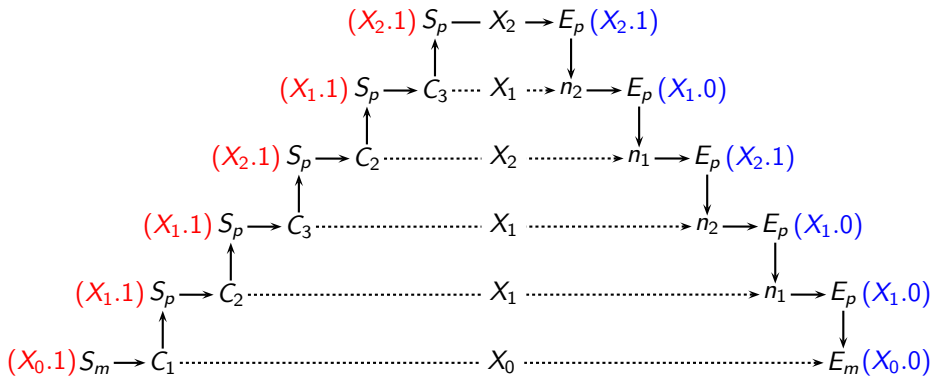
Innermost Recursion Along the Call at C_3

exitValue of X_0 is 1 (after merging with previous value 0)



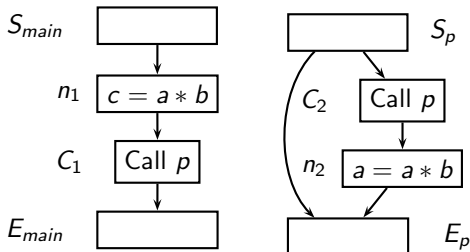
Innermost Recursion Along the Call at C_3

Again, the innermost call determines the *exitValue* of contexts
The final values at the entry of C_3 are 1 (union of 1 and 0)



Tutorial Problem #1 for Value Contexts

```
1. int a,b,c;  
2. void main()  
3. {   c = a*b;  
4.     p();  
5. }  
6. void p()  
7. {   if (...)  
8.     { p();  
9.       Is a*b available?  
10.      a = a*b;  
11.    }  
12. }
```



Tutorial Problem #2 for Value Contexts

Perform interprocedural live variables analysis using value contexts

<pre>main() { p(); }</pre>	<pre>p() { while (...) { printf ("%d\n",a); p(); } }</pre>
--------------------------------	--

Observe the change in edges in the transition diagram



Tutorial Problem #3 for Value Contexts

Perform interprocedural available expressions analysis using value contexts

<pre>main() { c = a*b; p(); }</pre>		<pre>p() { while (a > b) { p(); a = a*b; } }</pre>
---	--	---

Observe the change in edges in the transition diagram



Tutorial Problem #4 for Value Contexts

Perform interprocedural available expressions analysis using value contexts

```
1.  main()
```

```
2.  {
```

```
3.      c = a*b;
```

```
4.      p();
```

```
5.      a = a*b;
```

```
6.  }
```

```
7.  p()
```

```
8.  {    if (...)
```

```
9.      {    a = a*b;
```

```
10.         p();
```

```
11.     }
```

```
12.     else if (...)
```

```
13.     {    c = a * b;
```

```
14.         p();
```

```
15.         c = a;
```

```
16.     }
```

```
17.     else
```

```
18.         ; /* ignore */
```

```
19. }
```



Tutorial Problem #5 for Value Contexts

Perform interprocedural live variables analysis using value contexts

<pre>main() { a = 5; b = 3; c = 7; d = 2; p(); a = a + 2; e = c+d; d = a*b; q(); print a+c+e; }</pre>		<pre>p() { b = 2; if (b<d) c = a+b; else q(); print c+d; }</pre>		<pre>q() { a = 1; p(); a = a*b; }</pre>
---	--	---	--	---

Context sensitivity: e is live on entry to p but not before its call in main



Result of Tutorial #5

```
main()
{
    a = 5; b = 3;
    c = 7; d = 2;
    /*{a,d}*/
    p();
    /*{a,b,c,d}*/
    a = a + 2;
    e = c+d;
    /*{a,b,e}*/
    d = a*b;
    /*{d,e}*/
    q();
    /*{a,c,e}*/
    print a+c+e;
}
```

```
p()
{ /*{a,d,e}*/
    b = 2;
    if (b<d)
        /*{a,b,d,e}*/
        c = a+b;
    else
        /*{d,e}*/
        q();
    /*{a,b,c,d,e}*/
    print c+d;
}
```

```
q()
{
    /*{d,e}*/
    a = 1;
    /*{a,d,e}*/
    p();
    /*{a,b,c,d,e}*/
    a = a*b;
}
```

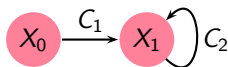


Tutorial Problem #6: Interprocedural Points-to Analysis

```
main()
{
  x = &y;
  z = &x;
  y = &z;
  p(); /* C1 */
}

p()
{
  if (...)
  {
    p(); /* C2 */
    x = *x;
  }
}
```

Value contexts method requires three contexts as shown below in the transition diagram



Reaching Definitions Analysis in GCC 4.0

Program	LoC	#F	#C	3K length bound				Proposed Approach		
				K	#CS	Max	Time	#CS	Max	Time
hanoi	33	2	4	4	100000+	99922	3973×10^3	8	7	2.37
bit_gray	53	5	11	7	100000+	31374	2705×10^3	17	6	3.83
analyzer	288	14	20	2	21	4	20.33	21	4	1.39
distray	331	9	21	6	96	28	322.41	22	4	1.11
mason	350	9	13	8	100000+	22143	432×10^3	14	4	0.43
fourinarow	676	17	45	5	510	158	397.76	46	7	1.86
sim	1146	13	45	8	100000+	33546	1427×10^3	211	105	234.16
181_mcf	1299	17	24	6	32789	32767	484×10^3	41	11	5.15
256_bzip2	3320	63	198	7	492	63	258.33	406	34	200.19

- LoC is the number of lines of code,
- #F is the number of procedures,
- #C is the number of call sites,
- #CS is the number of call strings
- Max denotes the maximum number of call strings reaching any node.
- Analysis time is in milliseconds.

(Implementation was carried out by Seema Ravandale.)



Some Observations

- Compromising on precision may not be necessary for efficiency.
- Separating the necessary information from redundant information is much more significant.
- Data flow propagation in real programs seems to involve only a small subset of all possible values.

Much fewer changes than the theoretically possible worst case number of changes.

- A precise modelling of the process of analysis is often an eye opener.



Some Observations

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Much fewer changes than the theoretically possible worst case number of changes.

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distinct tagged values =

Min (# actual contexts, # actual data flow values)

